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On W_3 strings

C.N. Pope,[1)] L.J. Romans[2)] and K.S. Stelle[3)]

CERN—Geneva

ABSTRACT

We construct a non-chiral anomaly-free theory of W_3 gravity and investigate its spacetime interpretation as a theory of critical W_3 strings.

1) Permanent address: Center for Theoretical Physics, Texas A&M University, College Station, TX 77843–4242, USA. Supported in part by the U.S. Department of Energy, under grant DE-FG05-91ER40633.

2) Present address: Physics Department, University of Southern California, Los Angeles CA 90089–0484, USA. Supported in part by the U.S. Department of Energy, under grant DE-FG03-84ER40168.

3) Permanent address: The Blackett Laboratory, Imperial College, London SW7 2BZ, England.

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The existence of an anomaly-free chiral theory of W_3 gravity in $d = 2$ dimensions [1] opens up a vista of possible generalisations of string theory based upon W symmetry. A first requirement for such a generalisation, since the spacetime coordinates of a string theory necessarily involve both left and right-moving modes, is the creation of a non-chiral anomaly-free theory of W_3 gravity. In this paper, we begin by extending our previous results to the non-chiral case. We then consider the classical dynamics of the W_3 string theory and show how the extra gauge symmetry effectively eliminates one specific coordinate. This "frozen" coordinate is the only one whose appearance in the W_3 current occurs other than through its stress tensor, and is thus the one characteristically "non-stringy" coordinate. This coordinate-freezing feature is obtained regardless of whether the extra dimension is spacelike or timelike, so that it is possible to have a positive-energy classical theory of W_3 strings even in a theory with two "times." Preliminary calculations in the quantised theory suggest that this "time without time" phenomenon should persist also at the quantum level.

The key ideas that allowed the construction of an anomaly-free theory of W_3 gravity in [1] are:

1) That the construction of a nilpotent BRST operator Q [2] leads directly to the formulation of the action of an anomaly-free quantum theory;
2) That the nilpotent BRST operator requires a quantum realisation $T_{\rm mat}, W_{\rm mat}$ of the W_3 algebra on the matter sector of the theory with central charge $c = 100$ and;
3) That such matter realisations exist with arbitrary numbers $n \ge 2$ of scalar fields [3].

These $c = 100$ matter realisations all involve background charges in $T_{\rm mat}, W_{\rm mat}$ for some of the scalar fields. The background charge terms may also be viewed as finite counterterms necessary to eliminate matter-dependent anomalies in the W_3 current algebra. A similar phenomenon was found in the renormalisation of classical w_∞ gravity into quantum W_∞ gravity [4]. The necessity of including background charges to cancel anomalies has also been discussed in refs. [5,6].

In order to build a non-chiral anomaly-free theory of W_3 gravity, we start from the formulation of classical non-chiral W_3 gravity with auxiliary fields [7], but in a version [8] in which the spin-2 and spin-3 gauge fields are treated on a similar footing. This will enable us to treat the non-chiral case essentially as the direct sum of the left and the right sectors. The classical non-chiral Lagrangian is [8]

$$\begin{aligned}\mathcal{L} = &-\tfrac12 \bar\partial\varphi^i\partial\varphi^i - J^i\widetilde{J}^i + \widetilde{J}^i\partial\varphi^i + J^i\bar\partial\varphi^i\\ &-\tfrac12 hJ^iJ^i - \tfrac13 Bd_{ijk}J^iJ^jJ^k - \tfrac12\tilde h\widetilde{J}^i\widetilde{J}^i - \tfrac13\widetilde{B}d_{ijk}\widetilde{J}^i\widetilde{J}^j\widetilde{J}^k.\end{aligned} \tag{1}$$

The equations of motion for the auxiliary fields are

$$\begin{aligned}J^i &= \partial\varphi^i - \tilde h\widetilde{J}^i - \widetilde{B}d_{ijk}\widetilde{J}^j\widetilde{J}^k,\\ \widetilde{J}^i &= \bar\partial\varphi^i - hJ^i - Bd_{ijk}J^jJ^k,\end{aligned} \tag{2}$$

which can be recursively solved to give J^i and $\widetilde{J}^i$ as non-polynomial expressions in φ^i and the gauge fields.

