$SU(5)$ and $SO(10)$ Models from F-Theory with Natural Yukawa Couplings

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Abstract

We construct the $SU(5)$ and $SO(10)$ models from F-theory. Turning on the $U(1)$ fluxes, we can break the $SU(5)$ gauge symmetry down to the Standard Model (SM) gauge symmetry, and break the $SO(10)$ gauge symmetry down to the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry. In particular, all the SM fermion Yukawa couplings preserve the enhanced $U(1)_a \times U(1)_b$ gauge or global symmetries at the triple intersections of the SM fermion and Higgs curves. And the SM fermion masses and mixings can be generated in the presence of background fluxes. In our models, the doublet-triplet splitting problem can be solved naturally. The additional vector-like particles can obtain heavy masses via the instanton effects or Higgs mechanism and then decouple at the high scale. The SM gauge couplings at the string scale, which are splitted due to the $U(1)$ flux effects, can be explained by considering heavy threshold corrections from the extra vector-like particles. Moreover, in the $SU(5)$ model, we have the Yukawa coupling unification for the bottom quark and tau lepton. In the $SO(10)$ models, we have the Yukawa coupling unification for the top and bottom quarks, and the Yukawa coupling unification for the tau lepton and tau neutrino.

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I. INTRODUCTION

The great challenge in string phenomenology is to construct realistic string models with moduli stabilization and without extra chiral exotics, and then make clean predictions that can tested at the Large Hadron Collider (LHC) and other future experiments. Previously, string model building has been studied extensively in the heterotic $E_8 \times E_8$ string theory and Type II string theories with D-branes.

Recently, semi-realistic Grand Unified Theories (GUTs) have been constructed locally in the F-theory with seven-branes, which can be considered as the strongly coupled formulation of ten-dimensional Type IIB string theory with a varying axion (a)-dilaton (ϕ) field $\tau = a + ie^{-\phi}$ [1–5] (For a briefly review, see Section II.). Then further model building and phenomenological consequences have been studied extensively [6–31]. As we know, the GUTs without additional chiral exotic particles are asymptotically free, and asymptotic freedom can be translated into the existence of a consistent decompactification limit. Moreover, the Planck scale $M_{\rm Pl}$ is about 10^{19} GeV while the GUT scale $M_{\rm GUT}$ is around 2×10^{16} GeV, so $M_{\rm GUT}/M_{\rm Pl}$ is indeed a small number around $10^{-3} - 10^{-2}$. Thus, from the effective field theory point of view in the bottom-up approach, it is natural to assume that $M_{\rm GUT}/M_{\rm Pl}$ is small, and then gravity can be decoupled. In the decoupling limit where $M_{\rm Pl} \to \infty$ while $M_{\rm GUT}$ remains finite, semi-realistic $SU(5)$ models and $SO(10)$ models without chiral exotic particles have been constructed locally. To decouple gravity and avoid the bulk matter fields on the observable seven-branes, we can show that the observable seven-branes should wrap a del Pezzo n surface dP_n with $n \geq 2$ for the internal space dimensions (For a review of del Pezzo n surfaces, see Appendix A.) [3, 4]. The GUT gauge fields are on the worldvolume of the observable seven-branes, while the matter and Higgs fields are localized on the codimension-one curves in dP_n.

A brand new feature is that the $SU(5)$ gauge symmetry can be broken down to the Standard Model (SM) gauge symmetry by turning on $U(1)_Y$ flux [3, 4], and the $SO(10)$ gauge symmetry can be broken down to the $SU(5) \times U(1)_X$ and $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetries by turning on the $U(1)_X$ and $U(1)_{B-L}$ fluxes, respectively [3, 4, 10, 13, 29]. In the $SU(5)$ models, to generate the up-type quark Yukawa couplings $\mathbf{10}_i\mathbf{10}_j\mathbf{5}$ at the triple intersections where the gauge symmetry is enhanced to E_6, the matter curves for $\mathbf{10}_i$ are pinched [3, 4]. To be concrete, the two matter curves for $\mathbf{10}_i$ and $\mathbf{10}_j$ that emanate from an E_6 point are acturally two different parts of a single curve which is pinched at the E_6 point. In other words, it can be viewed as starting with two distinct cuves which connect to each other some distance away from the E_6 point. This connection is not nicely captured by the local field theory description of the E_6 point in terms of the breaking $E_6 \to SU(5) \times U(1)_a \times U(1)_b$ as only a certain linear combintaion of $U(1)_a$ and $U(1)_b$ may

be realized as actual gauge symmetry. Simiar results hold for the $SO(10)$ models where the gauge symmetry is broken down to the flipped $SU(5) \times U(1)_X$ gauge symmetry by turning on the $U(1)_X$ flux [3, 4, 13, 29]. Although the SM fermion Yukawa martices are rank one, the SM fermion masses and mixings can be generated in the presence of H-fluxes [12, 31]. However, to construct the globally consistent models, one found that the problematic situation with two different $\mathbf{10}_i$ and $\mathbf{10}_j$ matter curves meeting at an E_6 point requires significant fine-tuning to achieve at all. What happens most generically in globally consistent models is that an E_6 point occurs when a single $\mathbf{10}$ matter curve meets a single $\mathbf{5}$ Higgs curve. Of the two $U(1)_a \times U(1)_b$ symmetries that one might expect in the small open neighborhood of the E_6 point, one linear combination of them is essentially projected out by a monodromy action. Because $\mathbf{10}_i$ and $\mathbf{10}_j$ carry the same charge under the other linear combination of $U(1)_a \times U(1)_b$ symmetries that is not broken in the small open neighborhood of the E_6 point, the up-type quark Yukawa couplings can be realized [18, 22, 30]. Moreover, in the $SU(5)$ models, the SM gauge couplings at the string scale are splitted due to the $U(1)_Y$ flux [5, 15], thus, it is pretty difficult to explain this splitting in the minimal $SU(5)$ models without extra vector-like particles. In the $SO(10)$ models where the gauge symmetry is broken down to the $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry by turning on the $U(1)_{B-L}$ flux [10], three right-handed lepton doublets, two left-handed quark doublets (the first two generations) and one right-handed quark doublet (the third generation) are on one matter curve, while three left-handed lepton doublets, two right-handed quark doublets (the first two generations), and one left-handed quark doublet (the third generation) are on the other matter curve. Although the correspoding $U(1)_a \times U(1)_b$ gauge or global symmetries at the triple intersections of the SM fermion and Higgs curves can be preserved explicitly, one indeed needs some mechanism to generate the small CKM quark mixings between the first two generations and the third generation.

In this paper, we first briefly review the F-theory model building. We construct one $SU(5)$ model and two $SO(10)$ models from F-theory. In our models, we seek to retain a number of $U(1)$ symmetries from the underlying E_8 structure while ensuring that all the possible SM fermion Yukawa couplings (*i.e.*, all the entries in all the SM fermion Yukawa matrices) are allowed to be non-zero by the corresponding selection rules. This is desirable from the low energy phenomenological point of view. In our models, we can break the $SU(5)$ gauge symmetry down to the SM gauge symmetry by turning on the $U(1)_Y$ flux, and break the $SO(10)$ gauge symmetry down to the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry by turning on the $U(1)_{B-L}$ flux. To preserve the explicit $U(1)_a \times U(1)_b$ gauge or global symmetries at the triple intersections of the SM fermion and Higgs curves, we put the left-handed quarks and the right-handed quarks on the different matter curves, and put the left-handed leptons and neutrinos and the right-handed leptons and neutrinos on

the different matter curves. Thus, in our models, all the SM fermion Yukawa couplings are invariant under the enhanced $U(1)_a \times U(1)_b$ gauge or global symmetries at the intersections of the SM fermion and Higgs curves. And then our models can be understood very well from the local field theory description. Although the SM fermion Yukawa matrices are rank one, the SM fermion masses and mixings can be generated in the presence of H-fluxes [31]. Moreover, we can solve the doublet-triplet splitting problem naturally. The extra vector-like particles can obtain the heavy masses via the instanton effects or Higgs mechanism and then decouple at the high scale. Therefore, we only have the supersymmetric Standard Model at the low energy. Similar to the $SU(5)$ models [5, 15], we show that the SM gauge couplings at the string scale are also splitted in our $SO(10)$ models due to the $U(1)_{B-L}$ flux. Interestingly, the extra vector-like particles in our $SU(5)$ and $SO(10)$ models can be considered as the heavy threshold corrections to the renormalization group equation (RGE) running for the SM gauge couplings, and then we can explain the SM gauge couplings at the string scale elegantly. Furthermore, in the $SU(5)$ model, we can have the Yukawa coupling unification for the bottom quark and tau lepton. In the $SO(10)$ models, we can have the Yukawa coupling unification for the top and bottom quarks, and the Yukawa coupling unification for the tau lepton and tau neutrino.

This paper is organized as follows. In Section II, we briefly review the F-theory model building. In Sections III and IV, we construct the $SU(5)$ model and the $SO(10)$ models, respectively. Our conclusions are in Section V. In Appendix A, we briefly review the del Pezzo surfaces.

