THE EFFECTS OF INCORPORATING DYNAMIC DATA ON ESTIMATES OF UNCERTAINTY

A Thesis

by

SHAHEBAZ HISAMUDDIN MULLA

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

December 2003

Major Subject: Petroleum Engineering

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Approved as to style and content by:

Duane A. McVay (Chair of Committee)	W. John Lee (Member)
Robert K. Popp (Member)	Hans Juvkam-Wold (Head of Department)

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ABSTRACT

The Effects of Incorporating Dynamic Data on Estimates of Uncertainty.

(December 2003)

Shahebaz Hisamuddin Mulla, B.S., MIT, Pune, India

Chair of Advisory Committee: Dr. Duane A. McVay

Petroleum exploration and development are capital intensive and smart economic decisions that need to be made to profitably extract oil and gas from the reservoirs. Accurate quantification of uncertainty in production forecasts will help in assessing risk and making good economic decisions.

This study investigates the effect of combining dynamic data with the uncertainty in static data to see the effect on estimates of uncertainty in production forecasting. Fifty permeability realizations were generated for a reservoir in west Texas from available petrophysical data. We quantified the uncertainty in the production forecasts using a likelihood weighting method and an automatic history matching technique combined with linear uncertainty analysis. The results were compared with the uncertainty predicted using only static data. We also investigated approaches for best selecting a smaller number of models from a larger set of realizations to be history matched for quantification of uncertainty.

We found that incorporating dynamic data in a reservoir model will result in lower estimates of uncertainty than considering only static data. However, incorporation of dynamic data does not guarantee that the forecasted ranges will encompass the true value. Reliability of the forecasted ranges depends on the method employed.

When sampling multiple realizations of static data for history matching to quantify uncertainty, a sampling over the entire range of realization likelihoods shows larger confidence intervals and is more likely to encompass the true value for predicted fluid recoveries, as compared to selecting the best models.

DEDICATION

I dedicate this work to my parents and my brother for their love and support.

ACKNOWLEDGEMENTS

I would like to express my sincere gratitude to my advisor Dr. Duane McVay for his guidance and financial assistance during my research, Dr. Robert Popp for serving on my committee and Dr. Lee for providing me with valuable advice during the weekly group meetings.

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CHAPTER I

INTRODUCTION

Overview of the problem

Petroleum exploration and development are capital intensive and smart economic decisions need to be made to profitably extract oil and gas from the reservoirs. Accurate quantification of reservoir uncertainty will help us in assessing the risks and making good economic decisions such that the outcome is profitable.

A number of studies, such as the PUNQ-S3[1-2] studies show that engineers tend to underestimate uncertainty of reservoir forecasts. Uncertainty in reservoir modeling arises primarily due to lack of data and due to measurement error. Often reservoir engineers use statistical techniques for incorporating uncertainty in reservoir models for production forecasting. Quantifications of uncertainty are represented with probability density functions or standard deviations. Uncertainty estimates Reservoir forecasts have been made using just static data,[3-4] while others incorporate dynamic data as well. A study done by Floris[2] lists numerous techniques used for quantifying reservoir uncertainty.

Background

Various geostatistical techniques are used to represent the uncertainty in reservoir properties in the form of equally probable spatial distributions, or realizations, which honor available data. Studies done by Mishra[3] and Ballin[4] used the uncertainty in static data for reservoir performance forecasting. In the Mishra[3] study, the uncertainty in permeability was modeled by making 50 realizations from available data using Gaussian simulation. Their aim was to best select a few realizations from a larger set of realizations to predict future performance and at the same time sample the uncertainty in all the models. They ranked the realizations based on their volumetric sweep efficiency using a streamline simulator. Models with probabilities of 0.9, 0.5 and 0.1 from the

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This thesis follows the style and format of *Society of Petroleum Engineers Reservoir Evaluation & Engineering*.

cumulative distribution function curve of volumetric sweep efficiency were selected and run using a finite-difference simulator to forecast future performance and quantify uncertainty. This study did not consider any dynamic data in its estimates of uncertainty is our motivation for our study in which we see the impact on uncertainty after adding dynamic data to a model.

Some studies incorporated dynamic data by history matching but used single models to quantify uncertainty. Lepine[5] performed a study in which perturbations around a single matched model were used for this purpose. This method assumes a linear relationship between predicted quantities and reservoir production parameters and neglects model error. The study does not sample the complete parameter space and thus may miss out on some of the uncertainty associated with the reservoir.

When history matching multiple realizations to quantify uncertainty, the best models are often used, since history matching multiple models requires a lot of computing time and it may not be possible to match all the models as in the Barker[1] study, they generated 23 porosity realizations for a reservoir. After history matching all the models using the pilot point technique they selected the 18 best-matched models for uncertainty quantification. Using only the best models the authors may not have sampled the complete parameter space for quantifying uncertainty. Thus it is possible that they may have underestimated the uncertainty.

Objectives

The objectives of our study are to

- Determine the impact of incorporating dynamic data on estimates of uncertainty based solely on static data.
- Determine how to best select a few models from a large set of realizations for history matching for the purpose of uncertainty quantification.

Brief Outline

In the remainder of the thesis we describe the two different techniques which were used to quantify uncertainty using static and dynamic data and a technique for best selecting few models for history matching and quantifying uncertainty. We then describe the application of the methodologies to a reservoir in west Texas. We provide and discuss the results and finally present our conclusions.

CHAPTER II

METHODOLOGY

Overview

Two approaches, an approach using the misfit in the history match and an automatic history matching technique that combined the uncertainty in static data with the dynamic data, were applied to see the effect of incorporating dynamic data on estimates of uncertainty.

We also investigated sampling techniques for best selecting a few models from multiple realizations of static data to history match for quantification of uncertainty.

Likelihood Weighting Method

This method uses the misfit in the history match to assign a weighting factor to the forecast of each model. The weighting factor is further used in the calculation of weighted means and standard deviations of fluid recoveries to quantify uncertainty. The resulting distributions are displayed as probability density functions.

The objective function (Eq.1) calculates the sum of the differences between the observed and simulated values of production variables like oil production, GOR, watercut and others. Each difference is normalized by the observed value for the variable. To avoid domination by frequently measured variables the normalized difference is divided by the number of observations. A weighting factor may be applied to the normalized difference on the basis of reservoir engineering judgement; e.g., a higher weighting factor may be assigned to water cut to give greater emphasis to matching water breakthrough in wells.

$$Q = \frac{1}{n_w}\sum_i \frac{1}{n_p}\sum_j \frac{1}{n_t}\sum_k \left(w_{ijk}\,\frac{o_{ijk}^{obs} - o_{ijk}^{sim}}{o_{ijk}^{obs}}\right)^2 \quad \text{(1)}$$

$$L = c\exp(-0.5 * Q) \quad \text{(2)}$$

The misfit is then used to calculate the likelihood (Eq. 2) which indicates the likelihood that the particular model is correct. The likelihood increases as the misfit decreases. The likelihood are further normalized and applied as weighting factors to the model forecasts. Weighted means and standard deviations of future fluid recoveries are calculated using Eqs. 3 and 4. Probability distribution functions and cumulative distribution functions plots were generated for weighted and unweighted distributions and used to determine the impact of adding dynamic data on uncertainty.

$$\sigma_{wtd} = \sqrt{\frac{\sum w(x - x_{avg})^2}{(n-1)\sum w}} \quad \text{...} \quad (3)$$

$$x_{avg} = \frac{\sum (w \times x)}{\sum w} \quad \text{...} \quad (4)$$

Automatic History Matching

In this method we used an automatic gradient-based history matching technique and linear uncertainty analysis to estimate the uncertainty in the production variables. An automatic history matching module called Simopt[6] was used for this purpose. The following sections present the theoretical basis for the technology. [5,6]

Objective Function

Matching of the production data by the reservoir model is accomplished by minimizing the objective function given below.

$$f = \frac{1}{2}\sum_{d,i} r_{di}^2 \quad \text{...} \quad (5)$$

$$r_{di} = w_d w_{di} \frac{(o_{di} - c_{di})}{\sigma_d} \quad \text{.. (6)}$$

$$RMS = \sqrt{\frac{2f}{m}} \quad \text{.. (7)}$$

The objective function is similar to the one used in the likelihood method. The difference is that here the misfit between the observed and simulated values is normalized with the measurement error. Weighting factors w_d and w_{di} are used to give weight to the data based on reservoir engineering considerations. The overall measure of a history match is expressed using the Root Mean Square (RMS) index (Eq. 7), where f is the objective function and m is the total number of observations over which the index is formed.

Parameterization

The objective of history matching is to reproduce the observed data by conditioning the reservoir model. Since reservoir horizontal permeability was the only uncertain variable in our case we adjusted the spatial distribution of permeability to minimize the objective function. In history matching parameterization can be done in a number of ways. One is to use individual grid blocks, but this approach ends up with a large number of parameters. Another way is to use homogeneous regions, which reduces the number of parameters. Although there are some limitations with this technique, it is the most widely used approach for parameterization. Other methods like the pilot point technique are also being used with history matching. Each parameter is assigned a modifier which varies normally or log-normally during regression to obtain a history match.

Optimization Using Gradients

A number of papers[5,7-10] spell out the details of gradient-based history matching. The objective function is minimized using an algorithm that changes the uncertain parameters and measures how the match between observed and the simulated data changes. Simopt[6] does this with the help of the Levenberg-Marquardt algorithm, which is a combination of the Newton method and the steepest decent approach. If the current vector of the normalized parameter modifier values is denoted by v^k, then the algorithm estimates the step $dv^k(\mu)$ that minimizes the objective function as:

$$dv^k(\mu) = (H + \mu I)^{-1} \nabla f(v^k) \,. \qquad (8)$$

The parameter μ is free and varies such that away from the solution it takes large values so that the bias of the step is towards the steepest descent direction, while near the solution it takes small values such that the fast convergence rate of the Newton step is used. In solving the above equation, first and second derivatives (gradients) of the objective function with respect to the normalized parameter modifiers are required. Once the gradients are calculated a correlation matrix between parameter regions is used to remove anti-correlated regions, which cause problems during regression. The model can then be regressed to obtain a history match.

Linear Uncertainty Analysis

This technique was used in a study done by Lepine.[5] This method proposes that once a model is history matched, further prediction in the uncertainty of a reservoir can be done by using sensitivity analysis. This method assumes that, once the parameter values are obtained that give us an acceptable match, it is possible to perturb these values slightly and still have a match that would be considered acceptable. If the perturbations are sufficiently small, a linear perturbation analysis can be used to derive confidence intervals for future production performance.

Consider a history matched model in which we matched production data using n parameters, v, and want to predict future values of m production values, c. We assume the relationship between them is linear and then the confidence interval on the calculated values is related to confidence interval on the parameters by

$$\Delta c = A\Delta v \,. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

$$[A]_{ij} = \frac{dc_i}{dv_j} \,. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

Assuming Δv has a normal distribution with mean zero, then Δc also has a normal distribution with covariance matrix

$$C_c = AC_v A^T \,. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

Where C_v is the covariance matrix of the parameters, which is obtained by inverting the Hessian matrix for the history period, hence

$$C_c = AH_{hist}^{-1} A^T \,. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

The standard deviations of the predicted values are approximated by the square roots of the elements of this covariance matrix, that is

$$\sigma_c = \sqrt{diag(C_c)} \,. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (13)$$

The distribution of the confidence interval for the production values has been assumed to have a normal distribution. Thus the standard deviations for the production values can be converted into confidence intervals using the confidence coefficient, *conf*, for the required level (%). This confidence coefficient is determined by solving the equation given below. Typical values of confidence coefficient for different confidence levels are shown in Table 1.

$$\frac{level}{100} = erf\left(\frac{conf}{\sqrt{2}}\right). \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(14)}$$

Table 1: Typical confidence limits

(Source: Simopt manual)

Confidence level (%)	Confidence coefficient
68.3	1.000
90.0	1.645
95.4	2.000
99.73	3.000

Hence the confidence interval for the production value is given by

$$\Delta c = conf\sqrt{diag\left(AH_{hist}^{-1}A^{T}\right)}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(15)}$$

Assumptions and Limitations

The assumptions and limitations in using the linear uncertainty analysis technique are listed below.

- Does not sample the posterior completely.
- This uncertainty estimation assumes a linear relationship between predicted and reservoir model parameters.
- It neglects model error.
- Reservoir model parameters are normally or log-normally distributed.
- Measurement errors are independent and normally distributed with mean zero.

Due the above limitations in this technique some of the results obtained can have significant errors.

Selection of History Matching Models

One of the best ways to quantify uncertainty would be to generate many realizations of static data, history match all of them, then forecast production with all of the models to quantify the uncertainty. However, history matching of multiple reservoir models is time consuming. Selecting a few models from a large set of models can save time and money. The problem is how to select a subset of models for uncertainty quantification. In the literature we have seen that selecting a few best models is sometimes used for quantification of uncertainty. The best models may have the best combinations of distributions of the various unknown parameters for the reservoir. However using only those we may miss some uncertainty associated with the reservoir. Thus we believe that the method of using the best models may not be able to capture the uncertainty in all the models and other means of sampling must be used to capture the entire range of uncertainty present in all the models. In our approach we sampled from a cumulative distribution function of the normalized likelihood values (weighting factors) estimated from the misfit in the history match (Fig.1). We compared uncertainty estimates derived from selection of the three best models to uncertainty estimates derived from a selection of models throughout the likelihood distribution (P90, P50 and P10).

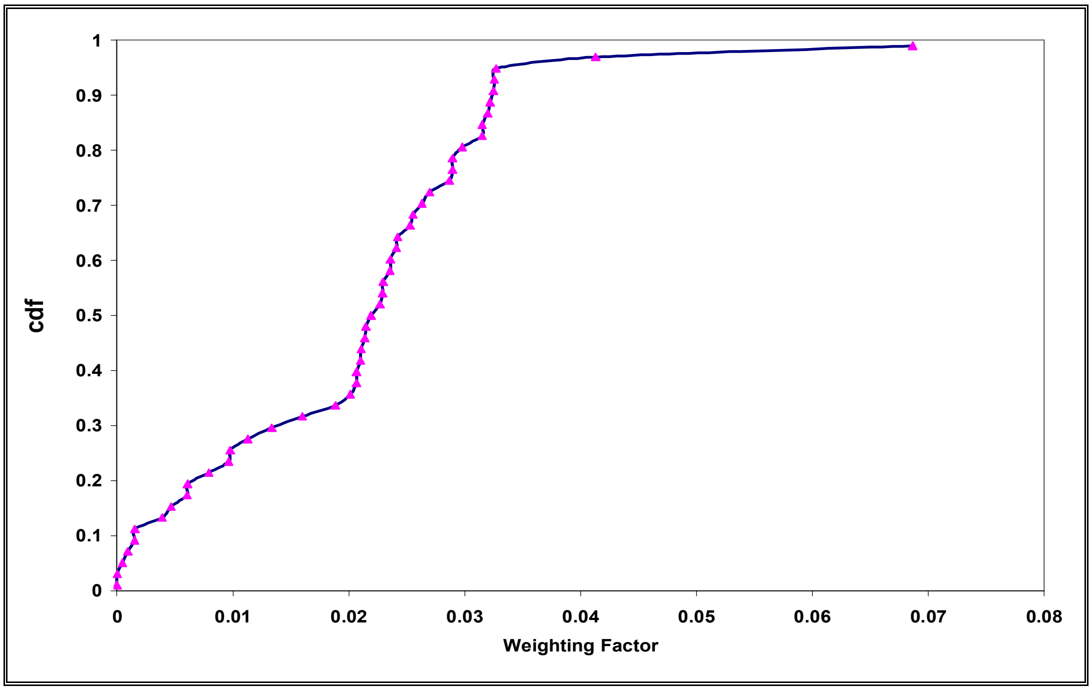

Fig. 1: Cumulative distribution function of weighting factors used to select the models for history matching

CHAPTER III

RESULTS AND DISCUSSION

Reservoir Application

The above methodology was applied to a synthetic model based on a heterogeneous carbonate reservoir, the North Robertson unit located in west Texas. The reservoir has as a low matrix porosity of about 10%. It covers an area of 5,633 acres and has 144 active producing wells, 109 injection wells and 6 water-supply wells. Sections 326-327 (Fig.2) were used for this study.

