(12) **United States Patent**
Liu et al.

(10) **Patent No.:** US 7,295,137 B2
(45) **Date of Patent:** *Nov. 13, 2007

(54) **DATA ENCODING AND DECODING USING SLEPIAN-WOLF CODED NESTED QUANTIZATION TO ACHIEVE WYNER-ZIV CODING**

(75) Inventors: **Zhixin Liu**, College Station, TX (US);
Samuel S. Cheng, Houston, TX (US);
Angelos D. Liveris, Stafford, TX (US);
Zixiang Xiong, Spring, TX (US)

(73) Assignee: **The Texas A&M University System**, College Station, TX (US)

(*) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 0 days.

This patent is subject to a terminal disclaimer.

(21) Appl. No.: **11/086,778**

(22) Filed: **Mar. 22, 2005**

(65) **Prior Publication Data**

US 2006/0197690 A1 Sep. 7, 2006

Related U.S. Application Data

(63) Continuation-in-part of application No. 11/068,737, filed on Mar. 1, 2005.

(60) Provisional application No. 60/657,520, filed on Mar. 1, 2005.

(51) **Int. Cl.**
H03M 7/34 (2006.01)

(52) **U.S. Cl.** **341/51**; 341/56; 341/67

(58) **Field of Classification Search** 341/155, 341/156, 50, 143, 51, 67, 107, 56; 375/141, 375/240, 340
See application file for complete search history.

(56) **References Cited**

U.S. PATENT DOCUMENTS

5,586,331 A * 12/1996 Levenstein 710/200

6,263,029	B1 *	7/2001	Alard et al.	375/340
6,441,764	B1 *	8/2002	Barron et al.	341/155
2002/0110206	A1 *	8/2002	Becker et al.	375/346
2002/0176494	A1 *	11/2002	Zhao et al.	375/240
2004/0119616	A1 *	6/2004	Han et al.	341/50
2005/0062623	A1 *	3/2005	Lee et al.	341/107
2006/0048038	A1 *	3/2006	Yedidia et al.	714/793

OTHER PUBLICATIONS

"wyner_Ziv Coding Based on TCQ and LDPC Codes" Yang Yang et al., IEEE 2003.*

Aaron D. Wyner and Jacob Ziv; "The Rate-Distortion Function for Source Coding with Side Information at the Decoder"; IEEE Transactions on Information Theory; Jan. 1976; pp. 1-10; vol. IT-22, No. 1.

A. D. Wyner; "The Rate-Distortion Function for Source Coding with Side Information at the Decorder-II: General Sources", Information and Control; 1978; pp. 60-80; vol. 38.

(Continued)

Primary Examiner—Linh Nguyen

(57) **ABSTRACT**

A system and method for realizing a Wyner-Ziv encoder may involve the following steps: (a) apply nested quantization to input data from an information source in order to generate intermediate data; and (b) encode the intermediate data using an asymmetric Slepian-Wolf encoder in order to generate compressed output data representing the input data. Similarly, a Wyner-Ziv decoder may be realized by: (1) applying an asymmetric Slepian-Wolf decoder to compressed input data using side information to generate intermediate values, and (b) jointly decoding the intermediate values using the side information to generate decompressed output data.
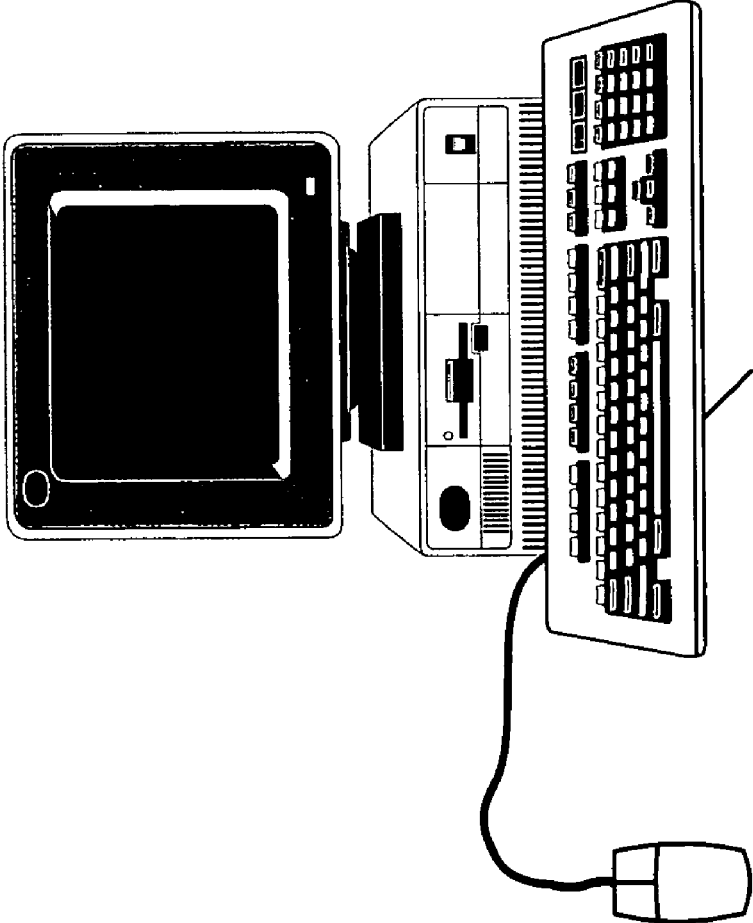
32 Claims, 14 Drawing Sheets
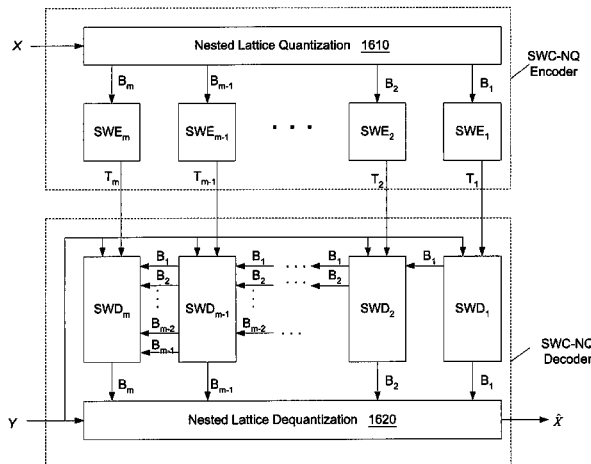
OTHER PUBLICATIONS

Sergio D. Servetto; "Lattice Quantization with Side Information,"; Data Compression Conference Proceedings; 2000; pp. 1-10.

Xin Wang and Michael T. Orchard; "Design of Trellis Codes for Source Coding with Side Information at the Decoder"; Data Compression Conference Proceedings; 2001; pp. 361-370.

Patrick Mitran and Jan Bajcsy; "Coding for the Wyner-Ziv Problem with Turbo-Like Codes," IEEE International Symposium on Information Theory; Jun./Jul. 2002; p. 91.

Anne Aaron, Rui Zhang and Bernd Girod; "Wyner-Ziv Coding of Motion Video"; Conference Record of the 36th Asilomar Conference on Signals, Systems and Computers; Nov. 2002; pp. 240-241; vol. 1.

S. Sandeep Pradhan and Kannan Ramchandran; "Distributed Source Coding Using Syndromes (DISCUS): Design and Construction"; IEEE Transactions on Information Theory; Mar. 2003; pp. 626-643; vol. 49, No. 3.

David Rebollo-Monedero, Rui Zhang, and Bernd Girod; "Design of Optimal Quantizers for Distributed Source Coding"; Data Compression Conference Proceedings; Mar. 2003; pp. 13-22.

Jim Chou, S. Sandeep Pradhan, and Kannan Ramchandran; "Turbo and Trellis-Based Constructions for Source Coding with Side Information"; Data Compression Conference Proceedings; Mar. 2003; pp. 33-42.

Angelos Liveris, Zixiang Xiong and Costas N. Georghiades; "Nested Convolutional/Turbo Codes for the Binary Wyner-Ziv Problem," International Conference on Image Processing Proceedings; Sep. 2003; pp. I-601-I-604; vol. 1.

Zixiang. Xiong, Angelos D. Liveris, Samuel Cheng, and Zhixin Liu; "Nested Quantization and Slepian-Wolf Coding: A Wyner-Ziv Coding Paradigm for I.I.D. Sources"; IEEE Workshop on Statistical Signal Processing; Sep./Oct. 2003; pp. 399-402.

Yang Yang, Samuel Cheng, Zixiang Xiong, and Wei. Zhao; "Wyner-Ziv Coding Based on TCQ and LDPC Codes"; Conference Record of the 37th Asilomar Conference on Signals, Systems and Computers; Nov. 2003; pp. 825-829; vol. 1.

Zhixin Liu, Samuel Cheng, Angelos Liveris, and Zixiang Xiong; "Slepian-Wolf Coded Nested Quantization (SWC-NQ) for Wyner-Ziv Coding: Performance Analysis and Code Design," Data Compression Conference Proceedings; Mar. 2004; pp. 322-331.

Gottfried Ungerboeck; "Channel Coding with Multilevel/Phase Signals"; IEEE Transactions on Information Theory; Jan. 1982; pp. 55-67; vol. IT-28, No.1.

Michael W. Marcellin and Thomas R. Fischer; "Trellis Coded Quantization of Memoryless and Gauss-Markov Sources"; IEEE Transactions on Communications; Jan. 1990; pp. 82-93; vol. 38, No. 1.

Ram Zamir and Shlomo Shamai, "Nested Linear/Lattice Codes for Wyner-Ziv Encoding"; Information Theory Workshop; Jun. 1998; pp. 92-93.

J. H. Conway, E. M. Rains and N. J. A. Sloane; "On the Existence of Similar Sublattices"; Canadian Journal of Mathematics; 1999; pp. 1300-1306; vol. 51, No. 6.

Ram Zamir, Shlomo Shamai, and Uri Erez; "Nested Linear/Lattice Codes for Structured Multiterminal Binning"; IEEE Transactions on Information Theory; Jun. 2002; pp. 1250-1276: vol. 48, No. 6.

M. Vedat Eyuboglu and G. David Forney Jr.; "Lattice and Trellis Quantizations with Lattice- and Trellis-Bounded Codebooks- High-Rate Theory for Memoryless Sources"; IEEE Transactions on Information Theory; Jan. 1993; pp. 46-59; vol. 39, No. 1.

David J. C. MacKay; "Good Error-Correcting Codes Based on Very Sparse Matrices"; IEEE Transactions on Information Theory; Mar. 1999; pp. 399-431; vol. 45, No. 2.

D. J. C. MacKay and R. M. Neal; "Near Shannon limit performance of low density parity check codes"; Electronics Letters; Mar. 13, 1997; pp. 457-458; vol. 33, No. 6.

David Rebollo-Monedero, Anne Aaron, and Bernd Girod "Transforms for High-Rate Distributed Source Coding"; Conference Record of the 37th Asilomar Conference on Signals, Systems and Computers; Nov. 2003; pp. 850-854; vol. 1.

David Slepian and J.K. Wolf; "Noiseless Coding of Correlated Information Sources"; IEEE Transactions on Information Theory; Jul. 1973; pp. 471-480; vol. 19.

Ram Zamir; "The Rate Loss in the Wyner-Ziv Problem"; IEEE Transactions on Information Theory; Nov. 1996; pp. 2073-2084; vol. 42, No. 6.

Vahid Tarokh, Alexander Vardy, and Kenneth Zeger; "Universal Bound on the Performance of Lattice Codes," IEEE Transactions on Information Theory, Mar. 1999; pp. 670-681; vol. 45, No. 2.

Lori A. Dalton; "Analysis of 1-D Nested Lattice Quantization and Slepian-Wolf Coding for Wyner-Ziv Coding of i.i.d. Sources", Project report for ELEN 663, Texas A&M University, May 2003, 22 pages.

G. David Forney, Jr.; "Coset Codes—Part II: Binary Lattices and Related Codes"; IEEE Transactions on Information Theory; Sep. 1988; pp. 1152-1187; vol. 34, No. 5.

Aneglos Liveris, Zixiang Xiong and Costas N. Georghiades; "Compression of Binary Sources With Side Information at the Decoder Using LDPC Codes," IEEE Communications Letters; Oct. 2002; pp. 440-442, vol. 6, No. 10.
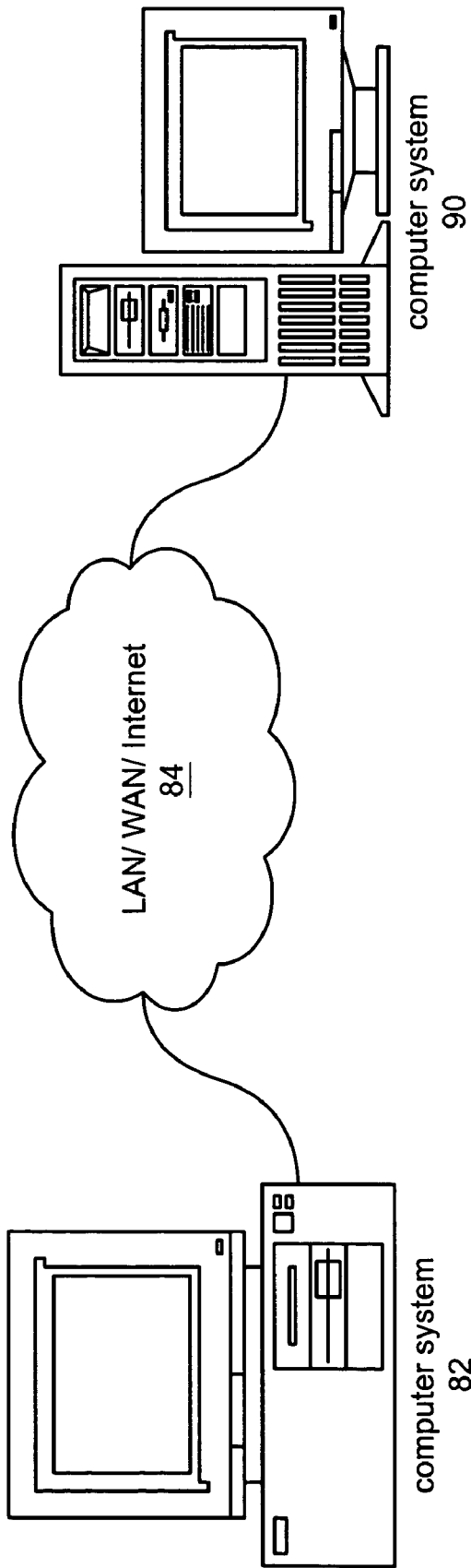
* cited by examiner

Computer System
82

Figure 1A

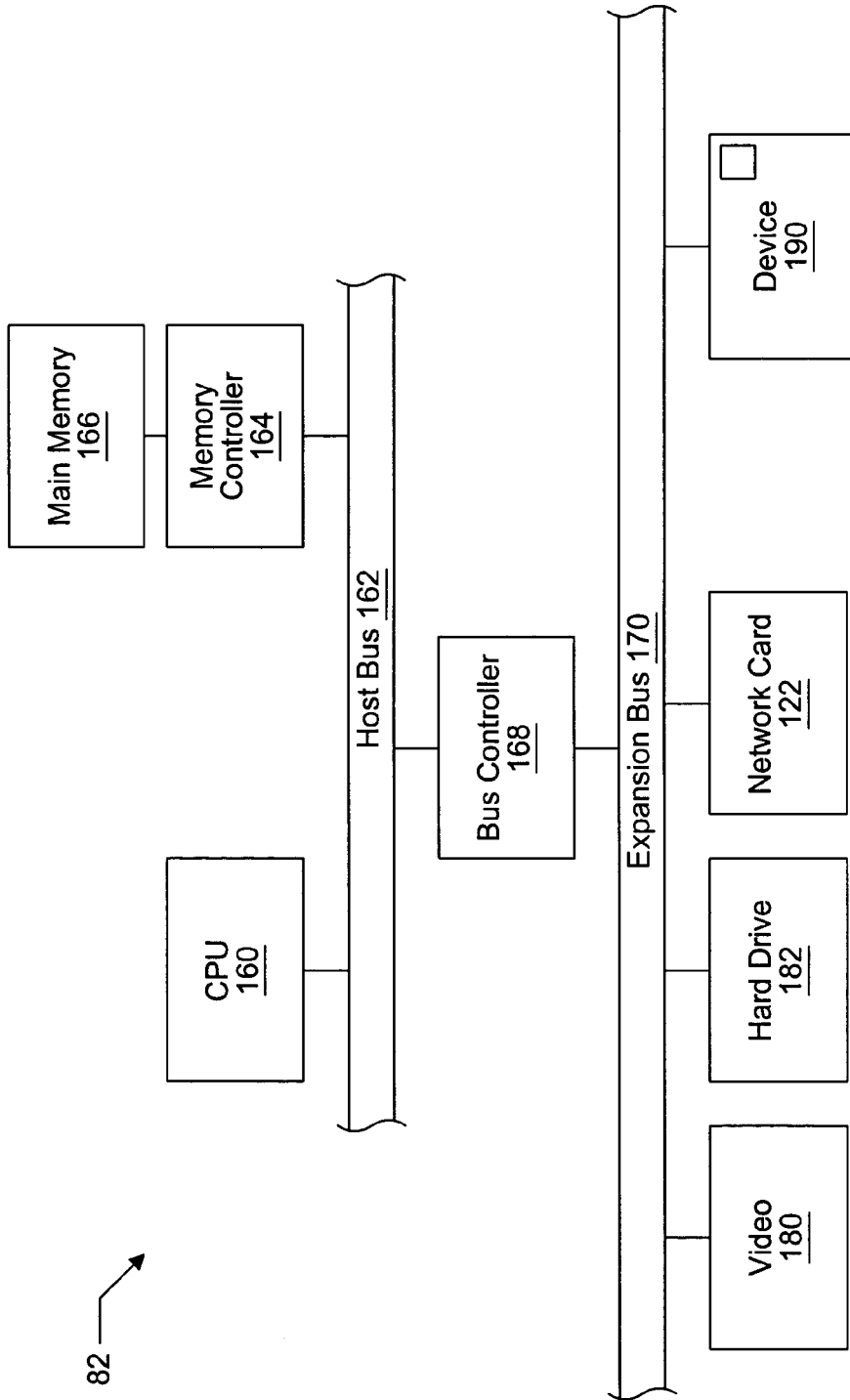computer system
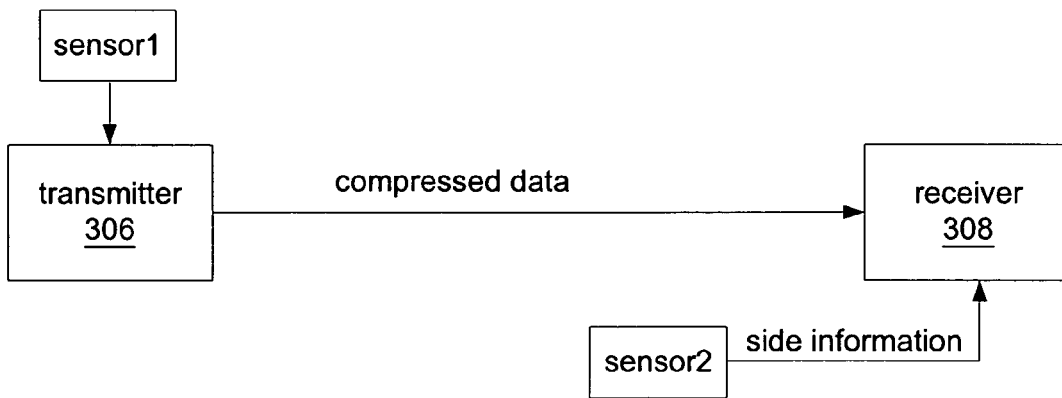90

