DYNAMIC PRICING PROBLEMS ARISING IN THE ADOPTION OF RENEWABLE ENERGY

A Dissertation

by

KE MA

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ABSTRACT

There are two problems at the interface of electrical power and economics that are examined in this thesis. The first problem addresses the issue of optimally operating electric vehicle (EV) charging stations, where price as well as scheduling of purchasing, storing, and charging play key roles. The second problem addresses the challenge faced by electric power system operators who have to balance power generation and demand at all times, and are faced with the task of maximizing the social welfare of all affected entities comprised of producers, consumers and prosumers (e.g., homes with solar panels who may be producers at some times and consumers at other times).

For the first problem, we have developed a layered decomposition approach that permits a holistic solution to solving the scheduling, storage and pricing problems of charging stations. The key idea is to decompose problems by time-scale.

For the second problem, we have shown that for the special case of LQG agents, by careful construction of a sequence of layered VCG payments over time, the intertemporal effect of current bids on future payoffs can be decoupled, and truth-telling of dynamic states is guaranteed if system parameters are known and agents are rational. We have also shown that a modification of the VCG payments, called scaled-VCG payments, achieves Budget Balance and Individual Rationality for a range of scaling, under a certain identified Market Power Balance condition.

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Contributors

This work was supported by a dissertation committee consisting of Professor P. R. Kumar, Professor Le Xie and Professor Srinivas Shakkottai of the Department of Electrical & Computer Engineering and Professor Steven Puller of the Department of Economics.

The algorithms in Chapter 3 were developed in collaboration with Dr. Rahul Singh.

All other work conducted for the dissertation was completed by the student independently.

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1. INTRODUCTION

There are two problems at the interface of electrical power and economics that are examined in this thesis. The first problem addresses the issue of optimally operating electric vehicle (EV) charging stations, where price as well as scheduling of purchasing, storing, and charging play key roles. The second problem addresses the challenge faced by electric power system operators who have to balance power generation and demand at all times, and are faced with the task of maximizing the social welfare of all affected entities comprised of producers, consumers and prosumers (e.g., homes with solar panels who may be producers at some times and consumers at other times).

Price is the “amount of money expected, required, or given in payment for something" [1]. In economics, price in a free market is arrived at after the interaction between supply and demand: the price is set such that quantity supplied equals quantity demanded. However, when the seller has great market power, it can generate more profit by dividing consumers into groups with separate demand curves and charging different prices to each group. The idea of *price discrimination* plays a fundamental role in determining the prices of EV charging in Chapter 2. Also, since the price of electricity is time-varying, the operator has to optimize when to purchase power, how much to store, and how to recharge customers with time deadlines. We present a layered decomposition approach and supporting theory that permit a holistic solution to the storage, scheduling and pricing problems of Electric Vehicle (EV) Charging Stations.

In Chapter 3, we are motivated by the problems faced by Independent System Operators (ISO) in allocating power generations and demands for agents in a power network such that social welfare is maximized, while balancing supply and demand, and satisfying network constraints. One centralized way to determine prices and make allocations to balance supply and demand is through calculating the values of Lagrange multipliers in an optimization problem. However, since the central agent, the ISO, does not know the details of the producers and consumers, it cannot formulate the optimization problem. Hence the solution will need to be based on an interaction between the ISO and the producers and consumers. We address a key issue of designing the market bidding structure that induces dynamic cooperation among agents in a dynamic decentralized fashion.

A key problem in connection with bidding is that when agents attempt to anticipate the effects of their actions on prices, strategic bidding arises, and the ability to converge to a social welfare optimal solution, i.e., to attain efficiency of the system, degrades. The problem of designing mechanisms so that truth telling is optimal for agents, called incentive compatibility, has been well examined for *static* agents who only have to bid once [2]. However agents in power systems are governed a dynamic system that is subject to stochastic uncertainties. In Chapter 4, for a set of linear quadratic Gaussian (LQG) [3] agents, we propose a modified layered version of the Vickrey-Clarke-Groves (VCG) [2] mechanism for payments that decouples the intertemporal effect of current bids on future payoffs, and establish that truth-telling of dynamic states forms a dominant strategy if system parameters are known and agents are rational. One would also like to ensure that the mechanism is “fair" in the sense that it charges each customer a fair “price". We further address the issue of ensuring budget balance, which ensures that the ISO does not have to subsidize the market, and individual rationality, which ensures that agents will not drop put of the market. We propose a modified Scaled VCG (SVCG) mechanism that satisfies incentive compatibility, social efficiency, budget balance and individual rationality for a power market consisting of LQG type agents.

2. A LAYERED ARCHITECTURE FOR EV CHARGING STATION BASED ON TIME-SCALE DECOMPOSITION *

2.1 Introduction

Infrastructures such as fast charging stations, shown in Fig. 2.1, will be crucial for the proliferation of Electric Vehicles (EVs). They play a psychological role, alleviating range anxiety of EV drivers, and also a functional role, providing incentives for higher utilization of infrastructures. There are two issues to consider in the design and operation of such charging stations. One concerns infrastructure, i.e., what kinds of devices are required, and what are the optimal quantities and sizes of these devices from both technological and economic perspectives. The other is operational, i.e., how to set prices, and schedule power purchase, storage and charging, such that profit is maximized while guaranteeing quality of service to arriving EVs. We focus on the latter issue in this chapter

We consider a single charging station connected to the electricity grid. While supply of power is thereby guaranteed, the price at which wholesale level grid power can be bought by a charging station is variable and greatly impacts its economic operation. One solution is to introduce on-site energy storage. Such storage also increases the number of EVs that can be charged simultaneously by increasing the peak constraint on grid power drawn, thereby enhancing economic viability of charging stations. Bae and Kwasinski [4] estimates charging demand for a station near a highway exit and conclude that adequate energy storage can reduce charging price. Bayram et al. [5, 6] demonstrates that storage capacity is key to minimizing charging cost while guaranteeing system performance. To alleviate CO_2 emissions it is desirable if renewable energy supplies are employed to meet the increased demand of EV charging stations [7, 8].

So motivated, we analyze charging stations as in Fig. 2.2 with sufficient energy storage capacity, connected to the grid from which they can buy electric power, as well as access to a source of

*Part of this Chapter is reprinted with permission from "A Layered Architecture for EV Charging Stations based on Time-Scale Decomposition" by Ke Ma, Le Xie, and P. R. Kumar, in 2014 IEEE International Conference on Smart Grid Communications (SmartGridComm), pp. 674-679, Nov 2014.

Figure 2.1: Solar Electric Vehicle Charging Station

renewable power.

There has been growing work on the problem of scheduling EVs. Zhang et al. [9] incorporates renewable sources in the station and models it as a Markov decision process with uncertainty in both grid power price and EV arrivals. It aims at minimizing total cost but does not consider the influence of price on arrival rates of EVs. Chen et al. [10, 11] formulate charging as a deadline scheduling problem and propose an on-line algorithm with admission control to achieve the maximum competitive ratio. They introduce a pricing function, tied to the individual utility of each charging request, to provide economic incentives for customers to relax their deadlines. The papers utilize price to control the arrival of EVs, but do not aim at maximizing running profit. There does not appear to exist any work that holistically addresses the operation of charging stations taking into account the optimal utilization of battery storage, the availability of renewable energy, the fluctuation in grid power price, and the stochastic EV arrivals with rate modulated by the pricing scheme. Our goal is to address this overall problem incorporating all of these coupled features.

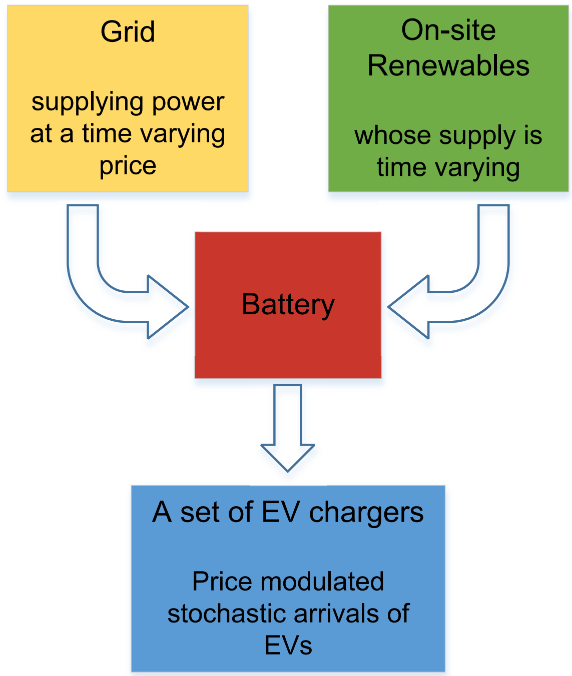

Figure 2.2: Envisioned infrastructure for charging station

We propose a decomposition that is layered by time-scale to handle this overall problem. In the top layer, a deterministic pricing scheme for EV customers is developed by considering the long-term average grid power price and availability of renewable energy, plus price-demand curves for different types of energy requests with different relative deadlines, i.e., remaining times to the absolute deadlines [12]. In the middle layer, we develop an optimal policy to determine the amounts of energy to buy from the grid and to use for charging in successive time intervals. We model customer arrivals as a stochastic process modulated by the price determined by the top layer and announced to customers. In the bottom layer, we fulfill all charging requests without missing deadlines by prioritizing requests with earlier deadlines, and using the optimal charging scheme derived in the middle layer. This three layer decomposition provides a tractable and holistic solution, illustrated by examples in Section 2.3 – 2.5. By using wholesale electricity data from Electric Reliability Council of Texas (ERCOT), we also show that it is beneficial to incorporate storage devices in the charging station in the long run. We demonstrate that the architectural solution proposed does not incur any significant loss of optimality [13].

The rest of the chapter is organized as follows. In Section 2.2, the charging station model is described, followed by the layered decomposition approach. In Sections 2.3, 2.4 and 2.5, we formulate the mathematical model of each layer and provide a solution. In Section 2.6, numerical examples for each layer are provided. In Section 2.7, we discuss the complexity of the algorithms, followed by the discussion of optimality and the value of a battery in Section 2.8.

2.2 Model

We assume that there are two sources of power available, as in Fig. 2.2. Grid power is available at a price that fluctuates with time, which may be partially predictable. Renewable power is inexpensive, but its availability fluctuates with time. Energy storage that is co-located with the charging station allows the station operator to buy power from the grid and store it when the price is low, and it also mitigates fluctuations of power generated by the renewable resource. Customer arrivals, energy requirements and relative deadlines are random, and are modulated by the prices announced by the charging station. Although the price-demand curve, which we will formally introduce in Section 2.3, is sufficient to capture the aggregate behavior of customers, the probabilistic characteristics of customers may depend on the price being charged for various services by the charging station since individual response of any customer is random. All the above give rise to an overall real-time stochastic scheduling problem.

To deal with the overall complexity, we introduce a layered decomposition approach by exploiting the time scales involved. Wholesale electricity price changes every 15 minutes. If we limit our focus to this time scale, then the individual demand coming from each customer is random, grid power price fluctuates randomly, and renewable supply also varies randomly. However, when we consider a longer time scale, for instance one day, the aggregate total demand of all customers in a day is relatively predictable. Fluctuations of grid power price and renewable energy averaged over a day are also comparatively smaller with respect to historical data. It is thus reasonable to model grid power price and renewable power supply on a daily basis as deterministic based on historical data. Total aggregate demand can also be regarded as a deterministic function of the price as given by the price-demand curve. Thus, we address the problem of how to set prices for different customers requesting different amounts of energy with different deadlines in a deterministic fashion.

Subsequently, we address the decisions to be made at a shorter time scale of 15 minutes. Over such a short time period, the charging price is constant and we can model the number of customers arriving in each 15 minute time slot, with a specific energy requirement and deadline, as a random variable with mean determined by the price from the demand curve. Modeling the fluctuating grid power price also as a random process, stochastic optimal control is used to determine how much to buy and use in each 15 minute time slot.

Finally, we consider real-time EV arrivals. We solve the scheduling problem of meeting customer requests by simply giving priority to customers with earlier deadlines.

Combining all the algorithms in the three-layer decoupled manner, we obtain the overall solution shown in Fig. 2.3.

2.3 Top-layer

At the topmost layer, we employ a deterministic model of customer response, and determine the optimal pricing scheme to control the total demand from customers. The relationship between aggregate (not short-term dynamic) demand and price is captured by the price-demand curve, as is standard in microeconomics. For each value of price p, there is a long-term asymptotic stable total demand q from all the customers, given by the strictly monotone decreasing function $p = P(q)$.

At a finer granularity, on the "service" side, the charging station problem can be treated as a multi-server, multi-class, preemptive queuing system with unlimited buffer, and balking and reneging of customers. It is preemptive since the interruption and rescheduling of charging can be made in a manner that is transparent to customers. We use the word "class" to refer to the combination of charging quantity and relative deadline, where relative deadline is the time difference between the deadline and the current time, i.e., the remaining time till the deadline expires. When a vehicle arrives at the charging station, it observes that the queue length is x. We allow for balking

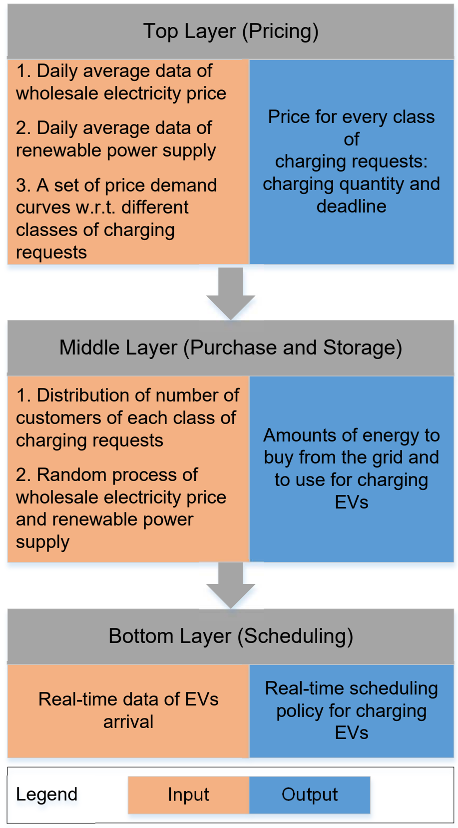

Figure 2.3: Layered decomposition approach

by supposing that it enters the queue with probability $\alpha(x)$, or, balks with probability $1 - \alpha(x)$. Moreover, after a customer has been in the queue for t units of time, it may continue to remain in the queue with probability $\beta(t)$, or, renege with probability $1 - \beta(t)$. Here α and β are nonincreasing functions. If a relative priority is determined for each group, then one obtains a Markov chain on a high dimensional state space. The total long-term expected revenue can be determined as a function of price by calculating the steady state distribution of the Markov chain and utilizing the demand curve. Such an approach however does not lead to a tractable formulation. Hence we develop an alternative layered model and architecture that is both tractable and implementable. Before introducing this model, we consider how price discrimination, a common technique used to capture consumer surplus, works.

