Spin resonance in Luttinger liquid with spin-orbit interaction

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Spin-orbit interaction in quantum wires leads to a spin resonance at low temperatures, even in the absence of an external dc magnetic field. We study the effect of electron-electron interaction on the resonance. This interaction is strong in quantum wires. We show that the electron-electron interaction changes the shape of the resonance curve and produces an additional cusp at the plasmon frequency. However, except for very strong electron-electron interaction these changes are weak since this interaction by itself does not break the spin-rotation symmetry that is violated weakly by the spin-orbit interaction and external magnetic field.

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A recent theoretical work [1] has predicted that in one-dimensional (1D) quantum wires with the spin-orbit interaction (SOI) [2] it is possible to observe a relatively sharp electronic spin resonance (ESR) in terahertz range. An external magnetic field, which is perpendicular to the internal SOI induced field, enhances the resonance absorption by several orders of magnitude leaving the resonance frequency almost unchanged. It occurs because this field violates a symmetry forbidding the electric dipolar mechanism of the spin-flip transition. The magnetic field oriented along the SOI field separates the resonance frequencies of the left- and right-moving electrons and generates the permanent electric current and dynamic magnetization.

In the work [1] the electrons have been treated as non-interacting particles. However, in 1D systems the electron-electron interaction is known to be strong $V/\epsilon_F \sim |\ln(na)|/(na)$, where ϵ_F is the Fermi energy, n is the 1D electron density, $a = \hbar^2\kappa/(me^2)$ is the Bohr's radius in the material, $m = 0.05\, m_e$ is the effective electron mass, and κ is the dielectric constant. For typical values $n \sim 10^6$ cm^{-1} and $\kappa \sim 20$ the ratio $V/\epsilon_F \simeq 1$. Therefore, it is important to study the effect of interaction on electron spin resonance in a quantum wire with the SOI. This is the main goal of this paper. The ESR in the Luttinger electron liquid is the excitation of a spin wave by external ac electromagnetic field. This resonance would have a simple Lorentzian shape in the absence of interaction.

Since the electron-electron interaction is strong, the fermionic excitations do not exist in 1D systems. They are replaced by bosonic collective excitations: charge and spin waves. In the framework of Luttinger model [3, 4] that neglects the SOI and the deviation of the electronic spectrum near the Fermi points from the linear behavior, the charge and spin degrees of freedom do not interact (this is the so-called spin-charge separation). The SOI separates the Fermi points for different spin projections and makes possible the resonant spin-flip processes. It was shown that the interplay of magnetic field, SOI, and electron-electron interaction leads to the formation of spin-density wave state when magnetic field is perpendicular to the effective SOI magnetic field [5]. In this Letter we assume that the magnetic field has nonzero component along the SOI field. Such a field terminates the spin-density wave instability [5] and simultaneously separates the spin resonances for left and right movers [1]. The Coulomb interaction is expected to change the shape of the spin resonance line from simple Lorentzian to a power-like one which is characteristic for the Luttinger liquid [3, 4, 6]. The SOI also violates spin-charge separation and thus enables the excitation of the charge waves at spin reversal. It can be seen as a weak resonance at a plasmon frequency instead of the spin-wave frequency. As we show below, both of these effects really take place, though both are weak for not too strong electron-electron interaction.

We consider a nanowire with a cross-section so small that electrons fill partially only the lowest band of the transverse motion (one channel). In this case the Tomonaga-Luttinger model is applied. The standard Luttinger liquid theory starts from the fermionic Hamiltonian with the linearized dispersion [4, 6] to which we add the Rashba SOI, H_R:

$$H_0 = -iv_F \sum_\sigma \int dx\, (\psi^\dagger_{R,\sigma}\partial_x\psi_{R,\sigma} - \psi^\dagger_{L,\sigma}\partial_x\psi_{L,\sigma}) + H_{int} + H_R. \quad (1)$$

Here the x-axis is taken along the quantum wire; $\partial_x = \partial/\partial x$; v_F is the Fermi velocity; R, L labels the right and left moving fermions; and $\sigma = \uparrow, \downarrow$ are the spin projections. The interaction part of the Hamiltonian, H_{int}, contains terms $\rho_{R(L)}(q)\rho_{R(L)}(q)$ quadratic in charge densities $\rho_{R(L)}(q)$ of left and right movers and the terms quadratic in spin densities such that the total spin is conserved, e.g. $:\psi^\dagger_{R,\uparrow}(x)\psi_{L,\uparrow}(x)::\psi^\dagger_{L,\downarrow}(x')\psi_{R,\downarrow}(x'):$.

With $H_R = 0$ the Hamiltonian (1) has an obvious $SU(2)$ symmetry of rotations in the spin space. The term $H_R = \alpha\int\psi^\dagger p_x\sigma_z\psi\, dx$ in the Hamiltonian H_0 represents the Rashba SOI [2] that splits Fermi momenta of up and down spins so that four Fermi points $p_{\rho,\sigma} = \rho p_F - \sigma\alpha m$ appear, but it leaves Fermi velocities unchanged. The Rashba SOI constant α has dimensionality of velocity and we assume $\alpha \ll v_F$; σ_z is the Pauli matrix; p_F is the Fermi momentum at $\alpha = 0$; and $\rho, \sigma = \pm 1$ correspond to right (left) movers and up

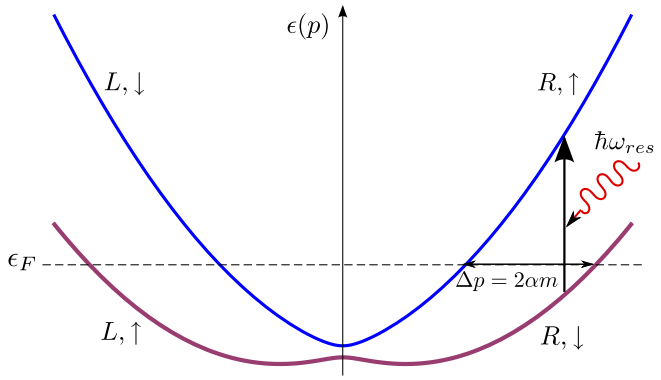

FIG. 1. (Color online) The up-spin and down-spin branches of the electron spectrum with nonzero Rashba spin-orbit interaction α and magnetic field $B_\perp$.

