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Flat Directions in Flipped $SU(5)$ I: All-Order Analysis

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Abstract

We present a systematic classification of field directions for the string–derived flipped $SU(5)$ model that are $D-$ and $F-$flat to all orders. Properties of the flipped $SU(5)$ model with field values in these directions are compared to those associated with other flat directions that have been shown to be $F-$flat to specific finite orders in the superpotential. We discuss the phenomenological Higgs spectrum, and quark and charged–lepton mass textures.

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1 Introduction

Over its approximately thirteen–year history [1, 2, 3], the string–derived supersymmetric flipped $SU(5)$ has become one of the more developed perturbative heterotic string models [4, 5, 6, 7, 8], and has achieved several phenomenological successes. Much of the strength and uniqueness of (supersymmetric) flipped $SU(5)$ lies in the fact that, unlike conventional GUT models based on E_6, $SO(10)$, or $SU(5)$ gauge groups, it can be broken to the Standard Model $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group without the need of adjoint or larger Higgs representations. This is important because it was proven long ago that the presence of massless adjoint or larger scalar multiplets was inconsistent with $N = 1$ or 0 spacetime supersymmetry in string models with an underlying level–1 Kač–Moody algebra [9]. In level–1 $SU(5)$, or flipped $SU(5)$, the only allowed massless representations are **1**, **5**, $\bar{\mathbf{5}}$, **10**, and $\bar{\mathbf{10}}$. These are not sufficient to break $SU(5) \to SU(3)_C \times SU(2)_L \times U(1)_Y$, but are sufficient to break flipped $SU(5) \to SU(3)_C \times SU(2)_L \times U(1)_Y \to SU(3)_C \times U(1)_{EM}$ [1].

In either conventional or flipped $SU(5)$ [10], a single generation of 16 matter fields (including a singlet 'right–handed' neutrino) can be accomodated by a set of **1**, $\bar{\mathbf{5}}$, and **10** representations. However, the flipped and standard versions of $SU(5)$ differ in how the 16 matter fields of each generation are embedded in these representations. Flipped $SU(5)$ received its name from the exchanges in the assignments of the fields: up–like and down–like fields are exchanged, as are electron–like with neutrino–like, as well as their anti–particle companions. Thus, in flipped $SU(5)$, the 16 components of a given generation are distributed as follows among a set of **1**, $\bar{\mathbf{5}}$, **10** representations: $\mathbf{1}_i = e^c_i$, $\bar{\mathbf{5}}_i = \{u^c_i, L_i\}$, $\mathbf{10}_i = \{Q_i, d^c_i, N^c_i\}$, where $i = 1, 2, 3$. This allows Higgs decuplets to include an electroweak singlet, and the appearance of a vacuum expectation value (VEV) for this singlet then breaks $SU(5)$ to $SU(3)_C \times SU(2)_L \times U(1)_Y$, rendering unnecessary adjoint or larger Higgs fields. However, we recall that the electroweak–doublet Higgs fields h_u and h_d of flipped $SU(5)$ appear in standard **5** and $\bar{\mathbf{5}}$ representations.

String–derived flipped $SU(5)$ was constructed in the free–fermion formulation [11] of the perturbative heterotic string. In principle, the superpotential terms in flipped $SU(5)$ or a similar free–fermion model can be calculated to any finite order, using the free–fermionic rules for level–one world–sheet field couplings that were developed some time ago [12, 13]. This has enabled the phenomenology of flipped $SU(5)$ to be studied in substantial detail in this perturbative regime. String–derived flipped $SU(5)$ has a characteristic that is generic to (quasi)–realistic $SU(3)_C \times SU(2)_L \times U(1)_Y$ or GUT models with three chiral generations that are of free–fermion, free–boson, or orbifold construction. Namely, the model contains several supplementary gauged Abelian symmetries, one of which, denoted by $U(1)_A$, is anomalous [14, 15, 16]. The anomaly appears because the trace of the $U(1)_A$ charge operator over the massless fields is non–zero: $\mathrm{Tr}\, Q^{(A)} \neq 0$.

The appearance of such an anomalous $U(1)_A$ has profound phenomenological effects. For instance, in a generic flipped $SU(5)$ model, such a $U(1)_A$ imposes constraints on fermion masses, R–violating couplings, and proton decay operators [17]. Much of the influence of a $U(1)_A$ in string models results as a by–product of the Green–Schwarz anomaly–cancellation mechanism and the retention of space–time supersymmetry following the cancellation. The latter requires several fields with anomalous charges to acquire VEVs along a 'flat direction', i.e., a direction in field space with vanishing scalar potential. This alters the classical vacuum of the model and hence the phenomenology [15, 16]. In this paper we explore field directions of the flipped $SU(5)$ model that are flat to all orders in the higher–order superpotential terms, and discuss various issues in their associated phenomenology. In Section 2 we briefly review the meaning of flat–direction VEVs and their associated D– and F–flatness constraints. Then, in Section 3 we present the set of all–order flat directions we have found for string–derived flipped $SU(5)$, along with a discussion how they were generated. In Section 4 we consider phenomenological features of these directions, and compare them with those of other field directions, whose flatness was proven only up to a finite order. We conclude our discussion in Section 5.

2 Generic Flat Directions

2.1 Constraints from D– and F–Flatness

In globally supersymmetric theories, such as the effective field theories derived from superstring models, there are both D terms, D^α_a, and F terms, F_{Φ_m}, contributing to the scalar potential:

$$V(\varphi) = \tfrac{1}{2}\sum_\alpha g_\alpha (\sum_{a=1}^{\dim(\mathcal{G}_\alpha)} D^\alpha_a D^\alpha_a) + \sum_m |F_{\Phi_m}|^2 \; . \tag{2.1}$$

There is a D term corresponding to each gauge group factor $\mathcal{G}_\alpha$, and the D^α_a in (2.1) have the general form

$$D^\alpha_a \;\equiv\; \sum_m \varphi^\dagger_m T^\alpha_a \varphi_m \; , \tag{2.2}$$

where T^α_a is a matrix generator of the gauge group $\mathcal{G}_\alpha$ for the representation φ_m. For an Abelian gauge group, (2.2) simplifies to

$$D^i \;\equiv\; \sum_m Q^{(i)}_m |\varphi_m|^2 \tag{2.3}$$

where $Q^{(i)}_m$ is the $U(1)_i$ charge of φ_m. We recall that D terms originate in the kinetic part of a supersymmetric lagrangian.

We also recall that there is an F term in (2.1) for each superfield Φ_m appearing in the superpotential:

$$F_{\Phi_m} \;\equiv\; \frac{\partial W}{\partial \Phi_m} \; . \tag{2.4}$$

Here, the φ_m are the scalar–field superpartners of the chiral spin–$\frac{1}{2}$ fermions ψ_m, which together form a superfield Φ_m.

We recall that, in such a globally supersymmetric theory, $\langle V\rangle > 0$ implies the breaking of space–time supersymmetry. Thus, since all of the D and F contributions to (2.1) are positive semi–definite, each must have a zero expectation value in order that $\langle V\rangle = 0$ and supersymmetry remains unbroken down to a relatively low mass scale.

An anomalous $U(1)_A$ makes its presence known in the low–energy effective field theory of a string model via triangle diagrams with gauge fields on all three external legs. Anomalies may appear in these triangle diagrams when either one or three of the external legs are associated with gauge bosons of the anomalous $U(1)_A$. In heterotic strings, the entire set of anomalous triangle diagrams is cancelled by an additional diagram generated by the VEV of the dilaton. This also adds to the D term of the anomalous $U(1)_A$ a Fayet–Iliopoulos (FI) term:

$$D^{(A)} \equiv \sum_m Q_m^{(A)}|\varphi_m|^2 + \epsilon\,; \quad \epsilon \;\equiv\; \frac{g^2_{\rm string}M_P^2}{192\pi^2}{\rm Tr}\,Q^{(A)}\,, \tag{2.5}$$

where g_s is the string coupling and M_P is the reduced Planck mass: $M_P \equiv M_{Planck}/\sqrt{8\pi} \approx 2.4\times 10^{18}$ GeV. By itself, the FI term would make a positive–definite contribution to the scalar potential: $\langle V\rangle \sim \frac{1}{2}g_A\epsilon^2$, and would break space–time supersymmetry at a scale $\sqrt{\epsilon}$. The recovery of supersymmetry requires a set of scalars to receive VEVs, in such a way that the total scalar VEV contribution to the anomalous D term cancels the FI contribution:

$$\langle D^{(A)}\rangle \;\equiv\; \sum_m Q_m^{(A)}|\langle\varphi_m\rangle|^2 + \epsilon = 0\,. \tag{2.6}$$

An anomalous $U(1)_A$ therefore induces a shift in the classical vacuum, while retaining flatness for the the non–anomalous Abelian D terms, the non–Abelian D terms, the F terms, and the superpotential as a whole:

$$\langle D^i\rangle = \langle D^\alpha_a\rangle = 0; \quad \langle F_{\Phi_m}\rangle = 0; \quad \text{and} \quad \langle W\rangle = 0\,. \tag{2.7}$$

The constraints (2.7) severely limit the set of scalars that could possibly be chosen non–perturbatively so as to satisfy (2.6).