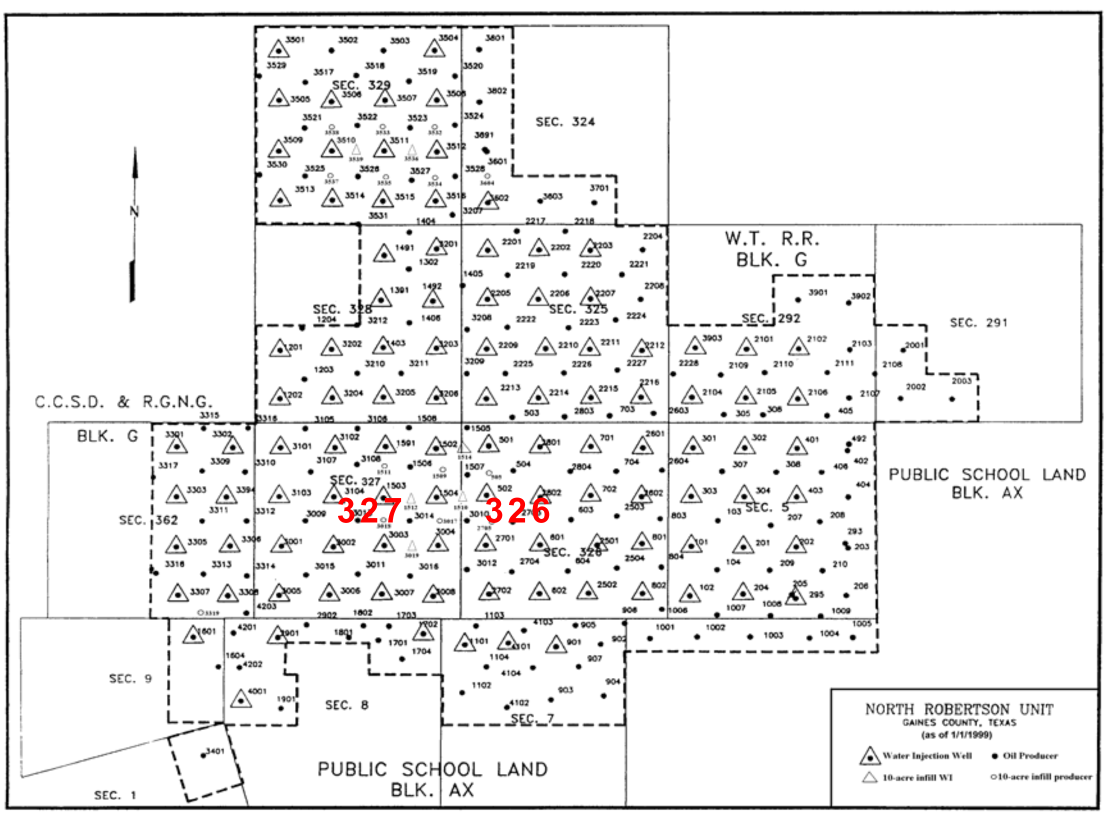

Fig 2: North Robertson Unit map

A simulation grid of 50 x 25 x 12 was used. Each grid block has a length and width of 262.5 feet. A thickness map was created using log data from a study done by Idrobo.[11] The thickness (Fig. 3) of the reservoir model used in this study is approximately 1300 feet. A net-to-gross ratio of 1 was used in the simulation.

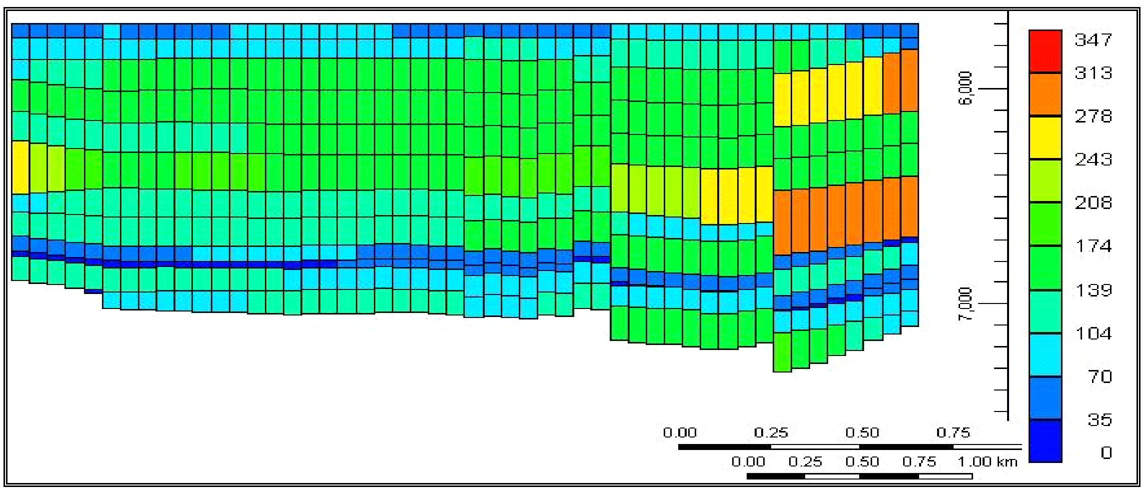

Fig. 3: Cross section of NRU reservoir and its thickness

Permeability-thickness data for 27 producing wells was taken from an earlier study by Idrobo[11] and used to generate fifty equiprobable permeability realizations using Gaussian simulation. Fig. 4 shows the distribution of permeability in the top layer of one of the realization models.

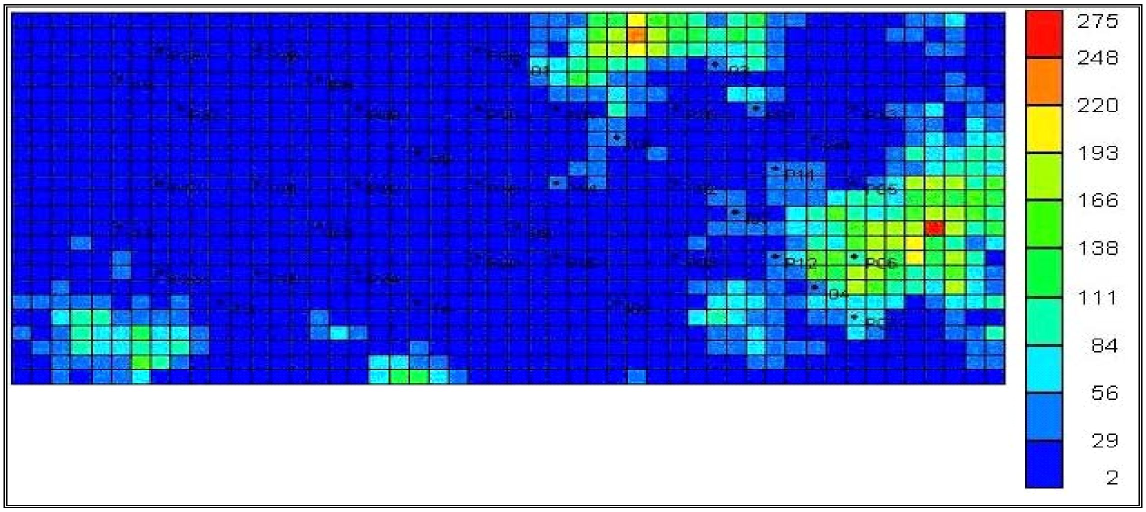

Fig. 4: Areal view of a model and its permeability distribution in the plan view

The reservoir was modeled using PVT and relative permeability data (Fig. 5) from the SPE 9[12] test case study. There are 27 producing wells and 15 injection wells, each completed in all twelve layers. The producers operate at a constant oil rate until the following bottomhole pressure drops to 500 psi, at which time the operating constraint is switched from constant rate to a constant flowing bottomhole pressure of 500 psi. The injection wells operate at a constant rate of 4000 stb/day with a maximum pressure of 8000 psi. The reservoir is at the bubble point pressure of 3600 psi at the start of the simulation. There is no gas cap and no OWC in the reservoir. Major drive mechanisms are solution-gas drive and water flooding.

Seven years of observed data were synthetically generated using the fiftieth permeability realization. We used cumulative GOR, cumulative oil production, flowing bottomhole pressures and water cut for all 27 producing wells in our objective function to calculate the misfits and the weighting factors.

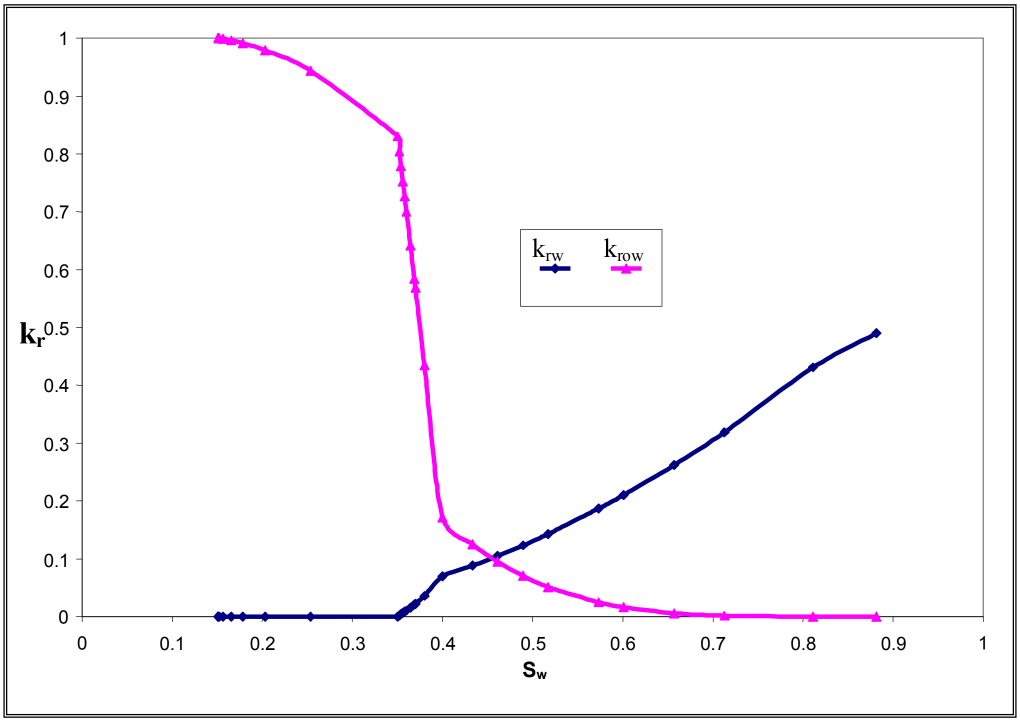

Fig. 5: Oil-water relative permeability data used in the model
(Source: SPE 9 Test case)

Quantification of Uncertainty (Likelihood Weighting method)

Our first objective was to determine the effect of incorporating dynamic data on estimates of uncertainty. We used the four performance variables to calculate misfits and likelihoods using Eqs. 1 and 2. Each forecast was weighted based on how well the model matched the production history. Models which poorer matches of the observed data had lower weighting factors (Fig. 6). The means and standard deviations for predicted oil and recoveries after 30 years of production were estimated using these weighting factors.

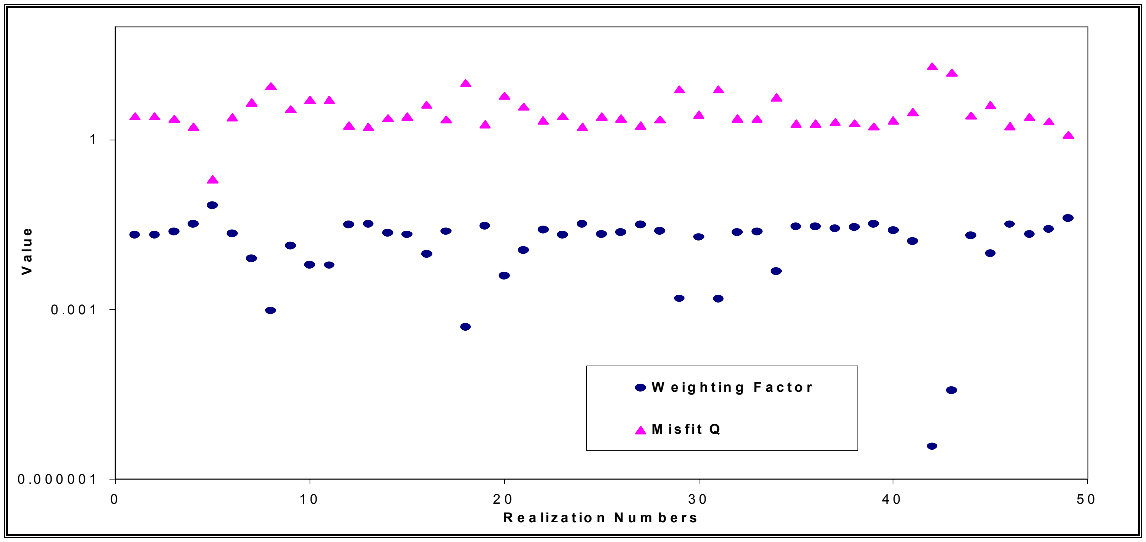

Fig. 6: Data misfit and the weighting factors for the models show that higher the misfit, lower the weighting factor of the model

We first calculated means and deviations without weighting by likelihood. A mean of 204 million bbls with standard deviation of 3.6 million bbls is predicted for future oil recovery and a mean of 1,161 Bcf with a standard deviation of 98 million scf is predicted for gas recovery after 30 years of production.

For the weighted models, the means and deviations were calculated using Eqs. 3 and 4. The weighted models predict a mean future oil recovery of 205 million STB with a standard deviation of 0.42 million STB. The mean of future gas recovery is 1,148 Bcf with a standard deviation of 13 Bcf after 30 years of production. The uncertainties are displayed and compared with pdf plots (Figs. 7 and 8), P90-P10 ranges (Figs. 9 and 10) and cdf plots (Figs. 11 and 12) using the means and standard deviations obtained. We see that the mean values were close between the two methods, but the unweighted models result in much larger ranges of uncertainty.