(down) spin projections, respectively. The momenta splitting can be removed by a single-particle unitary transformation $U = \exp(-i\sigma_z \alpha m x/\hbar)$ which shifts the momenta by $\pm\alpha m$. After this transformation the electronic spectrum becomes the same as without the SOI and the $SU(2)$ symmetry is restored.

An external permanent magnetic field breaks this symmetry. It leads to additional splitting of the Fermi points and to a difference in Fermi velocities for up and down spins, which cannot be compensated by this unitary transformation. We consider in this Letter only the magnetic field, $B_\perp$, perpendicular to the Rashba field (along z-axis) and apply it for definiteness along x-axis. The corresponding Zeeman Hamiltonian reads:

$$H_Z = -\frac{g\mu_B B_\perp}{2} \sum_{\rho,\sigma,\sigma'} \int dx \psi^\dagger_{\rho\sigma}(\sigma_x)_{\sigma\sigma'}\psi_{\rho\sigma'}, \qquad (2)$$

where μ_B is the Bohr magneton and g is the electron g-factor. Figure 1 schematically shows the electron energy as a function of momentum in the presence of the transverse magnetic field. We assume that the magnetic field is weak, $g\mu_B B_\perp \ll \alpha p_F$, and further consider it perturbatively. The residual symmetry in the perpendicular field is the combined reflection $p, \sigma \to -p, -\sigma$. It ensures that the right movers with the spin projection σ along z-axis have the same velocity as the left movers with the same energy and the opposite spin projection $v_{R,\sigma} = v_{L,-\sigma}$, but $v_{R,\sigma} \neq v_{R,-\sigma}$. Moroz *et al.* [7] have shown that a velocity difference $\delta v = v_{R,\uparrow} - v_{R,\downarrow} = v_{L,\downarrow} - v_{L,\uparrow}$ appears also due to the Rashba SOI in the wires of finite width. The curvature of the bands near Fermi level [8–13] can also be effectively taken into account by the nonzero velocity difference δv on the upper and lower branches of the energy spectrum. The later effect has a relative value of at most $\sim \alpha/v_F$.

We aim to calculate the absorption power of the resonant ac electromagnetic field for the spin-flip processes. The interaction of electromagnetic field with electrons is described by the Hamiltonian $H_{em} = -(1/c)\int jA_x dx$, where $j = e\psi^\dagger(x)\hat{v}\psi(x)$ is the current, $\hat{v} = \hat{p}/m + \alpha\sigma_z$ is the velocity operator, and A_x denotes the x-component of the vector potential of the ac field. We use the Coulomb gauge, $\boldsymbol{\nabla}\cdot\mathbf{A} = 0$, where the scalar potential is zero, and therefore the electric field is $\mathbf{E} = -(1/c)\partial\mathbf{A}/\partial t$. The part of the electric current responsible for the spin-flip processes is

$$j_s(x) = e\alpha\psi^\dagger(x)\sigma_z\psi(x). \qquad (3)$$

The absorption power of electromagnetic field is determined by the real part of the conductivity σ_ω at the frequency ω of the field multiplied by the square of the field's amplitude $|E_x(\omega)|^2$. We employ the Kubo formula for the conductivity:

$$\sigma_\omega = -\frac{1}{\hbar\omega l}\int_0^l dx \int_0^l dx' \int_{-\infty}^{\infty} dt' \theta(t-t') e^{i\omega(t-t')} \times \langle [j_s(x,t), j_s(x',t')]\rangle, \qquad (4)$$

where l is the length of the wire. According to Eq. (3), $j_s(x)$ is proportional to the density of z-component of the spin. Therefore, the spin-flip conductivity (4) can be represented as

$$\sigma_\omega = -\frac{4(e\alpha)^2}{\hbar\omega l}\int_{-\infty}^{t} \langle [S_z(t), S_z(t')]\rangle e^{i\omega(t-t')} dt', \qquad (5)$$

where $S_z(t)$ is the operator of the total spin projection at the moment of time t. In the absence of magnetic field $B_\perp$ the z-component of the total spin is conserved. Therefore, $[S_z(t), S_z(t')] = 0$ and the conductivity associated with the spin flip is zero. The violation of this conservation law at small $B_\perp$ in the first-order approximation of the time-dependent perturbation theory leads to

$$\delta S_z(t) = -\frac{i}{\hbar}\int_{-\infty}^{t} [V_I(t'), S_z(t')]\, dt', \qquad (6)$$

where $V_I(t) = U_0^{-1}(t)(-g\mu_B B_\perp S_x)U_0(t)$ with $U_0(t) = \exp(-iH_0 t/\hbar)$ being the evolution operator in the absence of magnetic field, and S_x is the projection of the total spin on the x-axis. It is convenient to write the Rashba Hamiltonian as a sum over electrons: $H_R = \sum_i \alpha p_i \sigma_{z,i}$. The kinetic and interaction energies commute with S_x, and therefore the perturbation operator $V_I(t)$ becomes

$$V_I(t) = -\frac{g\mu_B B_\perp}{2}\sum_i \left(\sigma_{x,i}\cos\omega_i t + \sigma_{y,i}\sin\omega_i t\right),$$

where $\omega_i = 2\alpha p_i/\hbar$. Substituting this expression into Eq. (6), we obtain

$$\delta S_z = \frac{g\mu_B B_\perp}{2\alpha}\sum_i \frac{1}{p_i}\left(\sigma_{+,i}e^{-i\omega_i t} + \sigma_{-,i}e^{i\omega_i t}\right), \qquad (7)$$

where $\sigma_\pm = \sigma_x \pm i\sigma_y$. Condition $\alpha \ll v_F$ makes it possible to replace the factor $1/p_i$ in Eq. (7) by $\pm 1/p_F$. Then the expression for δS_z becomes proportional to the sum of the operators $\sigma_{\pm,i}(t) = \sigma_{\pm,i}\exp(\mp i\omega t)$. In terms of secondary quantized operators it reads (we keep here only right movers):

$$\delta S_z(t) = \frac{g\mu_B B_\perp}{2\alpha p_F}\int \psi^\dagger_{R,\uparrow}(x,t)\psi_{R,\downarrow}(x,t)\, dx + \text{h.c.}\,. \qquad (8)$$

