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HETEROTIC PHASE TRANSITIONS AND SINGULARITIES OF THE GAUGE DYONIC STRING

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ABSTRACT

Heterotic strings on $R^6 \times K3$ generically appear to undergo some interesting new phase transition at that value of the string coupling for which the one of the six-dimensional gauge field kinetic energies changes sign. An exception is the $E_8 \times E_8$ string with equal instanton numbers in the two E_8's, which admits a heterotic/heterotic self-duality. In this paper, we generalize the dyonic string solution of the six-dimensional heterotic string to include non-trivial gauge field configurations corresponding to self-dual Yang-Mills instantons in the four transverse dimensions. We find that vacua which undergo a phase transition always admit a string solution exhibiting a naked singularity, whereas for vacua admitting a self-duality the solution is always regular. When there is a phase transition, there exists a choice of instanton numbers for which the dyonic string is tensionless and quasi-anti-self-dual at that critical value of the coupling. For an infinite subset of the other choices of instanton number, the string will also be tensionless, but all at larger values of the coupling.

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The purpose of this paper is to connect two hitherto unrelated phenomena appearing in six-dimensional heterotic string theory: phase transitions [1, 2, 3] and the dyonic string soliton [4]. First we extend the *neutral* dyonic string solution to the *gauge* solution by including non-trivial gauge field configurations corresponding to self-dual Yang-Mills instantons in the four dimensions transverse to the string. This is similar to the way that the gauge fivebrane [5] is related to the neutral fivebrane [6, 7] in ten dimensions. Then we show that vacua which undergo a phase transition always admit a gauge dyonic string solution exhibiting a naked singularity, whereas for vacua admitting a heterotic/heterotic self-duality [1] the solution is always regular. We find that when there is a phase transition, there exists a choice of instanton numbers for which the dyonic string is tensionless and quasi-anti-self-dual at that critical value of the coupling. For an infinite subset of the other choices of instanton number, the string will also be tensionless, but all at larger values of the coupling.

Let us begin by recalling the evidence for phase transitions in the six-dimensional heterotic string [1]. Before the recent interest in a duality between heterotic and Type IIA strings, it was conjectured that in $D \leq 6$ dimensions there ought to exist a duality between one heterotic string and another [10]. A comparison of the (purely "electric") fundamental string solution [11] and the (purely "magnetic") dual solitonic string solution [8, 9] suggests the following $D = 6$ duality dictionary: the dilaton $\tilde{\phi}$, the canonical metric $\tilde{g}_{MN}$ and 3-form field strength $\tilde{H}$ of the dual string are related to those of the fundamental string, ϕ, g_{MN} and H by the replacements $\phi \to \tilde{\phi} = -\phi$, $g_{MN} \to \tilde{g}_{MN} = g_{MN}$, $H \to \tilde{H} = e^{-2\phi}\, {}^*H$, where * denotes the Hodge dual. In going from the fundamental string to the dual string, one also interchanges the roles of worldsheet and spacetime loop expansions. Moreover, since the dilaton enters the dual string equations with the opposite sign to the fundamental string, the strong coupling regime of the string should correspond to the weak coupling regime of the dual string [8, 9]: $\lambda_6 = < e^{\phi} > = 1/\tilde{\lambda}_6$ where λ_6 and $\tilde{\lambda}_6$ are the fundamental string and dual string coupling constants. Because this duality interchanges worldsheet and spacetime loop expansions, it exchanges the tree level Chern-Simons contributions to the Bianchi identity

$$dH = \frac{\alpha'}{4}(-\mathrm{tr}\, R \wedge R + \sum_\alpha v_\alpha \mathrm{tr}\, F_\alpha \wedge F_\alpha) \ , \tag{1}$$

with the one-loop Green-Schwarz corrections to the field equations

$$d\widetilde{H} = \frac{\alpha'}{4}(-\mathrm{tr}\, R \wedge R + \sum_\alpha \tilde{v}_\alpha \mathrm{tr}\, F_\alpha \wedge F_\alpha) \ . \tag{2}$$

Here F_α is the field strength of the α'th component of the gauge group, tr denotes the trace in the fundamental representation, and $v_\alpha, \tilde{v}_\alpha$ are constants. (As explained in [1], we may, without loss of generality, choose the string tension measured in the string metric and the dual string tension measured in the dual string metric to be equal.) In fact, the Green-Schwarz anomaly cancellation mechanism in six dimensions requires that the anomaly eight-form I_8 factorize as a product of four-forms, $I_8 \sim dH \wedge d\tilde{H}$, and a six-dimensional string-string duality with the general features summarized above would exchange the two factors.