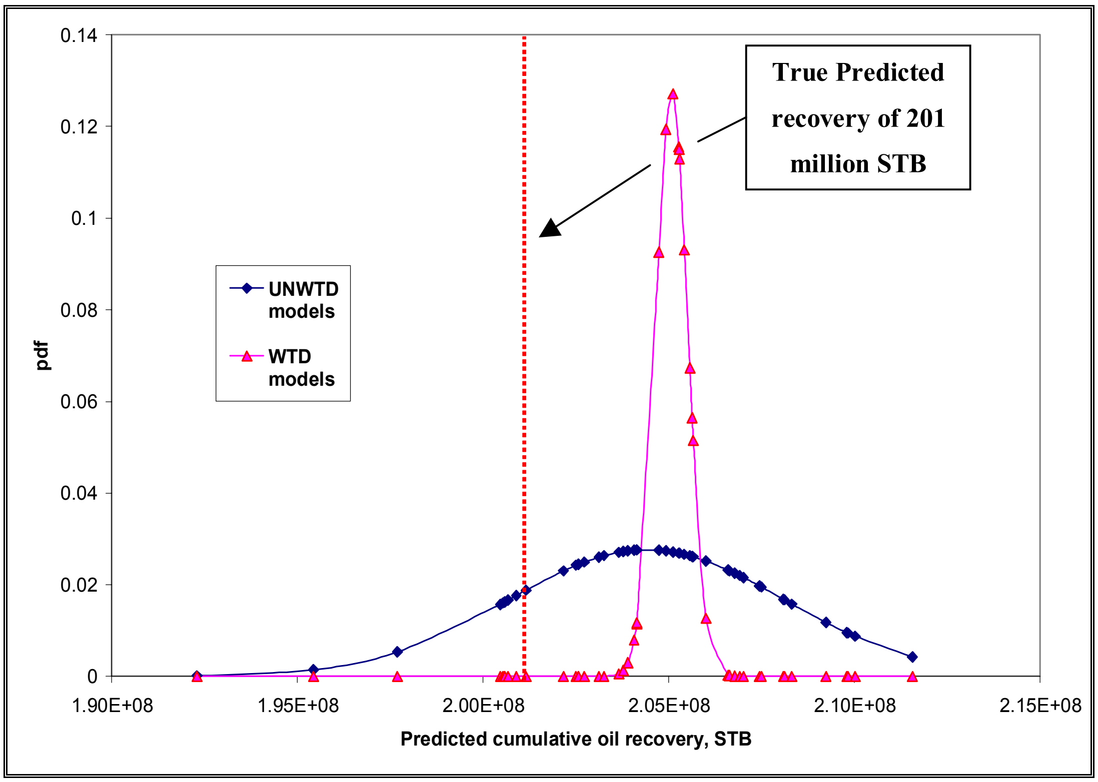

Fig. 7: PDF of cumulative oil recovery using weighted and unweighted models. Values show that the weighted models have a lower standard deviation resulting in a lower uncertainty range

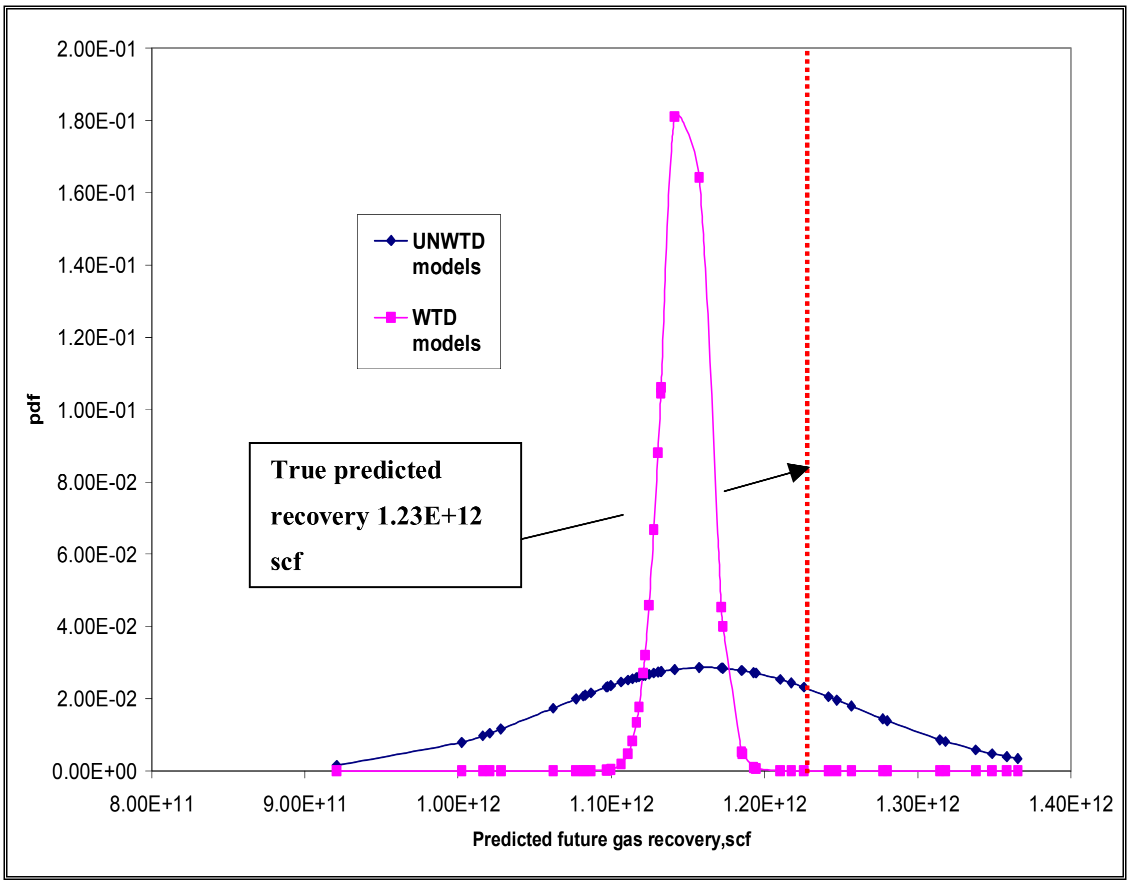

Fig. 8: PDF of cumulative gas recovery using weighted and unweighted values confirms that the weighted models with dynamic data have a lower standard deviation resulting in lower uncertainty

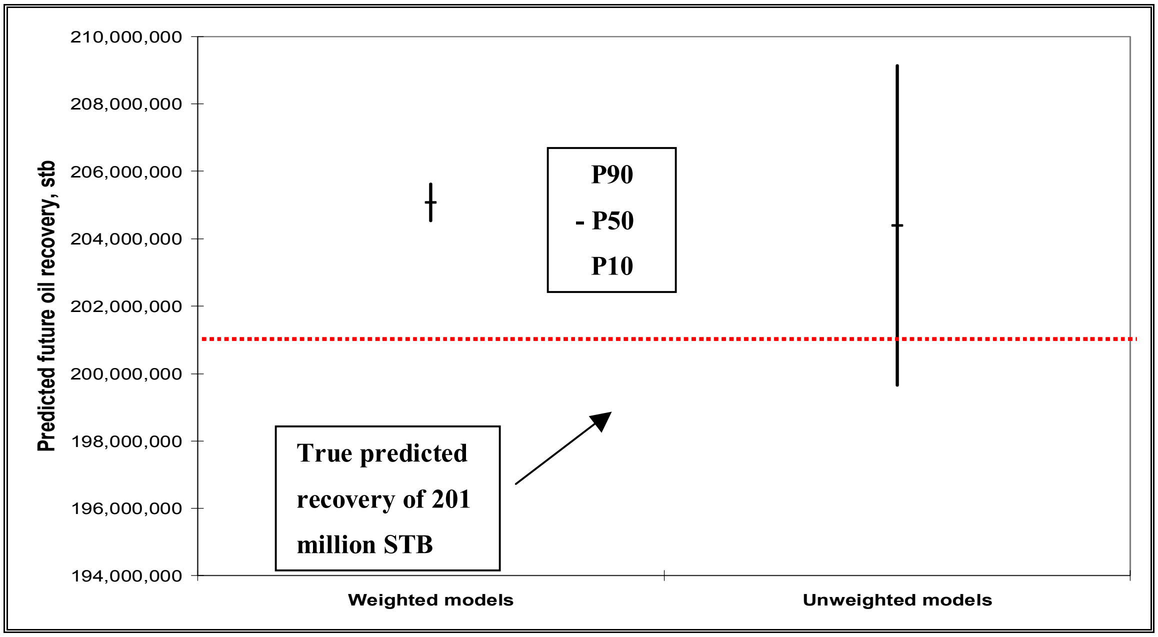

Fig. 9: 80 % confidence intervals of cumulative oil recovery using weighted and unweighted values

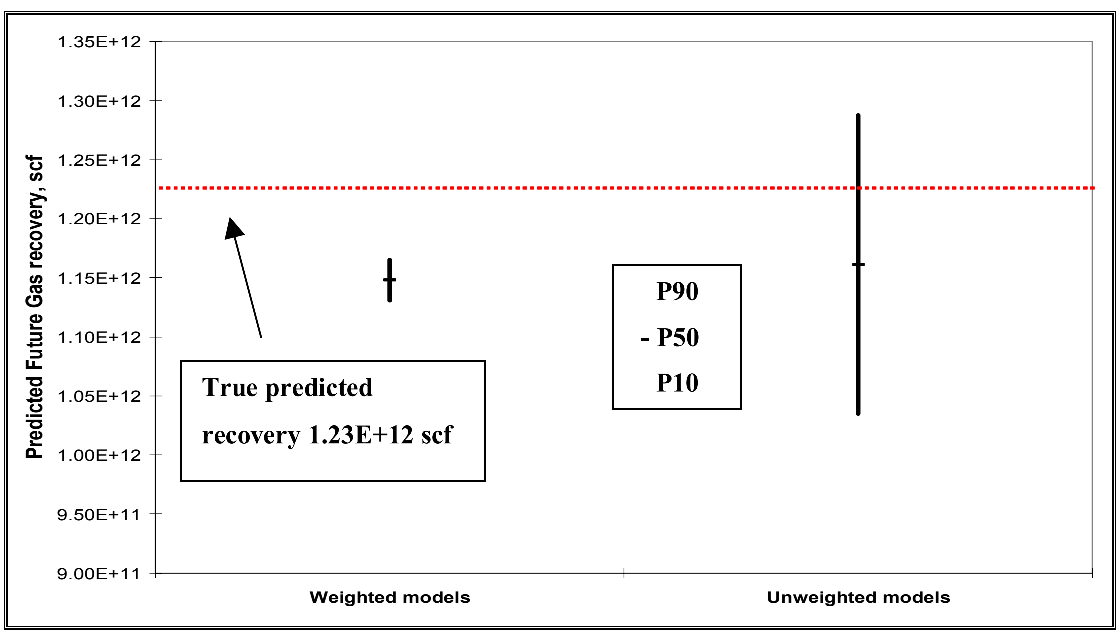

Fig. 10: 80% confidence interval of predicted cumulative gas recovery using weighted and unweighted values

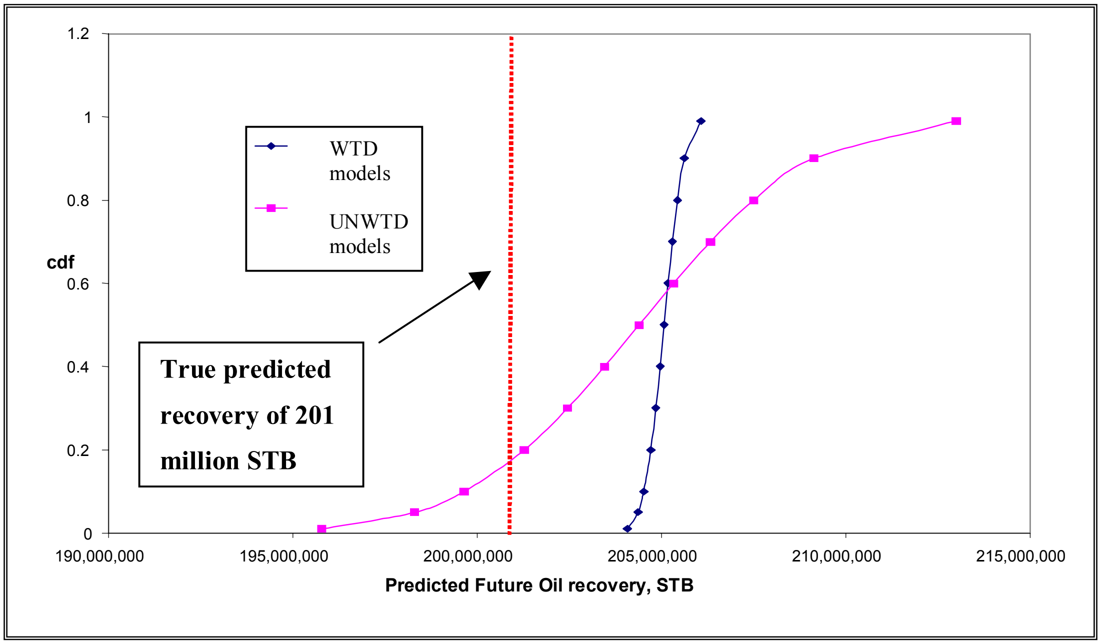

Fig. 11: CDF of cumulative oil recovery using weighted and unweighted values

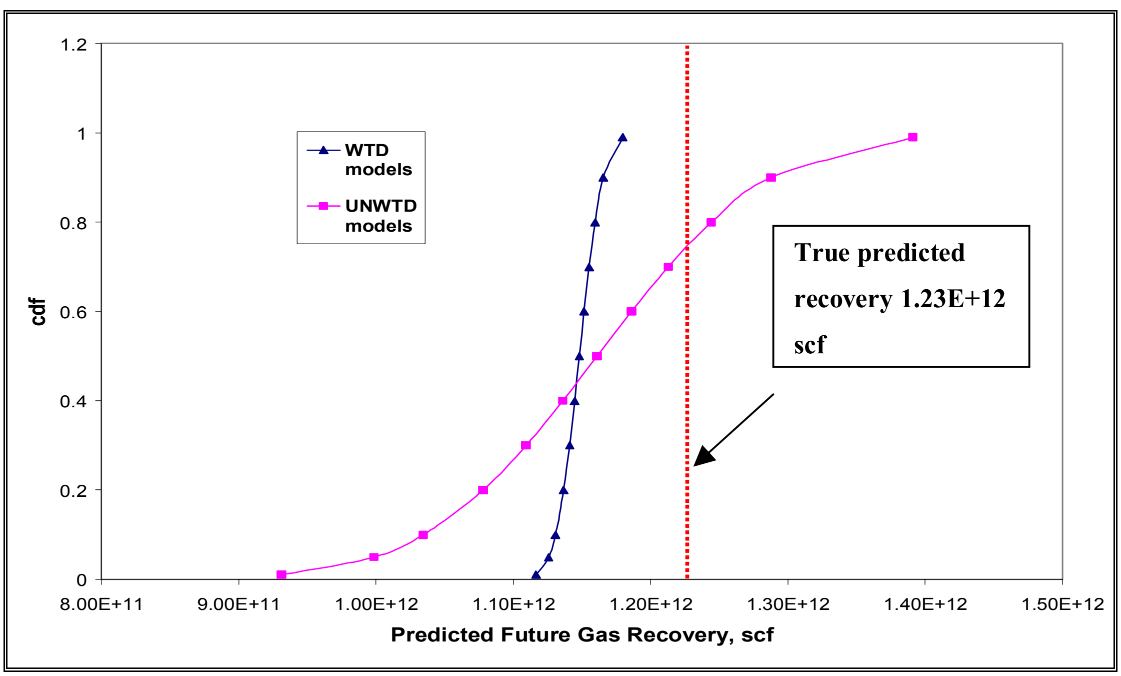

Fig. 12: CDF of cumulative gas recovery using weighted and unweighted values

Table 2 lists the predicted 80 % ranges of oil and gas after 30 years of reservoir life. The unweighted models predicted significantly higher ranges. Two significant results can be seen from this study. The first is that using only static data for quantification of uncertainty results in larger estimates of uncertainty in oil and gas recoveries. When dynamic data are incorporated uncertainty estimates are reduced. The second result is that incorporation of dynamic data does not guarantee that the uncertainty estimate will encompass the true value. The true values for recovery are 201 million STB of oil and 1,230 bcf of gas. The weighted models did not encompass the true value which may be because without history matching none of the models were sufficiently close to the true model. This is illustrated in the relatively uniform distribution of misfits and likelihoods (Fig. 6). We found that all the uncertainty estimates in this study are relatively narrow for fluid recoveries, around 5 % of the future recovery. This is likely attributed to the way the realizations were generated. The cumulative fluid productions for all the models fell in a narrow range of recovery, resulting in the low ranges of uncertainty.

Table 2: Uncertainty in predicted oil recovery of models using only static data as compared to models in which dynamic data is incorporated (Likelihood Method)

	80 % Confidence Intervals		
	Models in which dynamic data is incorporated	Models using only static data	True value
Predicted future recovery of oil (10^6stb)	204-205	199-209	201
Predicted future recovery of gas, (10^{12} scf)	1.13-1.16	1.034-1.28	1.23

Best Selection of Models for History Matching

The second objective was to determine how to best select a few models for history matching to quantify uncertainty. Six models were selected from a cdf plot of likelihood values (Fig.1). The top three models, corresponding to probability values of P98, P96 and P94 were compared to sampled models with probabilities of P90, P50 and P10. After history matching, we compared the uncertainty in the best models and the P90-P10 models using linear uncertainty analysis to see which group performed better.

Parameterization was done using Voronoi regions around the wells. The Voronoi region for a well is the set of blocks closer to the well than to any other wells. A total of 167 regions (Fig.13) were defined with each region extending over 3 layers of the reservoir. Since the reservoir is isotropic, combined X-Y permeability modifiers were regressed on to obtain a history match.