LAN/ WAN/ Internet
84

computer system
82

Figure 1B

82

Main Memory
166

Memory
Controller
164

CPU
160

Host Bus 162

Bus Controller
168

Expansion Bus 170

Device
190

Network Card
122

Hard Drive
182

Video
180

Figure 2

sensor1

transmitter
306

compressed data →

receiver
308

sensor2 | side information →

Figure 3A

video
source1

transmitter
316

compressed data →

receiver
318

video
source2 | side information →

Figure 3B

source1

encoder
326

compressed data →

memory
medium
327

compressed data →

decoder
328

side information

source2

Figure 3C

receive input data from an information source
405

apply nested quantization to the input data from an information source in order to generate intermediate encoded data
410

encode the intermediate data using an asymmetric Slepian-Wolf encoder in order to generate compressed output data representing the input data
420

perform at least one of: storing the compressed output data, and, transferring the compressed output data.
425

Fig. 4

receive compressed input data representing a block of samples of a first source X
510

receive a block of samples of a second source Y
512

apply an asymmetric Slepian-Wolf decoder to the compressed input data using the block of samples of the second source Y, in order to generate a block of intermediate values
514

perform joint decoding on each intermediate value and a corresponding sample of the block of second source samples to obtain a corresponding decompressed output value, where said performing joint decoding includes determining an estimate of a centroid of a function restricted to a region of space corresponding to the intermediate value.
516

Figure 5

compute a realization z of a first random vector;   610

compute a realization y of a second random vector   615

add z and y to determine a realization x of a source vector   620

quantize the realization x to a point in a fine lattice   625

compute an index J identifying a coset of a coarse lattice in the fine lattice based on the fine lattice point   630

add the realization x to a cumulative sum corresponding to the index J and the realization y   635

increment a count value corresponding to the index J and the realization y   640

repeat a number of times   645

divide the cumulative sums by their corresponding count values to obtain resultant values   650

store the resultant values in a memory medium.   655

Figure 6

$Q_{\Lambda_2}(x_{Q_{\Lambda_1}})$

x $x_{Q_{\Lambda_1}} = Q_{\Lambda_1}(x)$

Fig. 7

Encoder   Decoder

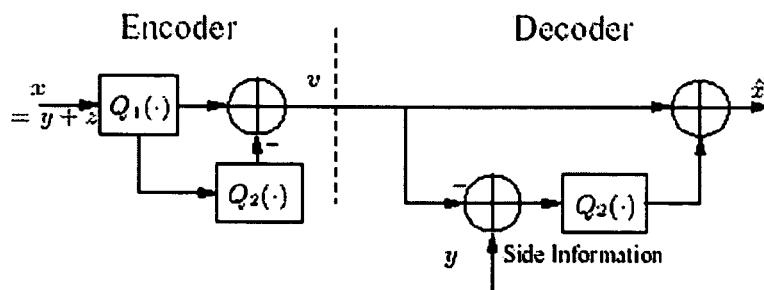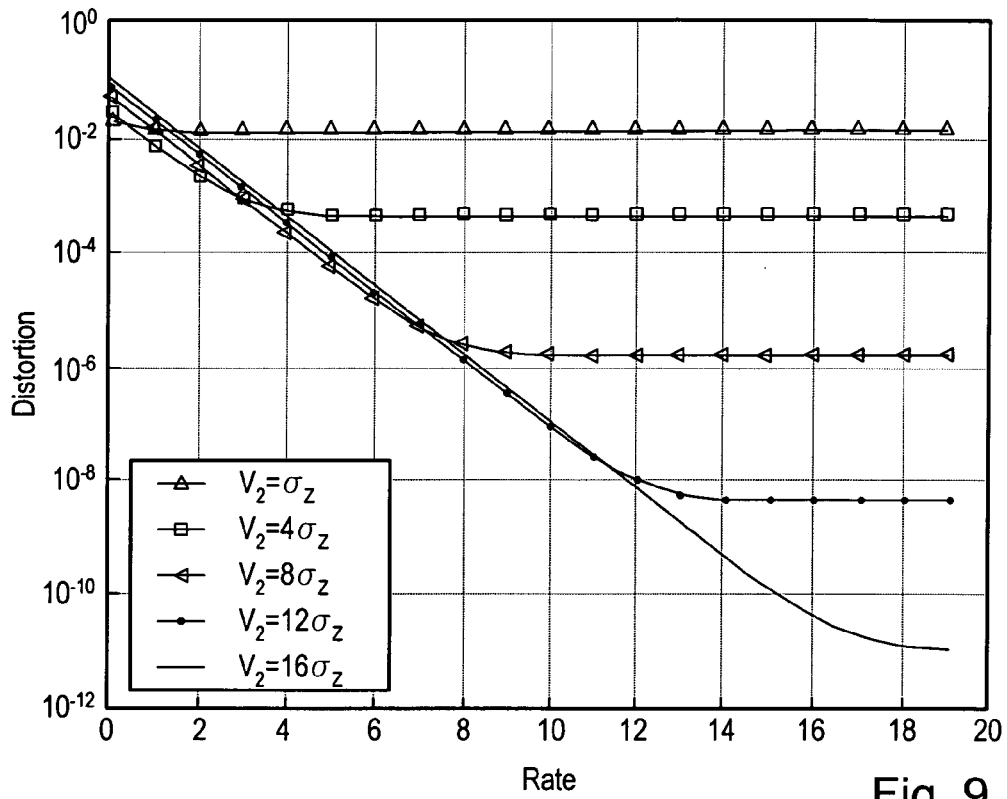$x = y + z$ $Q_1(\cdot)$ v $\hat{x}$

$Q_2(\cdot)$ $Q_2(\cdot)$

y Side Information

Fig. 8

Fig. 9

Fig. 10

Fig. 11

Theoretical Analyses of Rate-Distortion function of
Wyner-Ziv coding using nested lattice quantization

Fig. 12

Rate-Distortion of SWC-NQ with different V_2

Legend:
- $V_2=6\sigma_z$
- $V_2=9\sigma_z$
- $V_2=12\sigma_z$
- $V_2=15\sigma_z$
- $V_2=18\sigma_z$
- Wyner-Ziv limit

Fig. 13

$$\sigma\frac{2}{y}=1, \sigma\frac{2}{z}=0.01, X=Y+Z$$

Legend:
- Nested Quantization without SWC using linear estimator
- Nested Quantization without SWC using non-linear estimator
- Nested Quantization with ideal SWC using linear estimator
- Nested Quantization with odeal SWC using non-linear estimator
- Wyner-Ziv distortion-rate function for coding X

Fig. 15

$X = Y + Z, \sigma^2_{\frac{}{z}} = 0.01$

(b)

(a)

Fig. 14

Fig. 16

$$X = Y + Z, \sigma\frac{2}{y} = 1, \sigma\frac{2}{z} = 0.01$$

Distortion-rate function for coding X alone
WZC scheme without SWC (simulation)
WZC scheme without SWC (high-rate analysis)
WZC scheme with practical SWC
WZC scheme with ideal SWC (simulation)
WZC scheme with ideal SWC (high-rate analysis)
Wyner-Ziv distortion-rate function for coding X

1.53dB

rate in bits per sample

Fig. 17

$$X = Y + Z, \sigma\frac{2}{y} = 1, \sigma\frac{2}{z} = 0.01$$

Distortion-rate function for coding X alone
WZC scheme without SWC (simulation)
WZC scheme without SWC (high-rate analysis)
WZC scheme with practical SWC
WZC scheme with ideal SWC (simulation)
WZC scheme with ideal SWC (high-rate analysis)
Wyner-Ziv distortion-rate function for coding X
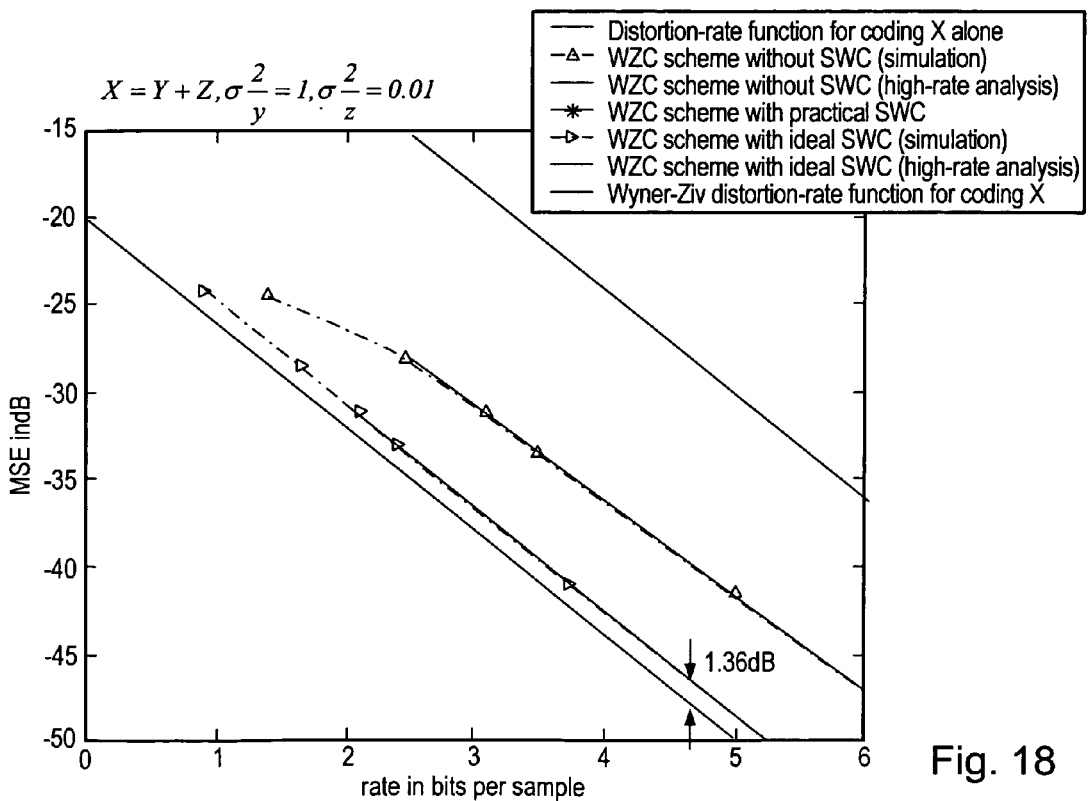
1.36dB

rate in bits per sample

Fig. 18

DATA ENCODING AND DECODING USING SLEPIAN-WOLF CODED NESTED QUANTIZATION TO ACHIEVE WYNER-ZIV CODING

PRIORITY DATA AND CONTINUATION DATA

This application claims the benefit of U.S. Provisional Application No. 60/657,520, filed on Mar. 1, 2005, entitled "Multi-Source Data Encoding, Transmission and Decoding", invented by Vladimir M. Stankovic, Angelos D. Liveris, Zixiang Xiong, Costas N. Georghiades, Zhixin Liu and Samuel S. Cheng, including Appendices A-H.

This application is a continuation in part of U.S. patent application Ser. No. 11/068,737, filed on Mar. 1, 2005, entitled "Data Encoding and Decoding Using Slepian-Wolf Coded Nested Quantization to Achieve Wyner-Ziv Coding", invented by Zhixin Liu, Samuel S. Cheng, Angelos D. Liveris and Zixiang Xiong, including Appendices A-H.

STATEMENT OF U.S. GOVERNMENT LICENSING RIGHTS

The U.S. Government has a paid-up license in this invention and the right in limited circumstances to require the patent owner to license others on reasonable terms as provided for by the terms of grant number CCR-01-04834 awarded by the National Science Foundation (NSF).

FIELD OF THE INVENTION

The present invention relates to the field of information encoding/decoding, and more particularly to a system and method for realizing a Wyner-Ziv code using nested quantization and Slepian Wolf coding.

DESCRIPTION OF THE RELATED ART

In 1976, Wyner and Ziv [1] established a theorem regarding the best possible source coding performance given distortion under the assumption that the decoder has access to side information. Unfortunately, codes realizing or approaching this best possible performance have not heretofore been demonstrated. Thus, it would be greatly desirable to be able to design codes (especially practical codes) realizing or approaching this best possible performance, and, to deploy such codes for use in encoders and decoders.

SUMMARY

In one set of embodiments, a system and method for generating compressed output data may involve:

(a) receiving input data from an information source;

(b) applying nested quantization to the input data in order to generate intermediate data;

(c) encoding the intermediate data using an asymmetric Slepian-Wolf encoder in order to generate compressed output data representing the input data; and

(d) performing at least one of storing the compressed output data, and, transferring the compressed output data.

The values of the input data may be interpreted as vectors in an n-dimensional space, where n is greater than or equal to one.

The information source may be a continuous source or a discrete source. A discrete source generates values in a finite set. A continuous source generates values in a continuum.

The operations (b) and (c) may be arranged so as to realize the encoder portion of a Wyner-Ziv code.

The compressed output data may be stored in a memory medium for future decompression. Alternatively, the compressed output data may be transferred to a decoder for more immediate decompression.

The process of applying nested quantization to the input data may include: quantizing values of the input data with respect to a fine lattice to determine corresponding points of the fine lattice; and computing indices identifying cosets of a coarse lattice in the fine lattice corresponding to the fine lattice points. The intermediate data include said indices. The coarse lattice is a sublattice of the fine lattice.

In any given dimension, some choices for the fine lattice and coarse lattice may lead to better performance than others. However, the principles of the present invention may be practiced with non-optimal choices for the fine lattice and coarse lattice as well as with optimal choices.

In another set of embodiments, a system and method for recovering information from compressed input data may involve:

(a) receiving compressed input data, wherein the compressed input data is a compressed representation of a block of samples of a first source X;

(b) receiving a block of samples of a second source Y;

(c) applying an asymmetric Slepian-Wolf decoder to the compressed input data using the block of samples of the second source Y, wherein said applying generates a block of intermediate values;

(d) performing joint decoding on each intermediate value and a corresponding sample of the block of second source samples to obtain a corresponding decompressed output value.

The operations (c) and (d) may be arranged so as to realize the decoder portion of a Wyner-Ziv code.

The joint decoding may involve determining an estimate of a centroid of a function restricted to a region of space corresponding to the intermediate value. The function may be the conditional probability density function of the first source X given said corresponding sample of the second source block. The centroid estimate may be (or may determine) the decompressed output value.

The region of space is a union of cells (e.g., Voronoi cells) corresponding to a coset of a coarse lattice in a fine lattice, wherein the coset is identified by the intermediate value.

In yet another set of embodiments, a system and method for computing a table representing a nested quantization decoder may involve:

(a) computing a realization z of a first random vector;

(b) computing a realization y of a second random vector;

(c) adding z and y to determine a realization x of a source vector;

(d) quantizing the realization x to a point in a fine lattice;

(e) computing an index J identifying a coset of a coarse lattice in the fine lattice based on the fine lattice point;

(f) adding the realization x to a cumulative sum corresponding to the index J and the realization y;

(g) incrementing a count value corresponding to the index J and the realization y;

(h) repeating operations (a) through (g) a number of times;

(i) dividing the cumulative sums by their corresponding count values to obtain resultant values; and

(j) storing the resultant values in a memory.

In one set of embodiments, a system for generating compressed output data may include a memory and a processor. The memory is configured to store data and program instructions. The processor is configured to read and execute the program instructions from the memory. In response to execution of the program instructions, the processor is operable to: (a) receive input data from an information source; (b) apply nested quantization to the input data in order to generate intermediate data; (c) encode the intermediate data using an asymmetric Slepian-Wolf encoder in order to generate compressed output data representing the input data; and (d) perform at least one of: storing the compressed output data; and transferring the compressed output data.