For each class of customer, the company would like to charge a different price. A company can do this because it has some kind of monopoly power. The charging station market is not pure competitive as a charging station can provide better service than other charging stations (perhaps through providing faster charging) and not all charging stations need to be price-takers. Thus, companies can create several consumer classes and charge different prices to different classes. In our case, and for simplicity of exposition, let us suppose that consumers can be divided into two classes: those with a short deadline and those with a longer deadline, both with the same energy requirement. Customers with long deadlines are more patient than those with early deadlines. If the charging station offers a price higher than they expect, EVs can simply go to another charging station. As a result, customers with longer deadlines have more price-elastic demand. On the other hand, customers with shorter deadlines are not very price-elastic, but could be regarded as being more deadline-elastic. A common strategy called *price discrimination* for a profit-maximizing company is the following: charge a higher price to consumers with smaller price elasticity, and lower price to consumers more sensitive to price.

The prices could be advertised in various ways, e.g., displayed on a big screen as gas stations do, thus generating a commensurate total demand since prices are relatively static and not changed at a high rate.

More generally there can be several customer classes differentiated by charging amounts and deadlines, with a matrix $[P_{ij}]$ denoting the price for customers with energy requirement L_i and relative deadline d_j.

Consider the multi-stage model, where decisions are made on a daily basis. There are two sources of cost, from the purchase of grid power, and the efficiency loss of the battery. Currently, most batteries are deep cycle having round-trip efficiencies of about 70-80% [15]. Hence the very act of storage incurs a cost. Consider the t-th time interval $[t, t+1]$. Let $S(t)$ denote the energy level in the battery at the beginning of the t-th interval, $R(t)$ the total amount of energy generated by the renewable resource during the interval, $G(t)$ the total amount of energy bought from the grid, $q(t)$ the total amount of energy used to charge the EVs (assumed equal at this layer to the total energy demand of consumers), $PG(t)$ the price of grid power, and $P_1(t)$ and $P_2(t)$ the prices charged to customers of the classes. There is a marginal storage cost c associated with the cost of storing electrical energy in the battery. If we assume constant round-trip efficiency loss and factor this loss into c, we can reasonably assume c is constant [16] . Also let s_0 and s_{max} denote the initial and maximum levels of the battery. Since $S(t)$ is known causally at the beginning of this time interval, the goal is to determine how much energy to buy from the grid, as well as how services to the customer classes are to be priced, so that for a finite time horizon T, the total profit over the time interval $[0, T]$ is maximized. We impose a steady-state condition $S(T) = s_0$ so that in the long run one does not get infinite energy from the battery as this optimization problem is repetitively used. One can replace this with any other suitable constraints. In particular one can replace the lower bound of $S(t)$ with s_{min} if that is deemed preferable.

We assume that the battery capacity is larger than the total possible energy demand [17], the maximum discharging rate of the battery is bigger than the peak demand rate, and that the marginal storage cost is less than the wholesale electricity price. We do not impose an upper bound on $G(t)$ because a typical distribution line has a capacity of 10 MVA [18], which is much bigger than the maximum possible rate at which the station draws power from the grid (See Section 2.6 for details). We also assume that price-demand curves for the two classes are both linear: $P_j = a_j + Q_j/b_j$ for $j = 1, 2$. Class 1 comprises of those customers with short deadline, i.e., $a_1 > a_2 > 0$ and $b_2 < b_1 < 0$.

The resulting discrete optimal control problem [19] is:

$$\max \sum_{t=0}^{T-1} \sum_{j=1}^{2} b_j P_j(t) \left(P_j(t) - a_j\right) - c \cdot S(t) - PG(t) \cdot G(t)$$

subject to (for $t = 0, 1, ..., T - 1$):

$$S(t+1) - S(t) = G(t) + R(t) - \sum_{j=1}^{2} b_j \left(P_j(t) - a_j\right), \tag{2.3.0.1}$$

$$S(0) = S(T) = s_0,$$

$$0 \le S(t) \le s_{max} \text{ and}$$

$$0 \le P_1(t) \le a_1,\ 0 \le P_2(t) \le a_2,\ G(t) \ge 0.$$

with $u(t) := [P_1(t),\ P_2(t),\ G(t)]^{'}$, let us denote

$$f_0(S(t), u(t)) := \sum_{j=1}^{2} b_j P_j(t) \left(P_j(t) - a_j\right) - c \cdot S(t) - PG(t) \cdot G(t),$$

$$f(S(t), u(t)) := G(t) + R(t) - \sum_{j=1}^{2} b_j \left(P_j(t) - a_j\right).$$

We can replace the lower bound of $S(t)$ with s_{min} if needed for practical reason.

To solve this, we construct the Lagrangian:

$$\begin{aligned} L &\left(S(0), ..., S(T); u(0), ..., u(T-1); p(1), ...p(T); \lambda^0, ..., \lambda^T; \alpha^0, \alpha^T; \gamma^0, ..., \gamma^T\right) \\ &:= \sum_{t=0}^{T-1} f_0\left(S(t), u(t)\right) - \left\{ \sum_{t=0}^{T-1} p(t+1) \left(S(t+1) - S(t)\ - f(S(t), u(t))\right) \right. \\ &\quad \left. + \sum_{t=0}^{T} (\lambda^t)^{'} \cdot q_t(S(t)) + \alpha^0 (S(0) - s_0) + \alpha^T (S(T) - s_0) + \sum_{t=0}^{T-1} (\gamma^t)^{'} h_t(u(t)) \right\}, \end{aligned}$$

where $q_t(S(t)) := [-S(t),\ S(t) - s_{max}]^{'}$,

$$h_t(u(t)) := [-P_1(t),\ -P_2(t),\ P_1(t) - s_{max},\ P_2(t) - s_{max},\ G(t)]^{'}.$$

Suppose $S^*(0), ..., S^*(T); u^*(0), ..., u^*(T-1)$ is optimal. Then, from the Karush-Kuhn-Tucker (KKT) condition, there exist $p^*(t) \in R^1$,$1 \le t \le T$, $(\lambda^t)^* \in R^2, 0 \le t \le T$, $(\alpha^t)^* \in R^1$, $t = 0, T$, $(\gamma^t)^* \in R^5, 0 \le t \le T-1$, such that

$$\frac{\partial L}{\partial S(t)} = 0, \text{ and } \frac{\partial L}{\partial u(t)} = 0. \tag{2.3.0.2}$$

Denoting $\lambda^t = [\lambda_1^t,\ \lambda_2^t]'$ and $\gamma^t = [\gamma_1^t,\ \gamma_2^t,\ \gamma_3^t,\ \gamma_4^t,\ \gamma_5^t]'$, from complementary slackness:

$$\lambda_1^t S(t) = \lambda_2^t \left(S(t) - s_{max}\right) = 0,\ t = 0, ..., T.$$

$$\gamma_1^t P_1(t) = \gamma_2^t P_2(t) = \gamma_3^t \left(P_1(t) - a_1\right) = \gamma_4^t \left(P_2(t) - a_2\right) = \gamma_5^t G(t) = 0,$$
$$t = 0, ..., T-1. \quad (2.3.0.3)$$

Noting that f_0 is a quadratic concave function since b_1 and b_2 are negative, and that all constraints are linear, the solution is unique and a global maximum.

We now show that the earlier method for price discrimination in a static problem (shown in Fig. **??**) also works at every stage. For each stage t, the marginal revenue for class j is $MR_j(t) = dTR_j(t)/dQ_j(t) = a_j + 2Q_j(t)/b_j = 2P_j(t) - a_j$ [14]. The total cost incurred in stage t is $TC(t) = c \cdot S(t) + PG(t)G(t) = c \cdot S(t) + PG(t) \cdot (S(t+1) - S(t) - R(t) + Q(t))^+$, where $(\cdot)^+ := \max(\cdot, 0),\ Q(t) := \sum_{j=1}^{2} Q_j(t)$, with $Q_1(t)$ and $Q_2(t)$ denoting the quantities corresponding to $P_1(t)$ and $P_2(t)$ on the demand curves for classes 1 and 2. Thus the marginal cost is $MC(t) = d(TC(t))/dQ(t) = PG(t) \cdot 1_{Q(t) > S(t) + R(t) - S(t+1)} = PG(t) \cdot 1_{G(t) > 0}$, where $1_{(\cdot)}$ is the characteristic function.

Theorem 1. *Suppose* $0 < P_j(t) < a_j$, *for* $j = 1, 2$. *Then for each stage* t, $MR_1(t) = MR_2(t) = MC(t)$ *if a strictly positive amount of grid power is bought, i.e.,* $G(t) > 0$.

Proof. We expand $\dfrac{\partial L}{\partial u(t)} = 0$ from (2.3.0.2). For $j = 1, 2$

$$\frac{\partial L}{\partial P_j(t)} = 2b_j P_j(t) - a_j b_j - b_j p(t+1) + \gamma_j^t - \gamma_{j+2}^t = 0,$$

$$\frac{\partial L}{\partial G(t)} = -PG(t) + p(t+1) - \gamma_5^t = 0. \quad (2.3.0.4)$$

Under the assumptions $0 < P_j(t) < a_j$ and $G(t) > 0$, $\gamma_1^i = \gamma_2^i = \gamma_3^i = \gamma_4^i = \gamma_5^i = 0$. Then from the above, $2P_1(t) - a_1 = 2P_2(t) - a_2$, which is the same as $MR_1(t) = MR_2(t)$. Also,

$2P_1(t) - a_1 = PG(t)$, which is the same as $MR_1(t) = MC(t)$. □

From the Lagrange multiplier, we observe the dependence of battery state on the variation of grid power price. Also, the following theorem shows that when the increase in price is expected to be greater than the marginal storage cost, then the battery should be charged to the full.

Theorem 2. *If $S(t-1) = 0$ and $PG(t) - PG(t-1) > c$, then $S(t) = s_{max}$.*

Proof. First we notice that if $S(t-1) = 0$, then from the previous assumption, the renewable energy is little and not enough to satisfy the total demand. From the system dynamics equation (2.3.0.1), we see $G(t) > 0$. By (2.3.0.3), $\gamma_5^{t-1} = 0$. From (2.3.0.4), we see $PG(t-1) + \gamma_5^t = p(t)$. Expanding (2.3.0.2) and substituting in $PG(t-1) = p(t)$, we have

$$PG(t) - PG(t-1) + \gamma_5^t + \lambda_1^t - \lambda_2^t = c.$$

Clearly, when $PG(t) - PG(t-1) - c > 0$, $\lambda_1^t - \lambda_2^t < 0$. By complementary slackness, $S(t) = s_{max}$. □

Section 2.6 presents a numerical example using data from the Electric Reliability Council of Texas (ERCOT).

2.4 Middle-layer

Now we consider the middle layer where decisions are made every 15 minutes. The total expected revenue is fixed since the prices have been fixed at the top layer and one day is long enough for the demand to reach an equilibrium. Therefore to maximize profit, we only need to minimize expected cost at the middle layer. The number of customers arriving in each stage is a random variable, with the mean determined by the price-demand curve from the prices announced at the top layer. For consistency, we assume there are two classes of customers with different relative deadlines, both requiring the same amount of energy; this can be generalized. It should be noted that the top layer only determines how much electricity on average, $G(t)$, the charging station should buy from the grid and how much to consume from the battery, $Q(t)$. Since a battery can be severely damaged if it is overdrawn or overcharged, a 15-minute charging and discharging policy needs to be specified. Thus at the middle layer, we determine the optimal policy for 15-minute operation of the battery, which specifies the amounts to charge (i.e., purchase from the grid) and discharge (i.e., use for charging EVs).

We adopt the same notation as for the top layer. Let $S(t)$ denote the energy level in the battery, and $X(t)$ and $Y(t)$ the numbers of customers with relative deadlines of one and two time slots. Let $M(t)$ denote the random number of customers that arrive in the t-th interval with deadline at the end of the interval, and $N(t)$ the random number of customers with deadline at the end of the $(t+1)$-th interval. Let $PG(t)$ denote the random wholesale price in the t-th interval, $H(t)$ the historical average wholesale electricity price for the t-th interval; $G(t)$ the amount of energy to buy in the t-th interval, $W(t)$ the amount to discharge, i.e., to use from the battery in the t-th interval. We assume our charging policy is work-conserving, i.e., $W(t)$ satisfies $L \cdot X(t) \leq W(t) \leq L \cdot (X(t) + Y(t))$.

The resulting multi-stage stochastic control problem is

$$\min \ \mathbb{E} \left\{ \sum_{t=0}^{T-1} PG(t) \cdot G(t) + c \cdot S(t) \right\}$$

subject to

$$S(t+1) = S(t) + G(t) - W(t), t = 0, 1, ..., T-1,$$

$$X(t+1) = X(t) + Y(t) - W(t)/L + M(t+1), t = 0, 1, ..., T-2,$$

$$Y(t) = N(t), t = 1, 2, ..T-1,$$

$$M(T-1) = 0 \text{ and } N(T-1) = 0,$$

$$S(0) = S(T) = s_0 \text{ and } 0 \leq S(t) \leq s_{max}, t = 0, ..., T,$$

$$G(t) \geq 0, t = 0, ..., T-1,$$

$$L \cdot X(t) \leq W(t) \leq L \cdot (X(t) + Y(t)), t = 0, ..., T-1,$$

$$M(t) \text{ is i.i.d, with } \mathbb{E}\left[M(t)\right] = b_2(P_2 - a_2)/96,$$

$$N(t) \text{ is i.i.d, with } \mathbb{E}\left[N(t)\right] = b_1(P_1 - a_1)/96,$$

$$\mathbb{E}\left[PG(t)\right] = H(t).$$

The minimization above is over all Markov policies [3].

Let us define: state space $\mathcal{S} := \{(S, X, Y, PG)\}$, action space $\mathcal{A} := \{(G, W)\}$ and $\mathcal{T} := \{0, 1, ..., T-1\}$. Our goal at this layer is to find a policy $\pi : \mathcal{S} \times \mathcal{T} \to \mathcal{A}$ such that for every $s \in \mathcal{S}$ and $t \in \mathcal{T}$, $\pi_t(s)$ specifies an action in $\mathcal{A}$ which minimizes the expected value of the cost function.

For computational purposes we assume that the number of values that M, N and PG can take is finite. For simplicity, we assume here that M, N, PG are independent; M is i.i.d. with mean depending on the total quantity associated with the price announced from the top layer; N is similar to M; PG's are independent, with symmetric probability mass functions, each centered at $H(t)$. However, the general problem is solved similarly. Using dynamic programming [3], the value function is

$$V_i(S, X, Y, PG) = \min_{(G,W)\in\mathcal{A}} \Big[PG \cdot G(t) + c \cdot S + \sum_{h\in\mathcal{H}} \sum_{n\in\mathcal{N}} \sum_{m\in\mathcal{M}} p_M(m) p_N(n) p_{PG}(h) \cdot V_{i+1}\left(S + G - W, X + Y - W/L + m, n, h\right) \Big], \quad (2.4.0.1)$$

$$V_T(S, X, Y, PG) = c \cdot S.$$

where p_M, p_N and p_{PG} are probability mass functions of M, N and PG, respectively; $\mathcal{M}$, $\mathcal{N}$ and $\mathcal{H}$ are the finite sets of values that M, N and PG can take, respectively.