To see where the phase transition makes its appearance, let us recall that in [12] corrections to the Bianchi identities of the type (1) and to the field equations of the type (2) were shown to be entirely consistent with supersymmetry, with no restrictions on the constants v_α and $\tilde{v}_\alpha$. Moreover, supersymmetry relates these coefficients to the gauge field kinetic energy. In the canonical metric, the dilaton dependence of the kinetic energy of the gauge field $F_{\alpha MN}$ is

$$L_{\rm gauge} \sim \sqrt{-g}\sum_\alpha \left(v_\alpha e^{-\phi} + \tilde{v}_\alpha e^{\phi}\right) {\rm tr} F_{\alpha MN} F_\alpha{}^{MN} \ . \tag{3}$$

Furthermore, $N=1$, $D=6$ supersymmetry guarantees that there are no higher (≥ 2) loop contributions to the gauge field kinetic energy. Positivity of the kinetic energy for all values of ϕ thus implies that v_α and $\tilde{v}_\alpha$ should both be non-negative, and at least one should be positive. Otherwise, some interesting new phase transition must occur at the value of ϕ at which the gauge field coupling constant changes sign, preventing the extrapolation from weak to strong coupling. Since v_α is essentially the Kac-Moody level [13, 9, 1] and is therefore non-negative, the problem devolves upon $\tilde{v}_\alpha$.

For $N=2$ heterotic strings obtained by compactification on T^4, this is never a problem because the theory is non-chiral and the Green-Schwarz $d\tilde{H}$ vanishes. This can also be seen by noting that this theory is dual to the Type IIA string compactified on $K3$ [14] which has no Chern-Simons terms. For $N=1$ heterotic strings obtained by compactification on $K3$, however, both dH and $d\tilde{H}$ are non-vanishing[1] and it is the rule, rather than the exception, that some of the $\tilde{v}_\alpha$ are negative! Consider, for example, the SO(32) string with a $k=24$ $SU(2)$ instanton embedded in the $SO(32)$. The resulting $D=6$ gauge group is $SO(28)\times SU(2)$ and the anomaly eight-form is given by [13]

$$I_8 \sim [{\rm tr}R^2 - {\rm tr}{F_{SO(28)}}^2 - 2{\rm tr}{F_{SU(2)}}^2][{\rm tr}R^2 + 2{\rm tr}{F_{SO(28)}}^2 - 44{\rm tr}{F_{SU(2)}}^2] \ , \tag{4}$$

and the $SO(28)$ coefficient in the second factor enters with the wrong sign [9, 1]. A similar problem arises for $E_8\times E_8$ where one embeds a k_1 $SU(2)$ instanton in one E_8 and a

[1]This was the reason for the speculation in [4] that the dyonic string might be relevant to heterotic/heterotic duality.

$k_2 = 24 - k_1$ $SU(2)$ instanton in the other, except in the case of symmetric embedding where both $k_i = 12$. The resulting $D = 6$ gauge group is $E_7 \times E_7$ and for generic embeddings one finds that $v_i = 1/6$ but

$$\tilde{v}_i = \frac{1}{12}(k_i - 12) \ . \tag{5}$$

Since we require $k_1 + k_2 = 24$, one factor will always have the wrong sign except for the $k = 12$ case discussed in [1], for which the $\tilde{v}_i$ both vanish. The anomaly eight-form is thus given by

$$I_8 \sim [\mathrm{tr}R^2 - \frac{1}{6}\mathrm{tr}F^2_{E_7} - \frac{1}{6}\mathrm{tr}F^2_{E_7}][\mathrm{tr}R^2] \ . \tag{6}$$

Since $\tilde{v}_\alpha = 0$ there is no wrong-sign problem and presumably one can extrapolate to strong coupling without encountering a phase transition. Qualitatively similar results hold for any other unbroken subgroup of $E_8 \times E_8$. Note, however, that since $v_\alpha \neq \tilde{v}_\alpha$, there is no manifest self-duality. In [1], however, the duality was deduced by looking in two different ways at eleven-dimensional M-theory compactified on $K3 \times S^1/Z_2$. Consequently, one is led to assume that the duality interchanges perturbative gauge fields ($v_\alpha = 0, \tilde{v}_\alpha < 0$), with non-perturbative gauge fields ($\tilde{v}_\alpha = 0, v_\alpha < 0$). We shall return to this special case later, but for the most part the present paper will be concerned with the generic case where a phase transition seems unavoidable.[2]