In another set of embodiments, a system for decoding compressed data may include a memory and processor. The memory is configured to store data and program instructions. The processor is configured to read and execute the program instructions from the memory. In response to execution of the program instructions, the processor is operable to: (a) receive compressed input data, wherein the compressed input data is a compressed representation of a block of samples of a first source X; (b) receive a block of samples of a second source Y; (c) apply an asymmetric Slepian-Wolf decoder to the compressed input data using the block of samples of the second source Y, wherein said applying generates a block of intermediate values; (d) perform joint decoding on each intermediate value and a corresponding sample of the block of second source samples to obtain a corresponding decompressed output value, wherein said performing joint decoding includes determining an estimate of a centroid of a function restricted to a region of space corresponding to the intermediate value, wherein said estimate determines the decompressed output value. The function is the conditional probability density function of the first source X given said corresponding sample of the second source block.

In yet another set of embodiments, a system for computing a table representing a nested quantization decoder may include a memory and processor. The memory is configured to store data and program instructions. The processor is configured to read and execute the program instructions from the memory. In response to execution of the program instructions, the processor is operable to: (a) computing a realization z of a first random vector; (b) computing a realization y of a second random vector; (c) adding z and y to determine a realization x of a source vector; (d) quantizing the realization x to a point in a fine lattice; (e) computing an index J identifying a coset of a coarse lattice in the fine lattice based on the fine lattice point; (f) adding the realization x to a cumulative sum corresponding to the index J and the realization y; (g) incrementing a count value corresponding to the index J and the realization y; (h) repeating operations (a) through (g) a number of times; (i) dividing the cumulative sums by their corresponding count values to obtain resultant values; and (j) storing the resultant values in a memory medium.

We propose a practical scheme that we refer to as Slepian-Wolf coded nested quantization (SWC-NQ) for Wyner-Ziv coding that deals with source coding with side information under a fidelity criterion. The scheme utilizes nested lattice quantization with a fine lattice for quantization and a coarse lattice for channel coding. In addition, at low dimensions (or block sizes), an additional Slepian-Wolf coding stage is added to compensate for the weakness of the coarse lattice channel code. The role of Slepian-Wolf coding in SWC-NQ

is to exploit the correlation between the quantized source and the side information for further compression and to make the overall channel code stronger.

The applications of this proposed scheme are very broad; it can be used in any application that involves lossy compression (e.g., of speech data, audio data, image data, video data, graphic data, or, any combination thereof).

We show that SWC-NQ achieves the same performance of classic entropy-constrained lattice quantization. For example, 1-D/2-D SWC-NQ performs 1.53/1.36 dB away from the Wyner-Ziv rate distortion (R-D) function of the quadratic Gaussian source at high rate assuming ideal Slepian-Wolf coding. In other words, the scheme may be optimal in terms of compression performance, at least in some embodiments. We also demonstrate means of achieving efficient Slepian-Wolf compression via multi-level LDPC codes.

BRIEF DESCRIPTION OF THE DRAWINGS

A better understanding of the present invention can be obtained when the following detailed description of the preferred embodiment is considered in conjunction with the following drawings, in which:

FIG. 1A illustrates one embodiment of a computer system that may be used for implementing various of the method embodiments described herein;

FIG. 1B illustrates one embodiment of a communication system including two computers coupled through a computer network;

FIG. 2 is a block diagram for one embodiment of a computer system that may be used for implementing various of the method embodiments described herein;

FIG. 3A illustrates one embodiment of a sensor system as a possible application of the inventive principles described herein;

FIG. 3B illustrates one embodiment of a video transmission as another possible application of the inventive principles described herein;

FIG. 3C illustrates a system that compressed source information and stored the compressed information in a memory medium for later retrieval and decompression;

FIG. 4 illustrates one embodiment of a method for encoding data;

FIG. 5 illustrates one embodiment of a method for decoding data using side information;

FIG. 6 illustrates one embodiment of a method for computing a table that represents an nested quantization decoder.

FIG. 7 illustrate an example of a fine lattice, coarse lattice, coset leader vector v and region R(v) in dimension $n=2$;

FIG. 8 illustrate a simplified nested quantization ender and decoder;

FIG. 9 shows $\delta_2(R)$ with different V_2's using nested A_2 lattices (i.e., hexagonal lattices) in dimension $n=2$;

FIG. 10 show $\overline{D}_2(R)$ as the convex hull of $\delta_2(R)$ with different V_2;

FIG. 11 shows the granular and boundary components of distortion with different V_2's;

FIG. 12 plots $\overline{D}_n(R)$ for $n=1, 2, 4, 8$ and 24 with $\sigma_z^2=0.01$;

FIG. 13 shows the lower bound of D(R) with different V_2's in the 1-D case;

FIGS. 14(a) and (b) plot the optimal V^*_2 (scaled by σ_z) as a function of R for the 1-D ($n=1$) and 2-D ($n=2$) cases;

FIG. 15 shows the improvement gained by using the optimal (non-linear) estimator at low rates, for $n=2$ and $\sigma_z^2=0.01$;

FIG. **16** illustrates one embodiment of a multi-layer Slepian Wolf coding scheme;

FIG. **17** shows results based on 1-D nested lattice quantization both with and without Slepian Wolf coding (SWC); and

FIG. **18** shows results based on 2-D nested lattice quantization both with and without Slepian Wolf coding (SWC).

While the invention is susceptible to various modifications and alternative forms, specific embodiments thereof are shown by way of example in the drawings and are herein described in detail. It should be understood, however, that the drawings and detailed description thereto are not intended to limit the invention to the particular form disclosed, but on the contrary, the intention is to cover all modifications, equivalents and alternatives falling within the spirit and scope of the present invention as defined by the appended claims.

DETAILED DESCRIPTION OF THE PREFERRED EMBODIMENTS

Incorporation by Reference

The following patent application documents are hereby incorporated by reference in their entirety as though fully and completely set forth herein:

U.S. Provisional Application Ser. No. 60/657,520, titled "Multi-Source Data Encoding, Transmission and Decoding", filed Mar. 1, 2005, whose inventors are Vladimir M. Stankovic, Angelos D. Liveris, Zixiang Xiong, Costas N. Georghiades, Zhixin Liu, Samuel S. Cheng, and Qian Xu, including Appendices A through H;

U.S. patent application Ser. No. 11/069,935, titled "Multi-Source Data Encoding, Transmission and Decoding Using Slepian-Wolf Codes Based On Channel Code Partitioning", filed Mar. 1, 2005, whose inventors are Vladimir M. Stankovic, Angelos D. Liveris, Zixiang Xiong, and Costas N. Georghiades, including Appendices A through H; and

U.S. patent application Ser. No. 11/068,737, titled "Data Encoding and Decoding Using Slepian-Wolf Coded Nested Quantization to Achieve Wyner-Ziv Coding", filed Mar. 1, 2005, whose inventors are Zhixin Liu, Samuel S. Cheng, Angelos D. Liveris, and Zixiang Xiong, including Appendices A through H.

The following publications are referred to herein and are incorporated by reference in their entirety as though fully and completely set forth herein:

[1] A. Wyner and J. Ziv, "The rate-distortion function for source coding with side information at the decoder," IEEE Trans. Inform. Theory, vol. IT-22, pp. 1-10, January 1976.

[2] A. Wyner, "The rate-distortion function for source coding with side information at the decoder-II: general sources," Inform. Contr., vol. 38, pp. 60-80, 1978.

[3] S. Servetto, "Lattice quantization with side information," in Proceedings of the IEEE Data Compression Conference, DCC2000, Snowbird, Utah, March 2000.

[4] X. Wang and M. Orchard, "Design of trellis codes for source coding with side information at the decoder," in Proc. DCC'01, Snowbird, Utah, March 2001.

[5] P. Mitran and J. Bajcsy, "Coding for the Wyner-Ziv problem with turbo-like codes," in Proc. ISIT'02, Lausanne, Switzerland, June 2002.

[6] R. Zhang A. Aaron and B. Girod, "Wyner-Ziv coding of motion video," in Proc. 36th Asilomar Conf., Pacific Grove, Calif., November 2002.

[7] S. Pradhan and K. Ramchandran, "Distributed source coding using syndromes (DISCUS): Design and construction," IEEE Trans. Inform. Theory, vol. 49, pp. 626-643, March 2003.

[8] D. Rebollo-Monedero, R. Zhang, and B. Girod, "Design of optimal quantizers for distributed source coding," in Proc. DCC'03, Snowbird, Utah, March 2003.

[9] J. Chou, S. Pradhan, and K. Ramchandran, "Turbo and trellis-based constructions for source coding with side information," in Proc. DCC'03, Snowbird, Utah, March 2003.

[10] A. Liveris, Z. Xiong and C. Georghiades, "Nested convolutional/turbo codes for the binary Wyner-Ziv problem," in Proc. ICIP'03, Barcelona, Spain, September 2003.

[11] Z. Xiong, A. Liveris, S. Cheng, and Z. Liu, "Nested quantization and Slepian-Wolf coding: A Wyner-Ziv coding paradigm for i.i.d. sources," in Proc. IEEE Workshop on Statistical Signal Processing, St. Louis, Mo., September 2003.

[12] Y. Yang, S. Cheng, Z. Xiong, and W. Zhao, "Wyner-Ziv coding based on TCQ and LDPC codes," in Proc. 37th Asilomar Conf., Pacific Grove, Calif., November 2003.

[13] J. H. Conway and Neil J. A. Sloane, "Sphere Packings, Lattices and Groups", Springer, N.Y., 1998.

[14] G. Ungerboeck, "Channel coding with multilevel/phase signals," IEEE Trans. Inform. Theory, vol. 28, pp. 55-67, January 1982.

[15] M. Marcellin and T. Fischer, "Trellis coded quantization of memoryless and Gaussian-Markov sources," IEEE Communications, vol. 38, pp. 82-93, January 1990.

[16] R. Zamir and S. Shamai, "Nested linear/lattice codes for Wyner-Ziv encoding," in Proc. IEEE Information Theory Workshop, Killarney, Ireland, June 1998, pp. 92-93.

[17] J. Conway, E. Rains, and N. Sloane, "On the existence of similar sublattices," Canadian J. Math., vol. 51, pp. 1300-1306, 1999.

[18] R. Zamir, S. Shamai, and U. Erez, "Nested linear/lattice codes for structured multiterminal binning," IEEE Trans. Inform. Theory, vol. 48, pp. 1250-1276, June 2002.

[19] M. V. Eyuboglu and G. D. Forney, Jr., "Lattice and trellis quantization with lattice- and trellis-bounded codebooks—high-rate theory for memoryless sources," IEEE Trans. Information Theory, vol. 39, pp. 46-59, January 1993.

[20] Robert G. Gallager, Low Density Parity Check Codes, MIT Press, 1963, ISBN: 0262571773.

[21] D. MacKay, "Good error-correcting codes based on very sparse matrices," IEEE Trans. Information Theory, vol. 45, pp. 399-431, March 1999.

[22] D. MacKay and R. Neal, "Near shannon limit performance of low density parity check codes," Electron. Lett., vol. 33, pp. 457458, March 1997.

[23] D. Rebollo-Monedero, A. Aaron, and B. Girod, "Transforms for high-rate distributed source coding," in Proc. 37th Asilomar Conf., Pacific Grove, CA, November 2003.

[24] D. Slepian and J. K. Wolf, "Noiseless coding of correlated information sources," IEEE Trans. Inform. Theory, vol. 19, pp. 471-480, July 1973.

[25] R. Zamir, "The rate loss in the Wyner-Ziv problem," IEEE Trans. Inform. Theory, vol. 42, pp. 2073-2084, November 1996.

[26] V. Tarokh, A. Vardy, and K. Zeger, "Universal bound on the performance of lattice codes," IEEE Trans. Inform. Theory, vol. 45, pp. 670-681, March 1999.

7

8

[27] Lori A. Dalton, "Analysis of 1-D nested lattice quantization and Slepian-Wolf coding for Wyner-Ziv coding of i.i.d. sources," May 2002, Technical report, Texas A&M University.

[28] G. D. Forney Jr., "Coset codes-Part II: Binary lattices and related codes," IEEE Trans. Inform. Theory, vol. 34, pp. 1152-1187, 1988.

[29] A. Liveris, Z. Xiong and C. Georghiades, "Compression of binary sources with side information at the decoder using LDPC codes," IEEE Communications Letters, vol. 6, pp. 440-442, October 2002.

[30] T. M. Cover and J. A. Thomas, Elements of Information Theory, Wiley Interscience, 1991.

[31] R. G. Gallager, Information Theory and Reliable Communication, New York: Wiley, 1968.

Terminology

The following is a glossary of terms used in the present application:

Memory Medium—Any of various types of memory devices, storage devices, or combinations thereof. The term "memory medium" is intended to include: CD-ROM, any of various kinds of magnetic disk (such as floppy disk or hard disk), any of various kinds of magnetic tape, optical storage, and bubble memory; any of various kinds of read only memory (ROM); any of various kinds of random access memory (RAM) such as DRAM, DDR RAM, SRAM, EDO RAM, Rambus RAM, etc.

Carrier Medium—a memory medium as described above, or, a communication medium on which signals are conveyed, e.g., signals such as electrical, electromagnetic, acoustic, optical signals.

Programmable Hardware Element—includes various types of programmable hardware, reconfigurable hardware, programmable logic, or field-programmable devices (FPDs), such as one or more FPGAs (Field Programmable Gate Arrays), or one or more PLDs (Programmable Logic Devices), or other types of programmable hardware. A programmable hardware element may also be referred to as "reconfigurable logic".

Program—the term "program" is intended to have the full breadth of its ordinary meaning. The term "program" includes 1) a software program which may be stored in a memory and is executable by a processor or 2) a hardware configuration program useable for configuring a programmable hardware element.

Software Program—the term "software program" is intended to have the full breadth of its ordinary meaning, and includes any type of program instructions, code, script and/or data, or combinations thereof, that may be stored in a memory medium and executed by a processor. Exemplary software programs include programs written in text-based programming languages, such as C, C++, Pascal, Fortran, Cobol, Java, assembly language, etc.; graphical programs (programs written in graphical programming languages); assembly language programs; programs that have been compiled to machine language; scripts; and other types of executable software. A software program may comprise two or more components that interoperate.

Hardware Configuration Program—a program, e.g., a netlist or bit file, that can be used to program or configure a programmable hardware element.

Computer System—any of various types of computing or processing systems, including a personal computer system (PC), mainframe computer system, workstation, network appliance, Internet appliance, personal digital assistant (PDA), television system, grid computing system, or other device or combinations of devices. In general, the term "computer system" can be broadly defined to encompass any device (or combination of devices) having at least one processor that executes instructions from a memory medium.

FIG. 1A—Computer System

FIG. 1A illustrates a computer system 82, according to one set of embodiments, operable to execute a set of programs. The programs may be configured to implement any or all of the method embodiments described herein. The computer system 82 may include one or more processors, memory media, and one or more interface devices. The computer system 82 may also include input and output devices. The memory media may include various well known systems and devices configured for the storage of data and computer programs. For example, the memory media may store one or more programs which are executable to perform the methods (or some subset of the methods) described herein. The memory medium may also store operating system software, as well as other software for operation of the computer system. In various embodiments, the computer system 82 may be a personal computer, a notebook computer, a workstation, a server, a router, a computer implemented on a card, etc.

FIG. 1B—Computer Network

FIG. 1B illustrates a communication system including a first computer system 82 and a second computer system 90, according to one set of embodiments. The first computer system 82 couples to the second computer system 90 through a network 84 (or, more generally, any of various known communication mechanisms). The first and second computer systems may each be any of various types, as desired. The network 84 can also be any of various types, including a LAN (local area network), WAN (wide area network), the Internet, or an Intranet.

Each of the computer systems may be configured with programs implementing any or all of the method embodiments described herein. In one embodiment, the first and second computer systems are each configured with software for encoding and decoding data as described variously herein.

It is noted that computer system 82 and computer system 90 may be configured according to any of various system architectures.

FIG. 2—Computer System Block Diagram

FIG. 2 is a block diagram representing one embodiment of computer system 82 and/or computer system 90.

The computer system may include at least one central processing unit CPU 160 which is coupled to a host bus 162. The CPU 160 may be any of various types, including, but not limited to, an x86 processor, a PowerPC processor, a CPU from the SPARC family of RISC processors, as well as others. A memory medium, typically comprising RAM, and referred to as main memory 166, is coupled to the host bus 162 by means of memory controller 164. The main memory 166 may store programs operable to implement encoding and/or decoding according to any (or all) of the various embodiments described herein. The main memory may also store operating system software, as well as other software for operation of the computer system.

The host bus 162 couples to an expansion or input/output bus 170 through a bus controller 168 or bus bridge logic. The expansion bus 170 may be the PCI (Peripheral Component Interconnect) expansion bus, although other bus types can be used. The expansion bus 170 includes slots for various

devices such as a video card **180**, a hard drive **182**, a CD-ROM drive (not shown) and a network interface **122**. The network interface **122** (e.g., an Ethernet card) may be used to communicate with other computers through the network **84**.

In one embodiment, a device **190** may also be connected to the computer. The device **190** may include an embedded processor and memory. The device **190** may also or instead comprise a programmable hardware element (such as an FPGA). The computer system may be operable to transfer a program to the device **190** for execution of the program on the device **190**. The program may be configured to implement any or all of the encoding or decoding method embodiments described herein.

In some embodiments, the computer system **82** may include input devices such as a mouse and keyboard and output devices such a display and speakers.

FIGS. **3A**, **3B** & **3C**—Exemplary Systems

Various embodiments of the present invention may be directed to sensor systems, wireless or wired transmission systems, or, any other type of information processing or distribution system utilizing the coding principles described herein.

For example, as FIG. **3A** shows, a sensor system may include a first sensor (or set of sensors) and a second sensor (or set of sensors). The first sensor may provide signals to a transmitter **306**. The sensors may be configured to sense any desired physical quantity or set of physical quantities such as time, temperature, energy, velocity, flow rate, displacement, length, mass, voltage, electrical current, charge, pressure, etc. The transmitter **306** may receive the signals, digitize the signals, encode the signals according the inventive principles described herein, and transmit the resulting compressed data to a receiver **308** using any of various known communication mechanism (e.g., a computer network). The receiver **308** receives the compressed data from the transmitter as well as side information from a second sensor. The receiver **308** decodes the compressed data, according to the inventive principles described herein, using the side information, and thereby, generates decompressed output data. The decompressed output data may be used as desired, e.g., displayed to a user, forwarded for analysis and/or storage, etc.

