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Soliton-pair Propagation under Thermal Bath Effect

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Abstract. We consider two atomic transitions excited by two variable laser fields in a three-level system. We study the soliton-pair propagation out of resonance and under thermal bath effect. We present general analytical implicit expression of the soliton-pair shape. Furthermore, we show that when the coupling to the environment exceeds a critical value, the soliton-pair propagation through three-level atomic system will be prohibited.

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1. Introduction

Recently the interaction of atoms or similar systems with electromagnetic fields has raised a lot of interest [6, 7, 11, 13–18, 22, 25, 26, 32]. It leads to interesting quantum features such as, entanglement [33], anti-bunching [19], squeezing [30], bistability [3] and optical soliton propagation [20, 21]. Solitons arise as the solutions of a widespread class of weakly nonlinear dispersive partial differential equations describing physical systems. They are an essential nonlinear kind of wave-like excitation, caused by a cancellation of nonlinear and dispersive effects in the medium and they have particle-like properties. Solitons have important applications in many branches of physics, from high energy and condensed matter physics to astrophysics and cosmology as well as in biology and telecommunication [4, 9, 10, 27, 29, 34]. Early work, have been investigated [8, 20, 31] to computed the properties of solitons.

In the Λ configuration, a pair of optical pulses propagate without absorption. This medium can be made experimentally [5] if two lasers are applied to a three-level system, the atoms will be driven to a population trapped state, and a medium that is opaque to a probe laser can, by applying both lasers simultaneously, be made transparent [2, 23]. In previous work [8, 20, 21], we derived analytical solutions of solitons and pair of solitons in dissipative media.

In this paper we investigate the Soliton-pair propagation in the three-level dissipative media out of resonance, under thermal bath effect.

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2. Model

Let us consider a three-level system in lambda configuration (three-level atom) interacting with two non resonant electromagnetic fields. The medium is excited by two laser fields one applied on the stokes transition and the second on the pump transition.

This system is described with three energy levels $|0\rangle$, $|1\rangle$ and $|2\rangle$. The transitions $|0\rangle \leftrightarrow |1\rangle$ and $|0\rangle \leftrightarrow |2\rangle$ are possible whereas the levels $|1\rangle$ and $|2\rangle$ are supposed to be decoupled (transition $|1\rangle \leftrightarrow |2\rangle$ negligible). The reason of the choice of this model is the fact that a free atom has at least two states at same parity between which an electrical dipole transition is not allowed. The restriction to two lower energy level is valid if the frequency of the interacting waves are distant enough to all other frequencies. In this model we take into account the rates $\gamma_{1,2}$ of radiative decay from the higher level $|0\rangle$ to the levels $|1\rangle$ and $|2\rangle$ and neglecting the other dissipation effects.

This three-level system is irradiated by a light beam propagating along an arbitrary direction x, with polarization adequate to couple the two optical transitions, and containing two monochromatic fields. This light beam is classically described as follows:

$$E^{'}(x,t) = E_1(x,t) + E_2(x,t) = \bar{E}_1(x,t)\exp(-i\omega_1 t) + \bar{E}_2(x,t)\exp(-i\omega_2 t). \tag{2.1}$$

$\bar{E}_1$ and $\bar{E}_2$ are the amplitudes of the two waves. $\bar{E}_1$ and $\bar{E}_2$ are assumed to be slowly varying functions in the sense [12], [28], [1]: $\frac{1}{\omega_2}\left|\frac{\partial \bar{E}_2}{\partial t}\right| << \left|\bar{E}_2\right|$ and $\frac{1}{\omega_1}\left|\frac{\partial \bar{E}_1}{\partial t}\right| << \left|\bar{E}_1\right|$.