To get a closed form solution of the value function V is difficult. As we recurse backwards from stage T, we notice that to satisfy (2.4.0.1), the value function V is piecewise, but the number of pieces of V grows. As a consequence, instead of deriving the necessary condition for the optimal policy by exploiting the properties of V, we employ a numerical algorithm to derive the optimal policy, illustrated in Section 2.6 for a numerical example following the results from top-layer.

2.5 Bottom-layer

In the bottom layer, the lookup table obtained at the middle layer, which consists of the optimal actions (G, W) for every possible state (S, X, Y, PG) at each time t, is used as a guideline for the scheduling problem. The amount to use from the battery W in each time interval, which is derived from the middle layer, equals the total amount of energy discharged from the battery during that interval. This sets an equality constraint for the bottom layer. We seek a real-time scheduling policy which explicitly prioritizes different classes of customers while satisfying the total energy consumption constraints and ensuring that no deadlines are missed. While finding the optimal real-time scheduling policy is not trivial, it is straightforward if the following assumptions hold:

- The charging rate for each charger is large enough that all requests can always be finished within one time slot. This is true if the discharging rate of the battery is large enough.
- The number of chargers is large enough so that whenever it is decided to charge a EV, there is an empty charger. This is feasible if the former assumption is made.
- The deadline for EVs that arrived in $(t-1, t]$ with a relative deadline of d is $t + d$, for $d = 1, 2$.

Wholesale electricity prices are typically announced every 15 minutes, as is done by ERCOT in Texas for example. Neglecting the initial opening period for the operation of the charging station, the prescription for the middle layer can be implemented in the following manner to yield the policy for the bottom layer:

During $(t-1, t]$:

- Keep track of EVs that arrived in $(t-1, t]$.

At time t:

1. Obtain the latest grid power price on the website for the period $(t-1, t]$.
2. Read the current energy level of the battery $S(t)$.

3. Determine the optimal action $W(S(t), X(t), Y(t), PG(t), t)$ from the middle layer and follow it to charge the EVs that arrived in $(t-1, t]$, i.e., if $W = X$, charge all the EVs with one time slot relative deadline; if $W = X + Y$, charge the EVs with a relative deadline of either one slot or two slots at the same time; if $X < W < X + Y$, charge all the EVs with one slot relative deadline and then charge $(W - X)/L$ EVs with two slots relative deadline.

2.6 Numerical Results

2.6.1 Top-layer

We use data from ERCOT [20] and the wholesale electricity prices of Houston, from Jan. 1 – 30, 2012, as $PG(t)$. From [20, 21], onsite renewable energy generation is random and only accounts for 5 to 10 percent of the total demand. Therefore, we generate a random vector in that range and fix it throughout the example. Moreover, we assume the typical battery capacity of an EV is 16 kWh [22] and that every consumer arrives with an empty battery and requires a full charge. For gasoline demand, the typical short term elasticity lies between 0.12 to 0.25 [14]; so we use 0.15 and 0.25 as the elasticities of demand for urgent consumers and non-urgent consumers.

We present the results in Fig. 2.4. All the curves are normalized so they do not overlap with each other. Compared with the daily average wholesale electricity price $PG(t)$, the announced prices $P_1(t)$ and $P_2(t)$ have relatively less fluctuation. This is because, from Theorem 1, the marginal revenues of the two classes are both equal to the marginal cost, which is the grid power price. It turns out that the charging price lies very close to the average of the highest possible price that can be charged and the wholesale electricity price. In Fig. 2.5 we present the storage level as a percentage of the battery's capacity. We see that whenever the grid power price is known to rise in the next period, a large amount of power is bought from the grid, resulting in a full battery in the next period. Then, in the next period, all the charging requirement is fulfilled by the battery, that is, the amount bought from the grid is zero.

With the development of smart phones, it is relatively easy for an EV driver to have access to charging station prices in real-time. Therefore, the old fashioned gasoline pricing scheme, where price is changed on a daily basis, needs to be reexamined. Using the same data, we show in Fig.

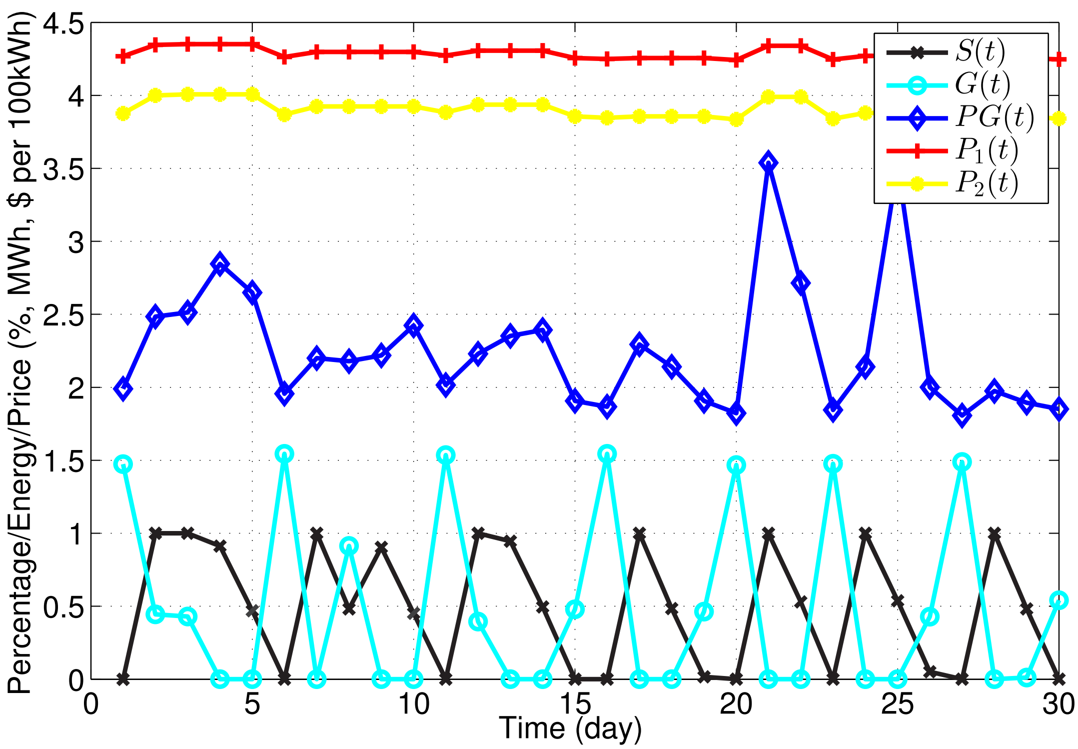

Figure 2.4: Numerical result of top layer

2.6 how the total profit changes with respect to the frequency at which prices are announced by the charging station. The horizontal axis represents the price change period, i.e., the x-coordinate 12 represents the total profit in January if price is changed every 12 hours. The total profit is of course a decreasing function of changing price period; the total profit obtained by announcing the price every 2 hours is larger than the profit obtained by changing the price daily, because the daily average of wholesale electricity price cancels out the possible volatility of price during the day. A more frequent change of pricing scheme results in a better utilization of the battery and thus earns a larger profit.

2.6.2 Middle-layer

In the example, we assume that the numbers of customers arriving with relative deadlines of one and two slots are both uniformly distributed. We consider the probability mass function of the

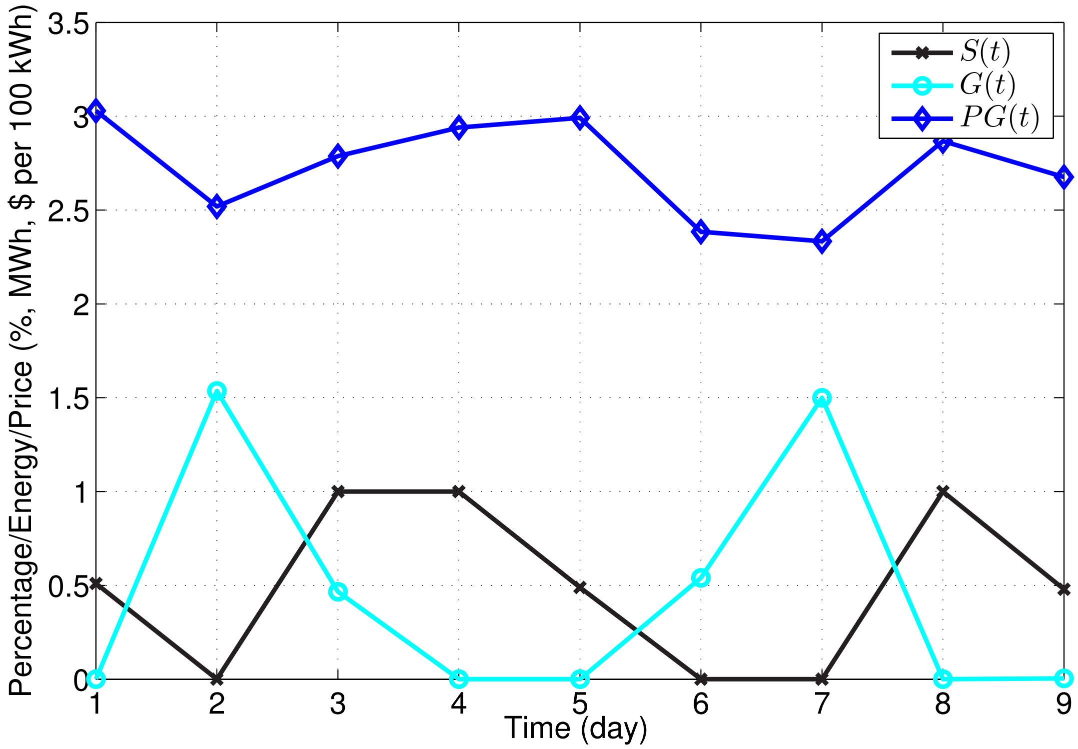

Figure 2.5: Battery level, grid power price and amount of grid power purchase for days $10 - 18$.

Table 2.1: Probability mass function of grid power price in each interval

Value	$H-2$	$H-1$	H	$H+1$	$H+2$
Probability	0.1	0.2	0.4	0.2	0.1

wholesale electricity price shown in Table 2.1. We take $|S| = |\mathcal{H}| = 20$, $|\mathcal{M}| = |\mathcal{N}| = 5$. Because we require that no deadlines should be missed, the size of the action space is $|\mathcal{A}| \leq |\mathcal{N}||S|$. We use discrete dynamic programming to solve this problem. Going backwards, we obtain a look-up table containing all the optimal actions - how much to buy from grid and how much to use to charge the EVs - specified for every possible state in each stage. This optimization has a time horizon of one day and needs to run on a daily basis after the price is determined from the top layer. For illustration purposes, we analyze eight 15-minute intervals, from Jan. 9, 9am to 11am. We use the wholesale electricity price data from ERCOT, Houston.

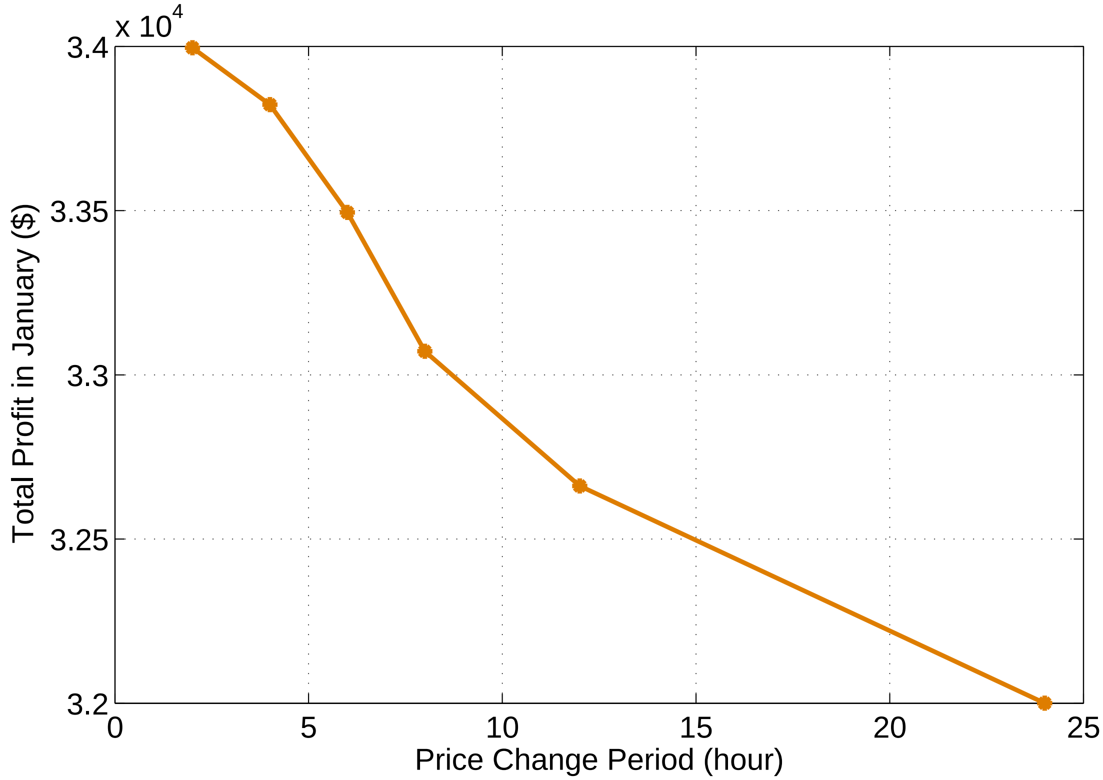

Figure 2.6: Total profit in January as a function of the periodicity of price change.

To illustrate the optimal policy, we exhibit a single sample path in Fig. 2.7. In the figure, $D(t)$ is the total charging requirement – the sum of energy required by customers with relative deadlines of one time slot and two slots. On this sample path, the maximum rate at which the station draws power from the grid is 2 MWh, which happens at $t = 1$. This justifies the assumption that the upper bound on $G(t)$ can be ignored.