As another example, a first video source may generate video signals as shown in FIG. **3B**. A transmitter **316** receives the video signal, encodes the video signals according the inventive principles described herein, and transmits the resulting compressed data to a receiver **318** using any of various known communication mechanism (e.g., a computer network). The receiver **318** receives the compressed data from the transmitter as well as side information from a second sensor. The receiver **318** decoders the compressed data, according to the inventive principles described herein, using the side information.

As yet another embodiment, a encoder **326** may receive signals from a first source and encode the source signals according to the inventive principles described herein, and store the resulting compressed data onto a memory medium **327**. At some later time, an encoder **328** may read the compressed data from the memory medium **327** and decode the compressed data according to the inventive principles described herein.

It is noted that embodiments of the present invention can be used for a plethora of applications and is not limited to the above applications. In other words, applications discussed in the present description are exemplary only, and the present

invention may be used in any of various types of systems. Thus, the system and method of the present invention is operable to be used in any of various types of applications, including audio applications, video applications, multimedia applications, any application where physical measurements are gathered, etc.

FIG. **4** illustrates one embodiment of a method for decoding data. In step **405**, input data is received from an information source.

In step **410**, nested quantization as described herein is applied to the input data in order to generate intermediate data.

In step **420**, the intermediate data is encoded using an asymmetric Slepian-Wolf encoder as described herein, in order to generate compressed output data representing the input data.

The nested quantization and asymmetric Slepian-Wolf encoder may be configured so that the combination of steps **410** and **420** realizes the encoder portion of a Wyner-Ziv code.

In step **425**, the compressed output data may be stored and/or transferred. In one embodiment, the compressed output data may be stored onto a memory medium for decompression at some time in the future. In another embodiment, the compressed output data may be transferred, e.g., to a decoder device.

The information source may be a continuous source or a discrete source. A discrete source generates values in a finite set. A continuous source generates values in a continuum. The values of the input data may be interpreted as vectors in an n-dimensional space, where n is greater than or equal to one.

The process of applying nested quantization to the input data may include: quantizing values of the input data with respect to a fine lattice to determine corresponding points of the fine lattice; and computing indices identifying cosets of a coarse lattice in the fine lattice corresponding to the fine lattice points. The intermediate data include said indices. The coarse lattice is a sublattice of the fine lattice.

In any given dimension, some choices for the fine lattice and coarse lattice may lead to better performance than others. However, the principles of the present invention may be practiced with non-optimal choices for the fine lattice and coarse lattice as well as with optimal choices.

In various embodiments, the information source may be a source of audio information, a source of video information, a source of image information, a source of text information, a source of information derived from physical measurements (e.g., by a set of one or more physical sensors), or, any combination thereof.

As discussed in reference [29], one way to do asymmetric Slepian-Wolf encoding is by means of syndrome forming, which involves a modification of classical channel encoding. This type of Slepian-Wolf encoding is used to generate the simulation results described in this paper. However, the general method of Slepian-Wolf coded nested quantization disclosed in this paper can also be performed with other forms of Slepian-Wolf encoders.

In some embodiments, the asymmetric Slepian-Wolf encoder may be a low density parity check syndrome former or a turbo syndrome former.

In one embodiment, the asymmetric Slepian-Wolf encoder may be configured as a multi-layered encoder as described herein.

An encoder system may be configured to implement any embodiment of the method illustrated and described above in connection with FIG. **4**. The encoder system may include

one or more processors or programmable hardware elements, and/or, dedicated circuitry such as application specific integrated circuits. In one embodiment, the encoder system includes a processor (e.g., a microprocessor) and memory. The memory is configured to store program instructions and data. The processor is configured to read and execute the program instructions from the memory to implement any embodiment of the method illustrated and described above in connection with FIG. **4**.

Furthermore, a computer-readable memory medium may be configured to store program instructions which are executable by one or more processors to implement any embodiment of the method illustrated and described above in connection with FIG. **4**.

FIG. **5** illustrates one embodiment of a method for decoding data. In step **510**, compressed input data is received. The compressed input data is a compression representation of a block of samples of a first source X. In step **512**, a block of samples of a second source Y is received. Steps **510** and **512** need not be performed in any particular order. In one embodiment, steps **510** and **512** may be performed in parallel, or, at least in a time overlapping fashion. The first source X and the second source Y may be statistically correlated.

In step **514**, an asymmetric Slepian-Wolf decoder as described herein is applied to the compressed input data using the block of samples of the second source Y. This application of the asymmetric Slepian-Wolf decoder generates a block of intermediate values.

In step **516**, joint decoding is performed on each intermediate value and a corresponding sample of the block of second source samples to obtain a corresponding decompressed output value. The joint decoding may include determining an estimate of a centroid of a function restricted to a region of space corresponding to the intermediate value. The function may be the conditional probability density function of the first source X given said corresponding sample of the second source block. The centroid estimate may be (or may determine) the decompressed output value. The resulting block of decompressed output values may be used in any of various ways as desired. For example, the block of decompressed output values may be displayed to a user, forwarded for analysis and/or storage, transmitted through a network to one or more other destinations, etc.

The steps **514** and **516** may be configured so as to realize the decoder portion of a Wyner-Ziv code.

The region of space is a union of cells (e.g., Voronoi cells) corresponding to a coset of a coarse lattice in a fine lattice, wherein the coset is identified by the intermediate value.

The centroid estimate may be determined by reading the centroid estimate from a table stored in a memory medium using said corresponding sample of the second source block and the intermediate value as addresses. The table may be computed in at a central code design facility, and, then deployed to a decoder system through any of various known means for data distribution. The table may be stored in a memory medium of the decoder system. The decoder system may accessing the table to determine the centroid estimate in real time.

In one alternative embodiment, the centroid estimate may be determined by performing a Monte Carlo iterative simulation at decode time.

The intermediate values generated in step **514** may specify cosets of a coarse lattice in a fine lattice. The coarse lattice may be a sublattice of the fine lattice.

The asymmetric Slepian-Wolf decoder may be a multi-layered decoder. Furthermore, the asymmetric Slepian-Wolf decoder may be a low density parity check decoder or a turbo decoder.

A decoder system may be configured to implement any embodiment of the method illustrated and described above in connection with FIG. **5**. The decoder system may include one or more processors or programmable hardware elements, and/or, dedicated circuitry such as application specific integrated circuits. In one embodiment, the decoder system includes a processor (e.g., a microprocessor) and memory. The memory is configured to store program instructions and data. The processor is configured to read and execute the program instructions from the memory to implement any embodiment of the method illustrated and described above in connection with FIG. **5**.

Furthermore, a computer-readable memory medium may be configured to store program instructions which are executable by one or more processors to implement any embodiment of the method illustrated and described above in connection with FIG. **5**.

FIG. **6** illustrates one embodiment of a method for computing a table representing a nested quantization decoder by Monte Carlo simulation. The method may be implemented by executing program instructions on a computer system (or a set of interconnected computer systems). The program instructions may be stored on any of various known computer-readable memory media.

In step **610**, the computer system may compute a realization z of a first random vector (the auxiliary vector), e.g., using one or more random number generators. In step **615**, the computer system may compute a realization y of a second random vector (the side information), e.g., using one or more random number generators. Steps **610** and **615** need not be performed in any particular order.

In step **620**, the computer system may add the realization y and the realization z to determine a realization x of a source vector.

In step **625**, the computer system may quantize the realization x to a point p in a fine lattice as described herein.

In step **630**, the computer system may compute an index J identifying a coset of a coarse lattice in the fine lattice based on the fine lattice point p. The coarse lattice is a sublattice of the fine lattice.

The computer system may maintain a set of cumulative sums, i.e., one cumulative sum for each possible pair in the Cartesian product (CPR) of the set of possible indices and the set of possible realizations of the second random vector (the side information). The cumulative sums may be initialized to zero. Furthermore, the computer system may maintain a set of count values, i.e., one count value for each possible pair in the Cartesian product CPR.

In step **635**, the computer system may add the realization x to a cumulative sum corresponding to the index J and the realization y. In step **640**, the computer system may increment a count value corresponding to the index J and the realization y. Steps **635** and **640** need not be performed in any particular order.

The computer system may repeat steps **610** through **640** a number of times as indicated in step **645**. In one embodiment, the number of repetitions may be determined by input provided by a user.

In step **650**, the computer system may divide the cumulative sums by their corresponding count values to obtain resultant values. The resultant values may be interpreted as being the centroid estimates described above in connection with FIG. **5**.

In step **655**, the computer system may store the resultant values as a table in a memory associated with the computer system, e.g., onto hard disk.

The table may be distributed (e.g., with decoding software configured according to any of the various method embodiments described herein) to decoder systems by any of various means. In one embodiment, the table may be downloaded to decoder systems over a network such as the Internet. In another embodiment, the table may be stored on a computer-readable memory media (such as CD-ROM, magnetic disk, magnetic tape, compact flash cards, etc.) and the memory media may be provided (e.g., sold) to users of decoder systems for loading onto their respective computer systems.

In one embodiment, system for computing a table representing a nested quantization decoder may be configured with a processor and memory. The memory is configured to store program instructions and data. The processor is configured to read and execute the program instructions from the memory to implement any embodiment of the method illustrated and described above in connection with FIG. **6**.

The Wyner-Ziv coding problem deals with source coding with side information under a fidelity criterion. The rate-distortion function for this setup, R*(D), is given by [1]:

$$R^*(D) = \min_{p(u|x)} \min_{f: A_U \times A_Y \to A_{\hat{X}}} [I(U; X) - I(U; Y)], \qquad (1)$$

where the source X (with an alphabet A_X), the side information Y (with an alphabet A_Y) and the auxiliary random variable U (with an alphabet A_U) form a Markov chain as Y $\leftrightarrow$ X $\leftrightarrow$ U, with the distortion constraint E[d(X,f(U,Y),Y)] $\leq \hat{D}$. The function I(•) denotes the Shannon mutual information as defined in [3]. The function p(u|x) is the conditional probability of U given X. The function f represents the mapping from the possible auxiliary variable and side information to a reconstructed value of X.

Although the theoretical limits for the rate-distortion function have been known for some time [1], [2], practical approaches to binary Wyner-Ziv coding and continuous Wyner-Ziv coding have not appeared until recently [3], [4], [5], [6], [7], [8], [9], [10], [11], [12]. A common context of interest for continuous Wyner-Ziv coding is code design for the quadratic Gaussian case, where the correlation between the source X and the side information Y is modeled as an additive white Gaussian noise (AWGN) channel as X=Y+Z, Z~N(0, σ_Z^2), with a mean-squared error (MSE) measure. For this case, one can first consider lattice codes [13] or trellis-based codes [14], [15] that have been used for both source and channel coding in the past, and focus on finding good nesting codes among them. Following Zamir et al's nested lattice coding scheme [16], Servetto [3] proposed explicit nested lattice constructions based on similar sublattices [17] with the assumption of high correlation. Research on trellis-based nested codes as a way of realizing high-dimensional nested lattice codes has just started recently [7]. For example, in DISCUS [7], two source codes (scalar quantization and trellis coded quantization—TCQ) and two channel codes (scalar coset code and trellis-based coset code [14]) are used in source-channel coding for the Wyner-Ziv problem, resulting in four combinations. One of them (scalar quantization with scalar coset code) is nested scalar quantization and another one (TCQ with trellis-based coset code, also suggested in [4]) can effectively be considered as nested TCQ.

Zamir et al. [18], [16] first outlined some theoretical constructions using a pair of nested linear/lattice codes for binary/Gaussian sources, where the fine code in the nested pair plays the role of source coding while the coarse code does channel coding. They also proved that, for the quadratic Gaussian case, the Wyner-Ziv rate-distortion (R-D) function is asymptotically achievable using nested lattice codes, with the assumption that the lattice is ideally sphere-packed as the lattice dimension goes to infinity.

The performance of a nested lattice quantizer can approach the Wyner-Ziv limit at high rate when high-dimensional lattices are used, because both the granular gain and boundary gain reach their ultimate values [19] when the dimension n→∞. Nevertheless, lattice coding and code design with high dimensionality are difficult in practice.

For a nested lattice quantizer using low-to-moderate dimensional lattices, a pragmatic approach to boost the overall performance is to increase the boundary gain with a second stage of binning, without increasing the dimensionality of the lattices. Suppose a second stage of binning is applied without introducing extra overload probability P_{ol}, and the binning scheme partitions the support region of fine lattice (actually the Voronoi region of the coarse lattice for nested lattice quantizer) into m cosets. Thus the volume of the support region decreases by a factor of N/m while the overload probability stays fixed, where N is the nesting ratio. From the definition of boundary gain [19], the boundary gain increases without changing the dimension of the lattices. Since various possible boundary gains are realizable using the second-stage of binning as discussed above, there is only maximally 1.53 dB (decibels) of granular gain left unexploited by the quantizer. Thus the second stage of binning allows us to show the theoretical performance limits at high rates with low-to-moderate dimensional source codes.

In this paper, we introduce a new framework for the continuous Wyner-Ziv coding of independent and identically distributed (i.i.d.) sources based a combination of Slepian-Wolf coding (SWC) and nested quantization (NQ). In this framework, which we refer to as SWC-NQ, the role of Slepian-Wolf coding, as a second-stage of binning which increases the boundary gain of source coding, is to exploit the correlation between the quantized source and the side information for further compression and by making the overall channel code stronger. SWC-NQ connects network information theory with the rich areas of (a) lattice source code designs (e.g., [13]) and (b) channel code designs (e.g., LDPC codes [20], [21] and [22]), making it feasible to devise codes that can approach the Wyner-Ziv rate-distortion function. LDPC is an acronym for "low density parity check".

For the quadratic continuous case, we establish the high-rate performance of SWC-NQ with low-to-moderate dimensional nested quantization and ideal SWC. We show that SWC-NQ achieves the same performance of classic entropy-constrained lattice quantization as if the side information were also available at the encoder. For example, 1-D/2-D SWC-NQ performs 1.53/1.36 dB away from the Wyner-Ziv R-D function of the quadratic continuous source at high rate assuming ideal SWC.

A recent work, [23], starts with non-uniform quantization with index reuse and Slepian-Wolf coding and shows the same high-rate theoretical performance as ours when the quantizer becomes an almost uniform one without index reuse. This agrees with our assertion that at high rates, the nested quantizer asymptotically becomes a non-nested regular one so that strong channel coding is guaranteed.

We also implement 1-D and 2-D nested lattice quantizers in the rate range of 1-7 bits per sample. Although our analysis shows that nesting does not help at high rate, experiments using nested lattice quantizers together with irregular LDPC codes for SWC obtain performances close to the corresponding limits at low rates. Our work thus shows that SWC-NQ provides an efficient scheme for practical Wyner-Ziv coding with low-dimensional lattice quantizers at low rates.

Although the theoretical analyses are taken under the assumption of high rate, the rate-distortion performance at low rate is still consistent with the one at high rate, i.e., SWC-NQ achieves the same performance of classic entropy coded quantization (ECQ) as if the side information were also available at the encoder even at low rate, when a non-linear estimator is applied at the decoder. This non-linear estimator, as we present in this paper, is the optimal one in the sense of the MSE measurement. At high rates, the non-linear estimator reduces to the linear one analyzed in this paper.

We note that the non-linear estimation in the decoder can yield significant gains for low rates and for high rates it cannot help noticeably. This is confirmed by the agreement of the high rate analysis results in this paper, which assume that the linear estimation is used, with the high rate simulation results, for which the non-linear estimation method is always used.

The following is a list of some of the contents of this paper:

1. A theoretical analysis and simulation for low-to-moderate dimensional nested lattice quantization at high rates. The rate-distortion function for general continuous sources with arbitrary probability density function (PDF) and MSE measurement, and a theoretical lower bound of rate-distortion function for the quadratic Gaussian case, are presented.

2. An analysis of the granular and boundary gains of the source coding component of nested lattice quantization. This analysis explains the phenomenon of an increasing gap of the rate-distortion function of nested lattice quantization at low-to-moderate dimension, with respect to the Wyner-Ziv limit, as we observe in the simulation.

3. A new Wyner-Ziv coding framework using nested lattice quantization and Slepian-Wolf coding, which we refer to as SWC-NQ, is introduced. The SWC-NQ rate-distortion function for general continuous sources with arbitrary PDF and MSE measurement is presented, and is in agreement with the performance of entropy-constrained lattice quantization as if the side information were available at the encoder.

4. A non-linear estimator for the decoder corresponding to the nested quantizer is presented, and is proved to be optimal in sense of MSE measurement. This estimator helps to improve the performance of SWC-NQ at low rates, and is consistent with the analytical performance at high rates.

5. Examples of practical code design using a 1-D (scalar) lattice and 2-D (hexagonal) lattice, and multi-layer irregular LDPC codes, are given in this paper.

Some Background on Wyner-Ziv Coding

In this section, we briefly review the basic concepts and milestone theorems of Wyner-Ziv coding. Wyner and Ziv [1], [2] present the limit of rate-distortion performance for lossy coding with side information, for both Gaussian and general sources.

