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WORMHOLE EFFECTS ON THE MASSES OF SPIN-0 BOSONS AND SPIN-½ FERMIONS

John Ellis and E. Floratos
CERN - Geneva

and

D.V. Nanopoulos
Center for Theoretical Physics, Physics Dept.
Texas A&M University
College Station, TX 77843, U.S.A.

ABSTRACT

We give a unified derivation of the large-volume corrections to the gravitational action due to spin-0 bosons and spin-½ fermions. We use these results to give a critical discussion of previous analyses of wormhole effects on the pion and neutrino masses. We formulate plausible hypotheses leading to the prediction $m_u/m_d = 44/130$ for the ratio of up and down quark masses.

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For some time it has been speculated that quantum gravitational effects might have significance even for the physics observable at energies far below the Planck mass $m_P \sim 10^{19}$ GeV. In particular, it has been suggested that quantum fluctuations in the topology of space-time might be important. Calculations have been made of their possible modifications to elementary particle masses [1], and it has been conjectured that they might engender information loss leading to the breakdown of quantum coherence [2]. Until recently, the relevance of these speculations has been difficult to assess, because a consistent formulation of quantum gravity has not been available. However, this handicap has now been partly overcome by two technical developments. One is the realization that string theory may furnish a consistent and finite quantum theory of gravity, and the other is the development [3] of a phenomenological "wormhole calculus" suitable for discussing some quantum topological effects. These cannot yet be discussed precisely in string theory, although this may be used to justify some of the assumptions made in "deriving" the wormhole calculus.

Dramatic consequences have been claimed for the wormhole calculus, including an intrinsic uncertainty in the fundamental constants of physics [3], a modification of conventional quantum mechanics [2] and the vanishing of the cosmological constant [4]. It should be stressed immediately that there are several theoretical and practical questions about the latter argument. These include sign questions related to the rotation between Lorentzian and Euclidean metrics [5,6], the suppression of large wormholes [7], the extremization of Newton's constant [8], and the behaviour of particle masses [9-12]. Whilst admitting the pertinence of these questions, we are sufficiently excited by the claimed success with the cosmological constant [4] to re-explore implications of the wormhole calculus for particle masses.

Various authors [9-12] have calculated the logarithmic corrections to the effective action associated with the masses of elementary spin-0 bosons and spin-$\frac{1}{2}$ fermions. On the basis of these calculations, it has been argued [9-12] that scalar masses m_0 such as the pion mass are driven to zero, and fermion masses $m_{\frac{1}{2}}$ such as the neutrino mass to infinity, by wormhole dynamics. These arguments are subject to two types of objection. One is that the arguments have been couched in terms of ultra-violet logarithms $\log(m^2/\Lambda^2_{uv})$, whereas it is more natural to consider infra-red logarithms $\log(m^2r^2)$ in the large-volume limit $r \to \infty$. The second objection is that full attention has not been paid to the requirements of chiral symmetry. On the one hand, this constrains the form of the effective action and forbids certain troublesome terms. On the other hand, it gives relations between pseudoscalar meson and quark masses: $m^2_\pi \sim \Lambda_{QCD} m_q$, in apparent conflict with the expectations that $m_0 \to 0$ and $m_{\frac{1}{2}} \to \infty$.

In this paper, we first address the technical problem of the mass-dependent logarithms. We calculate them consistently in the infra-red limit for both spin-0 bosons and spin-½ fermions, and recover the results previously obtained from a trace anomaly argument [11]. Then we re-examine the argument that wormholes drive $m_\pi \to 0$, and confirm that this occurs logarithmically [9-11], with appropriate chiral symmetry preventing the power-law disappearance suggested in Ref. [12]. We do not see how the pion's composite nature [11] could evade this conclusion, but instead formulate hypotheses that lead to the prediction

$$m_u / m_d = 44/130 \; . \tag{1}$$

Our hypotheses are that (1) $m_\pi^2 \simeq \Lambda_{QCD}(m_u + m_d)$ as conventionally believed, (2) m_e/m_d is fixed, as in many Grand Unified Theories (GUTs) [13], and (3) the first-generation neutrino mass $m_\nu \simeq m_u^2/M$ where M is some large independent mass scale, as also occurs in many GUTs [14].