2.2 Stringent F–Flatness and Non–Abelian Self–Cancellation

A given F term F_{Φ_m} may contain several components of similar or various orders n_i:

$$\langle F_{\Phi_m}\rangle \;\sim\; \sum_i \lambda_{n_i}\langle\varphi\rangle^2(\frac{\langle\varphi\rangle}{M_{string}})^{n_i-3}\,. \tag{2.8}$$

For a generic D–flat set of scalar VEVs, the resulting contributions to a given F–term will cancel among themselves only up to a given order $n_{i'}$. Then F–flatness, and thus supersymmetry, may in general be broken at order $n_{i'+1}$. In a particular model, F–flatness can often be verified up to a given order n_i for all F terms, but the exact order at which F–flatness disappears usually remains undetermined. It is clear that, the higher the order to which F–flatness is demanded, the fewer the D–flat directions that remain.

It is also clear that, the lower the order of an F–breaking term, the closer is the scale of supersymmetry breaking to the string scale. Since the FI scale is about an order of magnitude below the Planck scale, retention of space–time supersymmetry down to the electroweak scale in the observable sector probably requires F–flatness up to about the 17^{th} order in the weak–coupling limit, and to even higher orders as the coupling strength increases. For a generic D–flat direction, the flatness of each F term to such a high order would be extremely difficult to show if component cancellation is involved. However, for a subset of D–flat directions this can be avoided, and F–flatness can be shown to all finite orders. We term this subset of directions 'stringently' F–flat.

To be stringently F–flat means that each $\langle F_{\Phi_m} \rangle$ is zero, not because different components cancel among themselves, but because *each component* in $\langle F_{\Phi_m} \rangle$ is individually zero. For an F term containing only fields with Abelian charges, stringent flatness holds if each component of F_{Φ_m} has one or more fields that do not acquire VEVs. For an F term containing non–Abelian fields, this requirement can be relaxed slightly. Because non–Abelian fields contain more than one field component, *self–cancellation* [18] of a dangerous F term can sometimes occur along non–Abelian directions. That is, a contraction of two non–Abelian field VEVs may still be zero. Thus, for some directions it may be possible to maintain 'stringent' F–flatness even when dangerous F–breaking terms appear in the superpotential derived from string theory.

3 Flat Directions in Flipped $SU(5)$

In this Section, we investigate both Abelian (singlet) and non–Abelian stringently flat directions, along with 'self–cancelling' non–Abelian flat directions. We start by discussing the retention of F–flatness by self–cancellation in the flipped $SU(5)$ model, and determine means by which this might be implemented,* before moving on to investigate stringent flatness.

*Previous flipped $SU(5)$ investigations, such as [5] postulated the self–cancellation of otherwise dangerous F terms.

3.1 Self–Cancellation

The full gauge group of the string–derived flipped $SU(5)$ model is

$$[SU(5) \times U(1) \times \prod_{i=1}^{4} U(1)_i]_{\rm obs} \times [SO(10) \times SO(6)]_{\rm hid} \,. \tag{3.1}$$

Flat directions that cancel the FI term can be formed from Abelian fields carrying only $U(1)_i$ charges or from $SO(10)_{\rm hid}$ and $SO(6)_{\rm hid}$ fields that are also $SU(3)_C \times SU(2)_L \times U(1)_Y \in SU(5)$ singlets. Since we shall need many of its aspects, for convenience and completeness, the field content of the string–derived flipped $SU(5)$ model is displayed in Tables 1 and 2. The massless fields of $SO(10)_{\rm hid}$ are five fundamental vector $\mathbf{10}$'s, denoted by $T_{i=1\,{\rm to}\,5}$, while those of $SO(6)_{\rm hid}$ are five fundamental vector $\mathbf{6}$'s, denoted by $\Delta_{j=1\,{\rm to}\,5}$, and six pairs of $\mathbf{4}$ and $\bar{\mathbf{4}}$ spinors, denoted by $a_{k=1\,{\rm to}\,6}$ and $\bar{a}_{k=1\,{\rm to}\,6}$. Whilst the $\mathbf{10}$'s and $\mathbf{6}$'s are $SU(5) \times U(1)$ singlets, the $\mathbf{4}$ and $\bar{\mathbf{4}}$'s carry $U(1)$ charge $Q = \pm 5/4$, resulting in electric charges $Q_{\rm E} = \pm\frac{1}{2}$. Thus, the $\mathbf{4}$ and $\bar{\mathbf{4}}$'s cannot appear in FI–cancelling flat directions. Rather, it is expected that they form $Q_{\rm E} = Q = 0$ condensates at an intermediate scale. In our treatment of effective bilinear and trilinear terms containing $\mathbf{4} \cdot \bar{\mathbf{4}}$ condensates we assume the condensation scale to be no higher than $\mathcal{O}(10^{13}$ GeV), and most likely lower, as we discuss later.

For the fundamental vector representation of any $SO(2n)$ algebra, the $n(2n-1)$ generators of the algebra are imaginary antisymmetric matrices $M_{a,b}$, with $a, b \in$ 1 to $2n$ and $a < b$, of the form:

$$(M_{a,b})_{j,k} = -i(\delta_{a,j}\delta_{b,k} - \delta_{b,j}\delta_{a,k}) \tag{3.2}$$

with commutation relations:

$$[M_{a,b}, M_{c,d}] = -i(\delta_{b,c}M_{a,d} - \delta_{a,c}M_{b,d} + \delta_{a,d}M_{b,c} - \delta_{b,d}M_{a,c}) \,. \tag{3.3}$$

The Cartan generators form an n–dimensional subset of matrices $M_{2c-1,2c}$. Generic fundamental vector solutions of the entire set of non–linear $SO(2n)$ D–flat constraints,

$$< D_{a,b}^{SO(2n)} > \;\; \equiv \;\; < \sum_m \varphi_m^{\dagger} M_{a,b} \varphi_m >= 0 \;, \tag{3.4}$$

correspond to gauge–invariant products of the vector fields [19]. For example, we note the following tensor product rules for low–dimensional representations of $SO(10)$:

$$\begin{aligned}
\mathbf{10} \times \mathbf{10} &= \mathbf{1} \oplus \mathbf{45} \oplus \mathbf{54} && (3.5)\\
\mathbf{10} \times \mathbf{45} &= \mathbf{10} \oplus \mathbf{120} \oplus \mathbf{320} && (3.6)\\
\mathbf{10} \times \mathbf{54} &= \mathbf{10} \oplus \mathbf{210'} \oplus \mathbf{320} && (3.7)\\
\mathbf{45} \times \mathbf{45} &= \mathbf{1} \oplus \mathbf{45} \oplus \mathbf{54} + \ldots && (3.8)\\
\mathbf{45} \times \mathbf{54} &= \mathbf{45} \oplus \mathbf{54} \oplus \mathbf{210} + \ldots && (3.9)\\
\mathbf{54} \times \mathbf{54} &= \mathbf{1} \oplus \mathbf{45} \oplus \mathbf{54} + \ldots && (3.10)
\end{aligned}$$