During the first five intervals, $D(t)$ and $W(t)$ coincide, which means that all the requests are fulfilled even if the relative deadline is two time slots. Consider for example $t = 3$. The battery is full, responding to the rise of grid power price during the first two intervals. There is no need to buy from the grid and the best one can do is to fulfill all the pending requests. However at $t = 6$, $W(t) < D(t)$, which means that requests of customers with a relative deadline of two slots are deferred to the next interval. This may result from the fact that the grid power price is high but the battery level is low, since fulfilling the requests of both classes will necessitate buying a lot of

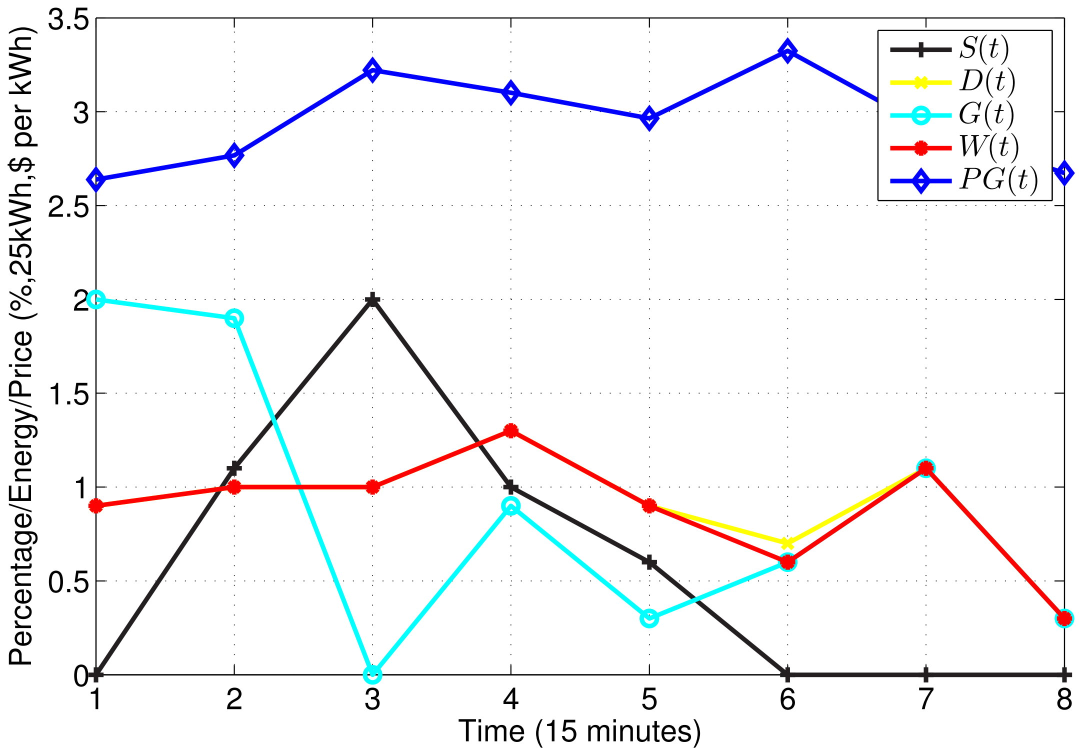

Figure 2.7: The evolution of the optimal policy for a specific sample path

Table 2.2: Average costs of different policies

Policy Name	Conservative	Radical	Optimal
Average Cost ($)	156.9	157.8	152.8

expensive energy from the grid.

We also compare the performance of the optimal policy against two simple policies, one "conservative" and the other "radical". The conservative policy always fulfills all charging requests immediately, even those with a relative deadline of two time slots. The radical policy fulfills only those requests needing to be attended to at the moment – deferring all the requests with relative deadline of two time slots to the next interval. Table 2.2 shows the average costs incurred by the three policies. Clearly, by incorporating the anticipation of future wholesale electricity prices and number of customers arriving into its planning, the optimal policy is able to do the best job.

2.7 Computation complexity

We now examine the computational complexity of the above approach in the general setting. In the top layer, there are altogether $(3K+6)N+5$ variables to solve, where $K = |I|\times|J|$ denotes the number of entries in the price matrix P. The resulting quadratic programming problem is computationally feasible for reasonable values of K and N, using CPLEX [23]. At the middle layer, the number of discrete values for each variable directly influences computational complexity. The time complexity is $\mathcal{O}\Big((n \times |S|^2 \times |\mathcal{H}| \times \prod_{j=1}^{J} \prod_{i=1}^{I} c_{ij})N\Big)$. Here I and J are the numbers of rows and column in the price matrix P respectively, with $|I|\times|J|= K$; $\mathcal{C}_{ij}$ is the the set of values that the number of ij-th class customers' arrivals can take and $c_{ij} = |\mathcal{C}_{ij}|$; $n = \sum_{j=1}^{J} \sum_{i=1}^{I} max_{p\in\mathcal{C}_{ij}} \{p\}$. Clearly, this is polynomial in the number of stages N; however, it is extremely sensitive to how finely we discretize the state and action spaces, as one may expect.

2.8 The Cost of the Layered Policy and the Value of Battery

The above analysis guarantees that the architectural solution is implementable in the real-time market operation of an EV charging station. However, as the time-scale decomposition is only a suboptimal solution of the overall stochastic scheduling problem, another important aspect is to evaluate the performance of the approach, and to determine if the architectural decomposition incurs a significant loss of profit. An upper bound on the total profit can be obtained by optimizing the top layer assuming full future information, including wholesale electricity price and renewable generation, and changing the announced price every 15 minutes. Running the top layer once on Jan. 1 with a horizon of 14 days, middle layer at 12:00 am everyday with a horizon of 1 day, and bottom layer every 15 minutes shows that the total profit obtained by our top layer achieves 90.2% of the upper bound. The middle and bottom layer together however reduce the cost and improve the ratio to 92.6%. We also run the above algorithm from Jan. 2 to Jan. 10. The results are shown in Fig. 2.8. It can be seen that the layered algorithm achieves 90% of the upper bound in all cases. A higher ratio is realized on days when wholesale electricity price has small variance. This is true if we look at the probability mass function of electricity price in our simulation. As a result, it can

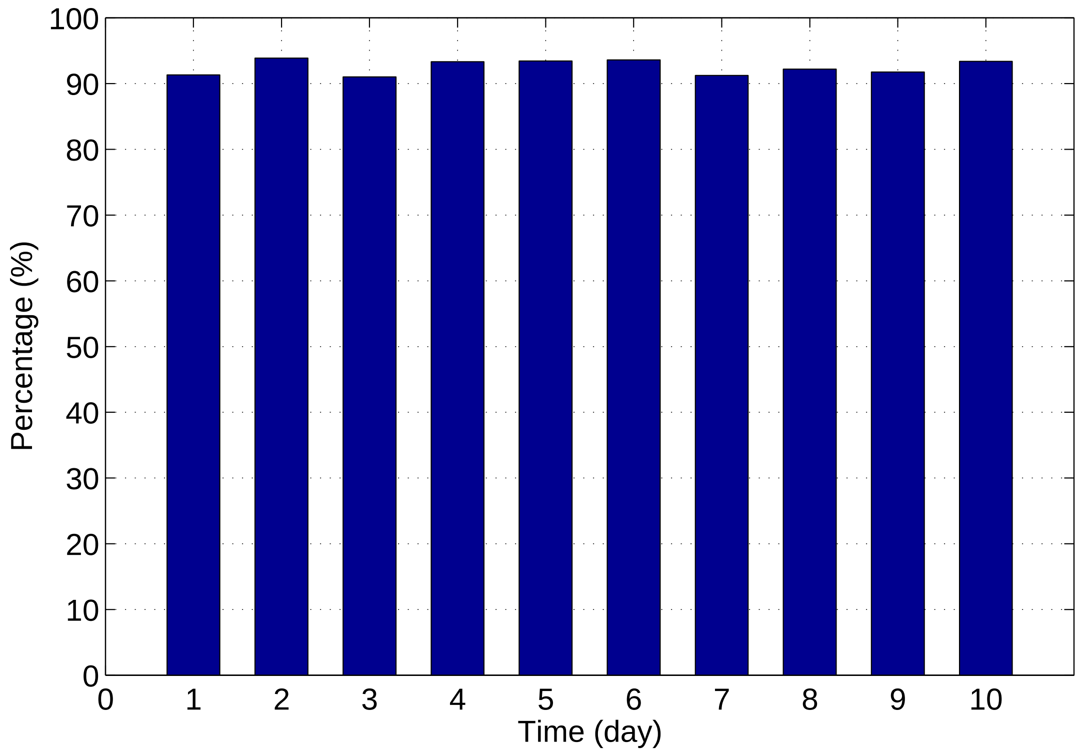

Figure 2.8: The ratio of total profit obtained by the layered algorithm to its upper bound

be concluded that the architectural solution, with three layers serving for monthly planning, daily adjustment and real-time scheduling respectively, does not lose much with respect to optimality.

The fixed cost and the operating cost of the battery are both high. As a result, we need to justify the benefit of introducing a battery in the charging station. We first set $s_{max} = 0$ and calculate the upper bound of the total profit in January in the same way as we do in the optimality test above. This gives us the maximum possible total profit in January without the battery. Not surprisingly, the total profit obtained by our algorithm in the presence of a battery is 30.2% more than the maximal achievable profit without a battery. Similarly, we run the simulation for February and all the way to December. The results are shown in Fig. 2.9. From the figure we can see that the maximal achievable profit without a battery is less than the profit with a battery for every month of 2012. During the months of July and August (summer in Texas), the difference is even bigger because wholesale electricity has bigger fluctuations during that time. These differences, as a consequence,

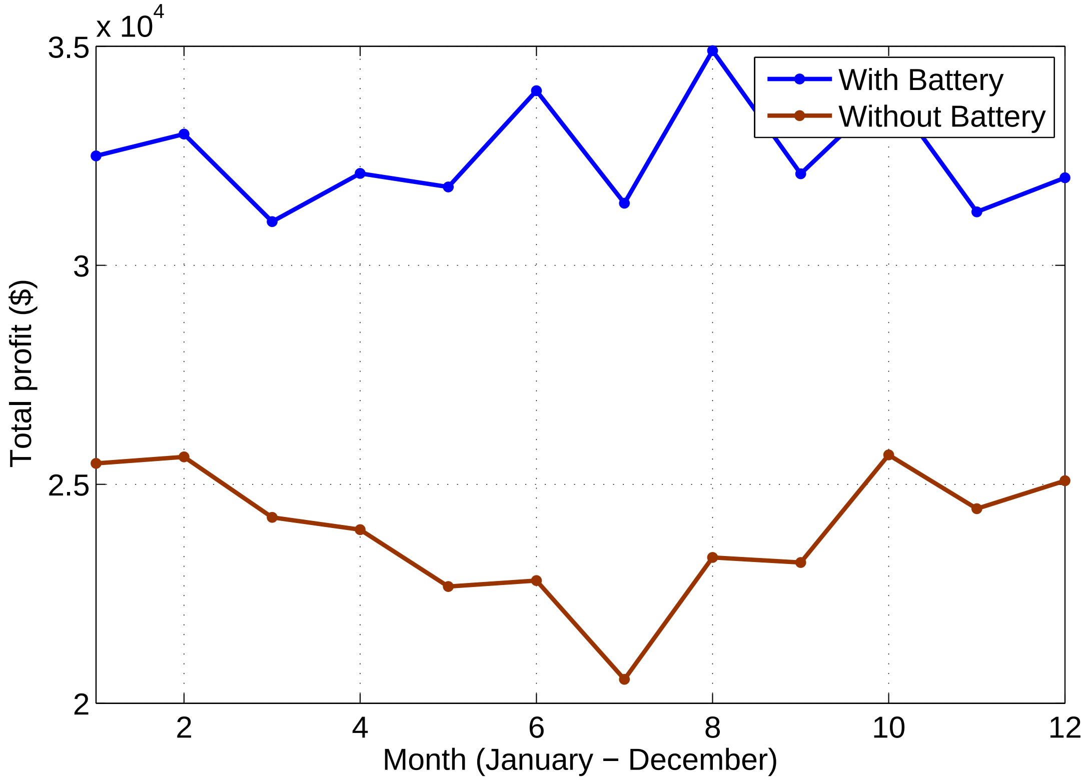

Figure 2.9: Total profit comparisons with and without battery

will definitely cover the fixed cost of the battery in the long run and therefore we conclude that it is beneficial for the charging station owner to introduce a battery into the system.

We also obtain numerical results of how total profit in January changes as battery capacity increases. The results are shown in Fig. 2.10. We see that as capacity increases, marginal benefit decreases. Thus if the fixed cost of the battery is factored in, we may expect an optimal battery capacity to be chosen at the level that maximizes rate of return.

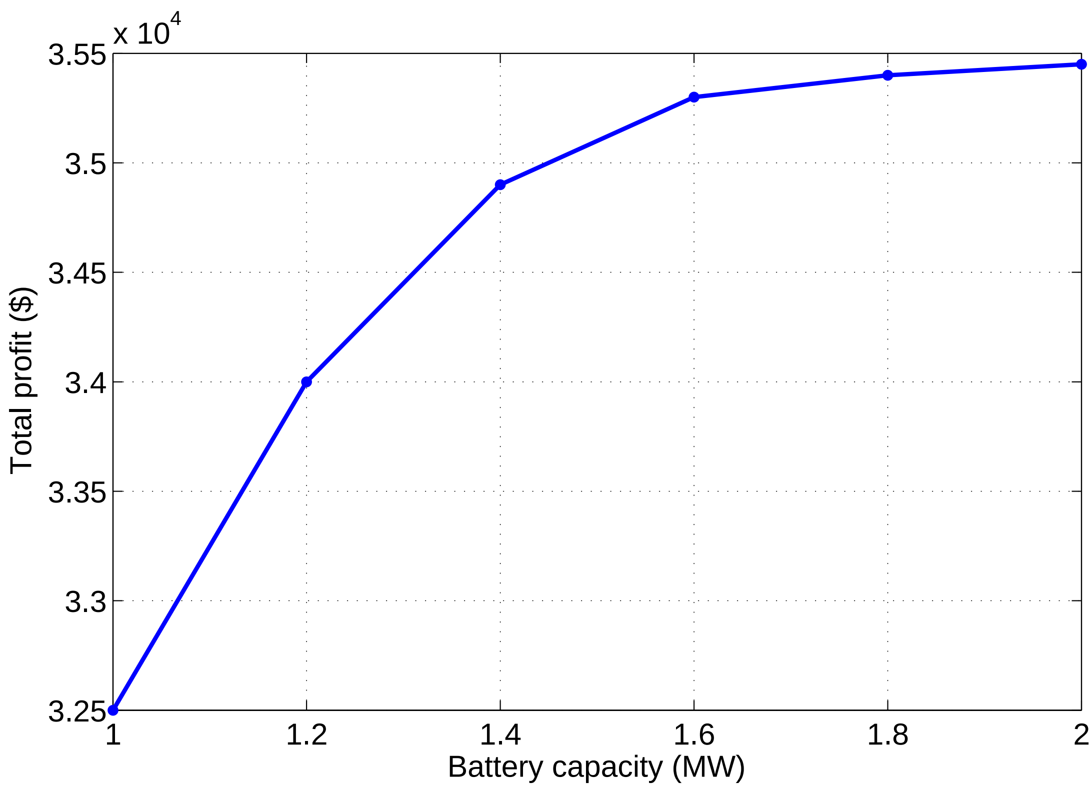

Figure 2.10: Total profit as a function of battery capacity

3. OPTIMAL MARKET OPERATION WITH DER FLEXIBILITIES: PROBLEM FORMULATION AND SOLUTION CONCEPT *

3.1 Introduction

In electric power systems, a key role is played by the grip operator, an entity called the System Operator or Independent System Operator (ISO). There are a number of unique challenges faced by the electricity grid operator: production has to be simultaneous with demand; the costs of different generating units vary significantly; and expected and unexpected conditions on the transmission network affect the selection of which generating units should be used to serve load reliably. Traditionally, given the demand of the loads (or a forecast), the role of the ISO, taking into account these factors, is to allocate the required power among generators such that power is reliably delivered to meet demand, and the cost of generation is minimized. This security constrained unit commitment (SCUC) [24] and economic dispatch (SCED) [25] problem can be solved by soliciting bids (marginal cost of production curve) from each producer and then, choosing for each hour which generator should be committed to be on-line and the output level of the corresponding on-line generators, typically for the next 24-hour period, such that the overall cost of producing the power demanded is minimized [26]. In real time, given the actual load and grid conditions, the ISO must decide the production level at which each available resource from the unit commitment stage should be operated such that overall production costs are minimized while maintaining reliability [27]. Adjustment of dispatch is needed because actual conditions may be different from those forecasted in the day-ahead commitment.