It has been pointed out that wormholes introduce a new uncertainty into the values of all physical parameters. Coleman, in his scenario for the vanishing of the cosmological constant, proposed the idea that it is possible to predict the most probable values of other constants of Nature, such as particle masses, couplings, mixing angles, etc. This "big fix" could be realized by maximizing the probability distribution, given by the double exponentials of the effective action, with respect to the parameters of the low-energy effective theory [4]. In the dilute gas approximation and neglecting any kind of wormhole self-interactions, the probability of finding the wormhole parameters of species i between the values α_i and $\alpha_i + d\alpha_i$, is given as

$$dP = N \cdot \prod_i d\alpha_i \; Z(\alpha) \, f(\alpha) \tag{2}$$

$$Z(\alpha) = \exp\Big(\sum_{topol} e^{-\Gamma_\alpha} \Big) \tag{3}$$

where, if the λ_i are the physical couplings and masses,

$$\Gamma_\alpha = \Gamma_{eff}(\lambda + \alpha) \tag{4}$$

and $f(\alpha)$ is determined by the boundary conditions used to define the wave function of the Universe [2].

For cosmological scales of size r, the leading dependence of the effective action comes from large and smooth background gravitational metrics

$$\Gamma_\alpha(g) \sim \int d^4x \sqrt{g}\, [\Lambda - (16\pi G)^{-1} R], \tag{5a}$$

while the next-to-leading term of order $O(R^2)$ has the general form

$$\Gamma_\alpha(g) \sim \int d^4x \sqrt{g}\, [\Lambda - (16\pi G)^{-1} R + \beta R_{\mu\nu\lambda\rho} R^{\mu\nu\lambda\rho} + \gamma R_{\mu\nu} R^{\mu\nu} + \delta R^2]. \tag{5b}$$

In this effective action, matter loops have been integrated out from the wormhole scale M_{WH} down to the cosmological scale, and so the cosmological constant Λ, Newton's constant $(16\pi G)^{-1}$, as well as the parameters β, γ and δ depend on the physical masses, couplings and whatever other parameters exist in a low-energy physical theory. On top of this, the crucial point is that they depend also on the wormhole parameters α_i. To make a prediction for the values of the particle masses m_i and the renormalized couplings λ_i, at some low-energy scale M_L, which we take to be of the order of r^{-1}, to be used in the expansion (5b), we must find the dependence of Λ, $(16\pi G)^{-1}$, β, γ, δ on $m_i(M_L)$ and $\lambda_i(M_L)$. This dependence is induced by quantum fluctuations of the matter fields with characteristic scales from r to $1/m_i$. This infra-red regime will give unique logarithmic contributions.

The contribution of matter loops to the effective action (5b) has been calculated by many authors in the past, but for different purposes [15-19]. Recently, Klebanov, Susskind and Banks [9] found unfortunate predictions from the wormhole scenario for particle masses. The pion mass was driven to zero, and Majorana neutrino masses induced by wormholes went to infinity. Wise and Grinstein also found that the masses of elementary scalar fields are driven to zero [10]. Later they simplified their calculation using the trace anomaly of the energy-momentum tensor [11], and showed how other couplings can in principle be determined. On the other hand, Preskill [8] presented detailed arguments that it is not possible to determine the most probable values of masses and couplings unless we know the details of the fundamental theory at the Planck scale. His arguments are based on two main observations. The first was that, since gravity has a universal coupling to all forms of matter, Newton's constant, $(16\pi G)^{-1}$, will necessarily depend on all couplings and masses through renormalization effects, such as matter loops, with the dominant contributions coming from close to the Planck scale. Secondly, he argued that the maximization condition for $(16\pi G)^{-1}$ must determine the values of all the species of wormhole parameters and hence the values of all constants of Nature, once the explicit dependence of $(16\pi G)^{-1}$ on the constants is known.

In the hope that there are other possibilities open for avoiding mass predictions for bosons and fermions [9,10] that conflict with the reality, we recall that above the wormhole scale the fundamental field theory presumably inherits broken or exact symmetries of a superstring theory. The effect of integrating over wormholes could, in principle, leave some remnants of these symmetries in the theory below the wormhole scale M_{WH}, and hence some degeneracy of the maximum of $(16\pi G)^{-1}$ in the wormhole parameter space. Thus the maximization of Newton's constant in the space of wormhole parameters, $\Lambda(\alpha) = 0$, should only reduce the dimensionality of this space, leaving the way for other physical constants to be determined. This indeterminacy in the wormhole parameter space also leaves open the way for a modification of conventional quantum mechanics, as argued in Ref. [2].