The Hamiltonian describing the interaction of the three level atom with the field has the expression:

$$H = \sum_{i=0}^{2}\varepsilon_i a_i^+ a_i + g_1\left(a_0^+ a_1 E_1 + a_1^+ a_0 E_1^*\right) + g_2\left(a_0^+ a_2 E_2 + a_2^+ a_0 E_2^*\right). \tag{2.2}$$

The first term of the Hamiltonian corresponds to the proper energies of the atom, the second and third terms of the Hamiltonian describe the interaction between the two fields and the atom: a_i, a_i^+ are respectively the annihilation and creation fermions operators of the atomic level i and ε_i represent the energy of the levels i. a_i, a_i^+ verify the anticommutation relation $\left[a_i, a_j^+\right]_+ = \delta_{ij}$. The two dipole transition matrix elements which are assumed to be real are denoted by g_1 and g_2.

To study the evolution of interaction between the atom and fields we use the density matrix formalism. The density matrix equation of motion is

$$\frac{d}{dt}\rho = \frac{1}{i\hbar}\left[H,\rho\right] + \frac{d}{dt}\rho_{irr}, \tag{2.3}$$

where $\frac{d}{dt}\rho_{irr}$ describes the dissipation in the total system and the coupling to the thermal reservoir.

$$\begin{aligned}\frac{d}{dt}\rho_{irr} = {} & \frac{\gamma_1}{2}(1+n_{th})(\left[a_1^+ a_0, a_0^+ a_1\rho\right] + \left[a_1^+ a_0\rho, a_0^+ a_1\right]) \\ & +\frac{\gamma_1}{2}n_{th}(\left[a_0^+ a_1, \rho a_1^+ a_0\right] + \left[a_0^+ a_1\rho, a_1^+ a_0\right]) \\ & \frac{\gamma_2}{2}(1+n_{th})(\left[a_2^+ a_0, a_0^+ a_2\rho\right] + \left[a_2^+ a_0\rho, a_0^+ a_2\right]) \\ & +\frac{\gamma_2}{2}n_{th}(\left[a_0^+ a_2, \rho a_2^+ a_0\right] + \left[a_0^+ a_2\rho, a_2^+ a_0\right]).\end{aligned} \tag{2.4}$$

So, we obtain the motion equation of the density matrix: where the elements ρ_{ij} are defined as $\rho = \sum\limits_{i,j=0}^{2} |i\rangle\,\rho_{ij}\,\langle j|$, ω_{10} and ω_{20} represent the two atomic transition frequencies $\omega_{10} = \frac{\varepsilon_0-\varepsilon_1}{\hbar}$ and $\omega_{20} = \frac{\varepsilon_0-\varepsilon_2}{\hbar}$ and $d_i = \frac{g_i}{\hbar}$ are the coupling constants. The diagonal elements of the density matrix ρ describe the level populations and determine the internal energy of the atom. The off-diagonal elements describe the atomic coherences. The ρ_{10} and ρ_{20} terms oscillate at the respective driving field frequency and the ρ_{21} oscillate with frequency differences of the two light fields. So, we can define the slowly varying amplitudes of the off-diagonal density matrix elements $\bar{\rho_{10}}$, $\bar{\rho_{20}}$ and $\bar{\rho_{21}}$ through the relations:

$$\rho_{j0} = \bar{\rho_{j0}} \exp(i\omega_{j0}t) \quad for\ j = 1, 2, \tag{2.5}$$
$$\rho_{21} = \bar{\rho_{21}} \exp(i\,(\omega_{20} - \omega_{10})\,t). \tag{2.6}$$

We decompose the off-diagonal elements into an imaginary part and a real part :

$$\bar{\rho_{j0}} = \chi_{j0} + i\psi_{j0}, \tag{2.7}$$
$$\bar{\rho_{21}} = \chi_{21} + i\psi_{21}. \tag{2.8}$$

The Hermitian propriety of the density matrix ensures that the diagonal elements $\rho_{11,}\rho_{22}$ and ρ_{00} must be real. δ_1 and δ_1 are the detunings between the laser frequencies and the atomic transitions frequencies: $\delta_1 = \omega_{10} - \omega_1$ and $\delta_2 = \omega_{20} - \omega_2$. The signal field E_2 and E_1 are described by the Maxwell equations for a slowly varying approximation (SVA) [2, 12]:

$$\frac{\partial \bar{E}_j}{\partial t} + c\frac{\partial \bar{E}_j}{\partial x} = ig_j^{'}\bar{\rho}_{j0}. \tag{2.9}$$