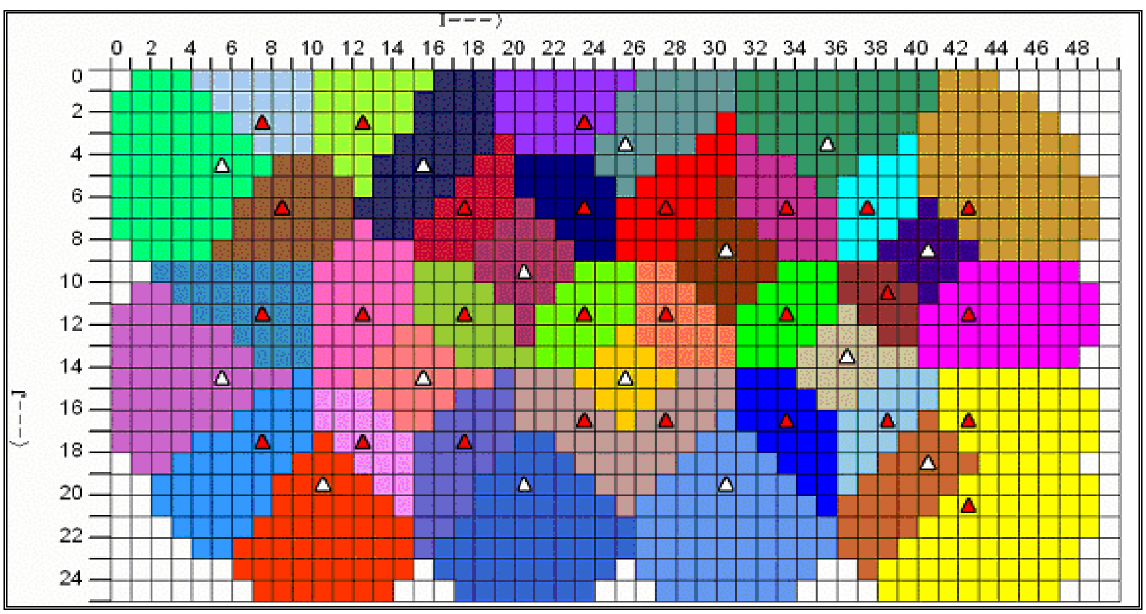

Fig. 13: Voronoi regions used for regression to obtain a history match

In this study we history matched cumulative oil production, cumulative GOR, flowing bottomhole pressures and watercut for a period of 7 years for all 27 producing wells. Measurement errors of 0.1 in watercut, 1 Mscf/stb in GOR, 100 psi in pressure and 200 STB for oil production were assumed. A weighting factor of unity was applied to all data and no *'a priori'* information was used in the objective function. Starting with the original permeability distributions of the models, which were generated using Gaussian simulation, we first estimated the misfit (RMS) for the model. After calculating gradients, we removed negatively correlated parameter regions and combined positively related regions, after which we regressed over permeability modifiers to obtain a history match. We successfully history matched all six models. Plots of the RMS values as the regression progresses are shown in Figs.14 and 15. In the first 10-12 iterations there was a significant drop in the RMS value, after which the changes were very small. A single iteration took around 45 minutes making the history matching a long and tedious process.

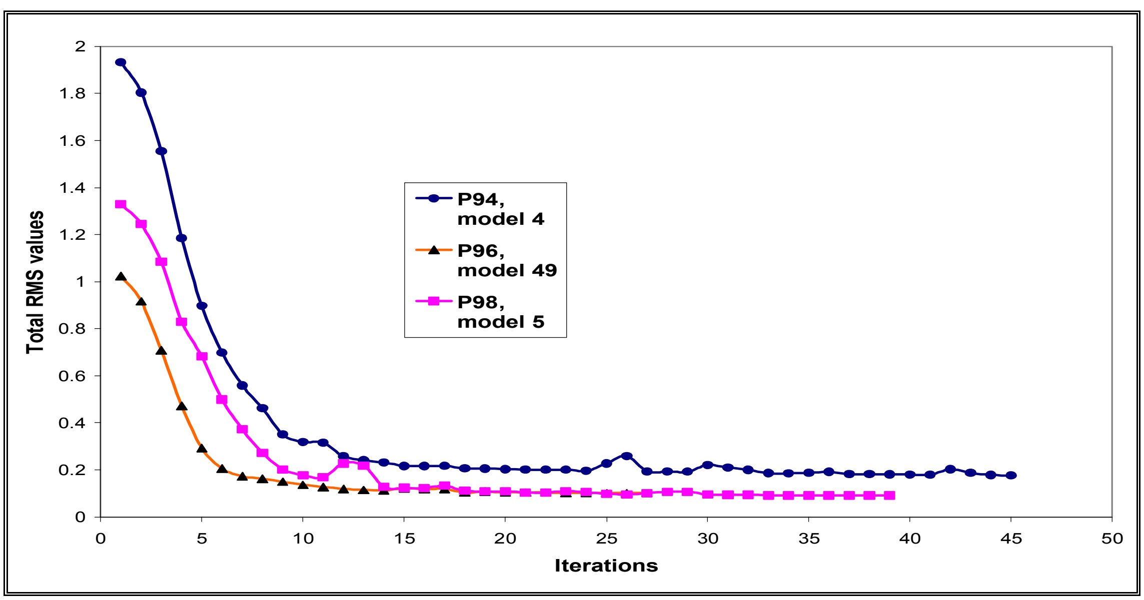

Fig. 14: Minimization of the objective functions and the number of iterations for the best models

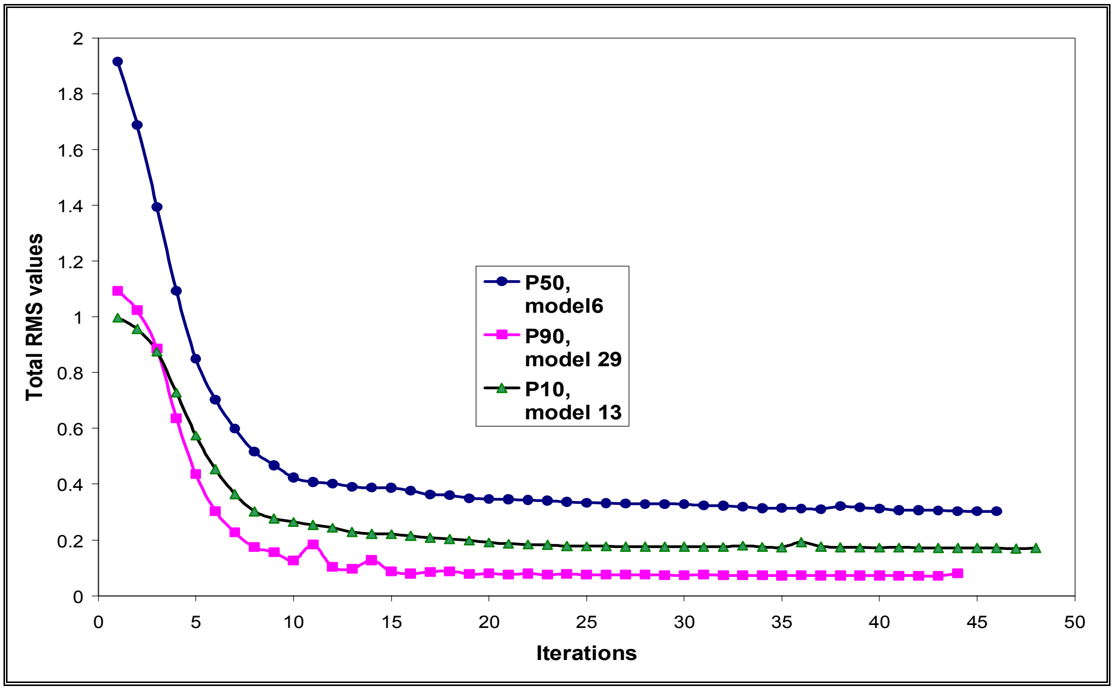

Fig.15: Minimization of the objective functions and the number of iterations for the P90-P10 models

After history matching we used linear uncertainty analysis to quantify 90% confidence intervals of future recoveries after 30 years of production for each model. Tables and plots of the 90% confidence intervals for different production variables for all the six models can be found in the appendix. After obtaining the 90% confidence intervals for the models, we combined the intervals for the three models in each group using SPSS statistical software. Fig. 16 shows how the uncertainty of the selected P90-P10 models compared to the selected best models. For oil recovery the best models predicted lower ranges as compared to our sampled models. For the best models, the average oil prediction was between 195-212 million STB, while for the sampled models it was between 189-221 million STB, a difference of about 10 million STB. It can also be seen that none of the best models encompassed the true value whereas two out of the three sampled models encompassed the true value for predicted future oil recovery.

Similar results can be seen in the case of gas predictions (Fig. 17). The best models predicted slightly lower ranges as compared to our sampled P90-P10 models. For the best models the average gas prediction was between 1,205-1,244 Bcf and for the sampled models it was 1,216-1,270 Bcf, a difference of about 10 Bcf. In the case of gas recovery both the best models and the sampled models were able to encompass the true value for predicted future gas recovery.

These results show that by using the best models we may underestimate the uncertainty in our predictions which may affect project economics. For purposes of quantifying uncertainty, we recommend sampling over the entire distribution of realization likelihoods rather than selecting the best models for history matching and uncertainty quantification.

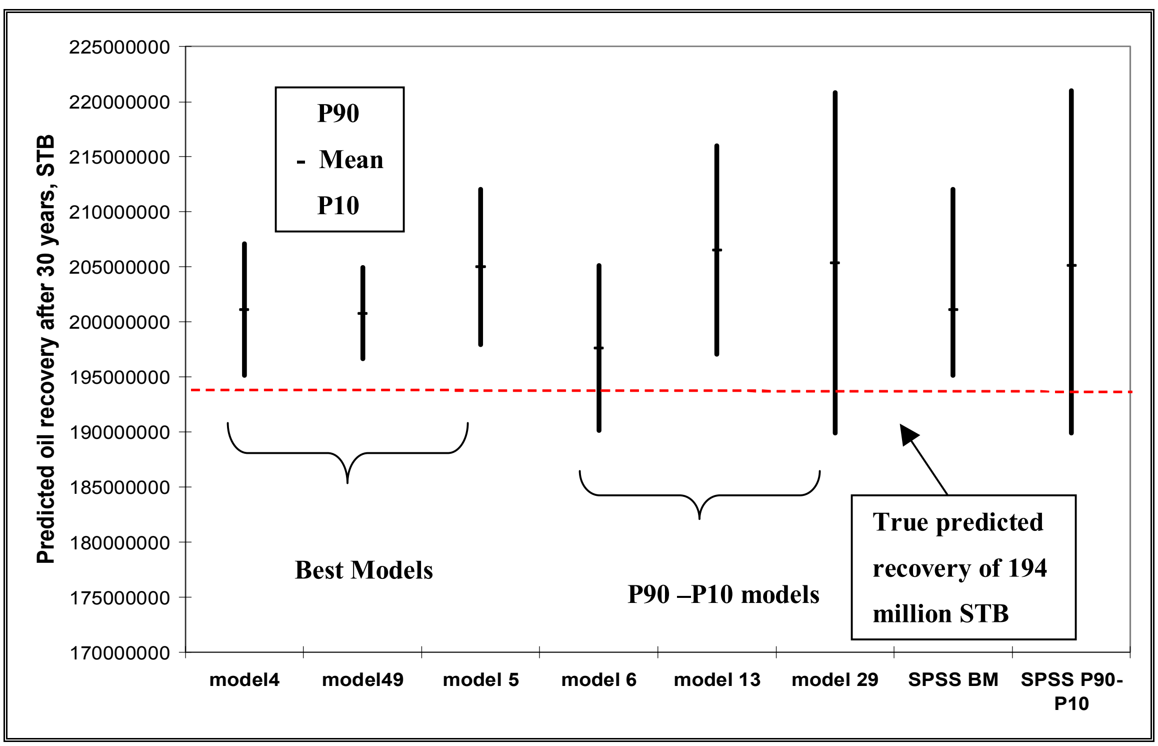

Fig. 16: P90-P10 models encompass the true value but show larger range of uncertainty as compared to the best models for oil recovery

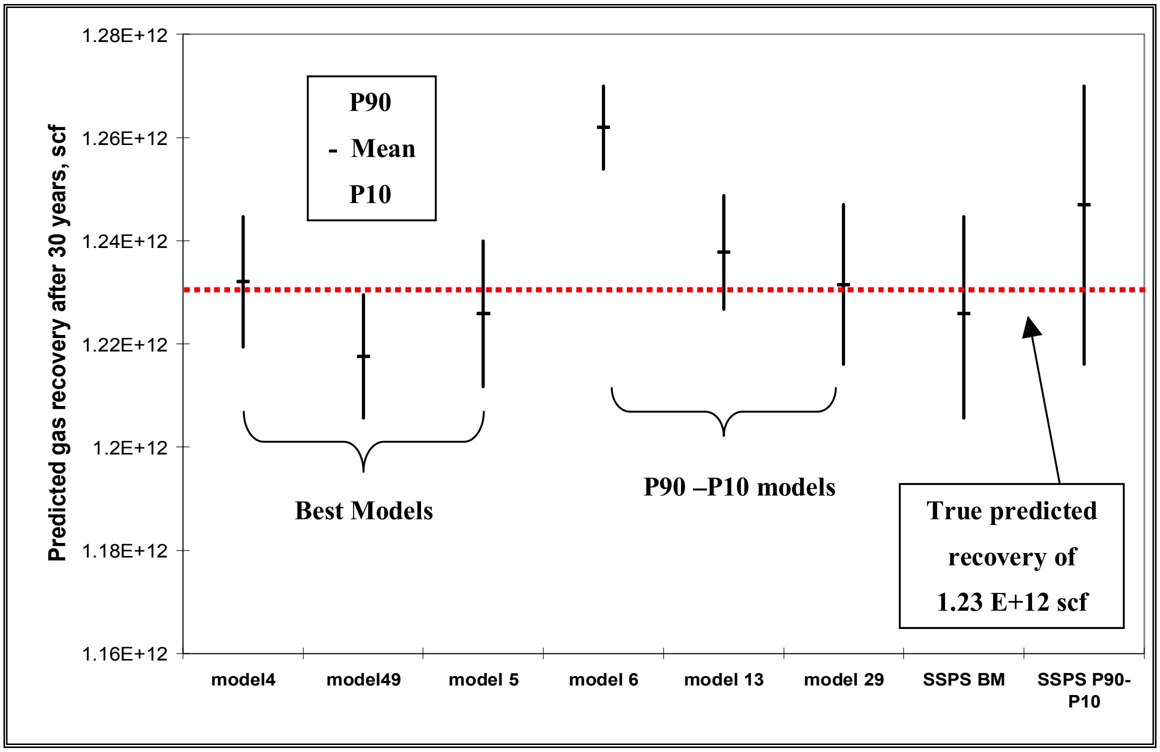

Fig. 17: Best models show a similar range of uncertainty as compared to the P90-P10 models at the same time encompassing the true value for gas recovery

CHAPTER IV

CONCLUSIONS AND FUTURE WORK

On the basis of my research, I have reached the following conclusions:

- Incorporating dynamic data in a reservoir model will result in lower estimates of uncertainty than considering only static data.
- Incorporation of dynamic data does not guarantee that the forecasted ranges will encompass the true value. Reliability of the forecasted ranges depends on the method employed.
- When sampling multiple realizations of static data for history matching to quantify uncertainty, a sampling over the entire range of realization likelihoods shows larger confidence intervals and is more likely to encompass the true value for predicted fluid recoveries, as compared to selecting the best models.

Future work

- Future work should consist of adding a prior term to the objective function to constrain the history matching variables from going to extreme values.
- There are a number of limitations in using the linear uncertainty analysis and any future work should investigate other techniques for quantifying uncertainty.
- This study should be carried out with real field production data to more rigorously test the techniques applied in this study.

