

Dissipative Liouville Cosmology: A Case Study

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Abstract

We consider solutions of the cosmological equations pertaining to a dissipative, dilaton-driven off-equilibrium Liouville Cosmological model, which may describe the effective field theoretic limit of a non-critical string model of the Universe. The non-criticality may be the result of an early-era catastrophic cosmic event, such as a big-bang, brane-world collision *etc.* The evolution of the various cosmological parameters of the model are obtained, and the effects of the dilaton and off-shell Liouville terms, including briefly those on relic densities, which distinguish the model from conventional cosmologies, are emphasised.

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The current astrophysical data [1–4] are capable of placing stringent constraints on the nature of the dark energy, whose equation of state may be determined by means of an appropriate global fit. Most of the analyses so far are based on effective four-dimensional Robertson-Walker Universes, which satisfy on-shell dynamical equations of motion of the Einstein-Friedmann form. Even in modern approaches to brane cosmology, which are described by equations that deviate during early eras of the Universe from the standard Friedmann equation (which is linear in the energy density), the underlying dynamics is assumed to be of classical equilibrium (on-shell) nature, in the sense that it satisfies a set of equations of motion derived from the appropriate minimisation of an effective space-time Lagrangian.

However, cosmology may not be an entirely classical equilibrium situation [5, 6]. The initial Big Bang or other catastrophic cosmic event, such as the collision of two brane worlds in the modern context of brane theories [7], which led to the initial rapid expansion of the Universe, may have caused a significant departure from classical equilibrium dynamics in the early Universe, whose signatures may still be present at later epochs including the present era. In [6, 8] there have been proposed specific models for the cosmological dark energy which are of this type, being associated with a rolling dilaton field that is a remnant of this non-equilibrium phase, described by a generic non-critical string theory [5, 9, 10]. The basic ingredient of this approach is the identification of target time with a local, dynamical (irreversible) renormalization group scale on the world sheet of the string [7, 11], being representing by the so-called Liouville mode [5, 10]. The consistency of the approach is guaranteed by the existence of solutions to the pertinent equations (“Liouville conditions”) for the various background target-space fields over which the non-critical Liouville-dressed [9] σ -model propagates. The latter express the restoration of conformal invariance conditions after Liouville dressing. We call this scenario ‘Q-cosmology’ [6].

It must be stressed that this Q-cosmology is physically very different from standard dilaton cosmologies in critical strings [12], where on-shell equations of motion for the background fields are satisfied. Our non-equilibrium, non-classical theory *is not described by the equations of motion derived by extremising an effective space-time Lagrangian*. One must use a more general formalism to make predictions that can be confronted with the current data. The approach we favour is formulated in the context of string/brane theory [7, 11], the best candidate theory of quantum gravity to date. Our approach is based on non-critical (Liouville) strings [5, 9, 10], which offer a mathematically consistent way of incorporating time-dependent backgrounds in string theory.

The basic idea behind such non-critical Liouville strings is the following. Usually,

in string perturbation theory, the target space dynamics is obtained from a stringy σ -model [11] that describes the propagation of strings in classical target-space background fields, including the space-time metric itself. Consistency of the theory requires conformal invariance on the world sheet, in which case the target-space physics is independent of the scale characterising the underlying two-dimensional dynamics. These conformal invariance conditions lead to a set of target-space equations for the various background fields, which correspond to the Einstein/matter equations derived from an appropriate low-energy effective action that is invariant under general coordinate transformations. Unfortunately, one cannot incorporate in this way time-dependent cosmological backgrounds in string theory, since, to low orders in a perturbative expansion in the Regge slope α' , the conformal invariance condition for the metric field would require a Ricci-flat target-space manifold, whereas a cosmological background necessarily has a non-vanishing Ricci tensor.

To be able to describe a time-dependent cosmological background in string theory, the authors of [5] suggested that a non-trivial rôle should be played by a time-dependent dilaton background. This approach leads to strings living in numbers of dimensions different from the customary critical number, and was in fact the first physical application of non-critical strings [9]. The approach of [5] was subsequently extended [10], [6], [8] to incorporate off-shell quantum effects and non-conformal string backgrounds describing other non-equilibrium cosmological situations, including catastrophic cosmic events such as the collision of two brane worlds, *etc.*.

In a recent work [13] we have discussed fits of such non-critical Liouville cosmological dark energy models to the available data on high-redshift supernovae [1] and baryon oscillations [14]. It was found in that analysis that a simple parametrisation of the full version of the Q-cosmology model that includes off-shell effects fits the data very well. Specifically, we have shown that, under certain approximations, which allowed for an analytic solution of the pertinent Liouville conditions, the Hubble parameter of this model, $H(z)$, where z is the redshift, can be expressed in the form:

$$H(z) = H_0 \left(\Omega_3(1+z)^3 + \Omega_\delta(1+z)^\delta + \Omega_2(1+z)^2 \right)^{1/2} , \quad (1)$$

with the subscript 0 denoting present-day values ($z = 0$) and

$$\Omega_3 + \Omega_\delta + \Omega_2 = 1 , \quad (2)$$

The exponent δ was treated in [13] as a fitting parameter.

The basic parameter used to fit the supernova data is the luminosity distance $d_L = c(1+z) \int_0^z \frac{dz'}{H(z')}$, which is connected to the apparent magnitude of the supernovae. The data seem to favour the value $\delta \sim 4$, which can also be explained theoretically. As discussed in [13], the “dust”-like contributions, Ω_3 , do not merely represent ordinary matter effects, but also receive contributions from the dilaton dark energy. In fact, the sign of Ω_3 depends on details of the underlying theory, and it could even be negative. For instance, as argued in [13], string loop corrections could lead to a negative Ω_3 . In addition, Kaluza-Klein graviton modes in certain brane-inspired models [15] also yield negative dust contributions. In a similar vein, the exotic contributions scaling as $(1+z)^\delta$ are affected by the off-shell Liouville terms of Q-cosmology. It is because of the similar scaling behaviours of dark matter and dilaton dark energy that we reverted to the notation Ω_i , $i = 2, 3, \delta$ in (1). More generally, one could have included a cosmological constant Ω_Λ contribution in (1), which may be induced in certain brane-world inspired models. We did not do so in [13], as our primary interest is to fit Q-cosmology models [6, 8], which are characterised by dark energy densities that relax to zero asymptotically in cosmic time.

