

The Black Branes of M-theory

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ABSTRACT

We present a class of black p -brane solutions of M-theory which were hitherto known only in the extremal supersymmetric limit, and calculate their macroscopic entropy and temperature.

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1 Introduction

There is now a consensus that the best candidate for a unified theory underlying all physical phenomena is no longer ten-dimensional string theory but rather eleven-dimensional *M-theory*. The precise formulation of M-theory is unclear but membranes and fivebranes enter in a crucial way, owing to the presence of a 4-form field strength F_4 in the corresponding eleven-dimensional supergravity theory [1]. The membrane is characterized by a tension T_3 and an “electric” charge $Q_3 = \int_{S^7} *F_4$. For $T_3 > Q_3$, the membrane is “black” [2], exhibiting an outer event horizon at $r = r_+$ and an inner horizon at $r = r_-$, where $r = \sqrt{y^m y_m}$ and where y^m , $m = 1, 2, \dots, 8$, are the coordinates transverse to the membrane. In the extremal tension=charge limit, the two horizons coincide, and one recovers the fundamental supermembrane solution which preserves half of the spacetime supersymmetries [3]. This supermembrane admits a covariant Green-Schwarz action [4]. Similar remarks apply to the fivebrane which is characterized by a tension T_6 and “magnetic charge” $P_6 = \int_{S^4} F_4$. It is also black when $T_6 > P_6$ and also preserves half the supersymmetries in the extremal limit [2]. There is, to date, no covariant fivebrane action, however. Upon compactification of M-theory to a lower spacetime dimension, a bewildering array of other black p -branes make their appearance in the theory, owing to the presence of a variety of $(p + 2)$ -form field strengths in the lower-dimensional supergravity theory [5, 6]. Some of these p -branes may be interpreted as reductions of the eleven-dimensional ones or wrappings of the eleven-dimensional ones around cycles of the compactifying manifold [7, 8, 9, 10]. In particular, one may obtain as special cases the four-dimensional black holes ($p = 1$). It has been suggested that, in the extremal limit, these black holes may be identified with BPS saturated string states [11, 12, 13, 14]. Moreover, it is sometimes the case that multiply-charged black holes may be regarded as bound states at threshold of singly charged black holes [11, 12, 15, 16]. Apart from their importance in the understanding of M-theory, therefore, these black p -branes have recently come to the fore as a way of providing a microscopic explanation of the Hawking entropy and temperature formulae [17-28] which have long been something of an enigma. This latter progress has been made possible by the recognition that some p -branes carrying Ramond-Ramond charges also admit an interpretation as Dirichlet-branes, or D -branes, and are therefore amenable to the calculational power of conformal field theory [29].

The compactified eleven-dimensional supergravity theory admits a consistent truncation to the following set of fields: the metric tensor g_{MN} , a set of N scalar fields $\vec{\phi} = (\phi_1, \dots, \phi_N)$,

and N field strengths F_α of rank n . The Lagrangian for these fields takes the form [30, 31]

$$e^{-1}\mathcal{L} = R - \frac{1}{2}(\partial\vec{\phi})^2 - \frac{1}{2n!} \sum_{\alpha=1}^N e^{\vec{a}_\alpha \cdot \vec{\phi}} F_\alpha^2, \quad (1.1)$$

where \vec{a}_α are constant vectors characteristic of the supergravity theory. The purpose of the present paper is to display a universal class of (non-rotating) black p -brane solutions to (1.1) and to calculate their classical entropy and temperature.