The problem of rate distortion with side information at the decoder asks the question of how many bits are needed to encode X under the constraint that $E[d(X,\hat{X})] \leq D$, assuming the side information Y is available at the decoder but not at the encoder. This problem generalizes the setup of [24] in that coding of X is lossy with respect to a fidelity criterion rather than lossless. For both discrete and continuous alphabets of A_X and general distortion metrics d(•), Wyner and Ziv [1] gave the rate-distortion function $R^*_{WZ}(D)$ for this problem as $R^*_{WZ}(D) = \inf I(X;Z|Y)$, where the infimum is taken over all random variables Z such that $Y \to X \to Z$ is a Markov chain and there exists a function $\hat{X} = X(Z,Y)$ satisfying $E[d(X,\hat{X})] \leq D$. According to [1],

$$R^*_{WZ}(D) \geq R_{X|Y}(D) = \inf_{\{\hat{X} \in A_X : E[d(X,\hat{X})] \leq D\}} I(X; \hat{X}|Y).$$

This means that usually there is a rate loss in the Wyner-Ziv problem. Zamir quantified this loss in [25]. In particular, Zamir showed a rate loss of less than 0.22 bit for a binary source with Hamming distance, and a rate loss of less than 0.5 bit/sample for continuous sources with MSE distortion.

Note that when D=0, the Wyner-Ziv problem degenerates to the Slepian-Wolf problem with $R^*_{WZ}(0) = R_{X|Y}(0) = H(X|Y)$. Another special case of the Wyner-Ziv problem is the quadratic Gaussian case when X and Y are zero mean and stationary Gaussian memoryless sources and the distortion metric is MSE. Let X_i denote the ith component of X, and Y_i denotes the ith component of Y, i=1, 2, . . . , n. Let the covariance matrix of (X_i, Y_i) be

$$cov(X_i, Y_i) = \begin{bmatrix} \sigma_X^2 & \rho\sigma_X\sigma_Y \\ \rho\sigma_X\sigma_Y & \sigma_Y^2 \end{bmatrix}$$

with $|\rho| < 1$ for all n, then

$$R^*_{WZ}(D) = R_{X|Y}(D) = \frac{1}{2}\log^+\left[\frac{\sigma_X^2(1-\rho^2)}{D}\right],$$

where $\log^+ x = \max\{0, \log x\}$. This case is of special interest in practice because many image and video sources can be modeled as jointly Gaussian (after mean subtraction) and Wyner-Ziv coding suffers no rate loss.

Lattices and Nested Lattices

In this section, we review the idea of lattice and nested lattices and introduce notation that will be used in our discussion.

For a set of n basis vectors $\{g_1, . . . ,g_n\}$ in R^n, an unbounded n-dimensional (n-D) lattice Λ is defined by

$$\Lambda = \{l = Gi : i \in Z^n\} \quad (2)$$

and its generator matrix

$$G = [g_1|g_2| . . . |g_n].$$

R donates the set of real numbers. R^n denotes n-dimensional Euclidean space. Z denotes the set of integers. Z^n denotes the Cartesian product of n copies of Z, i.e., the set of n-vectors whose components are integers.

The nearest neighbor quantizer $Q_\Lambda(x)$ associated with Λ is given by

$$Q_\Lambda(x) = \operatorname*{arg\,min}_{l \in \Lambda} \|x - l\|. \tag{3}$$

The notation "arg min" denotes the value of the argument (in this case l) where the minimum is achieved. Expression (3) is augmented with a set of "tie breaking" rules to decide the result in cases where two or more points of the lattice Λ achieve the minimum distance to vector x. Any of various sets of tie breaking rules may be used. For example, in dimension one (i.e., n=1) with lattice Λ being the integers, points of the form $k+(\frac{1}{2})$ with be equidistant to k and k+1. One possible tie-breaking rule would be to map such points up to k+1. In one set of embodiments, the nearest neighbor quantizer defined by (3) and a set of tie breaking rules has the property:

$$Q_\Lambda(x+l) = Q_\Lambda(x) + l, \ \forall l \in \Lambda.$$

The basic Voronoi cell of Λ, which specifies the shape of the nearest-neighbor decoding region, is

$$K = \{x : Q_\Lambda(x) = 0\}. \tag{4}$$

Associated with the Voronoi cell K are several important quantities: the cell volume V, the second moment σ^2 and the normalized second moment $G(\Lambda)$, defined by

$$V = \int_K dx, \tag{5}$$

$$\sigma^2 = \frac{1}{nV} \int_K |x|^2 \, dx, \tag{6}$$

$$G(\Lambda) = \frac{\sigma^2}{V^{2/n}}, \tag{7}$$

respectively. The minimum of $G(\Lambda)$ over all lattices in R^n is denoted as G_n. By [13],

$$G_n \geq 1/(2\pi e), \ \forall \, n \tag{8}$$

$$\lim_{n \to \infty} G_n = 1/(2\pi e) \tag{9}$$

The notation "$\forall$" is to be read as "for all". The constant e is Euler's constant.

A pair of n-D lattices (Λ_1, Λ_2) with corresponding generator matrices G_1 and G_2 is nested, if there exists an n×n integer matrix P such that

$$G_2 = G_1 \times P \text{ and}$$

$$\det\{P\} > 1,$$

where $\det\{P\}$ denotes the determinant of the matrix P. In this case V_2/V_1 is called the nesting ratio, and Λ_1 and Λ_2 are called the fine lattice and coarse lattice, respectively.

For a pair (Λ_1, Λ_2) of nested lattices, the points in the set $\Lambda_1/\Lambda_2 \equiv \{\Lambda_1 \cap K_2\}$ are called the coset leaders of Λ_2 relative to Λ_1, where K_2 is the basic Voronoi cell of Λ_2. The notation

"A≡B" means that A is being defined by expression B, or vice versa. For each $v \in \Lambda_1/\Lambda_2$ the set of shifted lattice points

$$C(v) \equiv \{v+l, \ \forall l \in \Lambda_2\}$$

is called a coset of Λ_2 relative to $A\Lambda_1$. The jth point of C(v) is denoted as $c_j(v)$. Then

$$C(0) = \{c_j(0), \ \forall \, j \in Z\} = \Lambda_2, \tag{10}$$

and

$$\bigcup_{v \in \Lambda_1/\Lambda_2} C(v) = \Lambda_1. \tag{11}$$

Since

$$c_j(v) \in \Lambda_1, \forall_j \in Z, \tag{12}$$

we further define

$$R_j(v) = \{x : Q_{\Lambda_1}(x) = c_j(v)\}$$

as the Voronoi region associated with $c_j(v)$ in Λ_1, and $R(v) = \bigcup_{j=-\infty}^{\infty} R_j(v)$. Then

$$\bigcup_{v \in \Lambda_1/\Lambda_2} R_0(v) = K_2, \tag{13}$$

and

$$\bigcup_{j=-\infty}^{\infty} \bigcup_{v \in \Lambda_1/\Lambda_2} R_j(v) = \bigcup_{v \in \Lambda_1/\Lambda_2} R(v) = \mathbb{R}^n. \tag{14}$$

FIG. 7 illustrates examples of v, C(v) and R(v). The fine lattice points are at the centers of the small hexagons. The coarse lattice points are at the centers of the large hexagons. R(v) is the union of the shaded hexagons. The coset C(v) is the set composed of the centers of the shaded hexagons. The fine lattice and coarse lattice may be generated by

$$G_1 = \begin{bmatrix} 2 & 1 \\ 0 & \sqrt{3} \end{bmatrix} \text{ and } G_2 = \begin{bmatrix} 5 & 1 \\ \sqrt{3} & 3\sqrt{3} \end{bmatrix},$$

respectively, and related by

$$P = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}.$$

Nested Lattice Quantization

Throughout this paper, we use the correlation model of X=Y+Z, where X, Y and Z are random vectors in R^n. X is the source to be coded, Y is the side information, and Z is the noise. Y and Z are independent. In this section we discuss the performance of nested lattice quantization for general sources where Y and Z are arbitrarily distributed with zero means, as well as for the quadratic Gaussian case where $Y_i \sim N(0, \sigma_Y^2)$ and $Z_i \sim N(0, \sigma_Z^2)$, i=1, 2, . . . , n, are Gaussian. For both cases, the mean squared error (MSE) is used as the distortion measurement.

Zamir et al.'s nested lattice quantization scheme [18], [16] works as follows: Let the pseudo-random vector U (also

referred to herein as the "dither"), known to both the quantizer encoder and the decoder, be uniformly distributed over the basic Voronoi cell K_1 of the fine lattice Λ_1. For a given target average distortion D, denote $\alpha\sqrt{1-}=D/\sigma_Z^2$ as the estimation coefficient. Given the realizations of the source, the side information and the dither as x, y and u, respectively, then according to [18], the nested quantizer encoder quantizes $\alpha x+u$ to the nearest point $x_{Q_{\Lambda 1}}=Q_{\Lambda_1}(\alpha x+u)$ in Λ_1, computes $x_{Q_{\Lambda 1}}-Q_{\Lambda_2}(x_{Q_{\Lambda 1}})$ which is the coset shift of $x_{Q_{\Lambda 1}}$ with respect to Λ_2, and t index corresponding to this coset shift.

The nested quantizer decoder receives the index, generates $x_{Q\Lambda 1}-Q_{\Lambda_2}(x_{Q_{\Lambda 1}})$ from the index, forms

$$w=x_{Q_{\Lambda 1}}-Q_{\Lambda 2}(x_{Q_{\Lambda 1}})-u-\alpha y$$

and reconstructs x as $\hat{x}=y+\alpha(w-Q_{\Lambda_2}(w))$ using linear combination and dithering in estimation.

It is shown in [18] that the Wyner-Ziv R-D function $D^{wz}(R)=\sigma_{X|Y}^2 2^{-2R}$ is achievable with infinite dimensional nested lattice quantization for quadratic Gaussian case. In this paper, we analyze the high-rate performance of low-dimensional nested lattice quantization, which is of more practical interest as high-dimensional nested lattice quantization is too complex to implement, for both general and Gaussian sources.

Our analysis is based on the high-resolution assumption, which means 1) V_1 is small enough so that the PDF of X, $f(x)$, is approximately constant over each Voronoi cell of Λ_1 and 2) dithering can be ignored. With the high-rate assumption, $\alpha\approx1$ and the encoder/decoder described above simplifies as follows:

The encoder quantizes x to $x_{Q_{\Lambda 1}}=Q_{\Lambda_1}(x)$, computes $v=x_{Q_{\Lambda 1}}-Q_{\Lambda_2}(x_{Q_{\Lambda 1}})$, and transmits an index corresponding to the coset leader v.

Upon receiving v, the decoder forms w=v−y and reconstructs x as $\hat{x}_v=y+w-Q_{\Lambda_2}(w)=v+Q_{\Lambda_2}(y-v)$.

This simplified nested lattice quantization scheme for high rate is shown in FIG. **8** and was also used in [3].

A. High Rate Performance for General Sources with Arbitrary Distribution

Theorem 4.1: If a pair of n-D nested lattices (Λ_1,Λ_2) with nesting ratio $N=V_2/V_1$ is used for nested lattice quantization, the distortion per dimension in Wyner-Ziv coding of X with side information Y at high rate is

$$D_n = G(\Lambda_1)V_1^{2/n} + \frac{1}{n}E_Z[\|Q_{\Lambda_2}(Z)\|^2]. \tag{15}$$

The notation $\|a\|$ denotes the length (or norm) of the vector a.

Proof: Since

$$\mathbb{R}^n = \bigcup_{j=-\infty}^{\infty} \bigcup_{v\in\Lambda_1/\Lambda_2} R_j(v), \tag{16}$$

the average distortion for a given realization of the side information Y=y is

$$D(y) = \int_{R^n} f(x|y)\|x-\hat{x}_v\|^2\,dx \tag{17}$$

$$= \sum_{v\in\Lambda_1/\Lambda_2}\sum_{j=-\infty}^{\infty}\int_{x\in R_j(v)} f(x|y)\|x-\hat{x}_v\|^2\,dx$$

$$= \sum_{v\in\Lambda_1/\Lambda_2}\sum_{j=-\infty}^{\infty}\int_{x\in R_j(v)} f(x|y)\|x-c_j(v)+c_j(v)-\hat{x}_v\|^2\,dx$$

$$= \sum_{v\in\Lambda_1/\Lambda_2}\sum_{j=-\infty}^{\infty}\int_{x\in R_j(v)} f(x|y)\left[\begin{array}{c}\|x-c_j(v)\|^2 + \|c_j(v)-\hat{x}_v\|^2 \\ +2<x-c_j(v),c_j(v)-\hat{x}_v>\end{array}\right]dx$$

$$\overset{(a)}{\approx} \sum_{v\in\Lambda_1/\Lambda_2}\sum_{j=-\infty}^{\infty}\left[\begin{array}{c}f(c_j(v)|y)\int_{x\in R_j(v)}\|x-c_j(v)\|^2\,dx + \\ \int_{x\in R_j(v)} f(x|y)\|c_j(v)-\hat{x}_v\|^2\,dx\end{array}\right]$$

$$\overset{(b)}{=} \sum_{v\in\Lambda_1/\Lambda_2}\sum_{j=-\infty}^{\infty}\left[\begin{array}{c}f(c_j(v)|y)nG(\Lambda_1)V_1^{1+(2/n)} + \int_{x\in R_j(v)} f(x|y)\|Q_{\Lambda_2}(c_j(v))- \\ Q_{\Lambda_2}(y-c_j(v)+Q_{\Lambda_2}(c_j(v)))\|^2\,dx\end{array}\right]$$

$$\overset{(c)}{\approx} nG(\Lambda_1)V_1^{\frac{2}{n}} + \sum_{j=-\infty}^{\infty}\sum_{v\in\Lambda_1/\Lambda_2}\int_{x\in R_j(v)} f(x|y)\|Q_{\Lambda_2}(x-y)\|^2\,dx$$

$$= nG(\Lambda_1)V_1^{\frac{2}{n}} + \int_{x\in R^n} f(x|y)\|Q_{\Lambda_2}(x-y)\|^2\,dx,$$

where (a) comes from the high rate assumption and

$$\int_{x\in R_j(v)} <x-c_j(v),c_j(v)-\hat{x}_v>\,dx = 0. \tag{18}$$

The latter is due to the fact that $x-c_j(v)$ is odd spherical symmetric for $x \in R_j(v)$ and both $c_j(v)$ and $\hat{x}_v$ are fixed for $x \in R_j(v)$ with given v and y. (b) is due to $c_j(v) = Q_{\Lambda_1}(x)$ for $x \in R_j(v)$, and

$$\hat{x}_v = c_j(v) - Q_{\Lambda_2}(c_j(v)) + Q_{\Lambda_2}(y - c_j(v) + Q_{\Lambda_2}(c_j(v))); \tag{19}$$

and (c) is due to

$$Q_{\Lambda_2}(a + Q_{\Lambda_2}(b)) = Q_{\Lambda_2}(a) + Q_{\Lambda_2}(b), \ \forall a,b \in R^n \tag{20}$$

and the high resolution assumption.

Therefore, the average distortion per dimension over all realizations of Y is

$$D_n = \frac{1}{n} E_Y[D(y)] \tag{21}$$

$$= G(\Lambda_1) V_1^{2/n} + \frac{1}{n} \int_x \int_y f(x, y) \|Q_{\Lambda_2}(x - y)\|^2 \, dx \, dy$$

$$= G(\Lambda_1) V_1^{2/n} + \frac{1}{n} \int_y f(y) \int_z f(z) \|Q_{\Lambda_2}(z)\|^2 \, dz \, dy$$

$$= G(\Lambda_1) V_1^{2/n} + \frac{1}{n} E_Z[\|Q_{\Lambda_2}(z)\|^2].$$

Remarks: There are several interesting facts about this rate-distortion function.

1) For a fixed pair of the nested lattices (Λ_1, Λ_2), D_n only depends on Z, i.e., the correlation between X and Y. D_n is independent of the marginal distribution of X (or Y).

2) The first term, $G(\Lambda_1) V_1^{2/n}$, in the expression for D_n is due to lattice quantization in source coding. It is determined by the geometric structure and the Voronoi cell volume V_1 of lattice Λ_1. The second term, $1/n E_Z[\|Q_{\Lambda_2}(z)\|^2]$, is the loss due to nesting (or the channel coding component of the nested lattice code). The second term depends on Voronoi cell volume V_2 and the distribution of Z. From another point of view, the first term is the granular component MSE_g with respect to the granular lattice Λ_1, and the second term is the overload component MSE_{ol} with respect to the lattice Λ_2 of the nested quantizer. $MSE^g = G(\Lambda_1) V_1^{2/n}$ is the same as the granular MSE for non-nested lattice quantizer [19].

Corollary 4.1: For the quadratic case, $D_n \to D_{WZ} = \sigma_{X/Y}^2 2^{-2R}$ as $n \to \infty$.

Proof Since the nested lattice quantizer is a fixed-rate quantizer with the rate

$$R = \frac{1}{n} \log\left(\frac{V_2}{V_1}\right), \text{ then (21) can be rewritten as} \tag{22}$$

$$D_n = G(\Lambda_1) V_2^{2/n} 2^{-2R} + \frac{1}{n} E_Z[\|Q_{\Lambda_2}(z)\|^2].$$

For the quadratic Gaussian case, according to equation (3.14) of [18],

$$\frac{1}{n} \log(V_2) \approx \frac{1}{2} \log(2\pi e \sigma_Z^2) \tag{23}$$

when n is sufficiently large, where $\sigma_Z^2 = \sigma_{X/Y}^2$ is the variance of the AWGN Z. Then

$$G(\Lambda_1) V_2^{2/n} 2^{-2R} \to \frac{1}{2\pi e} 2\pi e \sigma_Z^2 2^{-2R} \tag{24}$$

$$= \sigma_Z^2 2^{-2R}$$

$$= D_{WZ}.$$

At the same time, according to equation (3.12) of [18], $P_e = \Pr\{Z \notin K_2\} < \epsilon$, with any $\epsilon > 0$ and sufficiently large n, hence $1/n E_Z[\|Q_{\Lambda_2}(z)\|^2] \to 0$ as $n \to \infty$. Consequently, the performance becomes

$$D_n = G(\Lambda_1) V_2^{2/n} 2^{-2R} + \frac{1}{n} E_Z[\|Q_{\Lambda_2}(z)\|^2] \tag{25}$$

$$\to \sigma_{X/Y}^2 2^{-2R} = D_{WZ}$$

as $n \to \infty$, for the quadratic Gaussian case. This limit agrees with the statement in [18], which claims that the nested lattice quantization can achieve the Wyner-Ziv limit asymptotically as the dimensionality goes to infinity.