The above model with a fixed demand is insufficient for power systems nowadays when renewable power generation is integrated in the system. When employing renewable energy, such as solar and wind which vary unpredictably with time, demand needs to be adjusted accordingly to match the availability of renewable energy [28]. ISO allows this additional flexibility of loads

*Part of this Chapter is reprinted with permission from "A Theory for the Economic Operation of a Smart Grid with Stochastic Renewables, Demand Response and Storage" by Rahul Singh, Ke Ma, Anupam A. Thatte, P. R. Kumar, and Le Xie in in 2015 54th IEEE Conference on Decision and Control (CDC), pp. 3778-3785, Dec 2015.

to enter the system by introducing demand bids submitted by Load Serving Entities (LSEs) during the SCUC and SCED processes. LSEs, who have traditionally been electric utilities*, secure energy and transmission service (and related Interconnect Operations Services [30]) to serve the electrical demand and energy requirements of their end-users and wholesale customers. Similar to the dispatch problem stated above, there are two stages, or time periods, where bids from both generators and LSEs clear: day-ahead market and real-time market. The day-ahead energy market lets market participants commit to buy or sell wholesale electricity one day before the operating day, where SCUC and SCED are performed to satisfy energy demand bids and to ensure adequate scheduling of resources to meet next day's anticipated load. The real-time energy market lets market participants buy and sell wholesale electricity during the course of the operating day to balance the differences between day-ahead commitments and the actual real-time demand and production [31].

In reality, prices resulting from clearing the bids in both day-ahead and real-time markets are not identical across different locations. This is due to physical limits of the transmission systems, such as congestions and line losses. Prices at different locations, called Locational Marginal Prices (LMPs), reflect the operating characteristics of and the major constraints on the transmission system at different locations as well as losses resulting from physical limits of the transmission system. LMPs are widely used for collecting transmission congestion charges and determining compensation for holders of Financial Transmission Rights [31].

However, in the above design of both day-ahead market and real-time market, an important fact is neglected: power generators and loads are both dynamic systems with individual constraints. For example, one of the most important features of fossil fuel generators is a ramping constraint: there is a limit of increase or decrease of output level between two consecutive time intervals. Similarly, air conditioners can be deferred for a relatively short amount of time but not indefinitely, making loads dynamic systems. Hence both generators and loads, which may be aggregations of many small loads, need to be modeled as dynamic stochastic systems. All variables, including power

*This may change in the future as new business models arise and the traditional model of utilities also undergoes a transformation [29].

output and demand, are *functions of time*. Marketing clearing prices (LMPs), on the other hands, are *functions of both time and location*.

In this chapter, we consider the resulting problem faced by the ISO, called the ISO Problem. How should the ISO choose price as function of both time and locations, such that the sum of utilities of all agents in the system is maximized, while maintaining the balance of supply and demand and satisfying network constraints. The sum of the utilities of all agents is simply the benefit of power consumption by the consumers, minus the cost of power generated by the generators, and it is called the social welfare. Because of the competitive nature of commercial producers and the need to protect privacy of consumers, the desired solution needs to be *decentralized*: the ISO needs to operate without knowing the states/utilities/dynamics of the agents, and agents should also not need to know states/utilities/dynamics of each other. The only sharing of information happening between agents and the ISO and is restricted to the bidding process between the ISO and the agents in each time period (which could be a 15 minute period, or a 5 minute period, or a 24 hour period). This bidding process can be one-shot, or a sequence. The sequential case is considered in [32], where the ISO announces a sequence of tentative market clearing prices over time and locations, and agents respond back with their supply/consumption bids over time with respect to the prices. This iterative process continues until the price sequences converges, and this entire process is repeated in each time period. The complexity of iterating on price vector over time instead of price for one time instant is inevitable if agents do not share their states/utilities/dynamics with the ISO. It is worth noting that similar to the tatonnement process in general equilibrium theory [33], the system-level utility maximization problem faced by the ISO is conducted by agents in a distributed manner, coupled only by the prices announced by the ISO. We need to mention that in this Chapter we are only considering the non-strategic case, where agents always bid truthfully. In [34], we have considered the case where the network constraint are absent and thus prices are identical across different locations and in [32] the authors mention that linear constraints can be incorporated in the LQG case. We will consider here a complete system-wide dynamic optimization problem faced by the ISO with both energy balancing constraints and network constraints.

We examine both deterministic and stochastic models, which suit well the day-ahead and real-time markets, respectively. For deterministic model, our dynamic solution leads to a social welfare maximum under convexity assumptions of the utility functions. It is not restricted to linearity of system or quadraticity of costs. For the stochastic model, we investigate the case where agents are modeled as linear Gaussian systems and the cost functions are quadratic. It can be shown that a simple scheme yields the global optimum [35]. Under this policy, each agent i needs only to track its present state $X_i(t)$ instead of the entire history, as in the general case of decentralized stochastic control [36].

We compare our multi-period formulation with a single-period formulation via simulation. It is shown that our iterative bidding solution achieves a higher social welfare than the hour-by-hour (15-minute-by-15-minute) single-period bidding process in the day-ahead market (real-time market).

The rest of the chapter is organized as follows. In Section 3.2, a survey of related works is presented. This is followed by a complete description of the model and problem in Sections 3.3. Discussions of the iterative bidding scheme in a deterministic setting and a stochastic setting are presented in Section 3.4 and Section 3.5, respectively. Numerical results are provided in Section 3.6.

3.2 Related Works

There have been many papers addressing the problem of dynamic pricing for demand response assuming a known demand function. Borenstein, Jaske, and Rosenfield [37] present an overview and analysis of the possible approaches to bringing an active demand side into electricity markets. Borenstein [38] continues the study by focusing on the long-run efficiency gains from adopting real-time pricing (RTP) in a competitive electricity market. Using simple simulations with realistic parameters, the author demonstrates that the magnitude of efficiency gains from RTP is likely to be significant even if demand shows very little elasticity. Carrion, Conejo, and Arroyo propose a risk-constrained stochastic programming framework to decide which forward contracts the retailer should sign, and at which price it must sell electricity, so that its expected profit is maximized at a given risk level [39]. Conejo et al. [40] further address the optimal involvement in a futures electricity market of a power producer to hedge against the risk of pool price volatility. These results, however, assume implicitly that the demand function is known.

Closer to our work, there is an extensive line of work focusing on obtaining the optimal market prices which maximizes the social welfare of a collection of loads. Joo and Ilic [41] propose a distributed optimization algorithm to solve the social welfare maximization problem with a three-layer market structure (loads at the bottom, LSE at the middle, and the ISO at the top). By examining the relationships between the global objectives and the local objectives in different layers, the authors propose a set of conditions that guarantees the convergence of the algorithm. In [42], a dynamic model of the wholesale energy market that incorporates renewable resources and real-time pricing with demand response is proposed, and conditions under which stability of the market can be guaranteed are derived. Thomas and Tesfatsion [43] show that dynamic-price retail contracting can give rise to braided cobweb dynamics consisting of two interwoven cycles for power and price levels exhibiting either stability or instability depending on system conditions. The classic paper [44] develops a theory of pricing in electrical networks over space and time; however the system model does not incorporate dynamics of agents. To the authors' knowledge there do not appear to be any similar results for achieving optimal social welfare over a time period in a decentralized manner, in a smart grid consisting of network constraints and dynamic generators and loads with stochastic uncertainties. We show that our iterative bidding scheme achieves a higher social welfare than the single-period bidding scheme.

3.3 Problem Formulation

We consider a smart grid consisting of N agents, which may be either producers or consumers of electricity. We model time as consisting of discrete periods and each period corresponds to, say, a 1-hour interval of the day-ahead market. Since we consider the problem from the ISO level, each load bus (agent) is represented by an aggregate LSE model. Details of how to obtain such an aggregate model are not the focus of this thesis, and the reader is referred to [45], [46] and [47]. Each agent i is modeled as a dynamic system. The motivation is that generators typically have ramping constraints, and consumers behind each LSE may have similar ramping constraint as well as delays in load dynamics. The state of agent i at time t, denoted as $x_i(t)$, evolves as,

$$x_i(t+1) = f_i^t(x_i(t), u_i(t)),\ t = 0, 1, ..., T-1. \tag{3.3.0.1}$$

where $u_i(t)$ is the amount of energy (or equivalently, average interval power) each agent i supplies or obtains to the grid at time t. $u_i(t) > 0$ stands for supplying energy to the grid by agent i at time t, while $u_i(t) < 0$ signifies an energy consumption by agent i. A straightforward and important constraint is that there must be a power balance at each time over the grid: $\sum_{i=1}^{N} u_i(t) = 0$ for all t. In addition, network constraints posed by the underlying power-flow equations need to be incorporated. Here, for simplicity, we adopt the simplified direct-current (DC) power flow method, which yields fast estimate of line power flows on an AC power system, to specify the network constraints. A nonlinear model of the AC system is simplified to a linear model of DC power flow if the following assumptions hold [48]:

- Line resistances (active power losses) are negligible, i.e. $R \ll X$.
- Magnitudes of bus voltages are set to 1 per unit.
- Voltage angle differences are small, i.e. $\sin(\theta) \approx \theta$ and $\cos(\theta) \approx 1$.

Based on the above assumptions, voltage angles θ_i are the variables to solve given active power injections P_i in advance. Without loss of generality, we assume agents are geographically located at N different buses, which results in $P_i = u_i$. Under a DC flow assumption, the active power balance equations reduce to a set of linear equations:

$$u_i = \sum_{j=1}^{N} B_{ij}(\theta_i - \theta_j),$$

where B_{ij} is the susceptance between bus i and j, or the imaginary part of the bus admittance matrix Y_{ij}. As a result, active power flow through transmission line l_i between bus j and k can be calculated as:

$$P_{l_i} = \frac{\theta_j - \theta_k}{X_{l_i}}.$$

where X_{l_i} is the reactance of line l_i. DC power flow equations in matrix form and the corresponding matrix relation for flows through branches are represented in (3.3.0.2) and (3.3.0.3).

$$\Theta = B^{-1}U, \tag{3.3.0.2}$$

$$P_l = DA\Theta, \tag{3.3.0.3}$$

where

$U := (u_1, u_2, ..., u_N)^T$ is the vector of bus active power injections for buses $1, ..., N$,

$B \in \mathbb{R}^{N \times N}$ is a matrix whose non-diagonal elements are susceptances and diagonal elements are the sum of non-diagonal elements in the same row,

$\Theta \in \mathbb{R}^{N \times 1}$ is the vector of bus voltage angles for buses $1, ..., N$,

$P_l \in \mathbb{R}^{M \times 1}$ is the vector of branch flows (M is the number of branches),

$D \in \mathbb{R}^{M \times M}$ is a diagonal matrix with d_{kk} equal to the negative susceptance of line k,

$A \in \mathbb{R}^{M \times N}$ is the bus-branch incidence matrix.

Substituting (3.3.0.2) into (3.3.0.3), we have:

$$P_l = DAB^{-1}U = HU. \tag{3.3.0.4}$$

where $H := DAB^{-1}$ is an $M \times N$ matrix that maps active power injection at buses onto active power flow on branches. Here we note that the condition $\sum_{i=1}^{N} u_i = 0$ takes care of the issue of slack bus, since the slack bus is created solely to balance the active power in the system. A natural constraint on the active power flow on a transmission line is $HU \leq C$, where C is the vector of active power flow limits of the branches in the system. We note that this inequality takes care of the direction of flows since $|HU| \leq C$ can be equivalently written as $H'U \leq C$ where $H' = \begin{bmatrix} H \\ -H \end{bmatrix}$.

Note that in this chapter, we do not consider line losses, assuming losses are not significant enough to impact the economics of the system operation. However, our model could be generalized to include these constraints.

We suppose that each agent i has a stage-wise utility function $F_i(x_i(t), u_i(t))$. For producers, this could be the negative of the cost of production. We assume that $F_i(x_i(t), u_i(t))$ is the aggregate total utility of all the loads connected to the LSE. The total utility of agent i over the time horizon $\{0, ..., T-1\}$ is $\sum_{t=0}^{T-1} F_i(x_i(t), u_i(t))$. There could be constraints on input u_i for model (4.3.0.11), such as a ramp constraints $|u_i(t+1) - u_i(t)| \leq r_i$. In that case, these constraints are not dualized in the sequel, but carry over to the dual. For simplicity we will not explicitly consider this case here, but will incorporate such constraints in the numerical examples in Section 3.6.

With the above set-up, we are led to the following deterministic social welfare maximization problem (DA for day-ahead):

$$\max \sum_{i=1}^{N} \sum_{t=0}^{T-1} F_i(x_i(t), u_i(t))$$

subject to

$$\sum_{i=1}^{N} u_i(t) = 0, \text{ for } t = 0, ..., T-1,$$

$$HU(t) \leq C, \text{ for } t = 0, ..., T-1,$$

$$x_i(t+1) = f_i^t(x_i(t), u_i(t)), \text{ for } i = 1, ..., N, \ t = 0, ..., T-1.$$

where $U(t) := (u_1(t), u_2(t), ..., u_N(t))^T$. This model automatically takes care of renewable resources and uncontrollable loads since we can take $F_i \equiv 0$ and $u_i(t)$ as the prediction value over the next 24 hours.

Because commercial producers are competitive and the privacy of consumers must be protected, the desired solution needs to be decentralized. In Sections 3.4 and 3.5, for the deterministic case (day-ahead market) and stochastic case (real-time market) respectively, we will derive algorithms that satisfy information and action decentralization, with communications between ISO and agents restricted to ISO announcing a sequence of prices and agents responding with supply/consumption bids in response to prices.