The unitary transformation $U = \exp(-i\sigma_z \alpha m x/\hbar)$ that puts the split Fermi points together, modifies this equation by multiplying the integrand by factor $\exp(-2i\alpha m x/\hbar)$. As a result, we find for the conductivity (4) associated with the spin flip [14],

$$\sigma_\omega = -\frac{(eg\mu_B B_\perp)^2}{\hbar\omega l p_F^2}\int_0^l dx \int_0^l dx' \int_{-\infty}^{\infty} dt' \theta(t-t') e^{i\omega(t-t')} \times \langle [\psi^\dagger_{R,\uparrow}(x,t)\psi_{R,\downarrow}(x,t), \psi^\dagger_{R,\downarrow}(x',t')\psi_{R,\uparrow}(x',t')]\rangle \times e^{-2i\alpha m(x-x')/\hbar}. \quad (9)$$

An attempt to analyze the spin-flip process in Luttinger liquid has been made in Ref. 15. The authors assumed that Rashba SOI and longitudinal magnetic field only produce the violation of $SU(2)$ invariance and affect the velocity difference δv, but do not separate the Fermi points for spin up and down electrons. The physical origin of such a model where the Fermi points are not separated (as it should be for the realistic Rashba SOI) was not specified in Ref. 15.

Since the electrons propagating in one direction with the same velocity strongly interact, the bosonic fields ϕ_c, θ_c and ϕ_s, θ_s related to the charge and spin density waves, respectively, give a physically more adequate description of phenomena. The transformation from fermions to bosons (bosonization) reads:

$$\psi_{\rho,\sigma} = U_{\rho,\sigma}\frac{e^{i\rho k_F x}}{\sqrt{2\pi a_0}} e^{-i[\rho\phi_c(x) - \theta_c(x) + \rho\sigma\phi_s(x) - \sigma\theta_s(x)]/\sqrt{2}}, \quad (10)$$

where $U_{\rho,\sigma}$ are the Klein factors which ensure the proper anticommutation relations between the fermion, and a_0 is the ultraviolet cutoff length. The secondary quantized fermionic wavefunctions ψ_σ can be represented by the linear combinations of right-moving and left-moving fermions $\psi_{\rho,\sigma}$ with the momenta being close to $\pm k_F$, i.e., $\psi_\sigma = \psi_{R,\sigma} + \psi_{L,\sigma}$. The advantage of this model is that the interaction energy becomes quadratic in the charge and spin density bosonic operators. The density of fermions becomes linear in bosonic fields $\phi_{c,s}$,

$$\rho_{c,s}(x) = -\frac{\sqrt{2}}{\pi}\partial_x \phi_{c,s}(x). \quad (11)$$

As we have mentioned, the simplest modification of the fermionic Hamiltonian produced by the Rashba SOI in the absence of magnetic field can be removed by the unitary transformation. At nonzero magnetic field, the fermionic Hamiltonian $H = H_0 + H_Z$ after this transformation takes the form [16]:

$$H = -iv_1 \int dx\, (\psi^\dagger_{R,\uparrow}\partial_x \psi_{R,\uparrow} - \psi^\dagger_{L,\downarrow}\partial_x \psi_{L,\downarrow}) - iv_2 \int dx\, (\psi^\dagger_{R,\downarrow}\partial_x \psi_{R,\downarrow} - \psi^\dagger_{L,\uparrow}\partial_x \psi_{L,\uparrow}). \quad (12)$$

A difference of velocities $\delta v = v_1 - v_2$ can arise due to the SOI effect in a wire of finite width [1, 7, 16], magnetic field, and also quadratic corrections to the electronic dispersion. After bosonization (10) Hamiltonian (12) takes the form

$$H = \int \frac{dx}{2\pi}\left[v_c K_c (\partial_x\theta_c)^2 + \frac{v_c}{K_c}(\partial_x\phi_c)^2 + v_s K_s (\partial_x\theta_s)^2 + \frac{v_s}{K_s}(\partial_x\phi_s)^2 + \delta v\,(\partial_x\phi_c\partial_x\theta_s + \partial_x\phi_s\partial_x\theta_c)\right] \quad (13)$$

where v_c (v_s) is the velocity of plasmons (spinons). We have omitted the term $\int \cos[2\sqrt{2}\phi_s(x)]dx/(2\pi)$ as being irrelevant in the renormalization group procedure for the repulsive interactions ($K_c < 1$) [16].