Next, let us recall the *dyonic string soliton* [4] which carries both "electric" charge Q and "magnetic" charge P. In canonical metric, it takes the form

$$\phi = \phi_Q + \phi_P \ ,$$

$$\begin{aligned} ds^2 &= e^{(\phi_Q - \phi_P)} dx^\mu dx^\nu \eta_{\mu\nu} + e^{(\phi_P - \phi_Q)} dy^m dy^m \ , \\ e^{-2\phi_Q} &= e^{-\phi_0} + \frac{Q}{r^2}, \qquad e^{2\phi_P} = e^{\phi_0} + \frac{P}{r^2} \ , \\ H &= 2P\epsilon_3 + 2Qe^{2\phi}\, {}^*\epsilon_3 \ , \end{aligned} \tag{7}$$

where x^μ ($\mu = 0, 1$) are the coordinates of the string world volume, y^m ($m = 1, 2, 3, 4$) are the coordinates of the transverse space, $r = \sqrt{y^m y^m}$ and ϵ_3 is the volume form on S^3. Note that we have taken the metric to be asymptotically Minkowskian, and the asymptotic value for ϕ to be ϕ_0. The solution describes a single electric and single magnetic charge at $r = 0$ and it interpolates between the purely electric fundamental string of [11] and the purely

[2] It has been suggested in [3] that a phase transition can also be avoided in those cases where one can Higgs away the "wrong-sign" gauge groups, for example in the $(k_1, k_2) = (14, 10)$ compactification of the $E_8 \times E_8$ string.

magnetic solitonic string of [8, 9]. Its tension, or mass m per unit length, is given by

$$2\pi\alpha'^2\, m = Pe^{-\phi_0} + Qe^{\phi_0}\ . \tag{8}$$

In the present paper, we are interested in regarding this configuration as a solution of the chiral $N=1, D=6$ theory describing a graviton multiplet $(g_{MN}, \psi_M, B^+_{MN})$ coupled to a single tensor multiplet (B^-_{MN}, χ, ϕ), where the 2-forms B^+_{MN} and B^-_{MN} have 3-form field strengths that are self-dual and anti-self dual, respectively. As such, the solution preserves half of the spacetime supersymmetry for all values of P and Q.[3] In the self-dual limit $Pe^{-\phi_0} = Qe^{\phi_0}$, the contributions from the tensor multiplet become trivial. This self-dual string had already been found in [8] in the context of self-dual supergravity which describes only the graviton multiplet and no tensor multiplet. It is also interesting to consider the limit $Pe^{-\phi_0} = -Qe^{\phi_0}$ where the string becomes tensionless. We refer to this as the *quasi*-anti-self dual limit, because in this limit the graviton multiplet does not completely decouple, in that the spacetime is still curved and the self-dual part of the field strength is still non-vanishing.[4]

We now allow a further coupling to a Yang-Mills multiplet (A_M, λ) and turn to the discussion of the *gauge dyonic* string solution of this chiral $N=1$ theory. The bosonic equations of motion for the corresponding supergravity are [12]

$$\begin{aligned}
&\Box\phi = -\tfrac{1}{12}e^{-2\phi}H^2 + \frac{\alpha'}{16}\sum_\alpha (v_\alpha\, e^{-\phi} - \tilde v_\alpha\, e^{\phi})\mathrm{tr}(F_\alpha)^2\ ,\\
&R_{MN} = \partial_M\phi\partial_N\phi + \tfrac14 e^{-2\phi}(H^2_{MN} - \tfrac16 H^2 g_{MN})\\
&\qquad - \frac{\alpha'}{4}\sum_\alpha (v_\alpha\, e^{-\phi} + \tilde v_\alpha\, e^{\phi})\mathrm{tr}(F^2_{\alpha MN} - \tfrac18 F^2_\alpha g_{MN})\ ,\\
&D_M\Big((v_\alpha\, e^{-\phi} + \tilde v_\alpha\, e^{\phi})F^{MN}_\alpha\Big) - \tfrac12 v_\alpha\, e^{-\phi} H^N{}_{PQ}F^{PQ}_\alpha - \tfrac12 \tilde v_\alpha\, e^{\phi}\, {}^*H^N{}_{PQ}F^{PQ}_\alpha = 0\ ,\\
&dH = \frac{\alpha'}{4}\sum_\alpha v_\alpha \mathrm{tr}\, F_\alpha\wedge F_\alpha\ ,\qquad d\widetilde H = \frac{\alpha'}{4}\sum_\alpha \tilde v_\alpha \mathrm{tr}\, F_\alpha\wedge F_\alpha\ .
\end{aligned} \tag{9}$$

(It is not necessary to include the Lorentz Chern-Simons and Green-Schwarz terms in (1) and (2) since, in common with the gauge fivebrane [5], they will turn out to vanish for the gauge dyonic string solution.)