In momentum space, the kinetic terms for $(\varphi^i\ J^i,\ \widetilde{J}^i)$ become

$$\tfrac{1}{2}(\varphi^i,\quad J^i,\quad \widetilde{J}^i)\begin{pmatrix} p\bar{p} & \mathrm{i}\bar{p} & \mathrm{i}p \\ \mathrm{i}\bar{p} & 0 & -1 \\ \mathrm{i}p & -1 & 0 \end{pmatrix}\begin{pmatrix} \varphi^i \\ J^i \\ \widetilde{J}^i \end{pmatrix}, \tag{3}$$

and upon inversion we find the $(\varphi^i\ J^i,\ \widetilde{J}^i)$ propagator,

$$\frac{2}{p\bar{p}}\begin{pmatrix} -1 & -\mathrm{i}p & -\mathrm{i}\bar{p} \\ -\mathrm{i}p & p^2 & 0 \\ -\mathrm{i}\bar{p} & 0 & \bar{p}^2 \end{pmatrix}. \tag{4}$$

From (1) one can see that the only fields that can occur inside loop diagrams are the auxiliary fields J^i, $\widetilde{J}^i$ since these are the only fields that not only have propagators but also enter into the vertices. For the internal lines of Feynman diagrams, only the lower right 2×2 submatrix is relevant. Since this submatrix is diagonal, the entire diagrammatic perturbation theory cleaves into separate chiral and antichiral parts. In other words, although $\partial\varphi^i$ and $\bar{\partial}\varphi^i$ cannot be assigned purely to the chiral or antichiral sectors, the auxiliary fields J^i and $\widetilde{J}^i$ can. Concretely, from (4), we see that they satisfy the operator-product expansions

$$\begin{aligned} J^i(z)J^j(w) &\sim \frac{\hbar\delta^{ij}}{(z-w)^2}, \\ \widetilde{J}^i(\bar{z})\widetilde{J}^j(\bar{w}) &\sim \frac{\hbar\delta^{ij}}{(\bar{z}-\bar{w})^2}, \\ J^i(z)\widetilde{J}^j(\bar{w}) &\sim 0. \end{aligned} \tag{5}$$

The above observations make the construction of the non-chiral theory into a simple direct sum of left-moving and right-moving parts. In particular, the full non-chiral BRST operator is just

$$Q_{\mathrm{tot}} = Q + \widetilde{Q}, \tag{6}$$

where not only Q_{tot} but also Q and $\widetilde{Q}$ separately are nilpotent. Concentrating for now on the left-moving sector, the BRST operator Q may be written as [2]

$$Q = \oint dz\Big(c(T_{\mathrm{mat}} + \tfrac{1}{2}T_{\mathrm{gh}}) + \gamma(W_{\mathrm{mat}} + \tfrac{1}{2}W_{\mathrm{gh}})\Big), \tag{7}$$

where T_{mat} and W_{mat} must generate the W_3 algebra with central charge $c_{\mathrm{mat}} = 100$. The ghost currents T_{gh} and W_{gh} are given by [2]

$$T_{\mathrm{gh}} = -2b\,\partial c - \partial b\, c - 3\beta\,\partial\gamma - 2\partial\beta\,\gamma \tag{8a}$$

$$\begin{aligned} W_{\mathrm{gh}} = &-\partial\beta\, c - 3\beta\,\partial c - \tfrac{8}{261}\big[\partial(b\,\gamma\, T_{\mathrm{mat}}) + b\,\partial\gamma\, T_{\mathrm{mat}}\big] \\ &+ \tfrac{25}{6\cdot 261}\hbar\Big(2\gamma\,\partial^3 b + 9\partial\gamma\,\partial^2 b + 15\partial^2\gamma\,\partial b + 10\partial^3\gamma\, b\Big), \end{aligned} \tag{8b}$$

where the ghost-antighost pairs (c,b) and (γ,β) correspond respectively to the T and W generators in the left-moving sector. As a general rule, the matter currents of the left and right sectors of the non-chiral theory are constructed by replacing the quantities $\partial\varphi^i$ occuring in the purely chiral theory by J^i or $\widetilde{J}^i$ respectively, as, for example, in the classical currents appearing in (1). Thus following [3], we take for the W_3 currents in the left-moving sector

$$T_{\rm mat} = \tfrac{1}{2}J^iJ^i + \sqrt{\hbar}\alpha_i\partial J^i, \tag{9a}$$

$$W_{\rm mat} = \tfrac{1}{3}d_{ijk}J^iJ^jJ^k + \sqrt{\hbar}e_{ij}J^i\partial J^j + \hbar f_i\partial^2 J^i. \tag{9b}$$