3.4 Deterministic Case: Day-ahead Market

In the day-ahead (DA) market scenario, the ISO's task is to determinine the T-dimensional vectors $U_i := (u_i(0), u_i(1), ..., u_i(T-1))$, for $i = 1, 2, ..., N$, such that the social welfare $\sum_{i=1}^{N} \sum_{t=0}^{T-1} F_i(x_i(t), u_i(t))$ is maximized. We start by writing the Lagrangian for the DA problem:

$$\begin{aligned}\mathcal{L}(U_1, U_2, ..., U_N, \lambda, \mu) &:= \sum_{i=1}^{N} \sum_{t=0}^{T-1} F_i(x_i(t), u_i(t)) \\ + \sum_{t=0}^{T-1} \lambda(t) \left(\sum_{i=1}^{N} u_i(t) \right) &- \sum_{t=0}^{T-1} \mu^T(t) \left(HU(t) - C \right),\end{aligned} \tag{3.4.0.1}$$

where $\lambda := (\lambda(0), \lambda(1), ..., \lambda(T-1))$ are the Lagrange multipliers associated with energy balance constraints, and $\mu := (\mu(0), \mu(1), ..., \mu(T-1))$ are the Lagrange multipliers associated with the line constraints. The Lagrange dual function is,

$$\begin{aligned}D(\lambda, \mu) &:= \max_{U_1, U_2, ..., U_N} \mathcal{L}(U_1, U_2, ..., U_N, \lambda, \mu) \\ &= \max_{U_1, U_2, ..., U_N} \sum_{i=1}^{N} \Big(\sum_{t=0}^{T-1} F_i(x_i(t), u_i(t)) + \lambda(t) u_i(t) - \mu^T(t) H_i u_i(t) \Big) + \sum_{i=1}^{T} \mu^T(t) \cdot C \\ &= \max_{U_1, U_2, ..., U_N} \sum_{i=1}^{N} \Big(\sum_{t=0}^{T-1} F_i(x_i(t), u_i(t)) + (\lambda(t) - \mu^T(t) H_i) u_i(t) \Big) + \sum_{t=0}^{T-1} \mu^T(t) \cdot C,\end{aligned} \tag{3.4.0.2}$$

where $H_i \in \mathbb{R}^{M \times 1}$ is the i-th column of matrix H. The Lagrange dual function (3.4.0.2) can be decomposed agent-by-agent since they are only coupled by price $(\lambda(t) - \mu^T(t) H_i)$. Note that the obtained price coincides with the locational marginal price (LMP) defined in [31], with λ being the energy component and $\mu^T(t) H_i$ being the congestion component (the loss component is missing because we do not consider line losses in the model). Because of the existence of the $\mu^T(t) H_i$ term, different locations (agents) receive different prices. As a result, prices here are functions of both time and location. We consider the decomposed optimization problem faced by agent i:

$$\max_{U_i} \sum_{t=0}^{T-1} F_i(x_i(t), u_i(t)) + (\lambda(t) - \mu^T(t) H_i) u_i(t), \tag{3.4.0.3}$$

subject to:

$$x_i(t+1) = f_i^t(x_i(t), u_i(t)),\ t = 0, ..., T-1.$$

It maximizes agent i's total net utility, defined as the utility $F_i(x_i(t), u_i(t))$ plus the amount $(\lambda(t) - \mu^T(t)H_i)u_i(t)$ it pays/gets paid for electricity. The optimal cost is a function of the initial condition and the Lagrange multiplier sequence (λ, μ), and we denote it $V_i(x_i(0), \lambda, \mu)$. Therefore,

$$D(\lambda, \mu) = \sum_{i=1}^{N} V_i(x_i(0), \lambda, \mu) + C^T \mu \cdot e_T$$

where $e_T = (1, 1, ..., 1)^T$ is a T-dimensional vector. The term $C^T \mu \cdot e_T$ is commonly referred to as *ISO surplus* [49] or *congestion rent* [50]. When LMPs are different across the grid, the prices paid by wholesale consumers can diverge from the prices paid to generators. The difference between total consumer payments and total seller receipts is a net earnings stream collected and allocated by the ISO. When grids are modeled as lossless, LMP separation only arises in the presence of congestion: when at least one of the branches l_j reaches its capacity c_j (congestion happens) at time t, the corresponding Lagrange multiplier becomes positive with $\mu_j(t) > 0$, resulting in $C^T \mu \cdot e_T > 0$.

Since the dual function can be decomposed by agents, we observe that solving the dual problem leads us to a decentralized problem: The ISO first announces different price vectors to different agents with $(\lambda(t) - \mu^T(t)H_i)$, $t = 1, ..., T$ being sent to agent i. Each agent i optimizes its own objectives (3.4.0.3) by choosing the vector U_i. As a consequence, neither ISO nor the other agents need to know the utilities/states/dynamics of agent i. The dual problem is:

$$\min\ D(\lambda, \mu) \tag{3.4.0.4}$$

subject to

$$\mu(0), \mu(1), ...\mu(T-1) \geq 0.$$

We will assume strong duality holds, i.e., the optimal value of the DA problem and problem (3.4.0.4) are equal. One possible sufficient condition is that the function $\sum_{t=0}^{T-1} F_i(x_i(t), u_i(t))$ is *concave* in the input vector U_i, for $i = 1, ..., N$ and the feasible region of problem DA is nonempty. We require concavity only in the input vector U_i because $x_i(t)$ can be expressed in terms of the inputs $U_i = (u_i(0), ..., u_i(T-1))$ and thus the utility function $\sum_{i=1}^{N} \sum_{t=0}^{T-1} F_i(x_i(t), u_i(t))$ can also be expressed solely as a function of the inputs U_i, $i = 1, ..., N$.

The problem faced by the ISO is how to determine the optimal price vector (λ^*, μ^*) such that $D(\lambda, \mu)$ is minimized. Since $D(\lambda, \mu)$ is convex in λ and μ, we consider the use of subgradient for iterating on the price vector (λ, μ) in order to converge to the optimal (λ^*, μ^*) [51]:

$$\frac{\partial D}{\partial \lambda} = \left(\sum_{i=1}^{N} u_i^{(\lambda,\mu)}(0), \sum_{i=1}^{N} u_i^{(\lambda,\mu)}(1), ..., \sum_{i=1}^{N} u_i^{(\lambda,\mu)}(T-1) \right), \tag{3.4.0.5}$$

$$\frac{\partial D}{\partial \mu(t)} = -(U^{(\lambda,\mu)})^T(t) \cdot H^T + C^T. \tag{3.4.0.6}$$

where $U_i^{(\lambda,\mu)} := \left(u_i^{(\lambda,\mu)}(1), u_i^{(\lambda,\mu)}(2), ..., u_i^{(\lambda,\mu)}(T) \right)$ is the vector that achieves the maximal utility for the i-th agent for the price vector (λ, μ) in (3.4.0.3), $U^{(\lambda,\mu)}(t) := (u_1^{(\lambda,\mu)}(t), u_2^{(\lambda,\mu)}(t),$ $..., u_N^{(\lambda,\mu)}(t))$. Note that agent i will not receive the vector (λ, μ), but rather $(\lambda - \mu^T H_i)$. Here for simplicity of exposition, we denote $U_i^{\left(\lambda+\mu^T H_i\right)}$ as $U_i^{(\lambda,\mu)}$.

We thus obtains the price iteration Algorithm 1.

There are several choices for α_k, and corresponding convergence results for the resulting subgradient method [52].

Compared with the single-period scheme, our multi-period formulation achieves a higher social welfare because single-period scheme is mathematically equivalent to a multi-period formulation with additional constraints on $U(t)$, which results in a smaller feasible set.

Algorithm 1: Iterative bidding algorithm for DA problem

$k = 0$; Initialize $(\lambda, \mu)^k$ to some arbitrary value;
repeat

Each agent i solves the problem

$$\max_{U_i} \sum_{t=0}^{T-1} \left[F_i(x_i(t), u_i(t)) + (\lambda(t) - \mu^T(t)H_i)u_i(t)\right],$$

subject to

$$x_i(t+1) = f_i^t(x_i(t), u_i(t)),\ t = 0, ..., T-1.$$

and submit their bids $U_i^{(\lambda,\mu)^k}$, to the ISO.
ISO then updates the price vector, for $t = 0, ..., T-1$

$$\lambda^{k+1} = \lambda^k - \alpha_k \left(\sum_{i=1}^{N} U_i^{(\lambda,\mu)^k}\right),$$

$$\mu(t)^{k+1} = \max\left(\mu(t)^k + \alpha_k\left(HU^{(\lambda,\mu)^k}(t) - C\right), 0\right).$$

$k = k + 1$.
until $(\lambda, \mu)^k$ *converges to* $(\lambda, \mu)^*$;

3.5 Stochastic Case: Real-time Market

In the previous section, the dynamics of the agents (4.3.0.11) were assumed to be deterministic, i.e., system state at the next time instant $t + 1$ was completely determined by the state and input at time t. However, this might be unrealistic when considering the stochastic nature of renewable energy as well as consumer demands, especially in the real-time market context. Let $\omega_i = (\omega_i(1), \omega_i(2), ..., \omega_i(T))$ be the “private" stochastic process affecting only agent i's system via:

$$x_i(t+1) = f_i^t(x_i(t), u_i(t), \omega_i(t)),$$

The stochastic process ω_i is not completely observed by the other agents, and only agent i knows the law, i.e., the probability distribution, of ω_i. The social welfare maximization problem faced by the ISO (RT for real-time):

$$\max \ \mathbb{E} \sum_{i=1}^{N} \sum_{t=0}^{T-1} F_i(x_i(t), u_i(t))$$

subject to

$$\sum_{i=1}^{N} u_i(t) = 0, \text{ for } t = 0, ..., T-1,$$

$$HU(t) \leq C, \text{ for } t = 0, ..., T-1,$$

$$x_i(t+1) = f_i^t(x_i(t), u_i(t), w_i(t)), \text{ for } t = 0, ..., T-1.$$

If the goal of the ISO is to optimize social welfare over all decentralized policies, then the ISO needs to play a more active role in order to induce cooperation among agents: the ISO needs to know states/utility/dynamics of each individual agent i as well as the probability distribution of its privately observed process ω_i. Then the ISO could use dynamic programming to decide the optimal $U(t)$ for each t, as a function of the states of the entire system. As is well known, this method suffers from the curse of dimensionality as N increases. An optimal decentralized solution to the RT problem however, remains an open problem and [34] presents two approximation algorithms with reduced complexities.

In order to simplify the algorithm and at the same time maintain the iterative bidding structure, we consider the special case of the RT problem when all agents have linear dynamics, with Gaussian noises, and quadratic costs. The noises ω_i are i.i.d. Gaussian random variables with mean zero. Each agent i has a quadratic utility: $F_i(x_i(t), u_i(t)) = q_i x_i(t)^2 + r_i u_i(t)^2$ with $q_i \leq 0$ and $r_i < 0$. We have the constrained LQG (CLQG) problem:

$$\max \ \mathbb{E} \sum_{i=1}^{N} \sum_{t=0}^{T-1} \left(q_i x_i(t)^2 + r_i u_i(t)^2 \right)$$

subject to

$$\sum_{i=1}^{N} u_i(t) = 0, \text{ for } t = 0, ..., T-1, \tag{3.5.0.1}$$

$$HU(t) \leq C, \text{ for } t = 0, ..., T-1, \tag{3.5.0.2}$$

$$x_i(t+1) = a_i x_i(t) + b_i u_i(t) + \omega_i(t), \text{ for } t = 0, ..., T-1.$$

We will assume the same information sharing structure as in the DA problem: system dynamics given by (a_i, b_i) and cost functions given by (q_i, r_i) are all private to agent i and communications between ISO and agents are restricted to ISO announcing a sequence of prices and agents returning back with supply/consumption bids in response to prices.

Similar to the bidding solution of the DA problem, we propose an iterative bidding scheme for the CLQG problem: At time s, ISO first declares a price vector $(\lambda(t) - \mu^T(t)H_i)$ for agent i for times $t \geq s$. Agent i responds back with $u_i^{(\lambda,\mu)}$ for $t \geq s$. That is, at time s, each agent bids a vector of future supply/consumption in responses to future prices announced by the ISO, and ISO updates the prices in return, until convergence.

The key to showing the existence of such simple bidding scheme lies in the *certainty equivalence* property of unconstrained LQG systems [36]: A stochastic control problem is said to posses the property of certainty equivalence if the optimal policy for the stochastic control problem coincides with the optimal policy for the corresponding deterministic control problem in which the noise is absent. However, we need to be careful applying the property because certainty equivalence does not hold for generally constrained LQG problem. Fortunately, constraints (3.5.0.1) and (3.5.0.2) are both linear in U and as [53] and [54] point out, certainty equivalence continues to hold for LQG problems with linear constraints in U. Thus we obtain the iterative Algorithm 2.

The critical feature of Algorithm 2 is that there is an iteration of bids in response to future prices at each time s and once the price converges, the agents implement the control at only the *first* time instant.

It should be noted that since the current optimal supply/consumption depends on future prices, iteration of price at only one time instant is not sufficient to guarantee optimal decision when agents

Algorithm 2: Iterative bidding algorithm for CLQG problem

for *time* $s = 0$ **to** $T - 1$ **do**

$k = 0$; Initialize $(\lambda, \mu)^k$ for $t \geq s$ to some arbitrary value;

repeat

Each agent i solves the problem

$$\max \sum_{t=s}^{T-1} \Big[q_i x_i(t)^2 + r_i u_i(t)^2 + (\lambda(t) - \mu^T(t) H_i) u_i(t) \Big],$$

subject to

$$x_i(t+1) = f_i^t(x_i(t), u_i(t)),\ t = 0, ..., T-1.$$

and submit their bids $u_i^{(\lambda,\mu)^k}$, $t \geq s$ to the ISO.

ISO then updates the price vector

$$\lambda^{k+1} = \lambda^k - \alpha_k \left(\sum_{i=1}^{N} U_i^{(\lambda,\mu)^k} \right),\ t \geq s.$$

$$\mu(t)^{k+1} = \max \Big(\mu(t)^k + \alpha_k (H U^{(\lambda,\mu)^k}(t) - C), 0 \Big),\quad t \geq s.$$

$k = k + 1$.

until $(\lambda, \mu)^k$ *converges to* $(\lambda, \mu)^*$;

Implement $U^*(s)$

end

are dynamic systems. Please see Section 3.6 for detailed discussion.

3.6 Numerical Results

We illustrate the iterative Algorithms 1 and 2 by simple examples. We start with a deterministic case, followed by a stochastic case. We consider the 30-bus system detailed in [55], with generator data given in Appendix D.4 of [56] that includes data on the generators' cost coefficients and ramp rate limits. There are 9 generators and 21 LSEs in the system, which we assume are located at 30 distinct buses.

3.6.1 Deterministic dynamic case: An example

Without loss of generality, we assume linear dynamic systems and quadratic concave utilities for agents in the following example. For LSE i, we adopt the virtual battery model [57] and let $x_i(t)$ denote the state of charge (SOC) at time t that evolves as,

$$x_i(t+1) = \alpha_i x_i(t) + \beta_i h_i(t) - \gamma_i u_i(t),$$

where $h_i(t)$ denotes ambient heating (ambient temperature forecast) and $0 \leq x_i(t) \leq 1$. For a supplier i, $x_i(t)$ denotes the accumulated power production up to time t, and $u_i(t)$ denotes the power production level at time t. State $x_i(t)$ evolves as,

$$x_i(t+1) = \alpha_i x_i(t) + u_i(t),$$

$$\underline{u}_i \leq u_i(t) \leq \bar{u}_i, \text{ for } t = 0, ..., T-1.$$

$$-r_i \leq u_i(t+1) - u_i(t) \leq r_i, \text{ for } t = 0, ..., T-2,$$

where $\underline{u}_i$ and $\bar{u}_i$ are the minimum and maximum power production levels, respectively, and r_i is the maximal ramp rate allowed. For LSE i, let utility

$$F_i(x_i(t), u_i(t)) = -\left(x_i(t) - \frac{1}{2}(\phi_{1i} + \phi_{2i})\right)^2 + m_i,$$

where $[\phi_{1i}, \phi_{2i}]$ is the i-th LSE's "desired SOC range" and m_i's are constants. For suppliers, the one-step utility function is

$$F_i(x_i(t), u_i(t)) = -\left(a_i u_i^2(t) + b_i u_i(t) + c_i\right)$$

For renewable generation and uncontrollable loads, we set $F_i \equiv 0$ and let $u_i(t)$ be the forecast value. Data is from ERCOT [58].