The ansätze for the metric and the field strength H are given by

$$ds^2 \;=\; e^{2A}dx^\mu dx^\nu \eta_{\mu\nu} + e^{-2A}dy^m dy^m\ ,$$

[3] This is to be contrasted with $N=2$ and $N=4$ theories, where the purely electric or purely magnetic solutions preserve one half, but the dyonic solution only one quarter, of the supersymmetry.

[4] This tensionless string has been the subject of much discussion recently [15-23] but note that, contrary to some claims in the literature, it corresponds to the quasi-anti-self-dual limit of the dyonic string of [4] and *not* the self-dual string of [8]. The masslessness of the quasi-anti-self-dual dyonic string was first observed in [24].

$$H_{mnp} = \epsilon_{mnpq}\partial_q e^C \ , \qquad H_{\mu\nu m} = \epsilon_{\mu\nu}\partial_m e^{\widetilde{C}} \ , \tag{10}$$

The functions A, C and $\widetilde{C}$ depend only on $r = \sqrt{y^m y^m}$. Note that the ϵ symbols are purely numerical, and the contractions are performed in the Euclidean metric δ_{mn}. We shall first consider the case where the source for the solution is provided by a single $SU(2)$ self-dual Yang-Mills instanton in the 4-dimensional transverse space. It can be written as

$$F^a = \frac{2\rho^2}{(\rho^2+r^2)^2}\eta^a_{mn} dy^m \wedge dy^n \ , \tag{11}$$

where a is an adjoint $SU(2)$ index, η^a_{mn} are the 't Hooft symbols, and ρ is the scale size of the instanton. In order to solve the equations of motion (9), we take $C = \phi - 2A$ and $\widetilde{C} = \phi + 2A$. The equations are then all satisfied if C and $\widetilde{C}$ satisfy

$$\begin{aligned}
\partial_m\partial_m e^C &= \frac{\alpha'}{8} v\, e^{-4A}\, \mathrm{tr} F_{mn}F_{mn} = -\frac{48\alpha' v \rho^4}{(\rho^2+r^2)^4} \ , \\
\partial_m\partial_m e^{-\widetilde{C}} &= \frac{\alpha'}{8} \tilde{v}\, e^{-4A}\, \mathrm{tr} F_{mn}F_{mn} = -\frac{48\alpha' \tilde{v} \rho^4}{(\rho^2+r^2)^4} \ .
\end{aligned} \tag{12}$$

Thus we have

$$\begin{aligned}
e^{\phi-2A} &= e^C = e^{\phi_0} + \frac{P(2\rho^2+r^2)}{(\rho^2+r^2)^2} \ , \\
e^{-\phi-2A} &= e^{-\widetilde{C}} = e^{-\phi_0} + \frac{Q(2\rho^2+r^2)}{(\rho^2+r^2)^2} \ ,
\end{aligned} \tag{13}$$

where Q and P are the electric charge and magnetic charge, given by

$$Q \equiv \tfrac{1}{2\omega_3}\int_{S^3} \widetilde{H} = 2\alpha'\tilde{v} \ , \qquad P \equiv \tfrac{1}{2\omega_3}\int_{S^3} H = 2\alpha' v \ . \tag{14}$$

Here ω_3 denotes the volume of the unit three sphere S^3. The mass per unit length of this dyonic string is given by

$$2\pi\alpha'^2\, m = Pe^{-\phi_0} + Qe^{\phi_0} \ . \tag{15}$$

Note that since $\partial_m\partial_m r^{-2} = 0$, we could in principle add r^{-2} terms with arbitrary coefficients to the solutions for e^C and $e^{-\widetilde{C}}$. However, such terms would describe contributions to the string mass and charges coming from singular sources rather than from the instanton source that we are considering here. As one might expect, we recover the neutral dyonic solution (7) as we shrink the size of the instanton to zero.