To find the conductivity (9) we need to calculate the retarded correlation function $I^R_{\uparrow\downarrow,\downarrow\uparrow}(x,t) = -i\theta(t)\langle[\psi^\dagger_{R,\uparrow}(x,t)\psi_{R,\downarrow}(x,t), \psi^\dagger_{R,\downarrow}(0,0)\psi_{R,\uparrow}(0,0)]\rangle$ in the ground state of the Hamiltonian (13) with fermionic operators $\psi_{\rho\sigma}$ given by Eq. (10). Since the perturbation theory is developed for time-ordered averages in the imaginary time $\tau = -it$, it is instructive to go from $I^R(x,t)$ in the Kubo formula to the time-ordered product $I^T_{\uparrow\downarrow,\downarrow\uparrow}(x,\tau) = -\langle T_\tau \psi^\dagger_{R\uparrow}(x,\tau)\psi_{R\downarrow}(x,\tau)\psi^\dagger_{R\downarrow}(0,0)\psi_{R\uparrow}(0,0)\rangle$. Applying the Wick theorem, we obtain in terms of bosonic operators:

$$I^T_{\uparrow\downarrow,\downarrow\uparrow}(x,\tau) \propto -\frac{e^{g(x,\tau)}}{(2\pi a_0)^2}, \quad g(x,\tau) = \sum_{q,\omega}[1 - e^{i(\omega\tau - qx)}]\,\langle Y(q,\omega)Y(-q,-\omega)\rangle\,, \quad (14)$$

where we introduced $e^{Y(x,\tau)}/(2\pi a_0) = \psi^\dagger_{R\uparrow}(x,\tau)\psi_{R\downarrow}(x,\tau)$ so that $Y(x,\tau) = i\sqrt{2}[\phi_s(x,\tau) - \theta_s(x,\tau)]$ and $\tau > 0$. After obtaining $I^T_{\uparrow\downarrow,\downarrow\uparrow}(x,\tau)$, it can be converted into retarded correlator using $I^R_{\uparrow\downarrow,\downarrow\uparrow}(t) = i\theta(t)[I^T_{\uparrow\downarrow,\downarrow\uparrow}(t) - (I^T_{\downarrow\uparrow,\uparrow\downarrow}(-t))^*]$ [4].

To find the correlation functions of fields ϕ_s and θ_s in Eq. (14) we use the generating functional $\mathcal{Z}[\mathbf{J}]$:

$$\mathcal{Z} = \int \mathcal{D}\phi_i \mathcal{D}\theta_i \exp\left[\int d\tau \int dx \left(-\frac{1}{2}\overline{\Phi} M \Phi + \overline{J}\Phi\right)\right], \quad (15)$$

This expression is written in a matrix form with 4-vectors of the field $\overline{\Phi} = (\phi_c, \phi_s, \theta_c, \theta_s)$ and "current" $\overline{J} = (J_1, J_2, J_3, J_4)$. The 4×4 matrix M describes the system Lagrangian and is presented below. After the standard Gaussian integration we find

$$\mathcal{Z}[\mathbf{J}] = (\det M)^{-1/2} \exp\left(\frac{1}{2}\overline{J} M^{-1} J\right). \quad (16)$$

The bosonic correlation functions from Eq. (14) are represented in terms of the elements of matrix M as

$$\langle \Phi_i(x,\tau)\Phi_j(0,0)\rangle = \left.\frac{\delta^2 \ln \mathcal{Z}}{\delta J_i(x,\tau)\delta J_j(0,0)}\right|_{\mathbf{J}=0} = \int \frac{d\omega}{2\pi}\int \frac{dq}{2\pi} e^{iqx - i\omega\tau} M^{-1}_{ij}(q,\omega). \quad (17)$$

The matrix M is symmetric and has the following nonzero elements $M_{\phi_c\phi_c} = v_c q^2/(\pi K_c)$, $M_{\phi_s\phi_s} = v_s q^2/(\pi K_s)$,

$M_{\theta_c\theta_c} = v_c K_c q^2/\pi$, $M_{\theta_s\theta_s} = v_s K_s q^2/\pi$, $M_{\phi_c\theta_c} = M_{\phi_s\theta_s} = iq\omega/\pi$, and $M_{\phi_c\theta_s} = M_{\phi_s\theta_c} = \delta v q^2/(2\pi)$. With these expressions, $g(x,t)$ in Eq. (14) takes the form

$$g(x,\tau) = 2i\iint \frac{dqd\omega}{(2\pi)^2}(1-e^{iqx-i\omega\tau})[M^{-1}_{\phi_s\phi_s}(q,\omega) + M^{-1}_{\theta_s\theta_s}(q,\omega) - M^{-1}_{\phi_s\theta_s}(q,\omega) - M^{-1}_{\theta_s\phi_s}(q,\omega)]. \quad (18)$$

At zero SOI ($\alpha = 0$) and magnetic field ($B_\perp = 0$), the system has $SU(2)$ symmetry of spin rotation. This symmetry prevents the renormalization of the interaction constant in the spin channel and therefore $K_s = 1$ [13]. A weak SOI ($\alpha \ll v_F$) and magnetic field ($B_\perp \ll \alpha p_F/\mu_B$) only slightly violate the $SU(2)$ symmetry [17], so that $K_s - 1 \sim (\alpha/v_F)^2$ [17]. Therefore, in what follows we put $K_s = 1$ up to small corrections of order α^2. Thus, with this precision up to quadratic in δv terms we find

$$M^{-1}_{\phi_s\phi_s}(q,\omega) + M^{-1}_{\theta_s\theta_s}(q,\omega) - M^{-1}_{\phi_s\theta_s}(q,\omega) - M^{-1}_{\theta_s\phi_s}(q,\omega) \simeq \frac{2\pi i}{q(\omega+iv_sq)} - (\delta v)^2\frac{\pi q}{4K_c}\frac{\left(K_c^2+1\right)v_cq + 2iK_c\omega}{(\omega+iv_sq)^2(\omega^2+v_c^2q^2)}.$$

Performing the integration over frequencies, one finds the correlator as a function of imaginary time. Because of factor $e^{-i\omega\tau}$ only the poles in the lower half-plane of the complex plane ω contribute to the integral. After the integration and analytical continuation, expression (18) turns into a sum of logarithms of the type $C\ln(x \pm v_{c,s}t)$, where C is a constant. Inserting this result in Eq. (14) we find the corresponding time-ordered fermionic correlator but in real time. It can be converted into retarded correlation function as $I^R_{\uparrow\downarrow,\downarrow\uparrow}(t) = -2\theta(t)\mathrm{Im}\, I^T_{\uparrow\downarrow,\downarrow\uparrow}(t)$ [18], and using Eq. (9) we obtain

$$\sigma_\omega = \mathcal{A}\int_{-\infty}^{\infty}dx\int_0^{\infty}e^{i(\omega t-qx)}[K(t+i\delta)-K(t-i\delta)]dt, \quad (19)$$

$$K(t) = \frac{1}{(x-v_ct)^\lambda(x+v_ct)^\mu(x-v_st)^\nu}, \quad (20)$$

where the constant

$$\mathcal{A} = \frac{(eg\mu_B B_\perp)^2\, a_0^{\lambda+\mu+\nu-2}}{2\pi^2 p_F^3\alpha}. \quad (21)$$