Using the OPEs (5), these will generate a $c_{\rm mat} = 100$ realisation of the full quantum W_3 algebra provided that the coefficients α_i, d_{ijk}, e_{ij} and f_i satisfy certain algebraic conditions, which are given in [3].

The transformation rules corresponding to the left-moving BRST operator Q can be deduced from those in [1] by replacing $\partial\varphi^i$ by J^i:

$$\begin{aligned}
\delta\varphi^i &= c\,J^i + \gamma d_{ijk}\,J^jJ^k + \tfrac{8}{261}b\,\gamma\,\partial\gamma J^i\\
&\quad + \sqrt{\hbar}\Big(-\alpha_i\partial c + (e_{ij}-e_{ji})\gamma\partial J^j - e_{ji}\partial\gamma J^j - \tfrac{8}{261}\alpha_i\partial(b\gamma\partial\gamma)\Big) + \hbar f_i\partial^2\gamma,\\
\delta h &= \bar\partial c + c\,\partial h - \partial c\,h + \tfrac{4}{261}(\gamma\partial B - \partial\gamma B)J^iJ^i + \tfrac{8}{261}\sqrt{\hbar}(\gamma\partial B - \partial\gamma B)\alpha_i\partial J^i\\
&\quad + \tfrac{25}{6\cdot 261}\hbar\Big(2\gamma\,\partial^3 B - 3\partial\gamma\,\partial^2 B + 3\partial^2\gamma\,\partial B - 2\partial^3\gamma\,B\Big),\\
\delta B &= \bar\partial\gamma + c\,\partial B - 2\partial c\,B + 2\gamma\,\partial h - \partial\gamma\,h,\\
\delta c &= c\,\partial c + \tfrac{4}{261}\gamma\,\partial\gamma\,J^iJ^i + \tfrac{8}{261}\sqrt{\hbar}\alpha_i\gamma\,\partial\gamma\,\partial J^i + \tfrac{25}{6\cdot 261}\hbar(2\gamma\,\partial^3\gamma - 3\partial\gamma\,\partial^2\gamma)\\
\delta\gamma &= c\,\partial\gamma - 2\partial c\,\gamma,\\
\delta b &= \pi_b, \qquad\qquad \delta\pi_b = 0,\\
\delta\beta &= \pi_\beta, \qquad\qquad \delta\pi_\beta = 0.
\end{aligned} \tag{10}$$

Since we shall work in a 1.5-order formalism, the variations of the auxiliary fields J^i will not be relevant.

For the right-moving sector, we introduce ghost-antighost pairs ($\tilde c$,$\tilde b$) and ($\tilde\gamma$,$\tilde\beta$) for the right-moving W_3 symmetry. The right-moving ghost and matter currents will be analogous to ($8a,b$) and ($9a,b$), but with ∂ replaced by $\bar\partial$, and all ghosts and auxiliary fields replaced by their tilded versions. There will be analogous BRST transformation rules corresponding to $\widetilde{Q}$. Note that the total variation of φ^i will be the sum of $\delta\varphi^i$ as given in (10) and its counterpart $\tilde\delta\varphi^i$ from the right-moving sector.