II.   F-THEORY MODEL BUILDING

We first briefly review the F-theory model building [1–5]. The twelve-dimensional F theory is a convenient way to describe Type IIB vacua with varying axion-dilaton $\tau = a + ie^{-\phi}$. We compactify F-theory on a Calabi-Yau fourfold, which is elliptically fibered $\pi : Y_4 \to B_3$ with a section $\sigma : B_3 \to Y_4$. The base B_3 is the internal space dimensions in Type IIB string theory, and the complex structure of the T^2 fibre encodes τ at each point of B_3. The SM or GUT gauge theories are on the worldvolume of the observable seven-branes that wrap a complex codimension-one suface in B_3. Denoting the complex coordinate tranverse to these seven-branes in B_3 as z, we can write the elliptic fibration in Weierstrass form

$$y^2 = x^3 + f(z)x + g(z) \ , \tag{1}$$

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where $f(z)$ and $g(z)$ are sections of $K_{B_3}^{-4}$ and $K_{B_3}^{-6}$, respectively. The complex structure of the fibre is

$$j(\tau) = \frac{4(24f)^3}{\Delta} , \quad \Delta = 4f^3 + 27g^2 . \tag{2}$$

At the discriminant locus $\{\Delta = 0\} \subset B_3$, the torus T^2 degenerates by pinching one of its cycles and becomes singular. For a generic pinching one-cycle $(p, q) = p\alpha + q\beta$ where α and β are one-cylces for the torus T^2, we obtain a (p, q) seven-brane in the locus where the (p, q) string can end. The singularity types of the ellitically fibres fall into the familiar ADE classifications, and we identify the corresponding ADE gauge groups on the seven-brane world-volume. This is one of the most important advantages for the F-theory model building: the exceptional gauge groups appear rather naturally, which is absent in the perturbative Type II string theory. And then all the SM fermion Yuakwa couplings in the GUTs can be generated.

We assume that the observable seven-branes with GUT models on its worldvolume wrap a complex codimension-one suface S in B_3, and the observable gauge symmetry is G_S. When $h^{1,0}(S) \neq 0$, the low energy spectrum may contain the extra states obtained by reduction of the bulk supergravity modes of compactification. So we require that $\pi_1(S)$ be a finite group. In order to decouple gravity and construct models locally, the extension of the local metric on S to a local Calabi-Yau fourfold must have a limit where the surface S can be shrunk to zero size. This implies that the anti-canonical bundle on S must be ample. Therefore, S is a del Pezzo n surface dP_n with $n \geq 2$ in which $h^{2,0}(S) = 0$. By the way, the Hirzebruch surfaces with degree larger than 2 satisfy $h^{2,0}(S) = 0$ but do not define the fully consistent decoupled models [3, 4].

To describe the spectrum, we have to study the gauge theory of the worldvolume on the seven-branes. We start from the maximal supersymmetric gauge theory on $\mathbb{R}^{3,1} \times \mathbb{C}^2$ and then replace $\mathbb{C}^2$ with the Kähler surface S. In order to have four-dimensional $\mathcal{N} = 1$ supersymmetry, the maximal supersymmetric gauge theory on $\mathbb{R}^{3,1} \times \mathbb{C}^2$ should be twisted. It was shown that there exists an unique twist preserving $\mathcal{N} = 1$ supersymmetry in four dimensions, and chiral matters can arise from the bulk S or the codimension-one curve Σ in S which is the intersection between the observable seven-branes and the other seven-brane(s) [3, 4].

In order to have the matter fields on S, we consider a non-trivial vector bundle on S with a structure group H_S which is a subgroup of G_S. Then the gauge group G_S is broken down to $\Gamma_S \times H_S$, and the adjoint representation $\mathrm{ad}(G_S)$ of the G_S is decomposed as

$$\mathrm{ad}(G_S) \to \mathrm{ad}(\Gamma_S) \bigoplus \mathrm{ad}(H_S) \bigoplus_j (\tau_j, T_j) . \tag{3}$$

Employing the vanishing theorem of the del Pezzo surfaces, we obtain the numbers of the generations and anti-generations by calculating the zero modes of the Dirac operator on S

$$n_{\tau_j} = -\chi(S, \mathbf{T_j}), \quad n_{\tau_j^*} = -\chi(S, \mathbf{T_j}^*), \tag{4}$$

where $\mathbf{T_j}$ is the vector bundle on S whose sections transform in the representation T_j of H_S, and $\mathbf{T_j}^*$ is the dual bundle of $\mathbf{T_j}$. In particular, when the H_S bundle is a line bundle L, we have

$$n_{\tau_j} = -\chi(S, L^j) = -\left[1 + \frac{1}{2}\left(\int_S c_1(L^j)c_1(S) + \int_S c_1(L^j)^2\right)\right]. \tag{5}$$

In order to preserve supersymmetry, the line bundle L should satisfy the BPS equation [3]

$$J_S \wedge c_1(L) = 0, \tag{6}$$

where J_S is the Kähler form on S. Moreover, the admissible supersymmetric line bundles on del Pezzo surfaces must satisfy $c_1(L)c_1(S) = 0$, thus, $n_{\tau_j} = n_{\tau_j^*}$ and only the vector-like particles can be obtained. In short, we can not have the chiral matter fields on the worldvolume of the observable seven-branes.

Interestingly, the chiral superfields can come from the intersections between the observable seven-branes and the other seven-brane(s) [3, 4]. Let us consider a stack of seven-branes with gauge group $G_{S'}$ that wrap a codimension-one surface S' in B_3. The intersection of S and S' is a codimenion-one curve (Riemann surface) Σ in S and S', and the gauge symmetry on Σ will be enhanced to G_Σ where $G_\Sigma \supset G_S \times G_{S'}$. On this curve, there exist chiral matters from the decomposition of the adjoint representation adG_Σ of G_Σ as follows

$$\mathrm{ad}G_\Sigma = \mathrm{ad}G_S \oplus \mathrm{ad}G_{S'} \oplus_k (U_k \otimes U'_k). \tag{7}$$

Turning on the non-trivial gauge bundles on S and S' respectively with structure groups H_S and $H_{S'}$, we break the gauge group $G_S \times G_{S'}$ down to the commutant subgroup $\Gamma_S \times \Gamma_{S'}$. Defining $\Gamma \equiv \Gamma_S \times \Gamma_{S'}$ and $H \equiv H_S \times H_{S'}$, we can decompose $U \otimes U'$ into the irreducible representations as follows

$$U \otimes U' = \bigoplus_k (r_k, V_k), \tag{8}$$

where r_k and V_k are the representations of Γ and H, respectively. The light chiral fermions in the representation r_k are determined by the zero modes of the Dirac operator on Σ. The net number of chiral superfields is given by

$$N_{r_k} - N_{r_k^*} = \chi(\Sigma, K_\Sigma^{1/2} \otimes \mathbf{V_k}), \tag{9}$$

where K_Σ is the restriction of canonical bundle on the curve Σ, and $\mathbf{V_k}$ is the vector bundle whose sections transform in the representation V_k of the structure group H.

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In the F-theory model building, we are interested in the models where $G_{S'}$ is $U(1)'$, and H_S and $H_{S'}$ are respectively $U(1)$ and $U(1)'$. Then the vector bundles on S and S' are line bundles L and L'. The adjoint representation $\mathrm{ad}G_\Sigma$ of G_Σ is decomposed into a direct sum of the irreducible representations under the group $\Gamma_S \times U(1) \times U(1)'$ that can be denoted as $(\mathbf{r_j}, \mathbf{q_j}, \mathbf{q'_j})$

$$\mathrm{ad}G_\Sigma = \mathrm{ad}(\Gamma_S) \oplus \mathrm{ad}G_{S'} \oplus_j (\mathbf{r_j}, \mathbf{q_j}, \mathbf{q'_j}) \ . \tag{10}$$

The numbers of chiral supefields in the representation $(\mathbf{r_j}, \mathbf{q_j}, \mathbf{q'_j})$ and their Hermitian conjugates on the curve Σ are given by

$$N_{(\mathbf{r_j}, \mathbf{q_j}, \mathbf{q'_j})} = h^0(\Sigma, \mathbf{V}_j) \ , \quad N_{(\mathbf{\bar{r}_j}, -\mathbf{q_j}, -\mathbf{q'_j})} = h^1(\Sigma, \mathbf{V}_j) \ , \tag{11}$$

where

$$\mathbf{V}_j = K_\Sigma^{1/2} \otimes L_\Sigma^{q_j} \otimes L_\Sigma'^{q'_j} \ , \tag{12}$$

where $K_\Sigma^{1/2}$, $L_\Sigma^{r_j}$ and $L_\Sigma'^{q'_j}$ are the restrictions of canonical bundle K_S, line bundles L and L' on the curve Σ, respectively. In particular, if the volume of S' is infinite, $G_{S'} = U(1)'$ is decoupled. And then the index $\mathbf{q'_j}$ can be ignored.

Using Riemann-Roch theorem, we obtain the net number of chiral supefields in the representation $(\mathbf{r_j}, \mathbf{q_j}, \mathbf{q'_j})$

$$N_{(\mathbf{r_j}, \mathbf{q_j}, \mathbf{q'_j})} - N_{(\mathbf{\bar{r}_j}, -\mathbf{q_j}, -\mathbf{q'_j})} = 1 - g + \deg(\mathbf{V}_j) \ , \tag{13}$$

where g is the genus of the curve Σ.

Moreover, we can obtain the Yukawa couplings at the triple intersections of three curves Σ_i, Σ_j and Σ_k where the gauge group or the singularity type is enhanced further. To have the triple intersections, the corresponding homology classes $[\Sigma_i]$, $[\Sigma_j]$ and $[\Sigma_k]$ of the curves Σ_i, Σ_j and Σ_k must satisfy the following conditions

$$[\Sigma_i] \cdot [\Sigma_j] > 0 \ , \quad [\Sigma_i] \cdot [\Sigma_k] > 0 \ , \quad [\Sigma_j] \cdot [\Sigma_k] > 0 \ . \tag{14}$$

In this paper, we will construct the $SU(5)$ and $SO(10)$ models. For simplicity, we will consider S to be the del Pezzo 8 surface dP_8 which is wrapped by the observable seven-branes.