Sector	States	$SU(5)$	$SO(4)$	$SO(10)$	$U(1)$	U_1	U_2	U_3	U_4
0	$\Phi_{1\,\mathrm{to}\,5}$	1	1	1	0	0	0	0	0
	ϕ_{23}	1	1	1	0	0	-4	4	0
	$\overline{\phi}_{23}$	1	1	1	0	0	4	-4	0
	ϕ_{12}	1	1	1	0	-4	4	0	0
	$\overline{\phi}_{12}$	1	1	1	0	4	-4	0	0
	ϕ_{31}	1	1	1	0	4	0	-4	0
	$\overline{\phi}_{31}$	1	1	1	0	-4	0	4	0
	h_1	5	1	1	-4	4	0	0	0
	$\bar{h}_1$	-5	1	1	4	-4	0	0	0
	h_2	5	1	1	-4	0	4	0	0
	$\bar{h}_2$	-5	1	1	4	0	-4	0	0
	h_3	5	1	1	-4	0	0	4	0
	$\bar{h}_3$	-5	1	1	4	0	0	-4	0
$\mathbf{b}_1$	F_1	10	1	1	2	-2	0	0	0
	$\bar{f}_1$	-5	1	1	-6	-2	0	0	0
	l_1^c	1	1	1	10	-2	0	0	0
$\mathbf{b}_2$	F_2	10	1	1	2	0	-2	0	0
	$\bar{f}_2$	-5	1	1	-6	0	-2	0	0
	l_2^c	1	1	1	10	0	-2	0	0
$\mathbf{b}_3$	F_3	10	1	1	2	0	0	2	-2
	$\bar{f}_3$	-5	1	1	-6	0	0	2	2
	l_3^c	1	1	1	10	0	0	2	2
$\mathbf{b}_4$	F_4	10	1	1	2	-2	0	0	0
	f_4	5	1	1	6	2	0	0	0
	$\bar{l}_4^c$	1	1	1	-10	2	0	0	0
$\mathbf{b}_5$	$\bar{F}_5$	-10	1	1	-2	0	2	0	0
	$\bar{f}_5$	-5	1	1	-6	0	-2	0	0
	l_5^c	1	1	1	10	0	-2	0	0

Table 1: *Massless particle states in string-derived flipped $SU(5)$ [3]: $\mathbf{0}$, $\mathbf{b}_{1,2,3,4,5}$ sectors.*

These product rules indicate that several different types of invariants are possible for an even number of **10**'s. For two **10**'s, the only invariant in (3.5) is a trace product of the two **10**'s, $\mathbf{1} = \sum_{i=1}^{10} \mathbf{10}_i \mathbf{10}_i$. However, with four **10**'s, three different invariants can be formed from the tensor product of two right–hand sides of (3.5) since $\mathbf{1} \times \mathbf{1} = 1$, $\mathbf{45} \times \mathbf{45} = 1 + \ldots$, and $\mathbf{54} \times \mathbf{54} = 1 + \ldots$. Analogous invariants exist for any $SO(2n)$.

A dangerous F term containing VEVs of $SO(10)$ decuplets or $SO(6)$ sextets can sometimes be eliminated [5] for a given flipped $SU(5)$ non–Abelian D–flat direction. For example, a flat direction could contain four decuplets $\mathbf{10}^{a=1,4}$ where all VEV

Sector	States	$SU(5)$	$SO(4)$	$SO(10)$	$U(1)$	U_1	U_2	U_3	U_4
$\mathbf{S}+$	h_{45}	-5	1	1	4	2	2	0	0
$\mathbf{b}_4+\mathbf{b}_5$	h_{45}	5	1	1	-4	-2	-2	0	0
	ϕ_{45}	1	1	1	0	2	2	4	0
	$\overline{\phi}_{45}$	1	1	1	0	-2	-2	-4	0
	ϕ_1	1	1	1	0	2	-2	0	0
	ϕ_2	1	1	1	0	2	-2	0	0
	ϕ_3	1	1	1	0	2	-2	0	0
	ϕ_4	1	1	1	0	2	-2	0	0
	$\overline{\phi}_1$	1	1	1	0	-2	2	0	0
	$\overline{\phi}_2$	1	1	1	0	-2	2	0	0
	$\overline{\phi}_3$	1	1	1	0	-2	2	0	0
	$\overline{\phi}_4$	1	1	1	0	-2	2	0	0
	ϕ_+	1	1	1	0	2	-2	0	4
	$\overline{\phi}_+$	1	1	1	0	-2	2	0	-4
	ϕ_-	1	1	1	0	2	-2	0	-4
	$\overline{\phi}_-$	1	1	1	0	-2	2	0	4
$\mathbf{b}_i$	Δ_1	1	6	1	0	0	-2	2	0
$+2\alpha+(\mathbf{X})$	Δ_2	1	6	1	0	-2	0	2	0
	Δ_3	1	6	1	0	-2	-2	0	2
	Δ_4	1	6	1	0	0	-2	2	0
	Δ_5	1	6	1	0	2	0	-2	0
	T_1	1	1	10	0	0	-2	2	0
	T_2	1	1	10	0	-2	0	2	0
	T_3	1	1	10	0	-2	-2	0	-2
	T_4	1	1	10	0	0	2	-2	0
	T_5	1	1	10	0	-2	0	2	0
$\mathbf{b}_1\pm\alpha$	a_1	1	4	1	-5	-1	1	1	2
	a_2	1	4	1	-5	-1	1	1	-2
	a_3	1	4	1	-5	-1	1	1	-2
	a_4	1	4	1	-5	1	-1	1	-2
	a_5	1	4	1	5	-1	-1	1	-2
	a_6	1	4	1	-5	-3	1	-1	0
	$\bar{a}_1$	1	-4	1	5	1	-1	-1	-2
	$\bar{a}_2^{'}$	1	-4	1	5	-1	1	-1	-2
	$\bar{a}_3^{'}$	1	-4	1	5	-1	1	-1	-2
	$\bar{a}_4$	1	-4	1	5	-1	1	-1	2
	$\bar{a}_5$	1	-4	1	-5	1	1	-1	2
	$\bar{a}_6^{'}$	1	-4	1	5	-1	3	1	0

Table 2: *Massless particle states in string-derived flipped $SU(5)$ [3]: other sectors.*

components in $\mathbf{10}^{a=1,2}$ are $\langle\alpha\rangle$, while in $\mathbf{10}^{a=3,4}$ five components are $\langle\alpha\rangle$ and another five are $-\langle\alpha\rangle$. Self–cancellation would occur in any F–term containing exactly one of $\mathbf{10}^{a=1,2}$ and one of $\mathbf{10}^{a=3,4}$.

3.2 D– and F–Flat Singlet Directions

The flipped $SU(5)$ model contains 20 fields with non–trivial Abelian charges that are singlets of all the non–Abelian gauge group factors, as seen in Table 3 below. This set of 20 non–trivial singlets can be grouped into ten vector–like pairs, where the two members of each pair carry exactly opposite charges. Four of the 20 fields carry identical sets of $U(1)_i$ charges. Thus, for our purposes, the model contains fields with just seven distinct values of the $U(1)_i$ charges.

Vector–Like Singlets	U_A	U'_1	U'_2	U'_3
Φ_{12}	8	0	-16	-8
Φ_{23}	12	4	12	12
Φ_{31}	-20	-4	4	-4
ϕ_{45}	0	4	-4	24
$\phi_{1,2,3,4}$	-4	0	8	4
ϕ_+	-8	8	8	-4
ϕ_-	0	-8	8	12

Table 3: *The complete set of singlet fields with at least one non–zero $U(1)_i$ charge, but no non–Abelian charges. The normalization of the $U(1)_i$ charges in this paper is four times larger than that used in [3].*

The three independent non–anomalous D constraints result in the four–dimensional (when ϕ_i is fixed) non–trivial basis set of independent vector–like non–anomalous D–flat directions shown in Table 4. To each of these non–trivial directions, elements of a trivial basis set of D–flat directions may be added. This latter set is composed of the 10 pairs of vector–like fields, $(\Phi_{12},\overline{\Phi}_{12})$, $(\Phi_{23},\overline{\Phi}_{23})$, $(\Phi_{31},\overline{\Phi}_{31})$, $(\phi_{45},\overline{\phi}_{45})$, $(\phi_i,\overline{\phi}_i)$ for $i\in\{1,2,3,4\}$, $(\phi_+,\overline{\phi}_+)$, $(\phi_-,\overline{\phi}_-)$, the three $(\phi_j,\overline{\phi}_1)$ pairs for $j\in\{2,3,4\}$, and the five totally uncharged moduli fields, $\Phi_{1,2,3,4,5}$.