B. A Lower Bound of the Performance for Quadratic Case

The source-coding-loss in (21) has an explicit form, while the channel-coding loss is not so directly expressed. Among all the possible patterns of the additive channels, AWGN is of most interest. In such case Z is a Gaussian variable with zero mean and variance $\sigma_Z^2 = \sigma_{X/Y}^2$. From Theorem 4.1, we obtain a lower bound of the high-rate R-D performance of low-dimensional nested lattice quantizers for Wyner-Ziv coding, when Z is Gaussian, stated as the following corollary.

Corollary 4.2: For $X = Y + Z$, $Y \sim N(0, \sigma_Y^2)$ and $Z \sim N(0, \sigma_Z^2)$, the R-D performance of Wyner-Ziv coding for X with side information Y using n-D nested lattice quantizers is lower-bounded at high rate by

$$D_n(R) \geq \overline{D}_n(R) \equiv \min_{V_2 > 0} \delta_n(R), \tag{26}$$

where

$$\delta_n(R) \equiv G_n V_2^{2/n} 2^{-2R} + \frac{1}{n} \sum_{j=1}^{\infty} ((2j-1)\gamma_n V_2^{2/n}) u\left(\frac{j^2 V_2^{2/n} \Gamma\left(\frac{n}{2}+1\right)^{2/n}}{2\pi\sigma_Z^2}\right), \tag{27}$$

γ_n is the n-D Hermite's constant [13], [26], and $u(t)$ is defined in [26] as

$$u(t) \equiv \begin{cases} e^{-t}\left(1 + \frac{t}{1!} + \frac{t^2}{2!} + \ldots + \frac{t^{\frac{n}{2}-1}}{\left(\frac{n}{2}-1\right)!}\right) & \text{if } n \text{ is even} \\ e^{-t}\left(1 + \frac{t^{1/2}}{(1/2)!} + \frac{t^{3/2}}{(3/2)!} + \ldots + \frac{t^{\frac{n}{2}-1}}{\left(\frac{n}{2}-1\right)!}\right) & \text{if } n \text{ is odd} \end{cases}. \tag{28}$$

Specifically, when $n = 1$, the best possible high rate performance is

where

$$D_1(R) = \min_{V_2>0} \left\{ G_1 V_2^2 2^{-2R} + V_2^2 \sum_{j=0}^{\infty} (2j+1) Q\left(\frac{V_2}{\sigma_Z}\left(j+\frac{1}{2}\right)\right) \right\}, \qquad (29)$$

where

$$Q(t) = \frac{1}{\sqrt{2\pi}} \int_t^{\infty} e^{-\tau^2/2} \, d\tau. \qquad (30)$$

Proof: 1) Rate Computation: Note that the nested lattice quantizer is a fixed rate quantizer with rate

$$R = \frac{1}{n} \log_2\left(\frac{V_2}{V_1}\right).$$

2) Distortion computation: Define

$$L_2 = \frac{1}{2} \min_{\forall l,l' \in \Lambda_2, l \neq l'} \|l - l'\|, \qquad (31)$$

and

$$P_z(L) = Pr(\|Z\| > L). \qquad (32)$$

For the 1-D (scalar) case, P_Z can be expressed in terms of the Q function and $E_Z[\|Q_{\Lambda_2}(z)\|^2]$ simplifies to [27]

$$E_z[\|Q_{\Lambda_2}(z)\|^2] = V_2^2 \sum_{j=0}^{\infty} (2j+1) Q\left(\frac{V_2}{\sigma_z}\left(j+\frac{1}{2}\right)\right). \qquad (33)$$

For the n-D (with n>1) case, note that [28]

$$L_2^2 = \gamma(\Lambda_2) V(\Lambda_2)^{2/n}, \qquad (34)$$

and

$$\|Q_{\Lambda_2}(z)\|^2 \geq \|z\|^2 - \|z - Q_{\Lambda_2}(z)\|^2 \geq \|z\|^2 - L_2^2, \qquad (35)$$

where $\gamma(\Lambda_2)$ is the Hermite's constant of lattice Λ_2 [13], [26]. Then we get

$$\begin{aligned} E_Z[\|Q_{\Lambda_2}(z)\|^2] &= \sum_{j=1}^{\infty} \int_{((j-1)L_2 < \|z\| \leq jL_2)} f(z)\|Q_{\Lambda_2}(z)\|^2 \, dz \qquad (36) \\ &\geq \sum_{j=1}^{\infty} \int_{((j-1)L_2 < \|z\| \leq jL_2)} f(z)(\|z\|^2 - L_2^2) dz \\ &\geq \sum_{j=1}^{\infty} \int_{((j-1)L_2 < \|z\| \leq jL_2)} f(z)((j-1)^2 L_2^2 - L_2^2) dz \\ &= \sum_{j=1}^{\infty} ((j-1)^2 L_2^2 - L_2^2)[P_Z((j-1)L_2) - P_Z(jL_2)] \\ &= \sum_{j=1}^{\infty} ((2j-1)L_2^2) P_Z(jL_2) \\ &= \sum_{j=1}^{\infty} \left[(2j-1)\gamma(\Lambda_2) V_2^{\frac{2}{n}} \right] P_e\left(\frac{j^2 V_2^{\frac{2}{n}} \Gamma\left(\frac{n}{2}+1\right)^{\frac{2}{n}}}{2\pi\sigma_Z^2} \right), \end{aligned}$$

where

$$\Gamma(t) = \int_0^{\infty} u^{t-1} e^{-u} \, du$$

is Euler's gamma function, and $P_e(\cdot)$ is defined in [26] as the symbol error probability under maximum likelihood decoding while transmitting the lattice points over an AWGN channel. A lower bound of $P_e(\cdot)$ was also given in [26] as $P_e(t) \geq u(t)$

Then Theorem 4.1 and (36) give

$$D_n \geq \delta_n \equiv G_n V_1^{2/n} + \frac{1}{n} \sum_{j=1}^{\infty} \left((2j-1)\gamma_n V_2^{\frac{2}{n}} \right) u\left(\frac{j^2 V_2^{\frac{2}{n}} \Gamma\left(\frac{n}{2}+1\right)^{\frac{2}{n}}}{2\pi\sigma_Z^2} \right). \qquad (37)$$

Using

$$R = \frac{1}{n} \log_2\left(\frac{V_2}{V_1}\right),$$

we eliminate V_1 in D_n and obtain a lower bound on $D_n(R)$ as

$$D_n(R) \geq \overline{D}_n(R) \equiv \min_{V_2>0} \delta_n(R), \qquad (38)$$

where

$$\delta_n(R) = G_n V_2^{2/n} 2^{-2R} + \frac{1}{n} \sum_{j=1}^{\infty} ((2j-1)\gamma_n V_2^{2/n}) u\left(\frac{j^2 V_2^{\frac{2}{n}} \Gamma\left(\frac{n}{2}+1\right)^{\frac{2}{n}}}{2\pi\sigma_Z^2} \right). \qquad (39)$$

FIG. **9** shows $\delta_2(R)$ with different V_2's using nested Λ_2 lattices (i.e., hexagonal lattices) in 2-D with $\sigma_Z^2 = 0.01$. The lower bound $\overline{D}_2(R)$ is the lower convex hull of all operational R-D points with different V_2, as shown in FIG. **10**. We observe from FIG. **10** that the gap from $\overline{D}_n(R)$ to $D_{WZ}(R)$ in dBs keeps increasing as the rate increases with $\sigma_Z^2 = 0.01$. This increasing gap comes from the fact that, the granular MSE component

$$MSE_g \equiv G_n V_1^{2/n} = \frac{1}{12\gamma_g}\left(\frac{V_2}{N}\right)^{2/n} = \frac{1}{12\gamma_g} V_2^{2/n} 2^{-2R}$$

is away from the benchmark 2^{-2R} with an increasing gap as V_2 increases, where

$$\gamma_g \equiv \frac{\frac{1}{12}}{G_n}$$

is the granular gain [19] of lattice Λ_1, and

$$R = \frac{1}{n} \log N,$$

N is the nesting ratio, as shown in FIG. **11**.

FIG. **12** plots $\overline{D}_n(R)$ for n=1, 2, 4, 8 and 24 with σ_Z^2=0.01. We see that $\overline{D}_n(R)$ gets closer and closer to the Wyner-Ziv R-D function $D_{WZ}(R)=\sigma_{X/Y}^2 2^{-2R}$ as n goes to infinity.

C. Discussion of the Correlation-Asymptotical Property

As to the asymptotical property of the nested-lattice quantization for Wyner-Ziv coding, we have the following statement. Here asymptotical means that the correlation

$$\rho \equiv \frac{E[XY]}{\sqrt{E[X^2]E[Y^2]}}$$

between the source X and the side information Y goes to 1 asymptotically. If we fix σ_Y^2 then the asymptotical performance is the one when $\sigma_Z^2 \to 0$.

Corollary 4.3: The distortion of the nested lattice quantization maintains a constant gap (in dB) to the Wyner-Ziv bound for all $0 < \sigma_Z^2 < 1$.

Proof. Denote $s = V_2^{2/n}$,

$$t = \frac{j^2 \Gamma \left(\frac{n}{2} + 1 \right)^{2/n}}{2 \pi \sigma_Z^2} s, \text{ and } A = \Gamma \left(\frac{n}{2} + 1 \right)^{2/n}.$$

From Corollary 4.1, we get

$$\delta = G_n s 2^{-2R} + \frac{1}{n} \sum_{j=1}^{\infty} ((2j-1)\gamma_n s)u(t). \tag{40}$$

Fix rate R and dimensionality n (without loss of generality, assume n is even), and minimize δ with respect to s,

$$\frac{d\delta}{ds} = G_n 2^{-2R} + \frac{1}{n} \sum_{j=1}^{\infty} ((2j-1)\gamma_n(t)) + \frac{1}{n} \sum_{j=1}^{\infty} \left((2j-1)\gamma_n t \frac{du(t)}{dt} \right) \tag{41}$$

where

$$\frac{du(t)}{dt} = -e^{-t} \left(1 + \frac{t}{1!} + \ldots + \frac{t^{\frac{n}{2}-1}}{\left(\frac{n}{2} - 1 \right)!} \right) + \tag{42}$$

$$e^{-t} \left(1 + \frac{t}{1!} + \ldots + \frac{t^{\frac{n}{2}-2}}{\left(\frac{n}{2} - 2 \right)!} \right) = -e^{-t} \frac{t^{\frac{n}{2}-1}}{\left(\frac{n}{2} - 1 \right)!}$$

then

$$\frac{d\delta}{ds} = \tag{43}$$

$$G_n 2^{-2R} + \frac{1}{n} \sum_{j=1}^{\infty} ((2j-1)\gamma_n u(t)) - \frac{1}{n} \sum_{j=1}^{\infty} \left((2j-1)\gamma_n t e^{-t} \frac{t^{(n/2)-1}}{\left(\frac{n}{2} - 1 \right)!} \right).$$

Set

$$\frac{d\delta}{ds} = 0,$$

and denote the optimal s as s_0, and denote the corresponding t as

$$t_0 = \frac{j^2 A}{2 \pi \sigma_Z^2} s_0,$$

we get

$$G_n 2^{-2R} + \frac{1}{n} \sum_{j=1}^{\infty} ((2j-1)\gamma_n u(t_0)) = \tag{44}$$

$$\frac{1}{n} \sum_{j=1}^{\infty} \left((2j-1)\gamma_n t_0 e^{-t_0} \frac{t^{(n/2)-1}}{\left(\frac{n}{2} - 1 \right)!} \right),$$

hence

$$\delta^* = G_n s_0 2^{-2R} + \frac{1}{n} \sum_{j=1}^{\infty} ((2j-1)\gamma_n s_0 u(t_0)) \tag{45}$$

$$= \frac{1}{n} \sum_{j=1}^{\infty} \left((2j-1)\gamma_n s_0 t_0 e^{-t_0} \frac{t_0^{(n/2)-1}}{\left(\frac{n}{2} - 1 \right)!} \right)$$

$$= \frac{1}{n} \sum_{j=1}^{\infty} \left((2j-1)\gamma_n \frac{2 \pi \sigma_Z^2}{j^2 A} t_0^2 e^{-t_0} \frac{t_0^{(n/2)-1}}{\left(\frac{n}{2} - 1 \right)!} \right).$$

From (45) one can see that the optimal t_0 only depends on the rate R and the dimensionality n. The optimal t_0 stays unchanged with different σ_Z^2, thus the optimized distortion δ^* is a linear function of σ_Z^2, denoted as $D=\delta^*=B(R,n)\sigma_Z^2$. Since the Wyner-Ziv bound is $D_{WZ}=\sigma_Z^2 2^{-2R}$, the gap (in terms of dB) from the practical optimized distortion D to Wyner-Ziv bound D_{WZ} with fixed R and n is

$$\Delta D = 10 \log_{10} \frac{D}{D_{WZ}} = 10 \log_{10} \frac{B(R, n)}{2^{-2R}} \tag{46}$$

which stays constant for all $\sigma_Z^2 < 1$.

This result verifies our simulation results which show that the distortion of the nested lattice quantizer does NOT approach the Wyner-Ziv bound as the correlation between the source and the side information goes to 1 asymptotically.

Slepian-Wolf Coded Nested Lattice Quantization (SWC-NQ)

In this section, we evaluate the boundary gain of the source coding component of nested lattice quantization. Motivated by this evaluation, we introduce SWC-NQ and analyze its performance.

A. Motivation of SWC-NQ

From Theorem 4.1, the distortion per dimension of the nested lattice quantizer is $D_n=MSE_g+MSE_{ol}$, where $MSE_g=G(\Lambda_1)V_1^{2/n}$ is the granular component of the distortion, characterized by the granular gain

$$\gamma_g(\Lambda_1) = \frac{1/12}{G(\Lambda_1)},$$

while

$$MSE_{ol} = \frac{1}{n} E_z[\|Q_{\Lambda_2}(z)\|^2]$$

is the overload component of the distortion, characterized by the boundary gain $\gamma_b(\Lambda_2)$. The boundary gain $\gamma_b(\Lambda_2)$ is defined in [19] as follows. Suppose that Λ_2 is the boundary (coarse) lattice with its Voronoi region as the n-dimensional support region, and it has the same overload probability as a cubic support region of size αM centered at the origin. The boundary gain is then defined as the ratio of the normalized volume $(\alpha M)^2$ of the cubic support region to the normalized volume $V_2^{2/n}$, as

$$\gamma_b(\Lambda_2) = \frac{(\alpha M)^2}{V_2^{2/n}}. \tag{47}$$

Since

$$\begin{aligned} MSE_g &= G(\Lambda_1) V_1^{2/n} \\ &= \frac{1}{12\gamma_g(\Lambda_1)} V_2^{2/n} N^{-2/n} \\ &= \frac{1}{12\gamma_g(\Lambda_1)} \frac{\alpha M}{\gamma_b(\Lambda_2)} N^{-2/n} \end{aligned} \tag{48}$$

If the nesting ratio N stays constant (i.e., the codebook size N stays constant), then MSE_g will be reduced by a factor of $\gamma_b(\Lambda_2)$, without affecting MSE_{ol} because the overload probability stays unchanged.

To increase the boundary gain $\gamma_b(\Lambda_2)$, a second-stage of binning can be applied to the quantization indices. The essence of binning is a channel code which partitions the support region into several cosets. Assuming the channel code is strong enough so that there is no extra overload probability introduced (i.e., it is lossless coding for the indices without decoding error), and the channel code partitions the support region K_2 into m cosets, with the set composed of the coset leaders denoted as S, then #(S)=m and S is the support region for the quantization indices and hence the support region for the nested quantization, with Vol(S)=(m/N)V_2<V_2. Then the effective volume of the support region decreases by a factor of the coset size after the second stage of binning, and therefore, the boundary gain $\gamma_b(\Lambda_2)$ increases. The notation "#(A)" denotes the cardinality of (i.e., the number of elements in) the set A.

We thus propose a framework for Wyner-Ziv coding of i.i.d. sources based on SWC-NQ, which involves nested quantization (NQ) and Slepian-Wolf coding (SWC). The SWC operates as the second binning scheme. Despite the fact that there is almost no correlation among the nested quantization indices that identify the coset leaders $v \in \Lambda_1/\Lambda_2$ of the pair of nested lattices (Λ_1, Λ_2), there still remains correlation between v and the side information Y. Ideal SWC can be used to compress v to the rate of R=H(v|Y). State-of-the-art channel codes, such as LDPC codes, can be used

to approach the Slepian-Wolf limit H(v|Y) [29]. The role of SWC in SWC-NQ is to exploit the correlation between v and Y for further compression.

B. Uniform High Rate Performance

Let's evaluate the high rate performance for the quadratic Gaussian case first. For this case, a lower bound for the high-rate performance of SWC-NQ with a pair of arbitrary nested lattices (Λ_1, Λ_2) is given as

$$D_n(R) \geq G(\Lambda_1) 2^{(2/n)h'(X, \Lambda_2)} \sigma_{X|Y}^2 2^{-2R} + \tag{49}$$

$$\frac{1}{n} \sum_{j=1}^{\infty} ((2j-1)\gamma_n V_2^{2/n}) u\left(\frac{j^2 V_2^{2/n} \Gamma\left(\frac{n}{2}+1\right)^{2/n}}{2\pi\sigma_Z^2} \right)$$

where

$$h'(X, \Lambda_2) \equiv -\int_{x \in R^n} \bar{f}(x) \log_2 \left[\sum_{i=-\infty}^{\infty} \bar{f}\left(x + \frac{c_i(0)}{\sigma_{X|Y}}\right) \right] dx, \tag{50}$$

$\bar{f}(\bullet)$ is the PDF of an n-D i.i.d. Gaussian source with 0 mean and unit variance, u(t) is defined in (28), and $c_i(0)$ is defined above (in the section entitled "Lattices and Nested Lattices"), as the lattice points of Λ_2.