As discussed in section 2, it is also possible to make a further consistent truncation to a single scalar ϕ and single field strength F :

$$e^{-1}\mathcal{L} = R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2n!} e^{a\phi} F^2, \quad (1.2)$$

where the parameter a can be conveniently re-expressed as

$$a^2 = \Delta - \frac{2d\tilde{d}}{D-2}, \quad (1.3)$$

since Δ is a parameter that is preserved under dimensional reduction [32]. Special solutions of this theory have been considered before in the literature. Purely electric or purely magnetic black p -branes were considered in [5] for $D = 10$ dimensions and in [6] for general dimensions $D \leq 11$. All these had $\Delta = 4$. In the case of *extremal* black p -branes, these were generalized to other values of Δ in [32, 30]. Certain non-extremal non-dilatonic ($a = 0$) black p -branes were also obtained in [33]

A particularly interesting class of solutions are the *dyonic* p -branes. Dyonic p -brane occur in dimensions $D = 2n$, where the n -index field strengths can carry both electric and magnetic charges. There are two types of dyonic solution. In the first type, each individual field strength in (1.1) carries either electric charge or magnetic charge, but not both. A particularly interesting example, owing to its non-vanishing entropy even in the extremal limit [34], is provided by the four-dimensional dyonic black hole. This is the $a = 0$ (Reissner-Nordstrom) solution, recently identified as a solution of heterotic string theory [11], but known for many years to be a solution of M-theory [35, 36]. The construction of black dyonic p -branes of this type is identical to that for the solutions with purely electric or purely magnetic charges, discussed in section (3).

In section (4), we shall construct black dyonic p -branes of the second type, where there is one field strength, which carries both electric and magnetic charge. Special cases of these have also been considered before: the self-dual threebrane in $D = 10$ [5, 37], the extremal self-dual string [6] and extremal dyonic string in $D = 6$ [41], a black self-dual string in $D = 6$ [33, 19] and a different dyonic black hole in $D = 4$ [30]. See also [38] for the most general

spherically symmetric extremal dyonic black hole solutions of the toroidally compactified heterotic string.

Black multi-scalar p -branes, the extremal limits of which may be found in [31], are discussed in section (5).

The usual form of the metric for an isotropic p -brane in D dimensions is given by

$$ds^2 = e^{2A}(-dt^2 + dx^i dx^i) + e^{2B}(dr^2 + r^2 d\Omega^2) , \quad (1.4)$$

where the coordinates (t, x^i) parameterise the d -dimensional world-volume of the p -brane. The remaining coordinates of the D dimensional spacetime are r and the coordinates on a $(D - d - 1)$ -dimensional unit sphere, whose metric is $d\Omega^2$. The functions A and B depend on the coordinate r only, as do the dilatonic scalar fields. The field strengths F_α can carry either electric or magnetic charge, and are given by

$$F_\alpha = \lambda_\alpha * \epsilon_{D-n} , \quad \text{or} \quad F_\alpha = \lambda_\alpha \epsilon_n , \quad (1.5)$$

where ϵ_n is the volume form on the unit sphere $d\Omega^2$. The former case describes an elementary p -brane solution with $d = n - 1$ and electric charge $\lambda_\alpha = Q_\alpha$; the latter a solitonic p -brane solution with $d = D - n - 1$ and magnetic charge $\lambda_\alpha = P_\alpha$.

Solutions of supergravity theories with metrics of this form include extremal supersymmetric p -brane solitons, which saturate the Bogomol'nyi bound. The mass per unit p -volume of such a solution is equal to the sum of the electric and/or magnetic charges carried by participating field strengths. More general classes of “black” solutions exist in which the mass is an independent free parameter. In this paper, we shall show that there is a universal recipe for constructing such non-extremal generalisations of p -brane solutions, in which the metric (1.4) is replaced by

$$ds^2 = e^{2A}(-e^{2f} dt^2 + dx^i dx^i) + e^{2B}(e^{-2f} dr^2 + r^2 d\Omega^2) . \quad (1.6)$$

Like A and B , f is a function of r . The ansätze for the field strengths (1.5) remain the same as in the extremal case. Remarkably, it turns out that the functions A , B and $\vec{\phi}$ take exactly the same form as they do in the extremal case, but for rescaled values of the electric and magnetic charges. The function f has a completely universal form:

$$e^{2f} = 1 - \frac{k}{r^{\tilde{d}}} , \quad (1.7)$$

where $\tilde{d} = D - d - 2$. If k is positive, the metric has an outer event horizon at $r = r_+ = k^{1/\tilde{d}}$. When $k = 0$, the solution becomes extremal, and the horizon coincides with the location of the curvature singularity at $r = 0$.