We have generated D–flat directions $\mathbf{d}=\sum_x n_x\mathbf{b}_x$ for integer n_x in the range of -10 to 10 with the constraint that $n_{\overline{\phi}_{23}}+n_{\phi_{31}}>0$, so that $Q_A<0$. We tested each of these directions for 'stringent' F–flatness up to at least fifth order in the superpotential. Three directions passed this test, with each actually stringently F–flat to *all finite orders*, as can be shown simply by gauge invariance constraints. The three solutions $\mathbf{d}_1$, $\mathbf{d}_2$, and $\mathbf{d}_3$ are given in Table 5 below. We note that $\mathbf{d}_2$ is the 'root' of the flipped $SU(5)$ flat direction analyzed in [3], whilst $\mathbf{d}_3$ corresponds to the

Direction	Q_A	Φ_{12}	Φ_{23}	Φ_{31}	ϕ_{45}	ϕ_i	ϕ_+	ϕ_-
$\mathbf{b}_{\overline{\Phi}_{23}}$	-60	0	-3	0	1	0	3	2
$\mathbf{b}_{\Phi_{31}}$	-60	0	0	3	1	0	0	-1
$\mathbf{b}_{\Phi_{12}}$	0	1	0	0	0	0	1	1
$\mathbf{b}_{\phi_i}$	0	0	0	0	0	-2	1	1

Table 4: *Non–trivial basis set of singlet D–flat directions. The numerical entries specify the ratios of the norms of the VEVs of the fields. A negative entry indicates that the vector partner of the field, rather than the field, takes on the VEV. Accompanying the $\mathbf{b}_\Phi$ ($\mathbf{b}_\phi$) directions are the respective vector–partner directions, $\mathbf{b}_{\bar{\Phi}} = -\mathbf{b}_\Phi$ ($\mathbf{b}_{\bar{\phi}} = -\mathbf{b}_\phi$).*

alternative flat direction studied in [5]. Finally, $\mathbf{d}_3$ is a linear combination of the first two, $\mathbf{d}_3 = \mathbf{d}_1 + 3\mathbf{d}_2$ and actually represents an entire two–dimensional class of all–order flat directions, whose members are linear combinations $\alpha_1\mathbf{d}_1 + \alpha_2\mathbf{d}_2$, where α_1 and α_2 are real, positive coefficients. We recall that $\mathbf{d}_1$, $\mathbf{d}_2$, and $\mathbf{d}_3$ can be modified by allowing VEVs for any or all of the uncharged moduli fields, $\Phi_{i=1,2,4,5}$, whilst retaining F–flatness to all finite orders. The moduli can take on VEVs without harming flatness because the entire set of fields $\{\overline{\Phi}_{23}, \Phi_{31}, \phi_{45}, \overline{\phi}_-, \phi_+\}$ is linearly independent with regard to all $U(1)_i$ charges: no product of only these fields can ever appear in the superpotential. Note that Φ_3 cannot be appended to $\mathbf{d}_{1,2,3}$, because of the renormalizable terms $\Phi_3[\phi_+\overline{\phi}_+ + \phi_-\overline{\phi}_- + \phi_{45}\overline{\phi}_{45}]$. Moreover, it can be shown that a flat direction with simply $\mathbf{d}_1$ as its root would present some phenomenological problems. Thus, we make the significant observation that *the root–space of viable flipped $SU(5)$ singlet flat directions has been covered in the papers to date.*

Directions	$\langle\alpha\rangle$	Q_A	Φ_{12}	Φ_{23}	Φ_{31}	ϕ_{45}	ϕ_i	ϕ_+	ϕ_-
$\mathbf{d}_1$	9.2×10^{16} GeV	-60	0	0	3	1	0	0	-1
$\mathbf{d}_2$	9.2×10^{16} GeV	-60	0	-1	2	1	0	1	0
$\mathbf{d}_3$	4.6×10^{16} GeV	-240	0	-3	9	4	0	3	-1

Table 5: *The only D–flat directions, mod $\langle\Phi_{1,2,4,5}\rangle$, that are F–flat to at least fifth order in the superpotential. These three directions are actually flat to all finite order.*

The non–trivial set of singlet D–flat directions can be expanded by allowing hidden–sector non–Abelian fields also to acquire VEVs. This provides 14 additional basis directions that do not break the MSSM gauge group. However, we do not include $SO(6)$ $\langle a_i\bar{a}_j\rangle$ condensates among these additional directions, since their hidden–sector condensation scale should be significantly below the FI scale.

Table 6 displays the basis set of non–Abelian D–flat directions that leave the MSSM gauge group invariant. As seen in Table 6, the only components of F_i and $\bar{F}_5$

Dir.	Q_A	ϕ_i	ϕ_+	ϕ_-	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	T_1	T_2	T_3	T_4	T_5	F_1	F_2	F_3	F_4	F_5
$\mathbf{b}_{\Delta_1}$	60	-1	-3	-2	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\mathbf{b}_{\Delta_2}$	60	-1	0	1	0	6	0	0	0	0	0	0	0	0	0	0	0	0	0
$\mathbf{b}_{\Delta_3}$	60	2	-3	1	0	0	6	0	0	0	0	0	0	0	0	0	0	0	0
$\mathbf{b}_{\Delta_4}$	60	-1	-3	-2	0	0	0	6	0	0	0	0	0	0	0	0	0	0	0
$\mathbf{b}_{\Delta_5}$	-60	1	0	-1	0	0	0	0	6	0	0	0	0	0	0	0	0	0	0
$\mathbf{b}_{T_1}$	60	-1	-3	-2	6	0	0	0	0	6	0	0	0	0	0	0	0	0	0
$\mathbf{b}_{T_2}$	60	-1	0	1	0	6	0	0	0	0	6	0	0	0	0	0	0	0	0
$\mathbf{b}_{T_3}$	30	1	0	-1	0	0	6	0	0	0	0	3	0	0	0	0	0	0	0
$\mathbf{b}_{T_4}$	-60	1	3	2	0	0	0	6	0	0	0	0	6	0	0	0	0	0	0
$\mathbf{b}_{T_5}$	60	-1	0	1	0	0	0	0	6	0	0	0	0	6	0	0	0	0	0
$\mathbf{b}_{F_1}$	0	0	1	1	0	0	0	0	0	0	0	0	0	0	2	0	0	0	2
$\mathbf{b}_{F_2}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
$\mathbf{b}_{F_3}$	0	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	2
$\mathbf{b}_{F_4}$	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2

Table 6: *Basis set of non–Abelian D–flat directions that leave the MSSM gauge group invariant. The numerical entries have the same notation as in Table 4.*

that acquire VEVs in each of the $\mathbf{b}_{F_i}$ directions are the respective singlets ν_i^c and $\bar{\nu}_5^c$. These are the VEVs that break $SU(5) \to SU(3)_C \times SU(2)_L \times U(1)_Y$. We note that $SU(5)$ D–flatness requires equal ν_i^c and $\bar{\nu}_5^c$ VEVs. From Table 6 we see that Abelian D–flatness independently requires this VEV ratio. This implies that minimally one $\mathbf{b}_{F_i}$ basis direction must appear in a phenomenologically viable flat direction (with which $SU(5)$ is broken).

In order to obey the *stringent* F–flatness constraints, $\mathbf{b}_{F_2}$ and $\mathbf{b}_{F_3}$ are the only possible choices for an $SU(5)$–charged flat–direction component. First note that, in the contraction of two **10** (or two $\overline{\mathbf{10}}$) representations, there is an antisymmetrization factor ϵ^{ij}. This implies that $\langle F_i \cdot F_i \rangle = \langle \bar{F}_5 \cdot \bar{F}_5 \rangle = 0$. Hence trilinear terms like $F_1 F_1 h_1$ pose no threat to F–flatness. The only relevant trilinear term is $\bar{F}_5 \cdot F_4 \phi_3$, which prevents an F_4 VEV. Analogously, the non–renormalizable fifth–order term $(\bar{F}_5 \cdot F_1)^2 \overline{\Phi}_{12}$ prevents an F_1 VEV.