We recall that the wavevector q in the above integral is equal to $2\alpha m/\hbar$, cf. Eq. (9). The exact numerical factor $\mathcal{A}$ is obtained here from the comparison with the noninteracting result of Ref. 1, see supplementary material for details. The integrand in the integral over x has two singularities in the lower half-plane, at $x = v_st$ and $x = v_ct$. The expressions for the exponents λ, μ, and ν are as follows [19]

$$\lambda = (\delta v)^2\frac{(1+K_c)^2}{8K_c(v_c-v_s)^2}, \quad (22)$$

$$\mu = (\delta v)^2\frac{(K_c-1)^2}{8K_c(v_c+v_s)^2}, \quad (23)$$

$$\nu = 2-(\delta v)^2\frac{K_cv_c^2+\left(K_c^2+1\right)v_cv_s+K_cv_s^2}{2K_c(v_c^2-v_s^2)^2}. \quad (24)$$

To approximate σ_ω close to the spin resonance at frequency $\omega_{res} = v_sq = 2\alpha m v_s/\hbar$, we take $\gamma \le \mid \omega - \omega_{res} \mid \ll \omega_{res}$ with γ being the width of the resonance which we assume to be small [20]. In the limit $(v_c - v_s)\omega_{res}/(v_s\gamma) \gg 1$, we find [21]

$$\mathrm{Re}\,\sigma_\omega \simeq \mathcal{A}\frac{q^{\nu-1}}{(v_c-v_s)^\lambda(v_c+v_s)^\mu\Gamma(\nu)} \times \frac{\gamma}{[(\omega-\omega_{res})^2+\gamma^2]^{1-\frac{\lambda+\mu}{2}}}. \quad (25)$$

To evaluate σ_ω close to the other singularity, $\omega = 2\alpha m v_c/\hbar$, we use similar approximation and obtain

$$\mathrm{Re}\,\sigma_\omega \simeq \mathcal{A}\frac{2\pi\lambda q^{\lambda-1}}{(2v_c)^\mu(v_c-v_s)^\nu(2-\mu-\nu)} \times \frac{\gamma}{[(\omega-v_cq)^2+\gamma^2]^{1-\frac{\mu+\nu}{2}}}. \quad (26)$$

The plasmon singularity has a character of a weak cusp that can be detected only at large enough interaction.

Equations (25) and (26) are obtained under the assumption of well separated spinon and plasmon peaks, $(v_c - v_s)q \gg \gamma$ [22]. In the opposite case corresponding to the limit of non-interacting fermions, the peaks at $\omega = \omega_{res}$ and $\omega = v_cq$ merge. According to Eq. (19), the combined power of the merged peaks is $\lambda+\nu = 2+(\delta v)^2(1-K_c)^2/[8K_c(v_c+v_s)^2]$. In the limit of non-interacting fermions $v_c \to v_s$ and $K_c \to 1$, so that the power becomes 2 which corresponds to the Lorentzian shape of the spin resonance [23]. For small interaction g_0 between fermions, $K_c \simeq 1 - g_0/\pi$, $v_s = v_F$, and $v_c \simeq v_F(1+g_0/\pi)$, so that the power deviates from 2 by $\sim (\delta v)^2 g_0^2$. Therefore, in the framework of perturbation theory the shape of the resonance line near ω_{res} deviates slightly from Lorentzian. However, generally $g_0 = (e^2/\kappa\hbar v_F)|\ln qa|$ can be of the order of 1. For repulsive interactions $0 < K_c < 1$ and for strong fermionic interaction $K_c \to 0$. In this case the results (25) and (26) show that the shape of the absorption line may deviate significantly from Lorentzian at sufficiently strong interaction.

In conclusion, we have considered the electron spin-flip resonance caused by internal SOI field in the framework of Luttinger liquid theory. We have shown that the electron interaction incorporated does not destroy the resonance. In this theory it is treated as the excitation of a spin wave with the uniquely specified wavevector. We have found that the Luttinger liquid renormalizations almost do not change the Lorentzian shape of the resonance line at not too strong interaction. It occurs because the $SU(2)$ symmetry in the spin channel is only slightly violated by the spin-orbit interaction and weak dc magnetic field. The same small parameters ensure that the coupling of the spin flip to the charge channel and therefore the excitation of the plasma oscillations is weak at the same conditions. Nevertheless, since the Coulomb interaction in quantum wires is strong, it may be expected that the deviation from Lorentzian shape of the resonance can be

observed experimentally. In this work we considered a simplified model of the SOI whose only effect is the appearance of the difference between Fermi velocities of up and down spins. The exact consideration of the quadratic part of the dispersion is still an open problem.

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[19] They are presented here up to the quadratic order in small $\delta v/v_F$. There is also a contribution $\sim (x+v_s t)^{-\beta}$ in Eq. (19), but its exponent $\beta \sim (\delta v)^4$ is small and can be neglected to order $(\delta v)^2$.
[20] The resonance width γ is mostly due to electron-phonon interaction and can be estimated to be $\sim 10^9 - 10^{10}$ s^{-1} [1]. For realistic quantum wires $\gamma \ll \omega_{res} \sim 10^{12}$ s^{-1}.
[21] The details of this calculation are presented in the supplementary material.
[22] This result corresponds to the case of strong interaction g_0, i.e. $\delta v/v_F < g_0$. In the intermediate regime of weak interaction, $\delta v/v_F \gtrsim g_0$, there is also a strong mixing of charge and spin channels, however the entire Luttinger description becomes not suitable since it does not take into account the curvature effects which are important in this case.
[23] See the case of noninteracting electrons in the supplementary material.