III. $SU(5)$ **MODEL**

In this Section, we will construct the $SU(5)$ model from F-theory. First, let us briefly review the $SU(5)$ model and explain the convention. For convenience, we define the $U(1)_Y$ hypercharge generator in $SU(5)$ as follows

$$T_{U(1)_Y} = \mathrm{diag}\left(-2, -2, -2, 3, 3\right) \ . \tag{15}$$

Under $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry, the $SU(5)$ representations are decomposed as follows

$$\mathbf{5} = (\mathbf{3}, \mathbf{1}, -\mathbf{2}) \oplus (\mathbf{1}, \mathbf{2}, \mathbf{3}) \,, \tag{16}$$

$$\overline{\mathbf{5}} = (\overline{\mathbf{3}}, \mathbf{1}, \mathbf{2}) \oplus (\mathbf{1}, \mathbf{2}, -\mathbf{3}) \,, \tag{17}$$

$$\mathbf{10} = (\mathbf{3}, \mathbf{2}, \mathbf{1}) \oplus (\overline{\mathbf{3}}, \mathbf{1}, -\mathbf{4}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{6}) \,, \tag{18}$$

$$\overline{\mathbf{10}} = (\overline{\mathbf{3}}, \mathbf{2}, -\mathbf{1}) \oplus (\mathbf{3}, \mathbf{1}, \mathbf{4}) \oplus (\mathbf{1}, \mathbf{1}, -\mathbf{6}) \,, \tag{19}$$

$$\mathbf{24} = (\mathbf{8}, \mathbf{1}, \mathbf{0}) \oplus (\mathbf{1}, \mathbf{3}, \mathbf{0}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{0}) \oplus (\mathbf{3}, \mathbf{2}, -\mathbf{5}) \oplus (\overline{\mathbf{3}}, \mathbf{2}, \mathbf{5}) \,. \tag{20}$$

There are three families of the SM fermions whose quantum numbers under $SU(5)$ are

$$F_i = \mathbf{10}, \ \overline{f}_i = \overline{\mathbf{5}}, \ N_i^c = \mathbf{1} \,, \tag{21}$$

where $i = 1, 2, 3$, and N_i^c are the right-handed neutrinos. The SM particle assignments in F_i and $\bar{f}_i$ are

$$F_i = (Q_i, U_i^c, E_i^c) \,, \ \overline{f}_i = (D_i^c, L_i) \,, \tag{22}$$

where Q_i and L_i are respectively the left-handed quark and lepton doublets, and U_i^c, D_i^c, and E_i^c are the right-handed up-type quarks, down-type quarks, and charged leptons, respectively. In our model building, we will introduce the SM singlet fields $\overline{E}_i^c$ from $\overline{\mathbf{10}}$ whose SM quantum numbers are $(\mathbf{1}, \mathbf{1}, -\mathbf{6})$. Especially, $\overline{E}_i^c$ are Hermitian conjugate of E_i^c, and then they can form vector-like particles.

To break the electroweak gauge symmetry, we introduce one pair of Higgs fields whose quantum numbers under $SU(5)$ are

$$h = \mathbf{5} \,, \quad \overline{h} = \overline{\mathbf{5}} \,. \tag{23}$$

Explicitly, we denote the Higgs particles as follows

$$h = (D_h, D_h, D_h, H_u) \,, \ \overline{h} = (\overline{D}_{\overline{h}}, \overline{D}_{\overline{h}}, \overline{D}_{\overline{h}}, H_d) \,, \tag{24}$$

where H_u and H_d are one pair of the Higgs doublets in the Minimal Supersymmetric Standard Model (MSSM). In $SU(5)$ model, the superpotential for the SM fermion Yukawa couplings and the mass terms of the Higgs fields and right-handed neutrinos is

$$W_{\text{Yukawa}} = y_{ij}^U F_i F_j h + y_{ij}^{DE} F_i \overline{f}_j \overline{h} + y_{ij}^\nu N_i^c \overline{f}_j h + \mu h \overline{h} + m_{ij}^N N_i^c N_j^c \,, \tag{25}$$

where y_{ij}^U, y_{ij}^{DE}, y_{ij}^ν are Yukawa couplings, μ is the bilinear Higgs mass term, and m_{ij}^N are the Majorana masses for the right-handed neutrinos. So, we have the Yukawa coupling unification for the bottom quark and tau lepton.

Because we will construct $SU(5)$ model, we choose $G_S = SU(5)$ and $H_S = U(1)_Y$. The SM fermions $\overline{f}_i$, and the Higgs fields h and $\overline{h}$ are on the curves where the $SU(5)$ gauge symmetry is enhanced to $SU(6)$. Under $SU(5) \times U(1)$, the adjoint representation of $SU(6)$ is decomposed as follows

$$\mathbf{35} \;=\; (\mathbf{24}, \mathbf{0}) \oplus (\mathbf{1}, \mathbf{0}) \oplus (\mathbf{5}, \mathbf{6}) \oplus (\overline{\mathbf{5}}, -\mathbf{6}) \;. \tag{26}$$

Also, the SM fermions F_i are on the curves where the $SU(5)$ gauge symmetry is enhanced to $SO(10)$. Under $SU(5) \times U(1)$, the adjoint representation of $SO(10)$ is decomposed as follows

$$\mathbf{45} \;=\; (\mathbf{24}, \mathbf{0}) \oplus (\mathbf{1}, \mathbf{0}) \oplus (\mathbf{10}, \mathbf{4}) \oplus (\overline{\mathbf{10}}, -\mathbf{4}) \;. \tag{27}$$

In addition, the SM fermion Yukawa couplings $y_{ij}^{DE} F_i \overline{f}_j \overline{h}$ arise from the triple intersection where the gauge symmetry is enhanced to $SO(12)$. Under $SU(5) \times U(1)_1 \times U(1)_2$, the adjoint representation of $SO(12)$ is decomposed as follows

$$\begin{aligned}
\mathbf{66} \;=\;& (\mathbf{24}, \mathbf{0}, \mathbf{0}) \oplus (\mathbf{1}, \mathbf{0}, \mathbf{0}) \oplus (\mathbf{1}, \mathbf{0}, \mathbf{0}) \oplus (\mathbf{5}, \mathbf{2}, \mathbf{2}) \oplus (\mathbf{5}, \mathbf{2}, -\mathbf{2}) \oplus (\overline{\mathbf{5}}, -\mathbf{2}, \mathbf{2}) \\
&\oplus (\overline{\mathbf{5}}, -\mathbf{2}, -\mathbf{2}) \oplus (\mathbf{10}, \mathbf{4}, \mathbf{0}) \oplus (\overline{\mathbf{10}}, -\mathbf{4}, \mathbf{0}) \;.
\end{aligned} \tag{28}$$

We denote the $U(1)_1 \times U(1)_2$ generators in $SO(12)$ as t_1 and t_2. We emphasize that the field theory description in terms of $SU(5) \times U(1)_1 \times U(1)_2$ is valid only within a small open neighborhood of the $SO(12)$ point. Thus, to preserve the $U(1)_1 \times U(1)_2$ gauge or global symmetry for the down-type quark and lepton Yukawa couplings in the small open neighborhood of the $SO(12)$ point, we obtain that the SM fermions F_i, $\overline{f}_j$, and Higgs fields $\overline{h}$ should localize on the curves $t_1 = 0$, $t_1 + t_2 = 0$ (or $t_1 - t_2 = 0$), and $t_1 - t_2 = 0$ (or $t_1 + t_2 = 0$), respectively.

Also, the SM fermion Yukawa couplings $y_{ij}^U F_i F_j h$ arise from the triple intersection where the gauge symmetry is enhanced to E_6. Because in our models Q_i and U_i^c are not in the same F_i multiplets, we denote these Yukawa couplings as $y_{ij}^U \mathbf{10}_i \mathbf{10}'_j h$. Under $SU(5) \times U(1)_a \times U(1)_b$, the adjoint representation of E_6 is decomposed as follows

$$\begin{aligned}
\mathbf{78} \;=\;& (\mathbf{24}, \mathbf{0}, \mathbf{0}) \oplus (\mathbf{1}, \mathbf{0}, \mathbf{0}) \oplus (\mathbf{1}, \mathbf{0}, \mathbf{0}) \oplus (\mathbf{1}, \mathbf{5}, \mathbf{3}) \oplus (\mathbf{1}, -\mathbf{5}, -\mathbf{3}) \oplus (\mathbf{5}, -\mathbf{3}, \mathbf{3}) \\
&\oplus (\overline{\mathbf{5}}, \mathbf{3}, -\mathbf{3}) \oplus (\mathbf{10}, -\mathbf{1}, -\mathbf{3}) \oplus (\overline{\mathbf{10}}, \mathbf{1}, \mathbf{3}) \oplus (\mathbf{10}, \mathbf{4}, \mathbf{0}) \oplus (\overline{\mathbf{10}}, -\mathbf{4}, \mathbf{0}) \;. \tag{29}
\end{aligned}$$

We denote the $U(1)_a \times U(1)_b$ generators in E_6 as t_a and t_b. We emphasize that the field theory description in terms of $SU(5) \times U(1)_a \times U(1)_b$ is valid only within a small open neighborhood of the E_6 point. Thus, to preserve the $U(1)_a \times U(1)_b$ gauge or global symmetry for the SM up-type quark Yukawa couplings in the small open neighborhood of the E_6 point, we obtain that the SM fermions $\mathbf{10}_i$, $\mathbf{10}'_j$, and Higgs field h should localize on the curves $t_a = 0$ (or

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$t_a + 3t_b = 0$), $t_a + 3t_b = 0$ (or $t_a = 0$), and $t_a - t_b = 0$, respectively. Therefore, to have the up-type quark Yukawa couplings which are invariant under $U(1)_a \times U(1)_b$, we obtain that Q_i and U_j^c should localize on the matter curves $t_a = 0$ (or $t_a + 3t_b = 0$), and $t_a + 3t_b = 0$ (or $t_a = 0$), respectively.