Before we expose our ideas on the mass problem, we would like to describe in some detail a possibly new method of calculating the infra-red dependence of Λ, $(16\pi G)^{-1}$, β, γ and δ for a spherical universe of radius r, on the masses of particles with spin L. Although the results are known but scattered in various papers, for completeness we present them here in some detail. For a spherical universe of radius r, the action in relation (5b) becomes [15-17,10] ($\Omega_5 = 8\pi^2/3$):

$$\Gamma_\alpha(g) = \Omega_5 \left[\Lambda \cdot r^4 - (16\pi G)^{-1} 12 r^2 + c \right] \tag{6}$$

with

$$c = 24\beta + 36\gamma + 144\delta. \tag{7}$$

In the following we shall calculate the logarithmic infra-red dependence of Λ, $(16\pi G)^{-1}$ and c on particle masses and on the natural infra-red cut-off r, which, as can be shown by power-counting arguments, is unique. To this end we apply zeta function regularization to the ultra-violet divergences [15].

Here we shall only discuss massive particles of spin L = 0 and L = ½, as the cases L ≥ 1 require gauge fixing of the non-physical degrees of freedom and the calculation of induced Fadeev-Popov determinants*). The kinetic energy operators whose determinants will be calculated are:

$$Q_0 = -\Box + m^2 \tag{8}$$

and

$$Q_{1/2} = \gamma^\mu \partial_\mu - m \tag{9}$$

The renormalization of the effective action, in relation (6), is then given by

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*) A shortcut has been taken in [17] using index theorems, a unified gauge-fixing procedure for all spins and reducible SU(2) representations of tensors on curved backgrounds.

$$\delta\Gamma_0 = \frac{1}{2} \log \det Q_0 \tag{10}$$

and

$$\delta\Gamma_{1/2} = -\frac{1}{2} \log \det Q^+_{1/2} Q_{1/2} \,. \tag{11}$$

The kinetic operators for spin L have, for a spherical space-time, the following eigenvalues and degeneracies [17,19]:

$$\lambda_n = [\, n^2 + (2L+3)n + c_L \,] r^{-2} + m^2 \tag{12a}$$

$$g_n = \frac{2L+1}{3} (n+1)(n+2L+2)(n+L+\tfrac{3}{2}) \,, \quad n=0,1,2,.. \tag{12b}$$

and

$$c_{L=0} = 0 \quad , \quad c_{L=\frac{1}{2}} = 4 \,. \tag{13}$$

Then, introducing a renormalization scale [15] M, we obtain:

$$\delta\Gamma_0 = -\frac{1}{2} \zeta'_{L=0}(0) - \frac{1}{2} \log\left[\frac{\pi (Mr)^2}{4}\right] \cdot \zeta_{L=0}(0) \tag{14}$$

and

$$\delta\Gamma_{1/2} = \frac{1}{2} \zeta'_{L=\frac{1}{2}}(0) + \frac{1}{2} \log\left[\frac{\pi (Mr)^2}{4}\right] \cdot \zeta_{L=\frac{1}{2}}(0) \,, \tag{15}$$

where the ζ_L-function is defined by

$$\zeta_L(z) = \sum_{n=0}^{\infty} g_n (\lambda_n r^2)^{-z} \tag{16}$$

We extract next the large r behaviour of the ζ_L-function, keeping terms up to those linear in z, since we need only $\zeta_L(0)$ and $\zeta'_L(0)$. We use for this, and here we differ from other authors, the Plana summation formula, from which we find an integral representation of the ζ_L-function

$$\sum_{n=0}^{\infty} f(n) = \frac{1}{2} f(0) + \int_0^{\infty} d\zeta\, f(\zeta) + i \int_0^{\infty} \frac{d\zeta}{e^{2\pi\zeta}-1} \left[f(i\zeta) - f(-i\zeta) \right] . \qquad (17)$$

This relation holds for any function analytic in the right half-plane with slower than exponential growth for large $|\zeta|$.