Some remarks are now in order. First, we stress that the above formulae are valid for late eras, such as the ones pertinent to the supernova and other data ($0 \leq z \leq 2$) that we used in [13]. Moreover, in the context of Q-cosmology, the form (1), can only be obtained after making a series of approximations, which may not always be valid, as already stressed in [13]. It is the purpose of this paper to present a more complete, numerical treatment of the Liouville conditions of a case study in Q-cosmology, and derive the behaviour of the various cosmological parameters, including $H(z)$, with the cosmic time (or, equivalently, z). For the interested reader, the details and terminology of Q-cosmology can be found in the relevant literature [5, 6, 10].

We commence our analysis by considering a Liouville-dressed non-critical σ -model propagating in cosmological dilaton and graviton backgrounds. After the identification of the target time with the Liouville mode [10], [6], [8] the relevant Liouville equations in the Einstein frame [5], i.e. in the frame where the scalar curvature in the (off-shell) target space effective action assumes the canonical Einstein form to leading order in the

Regge-slope α' expansion, are given by

$$\begin{aligned}
3 H^2 - \tilde{\varrho}_m - \varrho_\phi &= \frac{e^{2\phi}}{2} \tilde{\mathcal{G}}_\phi \\
2 \dot{H} + \tilde{\varrho}_m + \varrho_\phi + \tilde{p}_m + p_\phi &= \frac{\tilde{\mathcal{G}}_{ii}}{a^2} \\
\ddot{\phi} + 3H\dot{\phi} + \frac{1}{4} \frac{\partial \hat{V}_{all}}{\partial \phi} + \frac{1}{2} (\tilde{\varrho}_m - 3\tilde{p}_m) &= -\frac{3}{2} \frac{\tilde{\mathcal{G}}_{ii}}{a^2} - \frac{e^{2\phi}}{2} \tilde{\mathcal{G}}_\phi.
\end{aligned} \tag{3}$$

As usual dots denote derivatives with respect Einstein time. The r.h.s of these equations constitute a manifestation of the non-critical string, *off-shell*, behaviour. Such terms are absent in critical string cosmologies, such as the models considered in [12]. The dependence of the off-shell terms \mathcal{G} on the cosmic scale factor, dilaton and (square root of the) central charge deficit Q , is as follows:

$$\begin{aligned}
\tilde{\mathcal{G}}_\phi &= e^{-2\phi} (\ddot{\phi} - \dot{\phi}^2 + Qe^\phi \dot{\phi}) \\
\tilde{\mathcal{G}}_{ii} &= 2 a^2 (\ddot{\phi} + 3H\dot{\phi} + \dot{\phi}^2 + (1-q)H^2 + Qe^\phi(\dot{\phi} + H)).
\end{aligned} \tag{4}$$

Notice the *dissipative* terms proportional to $Q\dot{\phi}$, which is responsible for the terminology ‘‘Dissipative Cosmology’’ used alternatively for Q-cosmology [6]. In these equations q is the deceleration $q \equiv -\ddot{a}/\dot{a}^2$ as function of the time ¹. The variation of the central charge deficit Q with the cosmic time is provided by the Curci - Paffuti equation [16], which expresses the renormalisability of the world-sheet theory. To leading order in an α' expansion, which we restrict ourselves in this work, this equation in the Einstein frame is given by

$$\frac{d\tilde{\mathcal{G}}_\phi}{dt_E} = -6 e^{-2\phi} (H + \dot{\phi}) \frac{\tilde{\mathcal{G}}_{ii}}{a^2}. \tag{5}$$

We remind the reader once more that all quantities in the above equations refer to the Einstein frame [5], and derivatives are with respect the Einstein time t_E which is related to the real time t by $t = \omega^{-1} t_E$. ω is an arbitrary dimensionful constant with units of inverse time.

The potential appearing above is defined by $\hat{V}_{all} = 2Q^2 \exp(2\phi) + V$ where in order to cover more general cases we have allowed for a potential term in the string action $-\sqrt{-G} V$ in addition to that dependent on the central charge deficit term. Although we have assumed a (spatially) flat Universe the terms on the r.h.s., which manifest departure

¹The function $\tilde{\mathcal{G}}_{00}$, which is the 00 component of $\tilde{\mathcal{G}}_{\mu\nu}$, is zero since the corresponding component of the metric is constant.

from the criticality, act in a sense like curvature terms as being non-zero at certain epochs. The dilaton energy density and pressure are

$$\begin{aligned}\varrho_\phi &= \frac{1}{2} (2\dot{\phi}^2 + \hat{V}_{all}) \\ p_\phi &= \frac{1}{2} (2\dot{\phi}^2 - \hat{V}_{all}) .\end{aligned}\tag{6}$$

The dilaton field is not canonically normalised in this convention and its dimension has been set to zero.

In the following we shall drop the explicit potential term V and keep only that term proportional to the central charge deficit squared Q^2 . This is the model considered in [6, 8, 10]. Note, however, that, as mentioned previously, string loop corrections, which contribute to V , might be significant at present eras, as seems to be indicated by the data [13]. Such an issue can be answered only after making detailed fits of the current numerical results to the actual data, along the lines of [13]. This is left for future work.