NOMENCLATURE

SYMBOL		Description
A	=	sensitivity matrix
c_{di}	=	simulated values
C_v	=	covariance matrix of the parameters
d	=	one set of observed data of a given type at a given well
dv^{k}	=	vector of current parameter normalized values
Erf	=	error function
f	=	objective function
H	=	hessian matrix
i	=	an individual data point for the d^{th} item of observed data
I	=	identity matrix
L	=	likelihood
m	=	total number of observations
n	=	number of observations
n_{w}	=	number of wells
n_{p}	=	number of production variables used
n_{t}	=	number of observed production data time series
o_{di}	=	observed values

o^{obs}	=	observed production data
o^{sim}	=	production data calculated for a particular model
Q	=	data misfit.
r_{di}	=	weighted difference between the observed and simulated values
w	=	weighting factor
w_d	=	overall weighting for d^{th} the data set
w_{di}	=	is a weighting for the i^{th} data point of the d^{th} data set
X_{avg}	=	mean value
x	=	value taken by a variable
σ_d	=	measurement error for the d^{th} data set

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APPENDIX

Table A1: Estimated Misfits, Likelihoods and Weighting Factors

Realization	Q	L	wt. factor	Realization	Q	L	wt. factor
1	2.59997	0.10873	0.020666928	25	2.53152	0.11251	0.021386558
2	2.56749	0.11051	0.021005398	26	2.39262	0.1206	0.022924612
3	2.33531	0.12411	0.023590988	27	1.75681	0.16574	0.031504021
4	1.68182	0.17207	0.032707753	28	2.28198	0.12746	0.024228565
5	0.19879	0.3612	0.06865714	29	7.77922	0.00816	0.00155102
6	2.48127	0.11537	0.021930734	30	2.78582	0.09908	0.018833063
7	4.51616	0.04171	0.007928324	31	7.79227	0.00811	0.001540933
8	8.76952	0.00497	0.000945313	32	2.38899	0.12082	0.022966324
9	3.47152	0.07032	0.01336665	33	2.33716	0.12399	0.023569259
10	5.04473	0.03202	0.006087013	34	5.56821	0.02465	0.004685229
11	5.06175	0.03175	0.00603542	35	1.92654	0.15225	0.028940843
12	1.75554	0.16584	0.031524045	36	1.92654	0.15225	0.028940843
13	1.6952	0.17092	0.0324896	37	2.06607	0.14199	0.026990537
14	2.4123	0.11942	0.022700141	38	1.94486	0.15087	0.028676959
15	2.56269	0.11077	0.021055862	39	1.71385	0.16934	0.032188132
16	4.12208	0.05079	0.009655083	40	2.19663	0.13302	0.02528485
17	2.29209	0.12682	0.024106405	41	3.1113	0.0842	0.016004501
18	10.0968	0.00256	0.000486814	42	19.7853	2E-05	3.83294E-06
19	1.86753	0.15681	0.029807375	43	15.2362	0.0002	3.72693E-05
20	5.93453	0.02052	0.003901093	44	2.65107	0.10598	0.02014562
21	3.8101	0.05937	0.011284978	45	4.09529	0.05148	0.009785263
22	2.17598	0.1344	0.025547325	46	1.72668	0.16825	0.031982267
23	2.59997	0.10873	0.020666928	47	2.52261	0.11301	0.021481994
24	1.69127	0.17126	0.032553566	48	2.11496	0.13856	0.026338729
				49	1.21494	0.21732	0.041307902

Table A2: Uncertainty Estimates Using the Automatic History Matching Technique With Linear Uncertainty Analysis

	Best Models			P90-P10 Models			
	Model 4	Model 49	Model 5	Model 6	Model 13	Model 29	True value
Predicted future recovery of oil (10^6stb)	195-207	196-204	197-212	190-205	197-215	189-221	194
Predicted future recovery of gas, (10^{12} scf)	1.21-1.24	1.20-1.22	1.21-1.24	1.25-1.27	1.22-1.24	1.21-1.24	1.23

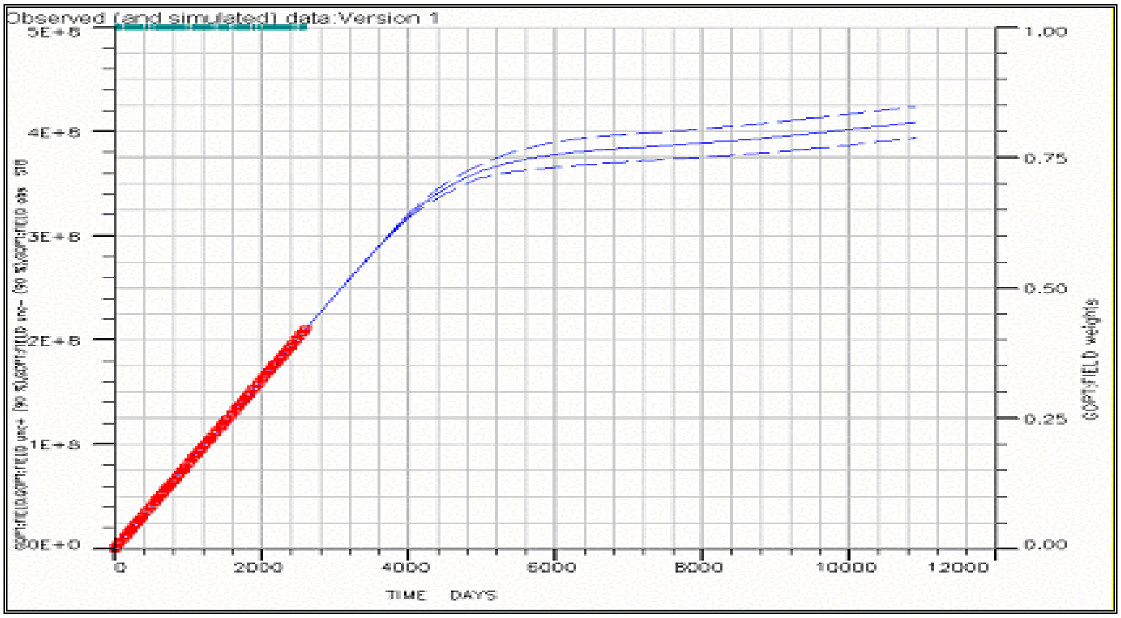


Figure A.1: Predicted uncertainty in oil recovery for model 6 after 30 years of production

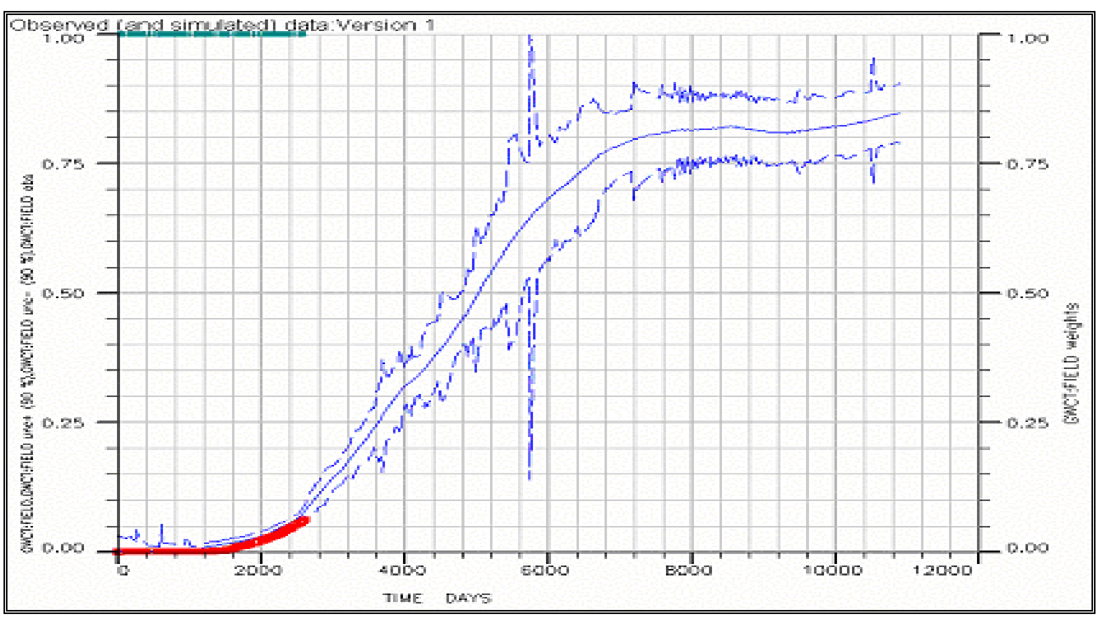


Figure A.2: Predicted uncertainty in watercut for model 6 after 30 years of production

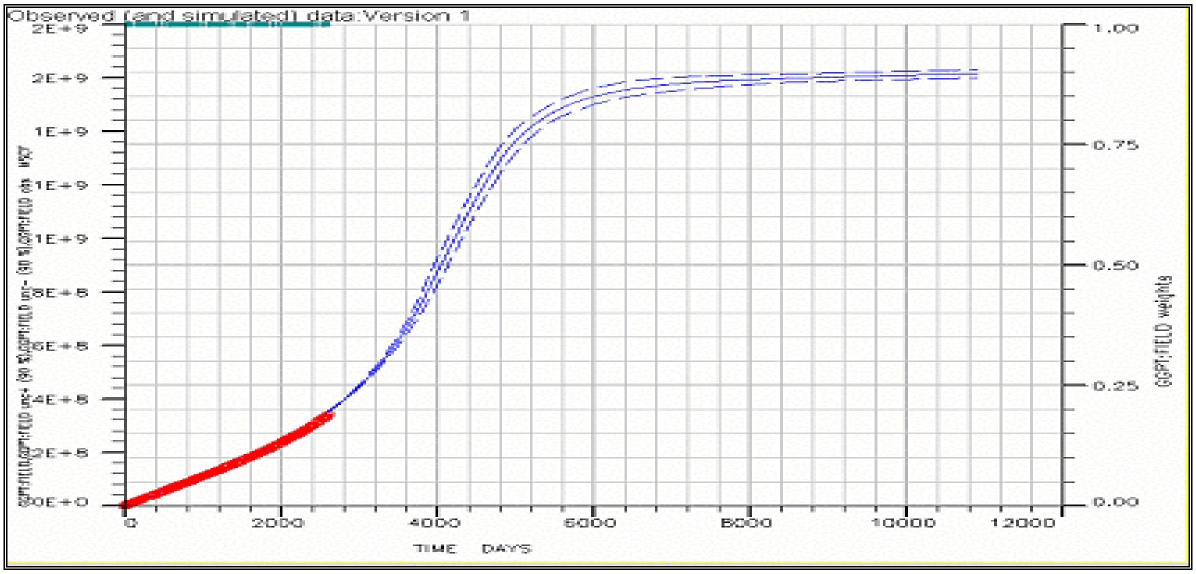


Figure A3: Predicted uncertainty in gas recovery for model 6 after 30 years of production

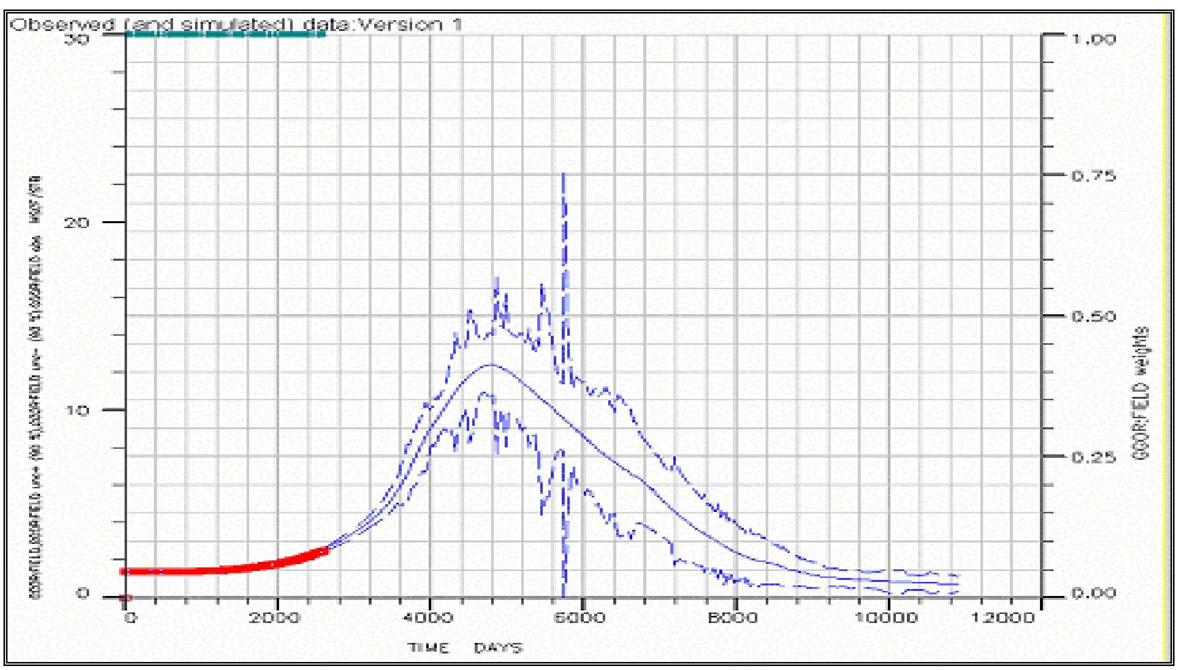
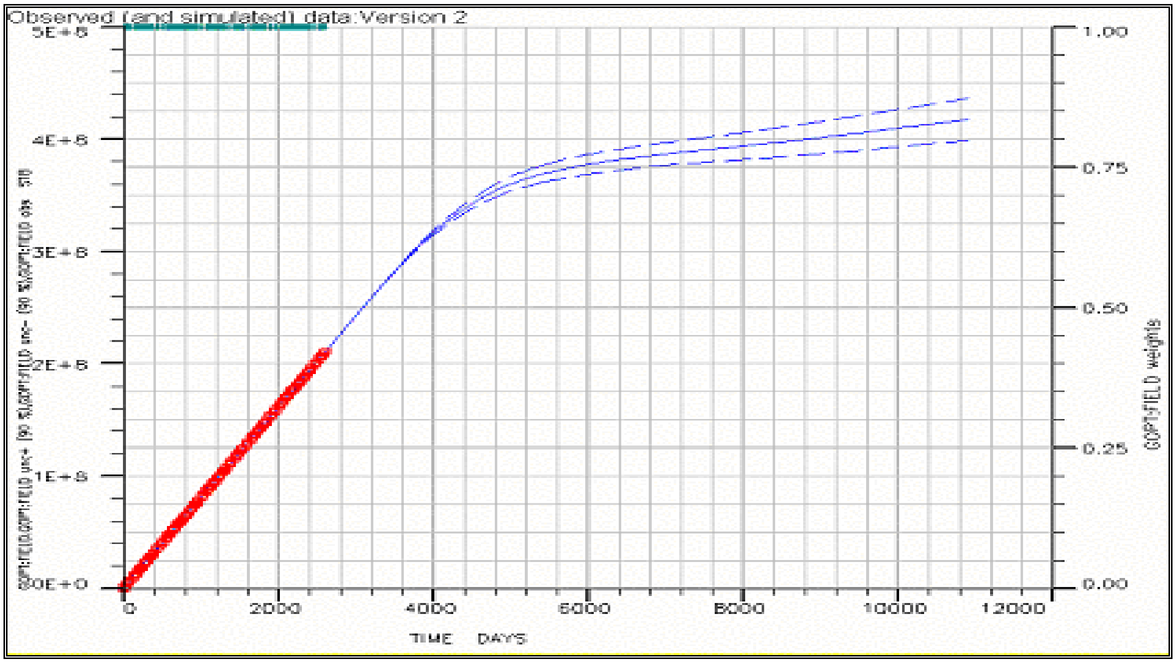
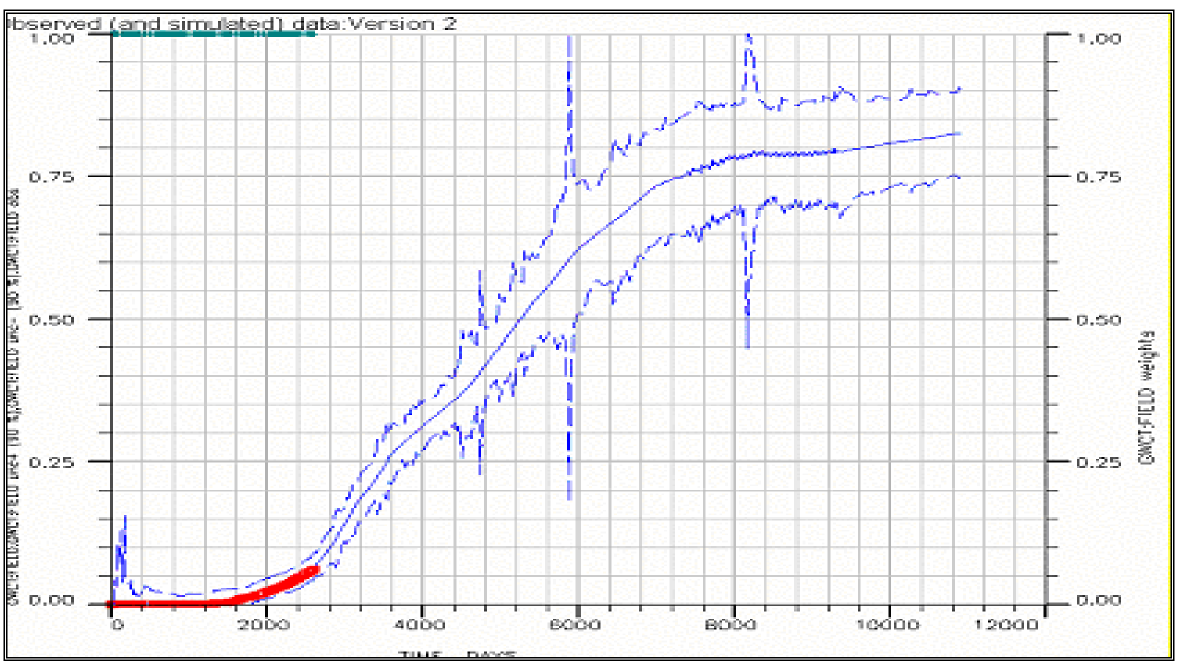