The Lagrangian for non-chiral W_3 gravity may now be written as

$$\begin{aligned}
\mathcal{L} &= -\tfrac{1}{2}\bar\partial\varphi^i\partial\varphi^i - J^i\widetilde{J}^i + \widetilde{J}^i\partial\varphi^i + J^i\bar\partial\varphi^i - hT_{\rm mat} - BW_{\rm mat} - \tilde h\widetilde{T}_{\rm mat} - \widetilde{B}\widetilde{W}_{\rm mat}\\
&\quad + \delta\Big(b(h - h_{\rm back}) + \beta(B - B_{\rm back})\Big) + \tilde\delta\Big(\tilde b(\tilde h - \tilde h_{\rm back}) + \tilde\beta(\widetilde{B} - \widetilde{B}_{\rm back})\Big)
\end{aligned} \tag{11}$$

Using (10), and the right-moving counterpart, we can write the Lagrangian as

$$
\begin{aligned}
\mathcal{L} = &-\tfrac{1}{2}\bar\partial\varphi^i\partial\varphi^i - J^i\widetilde{J}^i + \widetilde{J}^i\partial\varphi^i + J^i\bar\partial\varphi^i - b\bar\partial c - \beta\bar\partial\gamma - \tilde{b}\partial\tilde{c} - \tilde\beta\partial\tilde\gamma \\
&+ \pi_b(h - h_{\rm back}) + \pi_\beta(B - B_{\rm back}) - h(T_{\rm mat} + T_{\rm gh}) - B(W_{\rm mat} + W_{\rm gh}) \\
&+ \tilde\pi_b(\tilde h - \tilde h_{\rm back}) + \tilde\pi_\beta(\widetilde B - \widetilde B_{\rm back}) - \tilde h(\widetilde T_{\rm mat} + \widetilde T_{\rm gh}) - \widetilde B(\widetilde W_{\rm mat} + \widetilde W_{\rm gh}).
\end{aligned}
\qquad (12)
$$

As in the chiral W_3 gravity discussed in [1], the $\hbar$-independent terms in the Lagrangian and transformation rules describe the classical theory and its symmetries. The $\hbar$-dependent terms in the Lagrangian (12), where the currents are given by $(8a, b)$ and $(9a, b)$, correspond to counterterms needed for the explicit cancellation of anomalies. The $\hbar$-dependent terms in the transformation rules (10), and their right-moving counterparts, correspond to renormalisations that are also needed for anomaly cancellation. The resulting theory is free of all anomalies. Because of the diagonal nature of the propagators for the auxiliary fields J^i and $\widetilde{J}^i$, it follows that all connected Feynman diagrams with external gauge fields involve no mixing between left-moving and right-moving fields. In particular, the non-renormalisation theorem given in [1] for the chiral case, which excludes all but those infinities that may be removed by normal ordering, may now straightforwardly be carried over to the present non-chiral case. Moreover, because the OPEs (5) for the J^i currents have the same form as the OPEs for $\partial\varphi^i$ currents in the chiral case, and similarly for $\widetilde{J}^i$, the potential anomalies in each chiral sector are cancelled by the same mechanisms that were exhibited in [1] for the purely chiral theory.

Having established that the non-chiral W_3 gravity theory is anomaly free, it follows that one may, as in the classical theory, use the local spin-2 and spin-3 symmetries to choose gauges in which h, $\tilde h$, B and $\widetilde B$ are all zero. In this gauge, the equations of motion for J^i and $\widetilde{J}^i$ reduce to

$$
J^i = \partial\varphi^i, \qquad \widetilde{J}^i = \bar\partial\varphi^i. \qquad (13)
$$

The "Gauss' Law" constraints, given by the equations of motion for the gauge fields, become, at the classical level,

$$
\begin{aligned}
T &\equiv \tfrac{1}{2}\partial\varphi^i\partial\varphi^i = 0, \\
W &\equiv \tfrac{1}{3}d_{ijk}\,\partial\varphi^i\partial\varphi^j\partial\varphi^k = 0,
\end{aligned}
\qquad (14)
$$

together with the corresponding right-moving counterparts.

We now consider the physical consequences of the W_3 constraints. Classically, the totally-symmetric coefficients d_{ijk} are simply required to satisfy the conditions

$$
d_{(ij}{}^m d_{k\ell)m} = \lambda^2\delta_{(ij}\delta_{k\ell)}, \qquad (15)
$$

where λ is a constant. However, the requirement that the currents $(9a, b)$ should satisfy the W_3 algebra at the quantum level implies the full set of conditions given in [3], which include further restrictions on the coefficients d_{ijk}. In particular, it is shown in [3] that these restrictions on d_{ijk} may be solved, without loss of generality, by taking it to have the form

$$d_{111} = \lambda, \qquad d_{1ab} = -\lambda\delta_{ab}, \qquad 2 \le a \le n \tag{16}$$

with all other components not related to these by symmetry vanishing.