The matter - energy density $\tilde{\varrho}_m$ is dimensionless and is related to the actual density, occurring in the system of equations where all quantities have their proper dimensions, through

$$\tilde{\varrho}_m = \frac{8\pi G_N}{\omega^2} \varrho_m .$$

It is evident that a convenient choice for ω is $\omega = \sqrt{3} H_0$ where H_0 is the value of the Hubble constant. With this choice $\tilde{\varrho}_m$ become exactly the ratio of the actual density to the critical density in the conventional Cosmology ϱ_c usually denoted by Ω . We shall adhere to this choice of ω in the following.

All equations can be converted to the string frame which is characterised by a string time t_s . Since the equations do not explicitly depend on time we are allowed to perform a shift in the variable t_s and take the string time in such a way that $t_s = 0$ corresponds to the present time. The reader should recall that the Einstein time is related to the string time t_s through [5]

$$t_E = \int_{t_{sing}}^{t_s} e^{-\phi} dt_s + C .$$

with C an arbitrary constant having no physical significance either. The lower limit of the integration designates the point of the initial singularity where Q or ϕ become infinite. The arbitrary constant C is usually taken to be zero so that in the Einstein frame the origin of the time, $t_{Einstein} = 0$, is when the singularity is created.

In our numerical procedure we found it more convenient, although physicswise less transparent, to work directly in the string frame. In such an approach, all equations and

boundary conditions should be converted to string coordinates by converting derivatives with respect t_E to those with respect t_s and using, at the same time, the string cosmic scale factor related to the Robertson Walker Friedman (RWF) cosmic scale factor a by $a_s = e^\phi a$. In this frame, in order to solve the system of equations, one needs the initial values of the quantities involved at some point which we take to be $t_s = 0$, that is the present time. Using the measured values of the present-day Hubble constant H_0 and the deceleration q_0 today, the system has a unique solution provided that the value of the dilaton field, the matter-energy density and the central charge deficit Q are also given.

As we shall demonstrate later on, $Q(t)$, which in our approach is in general cosmic time dependent [10], [8], is determined in terms of the matter-energy density, the value of the dilaton field today, as well as the values of H_0, q_0 , through an algebraic equation which follows from the system of differential equations. Moreover, when we convert the system of equations to a system of first order differential equations using as independent variable the string time, the differential equations, as well as the boundary conditions, are manifestly invariant under a constant dilaton shift $\phi \rightarrow \phi + k$, followed by a rescaling of the density ², by $\tilde{\varrho}_m \rightarrow \tilde{\varrho}_m e^{2k}$.

This observation is crucial, since it actually implies that, if a solution with a given initial condition is found, then another solution of the same problem can be generated by merely shifting the dilaton and rescaling the density as above. Due to this, the available parameter space is significantly reduced. In fact if one considers values for the matter density in the range $a \leq \tilde{\varrho}_m \leq b$, then all values in the stripe $-\infty \leq \phi \leq +\infty$, $a \leq \tilde{\varrho}_m \leq b$ of the dilaton-energy-density plane should be considered *a priori*. However, due to the above property one has actually to consider only the line segment $\phi = 0, a \leq \tilde{\varrho}_m \leq b$, $\phi > 0, \tilde{\varrho}_m = a$ and $\phi < 0, \tilde{\varrho}_m = b$. Any other point in the allowed stripe can be projected to these segments. This reduces considerably the numerical effort. Moreover from the values of ϕ , large positive values are not allowed since they correspond to large values of the string coupling outside the perturbative regime for which our equations hold. Only $\phi \sim \mathcal{O}(1)$ can therefore be trusted. This reduces the parameter space even more.

In order to obtain the evolution of the matter-energy density with time it is convenient to use the continuity equation for matter, which follows by combining the set of equations (3),

$$\frac{d\tilde{\varrho}_m}{dt_E} + 3H(\tilde{\varrho}_m + \tilde{p}_m) + \frac{\dot{Q}}{2} \frac{\partial \hat{V}_{all}}{\partial Q} - \dot{\phi} (\tilde{\varrho}_m - 3\tilde{p}_m) = 6 (H + \dot{\phi}) \frac{\tilde{\mathcal{G}}_{ii}}{a^2}. \quad (7)$$

²We tacitly assume that pressure and density for all matter species are related by an equation of state so that a boundary condition for density yields automatically a boundary condition for the pressure too.

To proceed further one needs to make some extra assumptions. First we assume that the matter-energy density is split as $\tilde{\varrho}_m = \varrho_b + \varrho_r + \varrho_e$: the first term refers to as "dust", $w_b = 0$, and includes the baryonic matter and any other sort of matter, characterized by $w = 0$, which does not feel the effect of the non-critical terms. The second term refers to radiation $w_r = \frac{1}{3}$ and the third to an unknown sort of *exotic* matter which is characterised by an equation of state having a weight w_e which in our analysis we consider it to be an arbitrary parameter to be fitted by data ³. So far this does not sound much like an assumption. However we further impose that the exotic matter feels all the effect of the non-critical terms unlike the rest of the species which follow continuity equations given by

$$\begin{aligned} \frac{d\varrho_r}{dt_E} + 4H\varrho_r &= 0 \\ \frac{d\varrho_b}{dt_E} + 3H\varrho_b - \dot{\phi}\varrho_b &= 0. \end{aligned} \tag{8}$$

Notice that the first of these equations is the well-known continuity equation for radiation, while the second differs from the ordinary continuity equation only by the appearance of the dilaton dependent terms and is the same as that given in [12]. In the case described above the exotic matter satisfies the equation (7) and it is the only species assumed to be affected by the non-criticality. If one does not like the concept of the exotic matter, then (s)he can set $\varrho_b = 0$ and let ϱ_e play the rôle of dust matter, choosing simultaneously $w_e = 0$. Dark Matter or any sort of as yet undiscovered matter, such as possible supersymmetric particles or other matter species, can be included in either ϱ_b or ϱ_e , although it seems plausible to be accomodated within ϱ_b . This scheme covers the more general cases encountered in phenomenological applications.