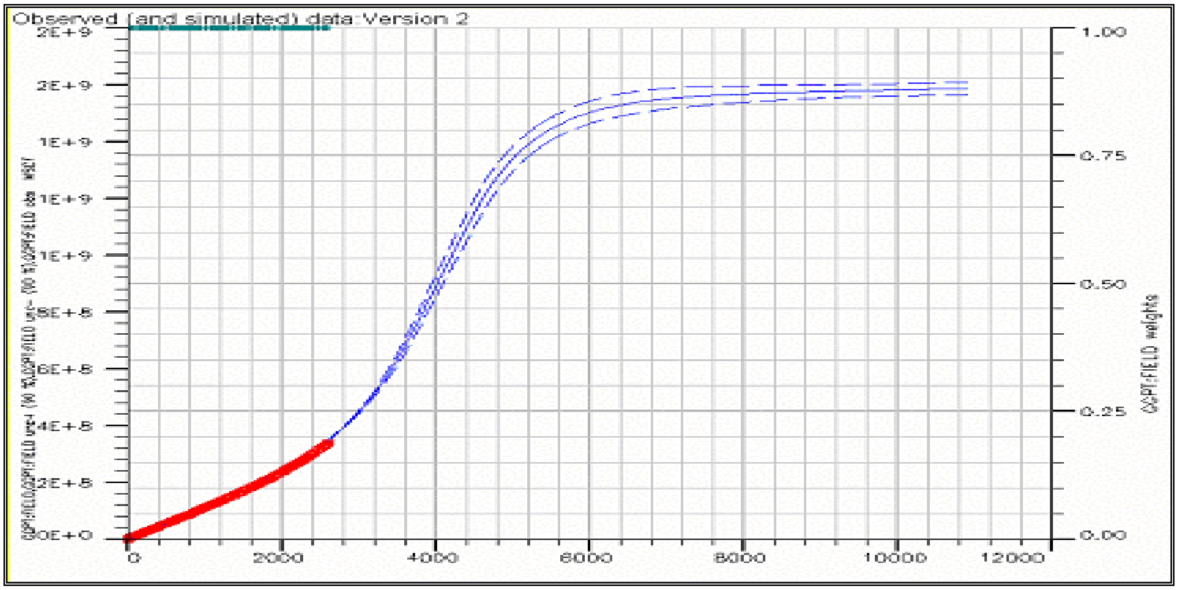
Figure A4: Predicted uncertainty in the GOR for model 6 after 30 years of production



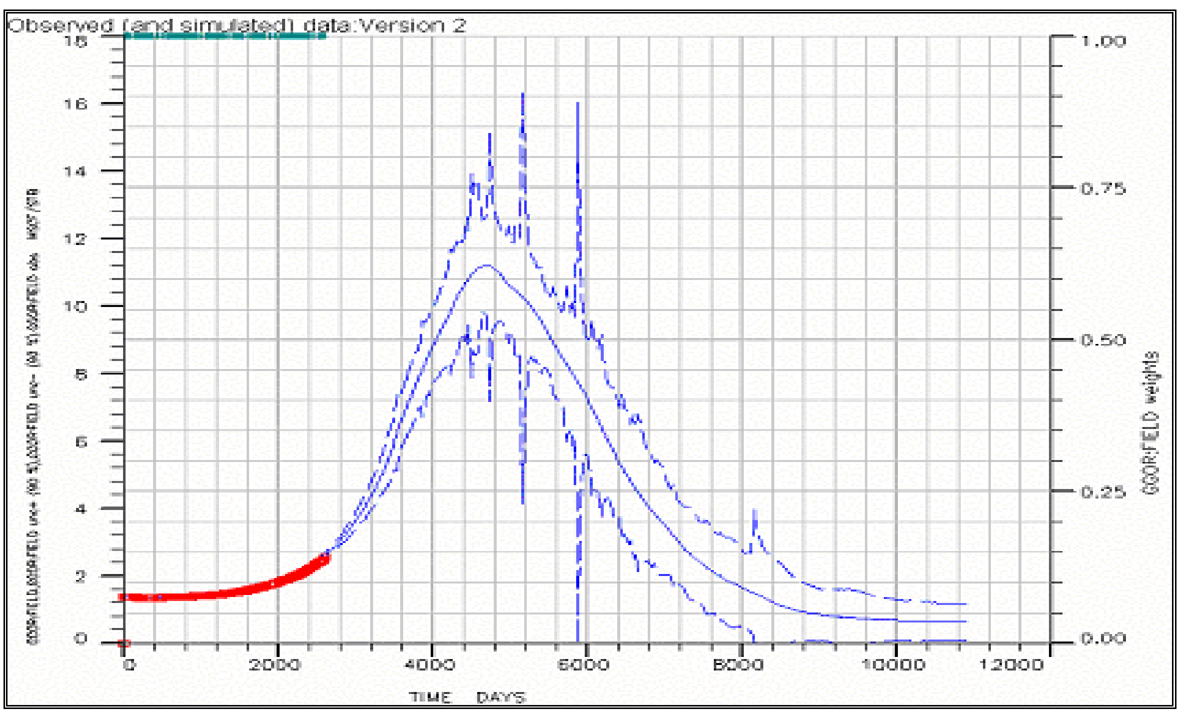
FigureA5: Predicted uncertainty in oil recovery for model 13 after 30 years of production



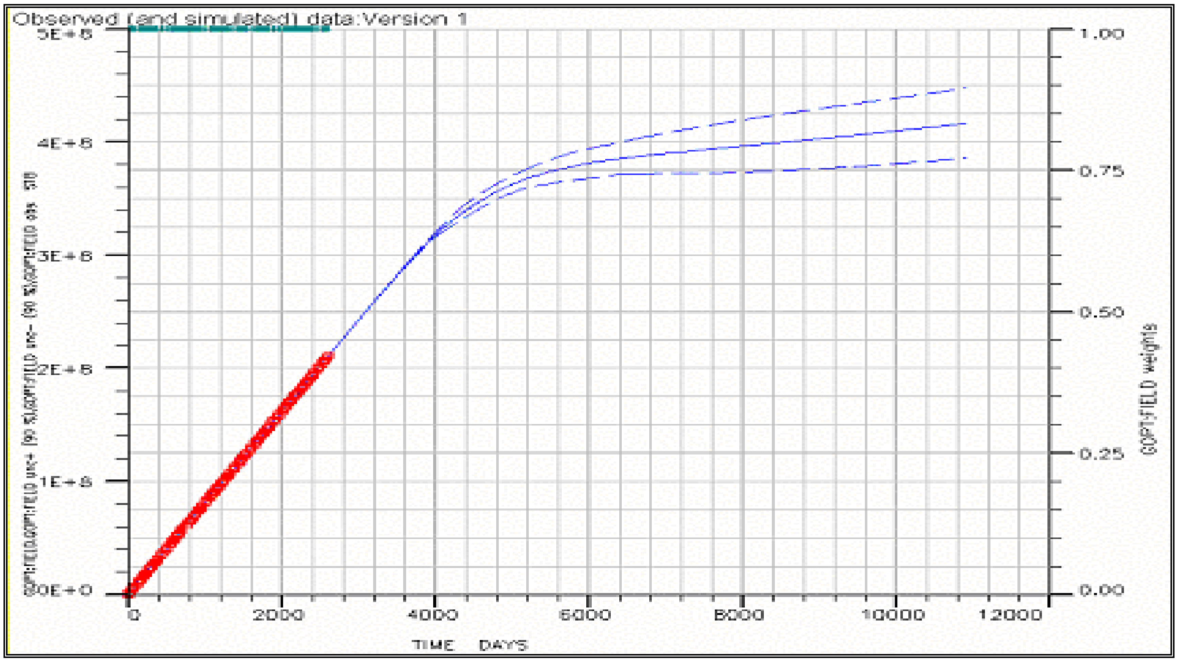
FigureA6: Predicted uncertainty in watercut for model 13 after 30 years of production



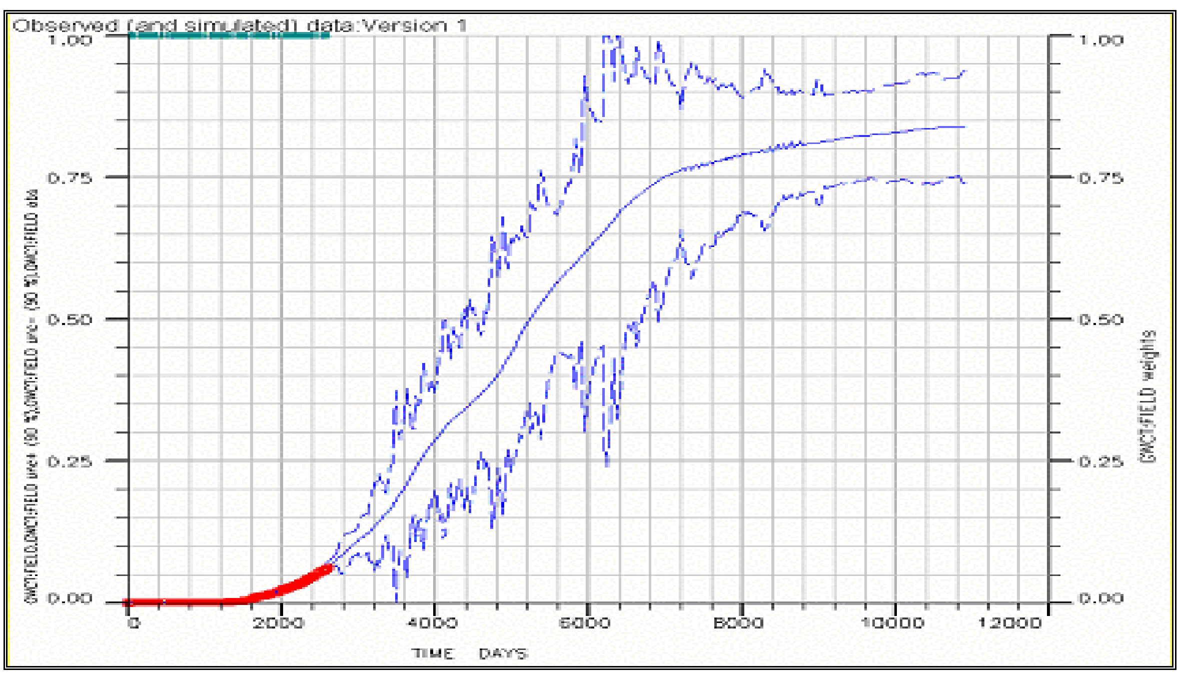
FigureA7: Predicted uncertainty in gas recovery for model 13 after 30 years of production



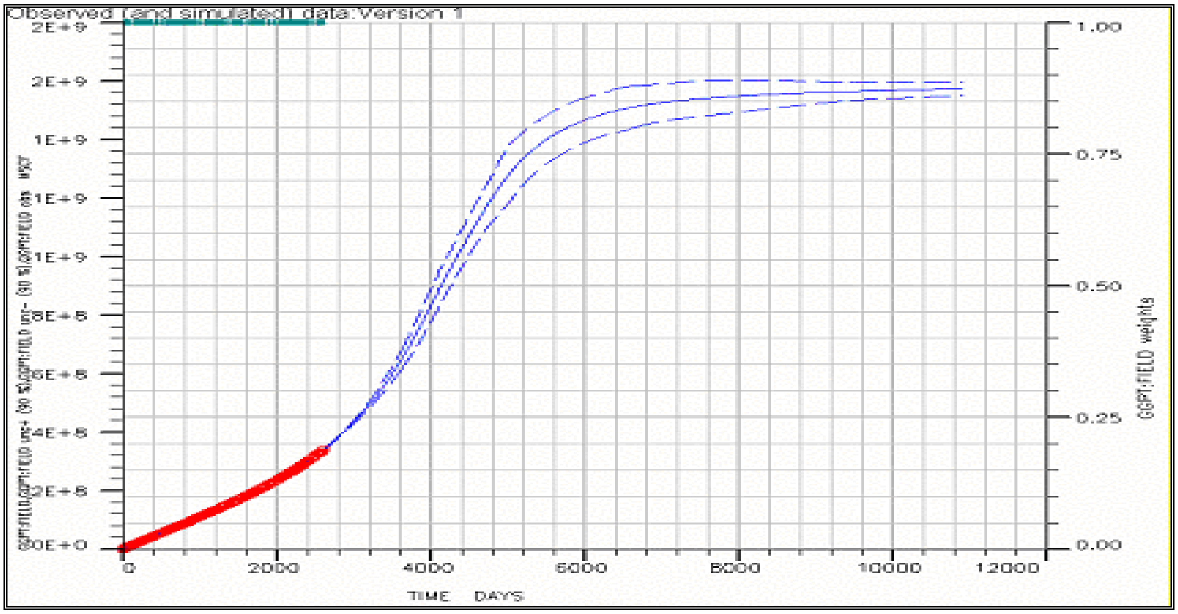
FigureA8: Predicted uncertainty in GOR for model 13 after 30 years of production



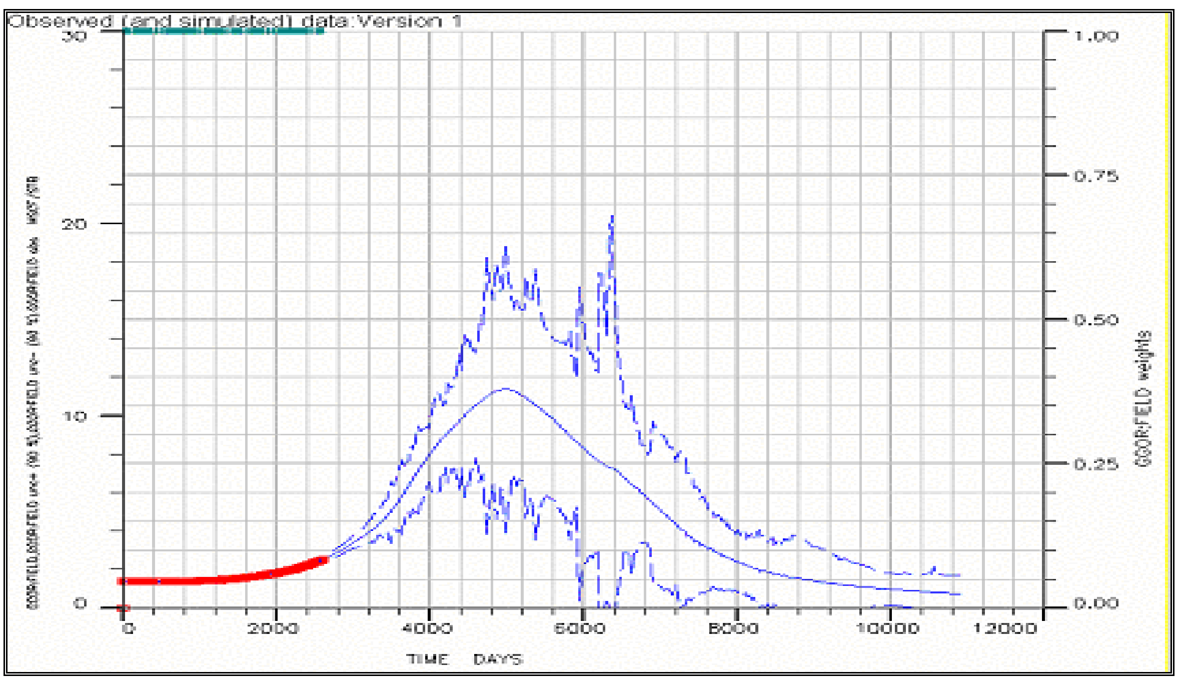
FigureA9: Predicted uncertainty in oil recovery for model 29 after 30 years of production