We may count the bosonic and fermionic zero modes by following the same procedure used for the gauge fivebrane [5]. For concreteness we consider the case where the $SU(2)$ is embedded minimally in an E_7 gauge group, *i.e.* $E_7 \longrightarrow SO(12) \times SU(2)$. First we count the bosonic modes: There will be 4 translations, 1 dilatation, 3 $SU(2)$ gauge rotations and

64 modes coming from the embedding, given by the dimension of the coset $E_7/(SO(12) \times SU(2))$. The counting of the fermionic zero modes is provided by the index theorem. Bearing in mind that the trace of $F \wedge F$ in the adjoint of $SU(2)$ is 4 times the trace in the doublet, and noting that the dual Coxeter number of E_7 is 18, we find $4 \times 18 = 72$ fermionic zero modes. Thus we have a total of $72 + 72$ bosonic and fermionic zero modes, which presumably corresponds to a non-critical string. Qualitatively similar results apply for other choices of gauge group.

This construction can be easily extended to the case where the source is provided by an $SU(2)$ multi-instanton configuration. This is particularly simple for the 't Hooft [25] and the Jackiw, Nohl and Rebbi [26] classes of multi-instanton solutions. In these solutions, $\mathrm{tr} F_{mn} F_{mn} = 4\partial^2\, \partial^2 \log f$, where $f = \epsilon + \sum_i \mu_i |\vec{y} - \vec{y}_i|^{-2}$, with $\epsilon = 1$ or 0 respectively. Thus the solutions for C and $\widetilde{C}$ are given by

$$e^{C} = \tfrac{1}{2}\alpha' v(\partial_m \partial_m f + h)\ , \qquad e^{-\widetilde{C}} = \tfrac{1}{2}\alpha' \tilde{v}(\partial_m \partial_m f + h)\ , \tag{16}$$

where h is a solution of the homogeneous equation $\partial_m \partial_m h = 0$, chosen so that the dyonic string solution has no other sources than those provided by the multi-instanton configuration. Note that these configurations break the isotropicity of the transverse space, and thus the functions C, $\widetilde{C}$ and hence A and ϕ depend non-isotropically on the coordinates y^m. The string solution has an electric charge and a magnetic charge, which are given by $Q = 2\alpha' n \tilde{v}$ and $P = 2\alpha' n v$ respectively, where n is the instanton number. The mass m per unit length is still given in terms of these charges by (15). If $SU(2)$ instantons in more than one of the components in the original Yang-Mills group are considered, the charges will be given by

$$Q = 2\alpha' \sum_\alpha n_\alpha \tilde{v}_\alpha\ , \qquad P = 2\alpha' \sum_\alpha n_\alpha v_\alpha\ , \tag{17}$$

where n_α is the instanton number for the α'th component of the Yang-Mills group.

Let us now consider the supersymmetry of the gauge dyonic string solutions. The transformation rules for the fermionic fields are

$$\begin{aligned}
\delta\psi_M &= D_M \epsilon + \tfrac{1}{48} e^{-\phi} H^{NPQ} \Gamma_{NPQ} \Gamma_M \epsilon\ , \\
\delta\chi &= \tfrac{\mathrm{i}}{2} \partial_M \phi \Gamma^M \epsilon + \tfrac{\mathrm{i}}{12} e^{-\phi} H_{MNP} \Gamma^{MNP} \epsilon\ , \\
\delta\lambda &= -\tfrac{1}{2\sqrt{2}} F_{MN} \Gamma^{MN} \epsilon\ .
\end{aligned} \tag{18}$$

It is straightforward to substitute the gauge dyonic string solutions into these transformation rules, and we find that the variations of the fermion fields vanish if

$$\epsilon = e^{\frac{1}{2}A} \epsilon_0\ , \qquad \Gamma_{01} \epsilon_0 = \epsilon_0\ , \tag{19}$$

where ϵ_0 is a constant spinor. Thus the gauge dyonic soliton also preserves half of the $N = 1$, $D = 6$ supersymmetry for all non-vanishing values of the electric charge Q or the magnetic charge P. Note that supersymmetry requires that the Yang-Mills configuration in the transverse space be self-dual, and therefore the instanton numbers must all be non-negative. If $Pe^{-\phi_0} = Qe^{\phi_0}$, the solution reduces to the gauge self-dual string, where the anti-self-dual component of H vanishes and the dilaton becomes constant, given by $\phi = \phi_0$, and hence the complete tensor multiplet decouples from the theory. On the other hand, if $Pe^{-\phi_0} = -Qe^{\phi_0}$ the string soliton becomes massless and quasi-anti-self-dual.