By the way, the simplest possibility is to put one complete **10** of the SM fermions on the matter curve $t_a = 0$ (or $t_a + 3t_b = 0$) and the remaining two complete **10**s of the SM fermions on the matter curve $t_a + 3t_b = 0$ (or $t_a = 0$), respectively. However, because the first **10** can only couple to the second and third **10**s but not to itself, and similarly, the second and third **10**s can couple only to the first **10**, we obtain the up-type quark Yukawa matrix

$$y^U \sim \begin{pmatrix} 0 & * & * \\ * & 0 & 0 \\ * & 0 & 0 \end{pmatrix} . \tag{30}$$

Note that there are two massless up-type quarks, this possibility is disfavored from the phenomenological point of view. Thus, we put the left-handed quark doublets Q_i and the right-handed up-type quarks U_j^c on the different matter curves.

Particles	Curve	Class	g_Σ	L_Σ	$L_\Sigma^{'n}$
H_u	Σ_h	$2H - E_1 - E_3$	0	$\mathcal{O}_{\Sigma_h}(1)^{1/5}$	$\mathcal{O}_{\Sigma_h}(1)^{2/5}$
H_d	$\Sigma_{\overline{h}}$	$2H - E_2 - E_3$	0	$\mathcal{O}_{\Sigma_{\overline{h}}}(-1)^{1/5}$	$\mathcal{O}_{\Sigma_{\overline{h}}}(-1)^{2/5}$
$\overline{f}_i$	Σ_f	$2H$	0	$\mathcal{O}_{\Sigma_f}$	$\mathcal{O}_{\Sigma_f}(-3p')$
$\left(E_{2i-1}^c,\ E_{2i}^c,\ Q_i\right)$	Σ_{Qi}	$2H - E_1 - E_{i+3}$	0	$\mathcal{O}_{\Sigma_{Qi}}(1)^{1/5}$	$\mathcal{O}_{\Sigma_{Qi}}(1)^{4/5}$
$\left(U_i^c,\ \overline{E}_i^c\right)$	Σ_{Ui}	$2H - E_2 - E_{i+3}$	0	$\mathcal{O}_{\Sigma_{Ui}}(-1)^{1/5}$	$\mathcal{O}_{\Sigma_{Ui}}(1)^{1/5}$

TABLE I: The particle curves and the gauge bundle assignments for each curve in the $SU(5)$ model. Here, i is the SM fermion family index, $i.e.$, $i = 1,\ 2,\ 3$.

In our $SU(5)$ model, we choose the line bundle $L = \mathcal{O}_S(E_1 - E_2)^{1/5}$, and present the particle curves with homology classes and the gauge bundle assignments for each curve in Table I. We do not have the vector-like particles from the bulk of the observable seven-branes since $\chi(S, L^5) = 0$. Note that the Higgs triplets in h and $\overline{h}$ do not have zero modes, we solve the doublet-triplet splitting problem. Also, we have six E_i^c fields and three $\overline{E}_i^c$ fields. Because they are vector-like, we indeed have three chiral E_i^c fields. In addition, we assume that the SM fermion and Higgs curves Σ_{Qi}, Σ_{Ui}, and Σ_h, intersect at one point in S where the gauge symmetry is enhanced to E_6. And we introduce the SM singlet Higgs fields X_k and $\overline{X}_k$ respectively with $SU(5) \times U(1)_a \times U(1)_b$ quantum numbers $(\mathbf{1}, \mathbf{5}, \mathbf{3})$ and $(\mathbf{1}, -\mathbf{5}, -\mathbf{3})$ at this E_6 point from the intersections of the other seven-branes. Moreover, we

assume that the SM fermion and Higgs curves Σ_{Qi}, Σ_f, and $\Sigma_{\overline{h}}$, intersect at the other point in S where the gauge symmetry is enhanced to $SO(12)$. In order to have the SM fermion Yukawa couplings which are invariant under $U(1)_1 \times U(1)_2$ and $U(1)_a \times U(1)_b$ symmetries, we choose that the Higgs curves Σ_h and $\Sigma_{\overline{h}}$ satisfy $t_1 - t_2 = 0$ in the small open neighborhood of the $SO(12)$ point and $t_a - t_b = 0$ in the small open neighborhood of the E_6 point, the SM fermion curve Σ_f satisfies $t_1 + t_2 = 0$ in the small open neighborhood of the $SO(12)$ point, the SM fermion curve Σ_{Qi} satisfies $t_1 = 0$ in the small open neighborhood of the $SO(12)$ point and $t_a = 0$ in the small open neighborhood of the E_6 point, and the SM fermion curve Σ_{Ui} satisfies $t_a + 3t_b = 0$ in the small open neighborhood of the E_6 point. Therefore, the superpotential in our $SU(5)$ model is

$$W_{SSM} = y_{ij}^D D_i^c Q_j H_d + y_{ij}^U U_i^c Q_j H_u + y_{ij}^E E_i^c L_j H_d + \lambda_1^{ijk} \overline{X}_i E_j^c \overline{E}_k^c + \mu H_d H_u , \qquad (31)$$

where y_{ij}^D, y_{ij}^U, y_{ij}^E, and λ_1^{ijk} are Yukawa couplings. μ term can be generated via the instanton effects or the Higgs mechanism since H_u and H_d can couple to the SM singlet Higgs fields from the intersection of the other seven-branes. Interestingly, we can also have the Yukawa coupling unification for the bottom quark and tau lepton at the GUT scale. Although the SM fermion Yukawa matrices are rank one, the SM fermion masses and mixings can be generated in the presence of H-fluxes [31]. Also, after $\overline{X}_i$ obtain the vacuum expectation values (VEVs), three linear combinations of the six E_i^c fields and three $\overline{E}_i^c$ fields will obtain the vector-like masses and then decouple at the high scale. Thus, only three linear combinations of six E_i^c fields will be massless, and then we have the supersymmetric Standard Model at the low energy.

In the $SU(5)$ models, the tree-level gauge kinetic functions with the $U(1)_Y$ flux contributions at the string scale are [5, 15]

$$f_{SU(3)_C} = \tau_o - \frac{1}{2}\tau \int_S c_1^2(L_o) , \qquad (32)$$

$$f_{SU(2)_L} = \tau_o - \frac{1}{2}\tau \int_S \left(c_1^2(L_o) + c_1^2(L^5) \right) , \qquad (33)$$

$$f_{U(1)_Y} = \tau_o - \frac{1}{2}\tau \int_S \left(c_1^2(L_o) + \frac{3}{5}c_1^2(L^5) \right) , \qquad (34)$$

where τ_o is the original gauge kinetic function of $SU(5)$, and L_o is the restriction of the line bundle on the internal Calabi-Yau manifold. Thus, we obtain the SM gauge coupling relation at the string scale [5, 15]

$$\alpha_1^{-1} - \alpha_3^{-1} = \frac{3}{5}(\alpha_2^{-1} - \alpha_3^{-1}) , \quad \alpha_3^{-1} < \alpha_1^{-1} < \alpha_2^{-1} , \qquad (35)$$

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where α_1, α_2, and α_3 are the gauge couplings for the $U(1)_Y$, $SU(2)_L$, and $SU(3)_C$, respectively. Interestingly, in our $SU(5)$ model, we have three linear combinations of the six E_i^c fields and three $\overline{E}_i^c$ fields that will obtain the heavy vector-like masses. If we consider these vector-like particles as heavy threshold corrections to the RGE running of the SM gauge couplings, we can explain the SM gauge couplings at the string scale. The detailed discussions will be given elsewhere.