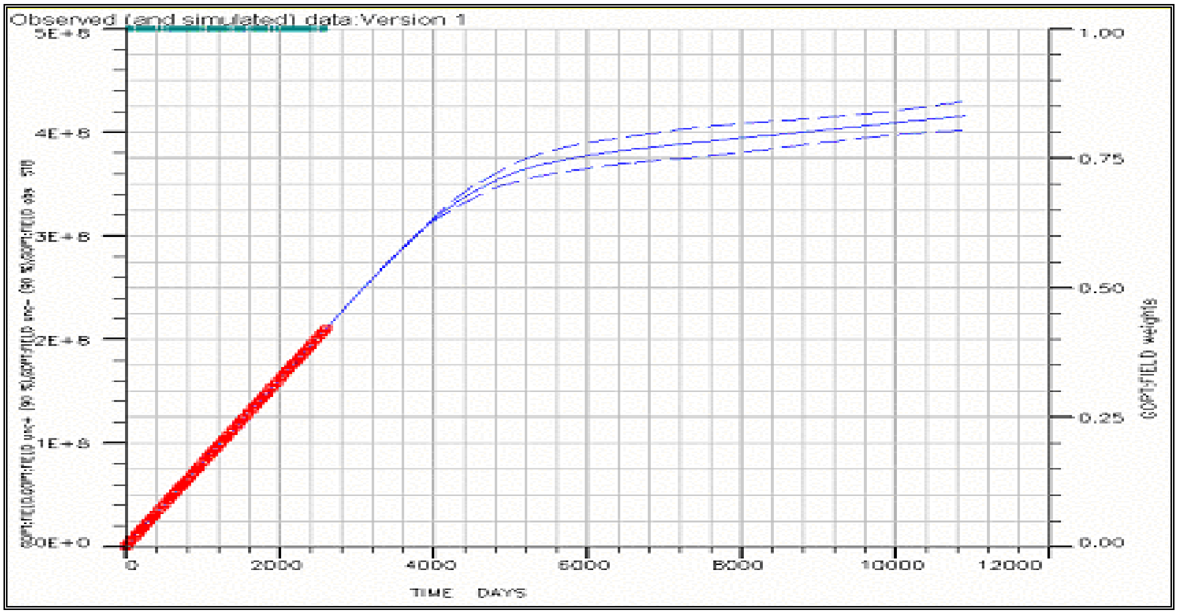
FigureA10: Predicted uncertainty in watercut for model 29 after 30 years of production



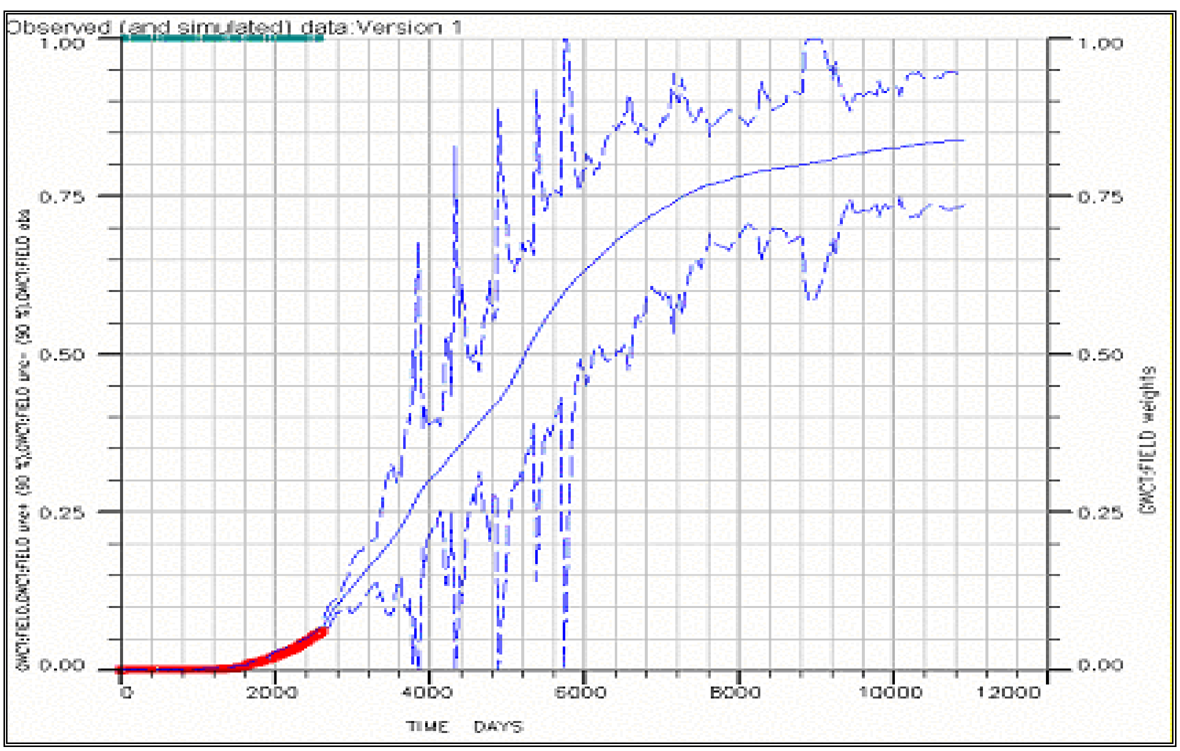
FigureA11: Predicted uncertainty in gas recovery for model 29 after 30 years of production



FigureA12: Predicted uncertainty in GOR for model 29 after 30 years of production



FigureA13: Predicted uncertainty in oil recovery for model 5 after 30 years of production



FigureA14: Predicted uncertainty in watercut for model 5 after 30 years of production

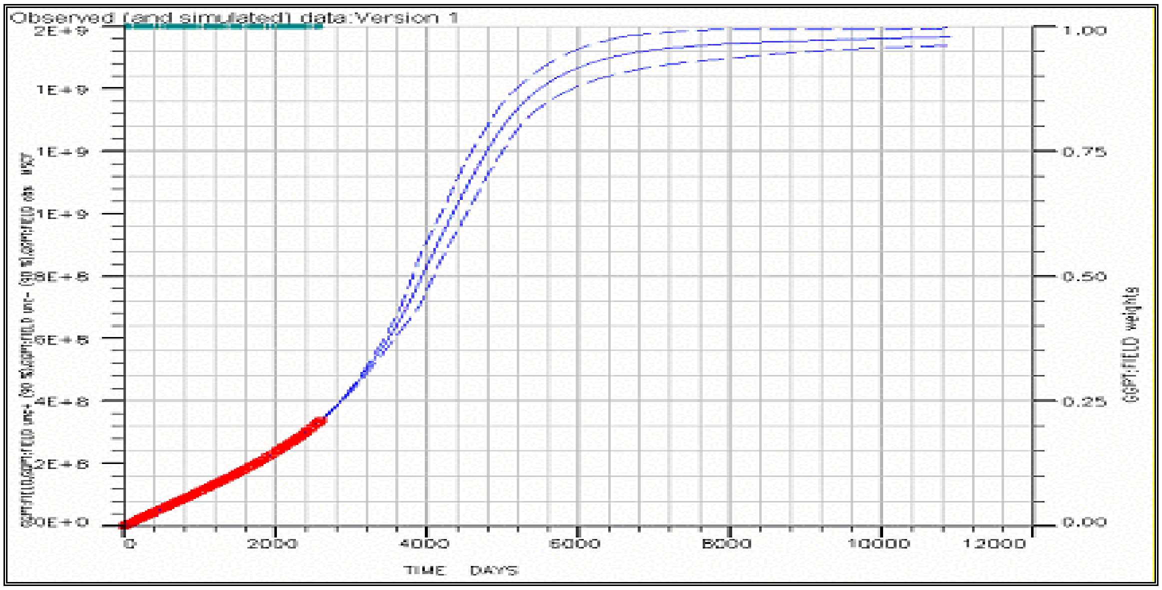
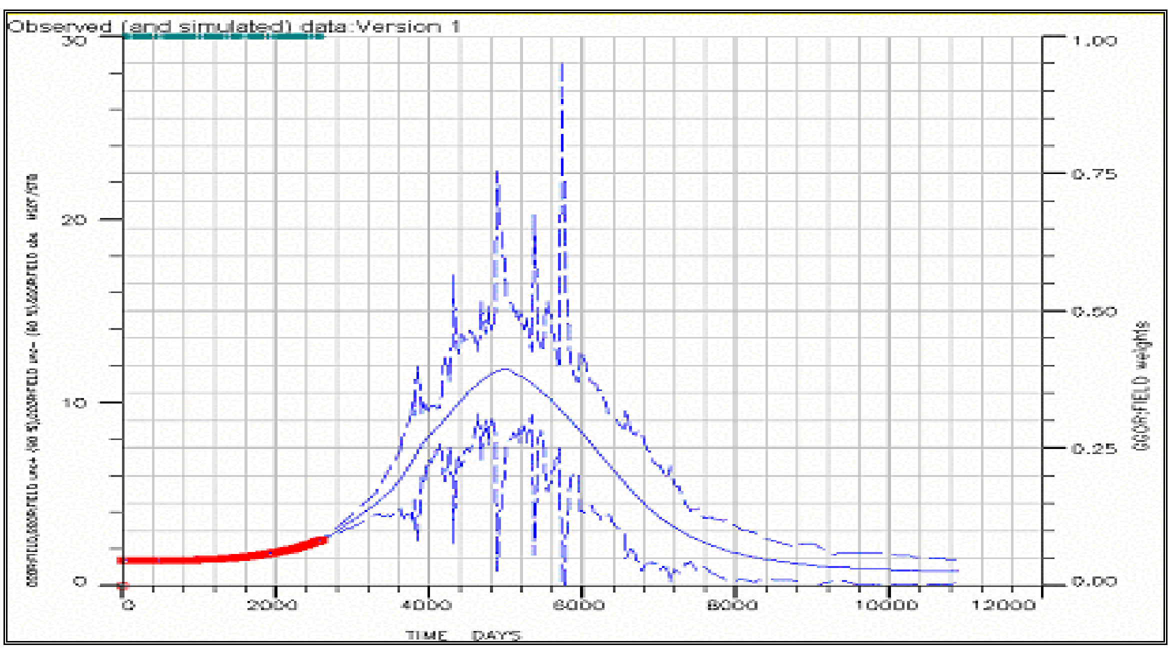
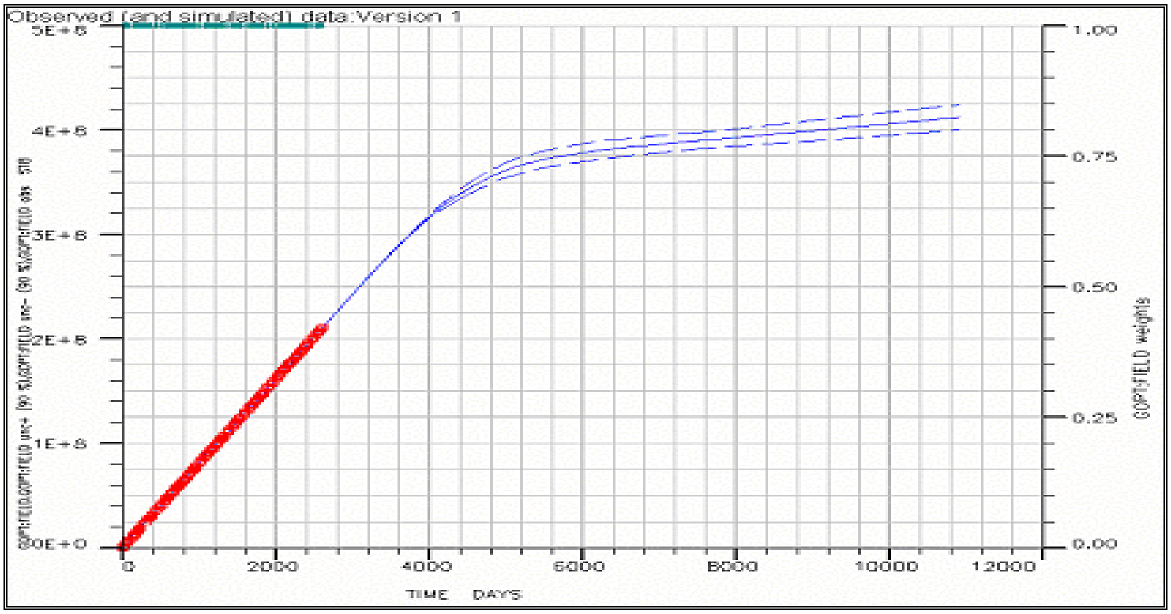


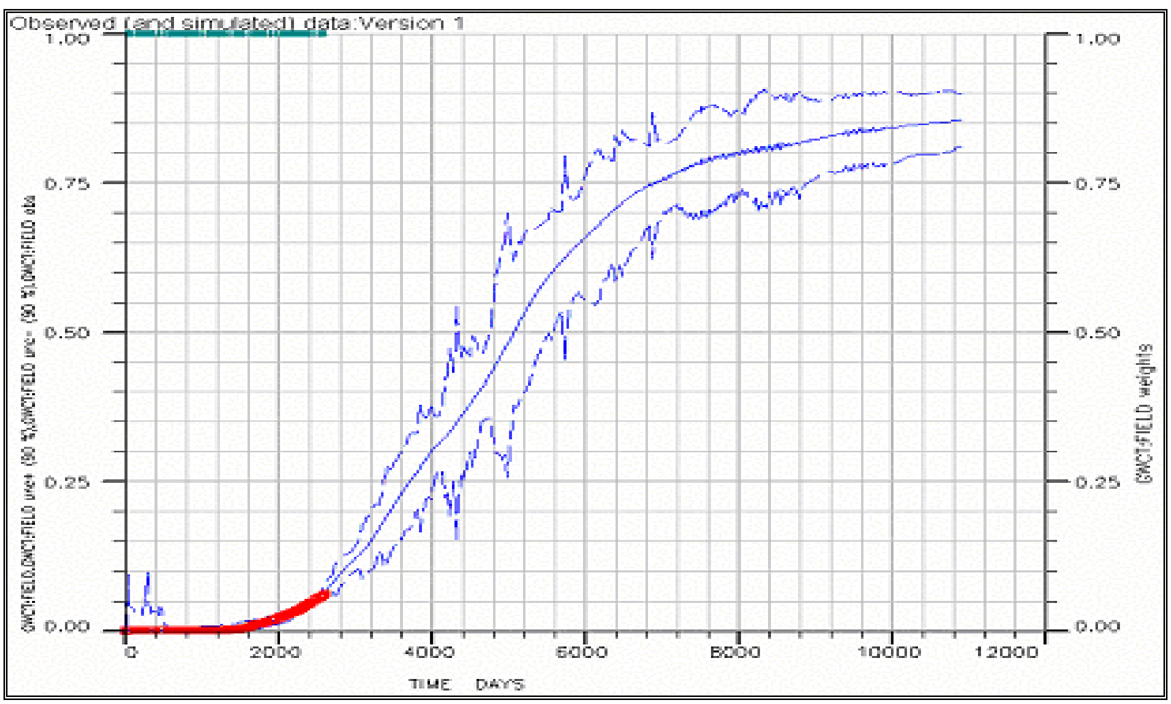
Figure A15: Predicted uncertainty in gas recovery for model 5 after 30 years of production