Having obtained the generic gauge dyonic string solutions, their connection with the phase transition now becomes apparent. As we stated earlier, it follows from (3) that positivity of the kinetic energy for all values of ϕ_0 requires that v_α and $\tilde{v}_\alpha$ should be non-negative, and at least one should be positive. In these vacua, the gauge dyonic string soliton has non-vanishing positive mass. It is in those string vacua where one or more of the $\tilde{v}_\alpha$ is negative that the possibility of the tensionless string arises, as can be seen from (15) and (17). As a concrete example, let us consider the case where just one of the $\tilde{v}_\alpha$ coefficients is negative, say $\tilde{v}_1$. Thus the phase transition takes place at the point $\phi_0 = \phi_0^{\rm cr}$, where

$$v_1 e^{-\phi_0^{\rm cr}} + \tilde{v}_1 e^{\phi_0^{\rm cr}} = 0 \ . \tag{20}$$

The tension, on the other hand, is given by

$$\pi\alpha' \, m = n_1(v_1 e^{-\phi_0} + \tilde{v}_1 e^{\phi_0}) + \sum_{\alpha\ge 2} n_\alpha (v_\alpha e^{-\phi_0} + \tilde{v}_\alpha e^{\phi_0}) \ . \tag{21}$$

Since the quantities in the summation over $\alpha \ge 2$ are all non-negative, the tension will be positive for $\phi_0 < \phi_0^{\rm cr}$. At the phase transition, the string tension remains positive for generic values of the instanton numbers, but becomes zero if $n_\alpha = 0$ for $\alpha \ge 2$. (There are further tensionless strings for other choices of instanton numbers if the resulting electric charge $Q = 2\alpha' \sum_{\alpha\ge 1} \tilde{v}_\alpha$ is negative. These all occur at larger values of the coupling, *i.e.* $\phi_0 > \phi_0^{\rm cr}$. With any further increase of the coupling, the tension of a tensionless string becomes negative. It should be emphasised that neither of these phenomena can occur in the $\phi_0 < \phi_0^{\rm cr}$ regime where the formalism can be trusted.)

The above discussion can be easily generalised to the cases where more than one $\tilde{v}_\alpha$ is negative. The phase transition occurs at the smallest value of the string coupling at which the coefficient of any of the Yang-Mills kinetic terms becomes zero. This value of the string coupling e^{ϕ_0} at which the phase transition takes place is always accompanied by a massless string. This phenomenon is reminiscent of the conifold transitions in Calabi-Yau compactification of the type II string induced by massless black holes [27]. However, in our case, for the larger values of the coupling when the kinetic energy of the gauge field $F_{\alpha MN}$ becomes negative, the mass of the dyonic string also becomes negative. As in the case where only one of the $\tilde{v}_\alpha$ coefficients is negative, here also there exists an infinite subset of the other choices of instanton number for which the value of e^{ϕ_0} where the string becomes massless does not correspond to the phase transition; in fact these tensionless dyonic strings all occur at larger values of the coupling e^{ϕ_0}. Thus all the string solutions have positive definite tension in the weak coupling regime before the onset of the phase transition, whilst all the tensionless solutions occur at or beyond the phase transition, and all the negative-tension string solutions occur beyond the phase transition.

Now we turn to the issue of naked singularities of the string solution. We can study this by looking at the scalar curvature. For the 6-dimensional metric $ds^2 = e^{2A} dx^\mu dx^\nu \eta_{\mu\nu} + e^{2B} dy^m dy^m$, it is given by $R = -e^{-2B}(e^{A-B}\,\partial_m\partial_m e^{B-A} + 5e^{-A-B}\,\partial_m\partial_m e^{A+B})$. It is easy to verify, for the canonical metric g_{MN} (10) and for the metrics $e^{\pm\phi} g_{MN}$ of the string and the dual string, that the scalar curvature diverges if either e^C or $e^{-\widetilde{C}}$, given by (13), vanishes. If this occurs for a positive value of r^2, the metric will have a naked singularity. We see from (13) and (17) that in order to avoid a naked singularity for all possible instanton numbers n_α and all possible values of the instanton sizes ρ_α, the coefficients v_α and $\tilde{v}_\alpha$ should both be non-negative. This is precisely the requirement that the theory not undergo a phase transition! We conclude, in particular, that naked singularities never arise in those vacua admitting a heterotic/heterotic self-duality.

As the present paper was nearing completion, we became aware of the paper by Seiberg and Witten [28], which also relates phase transitions to tensionless gauge dyonic strings.

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