Terms dangerous for F_2 and F_3 VEVs first appear at fifth and fourth order, respectively:

$$\{(\bar{F}_5 \cdot F_2)\Phi_{31} T_2 \cdot T_5,\ (\bar{F}_5 \cdot F_2)\phi_2 T_2 \cdot T_4 (1 + \Phi_1 + \Phi_5)\} \tag{3.11}$$

and

$$\begin{aligned} &\{(\bar{F}_5 \cdot F_3)^2 \overline{\phi}_{45} \phi_+,\ \bar{F}_5 \cdot F_3 \Delta_3 \cdot \Delta_4 (\overline{\Phi}_{23}\phi_3 + \Phi_{31}\overline{\phi}_3)\,, \\ &(\bar{F}_5 \cdot F_3)\Delta_3 \cdot \Delta_5 (1 + \sum_{i=1}^{5} \Phi_i \overline{\Phi}_i + \Phi_{12}\overline{\Phi}_{12} + \Phi_{23}\overline{\Phi}_{23} + \Phi_{31}\overline{\Phi}_{31} + \\ &\qquad \sum_{i=1}^{4} \phi_i \overline{\phi}_i + \phi_{45}\overline{\phi}_{45} + \phi_+ \overline{\phi}_+ + \phi_- \overline{\phi}_-)\}\,. \end{aligned} \tag{3.12}$$

Thus, requiring $< \bar{F}_5 \cdot F_2 > \neq 0$ along a stringent flat direction implies

- $< T_2 \cdot T_5 >= 0$ if $< \Phi_{31} >= 0$ or $< T_2 >=< T_5 >= 0$ if $< \Phi_{31} > \neq 0$, and
- $< T_2 \cdot T_4 >= 0$ if $< \phi_2 >= 0$ or $< T_2 >=< T_4 >= 0$ if $< \phi_2 > \neq 0$.

Similarly, $< \bar{F}_5 \cdot F_3 > \neq 0$ implies

- $< \overline{\phi}_{45} >=< \phi_+ >= 0$,
- $< \Delta_3 >=< \Delta_5 >= 0$, and
- $< \Delta_4 >= 0$ or $< \Phi_{31} >= 0$ or $< \overline{\phi}_3 >= 0$.

The D–flat basis direction $\mathbf{b}_{F_3}$ contains ϕ_+. This implies that some combination of $\mathbf{b}_{\Phi_{23}}$, $\mathbf{b}_{\overline{\Phi}_{12}}$, $\mathbf{b}_{\overline{\phi}_i}$, $\mathbf{b}_{\Delta_1,i}$, $\mathbf{b}_{\Delta_4,i}$, and $\mathbf{b}_{T_1,i}$ must be added to $\mathbf{b}_{F_3}$ to eliminate the ϕ_+ VEV.

At least 21 D–flat non–Abelian directions (and their primed associates) remain stringently F–flat to all finite order: see Table 7 below. One feature key to the all–order flatness of these directions is the specific set of world–sheet charges of the associated fields. The ϕ_{45}, ϕ_+, ϕ_-, Δ_3, T_3 (and their conjugates) are all Ramond fields carrying X_{56} charge, whilst F_2 and $\bar{F}_5$ are Ramond fields carrying X_{34} charge. The Φ_{23}, Φ_{31} (and their conjugates) are Neveu–Schwarz fields with X_{12} and X_{34} charges, respectively. For many of the 21 directions, several gauge–invariant terms of relatively low order (e.g., sixth through eighth) exist that might break F–flatness. However, only one of these terms, namely $\langle \Phi_{31}\phi_{45}\Delta_3 \cdot \Delta_3\rangle\langle a_2\bar{a}_2^{'}\rangle$, satisfies the picture–changed charge–conservation constraints. All the other terms contain too many X_{56} Ramond charges to satisfy the picture–changing constraint. It was shown in [12, 13] that the maximum number of identical Ramond $X_{i,i+1}$ charges that can appear is $n-2-n_{NS}$, where n is the order of the term and n_{NS} is the number of Neveu–Schwarz fields in the term. All but one of the potentially dangerous gauge–invariant terms contain more than $n-2-n_{NS}$ X_{56} Ramond fields.

Seven of the $\mathbf{d}$, and the corresponding $\mathbf{d}^{'}$, are flat to all orders, independent of any constraints. Following $\langle a_i\bar{a}_j\rangle$ condensation, F–flatness of the remaining directions (apart from $\mathbf{d}_{22}$) is threatened by the sixth–order term $\langle \Phi_{31}\phi_{45}\Delta_3 \cdot \Delta_3\rangle\langle a_2\bar{a}_2^{'}\rangle$. This term is of no concern if the condensation scale is around 10^{10} GeV or lower. However, if the condensation scale is above this, then we must require that

$$\langle d_3 \cdot d_3\rangle = 0\,, \tag{3.13}$$

as indicated in the last column of Table 7. However, our rough estimate for the condensation scale appears to be in the safe low–scale range, so that (3.13) is unnecessary.

With the exception of $\mathbf{d}_{22}$, for every direction not containing F_2 and $\bar{F}_5$ VEVs in Table 7, there is another that does. The corresponding directions are denoted $\mathbf{d}_i$ and $\mathbf{d}_i^{'}$, respectively, and are contained in the same row in Table 7. Each of these flat directions may additionally contain any or all of the uncharged moduli fields $\Phi_{1,2,4,5}$. The more realistic of our flat directions are clearly those in the $\mathbf{d}^{'}$ class, since the breaking of $SU(5)\times U(1)$ to the Standard Model requires at least one $\langle F_i\rangle \neq 0$ or $\langle \bar{F}_5\rangle \neq 0$, and $SU(5)$ D–flatness then requires $\langle \bar{F}_5\rangle = \langle F\rangle$, where $F \equiv \sum_{i=1}^{4}\alpha_i F_i$, for $|\vec{\alpha}| = 1$. Whilst an F_2 VEV was considered in [20], most recent papers have considered a VEV for F_1, rather than F_2. Thus, these F_2 directions possess somewhat different

Notation	order	Q_A	Φ_{23}	Φ_{31}	ϕ_{45}	ϕ_+	ϕ_-	Δ_3	T_3	F_2	F_3	$\bar{F}_5$	constraints
$\mathbf{d}_1^{(\prime)}$	∞	-60	0	3	1	0	-1	0	0	(1	0	1)	none
$\mathbf{d}_2^{(\prime)}$	∞	-60	-3	6	3	3	0	0	0	(1	0	1)	none
$\mathbf{d}_3^{(\prime)}$	∞	-240	-3	9	4	3	-1	0	0	(1	0	1)	none
$\mathbf{d}_4^{(\prime)}$	∞	-120	0	9	5	0	-5	0	6	(1	0	1)	none
$\mathbf{d}_6^{(\prime)}$	∞	-60	-1	3	2	0	0	2	0	(1	0	1)	$\langle \Delta_3 \cdot \Delta_3 \rangle_a = 0$
$\mathbf{d}_7^{(\prime)}$	∞	-120	-2	5	3	1	0	2	0	(1	0	1)	$\langle \Delta_3 \cdot \Delta_3 \rangle_a = 0$
$\mathbf{d}_8^{(\prime)}$	∞	-60	-1	4	3	-1	0	4	0	(1	0	1)	$\langle \Delta_3 \cdot \Delta_3 \rangle_a = 0$
$\mathbf{d}_9^{(\prime)}$	∞	-60	-3	3	4	0	2	6	0	(1	0	1)	$\langle \Delta_3 \cdot \Delta_3 \rangle_a = 0$
$\mathbf{d}_{10}^{(\prime)}$	∞	-240	-3	12	7	0	-1	6	0	(1	0	1)	$\langle \Delta_3 \cdot \Delta_3 \rangle_a = 0$
$\mathbf{d}_{11}^{(\prime)}$	∞	-60	-3	2	3	3	0	0	4	(1	0	1)	none
$\mathbf{d}_{12}^{(\prime)}$	∞	-60	-3	5	6	0	0	6	4	(1	0	1)	$\langle \Delta_3 \cdot \Delta_3 \rangle_a = 0$
$\mathbf{d}_{13}^{(\prime)}$	∞	-420	-3	18	10	3	-1	6	0	(1	0	1)	$\langle \Delta_3 \cdot \Delta_3 \rangle_a = 0$
$\mathbf{d}_{14}^{(\prime)}$	∞	-60	-3	6	7	-3	2	12	0	(1	0	1)	$\langle \Delta_3 \cdot \Delta_3 \rangle_a = 0$
$\mathbf{d}_{15}^{(\prime)}$	∞	-60	-3	3	4	3	-1	0	6	(1	0	1)	none
$\mathbf{d}_{16}^{(\prime)}$	∞	-60	-3	6	7	0	-1	6	6	(1	0	1)	$\langle \Delta_3 \cdot \Delta_3 \rangle_a = 0$
$\mathbf{d}_{17}^{(\prime)}$	∞	-60	-2	3	3	1	0	2	2	(1	0	1)	$\langle \Delta_3 \cdot \Delta_3 \rangle_a = 0$
$\mathbf{d}_{18}^{(\prime)}$	∞	-60	-2	5	5	-1	0	6	2	(1	0	1)	$\langle \Delta_3 \cdot \Delta_3 \rangle_a = 0$
$\mathbf{d}_{19}^{(\prime)}$	∞	-120	-6	9	11	0	1	12	6	(1	0	1)	$\langle \Delta_3 \cdot \Delta_3 \rangle_a = 0$
$\mathbf{d}_{20}^{(\prime)}$	∞	-60	-6	6	10	3	-1	6	12	(1	0	1)	$\langle \Delta_3 \cdot \Delta_3 \rangle_a = 0$
$\mathbf{d}_{21}^{(\prime)}$	∞	-60	-6	9	13	-3	2	18	6	(1	0	1)	$\langle \Delta_3 \cdot \Delta_3 \rangle_a = 0$
$\mathbf{d}_{22}$	8	-60	0	3	1	0	-1	0	0	0	2	2	none