IV. $SO(10)$ MODELS

In this Section, we will construct two $SO(10)$ models from F-theory. Turning on the $U(1)_{B-L}$ flux, we can break the $SO(10)$ gauge symmetry down to the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry. For convenience, we define the $U(1)_{B-L}$ generator in $SU(4)_C$ of $SO(10)$ as follows

$$T_{\mathrm{U(1)_{B-L}}} = \mathrm{diag}\,(1, 1, 1, -3) \ . \tag{36}$$

Under $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry, the $SO(10)$ representations are decomposed as follows

$$\mathbf{10} = (\mathbf{1, 2, 2, 0}) \oplus (\mathbf{3, 1, 1, -2}) \oplus (\mathbf{\overline{3}, 1, 1, 2}) \ , \tag{37}$$

$$\mathbf{16} = (\mathbf{3, 2, 1, 1}) \oplus (\mathbf{1, 2, 1, -3}) \oplus (\mathbf{\overline{3}, 1, 2, -1}) \oplus (\mathbf{1, 1, 2, 3}) \ , \tag{38}$$

$$\mathbf{45} = (\mathbf{8, 1, 1, 0}) \oplus (\mathbf{1, 3, 1, 0}) \oplus (\mathbf{1, 1, 3, 0}) \oplus (\mathbf{1, 1, 1, 0}) \oplus (\mathbf{3, 1, 1, 4})$$
$$\oplus (\mathbf{\overline{3}, 1, 1, -4}) \oplus (\mathbf{3, 2, 2, -2}) \oplus (\mathbf{\overline{3}, 2, 2, 2}) \ . \tag{39}$$

Under the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry, the SM fermions are

$$Q_i = (\mathbf{3, 2, 1, 1}) \ , \quad Q_i^R = (\mathbf{\overline{3}, 1, 2, -1}) \ , \quad L_i = (\mathbf{1, 2, 1, -3}) \ , \quad L_i^R = (\mathbf{1, 1, 2, 3}) \ , \tag{40}$$

where $i = 1, 2, 3$. The SM particle assignments in Q_i^R, and L_i^R are

$$Q_i^R = (U_i^c, D_i^c) \ , \quad L_i^R = (E_i^c, N_i^c) \ . \tag{41}$$

To break the $SU(2)_R \times U(1)_{B-L}$ gauge symmetry down to $U(1)_Y$ and to break the electroweak gauge symmetry, we introduce the following Higgs fields whose quantum numbers under the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry are

$$\Phi = (\mathbf{1, 1, 2, 3}) \ , \quad \overline{\Phi} = (\mathbf{1, 1, 2, -3}) \ , \quad H = (\mathbf{1, 2, 2, 0}) \ , \tag{42}$$

where Φ and $\overline{\Phi}$ are the Higgs fields to break the $SU(2)_R \times U(1)_{B-L}$ gauge symmetry, and H contains both H_u and H_d in the MSSM.

In the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model, the superpotential for the SM fermion Yukawa couplings and Higgs mass term is

$$W_{\text{Yukawa}} = y_{ij}^{UD} Q_i H Q_j^R + y_{ij}^{E\nu} L_i H L_j^R + \mu H H \ , \tag{43}$$

where y_{ij}^{UD} and $y_{ij}^{E\nu}$ are Yukawa couplings, and μ is the bilinear Higgs mass term. Thus, we obtain the Yukawa coupling unification for the SM quarks and the Yukawa coupling unification for the SM leptons and neutrinos at the GUT scale.

In our model building, we will also introduce the additional vector-like particles U_i' and $U_i'^c$ from the bulk S, and the particles $\overline{Q}_i$, $\overline{Q}_i^R$, $\overline{L}_i$ and $\overline{L}_i^R$ from $\overline{\mathbf{16}}$, whose quantum numbers under the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry are

$$
\begin{aligned}
U_i' &= (\mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{4}) \ , \quad U_i'^c = (\overline{\mathbf{3}}, \mathbf{1}, \mathbf{1}, -\mathbf{4}) \ , \quad \overline{Q}_i = (\overline{\mathbf{3}}, \mathbf{2}, \mathbf{1}, -\mathbf{1}) \ , \\
\overline{Q}_i^R &= (\mathbf{3}, \mathbf{1}, \mathbf{2}, \mathbf{1}) \ , \quad \overline{L}_i = (\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{3}) \ , \quad \overline{L}_i^R = (\mathbf{1}, \mathbf{1}, \mathbf{2}, -\mathbf{3}) \ .
\end{aligned}
\tag{44}
$$

Because we will construct the $SO(10)$ models, we choose $G_S = SO(10)$ and $H_S = U(1)_{B-L}$. The bidoublet Higgs field H is on the curve where the $SO(10)$ gauge symmetry is enhanced to $SO(12)$. Under $SO(10) \times U(1)$, the adjoint representation of $SO(12)$ is decomposed as follows

$$\mathbf{66} = (\mathbf{45}, \mathbf{0}) \oplus (\mathbf{1}, \mathbf{0}) \oplus (\mathbf{10}, \mathbf{2}) \oplus (\mathbf{10}, -\mathbf{2}) \ . \tag{45}$$

All the other fields in our models are on the curves where the $SO(10)$ gauge symmetry is enhanced to E_6. Under $SO(10) \times U(1)$, the adjoint representation of E_6 is decomposed as follows

$$\mathbf{78} = (\mathbf{45}, \mathbf{0}) \oplus (\mathbf{1}, \mathbf{0}) \oplus (\mathbf{16}, -\mathbf{3}) \oplus (\overline{\mathbf{16}}, \mathbf{3}) \ . \tag{46}$$

In addition, the SM fermion Yukawa couplings in $SO(10)$ models arise from the triple intersections where the gauge symmetry is enhanced to E_7. Under $SO(10) \times U(1)_a \times U(1)_b$, the adjoint representation of E_7 is decomposed as follows

$$
\begin{aligned}
\mathbf{133} = {}& (\mathbf{45}, \mathbf{0}, \mathbf{0}) \oplus (\mathbf{1}, \mathbf{0}, \mathbf{0}) \oplus (\mathbf{1}, \mathbf{0}, \mathbf{0}) \oplus (\mathbf{1}, \mathbf{0}, \mathbf{2}) \oplus (\mathbf{1}, \mathbf{0}, -\mathbf{2}) \oplus (\mathbf{10}, \mathbf{2}, \mathbf{0}) \\
& \oplus (\mathbf{10}, -\mathbf{2}, \mathbf{0}) \oplus (\mathbf{16}, -\mathbf{1}, \mathbf{1}) \oplus (\mathbf{16}, -\mathbf{1}, -\mathbf{1}) \oplus (\overline{\mathbf{16}}, \mathbf{1}, \mathbf{1}) \oplus (\overline{\mathbf{16}}, \mathbf{1}, -\mathbf{1}) \ . \tag{47}
\end{aligned}
$$

We denote the $U(1)_a \times U(1)_b$ generators in E_7 as t_a and t_b. We emphasize that the field theory description in terms of $SO(10) \times U(1)_a \times U(1)_b$ is valid only within a small open neighborhood of the E_7 point. Thus, to preserve the $U(1)_a \times U(1)_b$ gauge or global symmetry for the SM fermion Yukawa couplings in the small open neighborhood of the E_7 point, we obtain that the bidoublet Higgs field H localize on the curve $t_a = 0$, the SM quarks Q_i and Q_i^R should

localize respectively on the matter curves $t_a + t_b = 0$ (or $t_a - t_b = 0$) and $t_a - t_b = 0$ (or $t_a + t_b = 0$), and the SM leptons L_i and L_i^R should localize respectively on the matter curves $t_a - t_b = 0$ (or $t_a + t_b = 0$) and $t_a + t_b = 0$ (or $t_a - t_b = 0$).

Moreover, the Higgs Yukawa couplings in $SO(10)$ models arise from the Higgs curve intersection where the gauge symmetry is enhanced to $SO(14)$. Under $SO(10) \times U(1)_1 \times U(1)_2$, the adjoint representation of $SO(14)$ is decomposed as follows

$$\begin{aligned}
\mathbf{91} = &\ (\mathbf{45, 0, 0}) \oplus (\mathbf{1, 0, 0}) \oplus (\mathbf{1, 0, 0}) \oplus (\mathbf{10, 2, 0}) \oplus (\mathbf{10, -2, 0}) \oplus (\mathbf{10, 0, 2}) \\
&\oplus (\mathbf{1, 2, 2}) \oplus (\mathbf{1, -2, 2}) \oplus (\mathbf{10, 0, -2}) \oplus (\mathbf{1, 2, -2}) \oplus (\mathbf{1, -2, -2}) \ .
\end{aligned} \tag{48}$$

We denote the $U(1)_1 \times U(1)_2$ generators in $SO(14)$ as t_1 and t_2. We emphasize that the field theory description in terms of $SO(10) \times U(1)_1 \times U(1)_2$ is valid only within a small open neighborhood of the $SO(14)$ point. And we assume that the bidoublet Higgs field H localize on the curve $t_1 = 0$.

In this Section, we will take the line bundle as $L = \mathcal{O}_S(E_1 - E_2)^{1/2}$. Note that $\chi(S, L^4) = 3$, we have three pairs of vector-like particles U_i' and $U_i'^c$ from the bulk of the observable seven-branes. These vector-like particles U_i' and $U_i'^c$ can obtain masses via the instanton effects. Also, they can couple to the SM singlet Higgs fields from the intersections of the other seven-branes, and then obtain masses from the Higgs mechanism. For simplicity, in this paper, we will assume that the vector-like particles U_i' and $U_i'^c$ have masses around the GUT scale and then decouple. In the following two subsections, we will present two $SO(10)$ models.