The temperature of a black p -brane can be calculated by examining the behaviour of the metric (1.6) in the Euclidean regime in the vicinity of the outer horizon $r = r_+$. Setting $t = i\tau$ and $1 - kr^{-\tilde{d}} = \rho^2$, the metric (1.6) becomes

$$ds^2 = \frac{4r_+^2}{\tilde{d}^2} e^{2B(r_+)} \left(d\rho^2 + \frac{\tilde{d}^2}{4r_+^2} e^{2A(r_+) - 2B(r_+)} \rho^2 d\tau^2 + \dots \right). \quad (1.8)$$

We see that the conical singularity at the outer horizon ($\rho = 0$) is avoided if τ is assigned the period $(4\pi r_+ / \tilde{d}) e^{B(r_+) - A(r_+)}$. The inverse of this periodicity in imaginary time is the Hawking temperature,

$$T = \frac{\tilde{d}}{4\pi r_+} e^{A(r_+) - B(r_+)}. \quad (1.9)$$

We may also calculate the entropy per unit p -volume of the black p -brane, which is given by one quarter of the area of the outer horizon. Thus we have

$$S = \frac{1}{4} r_+^{\tilde{d}+1} e^{(\tilde{d}+1)B(r_+) + (d-1)A(r_+)} \omega_{\tilde{d}+1}, \quad (1.10)$$

where $\omega_{\tilde{d}+1} = 2\pi^{\tilde{d}/2+1} / (\frac{1}{2}\tilde{d})!$ is the volume of the unit $(\tilde{d} + 1)$ -sphere

In subsequent sections, we shall generalise various kinds of extremal p -brane solutions to obtain black single-scalar elementary and solitonic p -branes, black dyonic p -branes and black multi-scalar p -branes. The metric ansatz (1.6) gives rise to non-isotropic p -brane solutions for $d \geq 2$, in the sense that the Poincaré symmetry of the d -dimensional world volume is broken. When $d = 1$, however, the black hole solutions remain isotropic. In the extremal black hole solutions, the quantity $dA + \tilde{d}B$ vanishes, where A and B are defined in (1.4); whilst in the non-extremal cases, this quantity is non-vanishing. Isotropic p -brane solutions with $dA + \tilde{d}B \neq 0$ were discussed in [42].

2 Single-scalar black p -branes

The Lagrangian (1.1) can be consistently reduced to a Lagrangian for a single scalar and a single field strength

$$e^{-1} \mathcal{L} = R - \frac{1}{2} (\partial\phi)^2 - \frac{1}{2n!} e^{a\phi} F^2, \quad (2.1)$$

where a , ϕ and F are given by [30]

$$\begin{aligned} a^2 &= \left(\sum_{\alpha, \beta} (M^{-1})_{\alpha\beta} \right)^{-1}, & \phi &= a \sum_{\alpha, \beta} (M^{-1})_{\alpha\beta} \vec{a}_\alpha \cdot \vec{\phi}, \\ (F_\alpha)^2 &= a^2 \sum_{\beta} (M^{-1})_{\alpha\beta} F^2, \end{aligned} \quad (2.2)$$

and $M_{\alpha\beta} = \vec{a}_\alpha \cdot \vec{a}_\beta$. The parameter a can conveniently be re-expressed as

$$a^2 = \Delta - \frac{2d\tilde{d}}{D-2}, \quad (2.3)$$

where Δ is a parameter that is preserved under dimensional reduction [32]. Supersymmetric p -brane solutions can arise only when the value of Δ is given by $\Delta = 4/N$, with N field strengths participating in the solution. This occurs when the dot products of the dilaton vectors \vec{a}_α satisfy [31]

$$M_{\alpha\beta} = 4\delta_{\alpha\beta} - \frac{2d\tilde{d}}{D-2}. \quad (2.4)$$

An interesting special case is provided by the four-dimensional black holes with $a^2 = 3, 1, 1/3, 0$, i.e $N = 1, 2, 3, 4$ whose extremal limits admit the interpretation of 1, 2, 3, 4-particle bound states at threshold [11, 15, 16]. Their $D = 11$ interpretation has recently been discussed in [39, 40].