We assume that the propagation constants of the fields are given by $g_j' = \frac{2\pi}{\varepsilon_0} N g_j(\omega_j + \delta_j)$ where $j = 1, 2$, ε_0 is the vacuum electric constant, N the atomic dipole density and c is the light velocity. The condition for soliton-pair propagation is expressed as $\bar{E}_j\,(x, t) = \bar{E}_j\,(x - v_g t)$. We consider here two fields $\bar{E}_1$and $\bar{E}_2$ and we assume that they are real. The fact that $\bar{E}_2$ is real gives us $\chi_{20} = 0$. Then we introduce a moving coordinate which propagates with the pulses' velocities $z = x - v_g t$ which gives us $\frac{\partial}{\partial t} = -v_g\frac{\partial}{\partial z}$ and $\frac{\partial}{\partial x} = \frac{\partial}{\partial z}$ where v_g can be identified with the group velocity of the soliton-pair. The two spontaneous emission rates γ_1 and γ_2 are assumed to be approximately equal to γ. Finally, the complete set of the evolution equations for medium-fields interaction (Maxwell-Bloch equations) in the case of two photon resonance δ_1=δ_2 can be obtained from Maxwell equation and the system of evolution equations for the density matrix:

$$\begin{aligned}
\frac{d}{dz}\chi_{10} &= \frac{\delta}{v_g}\psi_{10} - \alpha_2\psi_{21} + \mathit{\Gamma}(1 + 3n/2)\chi_{10},\\
\frac{d}{dz}\psi_{10} &= \frac{\delta}{v_g}\chi_{10} - 2\alpha_1\chi_{11} - \alpha_2\chi_{21} + \mathit{\Gamma}(1 + 3n/2)\psi_{10},\\
\frac{d}{dz}\psi_{20} &= \alpha_2(-1 - \chi_{11}) - \alpha_1\chi_{21} + \mathit{\Gamma}(1 + 3n/2)\psi_{20},\\
\frac{d}{dz}\chi_{21} &= \alpha_1\psi_{20} + \alpha_2\psi_{10} + \mathit{\Gamma} n\psi_{21},\\
\frac{d}{dz}\psi_{21} &= \alpha_2\chi_{10} + \mathit{\Gamma} n\psi_{21},\\
\frac{d}{dz}\chi_{11} &= 2\,\alpha_1\psi_{10} + \mathit{\Gamma}\chi_{11}(1 + 2n),\\
\frac{d}{dz}\alpha_j &= -\frac{d_j g_j'}{v_g(c - v_g)}\psi_{j0} = -k_j\psi_{j0} \qquad j = 1, 2,\\
0 &= \frac{\delta}{v_g}\psi_{20} + \alpha_1\psi_{21}.
\end{aligned} \tag{2.10}$$

Where α_j are variables related to the field amplitudes by the following expressions $\alpha_j = \frac{d_j \tilde{E}_j}{v_g}$. $\mathit{\Gamma} = \frac{\gamma}{v_g}$ represents a new constant .

3. Soliton-pair shapes

Our interest is in studying the evolution of the fields α_2 and α_1 and we deal with the case of similar shape soliton-pair so we can write $\alpha_1 = A\alpha_2 = A\alpha$ (A is a real constant > 1). The fields 1 and 2 have a slowly varying amplitudes, in this case, we can neglect the variation of the curvature and we can assume that the third and the forth order of derivation are negligible. After algebraic manipulations and differentiation of the Maxwell-Bloch equations, we obtain a non-linear differential equation:

$$(-A^2E - F)\alpha^4 + M\alpha^2 = -B(1+\alpha)\,\frac{d\alpha}{dz}. \tag{3.1}$$