To be consistent with the day-ahead market context, let each time interval stand for 1 hour and let $T = 24$. We use QCQP (Quadratic Constrained Quadratic Programming) to solve each agent's individual optimal control problem (3.4.0.3). We first observe that when $h_i(t)$'s are small, no branch constraints are binding. This results in LMP's being identical across all buses. When we steadily increase $h_i(t)$ at bus 3, time $t = 4$, and hold other $h_i(t)$'s constant for all t, branches connected to this load (branch no. 2 and 4) become congested, resulting in LMP at bus 3 at time $t = 4$ being higher than LMP's at other buses at $t = 4$. Similar results are obtained when increasing $h_i(t)$ at bus 19 for $t = 6$. To better demonstrate the performance of the algorithm, we thus choose $h_3(t)$, $h_{19}(t)$ and $h_{10}(t)$ as in Fig. 3.1 such that there are 4 branches that are congested at least once in the 24-hour optimization window.

Fig. 3.2 plots the evolution of the price vector (λ, μ) for $t = 14$, where for simplicity of notation, from now to the end of the section, we use $\lambda_i(t)$ to denote the true LMP $\lambda(t) - \mu^T(t)H_i$ for agent i at time t. We see that the LMP for both congested buses and the uncongested bus converge quickly, in less than 15 iterations.

Fig. 3.3 shows the converged LMPs at different locations. LMPs at buses with relatively high ambient temperature are higher than those at buses with relatively low temperature, and all LMPs are positively correlated with ambient temperature (demand).

We compare our multi-period formulation with a single-period formulation. Since current supply/consumption depends on future prices, iteration at only one time instant is not sufficient to guarantee optimal solutions when agents have dynamic systems. We thus want to compare

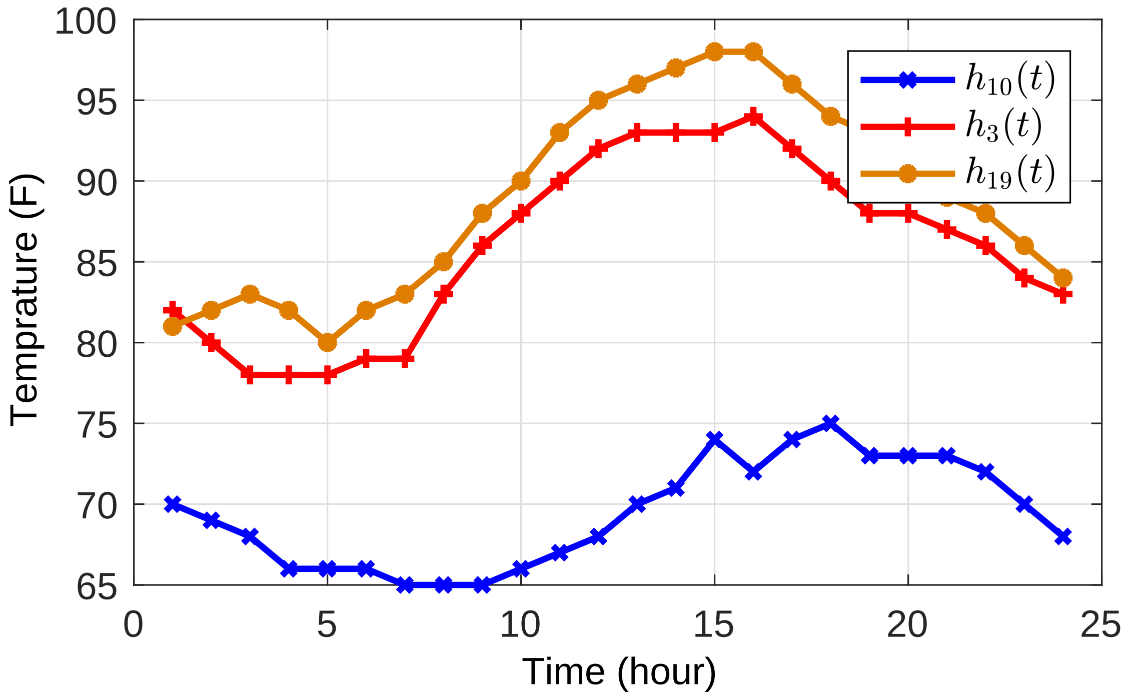

Figure 3.1: Ambient temperature

total utility obtained by our iterative procedure (on vector of prices) to total utility obtained by the single-period scheme. To simulate the single-period scheme, we repetitively run algorithm 1 with $T = 1$ and let initial state at time $t + 1$ equal to the final state at time t. Fig. 3.4 (additional temperature data is obtained from [59]) shows that in general, utility obtained by algorithm 1 is approximately 10% higher than utility obtained by the single-period scheme. We impose a steady state constraint $x_i(0) = x_i(T)$ in order for this optimization problem to be repetitively used. This results in a sub-optimal, or approximate value of the true maximal utility as $T \to \infty$. We thus calculate the total utility achieved by setting $T = 720$ (one month), and compare with utilities achieved by both Algorithm 1 and the single-period scheme. Results are summarized in Fig. 3.5. The differences in total utility between the cases having $T = 24$ and $T = 720$ are small compared to the differences between the $T = 1$ and $T = 24$ cases. Consequently, Fig. 3.5 demonstrates that iteration over a finite time horizon, though not optimal, attains higher total social welfare than the single-period scheme.

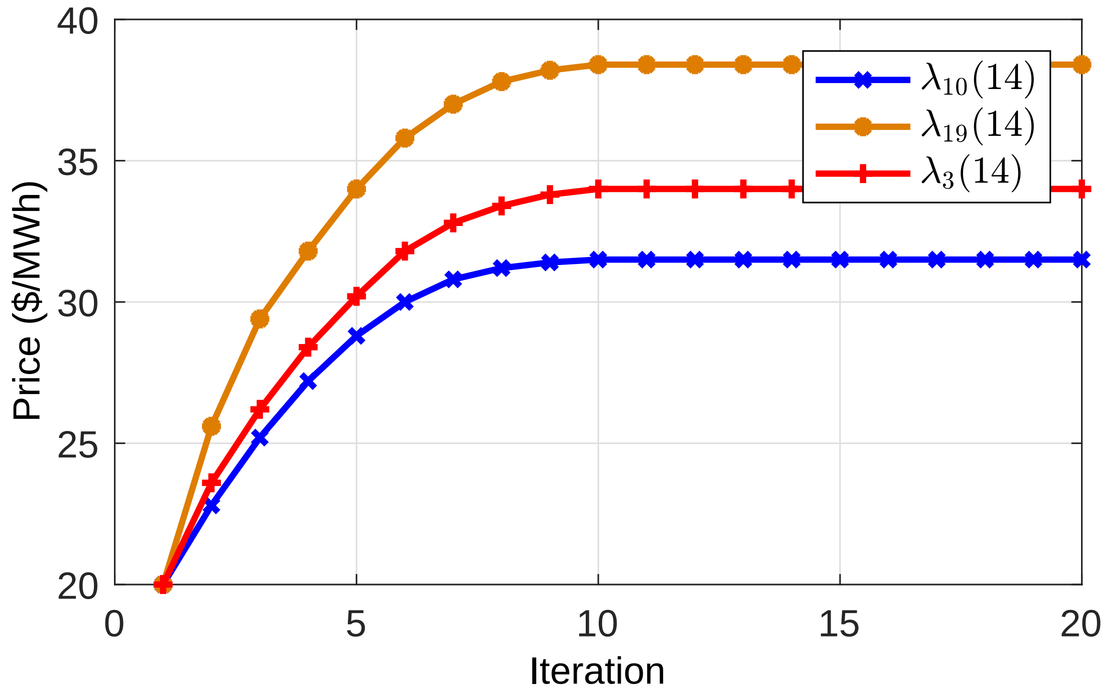

Figure 3.2: Convergence of LMPs at time $t = 14$

3.6.2 Stochastic case

We adopt the same notations as in the deterministic case, but modify the state equations by adding a Gaussian random variable ω_i incorporating the availability of renewables or stochasticity of demand. For simplicity we let $\omega_i(t) \sim \mathcal{N}(0, \sigma_i^2)$. To be consistent with the setting of real-time market, we let each time interval to be 15-minute and let $T = 8$.

The inner loop of Algorithm 2 is similar to Algorithm 1 and thus, rapid convergence of the price vector can be expected. It is shown in [60] that since the implementation of day-ahead market, the convergence between day-ahead prices and real-time LMPs has been narrowing in PJM market. Fig. 3.6 shows a similar trend as real-time prices $\lambda^R(t)$ track the day-ahead prices $\lambda^D(t)$ (with a time scale of 15-minute, day-ahead hourly prices are step functions) for both congested and uncongested buses.

In the presence of noise, it can be expected that the variance of prices $\lambda(t)$ is higher than the variance of prices in day-ahead market. It is shown in Fig. 3.7 that as variance of noise σ_i^2

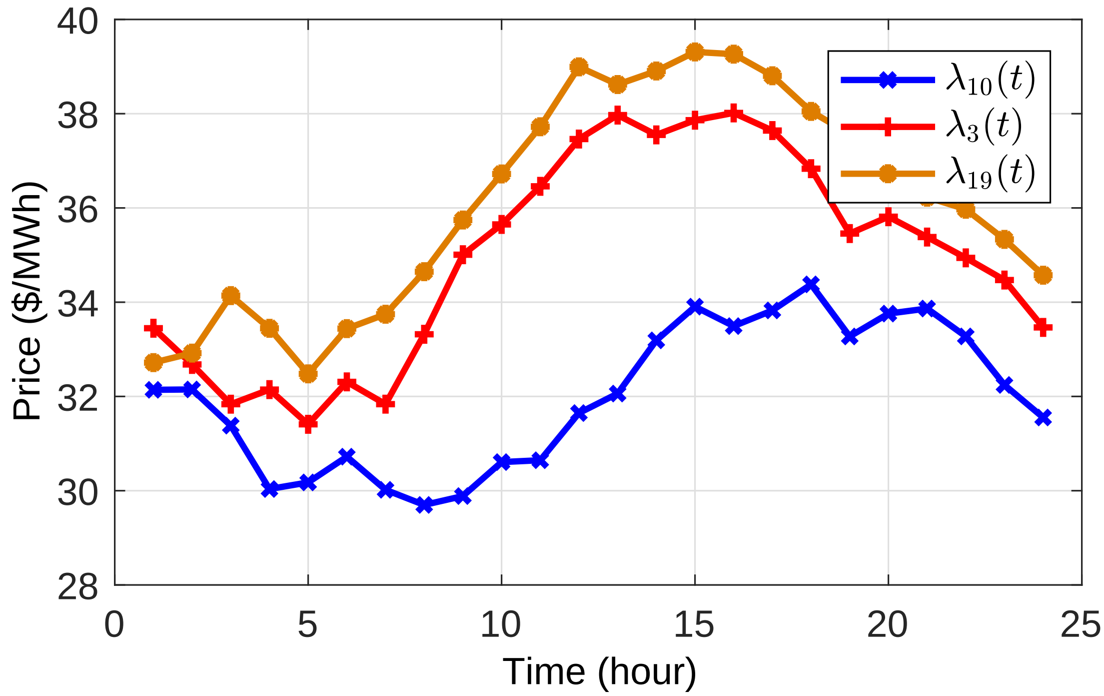

Figure 3.3: LMPs at different locations

increases, variance of LMPs obtained by Algorithm 2 and LMPs obtained by the single-period scheme both increase, and variance of congested LMPs increases faster than that of uncongested LMPs. However, compared to the variance of LMPs obtained by the single-period scheme, variance of LMPs determined by Algorithm 2 is smaller in both congested and uncongested cases. We also observe the change in total utility (over 4 hours) while increasing the variance of noise in Fig. 3.8.