The ζ_L-function can be written using (12a,b) as

$$\zeta_L(z) = \frac{2L+1}{6} \sum_{n=0}^{\infty} \left[f(n,z-1) + (2L+2-X_L) f(n,z) \right] , \qquad (18)$$

$$f(n,z) = \frac{2n+2L+3}{\left[n^2 + (2L+3)n + X_L \right]^z} \qquad (19)$$

with

$$X_L = m^2 r^2 + c_L \ . \qquad (20)$$

We apply (17) for the expression in relation (18),

$$\sum_{n=0}^{\infty} f(n,z) = \frac{2L+3}{2X_L^z} - \frac{1}{1-z}\,\frac{1}{X_L^{z-1}} - 2\int_0^{\infty} \frac{dt}{e^{2\pi t}-1} \left\{ \frac{t - t_c + \sqrt{-\Delta}}{(t_c - t)^z (t + t_c^*)^z} + \frac{t + t_c - \sqrt{-\Delta}}{(t_c^* - t)^z (t + t_c)^z} \right\} \qquad (21)$$

where

$$t_c = \sqrt{-\Delta} + (L + \tfrac{3}{2}) i \qquad (22a)$$

$$-\Delta = X_L - (L + \tfrac{3}{2})^2 \ . \qquad (22b)$$

The integrand in Eq. (21) in the region

$$|t_c| = \sqrt{X_L} < t < +\infty , \qquad (23)$$

is exponentially small ($\sim e^{2\pi\sqrt{x_L}}$) for large r. Expanding in powers of t/t_c in the region $0 < t < |t_c|$, up to terms linear in z, we find, up to exponentially small corrections, that

$$\delta\Gamma_0 = \Omega_5 \frac{1}{64\pi^2}\left\{ \xi^2 \log\xi - 4\xi\log\xi + 4\xi + \frac{58}{15}\log\xi \right\} - \frac{1}{2}\log\left[\frac{\pi}{4}(Mr)^2\right]\zeta_{L=0}(0) \tag{24}$$

$$\delta\Gamma_{1/2} = \Omega_5 \frac{1}{64\pi^2}\left\{ -2\xi^2\log\xi + 12\xi^2 - 4\xi\log\xi + 12\xi - \frac{11}{15}\log\xi\right\} + \frac{1}{2}\log\left[\frac{\pi}{4}(Mr)^2\right]\zeta_{L=\frac{1}{2}}(0), \tag{25}$$

where

$$\xi = m^2 r^2, \quad \Omega_5 = \frac{8\pi^2}{3}. \tag{26}$$

Since the energy scale for which the effective action is given by Eqs. (24) and (25) is of order r, we choose

$$M \sim r^{-1}. \tag{27}$$

Hence the $\zeta_L(0)$ terms do not contribute when we keep only terms that increase at least as $\sim \log \xi$.

So finally the cosmological constant Λ, Newton's constant $(16\pi G)^{-1}$ and the parameter c are renormalized in the infra-red as

$$\delta\Lambda = \frac{1}{64\pi^2} m^4 \log m^2 r^2 \tag{28a}$$

$$\delta(16\pi G)^{-1} = \frac{1}{192\pi^2} m^2(-1 + \log m^2 r^2) \tag{28b}$$

$$\delta c = \frac{1}{64\pi^2}\,\frac{58}{15}\log\xi, \tag{28c}$$

by massive particles of spin L = 0, and

$$\delta\Lambda = \frac{1}{64\pi^2} m^4 (12 - 2 \log m^2 r^2) \tag{29a}$$

$$\delta(16\pi G)^{-1} = \frac{1}{192\pi^2} m^2 (-3 + \log m^2 r^2) \tag{29b}$$

$$\delta c = \frac{1}{64\pi^2} \left(-\frac{11}{15}\right) \log m^2 r^2, \tag{29c}$$

by massive particles of spin $L = \frac{1}{2}$.

The r-independent m^4 and m^2 terms in (28a-c) and (29a-c) are non-universal, as has been observed by Preskill [8], because they receive contributions from gravitational and other interactions. From (28c) and (29c), we see that scalar particles maximize the probability distribution (2) when $m_0 \to 0$, while spin-$\frac{1}{2}$ particles maximize (2) when $m_{\frac{1}{2}} \to \infty$ [9,10].