It is worth pointing out that, due to the appearance of the dilaton dependent term, the density ϱ_b , does not scale with a^{-3} as in the conventional Cosmology [6, 13]. The exotic piece certainly does not scale as "dust", not only because of the dilaton term and the value of w_e which may differ from zero, but also because it is affected by the presence of the non-critical, off-shell terms \mathcal{G} . It should be stressed that no cosmological constant is introduced by hand as this is provided, as a relaxation effect, by the dilaton energy density.

Due to the dilaton dependence, ϱ_b follows a continuity equation which includes a $-\dot{\phi}\varrho_b$

³Recall that with the choice $\omega = \sqrt{3}H_0$ adopted $\varrho_{b,r,e}$ are the matter-energy densities in units of the critical density.

term. This term should be duly taken into account in calculating relic abundances and depending on its sign may lead to over or under-production in addition to the usual picture where changes in the relic abundances occur only through interactions with the cosmic plasma. This holds independently of the non-criticality.

In order to be more specific, in the absence of dilaton couplings, the number density n of particles of species X , assumed to be a Dark Matter candidate, changes according to the Boltzmann equation [17]

$$\frac{dn}{dt} = -3 H n - \langle \sigma v \rangle (n^2 - n_{eq}^2). \quad (9)$$

According to this equation, the relevant density is diluted because of the Universe expansion, $H = \dot{a}/a > 0$, but it also changes as a result of interactions, specified by the cross section term σ . Then the pertinent energy density is given by $\varrho_X = n m_X$, from which we can obtain the relic density of species X as $\Omega_X h_0^2$, after solving the Boltzmann equation for n as a function of the cosmic time, $n(t)$, from the equilibrium epoch, before the freezing-point, to today's values.

However, in dilaton-driven non-critical string cosmologies, as becomes clear from our continuity equation derived from the Liouville conditions above, the energy-density of X changes as

$$\frac{d\varrho_X}{dt} = -3 H \varrho_X + \dot{\phi} \varrho_X + \mathcal{G} \quad (10)$$

with \mathcal{G} denoting the off-shell Liouville terms.⁴

Comparing (10) with (9), we observe that the conventional Boltzmann equation needs to be modified in Q-cosmology, in order to incorporate consistently the effects of the dilaton *dissipative* pressure $\sim \dot{\phi}$ and the non-critical terms, " \mathcal{G} " in the calculation of the relic density. A convenient and mathematically consistent way to include such effects is to modify the Boltzmann equation as follows:

$$\frac{dn}{dt} = -3 H n - \langle \sigma v \rangle (n^2 - n_{eq}^2) + \dot{\phi} n + \mathcal{G}/m_X. \quad (11)$$

In this equation one should be cautious to avoid interpreting the last two terms as changing the number density of particles X . Actually they only change their energy as is apparent from (10). Their inclusion in (11) is merely a handy way to include both effects, that of

⁴Here, in order to cover more general situations, we adopt the point of view that Dark Matter feels the off-shell Liouville terms, that is it belongs to the exotic matter part discussed previously. Certainly one can ignore the effect of these terms and treat in this respect Dark Matter like ordinary matter.

the change of the number density and the energy due to the dilaton interaction, into a single equation, the Boltzmann equation, and calculate, after solving it, the relic density in the usual manner using the equation $\Omega_X h_0^2 = n m_X h_0^2$.

The non-critical/dilaton modifications may have dramatic consequences for SUSY predictions [18] since the effect of the term $\dot{\phi}$ is comparable to that of the expansion term proportional to H , over large periods during the evolution of the Universe. This fact can already be seen in critical string dilaton-driven cosmologies [12] where the off-shell terms \mathcal{G} are absent, but one still considers the effects of the dilaton dissipative terms $\dot{\phi}$ in (11).

Following an otherwise standard analysis [17], it can be shown [19] that in such a case the pertinent relic density is

$$\begin{aligned} \Omega_X h_0^2 &= (\text{no - dilaton case}) \times \left(\frac{\tilde{g}_*}{g_*}\right)^{1/2} \left(1 + \int_{\chi_0}^{\chi_f} \frac{\dot{\phi} H^{-1}}{\mathcal{S}(\chi)} d\chi\right), \\ \rho_\phi + \rho_{\text{matter}} &\equiv \frac{\pi^2}{30} T^4 \tilde{g}, \quad \mathcal{S}(\chi) \equiv \chi \exp\left(-\int_{\chi_0}^{\chi_f} \frac{\dot{\phi} H^{-1}}{\chi'} d\chi'\right). \end{aligned} \quad (12)$$

In these equations $\chi \equiv T/m_X$ and χ_f, χ_0 refer to values at the freeze-out and today's temperature respectively. The subscript \star indicates values at the freeze-out point, and above we assumed that the presence of dilaton dissipative “source terms” proportional to $\dot{\phi}$ in the Boltzmann equation does not affect the thermal equilibrium, that defines the effective number of degrees of freedom \tilde{g} . The untilded quantities g indicate parameters in the no-dilaton case.

Moreover, the freeze-out point itself is affected by the presence of the dilaton-source terms. In fact its value is shifted from the standard no-dilaton value $\chi_f^{\text{no dil}}$ as:

$$\chi_f^{-1} = (\chi_f^{\text{no dil}})^{-1} + \frac{1}{2} \ln\left(\frac{\tilde{g}_*}{g_*}\right) + \int_{\chi_0}^{\chi_f} \frac{\dot{\phi} H^{-1}}{\chi} d\chi \quad (13)$$

The presence of off-shell Liouville terms complicate the situation, since, unlike the dilaton dissipative terms considered above, they cannot be simply expressed as Γn source terms in the Boltzmann equation. A complete treatment will be discussed in a future work [19], where the relevant constraints imposed for the parameter space of the popular supersymmetric schemes will also be addressed in a quantitative manner.