FigureA16: Predicted uncertainty in GOR for model 5 after 30 years of production



FigureA17: Predicted uncertainty in oil recovery for model 4 after 30 years of production



FigureA18: Predicted uncertainty in water cut for model 4 after 30 years of production

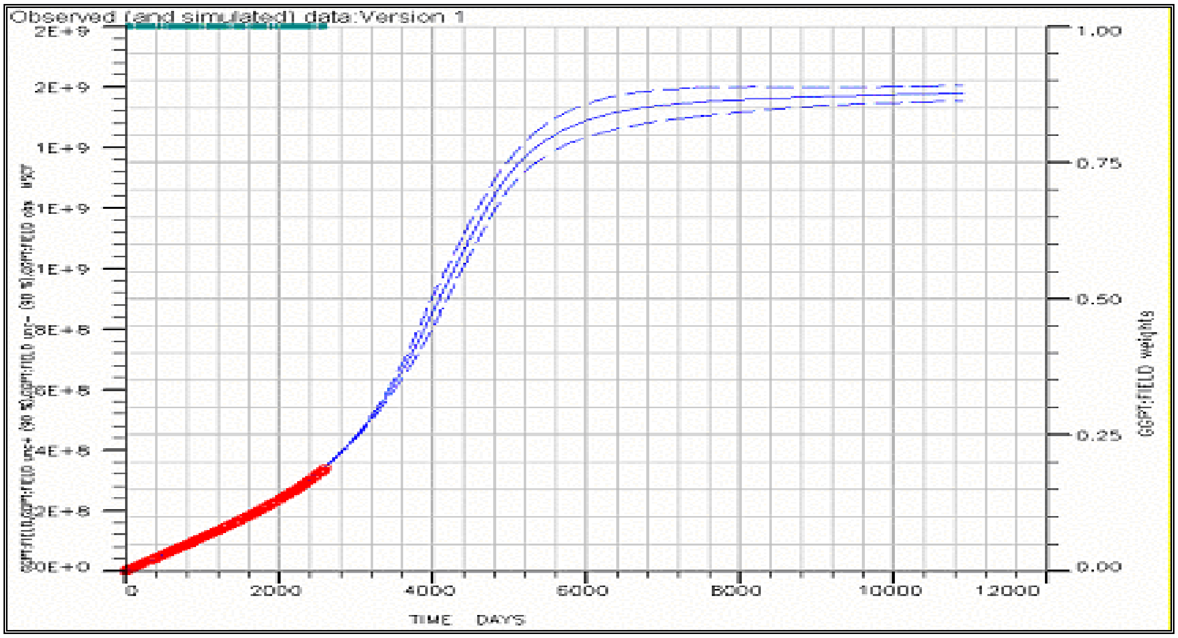
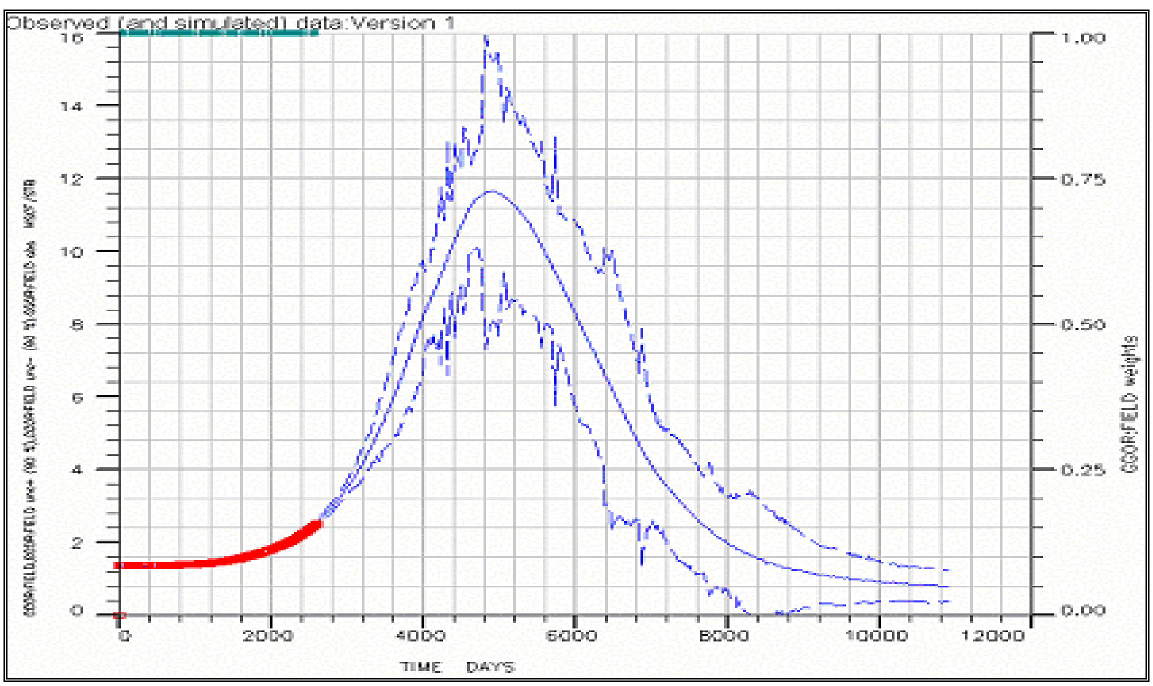
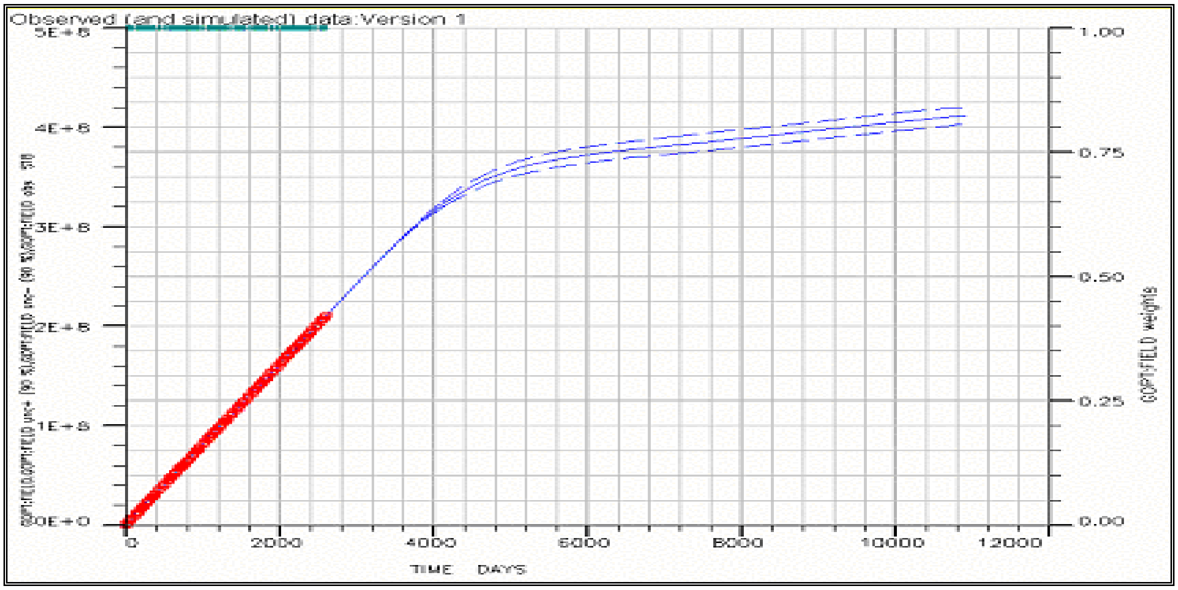


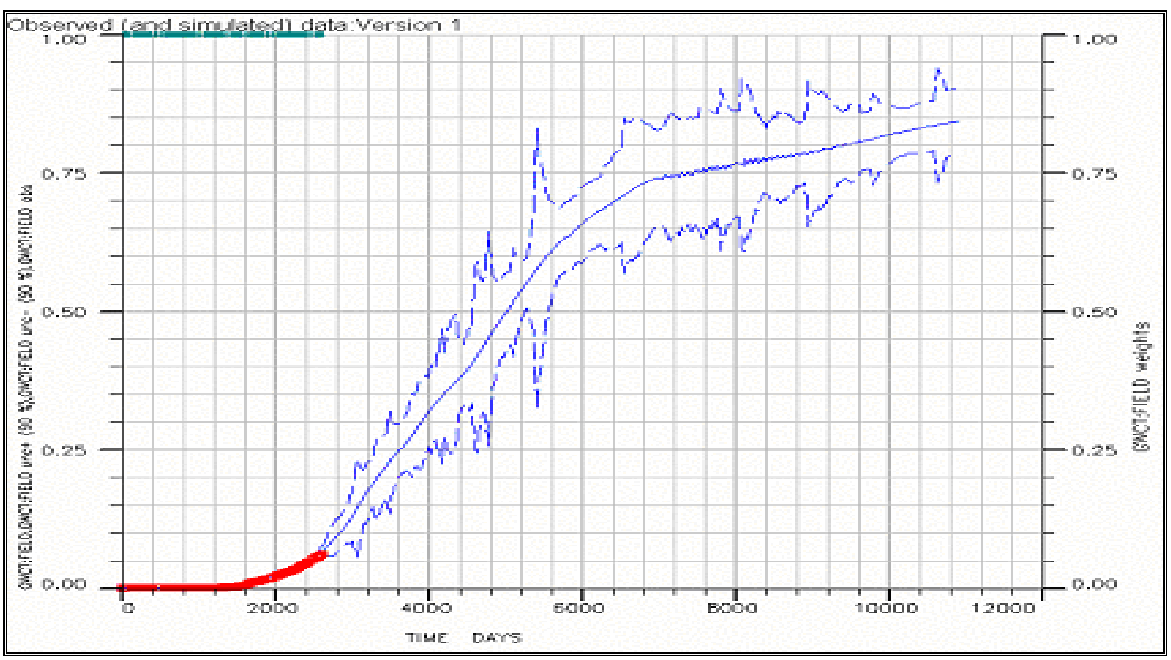
Figure A19: Predicted uncertainty in gas recovery for model 4 after 30 years of production



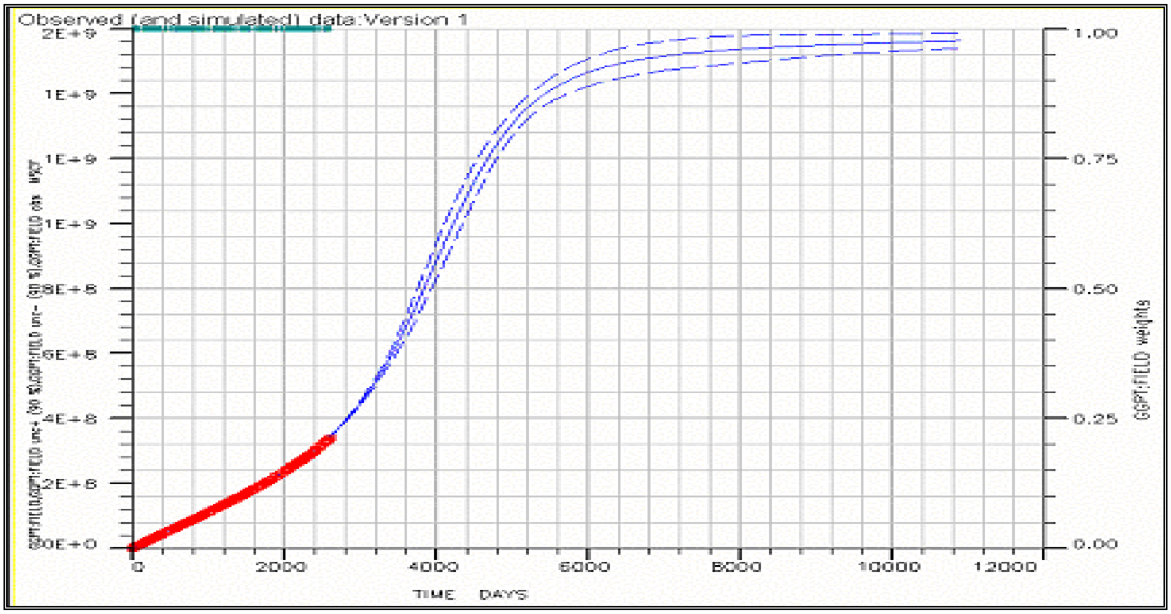
FigureA20: Predicted uncertainty in GOR for model 4 after 30 years of production



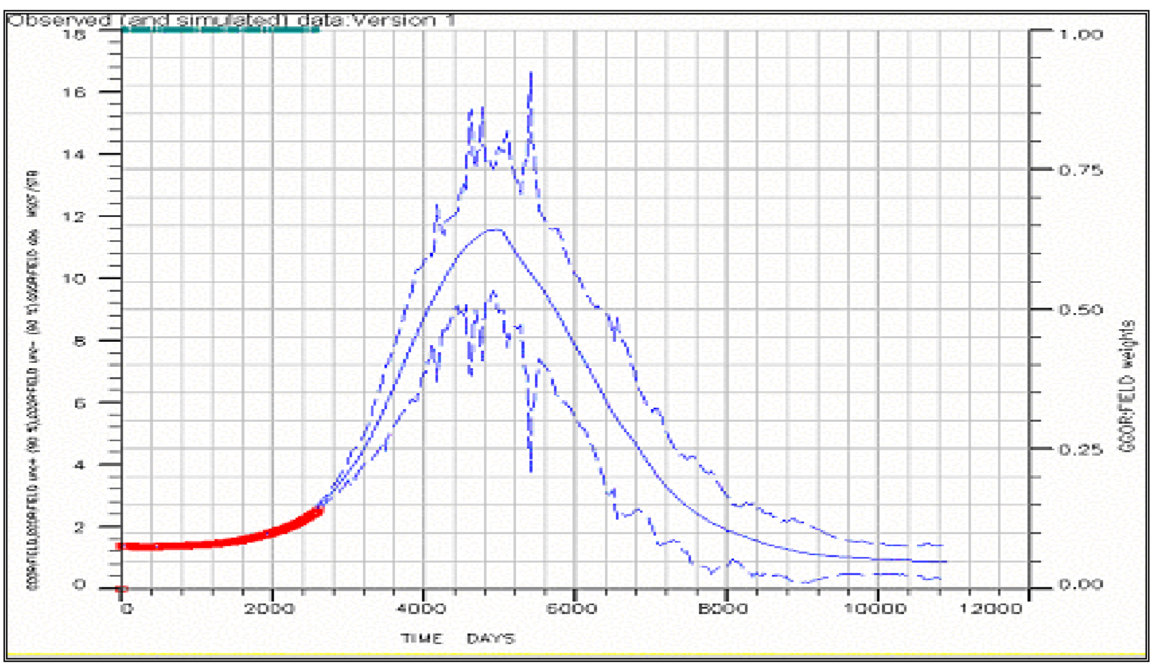
FigureA21: Predicted uncertainty in oil recovery for model 49 after 30 years of production



FigureA22: Predicted uncertainty in watercut for model 49 after 30 years of production



FigureA23: Predicted uncertainty in gas recovery for model 49 after 30 years of production



FigureA24: Predicted uncertainty in GOR for model 5 after 30 years of production

VITA

The Author was born in 1975 in Pune, India. He got his Bachelor's degree in petrochemical engineering from Maharashtra Institute of Technology, Pune, India in the year 1996. After finishing his BS studies the author worked for 4 years with various firms in Piping engineering and design before coming to the US for further studies. During his studies, he has also had two internships with Sperry-Sun Drilling Services, a subsidiary of Halliburton Energy Services in MWD/LWD. His permanent address is 111, Ghorpade Peth, Pune-411042, India.