In order to see the consequences of the constraints, we begin with the chiral case. From (14) and (16), the classical chiral T and W currents take the forms

$$T = \tfrac{1}{2}(\partial\varphi_1)^2 + t, \tag{17a}$$

$$W = \tfrac{1}{3}\lambda(\partial\varphi_1)^3 - 2\lambda\partial\varphi_1\, t, \tag{17b}$$

where t is the stress tensor for the $(n-1)$ scalar fields φ^a:

$$t = \tfrac{1}{2}\partial\varphi^a\partial\varphi^a. \tag{18}$$

The specific form of the W current (17b) allows it to be reorganised into the form*

$$W = \tfrac{4}{3}\lambda(\partial\varphi_1)^3 - 2\lambda\partial\varphi_1\, T. \tag{19}$$

Thus after the T constraint has been imposed, the new information contained in the W constraint is just

$$(\partial\varphi_1)^3 = 0, \tag{20}$$

and hence $\partial\varphi_1 = 0$. The W constraint has therefore singled out the φ_1 coordinate, and removed its z dependence.

In the non-chiral case we start from the classical terms in the T and W currents $(9a, b)$ and their right-moving counterparts, again with d_{ijk} taking the form (16). For the same reason as in the chiral case, the W current can be reorganised into

$$W = \tfrac{4}{3}\lambda(J_1)^3 - 2\lambda J_1 T, \tag{21}$$

and similarly for the $\widetilde{W}$ right-moving current. Consequently, the left-moving and right-moving constraints, taken together with the Virasoro constraints, imply $J_1 = \widetilde{J}_1 = 0$. Then, one may use the equations of motion for J_1 and $\widetilde{J}_1$ given in (2) to obtain

$$\begin{aligned} J_1 - \partial\varphi_1 + \tilde{h}\widetilde{J}_1 + \lambda\widetilde{B}\Big(2(\widetilde{J}_1)^2 - 2\widetilde{T}\Big) &= 0, \\ \widetilde{J}_1 - \bar{\partial}\varphi_1 + hJ_1 + \lambda B\Big(2(J_1)^2 - 2T\Big) &= 0. \end{aligned} \tag{22}$$

* This separation of the W current into a part involving only φ_1, and a remainder involving the total stress tensor T, persists at the quantum level [3].

Using the Virasoro constraints again, we find

$$\partial\varphi_1 = \bar{\partial}\varphi_1 = 0. \tag{23}$$

Thus in the non-chiral W_3 gravity theory, the W and $\widetilde{W}$ constraints imply classically that the φ_1 field is "frozen." Note that the derivation of (23) does not require any gauge choice to be made. Note also that the freezing of φ_1 occurs regardless of whether it is a spacelike or a timelike spacetime coordinate.

We now proceed to consider the quantum theory of W_3 strings. The fully renormalised matter currents [1,3] are, in the gauge $h = \tilde{h} = B = \widetilde{B} = 0$,

$$T_{\rm mat} = \mathcal{T} + \tfrac{1}{2}(\partial\varphi_1)^2 + \tfrac{1}{2}(\partial\varphi_2)^2 + \sqrt{\hbar}(\alpha_1\partial^2\varphi_1 + \alpha_2\partial^2\varphi_2) \tag{24a}$$

$$\begin{aligned} W_{\rm mat} = \frac{2}{\sqrt{261}}\Big\{ &\tfrac{1}{3}(\partial\varphi_1)^3 - \partial\varphi_1(\partial\varphi_2)^2 + \sqrt{\hbar}(\alpha_1\partial\varphi_1\partial^2\varphi_1 - 2\alpha_2\partial\varphi_1\partial^2\varphi_2 - \alpha_1\partial\varphi_2\partial^2\varphi_2) \\ &+ \hbar(\tfrac{1}{3}\alpha_1^2\partial^3\varphi_1 - \alpha_1\alpha_2\partial^3\varphi_2) - 2\partial\varphi_1\mathcal{T} - \alpha_1\sqrt{\hbar}\partial\mathcal{T}\Big\}, \end{aligned} \tag{24b}$$