E, F, B and M are constants depending of the system parameters

$$\begin{aligned} E &= \frac{\delta}{v_g K_1}, \\ B &= \frac{\delta}{v_g K_2}(1 - \mathit{\Gamma} n), \\ M &= \mathit{\Gamma}(1 + 3n/2)(1 - \mathit{\Gamma} n)\frac{\delta}{v_g K_2}, \\ F &= \frac{\delta}{v_g K_2}. \end{aligned} \tag{3.2}$$

by integrating the above equation, the soliton α verifies the following implicit equation

$$\eta e^{-2\frac{Mz}{b}} = e^{\frac{-1}{\alpha_2(z)}}\alpha_2(z)\left(\frac{1+\beta\alpha_2(z)}{1-\beta\alpha_2(z)}\right)^{1/2\beta}(-1+\beta^2\alpha_2^2(z))^{-1/2}\ , \tag{3.3}$$

where $\beta = \sqrt{\frac{A^2E+F}{M}} = \sqrt{\frac{A^2-1}{\mathit{\Gamma}(1+n3/2)(1-n\mathit{\Gamma})}}$ and η is a free constant that can be determined from the initial condition as following

$$\eta = \alpha_2(z_0)e^{2\frac{Mz_0}{b} - \frac{1}{\alpha_2(0)}}\left(\frac{1+\beta\alpha_2(z_0)}{1-\beta\alpha_2(z_0)}\right)^{1/2\beta}(-1+\beta^2\alpha_2^2(z_0))^{-1/2}. \tag{3.4}$$

This gives us a condition of existence of the signal field

$$\frac{-1}{\beta} < \alpha < \frac{1}{\beta} \text{ and } \alpha \neq 0. \tag{3.5}$$

In other words if the initial amplitude value of the signal α is out of the range $[\frac{-1}{\beta}, \frac{1}{\beta}]$ no more soliton-pair propagation is possible. Moreover the thermal bath coefficient n should be less than a critical value $n_c = 1/\mathit{\Gamma}$, otherwise if the coupling to the environment exceeds the critical value n_c then no more soliton-pair can be propagate. In Figure 1 we plot the soliton shape $\alpha(z)$ for $n = 1.5$, $\mathit{\Gamma} = 0.5$, $E = 1$ and $F = 1$ and for the initial condition $\alpha(0) = 0.05$.

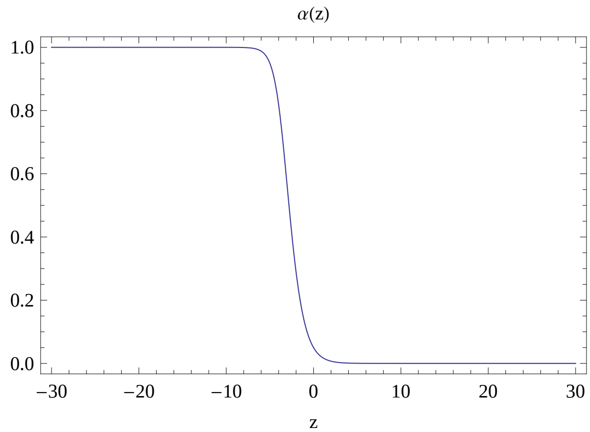

FIGURE 1. soliton shape for in $n = 1.5$, $\Gamma = 0.5$, $E = 1$, $F = 1$ and for the initial condition $\alpha(0) = 0.05$.

4. Conclusion

In summary, we have investigated a theoretical model describing a pair of solitons propagating through an absorbing three-level atoms, interacting with the environment through the thermal bath effect, out of resonance. We have derived an analytical implicit expression for the shape of the soliton-pair. We have shown that up a critical value of the thermal bath, the soliton-pair is not allowed to propagate through the three-level atomic system.

These results are useful in optical data communication, where the optical fibre can be modelled as an absorbing three level system [5]. The advantage of soliton in supporting data information is the invariance of their shape which minimizes the noise effect, this is usually the origin of the signal defects. Besides, solitons propagate without dispersion. Therefore, we can send the optical information with high bit rate.

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