We now comment on the phenomenological implications of the results (28), (29). Our first remark is that we do not think that it is correct, as was proposed in Ref. [11], to avoid the conclusion that $m_\pi \to 0$ by dissolving the pion into its constituent quarks. The relevant region for maximizing the probability is the far infra-red, $r \gg m_\pi^{-1}$, where the pion appears elementary. Our second remark concerns the comment in Ref. [12] that a term in the effective action of the form

$$\Gamma \ni \int d^4x \sqrt{g}\; U(\phi) R^2 \tag{30}$$

where ϕ is a spin-0 boson, would yield at the tree level an extremum

$$\Gamma_0 \simeq -24\pi^2 \frac{U^2(\phi)}{V(\phi)}, \tag{31}$$

where $V(\phi)$ is the conventional scalar potential for the ϕ field. In the chiral limit m_q, $m'_\pi = 0$, the function U is independent of the pion field π, like the potential V. Chiral symmetry allows $U,V \propto m_\pi^2 f(\pi^2)$, in which case the tree-level contribution (31) to the effective action vanishes $\propto m_\pi^2$, whereas the one-loop correction (28) only vanishes logarithmically, and hence dominates in the interesting limit of small m_π.

We have no further suggestion how to avoid the vanishing of m_π, but we can offer some suggestions on how to obtain interesting mass relations in the limit of small m_π. We include in the effective action all the stable hadrons and leptons associated with the lightest generation*):

$$\delta C \equiv 960\pi^2 \sum_{u,d,e,\nu_e} \delta c = 58.3 \log m_\pi^2 r^2 - 11.2 \log m_e^2 r^2$$

$$-11.2 \cdot (\log m_p^2 r^2 + \log m_n^2 r^2) - 11 \log m_{\nu_e}^2 r^2 . \tag{32}$$

For the hadrons in (32) we use the standard QCD expectations:

$$m_\pi^2 \simeq a \Lambda_{QCD} (m_u + m_d) , \tag{33a}$$

$$m_{p,n} \simeq b \Lambda_{QCD} + O(m_u, m_d), \tag{33b}$$

where a and b are numerical coefficients that are calculable in principle but unknown in practice**). In the interesting limit $m_{u,d} << \Lambda_{QCD}$, we may write

$$\begin{aligned} \delta C = & \ 174 \log (m_u + m_d) r + 86 \log (\Lambda_{QCD} r) \\ & -44 \log (m_e r) - 22 \log (m_{\nu_e} r) . \end{aligned} \tag{34}$$

We note that Λ_{QCD} would appear to be driven to zero by Eq. (34), but will not pursue this matter. Instead, we formulate some motivated hypotheses about the fundamental fermion masses in (34).

(1) We expect that m_e/m_d is fixed, as in many GUTs [13]. For example, in the minimal SU(5) GUT (which is not in conflict with a recent lattice estimate [20]), before renormalization m_e and m_d are given by a single SU(5)-invariant Higgs coupling and $m_e = m_d$. Since wormhole effects can only generate gauge-invariant operators, they can change the values of m_d and m_e, but not their ratio.

(2) We adopt the standard GUT seesaw formula [14] for the neutrino mass:

$$m_{\nu_e} = m_u^2 / M , \tag{35}$$

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*) Our results are not essentially changed by including heavier generations.

**) Similar results are obtained if baryons are dropped from Eq. (32).

where M is some large mass-scale from beyond the Standard Model.

Under these hypotheses, Eq. (34) becomes

$$\delta C = 174 \log (m_u + m_d) r - 44 \log (m_d r) - 44 \log (m_u r) + \cdots \tag{36}$$

Assuming that m_d is fixed by some other consideration, and treating $x \equiv m_u/m_d$ as variable, δC (36) is extremized when

$$174 / (x+1) = 44 / x \tag{37}$$

i.e.,

$$m_u / m_d = 44/130 \quad . \tag{38}$$

The best phenomenological estimates of m_u/m_d are around 0.56 [21], so the prediction (38) is not obviously successful*). Also, the wormhole calculus approach that we have used is questionable. However, we feel that the prediction (38) cannot yet be excluded. Moreover, we think that it is important to identify plausible hypotheses within the wormhole framework that lead to testable experimental predictions.

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*) See, however, Refs. [22] for some questioning of this conventional estimate of m_u/m_d, which leaves open the possibility that our prediction (38) might actually be correct.

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