Proof The proof to this lower bound is provided later.

For example, the lower bounds of D(R) for the 1-D case with different V_2 are plotted in FIG. **13**.

FIG. **13** gives us a hint that, intuitively, the best R-D function of SWC-NQ is the R-D function as if the side information were also available at the encoder, and maintains a constant gap of $2\pi e G_n$ from the Wyner-Ziv limit in dB. Here the best means that, for a given distortion D, the minimal achievable rate R over all possible V_2, or equivalently, the minimal achievable distortion D over all possible V_2 for a given rate R. This claim is stated and proved as follows. Let's start with the following lemma and then prove the main theorem.

Lemma 5.1: For nested lattice quantization, denote $W \equiv Q_{\Lambda_1}$ (X), and $V \equiv W - Q_{\Lambda_2}(W)$. At high rate, H(V/Y)$\approx$H(W|Y).

Proof: The proof is provided later.

Theorem 5.2: The optimal R-D performance of SWC-NQ for general sources using low-dimensional nested lattices for Wyner-Ziv coding at high rate is

$$D_n^*(R) \equiv \min_{V_2} D(R) = G_n 2^{(2/n)h(X|Y)} 2^{-2R}. \tag{51}$$

Proof 1) When $V_2 \to \sqrt{}$, $Q_{\Lambda_2}(Q_{\Lambda_1}(X))=0$ and $Q_{\Lambda_2}(Z)=0$, then

$$\begin{aligned} nR &= H(V|Y) = H(Q_{\Lambda_1}(X) - Q_{\Lambda_2}(Q_{\Lambda_1}(X))|Y) \tag{52} \\ &= H(Q_{\Lambda_1}(X)|Y) \\ &= h(X|Y) - \log(V_1) \end{aligned}$$

and $D_n(R)=G_nV_1^{2/n}$. Combine R and D_n through V_1 and we get the R-D function as

$$D_n(R)_{V_2 \to \infty} = G_n 2^{(2/n)h(X/Y)} 2^{-2R}. \tag{53}$$

Since

$$D_n^*(R) \equiv \min_{V_2} D(R) \leq D_n(R)_{V_2 \to \infty}, \text{ then} \tag{54}$$

$$D_n^*(R) \leq G_n 2^{(2/n)h(X|Y)} 2^{-2R}.$$

2) Denote $w \equiv Q_{\Lambda_1}(x)$, and $S_1 \equiv \{(x,\hat{x}):E[d(x,\hat{x})] \leq D\}$. The rate of Wyner-Ziv coding with respect to a given distortion D is [1]

$$nR^*(D) = \min_{p(v),p(\hat{x}|v,y),(x,\hat{x}) \in S_1} I(X;V|Y), \tag{55}$$

$$\overset{(a)}{=} \min_{p(v),p(\hat{x}|v,y),(x,\hat{x}) \in S_1} H(V|Y)$$

$$\overset{(b)}{\approx} \min_{p(v),p(\hat{x}|v,y),(x,\hat{x}) \in S_1} H(W|Y)$$

where (a) comes from H(V|X,Y)=0 and (b) comes from Lemma 5.1.

Define $S_2 \equiv \{(x,\hat{x}):E[d(x,w)] \leq D\}$. From Theorem 4.1,

$$E[d(x,\hat{x})] = G_n V_1^{2/n} + \frac{1}{n} E_Z[\|Q_{\Lambda_2}(z)\|^2] \tag{56}$$

$$= E[d(x,w)] + \frac{1}{n} E_Z[\|Q_{\Lambda_2}(z)\|^2] \geq$$

$$E[d(x,w)]$$

Then $\forall (x,\hat{x}) \in S_1$,

$$D \geq E[d(x,\hat{x})] \geq E[d(x,w)], \tag{57}$$

it means that $(x,\hat{x}) \in S_2$.

Then $S_1 \subseteq S_2$, and

$$nR^*(D) \approx \min_{p(v),p(\hat{x}|v,y),(x,\hat{x}) \in S_1} H(W|Y) \geq \min_{p(v),p(\hat{x}|v,y),(x,\hat{x}) \in S_2} H(W|Y). \tag{58}$$

Since $H(W|Y)=h(X|Y)-\log(V_1)$ and $E[d(x,\hat{x})|Y]=G_nV_1^{2/n}$, $R^*(D)$ can be calculated using Lagrangian method, as

$$nR^*(D) \geq \min_{p(v),p(\hat{x}|v,y),(x,\hat{x}) \in S_2} H(W|Y) = h(X|Y) - \frac{n}{2}\log\left(\frac{D}{G_n}\right). \tag{59}$$

Then

$$D^*(R) \geq G_n 2^{(2/n)h(X/Y)} 2^{-2R}. \tag{60}$$

From (54) and (60), it is proved that, at high rate, the best R-D function of SWC-NQ using low-dimensional lattices is

$$D^*(R)=G_n 2^{(2/n)h(X/Y)} 2^{-2R}. \tag{61}$$

Corollary 5.4: The optimal R-D performance of quadratic Gaussian SWC-NQ using low-dimensional nested lattices at high rate is

$$D^*_n(R)=2\pi e G_n \sigma_{X/Y} 2^{-2R}. \tag{62}$$

We thus conclude that at high rates, SWC-NQ performs the same as the traditional entropy-constrained lattice quantization with the side information available at both the encoder and decoder. Specifically, the R-D functions with 1-D (scalar) lattice and 2-D (hexagonal) lattice are 1.53 dB and 1.36 dB away from the Wyner-Ziv bound, respectively.

Remarks: We found that for finite rate R and small n (e.g., n=1 and 2), the optimal V_2, denoted as V^*_2, that minimizes the distortion $D_n(R)$ is also finite. FIGS. **14**(a) and (b) plot the optimal V^*_2 (scaled by σ_Z) as a function of R for the 1-D (n=1) and 2-D (n=2) case. We see that as R goes to infinity, V^*_2 also goes to infinity. We also observe that for fixed R and n, $D_n(R)$ stays roughly constant for $V_2 > V^*_2$.

Code Design and Simulation Results

In this section, the optimal decoder for nested quantizer at low rate is introduced, and the issue of code design is also discussed, along with simulation results.

A. The Optimal Decoder for Nested Quantizer at Low Rate

The optimal estimator for the decoder corresponding to the nested quantizer should minimize the distortion between X and the reconstructed $\hat{X}$. If mean squared error (MSE) is used as the distortion measure, $\hat{x}$ will be E[X|X, j,y], where j is the received bin index corresponding to the coset leader $v=x_{Q_{\Lambda_1}}-Q_{\Lambda_2}(x_{Q_{\Lambda_1}})$ Let Y and Z be independent zero mean Gaussian random variables with variances σ_Y^2 and σ_Z^2, then we have X|y~N(y,σ_Z^2).

When n=1, the optimal decoder for nested quantizer can be derived directly from E[X|j,y] as

$$\hat{x}(j,y) = \frac{1}{\sqrt{2\pi\sigma_Z^2}} \sum_{n=-\infty}^{\infty} \int_{jq+nQ}^{(j+1)q+nQ} x\exp\left(-\frac{\|x-y\|^2}{2\sigma_Z^2}\right) dx, \tag{63}$$

where q and Q are the uniform intervals of the two nested lattices used by the scalar quantizer, with Q/q=N, where N is the nesting ratio. At high rates, the rate distortion performance using this non-linear estimation matches our analysis in (29); at low rate, such estimation method helps to boost the performance.

When n>1, the optimal decoder for the nested quantizer is stated as follows.

Theorem 6.3: The optimal decoder for the nested quantizer in the sense of MSE is

$$\hat{x} = E[x|y,j] = \frac{\int_{R(v)} xf(x|y)dx}{\int_{R(v)} f(x|y)dx}. \tag{64}$$

Proof:

-continued

$$\hat{x} = E[x|y, j] \tag{65}$$

$$= \int_{R^n} x f(x|y, j) dx$$

$$= \int_{R^n} x \frac{f(x, j|y)}{P(j|y)} dx$$

$$= \frac{\int_{R^n} x f(x, j|y) dx}{P(j|y)}$$

$$= \frac{\int_{R^n} x f(x|y) P(j|x, y) dx}{\int_{R(v)} f(x|y) dx}$$

Note that j, x, y form a Markov chain as $y \leftrightarrow x \leftrightarrow j$, then and we get

$$P(j|x, y) = P(j|x) = \begin{cases} 0 & \text{if } x \notin R(v) \\ 1 & \text{if } x \in R(v) \end{cases}, \tag{66}$$

$$\hat{x} = \frac{\int_{R^n} x f(x|y) P(j|x) dx}{\int_{R(v)} f(x|y) dx} \tag{67}$$

$$= \frac{\int_{R(v)} x f(x|y) dx}{\int_{R(v)} f(x|y) dx}.$$

Estimation at the decoder plays an important role for low-rate implementation. We thus apply an optimal non-linear estimator at the decoder at low rates in our simulations.

Corollary 6.5: The optimal estimator stated in Theorem 6.3 degenerates to the linear one $\hat{x}=v+Q_{\Lambda_2}(y-v)$ at high rates as we discussed above in the section entitled "Nested Lattice Quantization" and in the section entitled "Slepian-Wolf Coded Nested Lattice Quantization".

Proof: At high rate,

$$\hat{x} = \frac{\int_{R(v)} x f(x|y) dx}{\int_{R(v)} f(x|y) dx} \tag{68}$$

$$= \frac{\sum_{i=-\infty}^{\infty} \int_{R_i(v)} x f(x|y) dx}{\sum_{i=-\infty}^{\infty} \int_{R_i(v)} f(x|y) dx}$$

$$\overset{(a)}{\approx} \frac{\sum_{i=-\infty}^{\infty} c_i(v) \int_{R_i(v)} f(x|y) dx}{\sum_{i=-\infty}^{\infty} \int_{R_i(v)} f(x|y) dx}$$

$$\overset{(b)}{\approx} \frac{v + Q_{\Lambda_2}(y-v) \int_{K(v+Q_{\Lambda_2}(y-v))} f(x|y) dx}{\int_{K(v+Q_{\Lambda_2}(y-v))} f(x|y) dx}$$

$$= v + Q_{\Lambda_2}(y-v),$$

which is the linear estimator, discussed above in the section entitled "Nested Lattice Quantization" and in the section entitled "Slepian-Wolf Coded Nested Lattice Quantization", for high rate performance. Steps (a) and (b) of (68) come from the high rate assumption.

Since the non-linear estimation is a definite integral of a simple function over a disconnected region which includes many isolated Voronoi cells, we choose the Monte Carlo method to do this integral. In one simulation, for each scaling factor, there are totally $10^4 \times 10^4 = 108$ pairs of $\{x,y\}$ to be simulated, and for each pair of $\{x,y\}$, there are 10^4 samples to calculate this definite integral.

FIG. **15** shows the improvement gained by using the optimal (non-linear) estimator at low rates, for n=2 and $\sigma_Z^2=0.01$.

B. Code Design of LDPC Codes

Let J ($0 \le J \le N-1$) denote the index of the coset leader v. The index J is coded using Slepian-Wolf codes with Y as the side information. Instead of coding J as a whole, we code J bit-by-bit using multi-layer Slepian-Wolf coding as follows.

Assume $J=(B_m B_{m-1} \dots B_2 B_1)_2$, where B_m is the most significant bit (MSB) of J, and B_1 is the least significant bit (LSB). A block of the indices may be collected. The first B_1 (i.e., a block of the first bits from the block of indices) is encoded at rate $R_1=H(B_1|Y)$ using a Slepian-Wolf code designed under the assumption that the corresponding decoder has only Y as side information; then the second bit B_2 (i.e., a block of the second bits from the block of indices) is encoded at rate $R_2=H(B_2|Y,B_1)$ using a Slepian-Wolf code designed under the assumption that the corresponding decoder has only Y and B_1 as side information; . . . ; finally, the last bit B_m (i.e., a block of the last bits from the block of indices) is encoded at rate $R_m=H(B_m|Y, B_1, B_2, \dots, B_{m-1})$ with a Slepian-Wolf code designed under the assumption that the corresponding decoder has side information $\{Y, B_1, B_2', \dots, B_{m-1}\}$. Hence the total rate of the Slepian-Wolf code is $H(J|Y)=H(v|Y)$.

Practically, strong channel codes such as LDPC or Turbo codes are applied as Slepian-Wolf codes. The first step in designing is to determine the rate of the channel code to be used. Since R_n is equivalent to the amount of syndromes to be sent per bit, the channel code rate is $1-R_n$. Thus the optimum rate at the nth layer that achieves Slepian-Wolf bound is $1-H(B_n|Y, B_1, \dots, B_{n-1})$. This multi-layer Slepian-Wolf coding scheme is shown in FIG. **16**.

As shown in FIG. **16**, one embodiment of an SWC-NQ encoder includes a nested lattice quantization unit **1610** and a set of Slepian-Wolf encoder SWE_1, SWE_2, . . . , SWE_m. The nested quantization unit **1610** operates on a value of the input source X and generates the bits B_1, B_2, . . . , B_{m-1}, B_m of the index J as described above. The nested quantization unit does this operation repeatedly on successive values of the input source, and thus, generates a stream of indices. Each of the Slepian-Wolf encoders SWE_n, n=1, 2, . . . , m, collects a block of the B, bits from the stream of indices and encodes this block, thereby generating an encoded block T_n. The encoded blocks T_1, T_2, . . . , T_m are sent to an SWC-NQ decoder.

As shown, one embodiment of the SWC-NQ decoder includes a set of Slepian-Wolf decoders SWD_1, SWD_2, . . . , SWD_m and a nested quantization decoder **1620**. Each Slepian-Wolf decoder SWD_n, n=1, 2, . . . , m, decodes the compressed block T, to recover the corresponding block of B_n bits. As noted above, decoder SWD, uses side information $\{Y, B_1, B_2, \dots, B_{n-1}\}$. The nested quantization decoder **1620** operates on the blocks generated by the

decoders using a block of the Y values, as described above, to compute a block of estimated values of the source.

C. Simulation Results

We carry out 1-D nested lattice quantizer design for different sources with 10^6 samples of X in each case. For $\sigma_Y^2=1$ and $\sigma_Z^2=0.01$, FIG. **17** shows results with nested lattice quantization alone and SWC-NQ. The former exhibits a 3.95-9.60 dB gap from $D_{WZ}(R)$ for R in the range from 1.0 to 7.0 bits/sample (b/s), which agree with the high rate lower bound of Theorem 1. At high rate, we observe that the gap between our results with ideal SWC (i.e., rate computed as H(J|Y) in the simulation) and $D_{WZ}(R)$ is indeed 1.53 dB. With practical SWC based on irregular LDPC codes of length 10^6 bits, this gap is 1.66-1.80 dB for R in the range from 0.93 to 5.00 b/s.

For 2-D nested lattice quantization, we use the A_2 hexagonal lattices again with $\sigma_Y^2=1$ and $\sigma_Z^2=0.01$. FIG. **18** shows results with nested lattice quantization alone and SWC-NQ. At high rate, the former case exhibits a 4.06-8.48 dB gap from $D_{WZ}(R)$ for R=1.40-5.00 b/s, again in agreement with the high rate lower bound of Theorem 1. We observe that the gap between our results with ideal SWC (measured in the simulation) and $D_{WZ}(R)$ is 1.36 dB. With practical SWC based on irregular LDPC codes (of length 10^6 bits), this gap is 1.67-1.72 dB for R=0.95-2.45 b/s.

We thus see that using optimal estimation as described herein, our simulation results with either 1-D or 2-D nested quantization (and practical Slepian-Wolf coding) are almost a constant gap away from the Wyner-Ziv limit for a wide range of rates.

In this paper, the high-rate R-D performance of the nested lattice quantization for the Wyner-Ziv coding is analyzed, with low dimensional lattice codes. The performance is away from the Wyner-Ziv bound with each specific lattice code, and exhibits an increasing gap from the Wyner-Ziv bound as the rate increases. The reason for the increase of the gap mainly comes from the fact that the granular component of the distortion is an increasing function of the rate. Therefore the Slepian-Wolf coding, as a second-layer binning scheme, is applied to the quantization indices for further compression. This Slepian-Wolf coded nested lattice quantization (SWC-NQ) performs at a constant gap from the Wyner-Ziv bound at high rates, and the constant gap is the same as the one from ECVQ (entropy coded vector quantization) to the ideal R-D function of source coding without the side information. Moreover, a non-linear estimator for the decoder is introduced, and proved to be optimal in the sense of the MSE measurement. This non-linear estimator helps at low-rates, and degrades to the linear one which is assumed in the theoretical analyses in this paper. Simulation results for 1-D and 2-D cases are in agreement with the theoretical analysis.

Proof of Lower Bound (49)

Proof to establish the lower bound for the performance of quadratic Gaussian SWC-NQ.