In the present work we shall limit our discussion to particular background solutions which at the present era describe a spatially flat and accelerating Universe as the cosmological data suggest [1, 2]. One can solve numerically the set of equations (3,7 and 5) in order to observe the time evolution of ϕ, a, Q and the densities ρ_b, ρ_r at various epochs,

as already discussed. Certainly not all of these equations are independent, as already remarked. For instance the continuity equation (7) follows from the set of equations (3) so that one of them is actually redundant.

In solving these equations we take as initial conditions the value of the Hubble constant H_0 , the deceleration parameter q_0 , at the present time, and also the values of the matter and radiation ϱ_b, ϱ_r as well as the value of the exotic matter ϱ_e and the value for w_e of its equation of state which we assumed constant. The radiation at present era is small and its value is known. For the other two it is reasonable to assume input values in the range $0.05 \leq \varrho_b + \varrho_e \leq 0.3$. The initial dilaton values can be non-vanishing but as already discussed the range of its allowed values is considerably reduced since we are forced to stay within the perturbation regime of the string theory. Thus the calculational task is easier than anticipated.

It should be remarked that the initial value of Q is not a free parameter. In principle it is calculated within the framework of the underlying conformal field theory model. From an effective theory point of view we adopt in this work, $Q(t)$ can be determined by combining appropriately the Liouville equations (3). The result is a quadratic algebraic equation

$$2 Q^2 - e^{-\phi} H Q + e^{-2\phi} \left(\dot{\phi}^2 - 8H^2 - 3H\dot{\phi} + \frac{5}{2}\varrho_b + \frac{8}{3}\varrho_r + \frac{5+w_e}{2}\varrho_e \right) = 0 ,$$

which relates $Q(t)$ to the remaining fields. One can be convinced of that by taking the differential of this equation which is proved to vanish if the set of equations (3,5) are satisfied.

This equation can then be used to obtain the value Q_0 of the central charge Q at zero string time, that is today, in terms of the remaining inputs. Using our numerical code we have shown that one of the roots of this equation leads to cosmologically sensible solutions, but the other fails to reproduce a reasonable picture of the evolution of the Universe.

The initial values H_0, q_0 by themselves determine the derivative of the dilaton field $\dot{\phi}_0$ today, as can be seen by combining the differential equations, and hence the dilaton kinetic energy. On the other hand from the previous equation it is obvious that the combination $\hat{Q} = Qe^\phi$ is uniquely determined for given densities and H_0, q_0 . As an effect the potential energy $2 Q^2 e^{2\phi}$ of the dilaton energy is also fixed. Therefore the dilaton energy and pressure today do not depend on the particular dilaton inputs and they are set once the Hubble constant, the deceleration and the today's values of the densities as well as the value of w_e are given.

In the following we present the results of our numerical analysis taking $H_0^{-1} = 13.4 \times 10^9$ yrs, corresponding to a rescaled Hubble constant $h_0 = 0.73$, $q_0 = -0.61$, $\varrho_r = 5.0 \times 10^{-5}$, which is today's value for the radiation density in units of the critical density, and various initial values for the "dust" and "exotic" densities, ϱ_b, ϱ_e . As already remarked, the value of w_e is also a free parameter which in our analysis we allow it to vary.

With these inputs we can solve the system of differential equations involved and follow the evolution of all parameters of interest to past and future times. In figures 1 to 3 we present sample outputs of our analysis for $\varrho_b = 0.238$, $\varrho_e = 0.0$ and $w_e = 0.5$. We have assumed that Dark Matter is contained in ϱ_b and the value 0.238 for ϱ_b corresponds to the observed central value $\Omega_{matter} h_0^2 = 0.127$, [2], with $h_0 = 0.73$. The value of the exotic matter density today is assumed vanishing and the dilaton initial value is taken $\phi_0 = 0.0$ as well. In the left panel of 1 we present the dilaton ϕ , the deficit Q and the ratio a/a_0 of the cosmic scale factor to its value today a_0 as functions of the Einstein time $t_{Einstein}$. The present time is located where $a/a_0 = 1$ and in the plot corresponds to $t_{today} \simeq 1.07$. The Bing Bang Singularity (BBS) is located at the point $t_{Einstein} = 0$. We have ignored at this level inflation. If inflation dynamics is included the solutions are expected to change slightly when we approach the time Universe exited from the inflation period. Also the string dynamics plays a significant rôle near BBS and on these grounds the solutions are expected to be modified near the origin of time. In fact in such early regimes our σ -model and $\mathcal{O}(\alpha')$ analysis breaks down. Notice that the solutions obtained for these inputs tend asymptotically in cosmic time [6,8] to the conformal invariant solutions found in [5]. This is a rather general feature. In the specific case considered here, these asymptotic forms are obtained for times $t \geq 1.3$ that is greater than the present time $t_{today} \sim 1.07$.

In all cases, in the present era the Universe has not reached, as yet, the asymptotic regime and the non-critical terms play a significant rôle. On the right panel of figure 1 the values of $\Omega_i \equiv \rho_i/\rho_c$ for the various species, $i = b, r, e$ and $i = \phi$ for the dilaton, as functions of $t_{Einstein}$ are shown. For comparison, the contribution of the non-critical terms to the Friedmann equation is also shown, labelled by Ω_{noncr} . Notice the behaviour of the exotic matter density which is slightly negative for times $t_{Einstein} > 1.0$, "dragged" mainly by the action of the non-critical terms.

The bulk of the total energy today is carried by the dilaton whose energy is almost four times the energy carried out by the dust and radiation together. This is more clearly seen on the left panel of figure 2 where the ratios of the dilaton, exotic matter and non-critical densities to the dust and radiation energy density are displayed. On the right panel of that figure the quantities $\rho_b a^3$, for "dust", $\rho_r a^4$ and $\rho_e a^2$ are shown. The dust

matter density deviates slightly from the law $\rho_b = \text{constant}/a^3$ at certain epochs due to its interaction with the dilaton. The exotic matter behaves almost as $\rho_e = \text{constant}/a^2$ asymptotically in agreement with the asymptotic solutions considered in [6].