where $\mathcal{T}$ is the stress tensor for the $D = n-2$ free scalar fields without background charges φ^μ, $\mu = 3, \ldots, n$:

$$\mathcal{T} = \tfrac{1}{2}\partial\varphi^\mu\partial\varphi^\mu, \tag{25}$$

together with their right-moving counterparts. The background charges α_1 and α_2 for φ_1 and φ_2 are given by* [1]

$$\begin{aligned} \alpha_1^2 &= -\tfrac{49}{8} \\ \alpha_2^2 &= \tfrac{1}{12}(D - \tfrac{49}{2}), \end{aligned} \tag{26}$$

giving a total matter central charge $c_{\rm mat} = 100$.

Up until now, we have not made any specific choices for the signature of our scalar-field metric, but have followed initially the Euclidean assignment of [1] (which actually corresponds to all fields initially being timelike). Although we shall not make a detailed analysis of the compactification problem in this paper, we note that since a background charge breaks Lorentz invariance in the dimension in which it occurs, the eventual recovery of lower-dimensional Lorentz invariance would require that the dimensions with background charges be compactified. In order to compactify such a dimension, with coordinate $\varphi_\star$, the background charge of this dimension should be imaginary [9], $\alpha_\star = {\rm i}Q_\star$. This is necessary in order that the path integral measure e^{-I} be invariant under constant shifts $\varphi_\star \to \varphi_\star + (2\pi/Q_\star)r$, $r \in \mathbf{Z}$ (as can be seen by considering the background charge to arise from a dilaton coupling linear in $\varphi_\star$), thus allowing compactification.

* Note that the specific details of the construction of the stress tensor $\mathcal{T}$ are not important for obtaining a realisation of W_3; any realisation of the Virasoro algebra with central charge D will suffice. In particular, we could choose a realisation with 24 free scalars and a real free fermion, thereby effectively having $D = \frac{49}{2}$ and hence giving $\alpha_2 = 0$. The 25 scalars φ_2, φ^μ would then occur on an equal footing, appearing only through their free unimproved stress tensor.

The φ_1 coordinate is the one that we have shown to be "frozen" at the classical level. We shall see below that this feature should be maintained at the quantum level. Thus, the issue of compactification does not seem to be necessary for φ_1. In this paper, we shall not make a definite choice of the metric signature for φ_1, but shall for simplicity stay with the initial timelike signature given in [1]. The situation for φ_2 is different: there is no constraint that specifically kills this coordinate, and so compactification is necessary if one wishes to have Lorentz invariance in the lower dimensions. From (26) we see that, relative to our initial signature choice, by choosing the dimension D appropriately one may make α_2 real (for $D \geq 25$) or imaginary (for $D \leq 24$). If one wants to keep the real "time" as one of the Lorentz-covariant dimensions φ^μ, then there is no room for another (unfrozen) time. Thus, φ_2 and $D-1$ of the φ^μ should be Wick-rotated into spatial directions:

$$\begin{aligned} &\varphi_2 \rightarrow \mathrm{i}Y \\ &\varphi^A \rightarrow \mathrm{i}X^{A-3} \qquad A = 4, \ldots, n. \end{aligned} \tag{27}$$

The coordinate φ^3 is then the real time coordinate,

$$\varphi^3 \rightarrow X^0. \tag{28}$$

In order for the background charge for Y to be imaginary so that it can be compactified, α_2 should be real, requiring $D \geq 25$.

In this paper, we shall not carry out a full quantum analysis of the spectrum and unitarity properties of the theory defined by the above choices. Some preliminary information on the spectrum may be derived, nonetheless, in the standard way by expanding the scalar fields in oscillators, picking a Fock vacuum and proceeding to the construction of states satisfying the T and W constraints. One could do this either using the full BRST formalism or by applying the $T_{\rm mat}$ and $W_{\rm mat}$ constraints to states with no ghost excitations. We have followed the latter procedure, imposing the L_m and W_m constraints for $m \geq 0$, where $T(z) = \sum_n L_n z^{-n-2}$ and $W(z) = \sum_n W_n z^{-n-3}$. Actually, the full content of these constraints is obtained simply by imposing L_0, L_1, L_2 and W_0 and their right-moving counterparts, since the rest of the imposed constraints may be obtained by commutation.