To begin, let us consider the more general metric

$$ds^2 = -e^{2u} dt^2 + e^{2A} dx^i dx^i + e^{2v} dr^2 + e^{2B} r^2 d\Omega^2. \quad (2.5)$$

It is straightforward to show that the Ricci tensor for this metric has the following non-vanishing components

$$\begin{aligned} R_{00} &= e^{2(u-v)} \left(u'' - u'v' + v'^2 + (d-1)u'A' + (\tilde{d}+1)u'(B' + \frac{1}{r}) \right), \\ R_{ij} &= -e^{2(A-v)} \left(A'' - A'v' + A'u' + (d-1)A'^2 + (\tilde{d}+1)A'(B' + \frac{1}{r}) \right) \delta_{ij}, \\ R_{rr} &= -u'' + u'v' - u'^2 - (d-1)A'' + (d-1)A'v' - (d-1)A'^2 - (\tilde{d}+1)B'' \\ &\quad + \frac{\tilde{d}+1}{r}v' - \frac{2(\tilde{d}+1)}{r}B' + (\tilde{d}+1)v'B' - (\tilde{d}+1)B'^2, \\ R_{ab} &= -e^{2(B-v)} \left(B'' + (B' + \frac{1}{r})[u' - v' + (d-1)A' + (\tilde{d}+1)(B' + \frac{1}{r})] - \frac{1}{r^2} \right) g_{ab} + \tilde{d}g_{ab}, \end{aligned} \quad (2.6)$$

where a prime denotes a derivative with respect to r , and g_{ab} is the metric on the unit $(\tilde{d}+1)$ -sphere. For future reference, we note that the ADM mass per unit p -volume for this metric is given by [43]

$$m = \left[(d-1)(e^{2A})'r^{\tilde{d}+1} + (\tilde{d}+1)(e^{2B})'r^{\tilde{d}+1} - (\tilde{d}+1)(e^{2v} - e^{2B})r^{\tilde{d}} \right] \Big|_{r \rightarrow \infty}. \quad (2.7)$$

The Ricci tensor for the metric (1.6) is given by (2.6) with $u = 2(A+f)$ and $v = 2(B-f)$. As in the case of isotropic p -brane solutions, the equations of motion simplify dramatically after imposing the ansatz

$$dA + \tilde{d}B = 0. \quad (2.8)$$

Furthermore, the structure of the equations of motion implies that it is natural to take

$$f'' + \frac{\tilde{d}+1}{r}f' + 2f'^2 = 0, \quad (2.9)$$

which has the solution given by (1.7). Note that we have chosen the asymptotic value of f to be zero at $r = \infty$. This is necessary in order that the metric (1.6) be Minkowskian at $r = \infty$. The equations of motion then reduce to the following three simple equations:

$$\begin{aligned} \phi'' + \frac{\tilde{d}+1}{r}\phi' + 2\phi'f' &= -\frac{\epsilon a}{2}s^2e^{-2f}, \\ A'' + \frac{\tilde{d}+1}{r}A' + 2A'f' &= \frac{\tilde{d}}{2(D-2)}s^2e^{-2f}, \\ d(D-2)A' + \frac{1}{2}\tilde{d}\phi'^2 + 2(D-2)A'f' &= \frac{1}{2}\tilde{d}s^2e^{-2f}, \end{aligned} \quad (2.10)$$

where s is given by

$$s = \lambda e^{-\frac{1}{2}\epsilon a\phi + dA} r^{-(\tilde{d}+1)}, \quad (2.11)$$