Supplementary material for "Spin resonance in Luttinger liquid with spin-orbit interaction"

Relation between $I^R_{\uparrow\downarrow,\downarrow\uparrow}(t)$ and $I^T_{\uparrow\downarrow,\downarrow\uparrow}(t)$

The perturbation theory is valid for time-ordered averages in imaginary time, whereas what we need to calculate is a retarded average $I^R_{\uparrow\downarrow,\downarrow\uparrow}(t)$. Therefore, we need a relationship between $I^R_{BA}(x,t) = -i\theta(t)\langle[B(x,t),A(0,0)]\rangle$ and $I^T_{BA}(x,\tau) = -\langle T_\tau B(x,\tau)A(0,0)\rangle$ for imaginary time τ where $B(x,t) = \psi^\dagger_{R,\uparrow}(x,t)\psi_{R,\downarrow}(x,t)$ and $A(0,0) = \psi^\dagger_{R,\downarrow}(0,0)\psi_{R,\uparrow}(0,0)$ are boson-like operators. These two types of averages are related by equality [4]:

$$I^R_{BA}(t) = i\theta(t)\left[I^T_{BA}(t) - \left(I^T_{A^\dagger B^\dagger}(-t)\right)^*\right], \qquad (27)$$

which follows from

$$I^R_{BA}(t) = -i\theta(t)\left[\langle B(t)A(0)\rangle - \langle A(0)B(t)\rangle\right], \qquad (28)$$
$$I^T_{BA}(t) = -\left[\theta(t)\langle B(t)A(0)\rangle + \theta(-t)\langle A(0)B(t)\rangle\right]. \qquad (29)$$

For positive time $t > 0$,

$$I^T_{BA} = -\langle B(t)A(0)\rangle, \qquad (30)$$
$$-\left(I^T_{A^\dagger B^\dagger}(-t)\right)^* = \left\langle B^\dagger(0)A^\dagger(-t)\right\rangle^*, \qquad (31)$$

and $\left\langle B^\dagger(0)A^\dagger(-t)\right\rangle^* = \langle A(-t)B(0)\rangle = \langle A(0)B(t)\rangle = \left\langle B^\dagger(t)A^\dagger(0)\right\rangle^*$. In our case, due to the above definitions of A and B, $\left\langle B^\dagger(t)A^\dagger(0)\right\rangle$ differs from $\langle B(t)A(0)\rangle$ by changing the spin components $\sigma \to -\sigma$. It is equivalent to $Y(x,t) \to -Y(x,t)$ since we introduced $e^{Y(x,\tau)}/(2\pi a_0) = \psi^\dagger_{R\uparrow}(x,\tau)\psi_{R\downarrow}(x,\tau)$. However, Y enters in all correlation functions quadratically, see Eq. (14) in the main part, and we conclude that this transformation does not change the correlator. Then $I^R_{BA}(t) = i\theta(t)[I^T_{BA}(t) - \left(I^T_{BA}(t)\right)^*]$, and we find

$$I^R_{BA}(t) = -2\theta(t)\mathrm{Im}\, I^T_{BA}(t). \qquad (32)$$

Evaluation of conductivity in Eqs. (25) and (26)

To find the absorption power of electromagnetic field we need to calculate $\mathrm{Re}\,(\sigma_\omega)$, where the conductivity is given by Eq. (19) in the main text.

The integral in Eq. (19) is

$$\sigma_\omega = \mathcal{A}\int_{-\infty}^{\infty}dx\int_0^\infty e^{i(\omega t - qx)}[K(t+i\delta) - K(t-i\delta)]dt, \qquad (33)$$

$$K(t) = \frac{1}{(x-v_c t)^\lambda (x+v_c t)^\mu (x-v_s t)^\nu}, \qquad (34)$$

where

$$\mathcal{A} = \frac{(eg\mu_B B_\perp)^2 a_0^{\lambda+\mu+\nu-2}}{\pi^2 p_F^3 \alpha}.$$

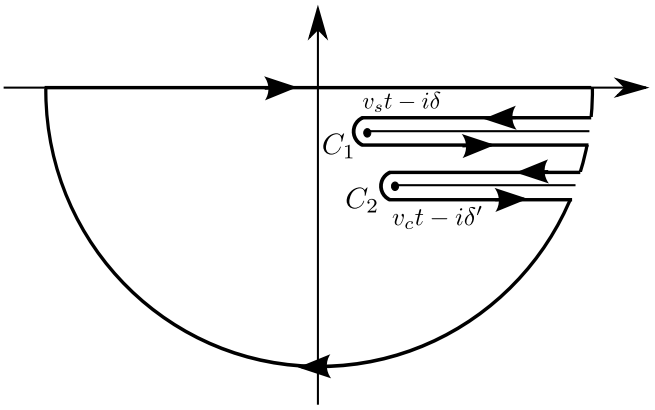

FIG. 2. The integration contour in the lower half-plane of the complex variable x.

Since q is positive, the exponent e^{-iqx} vanishes at large x in the lower half-plane of the complex variable x. Therefore, the integral over real axis x is equal to the sum of two contour integrals in the lower half-plane of x along contours winding around two branch cuts shown in Fig. 2. The contour C_1 winds around the branch cut from the point $x = v_s t - i\delta$ to $x = +\infty - i\delta$, and the contour C_2 winds around the branch cut from $x = v_c t - i\delta'$ to $x = +\infty - i\delta'$. (We ignore here the potential singularity at $x = -v_c t$ which is associated with the inverse processes of spin flip from up to down on the right branch. These processes should be suppressed in the approximation of small excited-state occupation numbers which we employ.) We can estimate the integrals over x around each branch cut separately. The conductivity σ_ω has two singularities: at $\omega = v_s q$ and at $\omega = v_c q$. As we show below, close to the singularity near $\omega = v_s q$ the main contribution to the integral comes from contour C_1, and the other (plasmon) singularity is dominated by the integral over C_2.