On the left panel of figure 3 we present the deceleration q and the dimensionless Hubble expansion rate. The deceleration today has been taken $q_0 = -0.61$ and stays negative for future times. It is worth pointing out that the change from the acceleration to the deceleration phase started at redshifts $z \approx 0.20$. This is a general characteristic and not a specific feature of the particular sample outputs. For all cases the transition to acceleration occurred at redshifts $z \sim \mathcal{O}(0.15 - 0.20)$. We shall return to this point later.

On the right panel of figure 3 we present the derivative of the dilaton value, which governs the non-dissipative pressure term in the continuity equation for matter and exotic matter, as well as its ratio to \hat{H} . $\dot{\phi}$ behaves like $-1/t_{Einstein}$ asymptotically tending to zero for very large times, not shown in the figure. Its ratio to \hat{H} tends to -1 for large times. Although we are not in the asymptotic regime for the values shown in the figure this tendency is clearly seen. Recall the \hat{H} is defined by $\hat{H} \equiv H/\sqrt{3}H_0$, that is its value today is $1/\sqrt{3}$.

The fact that this ratio is of order of unity for certain epochs shows that it contributes on equal footing with the expansion rate and cannot be neglected. In certain epochs it acts constructively (destructively) with the expansion rate depending whether it is negative (positive), tending to decrease (increase) the energy density diluted by the expansion. This is so because it appears with negative sign relative to the Hubble rate term in the continuity equation (7). This is responsible for the behaviour of the dust density ρ_b for large times, $t_{Einstein} > 1$, which decreases at a rate faster than a^{-3} . Notice that for the exotic matter the effect of this term may be reversed since the relative strength of $\dot{\phi}$ to \hat{H} is weighted by the factor $\frac{1-3w_e}{1+w_e}$, as be seen from (7), which depending on the value of w_e may be negative or positive.

For the same inputs and changing only the initial value of the dilaton to $\phi_0 = -1.0$ we display the dilaton, the cosmic scale factor and the charge Q in the left panel of figure 4. The entrance into the asymptotic regime is delayed slightly due to the behaviour of Q . For positive initial values for the dilaton field the situation is altered. However for positive initial values the string coupling becomes large and the perturbative solutions are not valid. On the right panel of the same figure we display the deceleration and the \hat{H} for comparison with the case considered previously. Notice that in this case too the deceleration behaves almost in the same way.

Another important feature of the Q-cosmology is the relation of the present- and late

-eras acceleration of the Universe to the string coupling $g_s = e^\phi$. In [6], based on the detailed, but matterless, Liouville model of [8], it has been argued that the acceleration of the Universe at late-eras, where matter effects are suppressed, is proportional to the square of the string coupling

$$-q = g_s^2 \tag{14}$$

This seems to be a general feature of Liouville or Q-cosmology, which asymptotically turn to the solutions of [5].

A detailed test of (14) is presented on the left panel of figure 5. The input data are as in the previous figures with the exception of the dilaton whose value is taken ≈ 0.25 so that $|q|/g_s^2$ is unity at present time. Notice that its value smoothly tends to unity for late eras too not deviating significantly from unity in intermediate epochs. The rapid change of this ratio, near redshift values $z \approx 0.16$, signals the change of the sign in q from positive to negative values and hence the entrance to acceleration era. The string coupling constant is less than unity, as is shown in the right panel, which demonstrates that we are indeed within the perturbative regime of the string theory and our solutions are valid.

In figure 6 we display the deceleration as function of the redshift values in the range 0.0 – 1.0 to cover the range of the high- z supernovae data which provides evidence on entrance to acceleration period at redshifts in the vicinity $z \approx 0.2$. The shape of the curve is strikingly similar to figure 7 of ref. [20], in which a fit to SNeIa data is attempted using conventional cosmological models.

With these remarks we conclude our analysis of this case study of Q-cosmology. The numerical solutions we have found are compatible in general terms with the current situation of an accelerating Universe, suggested by the astrophysical data. However, to complete the analysis it is necessary to perform detailed fits of these results to the actual astrophysical data, along the lines of [13]. From our analysis above it becomes clear that the naive power-law behaviour in the redshift $(1+z)$ (1) of the various components appearing in the Hubble parameter $H(z)$, which was used for late eras in the analysis of [13] is not valid in the present model. More complex z -dependence appears to characterise $H(z)$ in our case. It remains to be seen whether the data favour this model, as compared with the other models in the literature.

Moreover, it is also of outmost importance to study the effects of the off-shell and dilaton dissipative terms on the available parameter space of the relic densities in the context of supersymmetric dark matter models in the Q-cosmology framework. As dis-

cussed briefly in this work, it is expected that the current constraints [18] will change, rather significantly, in view of the results of our analysis according to which in the present (and earlier eras) the effects of the off-shell Liouville terms are important. This is left for future work [19] .

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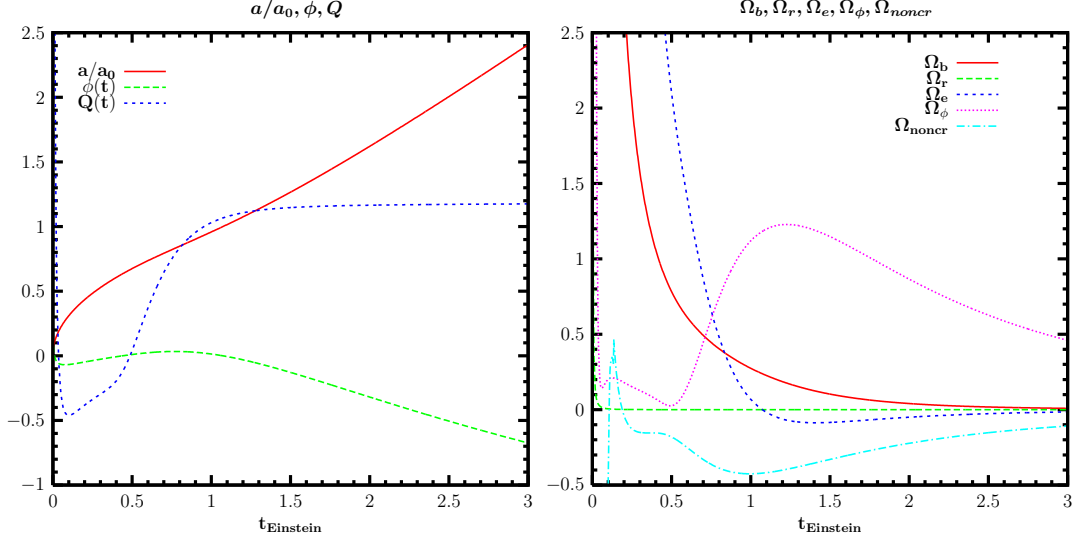