Table 7: *Non–Abelian D–flat directions, mod $\langle \Phi_{1,2,4,5} \rangle$, that are F–flat to all orders (with the exception of $\mathbf{d}_{22}$) in the superpotential. The scale of the VEV for each of these directions is $\langle \alpha \rangle / \sqrt{-Q_A/60}$, where $\langle \alpha \rangle \equiv 9.2 \times 10^{16}$ GeV. The primed and unprimed versions of a flat direction vary only by the presence or absence, respectively, of a $F_2 \cdot \bar{F}_5$ VEV component. In the constraints column, the subscript "a" denotes that $\langle \Delta_3 \cdot \Delta_3 \rangle = 0$ is required only if the hidden–sector $SO(6)$ condensation scale of quadruplet a fields occurs at or above $\sim 10^{10}$ GeV.*

phenomenology from those generally investigated, and we therefore consider them in the next section, and compare our results to those of [20].

We note that there appears to be no all–order flat direction containing F_3: for example, the flatness of $\mathbf{d}_{22}$ is broken at eighth order by the superpotential term $\langle \Phi_{31}\overline{\phi}_{45}\overline{\phi}_{-}(F_3 \cdot \bar{F}_5)^2 \rangle \Phi_{23}$. In any case, the flipped $SU(5)$ doublet–triplet splitting mechanism prevents F_3 from being alone among the F_i fields to acquire a VEV [3, 20].

4 Flat–Direction Phenomenology

From Table 7 we observe that each of the all–order flat directions $\mathbf{d}_1^{(\prime)}$ through $\mathbf{d}_{19}^{(\prime)}$ can be embedded in either $\mathbf{d}_{20}^{\prime}$ or $\mathbf{d}_{21}^{\prime}$. Thus, in the next subsection we examine the Higgs mass eigenstates and eigenvalues resulting from $\mathbf{d}_{20}^{\prime}$ or $\mathbf{d}_{21}^{\prime}$. The corresponding eigenstates and eigenvalues for the 19 embedded directions can be easily determined from these results. In the following subsection, we then examine for $\mathbf{d}_{20}^{\prime}$ and $\mathbf{d}_{21}^{\prime}$ the corresponding masses of the three Standard Model generations of quarks and leptons.

When we indicate the components of a mass matrix, we generally list only the leading term, or one representative of them if there are several leading–order terms. For terms involving $SO(6)$ condensates, $\langle a_i \bar{a}_j \rangle$, we assume a condensation scale no higher than 10^{13} GeV. Relatedly, we assume a suppression factor of $\sim \frac{1}{10^8}$ *or less*, rather than $\sim \frac{1}{100}$, for each condensate. We include up to eleventh (seventh) order terms in the mass matrices when condensates are absent (present).

4.1 Higgs Mass Textures

We first determine the Higgs mass eigenstates and eigenvalues produced by our all–order flat directions. As in [7], our 5×5 Higgs mass matrices contain terms for both the $SU(2)_L$ doublet and $SU(3)_C$ triplet components of the $SU(5)$ 5 and $\bar{5}$ Higgs representations. The 4×4 $SU(2)_L$ doublet Higgs matrix excludes the $\bar{F}_5$ and F_2 components, whilst the $SU(3)_C$ triplet matrix is the entire 5×5 matrix. In the absence of $SO(6)$ condensates, it takes the following form:

$$M_{11} = \begin{array}{c} \\ h_1 \\ \\ h_2 \\ h_3 \\ h_{45} \\ \bar{F}_5 \end{array} \begin{array}{c} \begin{array}{ccccc} \bar{h}_1 & \bar{h}_2 & \bar{h}_3 & \bar{h}_{45} & F_2 \end{array} \\ \left(\begin{array}{ccccc} & & & & < X^{15}_{(0)} > + \\ & & & & < X^{15}_{(1)} > \\ & & & & < F_2 > \\ < \Phi_{31} > & < \overline{\Phi}_{23} > & & & \\ & & < \phi_{45} > & & < X^{45} > \\ < X^{51} > + & < \bar{F}_5 > & & < X^{54}_{(0)} > + & \\ < X^{51}_{(0)} > & & & < X^{54}_{(1)} > & \end{array} \right) \end{array}, \tag{4.1}$$

where

$$\begin{aligned} X^{15}_{(0)} &= F_2 \overline{\Phi}_{23} \phi_{45} T_3 \cdot T_3 (\overline{\phi}_- + \overline{\Phi}_{23} \phi_{45} \Delta_3 \cdot \Delta_3) \\ X^{15}_{(1)} &= F_2 (\overline{\Phi}_{23} \phi_{45} \overline{\phi}_+ \Delta_3 \cdot \Delta_3 + \overline{\Phi}^2_{23} \phi^2_{45} \Delta_3 \cdot \Delta_3 T_3 \cdot T_3) \\ X^{45} &= F_2 \overline{\Phi}_{23} \phi_{45} \\ X^{51} &= \bar{F}_5 \Phi^2_{31} \phi^2_{45} \Delta_3 \cdot \Delta_3 T_3 \cdot T_3 \\ X^{51}_{(0)} &= \bar{F}_5 \Phi_{31} \phi_{45} \phi_+ \Delta_3 \cdot \Delta_3 T_3 \cdot T_3 \\ X^{54}_{(0)} &= \bar{F}_5 \phi_+ T_3 \cdot T_3 \\ X^{54}_{(1)} &= \bar{F}_5 \phi_- \Delta_3 \cdot \Delta_3 \, . \end{aligned} \tag{4.2}$$

The matrix (4.1) contains all condensate–free Higgs mass terms for flat directions $\mathbf{d}'_{20}$ and $\mathbf{d}'_{21}$. If no subscript appears on a term in (4.1), then that term is produced by both flat directions, On the other hand, a subscript (0) or (1) indicates a term that is produced only by $\mathbf{d}'_{20}$ or $\mathbf{d}'_{21}$, respectively. Although not explicitly shown, the appropriate $n^{\rm th}$–order coupling constants λ_n and related $1/M^{n-3}_{string}$ factors for each term are implied.