First, we estimate the integral I_1 over the contour C_1. After the change of variable $u = x - v_s t$ the contour that maps C_1 in a complex plane u winds around the branch cut from $u = 0$ to $u = +\infty$ and will be denoted by the same symbol C_1. Thus, the integral I_1 can be written as follows:

$$I_1 = \int_{C_1} du \frac{e^{-iqu - iv_s qt}}{u^\nu \left[u + (v_s - v_c)\,t\right]^\lambda \left[u + (v_s + v_c)\,t\right]^\mu}.$$

We aim to approximate σ_ω very close to the resonance at $\omega_{res} = v_s q$. The closeness is determined by inequalities relating to the detuning $\delta\omega = |\omega - \omega_{res}|$:

$$\gamma \ll \delta\omega \ll \omega_{res},$$

where γ is the attenuation rate mainly due to Cherenkov emission of phonons [1]. Then the time during which the resonance absorption is accumulated is large enough, $t \sim 1/\delta\omega$. In this limit, $(v_c - v_s)qt \gg 1$, we approximate the above integral as

$$I_1 = \frac{e^{-iv_s qt}(-iq)^{\nu-1}}{\left[(v_s - v_c)\,t\right]^\lambda \left[(v_s + v_c)\,t\right]^\mu}\int_C z^{-\nu}e^z dz, \qquad (35)$$

where we introduced new variable $z = -iqu$. As a result of this change of variables, the contour C_1 turns into contour C winding around a branch cut going from $z = 0$ to $z = -i\infty$. The contour C can be rotated clockwise together with the branch cut until the latter coincides with the left half of the real axis $z < 0$. Then the contour integral turns into the Hankel's representation of the inverse Gamma function and gives $2\pi i/\Gamma(\nu)$. After that we can integrate over time with the following final result:

$$I_{\omega,q} \simeq \mathcal{A}\frac{-2\pi i\Gamma(1-\lambda-\mu)q^{\nu-1}i^{-\nu-\lambda-\mu}}{(v_s-v_c)^{\lambda}(v_s+v_c)^{\mu}\Gamma(\nu)(\omega-v_sq+i\gamma)^{1-\lambda-\mu}}. \tag{36}$$

Then the real part of conductivity near $\omega = v_sq$ becomes

$$\mathrm{Re}\,\sigma_\omega \simeq \mathcal{A}\frac{\Gamma(1-\lambda-\mu)q^{\nu-1}}{(v_c-v_s)^{\lambda}(v_c+v_s)^{\mu}\Gamma(\nu)} \times\frac{\gamma}{[(\omega-\omega_{res})^2+\gamma^2]^{1-\frac{\lambda+\mu}{2}}}, \tag{37}$$

which for small λ and μ is approximated by Eq. (25). Here we used that $v_c \geq v_s$. The integral I_2 over the contour C_2 does not contribute to the singularity at $\omega = v_sq$ and therefore can be neglected.

In addition, there is also a small part of $\mathrm{Re}\,\sigma_\omega$ which is independent of γ:

$$\frac{\pi^2\mathcal{A}\Gamma(1-\lambda-\mu)q^{\nu-1}(\nu+\lambda+\mu-2)}{(v_c-v_s)^{\lambda}(v_c+v_s)^{\mu}\Gamma(\nu)|\omega-\omega_{res}|^{1-\lambda-\mu}}. \tag{38}$$

It is small in positive parameter $\nu+\lambda+\mu-2 = (\delta v)^2(K_c-1)^2/[4K_c(v_c+v_s)^2]$.

Nevertheless, I_2 contributes to a singularity at plasma frequency $\omega = v_cq$. Next we analyze this singularity. We consider the integral I_2 over contour C_2 similarly to what we did for I_1. We change variable in I_2 to $u = x - v_ct$. The mapped contour C_2 winds around the branch cut from $u = 0$ to $u = +\infty$. As a result of winding around the branch cut we obtain factor $\left(1-e^{-2\pi i(\lambda-1)}\right) \approx 2\pi i\lambda$ and the integral over u from 0 to infinity:

$$I_2 = 2\pi i\lambda\int_0^{\infty} du\frac{e^{-iq(u+v_ct)}}{u^{\lambda}(u+2v_ct)^{\mu}[u+(v_c-v_s)t]^{\nu}}. \tag{39}$$

At small detuning $\delta\omega = |\omega - v_cq|$ from the plasma resonance we expect that similarly to what we observed for I_1 the accumulation time for the resonance absorption is large, $t \sim 1/\delta\omega \gg 1/(v_cq)$, and therefore in the factors $u+2v_ct$, $u+(v_c-v_s)t$ it is possible to neglect $u \sim 1/q$. After this procedure the resulting integral over t diverges at $t = 0$. This divergence however is spurious. It has happened because at small $t < 1/[(v_c-v_s)q]$, the variable u cannot be neglected. It means that the integration over t is effectively cut off at $t_0 \sim 1/[(v_c-v_s)q]$. To estimate the singular part on the background of nonsingular contribution originated from small t, we represent the exponent $e^{i(\omega-v_cq)t}$ as a sum, $e^{i(\omega-v_cq)t} = [e^{i(\omega-v_cq)t}-1]+1$, and divide the integral over time into two parts:

$$\int_{t_0}^{\infty}\frac{\left[e^{i(\omega-v_cq)t}-1\right]dt}{t^{\mu+\nu}} + \int_{t_0}^{\infty}\frac{dt}{t^{\mu+\nu}}.$$

The second integral is approximately equal to $t_0^{1-\mu-\nu}/(\nu+\mu-1)$ and has no singularity. The first integral converges and can be extended to $t = 0$ if $\mu+\nu < 2$. This condition is satisfied in a broad range of not too strong interaction as it can be readily checked from Eqs. (22) – (24). The first integral after the change of variable $\tau = (\omega-v_cq)t$ turns into

$$(\omega-v_cq)^{\mu+\nu-1}\int_0^{\infty}\frac{\left(e^{i\tau}-1\right)d\tau}{\tau^{\mu+\nu}}. \tag{40}$$