Before we construct the $SO(10)$ models, let us consider the $U(1)_{B-L}$ flux contributions to the SM gauge couplings at the string scale. For $G = SO(10)$ gauge group, the generators T^a of $SO(10)$ are imaginary antisymmetric 10×10 matrices. In terms of the 2×2 identity matrix σ_0 and the Pauli matrices σ_i, they can be written as tensor products of 2×2 and 5×5 matrices, $(\sigma_0, \sigma_1, \sigma_3) \otimes A_5$ and $\sigma_2 \otimes S_5$ as a complete set, where A_5 and S_5 are the 5×5 real anti-symmetric and symmetric matrices [32]. The generators for $SU(4)_C \times SU(2)_L \times SU(2)_R$ are [32]

$$\begin{aligned}
(\sigma_0, \sigma_1, \sigma_3) \otimes A_3 \ , \quad (\sigma_0, \sigma_1, \sigma_3) \otimes A_2 \ , \\
\sigma_2 \otimes S_3 \ , \quad \sigma_2 \otimes S_2 \ ,
\end{aligned} \tag{49}$$

where A_3 and S_3 are respectively the diagonal blocks of A_5 and S_5 that have indices 1, 2, and 3, while the diagonal blocks A_2 and S_2 have indices 4 and 5. In addition, the generator for the $U(1)_{B-L}$ is

$$T_{B-L} = \frac{1}{\sqrt{3}} \sigma_2 \otimes \mathrm{diag}(1, 1, 1, 0, 0) \ . \tag{50}$$

The flux contributions to the gauge couplings can be computed by dimensionally reducing the Chern-Simons action of the observable seven-branes wrapping on S

$$S_{\mathrm{CS}} = \mu_7 \int_{S \times \mathbb{R}^{3,1}} a \wedge \mathrm{tr}(F^4) . \tag{51}$$

In our $SO(10)$ models, we choose the $U(1)_{B-L}$ flux as follows

$$\langle F_{\mathrm{U(1)_{B-L}}} \rangle = \frac{1}{2} V_{U(1)_{B-L}} \sigma_2 \otimes \mathrm{diag}(1,1,1,0,0) . \tag{52}$$

Let us noramlize the $SO(10)$ generators T^a as $\mathrm{Tr}(T^a T^b) = 2\delta_{ab}$. Then, we obtain the tree-level gauge kinetic functions with the $U(1)_{B-L}$ flux contributions for $SU(3)_C$, $U(1)_{B-L}$, $SU(2)_L$, and $SU(2)_R$ gauge symmetries at the string scale

$$f_{SU(3)_C} = f_{U(1)_{B-L}} = \tau_o - \frac{1}{2}\tau \int_S c_1^2(L^2) \equiv \tau_o + \tau , \tag{53}$$

$$f_{SU(2)_L} = f_{SU(2)_R} = \tau_o , \tag{54}$$

where τ_o is the original gauge kinetic function of $SO(10)$.

We will break the $SU(2)_R \times U(1)_{B-L}$ gauge symmetry at the string scale by Higgs mechanism in our $SO(10)$ models. Thus, we obtain the gauge kinetic function for $U(1)_Y$

$$f_{U(1)_Y} = \frac{3}{5}f_{SU(2)_R} + \frac{2}{5}f_{U(1)_{B-L}} = \tau_o + \frac{2}{5}\tau . \tag{55}$$

Therefore, we obtain the SM gauge coupling relation at the string scale

$$\alpha_1^{-1} - \alpha_3^{-1} = \frac{3}{5}(\alpha_2^{-1} - \alpha_3^{-1}) , \quad \alpha_2^{-1} < \alpha_1^{-1} < \alpha_3^{-1} . \tag{56}$$

We emphasize that although the SM gauge coupling relation at the string scale in the $SO(10)$ models is the same as that in the $SU(5)$ model, the order of the SM gauge couplings (α_i^{-1}) from small to large is reversed.

A. Type I $SO(10)$ Model

In Type I $SO(10)$ model, we present the particle curves with homology classes and the gauge bundle assignments for each curve in Table II. Note that the Higgs triplets in $\mathbf{10}$ of $SO(10)$ do not have zero modes, we solve the doublet-triplet splitting problem. Also, we have six L_i fields, six L_i^R fields, three $\overline{L}_i$ fields, and three $\overline{L}_i^R$ fields. Because $(L_i, \overline{L}_i)$ and $(L_i^R, \overline{L}_i^R)$ are vector-like, the net numbers of chiral L_i and L_i^R superfields are three. And we have two bidoublet Higgs fields H and H', which respectively come from the $(\mathbf{10}, \mathbf{2})$ and $(\mathbf{10}, -\mathbf{2})$ of the decompositions of the $SO(12)$ adjoint representation.

Particles	Curve	Class	g_Σ	L_Σ	$L_\Sigma'^n$
$(H,\ H')$	Σ_H (pinched)	$3H - E_1 - E_2$	1	$\mathcal{O}_{\Sigma_H}(p_1 - p_2)^{1/2}$	$\mathcal{O}_{\Sigma_H}$
$\left(\Phi,\ \overline{\Phi}\right)$	Σ_Φ (pinched)	$3H - E_1 - E_2 - E_8$	1	$\mathcal{O}_{\Sigma_\Phi}(p_1' - p_2')^{1/2}$	$\mathcal{O}_{\Sigma_\Phi}(p_1' - p_2')^{3/2}$
$\left(L_{2i-1}^R,\ L_{2i}^R,\ Q_i,\ \overline{L}_i\right)$	Σ_{Qi}	$2H - E_1 - E_{i+2}$	0	$\mathcal{O}_{\Sigma_{Qi}}(1)^{1/2}$	$\mathcal{O}_{\Sigma_{Qi}}(-1)^{1/2}$
$\left(L_{2i-1},\ L_{2i},\ Q_i^R,\ \overline{L}_i^R\right)$	Σ_{QRi}	$2H - E_2 - E_{i+2}$	0	$\mathcal{O}_{\Sigma_{QRi}}(-1)^{1/2}$	$\mathcal{O}_{\Sigma_{QRi}}(-1)^{1/2}$

TABLE II: The particle curves and the gauge bundle assignments for each curve in the Type I $SO(10)$ model. Here, $i = 1,\ 2,\ 3$.

In addition, we assume that the SM fermion and Higgs curves Σ_{Qi}, Σ_{QRi} and Σ_H intersect at one point in S where the gauge symmetry is enhanced to E_7. And we introduce the SM singlet Higgs fields X_k and $\overline{X}_k$ respectively with $SO(10) \times U(1)_a \times U(1)_b$ quantum numbers $(\mathbf{1}, \mathbf{0}, \mathbf{2})$ and $(\mathbf{1}, \mathbf{0}, -\mathbf{2})$ at this E_7 point from the intersection of the other seven-branes. Moreover, we assume that the SM fermion and Higgs curves Σ_{Qi} and Σ_Φ intersect at the other point in S where the gauge symmetry is enhanced to E_7 as well. And we introduce the SM singlet fields X_k' and $\overline{X}_k'$ respectively with $SO(10) \times U(1)_a \times U(1)_b$ quantum numbers $(\mathbf{1}, \mathbf{0}, \mathbf{2})$ and $(\mathbf{1}, \mathbf{0}, -\mathbf{2})$ at this E_7 point from the intersection of the other seven-branes. In order to have the SM fermion Yukawa couplings which are invariant under $U(1)_a \times U(1)_b$ symmetry, we choose that in the small open neighborhoods of these E_7 points, the Higgs curve Σ_H satisfies $t_a = 0$, the Higgs curve Σ_Φ satisfies $t_a - t_b = 0$, the SM fermion curve Σ_{Qi} satisfies $t_a + t_b = 0$, and the SM fermion curve Σ_{QRi} satisfies $t_a - t_b = 0$. Futhermore, the Higgs curve Σ_H is pinched at a point, where the gauge symmetry is enhanced to $SO(14)$. We assume that the Higgs curve Σ_H satisfies $t_1 = 0$ in the small open neighborhood of the $SO(14)$ point. And we can introduce the SM singlet Higgs fields ϕ_i^{++}, ϕ_i^{+-}, ϕ_i^{-+}, and ϕ_i^{--} respectively with $SO(10) \times U(1)_1 \times U(1)_2$ quantum numbers $(\mathbf{1}, \mathbf{2}, \mathbf{2})$, $(\mathbf{1}, \mathbf{2}, -\mathbf{2})$, $(\mathbf{1}, -\mathbf{2}, \mathbf{2})$, and $(\mathbf{1}, -\mathbf{2}, -\mathbf{2})$ on the $SO(14)$ point from the intersections of the other seven-branes. Therefore, the superpotential in Type I $SO(10)$ model is

$$
\begin{aligned}
W_{SSM} =\ & y_{ij}^{UD} Q_i H Q_j^R + y_{ij}^{E\nu} L_i H L_j^R + \lambda_1^{ijk} \overline{X}_i L_j \overline{L}_k + \lambda_2^{ijk} X_i L_j^R \overline{L}_k^R + \lambda_3^{ij} X_i' L_j^R \overline{\Phi} \\
& + \mu' H H' + \mu_\Phi \Phi \overline{\Phi} + \lambda_1'^{ij} \frac{1}{M_{\rm Pl}} H H \phi_i^{-+} \phi_j^{--} + \lambda_2'^{ij} \frac{1}{M_{\rm Pl}} H' H' \phi_i^{++} \phi_j^{+-} ,
\end{aligned}
\tag{57}
$$

where y_{ij}^{UD}, $y_{ij}^{E\nu}$, λ_1^{ijk}, λ_2^{ijk}, λ_3^{ij}, $\lambda_1'^{ij}$, and $\lambda_2'^{ij}$ are Yukawa couplings. Similar to the $SU(5)$ model, the μ' and μ_Φ terms can be generated via the instanton effects or Higgs mechanism. Interestingly, we can have the Yukawa coupling unification for the SM quarks and the Yukawa coupling unification for the SM leptons and neutrinos at the GUT scale. Also, after $\overline{X}_i$ and X_i obtain the VEVs, three linear combinations of the six L_i fields and three $\overline{L}_i$ fields will obtain the vector-like masses, and three linear combinations of the six L_i^R fields and three

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$\overline{L}_i^R$ fields will obtain the vector-like masses. And then these vector-like particles will be decouple around the GUT scale. Thus, only three linear combinations of six L_i fields and three linear combinations of six L_i^R fields will be massless below the GUT scale. In addition, after Φ and $\overline{\Phi}$ obtain VEVs, the right-handed neutrinos and some linear combinations of X_k' will obtain the Dirac masses. Thus, with three or more pairs of vector-like particles X_k' and $\overline{X}_k'$, we can generate the active neutrino masses via the double seesaw mechanism [33]. In short, although the SM fermion Yukawa matrices are rank one, the SM fermion masses and mixings can be generated in the presence of H-fluxes [31]. Furthermore, with small μ' term, we can make sure that the bidoublet Higgs field H' obtains mass at the high scale while the bidoublet Higgs field H remains light around the electroweak scale by choosing suitable VEVs for the SM singlet Higgs fields ϕ_i^{++}, ϕ_i^{+-}, ϕ_i^{-+}, and ϕ_i^{--}. Then the bidoublet Higgs field H' can be decoupled, and the μ term for the bidoublet Higgs field H can be around the TeV scale. Therefore, we can only have the supersymmetric Standard Model at the low energy. In particular, to explain the SM gauge couplings at the string scale, we can consider the heavy bidoublet Higgs field H' as the heavy threshold corrections to the RGE running of the SM gauge couplings. The detailed discussions will be given elsewhere.