1) Rate Computation:

The rate for SWC-NQ is:

$$R = \frac{1}{n} H(v \mid Y). \tag{69}$$

Since at high rate,

$$
\begin{aligned}
P(v \mid Y) &= \sum_{j=-\infty}^{\infty} \int_{x \in R_j(v)} f_{X\mid Y}(x)dx \tag{70}\\
&= \sum_{j=-\infty}^{\infty} \int_{x \in R_0(v)} f_{X\mid Y}(x + c_j(0))dx\\
&\approx \sum_{j=-\infty}^{\infty} f_{X\mid Y}(v + c_j(0))\, V_1\\
&\equiv g(v)\, V_1,
\end{aligned}
$$

where

$$g(x) \equiv \sum_{j=-\infty}^{\infty} f_{X\mid Y}(x + c_j(0)), \text{ and } X \mid Y \sim N(0, \sigma_{X\mid Y}^2).$$

Then the achievable rate of SWC-NQ is

$$
\begin{aligned}
nR &= H(v \mid Y)\\
&= -\sum_{v \in \Lambda_1/\Lambda_2} P(v \mid Y) \log_2[P(v \mid Y)]\\
&\approx -\sum_{v \in \Lambda_1/\Lambda_2} P(v \mid Y) \log_2[g(v) V_1]\\
&= -\sum_{v \in \Lambda_1/\Lambda_2} \sum_{j=-\infty}^{\infty} \int_{x \in R_0(v)} f_{X\mid Y}(x + c_j(0))dx \log_2 g(v) - \log_2 V_1\\
&\approx -\sum_{j=-\infty}^{\infty} \sum_{v \in \Lambda_1/\Lambda_2} \int_{x \in R_0(v)} f_{X\mid Y}(x + c_j(0)) \log_2 g(x)\, dx - \log_2 V_1\\
&\overset{(a)}{=} -\int_{x \in R^n} f_{X\mid Y}(x) \log_2 g(x)\, dx - \log_2 V_1
\end{aligned}
$$

where (a) comes from the periodic property of $g(\cdot)$, i.e., $g(x-l)=g(x), \forall l \in \Lambda_2$. Thus the achievable rate of SWC-NQ is

$$nR = H(v\mid Y) = h'(X, \Lambda_2) + \log_2 \sigma_{X\mid Y}^n - \log_2 V_1. \tag{71}$$

2) Distortion Computation: From Theorem 4.1, the average distortion of nested lattice quantization over all realizations of (X,Y) is

$$
\begin{aligned}
D_n &= G(\Lambda_1) V_1^{2/n} + \frac{1}{n} E_Z[\|Q_{\Lambda_2}(z)\|^2] \geq G(\Lambda_1) V_1^{2/n} + \tag{72}\\
&\quad \frac{1}{n} \sum_{j=1}^{\infty} ((2j-1)\gamma_n V_2^{2/n}) u\left(\frac{j^2 V_2^{2/n} \Gamma\left(\frac{n}{2}+1\right)^{2/n}}{2\pi\sigma_Z^2}\right).
\end{aligned}
$$

Because SWC is lossless, the distortion of SWC-NQ is also D_n. Combining D_n and R through V_1, we obtain the R-D performance of SWC-NQ with a pair of n-D nested lattices (Λ_1, Λ_2) as

$$
\begin{aligned}
D_n(R) &\geq G(\Lambda_1) 2^{(2/n)h'(X,\Lambda_2)} \sigma_{X\mid Y}^2 2^{-2R} + \tag{73}\\
&\quad \frac{1}{n} \sum_{j=1}^{\infty} ((2j-1)\gamma_n V_2^{2/n}) u\left(\frac{j^2 V_2^{2/n} \Gamma\left(\frac{n}{2}+1\right)^{2/n}}{2\pi\sigma_Z^2}\right).
\end{aligned}
$$

Proof of Lemma 5.1

This proof closely follows the remark 3) of [1] page 3, with some slight modifications. Let

$$\delta \equiv \min_{w \neq \hat{x}} d(w, \hat{x})_{|Y} > 0.$$

Here δ is actually the minimum of the distance between two lattice points of Λ_2. Thus if $(x,\hat{x}) \in S_1$,

$$\lambda = Pr\{W \neq \hat{X}\} < E[d(W,\hat{X})]/\delta^{(a)} < (E[d(W,\hat{X})]+E[d(X,\hat{X})])/\delta \tag{74}$$

where (a) comes from the triangle inequality. From Theorem 1,

$$D = E[d(X,\hat{X})] = MSE_g + MSE_{ol},$$

where $MSE_g = E[d(W,X)]$ is the granular component and MSE_{ol} is the overload component, then

$$\lambda \leq 2D/\delta. \tag{75}$$

Now since $\hat{X}$ is a function of V, Y, Fano's inequality [30], [31] implies that

$$H(W \mid V, Y) \leq -\lambda \log \lambda - (1-\lambda)\log(1-\lambda) + \lambda \log(|W|) \equiv \varepsilon(\lambda), \tag{76}$$

so that

$$H(V \mid Y) \geq I(W; V \mid Y) = \tag{77}$$
$$H(W \mid Y) - H(W \mid V, Y) \geq H(W \mid Y) - \varepsilon\left(\frac{2D}{\delta}\right).$$

Meanwhile, from data processing rule, we have $H(V|Y) \leq H(W|Y)$. At high rate, $D \to 0$, and

$$\varepsilon\left(\frac{2D}{\delta}\right) \to 0.$$

Thus at high rate, $H(V|Y)$ $H(W|Y)$.

This claim is also verified intuitively by FIG. **13**, where the slant part of each curve which corresponds to the R-D performance with a fixed V_2, or δ, approximately maintains a constant slope.

It is noted that any or all of the method embodiments described herein may be implemented in terms of program instructions executable by one or more processors. The program instructions (or subsets thereof) may be stored and/or transmitted on any of various carrier media. Furthermore, the data generated by any or all of the method embodiments described herein may be stored and/or transmitted on any of various carrier media.

Although the embodiments above have been described in considerable detail, numerous variations and modifications will become apparent to those skilled in the art once the above disclosure is fully appreciated. It is intended that the following claims be interpreted to embrace all such variations and modifications.

What is claimed is:

1. A method comprising:
   (a) receiving compressed input data, wherein the compressed input data is a compressed representation of a block of samples of a first source X;

   (b) receiving a block of samples of a second source Y;
   (c) applying an asymmetric Slepian-Wolf decoder to the compressed input data using the block of samples of the second source Y, wherein said applying generates a block of intermediate values; and
   (d) performing joint decoding on each intermediate value and a corresponding sample of the block of second source samples to obtain a corresponding decompressed output value, wherein said performing joint decoding includes determining an estimate of a centroid of a function restricted to a region of space corresponding to the intermediate value, wherein said estimate determines the decompressed output value.

2. The method of claim **1**, wherein the function is the conditional probability density function of the first source X given said corresponding sample of the second source block.

3. The method of claim **1**, wherein the region of space is a union of cells corresponding to a coset of a coarse lattice in a fine lattice, wherein the coset is identified by the intermediate value.

4. The method of claim **1**, wherein said determining the centroid estimate is performed by reading the centroid estimate from a table stored in memory using said corresponding sample of the second source block and the intermediate value as addresses.

5. The method of claim **1**, wherein said determining the centroid estimate comprises performing a Monte Carlo iterative simulation.

6. The method of claim **1**, wherein the intermediate values specify cosets of a coarse lattice in a fine lattice, wherein the coarse lattice is a sublattice of the fine lattice.

7. The method of claim **1**, wherein (a) and (b) are performed in parallel.

8. The method of claim **1**, wherein said asymmetric Slepian-Wolf decoder is a multi-layered decoder.

9. The method of claim **1**, wherein the asymmetric Slepian-Wolf decoder achieves a decoding rate that approaches close to the limit for Slepian-Wolf coding, wherein the decompressed output value is digital data.

10. The method of claim **1**, wherein said applying the asymmetric Slepian-Wolf decoder and said performing joint decoding are realized by any subset of:
    (a) one or more processors executing program instructions;
    (b) one or more programmable hardware elements; and
    (c) dedicated circuitry.

11. The method of claim **1**, wherein the asymmetric Slepian-Wolf decoder is an iterative decoder based on one or more LDPC codes.

12. The method of claim **1**, wherein the asymmetric Slepian-Wolf decoder is an iterative decoder based on one or more turbo codes.

13. A method for computing a table representing a nested quantization decoder, the method comprising:
    (a) computing a realization z of a first random vector;
    (b) computing a realization y of a second random vector;
    (c) adding z and y to determine a realization x of a source vector;
    (d) quantizing the realization x to a point in a fine lattice;
    (e) computing an index J identifying a coset of a coarse lattice in the fine lattice based on the fine lattice point;
    (f) adding the realization x to a cumulative sum corresponding to the index J and the realization y;
    (g) incrementing a count value corresponding to the index J and the realization y;
    (h) repeating operations (a) through (g) a number of times;

(i) dividing the cumulative sums by their corresponding count values to obtain resultant values; and

(j) storing the resultant values in a memory medium.

14. A system for decoding compressed data, the system comprising:

a memory configured to store data and program instructions; and

a processor configured to read and execute the program instructions from the memory, wherein in response to execution of the program instructions, the processor is operable to:

(a) receive compressed input data, wherein the compressed input data is a compressed representation of a block of samples of a first source X;

(b) receive a block of samples of a second source Y;

(c) apply an asymmetric Slepian-Wolf decoder to the compressed input data using the block of samples of the second source Y, wherein said applying generates a block of intermediate values; and

(d) perform joint decoding on each intermediate value and a corresponding sample of the block of second source samples to obtain a corresponding decompressed output value, wherein said performing joint decoding includes determining an estimate of a centroid of a function restricted to a region of space corresponding to the intermediate value, wherein said estimate determines the decompressed output value.

15. The system of claim **14**, wherein the function is the conditional probability density function of the first source X given said corresponding sample of the second source block.

16. The system of claim **14**, wherein the asymmetric Slepian-Wolf decoder achieves a decoding rate that approaches close to the limit for Slepian-Wolf coding, wherein the decompressed output value is digital data.

17. The system of claim **14**, wherein the asymmetric Slepian-Wolf decoder is an iterative decoder based on one or more LDPC codes.

18. The system of claim **14**, wherein the asymmetric Slepian-Wolf decoder is an iterative decoder based on one or more turbo codes.

19. A system for computing a table representing a nested quantization decoder, the system comprising:

a memory configured to store data and program instructions; and

a processor configured to read and execute the program instructions from the memory, wherein in response to execution of the program instructions, the processor is operable to:

(a) computing a realization z of a first random vector;

(b) computing a realization y of a second random vector;

(c) adding z and y to determine a realization x of a source vector;

(d) quantizing the realization x to a point in a fine lattice;

(e) computing an index J identifying a coset of a coarse lattice in the fine lattice based on the fine lattice point;

(f) adding the realization x to a cumulative sum conesponding to the index J and the realization y;

(g) incrementing a count value corresponding to the index J and the realization y;

(h) repeating operations (a) through (g) a number of times;

(i) dividing the cumulative sums by their conesponding count values to obtain resultant values; and

(j) storing the resultant values in a memory medium.

20. A computer-readable memory medium configured to store program instructions, wherein the program instructions are executable to implement:

(a) receiving compressed input data, wherein the compressed input data is a compressed representation of a block of samples of a first source X;

(b) receiving a block of samples of a second source Y;

(c) applying an asymmetric Slepian-Woif decoder to the compressed input data using the block of samples of the second source Y, wherein said applying generates a block of intermediate values; and

(d) performing joint decoding on each intermediate value and a corresponding sample of the block of second source samples to obtain a corresponding decompressed output value, wherein said performing joint decoding includes determining an estimate of a centroid of a function restricted to a region of space corresponding to the intermediate value, wherein said estimate determines the decompressed output value.

21. The computer-readable memory medium of claim **20**, wherein the function is the conditional probability density function of the first source X given said corresponding sample of the second source block.

22. The memory medium of claim **20**, wherein the asymmetric Slepian-Wolf decoder achieves a decoding rate that approaches close to the limit for Slepian-Wolf coding, wherein the decompressed output value is digital data.

23. The memory medium of claim **20**, wherein the asymmetric Slepian-Wolf decoder is an iterative decoder based on one or more LDPC codes.

24. The memory medium of claim **20**, wherein the asymmetric Slepian-Wolf decoder is an iterative decoder based on one or more turbo codes.

25. A computer-readable memory medium configured to store program instructions, wherein the program instructions are executable to implement:

(a) computing a realization z of a first random vector;

(b) computing a realization y of a second random vector;

(c) adding z and y to determine a realization x of a source vector;

(d) quantizing the realization x to a point in a fine lattice;

(e) computing an index J identifying a coset of a coarse lattice in the fine lattice based on the fine lattice point;

(f) adding the realization x to a cumulative sum corresponding to the index J and the realization y;

(g) incrementing a count value corresponding to the index J and the realization y;

(h) repeating operations (a) through (g) a number of times;

(i) dividing the cumulative sums by their corresponding count values to obtain resultant values; and

(j) storing the resultant values in a memory medium.

26. A system comprising:

a first means for decompressing compressed input data using side information, wherein the compressed input data is a compressed representation of a block of samples of a first source X, wherein the side information is a block of samples of a second source Y, wherein said applying generates a block of intermediate values;

a second means for performing joint decoding on each intermediate value and a corresponding sample of the block of second source samples to obtain a corresponding decompressed output value, wherein said performing joint decoding includes determining an estimate of

a centroid of a function restricted to a region of space corresponding to the intermediate value, wherein said estimate determines the decompressed output value.

27. A method comprising:

(a) receiving compressed input data, wherein the compressed input data is a compressed representation of a block of samples of a first source X;

(b) receiving a block of samples of a second source Y;

(c) applying an iterative decoder to the compressed input data using the block of samples of the second source Y, wherein said applying generates a block of intermediate values, wherein the iterative decoder is based on one or more low density parity check codes;

(d) performing joint decoding on each intermediate value and a corresponding sample of the block of second source samples to obtain a corresponding decompressed output value, wherein said performing joint decoding includes determining an estimate of a centroid of a function restricted to a region of space corresponding to the intermediate value, wherein said estimate determines the decompressed output value.

28. A method comprising:

(a) receiving compressed input data, wherein the compressed input data is a compressed representation of a block of samples of a first source X;

(b) receiving a block of samples of a second source Y;

(c) applying an iterative decoder to the compressed input data using the block of samples of the second source Y, wherein said applying generates a block of intermediate values, wherein the iterative decoder is based on one or more turbo codes;

(d) performing joint decoding on each intermediate value and a corresponding sample of the block of second source samples to obtain a corresponding decompressed output value, wherein said performing joint decoding includes determining an estimate of a centroid of a function restricted to a region of space corresponding to the intermediate value, wherein said estimate determines the decompressed output value.

29. A system for decoding compressed data, the system comprising:

a memory configured to store data and program instructions; and

a processor configured to read and execute the program instructions from the memory, wherein in response to execution of the program instructions, the processor is operable to:

(a) receive compressed input data, wherein the compressed input data is a compressed representation of a block of samples of a first source X;

(b) receive a block of samples of a second source Y;

(c) apply an iterative decoder to the compressed input data using the block of samples of the second source Y, wherein said applying generates a block of intermediate values, wherein the iterative decoder is based on one or more low density parity check codes;

(d) perform joint decoding on each intermediate value and a corresponding sample of the block of second source samples to obtain a corresponding decompressed output value, wherein said performing joint decoding includes determining an estimate of a centroid of a function restricted to a region of space corresponding to the intermediate value, wherein said estimate determines the decompressed output value.

30. A system for decoding compressed data, the system comprising:

a memory configured to store data and program instructions; and

a processor configured to read and execute the program instructions from the memory, wherein in response to execution of the program instructions, the processor is operable to:

(a) receive compressed input data, wherein the compressed input data is a compressed representation of a block of samples of a first source X;

(b) receive a block of samples of a second source Y;

(c) apply an iterative decoder to the compressed input data using the block of samples of the second source Y, wherein said applying generates a block of intermediate values, wherein the iterative decoder is based on one or more turbo codes;

(d) perform joint decoding on each intermediate value and a corresponding sample of the block of second source samples to obtain a corresponding decompressed output value, wherein said performing joint decoding includes determining an estimate of a centroid of a function restricted to a region of space corresponding to the intermediate value, wherein said estimate determines the decompressed output value.

31. A computer-readable memory medium configured to store program instructions, wherein the program instructions are executable to implement:

(a) receiving compressed input data, wherein the compressed input data is a compressed representation of a block of samples of a first source X;

(b) receiving a block of samples of a second source Y;

(c) applying an iterative decoder to the compressed input data using the block of samples of the second source Y, wherein said applying generates a block of intermediate values, wherein the iterative decoder is based on one or more low density parity check codes;

(d) performing joint decoding on each intermediate value and a corresponding sample of the block of second source samples to obtain a corresponding decompressed output value, wherein said performing joint decoding includes determining an estimate of a centroid of a function restricted to a region of space corresponding to the intermediate value, wherein said estimate determines the decompressed output value.

32. A computer-readable memory medium configured to store program instructions, wherein the program instructions are executable to implement:

(a) receiving compressed input data, wherein the compressed input data is a compressed representation of a block of samples of a first source X;

(b) receiving a block of samples of a second source Y;

(c) applying an iterative decoder to the compressed input data using the block of samples of the second source Y, wherein said applying generates a block of intermediate values, wherein the iterative decoder is based on one or more turbo codes;

(d) performing joint decoding on each intermediate value and a corresponding sample of the block of second source samples to obtain a corresponding decompressed output value, wherein said performing joint decoding includes determining an estimate of a centroid of a function restricted to a region of space corresponding to the intermediate value, wherein said estimate determines the decompressed output value.

* * * * *

UNITED STATES PATENT AND TRADEMARK OFFICE
CERTIFICATE OF CORRECTION

PATENT NO.　　　　: 7,295,137 B2　　　　　　　　　　　　　　　Page 1 of 1
APPLICATION NO. : 11/086778
DATED　　　　　　: November 13, 2007
INVENTOR(S)　　　: Liu et al.

It is certified that error appears in the above-identified patent and that said Letters Patent is
hereby corrected as shown below:

<u>Column 37,</u>

Line 61, please delete "conesponding" and substitute --corresponding--.

<u>Column 38,</u>

Line 1, please delete "conesponding" and substitute --corresponding--.

Signed and Sealed this

First Day of April, 2008

JON W. DUDAS
Director of the United States Patent and Trademark Office