The integral in Eq. (40) is a large number $\approx i(2-\mu-\nu)^{-1}$ proportional to $[\delta v/(v_c-v_s)]^{-2}$. The ratio of the first term to the second has the order of magnitude $[\delta v/(v_c-v_s)]^{-2}[|\omega-v_cq|/(v_cq)]^{\mu+\nu-1}$. Thus, the nonresonant contribution is comparable with the resonant one only in a narrow region close to the resonance $\delta\omega \leq v_cq[\delta v/(v_c-v_s)]^2$. Combining all the results, we arrive at the expression for the singularity due to spin-flip processes at the plasmon frequency:

$$I_{\omega,q} \simeq -\mathcal{A}i^{\lambda-1}q^{\lambda-1}\frac{2\pi\lambda\Gamma(1-\lambda)}{2-\mu-\nu}\frac{(\omega-v_cq+i\gamma)^{\mu+\nu-1}}{(2v_c)^{\mu}(v_c-v_s)^{\nu}}. \tag{41}$$

The calculation of the real part gives the following result:

$$\mathrm{Re}\,I_{\omega,q} \simeq \mathcal{A}\frac{2\pi\lambda\Gamma(1-\lambda)q^{\lambda-1}}{(2v_c)^{\mu}(v_c-v_s)^{\nu}(2-\mu-\nu)} \times\frac{\gamma}{[(\omega-v_cq)^2+\gamma^2]^{1-\frac{\mu+\nu}{2}}}, \tag{42}$$

c.f. (26) in the main text for $\mathrm{Re}\,\sigma_\omega$. The plasmon singularity has a character of a weak cusp that can be detected only at large enough interaction.

Case of noninteracting electrons

The rate w of the spin flips per one electron found in the work [1] reads:

$$w = \left(\frac{2e\alpha}{\hbar\omega_{res}}\right)^2\left(\frac{g\mu_BB}{2\alpha p_F}\right)^2 I_\omega, \tag{43}$$

where I_ω is the spectral density of the driving electromagnetic field. It is determined by

$$\overline{\mathbf{E}^*(t)\mathbf{E}(t')} = \int_{-\infty}^{\infty} I_\omega e^{i\omega(t-t')}\frac{d\omega}{2\pi}. \tag{44}$$

For the monochromatic field with the frequency ω_0 the spectral density is $I_\omega = 4\pi\delta(\omega-\omega_0)|\mathbf{E}_\omega|^2$, where $\mathbf{E}_\omega$ is the complex amplitude of the field. The energy absorption per particle per unit time is equal to $w\hbar\omega$. The Luttinger model neglects

the quadratic part of energy dispersion. From this point of view any particle in the single-occupied range has the same energy of the spin flip or equivalently the same spin-resonance frequency. To calculate the energy losses per unit wire length and time it is necessary to multiply the energy rate per particle by the density of electrons in the single-occupied ranges of the momentum. The latter is equal to $n_{sf} = \frac{2\alpha}{v_F} n$, where $n = 2p_F/(\pi\hbar)$ is the total one-dimensional density of electrons. As a result we find the absorption power per unit wire length:

$$P = w\hbar\omega n_{sf} = \frac{8e^2 \left(g\mu_B B\right)^2 |\mathbf{E}_\omega|^2}{\hbar p_F^2 v_F} \delta(\omega - \omega_{res}). \quad (45)$$

The real part of conductivity, $\mathrm{Re}\,\sigma_\omega$ for noninteracting electrons is equal to the power, Eq. (45), divided by $|\mathbf{E}_\omega|^2$:

$$\sigma_\omega = \frac{8e^2 \left(g\mu_B B\right)^2}{\hbar p_F^2 v_F} \delta(\omega - \omega_{res}). \quad (46)$$

The exact numerical factor for the conductivity of interacting electrons in Luttinger model is extracted by the matching it with the conductivity of noninteracting electrons (46).

In the case of noninteracting electrons $v_s = v_c$, $\mu = 0$, and $\lambda + \nu = 2$. Substituting it into Eqs. (19) – (20) of the main text we find $\mathcal{A} \to \mathcal{A}|_{\lambda+\mu+\nu=2}$ and Eq. (20) simplifies to

$$K(t) = \frac{1}{\left(x - v_s t\right)^2}. \quad (47)$$

Since q is positive, the exponent e^{-iqx} in Eq. (19) of the main text vanishes at large x in the lower half-plane of the complex variable x. Therefore, the integral over real axis x is equal to the integral in the lower half-plane of x. Therefore, only $K(t - i\delta)$ has nonzero contribution, and using the residue theorem,

$$\sigma_\omega = \mathcal{A} \int_0^\infty dt e^{i\omega(t-i\delta)} \times (-2\pi i) \mathrm{Res}_{x=v_s(t-i\delta)} \left[\frac{e^{-iqx}}{\left(x - v_s(t - i\delta)\right)^2} \right].$$

At a finite attenuation γ, the conductivity becomes

$$\sigma_\omega = \mathcal{A} \frac{-2\pi i q}{\omega - v_s q + i\gamma}. \quad (48)$$

Taking the real part we find in the limit $\gamma \to 0$,

$$\mathrm{Re}\,\sigma_\omega = 2\pi^2 \mathcal{A} q \delta\left(\omega - v_s q\right). \quad (49)$$

Thus, in the noninteracting limit the conductivity of Luttinger liquid coincides with the conductivity of free electrons provided a correct choice of the factor $\mathcal{A}$. Note that the cut-off length a_0 disappears from the factor $\mathcal{A}$ in this limit. At a finite interaction it enters in a small power $\propto (\delta v)^2$. Therefore it can be defined only by the order of magnitude.