B. Type II $SO(10)$ Model

Particles	Curve	Class	g_Σ	L_Σ	$L_\Sigma'^n$
$(H,\ H')$	Σ_H (pinched)	$3H - E_1 - E_2$	1	$\mathcal{O}_{\Sigma_H}(p_1 - p_2)^{1/2}$	$\mathcal{O}_{\Sigma_H}$
$(\Phi,\ \overline{\Phi})$	Σ_Φ (pinched)	$3H - E_1 - E_2 - E_8$	1	$\mathcal{O}_{\Sigma_\Phi}(p_1' - p_2')^{1/2}$	$\mathcal{O}_{\Sigma_\Phi}(p_1' - p_2')^{3/2}$
$\left(4L_i^R,\ 3Q_j,\ 2Q_k^R, L_5\right)$	Σ_{Q1}	$2H - E_1 - E_3$	0	$\mathcal{O}_{\Sigma_{Q1}}(1)^{1/2}$	$\mathcal{O}_{\Sigma_{Q1}}(-1)^{5/2}$
$\left(4L_i,\ 3Q_j^R,\ 2Q_k, L_5^R\right)$	Σ_{QR1}	$2H - E_2 - E_3$	0	$\mathcal{O}_{\Sigma_{QR1}}(-1)^{1/2}$	$\mathcal{O}_{\Sigma_{QR1}}(-1)^{5/2}$
$\left(2\overline{L}_l,\ \overline{Q}_1^R,\ L_6^R\right)$	Σ_{LR6}	$2H - E_1 - E_4$	0	$\mathcal{O}_{\Sigma_{LR6}}(1)^{1/2}$	$\mathcal{O}_{\Sigma_{LR6}}(1)^{1/2}$
$\left(2\overline{L}_l^R,\ \overline{Q}_1,\ L_6\right)$	Σ_{L6}	$2H - E_2 - E_4$	0	$\mathcal{O}_{\Sigma_{L6}}(-1)^{1/2}$	$\mathcal{O}_{\Sigma_{L6}}(1)^{1/2}$
$\left(2\overline{L}_m,\ \overline{Q}_2^R,\ L_7^R\right)$	Σ_{LR7}	$2H - E_1 - E_5$	0	$\mathcal{O}_{\Sigma_{LR7}}(1)^{1/2}$	$\mathcal{O}_{\Sigma_{LR7}}(1)^{1/2}$
$\left(2\overline{L}_m^R,\ \overline{Q}_2,\ L_7\right)$	Σ_{L7}	$2H - E_2 - E_5$	0	$\mathcal{O}_{\Sigma_{L7}}(-1)^{1/2}$	$\mathcal{O}_{\Sigma_{L7}}(1)^{1/2}$

TABLE III: The particle curves and the gauge bundle assignments for each curve in the Type II $SO(10)$ model. Here, $i = 1,\ 2,\ 3,\ 4$, $j = 1,\ 2,\ 3$, $k = 4,\ 5$, $l = 1,\ 2$, and $m = 3,\ 4$.

In Type II $SO(10)$ model, we present the particle curves with homology classes and the gauge bundle assignments for each curve in Table III. Note that the Higgs triplets in **10** of $SO(10)$ do not have zero modes, we solve the doublet-triplet splitting problem. Also, we

have seven L_i fields, seven L_i^R fields, five Q_i fields, five Q_i^R fields, four $\overline{L}_i$ fields, four $\overline{L}_i^R$ fields, two $\overline{Q}_i$ fields, and two $\overline{Q}_i^R$ fields. Thus, for the net numbers of the chiral superfields, we have three families of the SM fermions, *i.e.*, three L_i fields, three L_i^R fields, three Q_i fields, and three Q_i^R fields. In addition, we assume that the SM fermion and Higgs curves Σ_H, Σ_{Q1}, and Σ_{QR1} intersect at one point in S where the gauge symmetry is enhanced to E_7, and the SM fermion and Higgs curves Σ_{Q1} and Σ_Φ intersect at the other point in S where the gauge symmetry is enhanced to E_7 as well. On the second E_7 point, we introduce the SM singlet fields X_k' and $\overline{X}_k'$ respectively with $SO(10) \times U(1)_a \times U(1)_b$ quantum numbers $(\mathbf{1}, \mathbf{0}, \mathbf{2})$ and $(\mathbf{1}, \mathbf{0}, -\mathbf{2})$ from the intersection of the other seven-branes. In order to preserve the $U(1)_a \times U(1)_b$ symmetry for the SM fermion Yukawa couplings, we choose that in the small open neighborhoods of these E_7 points, the Higgs curve Σ_H satisfies $t_a = 0$, the Higgs curve Σ_Φ satisfies $t_a - t_b = 0$, the SM fermion curves Σ_{Q1}, Σ_{LR6} and Σ_{LR7} satisfy $t_a + t_b = 0$, and the SM fermion curves Σ_{QR1}, Σ_{L6}, and Σ_{L7} satisfy $t_a - t_b = 0$. For simplicity, we assume that one linear combination of the four L_i fields, the L_5, L_6, L_7 fields, and four $\overline{L}_i$ fields form vector-like particles, and obtain the vector-like masses at the GUT scale via the instanton effects or the Higgs mechanism due to their couplings to the SM singlet Higgs fields with $SO(10) \times U(1)_a \times U(1)_b$ quantum number $(\mathbf{1}, \mathbf{0}, -\mathbf{2})$ at the intersections of the other seven-branes. We assume that one linear combination of the four L_i^R fields, the L_5^R, L_6^R, L_7^R fields, and four $\overline{L}_i^R$ fields form the vector-like particles, and obtain the vector-like masses at the GUT scale via the instanton effects or the Higgs mechanism due to their couplings to the SM singlet Higgs fields with $SO(10) \times U(1)_a \times U(1)_b$ quantum number $(\mathbf{1}, \mathbf{0}, \mathbf{2})$ at the intersections of the other seven-branes. We assume that two Q_k fields and two $\overline{Q}_k$ fields form vector-like particles, and obtain the vector-like masses at the GUT scale via the instanton effects or Higgs mechanism. And we assume that two Q_k^R fields and two $\overline{Q}_k^R$ fields form vector-like particles, and obtain the vector-like masses at the GUT scale via the instanton effects or Higgs mechanism. Here we assume that $i = 1, 2, 3, 4$, and $k = 4, 5$. Thus, we have three families of the SM left-handed leptons and neutrinos from three linear combinations of four L_i fields and three families of the SM right-handed quarks on the curve Σ_{QR1}, and have three families of the SM right-handed leptons and neutrinos from three linear combinations of four L_i^R fields and three families of the SM left-handed quarks on the curve Σ_{Q1}.