We now exhibit numerical order–of–magnitude estimates of the entries in (4.1):

$$M^{oom}_{11} = \begin{array}{c} \\ h_1 \\ h_2 \\ h_3 \\ h_{45} \\ \bar{F}_5 \end{array} \begin{array}{c} \begin{array}{ccccc} \bar{h}_1 & \bar{h}_2 & \bar{h}_3 & \bar{h}_{45} & F_2 \end{array} \\ \left(\begin{array}{ccccc} 0 & 0 & 0 & 0 & 10^{-4} \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 10^{-1} \\ 10^{-4} & 1 & 0 & 10^{-2} & \end{array} \right) \end{array}, \tag{4.3}$$

whose consequences we explore later.

In the presence of $SO(6)$ condensates, there are additional Higgs mass terms, as follows:

$$
M_7^{a\bar{a}} = \begin{array}{c} \\ h_1 \\ h_2 \\ h_3 \\ h_{45} \\ \bar{F}_5 \end{array}
\begin{array}{c}
\begin{array}{ccccc} \bar{h}_1 & \bar{h}_2 & \bar{h}_3 & \bar{h}_{45} & F_2 \end{array} \\
\left(\begin{array}{ccccc}
<Y^{11}_{(0)}> & & & <Y^{14}> & <Y^{15}> \\
 & <Y^{22}> & & & \\
 & & <Y^{33}_{(0)}> & & <Y^{35}> \\
<Y^{41}> + <Y^{41}_{(0)}> & <Y^{42}> + <Y^{42}_{(0)}> & & <Y^{44}> + <Y^{44}_{(0)}> & \\
 & & & &
\end{array} \right)
\end{array}, \quad (4.4)
$$

where

$$
\begin{aligned}
Y^{11}_{(0)} &= \bar{F}_5 F_2 \phi_+ a_2 \bar{a}_3^{'} \\
Y^{14} &= \Delta_3 \cdot \Delta_3 a_2 \bar{a}_2^{'} \\
Y^{15} &= F_2 a_6 \bar{a}_6^{'} \\
Y^{22} &= \Phi_{31} a_1 a_1 a_5 a_5 \\
Y^{33}_{(0)} &= \bar{F}_5 F_2 \phi_+ a_2 \bar{a}_3^{'} \\
Y^{35} &= F_2 \bar{a}_2^{'} \bar{a}_2^{'} \bar{a}_5 \bar{a}_5 \\
Y^{41} &= \phi_{45} \bar{a}_1 \bar{a}_1 \bar{a}_5 \bar{a}_5 \\
Y^{41}_{(0)} &= \Phi_{31} \phi_{45} \phi_+ a_2 \bar{a}_2^{'} \\
Y^{42} &= \phi_{45} \bar{a}_2^{'} \bar{a}_2^{'} \bar{a}_5 \bar{a}_5 \\
Y^{42}_{(0)} &= \overline{\Phi}_{23} \phi_{45} \phi_+ a_2 \bar{a}_2^{'} \\
Y^{44} &= \Phi_{31} a_1 a_1 a_5 a_5 \\
Y^{44}_{(0)} &= \phi_+ a_2 \bar{a}_2^{'} \, .
\end{aligned}
$$

and the following is a numerical estimate of (4.1) and (4.4) combined:

$$
\begin{aligned}
M_{11}^{oom'} &= M_{11}^{oom} + M_7^{a\bar{a},oom} \\
&= \begin{array}{c} \\ h_1 \\ h_2 \\ h_3 \\ h_{45} \\ \bar{F}_5 \end{array}
\begin{array}{c}
\begin{array}{ccccc} \bar{h}_1 & \bar{h}_2 & \bar{h}_3 & \bar{h}_{45} & F_2 \end{array} \\
\left(\begin{array}{ccccc}
\lesssim 10^{-9}_{(0)} & 0 & 0 & \lesssim 10^{-8} & 10^{-4} \\
0 & 10^{-15} & 0 & 0 & 1 \\
1 & 2 & \lesssim 10^{-9}_{(0)} & 0 & \lesssim 10^{-15}_{(1)} \\
\lesssim 10^{-9}_{(0)} + 10^{-15}_{(1)} & \lesssim 10^{-9}_{(0)} + 10^{-15}_{(1)} & 2 & \lesssim 10^{-7}_{(0)} + 10^{-15}_{(1)} & 10^{-1} \\
10^{-5} & 1 & 0 & 10^{-2} &
\end{array} \right)
\end{array} .
\end{aligned}
$$

$$(4.5)$$

where we recall that a subscript (0) or (1) indicates a term that is produced only by $\mathbf{d}'_{20}$ or $\mathbf{d}'_{21}$, respectively.

The Higgs doublet matrix embedded in (4.3) has the generic form †:

$$M_{11}^{gen} = \begin{array}{c} \\ h_1 \\ h_2 \\ h_3 \\ h_{45} \end{array} \begin{array}{c} \begin{array}{cccc} \bar{h}_1 & \bar{h}_2 & \bar{h}_3 & \bar{h}_{45} \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ c & d & 0 & 0 \\ 0 & 0 & f & 0 \end{pmatrix} \end{array}, \tag{4.6}$$

for all flat directions in Table 7. We see that (4.6) produces two pairs of FI–scale massive doublet eigenstates, and two pairs of massless Higgs doublet eigenstates. The (unnormalized) massive Higgs eigenstates are $h_1^M \equiv h_3$ and $\bar{h}_1^M \equiv c\,\bar{h}_1 + d\,\bar{h}_2$ with $M^2 = c^2 + d^2$, and $h_2^M \equiv h_{45}$ and $\bar{h}_2^M \equiv \bar{h}_3$ with $M^2 = f^2$. The massless Higgs are $h_1, h_2, d\,\bar{h}_1 - c\,\bar{h}_2$, and $\bar{h}_{45}$.

In (4.5) we note that along the $\bar{F}_5$ row, and $\bar{h}_1$, $\bar{h}_2$, and $\bar{h}_{45}$ columns, and along the F_2 column, and h_1, h_2, and h_{45} rows, the triplet mass components are non–zero. A generic matrix of the form

$$M_{11}^{gen,3} = \begin{array}{c} \\ h_1 \\ h_2 \\ h_3 \\ h_{45} \\ \bar{F}_5 \end{array} \begin{array}{c} \begin{array}{ccccc} \bar{h}_1 & \bar{h}_2 & \bar{h}_3 & \bar{h}_{45} & F_2 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & k \\ 0 & 0 & 0 & 0 & l \\ c & d & 0 & 0 & 0 \\ 0 & 0 & f & 0 & m \\ g & i & 0 & j & 0 \end{pmatrix} \end{array}. \tag{4.7}$$

produces exactly one massless triplet/anti–triplet pair: $h^{[3],0} \equiv l\,h_1 - k\,h_2$ and $\bar{h}^{[3],0} \equiv d\,\bar{h}_1 - c\,\bar{h}_2 + \frac{-dg+ci}{j}\,\bar{h}_{45}$. Thus, additional terms beyond those in (4.1) must appear in the Higgs doublet matrix, to provide FI–scale masses for one additional pair of doublets and the remaining triplet/anti–triplet pair. As (4.5) and (4.4) indicate, our flat directions do indeed yield additional terms when terms containing $SO(6)$ condensates are included. However, the contributions from these terms are substantially smaller, since the condensate scale would most naturally be no higher than about 10^{13} GeV. That is, each condensate $\langle a_i \bar{a}_j \rangle$ contributes a suppression factor of order 10^{-8} or smaller. We have listed in (4.4) the related condensate terms through seventh order. We find these mass contributions associated with the flat direction $\mathbf{d}'_{20}$ have values $\lesssim 10^{-9}$ times the FI scale, namely $\sim 10^8$ GeV, except for a $h_1\bar{h}_{45}$ component, denoted Y^{14}, which is larger by an estimated factor ~ 10 and a $h_{45}\bar{h}_{45}$ component, denoted $Y^{44}_{(0)}$, which is estimated to be larger than these by a

†This mass texture is embedded in all of the flipped $SU(5)$ Higgs matrices in [7].

factor ~ 100. Whilst $\mathbf{d}^{'}_{21}$ also produces the term Y^{14}, the remaining $\mathbf{d}^{'}_{21}$ condensate terms only have values $\sim 10^{-15}$ times the FI scale, namely ~ 100 GeV. Thus, the condensate terms unique to the $\mathbf{d}^{'}_{21}$–class directions can clearly be ignored.