Figure 1: Left panel: The dilaton ϕ , the (square root of the) central charge deficit Q and the ratio a/a_0 of the cosmic scale factor as functions of the Einstein time $t_{Einstein}$. The present time is located where $a/a_0 = 1$ and in the figure shown corresponds to $t_{today} \simeq 1.07$. The input values for the densities are $\rho_b = 0.238$, $\rho_e = 0.0$ and w_e is 0.5. The dilaton value today is taken $\phi = 0.0$. Right panel: The values of $\Omega_i \equiv \rho_i/\rho_c$ for the various species as functions of $t_{Einstein}$.

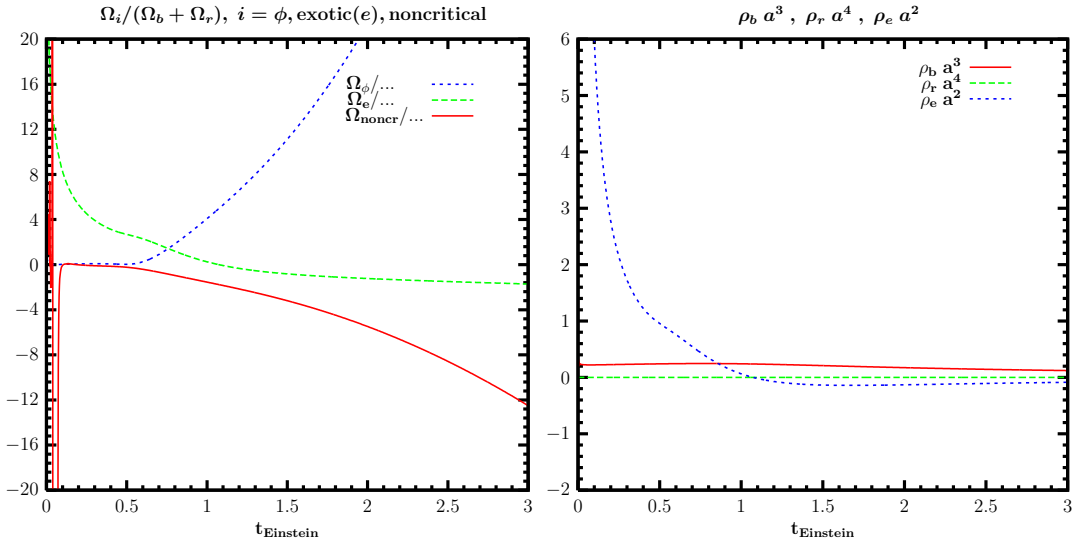


Figure 2: Left panel: Ratios of Ω 's for the dilaton (ϕ), exotic matter (e) and the non-critical terms ("noncrit") to the sum of "dust" and radiation $\Omega_b + \Omega_r$ densities. Right panel: The quantities $\rho_b a^3$, for "dust", $\rho_r a^4$ and $\rho_e a^2$ as functions of $t_{Einstein}$.

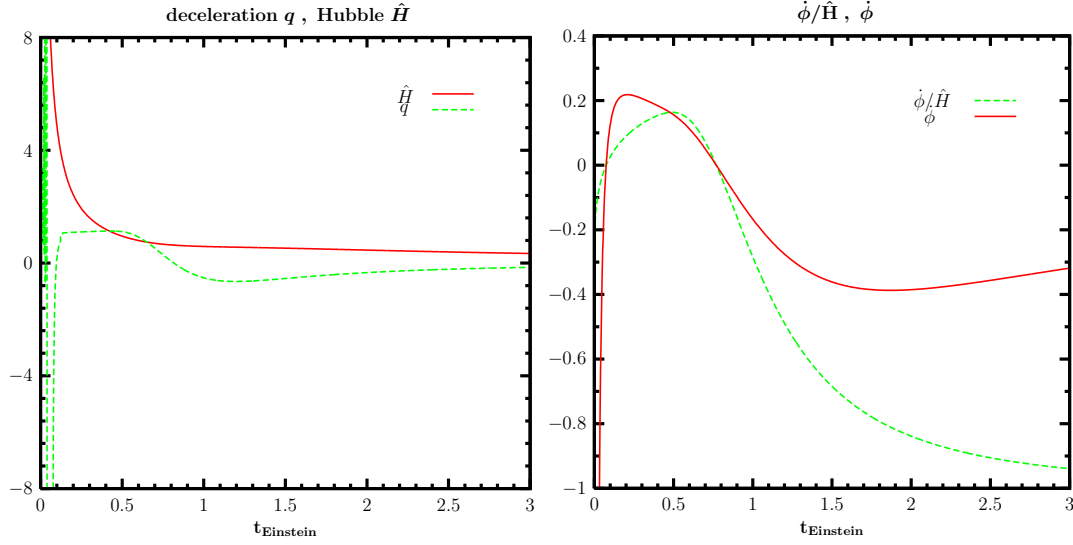


Figure 3: Left panel: The deceleration q and the dimensionless Hubble expansion rate $\hat{H} \equiv \frac{H}{\sqrt{3}H_0}$ as functions of $t_{Einstein}$. Right panel : The derivative of the dilaton and its ratio to the dimensionless expansion rate.