It is already known from consideration of the nilpotence of the BRST operator Q that the intercept for L_0 is -4 while that for W_0 is zero [2]. Thus, the constraints to be imposed are

$$L_0 - 4 = 0, \quad L_1 = 0, \quad L_2 = 0, \quad W_0 = 0, \tag{29a}$$

$$\widetilde{L}_0 - 4 = 0, \quad \widetilde{L}_1 = 0, \quad \widetilde{L}_2 = 0, \quad \widetilde{W}_0 = 0 \tag{29b}$$

acting on physical states. The incorporation of the background charges into this standard calculation goes straightforwardly, as in [9], and similarly for the imposition of the W constraints. The lowest-lying state satisfying the constraints is a scalar just as in ordinary bosonic string theory; for it the L_0 constraint gives the equation of motion:

$$\mathcal{M}^2 = (p^1 + Q_1)^2 - (p_2 + \alpha_2)^2 + (p^0)^2 - p^A p^A = \tfrac{1}{12}(2 - D), \tag{30}$$

where $\alpha_1 = iQ_1$. The only other non-trivial constraints at this level are the W constraints, which freeze the φ_1 field in a generalisation of our classical result (20):

$$(p^1 + Q_1)(p^1 + \tfrac{6}{7}Q_1)(p^1 + \tfrac{8}{7}Q_1) = 0. \tag{31}$$

If we pick the first root of (31), $p_1 = -Q_1$, then the (mass)^2 operator in the unfrozen directions takes the value $\frac{1}{12}(2 - D)$; the other two roots give a value $\frac{1}{24}(1 - 2D)$. Thus, with the first root of the W constraint, the scalar is massless for $D = 2$, whilst for $D > 2$ the scalar is tachyonic.

The next-to-lowest states are created by acting on the Fock vacuum with a linear combination of creation operators in each of the left and right sectors, making states with spins up to 2. They also have to satisfy the usual Virasoro $L_0 - \widetilde{L}_0 = 0$ constraint for closed string theories. As with the ordinary bosonic string theory, the holomorphic factorisation discussed earlier allows one to consider the effect of the constraints in each sector separately. The linear combination of creation operators applied to the Fock vacuum is specified by a polarisation vector ξ^i. One then finds from the L_0 constraint the mass-shell condition

$$\mathcal{M}^2 = (p^1 + Q_1)^2 - (p_2 + \alpha_2)^2 + (p^0)^2 - p^A p^A = \tfrac{1}{12}(26 - D), \tag{32}$$

and from L_1 one finds the generalised transversality condition

$$p^i \xi_i + 2(Q_1 \xi_1 + \alpha_2 \xi_2) = 0. \tag{33}$$

The W constraints at this level give results depending on whether or not ξ_1 vanishes. In the case $\xi_1 = 0$, one finds eqn. (31) once again, with no further conditions on the remaining components of the polarisation vector. If $\xi_1 \neq 0$, then one obtains $(p^1 + \frac{11}{7}Q_1)(p^1 + \frac{10}{7}Q_1) = 0$, again freezing the momentum p^1, and also a restriction on the polarisation vector, $\xi_a = p_a \xi_1 (3p^1 + 4Q_1)^{-1}$. Note that in this case there is only a single independent polarisation. It is interesting to note that if $\xi_1 = 0$ and if we pick the first root of eqn. (31), then there would be a massless spin-2 state for $D = 26$.

To conclude, we have constructed in this paper a non-chiral critical W_3 gravity theory and have begun the task of formulating a space-time interpretation of it as a W_3 string theory. At the classical level, the additional W and $\widetilde{W}$ constraints conspire to freeze out all the degrees of freedom in one of the coordinates, thus leaving an ordinary bosonic closed string theory in one dimension less. At the quantum level, we have found that this basic pattern persists in the first few levels that we have investigated.

Many important questions remain to be investigated before one can have a full picture of the properties of W_3 strings. In particular, we have not yet investigated the unitarity properties of the theory. Because of the anomaly freedom of the theory, one may expect to be able to impose a light-cone gauge choice where unitarity would be manifest.

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