We can effectively ignore all condensate terms in (4.5), except for Y^{14} and $Y^{44}_{(0)}$. The result of adding both of these terms to M_{11} for $\mathbf{d}^{'}_{20}$, or only the first for $\mathbf{d}^{'}_{21}$, is to produce a matrix of similar form to matrices (20) and (25) of [7]. The massless eigenstates for a matrix of the form

$$M^{gen'}_{11} \quad = \quad \begin{array}{c} \\ h_1 \\ h_2 \\ h_3 \\ h_{45} \end{array} \begin{array}{c} \begin{array}{cccc} \bar{h}_1 & \bar{h}_2 & \bar{h}_3 & \bar{h}_{45} \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & p \\ 0 & 0 & 0 & 0 \\ c & d & q & 0 \\ 0 & 0 & f & 0 \end{pmatrix} \end{array}, \tag{4.8}$$

are (assuming real VEVs) clearly: $h \equiv h_2$ and $\bar{h} \equiv d\,\bar{h}_1 - c\,\bar{h}_2$, Here $d = \langle\overline{\Phi}_{23}\rangle$ and $c = \langle\Phi_{31}\rangle$.

We note that $Y^{14} \sim 10^{-8}$ of the FI scale gives a formerly massless Higgs doublet pair an intermediate mass $\sim 10^{8 \text{ or } 9}$ GeV. Thus, depending on the condensate scale, it appears that a single massless Higgs doublet pair can be produced by some all–order flat directions. However, again comparing (4.8) with the matrices of [7], we see that the absence of a non–zero $h_1\bar{h}_1$ or $h_1\bar{h}_2$ term in (4.8) eliminates an $\bar{h}_{45}$ component in $\bar{h}$. This has profound phenomenological consequences that we shall discuss in the following subsection.

Recall that in Table 7 we indicated that for all–order flatness, directions $\mathbf{d}^{'}_{20}$ and $\mathbf{d}^{'}_{21}$ may require $\langle\Delta_3 \cdot \Delta_3\rangle = 0$, to eliminate the appearance of a possibly dangerous $\langle W\rangle$–term, $\langle\Phi_{31}\phi_{45}\Delta_3 \cdot \Delta_3\rangle\langle a_2\bar{a}^{'}_2\rangle$. Since the Y^{14} term contains $\Delta_3 \cdot \Delta_3$, this zero–VEV constraint has severe phenomenologically consequences. If $\langle\Delta_3 \cdot \Delta_3\rangle = 0$ then $Y^{14} = 0$ and, therefore, two Higgs doublets and one Higgs triplet would remain massless. Furthermore, note also that the potentially dangerous $\langle W\rangle$ term and Y^{14} have the same last four VEV components: $\langle\Delta_3 \cdot \Delta_3\rangle\langle a_2\bar{a}^{'}_2\rangle$. Thus, if $\langle a_2\bar{a}^{'}_2\rangle$ has a magnitude low enough that the W term can be ignored, then one would expect that the Higgs mass term Y^{14} can likewise be ignored. Alternatively, if $\langle a_2\bar{a}^{'}_2\rangle$ is too large then we must require $\langle\Delta_3 \cdot \Delta_3\rangle = 0$ and the $\langle W\rangle$–term and Y^{14} should both vanish.

We observe that, along the 21 flat directions, only one $SO(6)$ $\mathbf{4}$–$\bar{\mathbf{4}}$ pair gains a mass at the FI scale. For a generic $SU(N_c)$ gauge group containing $N_f < 2N_c$ flavors of massless matter states in vector–like pairings, $T_i\overline{T}_i$, $i = 1, \ldots N_f$, the gauge coupling g_s becomes strong at a condensation scale defined by $\Lambda = M_P e^{8\pi^2/\beta g_s^2}$, where the β–function is given by $\beta = -3N_c + N_f$. Thus, for $N_c = 4$ and $N_f = 5$, $\beta = -7$ and the $SO(6)_H$ condensate scale should be around $\Lambda = e^{-22.5} M_P \sim 4 \times 10^8$ GeV.

In the following subsection, however, we briefly explore some of the phenomenology resulting from the Higgs pair h_2 and $\langle\overline{\Phi}_{23}\rangle\,\bar{h}_1 - \langle\Phi_{31}\rangle\,\bar{h}_2$, under the assumption that they are the only masslesss Higgs doublets, concentrating on the textures of the quark and charged–lepton mass matrices.

4.2 Quark and Charged–Lepton Mass Textures

In combination with the flat direction $\mathbf{d}'_{20}$ or $\mathbf{d}'_{21}$, the massless pair of Higgs fields h_2 and $\langle\overline{\Phi}_{23}\rangle\bar{h}_1 - \langle\Phi_{31}\rangle\,\bar{h}_2$ produce several MSSM quark and lepton mass terms. However, most of these terms contain $SO(6)$ condensates, which would most likely result in over–suppression of the lower–generation masses (except perhaps for Dirac neutrino terms). Hence, for quark and lepton masses, we consider only mass terms for which condensates are absent. Through eighth order these terms are [‡]:

$$\text{up}: \quad \text{no terms}, \tag{4.9}$$

$$\text{down}: \quad (F_1F_1 + F_4F_4)h_2\langle\Phi_{31}\phi_{45}(\phi_+T_3T_3 + \phi_-\Delta_3\Delta_3)\rangle\,, \tag{4.10}$$

$$\text{electron}: \quad (\bar{f}_2 l_2^c + \bar{f}_5 l_5^c)h_2 + (\bar{f}_2 l_5^c + \bar{f}_5 l_2^c)h_2\langle\bar{F}_5\cdot F_2\rangle\,. \tag{4.11}$$

Clearly this is not a viable set: No up–quark mass terms appear below at least ninth order. Unsuppressed or slightly suppressed up–quark mass terms only appear when $\bar{h}$ contains a $\bar{h}_{45}$ component. Specifically, $\bar{h}_{45}$ produces viable top and charm masses from third– and fifth–order terms, respectively: $F_4\bar{f}_5\bar{h}_{45}$ and $F_4\bar{f}_2\bar{h}_{45}\langle\bar{F}_5\cdot F_2\rangle$. Along flat direction $\mathbf{d}'_{20}$, an $\bar{h}_{45}$ component could be possible only if condensates in some specific terms in (4.5) receive sufficiently large VEVs, e.g., of order 10^{-9} or greater. Note also that the down–quark mass matrix has two equivalent fifth–order mass terms, for F_1F_1 and F_4F_4. This produces the further phenomenological disaster of equal bottom and strange masses. A generic degeneracy of second– and third–generation down–quark masses for $\langle F_1\rangle = 0$, $\langle F_2\rangle \neq 0$ was first noted in [20].

5 Concluding Discussion

Our main result has been to demonstrate that in the flipped $SU(5)$ model Higgs mass textures produced by all–order stringently–flat directions, i.e., those where cancellations between components of a given F term are not postulated, are extremely constrained. Generally, two out of four pairs of MSSM Higgs doublets receive FI–scale masses and so decouple from the low–energy effective field theory. However, along some of our all–order flat directions it may be possible for three out of the four pairs of Higgs doublets h_i and $\bar{h}_i$ to gain FI–scale masses, while one combination remains massless. Whether or not one or two pairs of Higgs doublets remain massless appears to depend on the hidden sector $SO(6)$ condensate scale. We have also found that, along our all–order flat directions, the surviving $\bar{h}$ will not contain an $\bar{h}_{45}$ component, unless some terms containing $\langle a_i\bar{a}_j\rangle$ condensates appear in the mass

[‡]Recall that in our all–order flat directions F_2 is massive at the FI scale. Hence all terms containing F_2 decouple.

matrix. However, we recall that the presence in $\bar{h}$ of an $\bar{h}_{45}$ component is critical for a viable top-quark mass term.

The form of the quark and lepton mass matrices is heavily restricted for all–order stringent flat directions, and not very realistic. *This reinforces the phenomenological necessity of studying non–stringently flat directions, wherein supersymmetry is almost inevitably broken at some finite order.* This might even be a positive advantage, if the breaking occurs at a sufficiently high order. Thus, building on the analysis started here, in [21] we will review the non–stringently–flat directions investigated previously in [4, 5, 6, 7], and determine the respective orders at which F–flatness is broken for these directions, as well as address other phenomenological issues.

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