Moreover, the Higgs curve Σ_H is pinched at a point, where the gauge symmetry is enhanced to $SO(14)$. We assume that the Higgs curve Σ_H satisfies $t_1 = 0$ in the small open neighborhood of the $SO(14)$ point. Similar to the above subsection, we introduce the SM singlet Higgs fields ϕ_i^{++}, ϕ_i^{+-}, ϕ_i^{-+}, and ϕ_i^{--} respectively with $SO(10) \times U(1)_1 \times U(1)_2$ quantum numbers $(\mathbf{1}, \mathbf{2}, \mathbf{2})$, $(\mathbf{1}, \mathbf{2}, -\mathbf{2})$, $(\mathbf{1}, -\mathbf{2}, \mathbf{2})$, and $(\mathbf{1}, -\mathbf{2}, -\mathbf{2})$ on the $SO(14)$ point from the

intersections of the other seven-branes. Therefore, the superpotential in Type II $SO(10)$ model is

$$
\begin{aligned}
W_{SSM} &= y_{ij}^{UD} Q_i H Q_j^R + y_{ij}^{E\nu} L_i H L_j^R + \lambda^{ij} X_i' L_j^R \overline{\Phi} + \mu' H H' + \mu_\Phi \Phi \overline{\Phi} \\
&\quad + \lambda_1'^{ij} \frac{1}{M_{\mathrm{Pl}}} H H \phi_i^{-+} \phi_j^{--} + \lambda_2'^{ij} \frac{1}{M_{\mathrm{Pl}}} H' H' \phi_i^{++} \phi_j^{+-} ,
\end{aligned} \tag{58}
$$

where y_{ij}^{UD}, $y_{ij}^{E\nu}$, λ_3^{ij}, $\lambda_1'^{ij}$, and $\lambda_2'^{ij}$ are Yukawa couplings. μ' and μ_Φ terms can be generated via the instanton effects or Higgs mechanism. Interestingly, we can have the Yukawa coupling unification for the SM quarks, and the Yukawa coupling unification for the SM leptons and neutrinos at the GUT scale. Also, the SM fermion masses and mixings can be generated in the presence of H-fluxes [31]. In addition, after Φ and $\overline{\Phi}$ obtain VEVs, the right-handed neutrinos and some linear combinations of X_k' will obtain the Dirac masses. Thus, with three or more pairs of vector-like particles X_k' and $\overline{X}_k'$, we can explain the neutrino masses and mixings via the double seesaw mechanism [33]. Similar to the above subsection, we can decouple the bidoublet Higgs field H' at the high scale while keep H around the TeV scale. In short, at the low energy we obtain the supersymmetric Standard Model as well. In particular, to explain the SM gauge couplings at the string scale, we can consider the heavy bidoublet Higgs field H' as the heavy threshold corrections to the RGE running of the SM gauge couplings. The detailed discussions will be given elsewhere.

V. CONCLUSIONS

In this paper, we first briefly reviewed the F-theory model building. We constructed one $SU(5)$ model and two $SO(10)$ models from F-theory. The $SU(5)$ gauge symmetry can be broken down to the SM gauge symmetry by turning on the $U(1)_Y$ flux, and the $SO(10)$ gauge symmetry can be broken down to the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry by turning on the $U(1)_{B-L}$ flux. To preserve the $U(1)_a \times U(1)_b$ gauge or global symmetries at the triple intersections of the SM fermion and Higgs curves, we put the left-handed quarks and the right-handed quarks on the different matter curves, and put the left-handed leptons and neutrinos and the right-handed leptons and neutrinos on the different matter curves. Thus, in our models, all the SM fermion Yukawa couplings are indeed invariant under the enhanced $U(1)_a \times U(1)_b$ gauge or global symmetries at the triple intersections of the SM fermion and Higgs curves. And then our models can be understood very well from the local field theory description. Although the SM fermion Yukawa matrices are rank one, the SM fermion masses and mixings can be generated in the presence of H-fluxes. In addition, we can solve the doublet-triplet splitting problem naturally. The extra vector-like particles can obtain the heavy masses via the instanton effects or Higgs mechanism and then decouple

at the high scale. Therefore, we only have the supersymmetric Standard Model at the low energy. Similar to the $SU(5)$ models, we showed that the SM gauge couplings are also splitted in our $SO(10)$ models due to the $U(1)_{B-L}$ flux. Interestingly, we can explain the SM gauge couplings at the string scale by considering the heavy threshold corrections from the extra vector-like particles in our $SU(5)$ and $SO(10)$ models. Furthermore, in the $SU(5)$ model, we can have the Yukawa coupling unification for the bottom quark and tau lepton. In the $SO(10)$ models, we can have the Yukawa coupling unification for the top and bottom quarks, and the Yukawa coupling unification for the tau lepton and tau neutrino.

Acknowledgments

Appendix A: Breifly Review of del Pezzo Surfaces

The del Pezzo surfaces dP_n, where $n = 1,\ 2,\ ...,\ 8$, are defined by blowing up n generic points of $\mathbb{P}^1 \times \mathbb{P}^1$ or $\mathbb{P}^2$. The homological group $H_2(dP_n, Z)$ has the generators

$$H,\ E_1,\ E_2,\ ...,\ E_n\ , \tag{A1}$$

where H is the hyperplane class for P^2, and E_i are the exceptional divisors at the blowing up points and are isomorphic to $\mathbb{P}^1$. The intersecting numbers of the generators are

$$H \cdot H = 1\ ,\quad E_i \cdot E_j = -\delta_{ij}\ ,\quad H \cdot E_i = 0\ . \tag{A2}$$

The canonical bundle on dP_n is given by

$$K_{dP_n} = -c_1(dP_n) = -3H + \sum_{i=1}^{n} E_i. \tag{A3}$$

For $n \geq 3$, we can define the generators as follows

$$\alpha_i = E_i - E_{i+1}\ ,\quad \text{where}\ \ i = 1,\ 2,...,\ n-1\ , \tag{A4}$$

$$\alpha_n = H - E_1 - E_2 - E_3\ . \tag{A5}$$

Thus, all the generators α_i is perpendicular to the canonical class K_{dP_n}. And the intersection products are equal to the negative Cartan matrix of the Lie algebra E_n, and can be considered as simple roots.

The curves Σ_i in dP_n where the particles are localized must be divisors of S. And the genus for curve Σ_i is given by

$$2g_i - 2 \;=\; [\Sigma_i] \cdot ([\Sigma_i] + K_{dP_k}) \,. \tag{A6}$$

For a line bundle L on the surface dP_n with

$$c_1(L) = \sum_{i=1}^{n} a_i E_i, \tag{A7}$$

where $a_i a_j < 0$ for some $i \neq j$, the Kähler form J_{dP_n} can be constructed as follows [3]

$$J_{dP_k} = b_0 H - \sum_{i=1}^{n} b_i E_i, \tag{A8}$$

where $\sum_{i=1}^{k} a_i b_i = 0$ and $b_0 \gg b_i > 0$. By the construction, it is easy to see that the line bundle L solves the BPS equation $J_{dP_k} \wedge c_1(L) = 0$.

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[1] C. Vafa, Nucl. Phys. B **469**, 403 (1996).

[2] R. Donagi and M. Wijnholt, arXiv:0802.2969 [hep-th].

[3] C. Beasley, J. J. Heckman and C. Vafa, JHEP **0901**, 058 (2009).

[4] C. Beasley, J. J. Heckman and C. Vafa, JHEP **0901**, 059 (2009).

[5] R. Donagi and M. Wijnholt, arXiv:0808.2223 [hep-th].

[6] J. J. Heckman, J. Marsano, N. Saulina, S. Schafer-Nameki and C. Vafa, arXiv:0808.1286 [hep-th].

[7] J. Marsano, N. Saulina and S. Schafer-Nameki, arXiv:0808.1571 [hep-th].

[8] J. Marsano, N. Saulina and S. Schafer-Nameki, arXiv:0808.2450 [hep-th].

[9] J. J. Heckman and C. Vafa, arXiv:0809.1098 [hep-th].

[10] A. Font and L. E. Ibanez, JHEP **0902**, 016 (2009).

[11] A. P. Braun, A. Hebecker, C. Ludeling and R. Valandro, Nucl. Phys. B **815**, 256 (2009).

[12] J. J. Heckman and C. Vafa, arXiv:0811.2417 [hep-th].

[13] J. Jiang, T. Li, D. V. Nanopoulos and D. Xie, arXiv:0811.2807 [hep-th].

[14] A. Collinucci, arXiv:0812.0175 [hep-th].

[15] R. Blumenhagen, Phys. Rev. Lett. **102**, 071601 (2009).

[16] J. J. Heckman, A. Tavanfar and C. Vafa, arXiv:0812.3155 [hep-th].

[17] J. L. Bourjaily, arXiv:0901.3785 [hep-th].

[18] H. Hayashi, T. Kawano, R. Tatar and T. Watari, arXiv:0901.4941 [hep-th].

[19] B. Andreas and G. Curio, arXiv:0902.4143 [hep-th].

[20] C. M. Chen and Y. C. Chung, arXiv:0903.3009 [hep-th].

[21] J. J. Heckman, G. L. Kane, J. Shao and C. Vafa, arXiv:0903.3609 [hep-ph].

[22] R. Donagi and M. Wijnholt, arXiv:0904.1218 [hep-th].

[23] V. Bouchard, J. J. Heckman, J. Seo and C. Vafa, arXiv:0904.1419 [hep-ph].

[24] L. Randall and D. Simmons-Duffin, arXiv:0904.1584 [hep-ph].

[25] J. J. Heckman and C. Vafa, arXiv:0904.3101 [hep-th].

[26] J. Marsano, N. Saulina and S. Schafer-Nameki, arXiv:0904.3932 [hep-th].

[27] J. L. Bourjaily, arXiv:0905.0142 [hep-th].

[28] R. Tatar, Y. Tsuchiya and T. Watari, arXiv:0905.2289 [hep-th].

[29] J. Jiang, T. Li, D. V. Nanopoulos and D. Xie, arXiv:0905.3394 [hep-th].

[30] J. Marsano, N. Saulina and S. Schafer-Nameki, JHEP **0908**, 046 (2009).

[31] S. Cecotti, M. C. N. Cheng, J. J. Heckman and C. Vafa, arXiv:0910.0477 [hep-th].

[32] C. S. Huang, J. Jiang and T. Li, Nucl. Phys. B **702**, 109 (2004), and the references therein.

[33] L. Wolfenstein and D. Wyler, Nucl. Phys. B**218** (1983) 205; R. N. Mohapatra and J. W. F. Valle, Phys. Rev. D **34**, 1642 (1986); S. Nandi and U. Sarkar, Phys. Rev. Lett. **56**, 564 (1986); J. H. Kang, P. Langacker and T. Li, Phys. Rev. D **71**, 015012 (2005).