and $\epsilon = 1$ for elementary solutions and $\epsilon = -1$ for solitonic solutions. The last equation in (2.10) is a first integral of the first two equations, and hence determines an integration constant. The first two equations in (2.10) imply that we can naturally solve for the dilaton ϕ by taking $\phi = a(D-2)A/\tilde{d}$. The remaining equation can then be easily solved by making the ansatz that the function A takes the identical form as in the extremal case, but with a rescaled charge, *i.e.* it satisfies

$$A'' + \frac{\tilde{d}+1}{r}A' = \frac{\tilde{d}}{2(D-2)}\tilde{s}^2, \quad \text{with} \quad \tilde{s} = \tilde{\lambda} e^{-\frac{1}{2}\epsilon a\phi + dA} r^{-(\tilde{d}+1)}. \quad (2.12)$$

This has the solution $e^{-(D-2)\Delta A/(2\tilde{d})} = 1 + \tilde{\lambda}\sqrt{\Delta}/(2\tilde{d})r^{-\tilde{d}}$. Thus from (2.10) we have

$$2A'f' = (A'' + \frac{\tilde{d}+1}{r}A')(-1 + \frac{\lambda^2}{\tilde{\lambda}^2}e^{-2f}), \quad (2.13)$$

implying

$$\frac{\lambda^2}{\tilde{\lambda}^2} - e^{2f} = c(1 + \frac{\tilde{\lambda}\sqrt{\Delta}}{2\tilde{d}}r^{-\tilde{d}}), \quad (2.14)$$

where c is an integration constant. Substituting (1.7) into this, we deduce that

$$e^{-(D-2)\Delta A/(2\tilde{d})} = 1 + \frac{k}{r^{\tilde{d}}}\left(\frac{\lambda^2}{\tilde{\lambda}^2} - 1\right)^{-1}, \quad (2.15)$$

Thus it is natural to set $\tilde{\lambda} = \lambda \tanh \mu$, giving

$$e^{-(D-2)\Delta A/(2\tilde{d})} = 1 + \frac{k}{r^{\tilde{d}}}\sinh^2 \mu. \quad (2.16)$$

The blackened single-scalar p -brane solution is therefore given by

$$\begin{aligned}
ds^2 &= \left(1 + \frac{k}{r^{\tilde{d}}} \sinh^2 \mu\right)^{-\frac{4\tilde{d}}{\Delta(D-2)}} (-e^{2f} dt^2 + dx^i dx^i) \\
&\quad + \left(1 + \frac{k}{r^{\tilde{d}}} \sinh^2 \mu\right)^{\frac{4\tilde{d}}{\Delta(D-2)}} (e^{-2f} dr^2 + r^2 d\Omega^2), \\
e^{\frac{\epsilon\Delta}{2a}\phi} &= 1 + \frac{k}{r^{\tilde{d}}} \sinh^2 \mu, \quad e^{2f} = 1 - \frac{k}{r^{\tilde{d}}}, \tag{2.17}
\end{aligned}$$

with the two free parameters k and μ related to the charge λ and the mass per unit p -volume, m . Specifically, we find that

$$\lambda = \frac{\tilde{d}k}{\sqrt{\Delta}} \sinh 2\mu, \quad m = k \left(\frac{4\tilde{d}}{\Delta} \sinh^2 \mu + \tilde{d} + 1 \right). \tag{2.18}$$

The extremal limit occurs when $k \rightarrow 0$, $\mu \rightarrow \infty$ while holding $ke^{2\mu} = \sqrt{\Delta}\lambda/\tilde{d} = \text{constant}$.