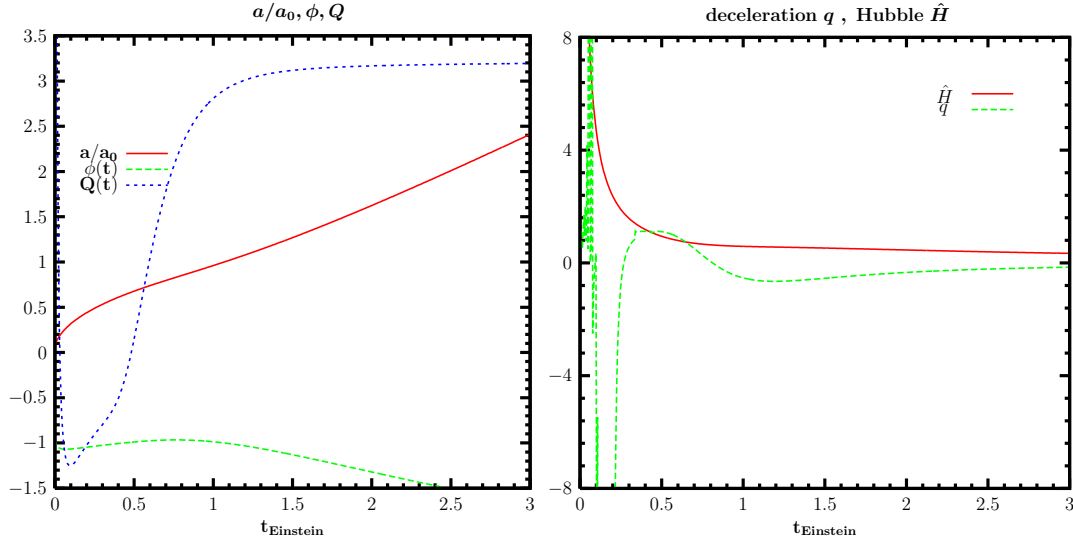


Figure 4: Left panel: The dilaton ϕ , the (square root of the) central charge deficit Q and the ratio a/a_0 of the cosmic scale factor as functions of the Einstein time $t_{Einstein}$. The inputs are as in figure 1 with only changing the dilaton to $\phi_0 = -1.0$. Right panel: The deceleration and \hat{H} for the same inputs.

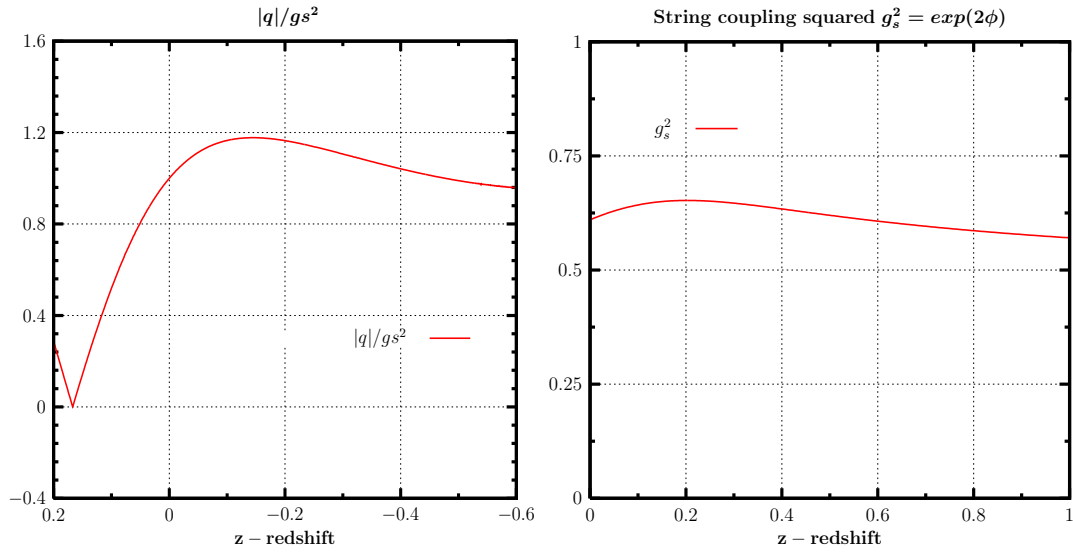


Figure 5: Left panel: The ratio $|q|/g_s^2$ as function of the redshift for z ranging from $z = 0.2$ to future values $z = -0.6$, for the inputs discussed in the main text. The rapid change near $z \approx 0.16$ signals the passage from deceleration to the acceleration period. Right panel: The values of the string coupling constant plotted versus redshift value in the range $z = 0.0 - 1.0$.

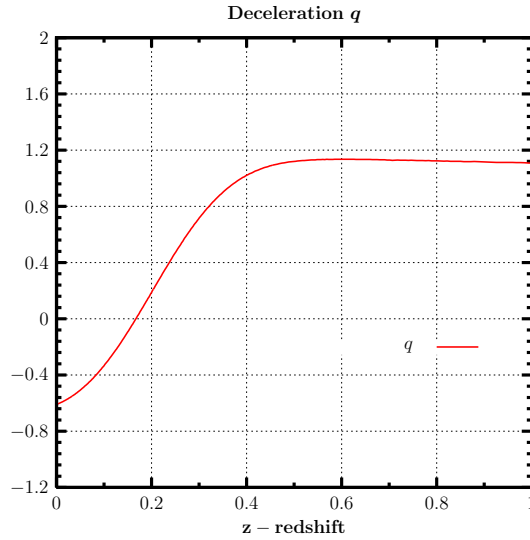


Figure 6: The deceleration as function of redshift values in the range $z = 0.0 - 1.0$. The inputs are as in figure 5.