If k is non-negative, the mass and charge satisfy the bound

$$m - \frac{2\lambda}{\sqrt{\Delta}} = \frac{k}{\Delta} \left[(\tilde{d} + 1)\Delta - 2\tilde{d} + 2\tilde{d}e^{-2\mu} \right] \geq \frac{2k\tilde{d}^2(d-1)}{\Delta(D-2)} \geq 0, \tag{2.19}$$

where the inequality is derived from $\Delta = a^2 + 2d\tilde{d}/(D-2) \geq 2d\tilde{d}/(D-2)$. The mass/charge bound (2.19) is saturated when k goes to zero, which is the extremal limit. In cases where $\Delta = 4/N$, the extremal solution becomes supersymmetric, and the bound (2.19) coincides with the Bogomol'nyi bound. Note however that in general there can exist extremal classical p -brane solutions for other values of Δ , which preserve no supersymmetry [30].

It follows from (1.9) and (1.10) that the Hawking temperature and entropy of the black p -brane (2.17) are given by

$$T = \frac{\tilde{d}}{4\pi r_+} \left(\cosh \mu \right)^{-\frac{4}{\Delta}}, \quad S = \frac{1}{4} r_+^{\tilde{d}+1} \omega_{\tilde{d}+1} \left(\cosh \mu \right)^{\frac{4}{\Delta}}. \tag{2.20}$$

In the extremal limit, they take the form

$$T \propto (e^\mu)^{2(a^2 - \frac{2\tilde{d}^2}{D-2})/(\Delta\tilde{d})}, \quad S \propto e^{\mu(4/\Delta - 2(\tilde{d}+1)/\tilde{d})}. \tag{2.21}$$

Thus the entropy becomes zero in the extremal limit $\mu \rightarrow \infty$, unless the constant a is zero and $d = 1$, since the exponent can be rewritten as $\mu(4/\Delta - 2(\tilde{d} + 1)/\tilde{d}) = -2\mu \left(2(d-1)\tilde{d}/(D-2) + (\tilde{d} + 1)a^2/\tilde{d} \right)/\Delta$. In these special cases the dilaton ϕ vanishes and the entropy is finite and non-zero. The situation can arise for black holes with $\Delta = 4/3$ in $D = 5$, and $\Delta = 1$ in $D = 4$. The temperature of the extremal p -brane is zero, finite and non-zero, or infinite, according to whether $(a^2 - \frac{2\tilde{d}^2}{D-2})$ is negative, zero or positive.

3 Black dyonic p -branes

Dyonic p -brane occur in dimensions $D = 2n$, where the n -index field strengths can carry both electric and magnetic charges. There are two types of dyonic solution. In the first type, each individual field strength in (1.1) carries either electric charge or magnetic charge, but not both. The construction of black dyonic p -branes of this type is identical to that for the solutions with purely electric or purely magnetic charges, which we discussed in the previous section.

In this section, we shall construct black dyonic p -branes of the second type, where there is one field strength, which carries both electric and magnetic charge. The Lagrangian is again given by (2.1), with the field strength now taking the form

$$F = \lambda_1 \epsilon_n + \lambda_2 * \epsilon_n . \quad (3.1)$$

As in the case of purely elementary or purely solitonic p -brane solutions, we impose the conditions (2.8) and (2.9) on B and f respectively. The equations of motion then reduce to

$$\begin{aligned} \phi'' + \frac{n}{r} \phi' + 2\phi' f' &= \frac{1}{2} a (s_1^2 - s_2^2) e^{-2f} , \\ A'' + \frac{n}{r} A' + 2A' f' &= \frac{1}{4} (s_1^2 + s_2^2) e^{-2f} , \\ d(D-2)A'^2 + \frac{1}{2} \tilde{d} \phi'^2 + 2(D-2)A' f' &= \frac{1}{2} \tilde{d} (s_1^2 + s_2^2) e^{-2f} , \end{aligned} \quad (3.2)$$

where

$$s_1 = \lambda_1 e^{\frac{1}{2} a \phi + (n-1)A} r^{-n} , \quad s_2 = \lambda_2 e^{-\frac{1}{2} a \phi + (n-1)A} r^{-n} . \quad (3.3)$$

We can solve the equations (3.2) for black dyonic p -branes by following analogous steps to those described in the previous section, relating the solutions to extremal dyonic solutions. In particular, we again find that the functions A , B and ϕ take precisely the same forms as in the extremal case, but with rescaled values of charges. Solutions for extremal dyonic p -branes are known for two values of a , namely $a^2 = n-1$ and $a = 0$ [30]. When $a^2 = n-1$, we find that the black dyonic p -brane solution is given by

$$e^{-\frac{1}{2} a \phi - (n-1)A} = 1 + \frac{k}{r^{n-1}} \sinh^2 \mu_1 , \quad e^{\frac{1}{2} a \phi - (n-1)A} = 1 + \frac{k}{r^{n-1}} \sinh^2 \mu_2 , \quad (3.4)$$

with f given by (1.7). The mass per unit volume and the electric and magnetic charges are given by

$$m = k(2 \sinh^2 \mu_1 + 2 \sinh^2 \mu_2 + 1) , \quad \lambda_\alpha = (ak/\sqrt{2}) \sinh(2\mu_\alpha) . \quad (3.5)$$

For the non-negative values of k , the mass and the charges satisfy the bound

$$m - (\lambda_1 + \lambda_2) = k(n-2 + e^{-2\mu_1} + e^{-2\mu_2}) \geq 0 . \quad (3.6)$$

The bound is saturated in the extremal limit $k \rightarrow 0$. The solution (3.4) corresponds to the black dyonic string with $n = 3$ and $\Delta = 4$ in $D = 6$, and the dyonic black hole with $n = 2$ and $\Delta = 2$ in $D = 4$. In both cases, the extremal solution is supersymmetric and the bound (3.6) coincides with the Bogomol'nyi bound. Using (1.9) and (1.10), we find that the Hawking temperature and entropy of the non-extremal solutions are given by

$$T = \frac{\tilde{d}}{4\pi r_+} \left(\cosh \mu_1 \cosh \mu_2 \right)^{-\frac{2}{n-1}}, \quad S = \frac{1}{4} r_+^n \omega_n \left(\cosh \mu_1 \cosh \mu_2 \right)^{\frac{2}{n-1}}. \quad (3.7)$$

When $a = 0$, the equations of motion degenerate and the dilaton ϕ decouples. We find the solution

$$\phi = 0, \quad e^{-(n-1)A} = 1 + \frac{k}{r^{n-1}} \sinh^2 \mu, \quad (3.8)$$

where again f is given by (1.7). The constant μ is related to the electric and magnetic charges by $\sqrt{\lambda_1^2 + \lambda_2^2} = k \sinh 2\mu$. In this case, unlike the $a^2 = n - 1$ case, the solution is invariant under rotations of the electric and magnetic charges, and hence it is equivalent to the purely electric or purely magnetic solutions we discussed in the previous section. Note that in the dyonic solution (3.4), when the parameter $\mu_1 = \mu_2$, *i.e.* the electric and magnetic charges are equal, the dilaton field also decouples. For example, this can happen if one imposes a self-dual condition on the 3-form field strength in the dyonic string in $D = 6$. However, this is a different situation from the $a = 0$ dyonic solution, since in the latter case the electric and magnetic charges are independent free parameters. In fact the $a = 0$ dyonic solution with independent electric and magnetic charges occurs only in $D = 4$.

4 Black multi-scalar p -branes

To describe multi-scalar p -brane solutions, we return to the Lagrangian (1.1) involving N scalars and N field strengths. As we discussed previously, it can be consistently truncated to the single-scalar Lagrangian (2.1), in which case all the field strengths F_α are proportional to the canonically-normalised field strength F , and hence there is only one independent charge parameter. In a multi-scalar p -brane solution, the charges associated with each field strength become independent parameters. After imposing the conditions (2.8) and (2.9), the equations of motion reduce to

$$\begin{aligned} \varphi''_\alpha + \frac{\tilde{d}+1}{r} \varphi'_\alpha + 2\varphi'_\alpha f' &= -\frac{1}{2} \epsilon e^{-2f} \sum_{\beta=1}^N M_{\alpha\beta} S_\beta^2, \\ A'' + \frac{\tilde{d}+1}{r} A' + 2A' f' &= \frac{\tilde{d}}{2(D-2)} e^{-2f} \sum_{\alpha=1}^N S_\alpha^2, \end{aligned} \quad (4.1)$$

$$d(D-2)A'^2 + \frac{1}{2}\tilde{d} \sum_{\alpha,\beta=1}^N (M^{-1})_{\alpha\beta} \varphi'_\alpha \varphi'_\beta + 2(D-2)A'f' = \frac{1}{2}\tilde{d}e^{-2f} \sum_{\alpha=1}^N S_\alpha^2,$$

where $\varphi_\alpha = \vec{a}_\alpha \cdot \vec{\phi}$ and $S_\alpha = \lambda_\alpha e^{-\frac{1}{2}\epsilon\varphi_\alpha + dA} r^{-(\tilde{d}+1)}$. We again find black solutions by taking A and φ_α to have the same forms as in the extremal case, with rescaled charges. Extremal solutions can be found in cases where the dot products of the dilaton vectors \vec{a}_α satisfy (2.4) [30]. Thus we find that the corresponding black solutions are given by

$$\begin{aligned} e^{\frac{1}{2}\epsilon\varphi_\alpha - dA} &= 1 + \frac{k}{r^{\tilde{d}}} \sinh^2 \mu_\alpha, & e^{2f} &= 1 - \frac{k}{r^{\tilde{d}}}, \\ ds^2 &= \prod_{\alpha=1}^N \left(1 + \frac{k}{r^{\tilde{d}}} \sinh^2 \mu_\alpha\right)^{-\frac{\tilde{d}}{D-2}} (-e^{2f} dt^2 + dx^i dx^i) \\ &+ \prod_{\alpha=1}^N \left(1 + \frac{k}{r^{\tilde{d}}} \sinh^2 \mu_\alpha\right)^{\frac{\tilde{d}}{D-2}} (e^{-2f} dr^2 + r^2 d\Omega^2). \end{aligned} \quad (4.2)$$

The mass per unit volume and the charges for this solution are given by

$$m = k(\tilde{d} \sum_{\alpha=1}^N \sinh^2 \mu_\alpha + \tilde{d} + 1), \quad \lambda_\alpha = \frac{1}{2}\tilde{d}k \sinh 2\mu_\alpha. \quad (4.3)$$

For non-negative values of k , the mass and charges satisfy the bound

$$m - \sum_{\alpha=1}^N \lambda_\alpha = \frac{1}{2}k\tilde{d} \sum_{\alpha=1}^N (e^{-2\mu_\alpha} - 1) + k(\tilde{d} + 1) \geq \frac{k\tilde{d}(d-1)}{d} \geq 0. \quad (4.4)$$

The bound coincides with the Bogomol'nyi bound. The Hawking temperature and entropy are given by

$$T = \frac{\tilde{d}}{4\pi r_+} \prod_{\alpha=1}^N (\cosh \mu_\alpha)^{-1}, \quad S = \frac{1}{4}r_+^{\tilde{d}+1} \omega_{\tilde{d}+1} \prod_{\alpha=1}^N (\cosh \mu_\alpha). \quad (4.5)$$

In the extremal limit $k \rightarrow 0$, the bound (4.4) is saturated, and the solutions become supersymmetric.

5 Conclusions

We have presented a class of black p -brane solutions of M-theory which were hitherto known only in the extremal supersymmetric limit and have calculated their macroscopic entropy and temperature. It would obviously be of interest to provide a microscopic derivation of the entropy and temperature using D -brane techniques and compare them with the macroscopic results found in this paper. Agreement would both boost the credibility of M-theory and further our understanding of black hole and black p -brane physics.

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