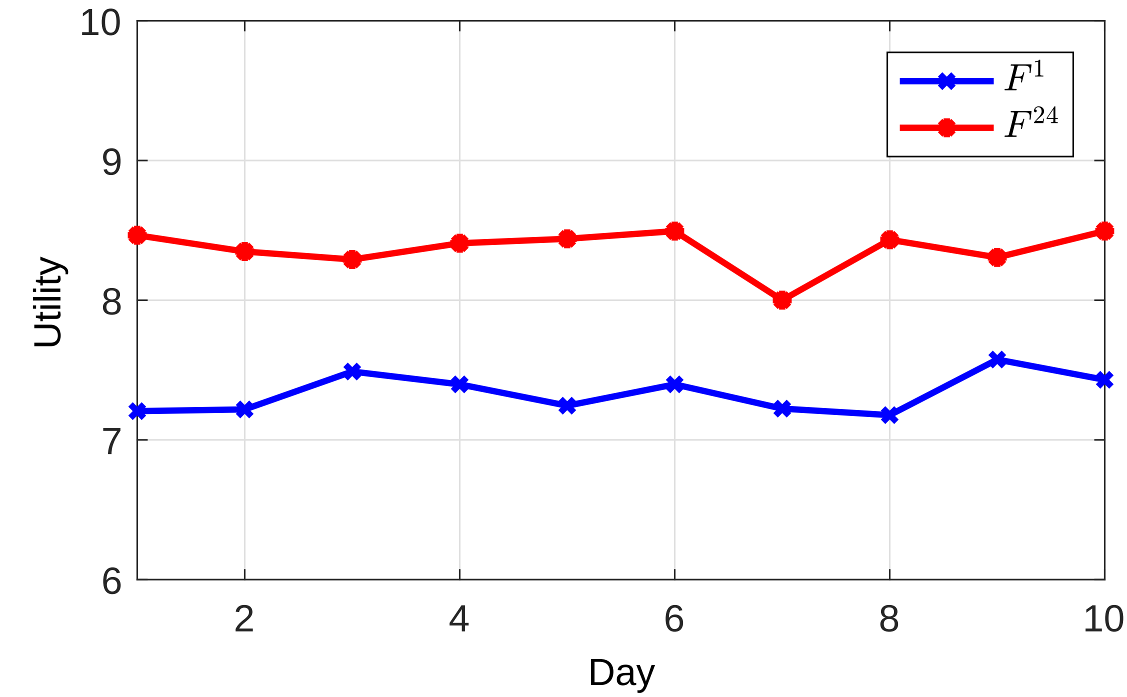

Figure 3.4: Comparison of total utility per day

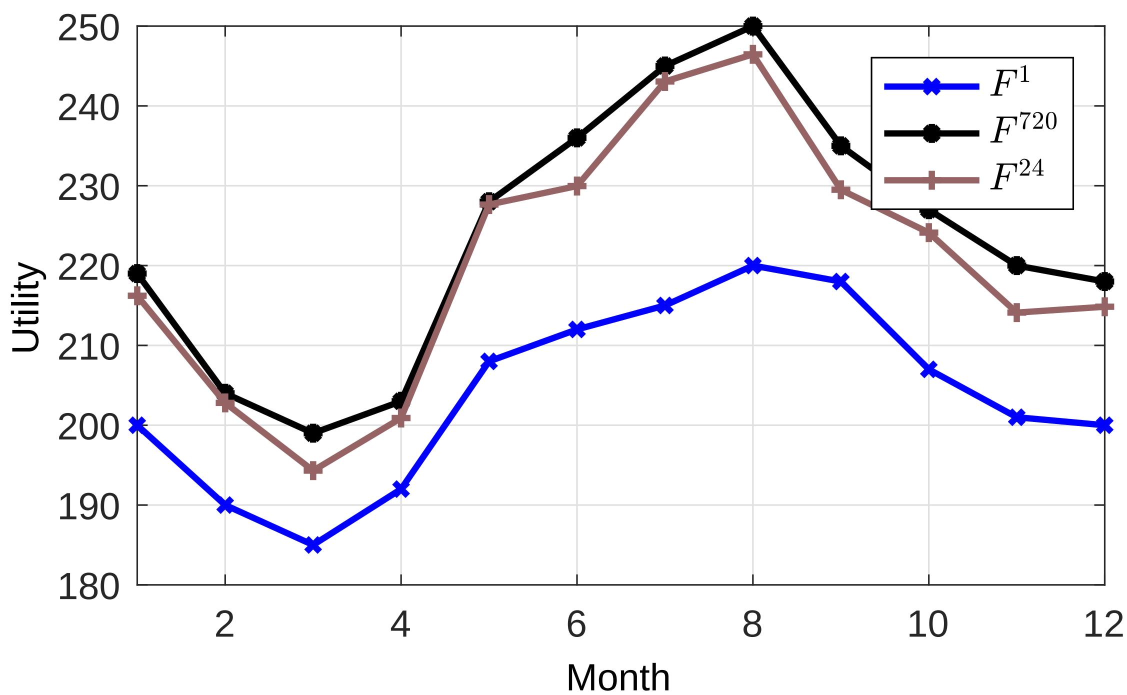

Figure 3.5: Comparison of total utility per month

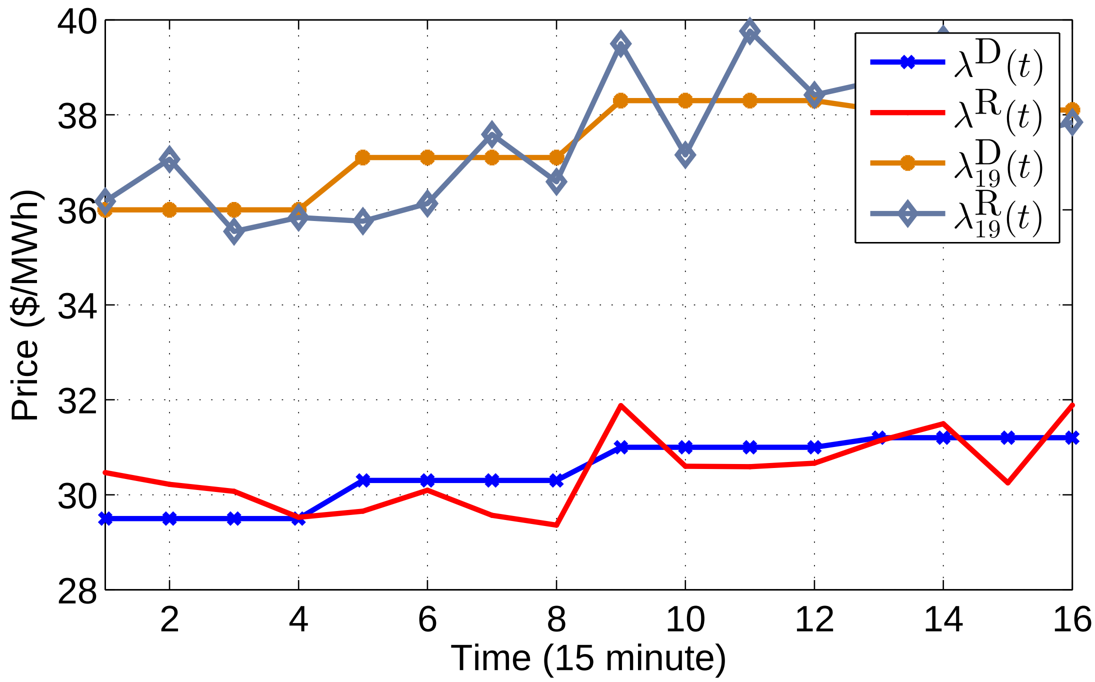

Figure 3.6: Comparison of Day-ahead LMP and Real-time LMP

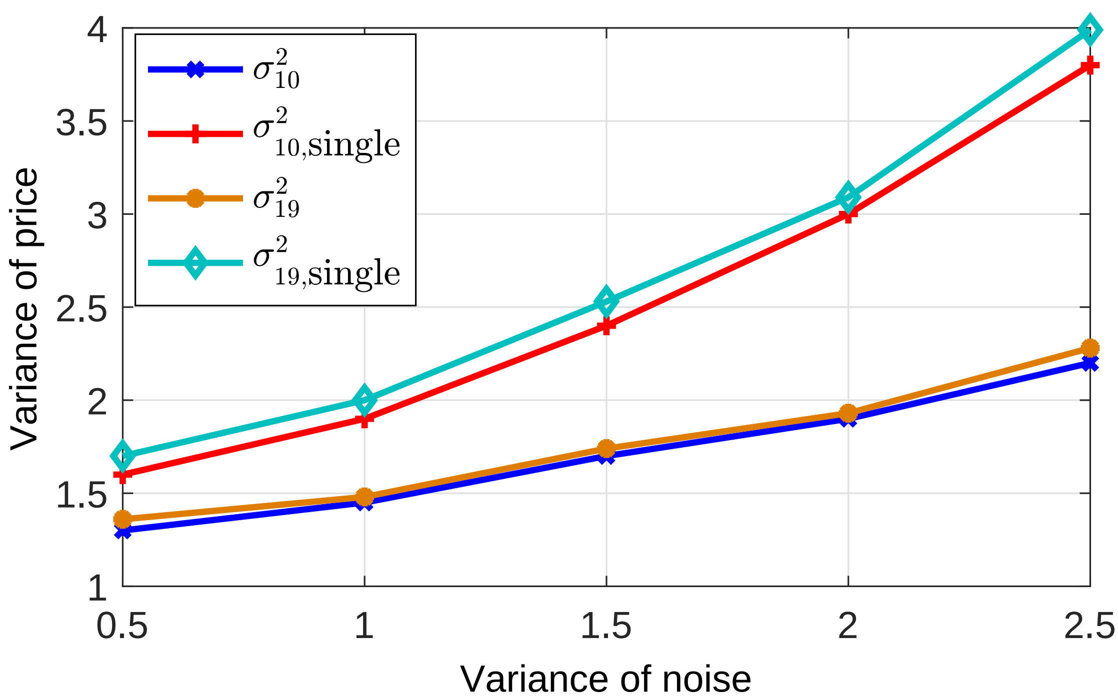

Figure 3.7: Variance of prices

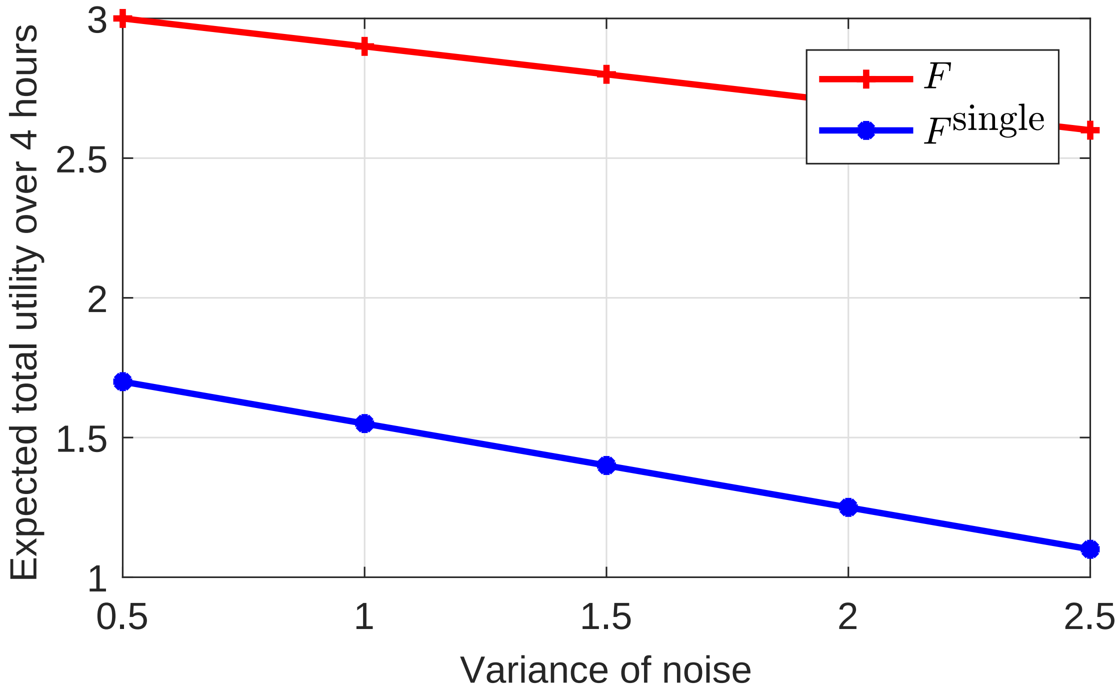

Figure 3.8: Changes in total utility as variance of noise varies

4. INCENTIVE COMPATIBILITY IN STOCHASTIC SYSTEMS

4.1 Introduction

Mechanism design is the sub-field of game theory that considers how to implement socially optimal solutions to problems involving multiple self-interested agents, each with a private utility function. A typical approach in mechanism design is to provide financial incentives such as payments to promote truth-telling of utility function parameters by agents. An important example is the Independent System Operator (ISO) problem of electric power systems in which the ISO aims to maximize social welfare and maintain balance of generation and consumption while each generator/load has a private utility function.

The classic Vickery-Clarke-Groves (VCG) mechanism [2] has played a central role in classic mechanism design since it ensures incentive compatibility, i.e., truth-telling of utility functions of all agents forms a dominant strategy, as well as social welfare optimality, i.e., the sum of utilities of all agents is maximized. The outcome generated by the VCG mechanism is stronger than a Nash equilibrium in the sense that it is *strategy-proof*, meaning that truth-telling of utility functions is optimal irrespective of what others are bidding. In fact, Green, Laffont and Holmstrom [61] show that VCG mechanisms are the *only* mechanisms that are both efficient and strategy-proof if payoffs are quasi-linear, i.e., linear in the amount of money.

While the VCG mechanism is applicable to a static one-shot game, it does not work for stochastic dynamic games. In a stochastic dynamic environment that unfolds over time, the agents' intertemporal payoffs depend on the expected future controls and payments, and a direct extension of the VCG mechanism does not guarantee incentive compatibility. A fundamental difference between dynamic and static mechanism design is that in the former, an agent can bid an untruthful utility function conditional on its past bids (which need not be truthful) and past allocations (from which it can make an inference about other agents' utility functions). For dynamic *deterministic* systems, by collecting the VCG payments as a lump sum of all the payments over the entire time horizon at the beginning, incentive compatibility is still assured. However, for a dynamic *stochastic* system, the states are private random variables and it is necessary to incentivize agents to bid their states truthfully. However, it does not appear to be feasible to construct mechanisms that ensure the dominance of dynamic truth-telling for agents comprised of general stochastic dynamic systems.

Nevertheless, for the special case of Linear-Quadratic-Gaussian (LQG) agents, where agents have linear state equations, quadratic utility functions and additive white Gaussian noise, we show that a dynamic stochastic extension of the VCG mechanism does exist, based on a careful construction of a sequence of layered payments over time. We propose a modified layered mechanism for payments that decouples the intertemporal effect of current bids on future payoffs, and prove that truth-telling of dynamic states forms a dominant strategy if system parameters are known and agents are rational. “Rational" means that an agent will adopt a dominant strategy if it is the unique one, and it will act on the basis that it and others will do so at future times.

An important example of a problem needing such optimal dynamic coordination of stochastic agents arises in the ISO problem of power systems. Renewable energy resources such as solar/wind are stochastic and dynamic in nature, as are consumptions by loads which are influenced by factors such as local temperatures and thermal inertias of facilities. In general, agents may have different approaches to responding to the prices set by the ISO. If each agent acts as a *price taker*, i.e., it honestly discloses its energy consumption at the announced prices, a *competitive equilibrium* would be reached among agents. However, if agents are *price anticipators*, then it is critical for the ISO to design a market mechanism that is strategy-proof (i.e., incentive compatible). The challenge for the ISO is to determine a bidding scheme between agents (producers and consumers) and the ISO that maximizes social welfare, while taking into account the stochastic dynamic models of agents. Currently, the ISO solicits bids from generators and Load Serving Entities (LSEs) and operates two markets: a day-ahead market and a real-time market. The day-ahead market lets market participants commit to buy or sell wholesale electricity one day before the operating day, to satisfy energy demand bids and to ensure adequate scheduling of resources to meet the next day’s anticipated load. The real-time market lets market participants buy and sell wholesale electricity during the course of the operating day to balance the differences between day-ahead commitments and the actual real-time demand and production [31]. Our layered VCG mechanism fits perfectly in the real-time market, as we will see in the sequel.

However, there is also a fatal downside for the VCG mechanism: in general, the sum of total payments collected by the ISO may be negative. In fact, when agents have quadratic utility functions, the total payments collected from consumers is indeed not enough to cover the total payments to the suppliers. In effect, in order to force agents to reveal their true utility functions, the ISO needs to subsidize the market. In this chapter we will also propose a solution to this problem. The VCG payment charges each agent i the difference between social welfare of others if agent i is absent and social welfare of others when agent i is present. In this chapter we will exhibit a solution for budget balance which consists of inflating the first term above in all the agents' VCG payments by a constant factor c. We argue that based on historic knowledge of the market, the ISO may be able to choose such a c that does not depend on any agent's tactical announcement. There are several additional issues to be addressed when proposing such a scheme. These concern the issue of individual rationality, and whether the solution is indeed Lagrange optimal for each agent. The magnitude of c is important; if the constant number c is chosen to be too large, an agent may simply opt out of the whole process and not even join the market. That is, the scheme is not individually rational. Moreover, even if a customer participates, the price and the net utility it obtains, which is the utility of energy consumption minus the amount it pays, need not be Lagrange optimal. We show that there is indeed a systematic way to choose this number c such that there is no budget deficit for the ISO, while at the same time guaranteeing that producers and consumers will actively participate in the market. Moreover, c can be chosen in a way such that the distortion between the VCG payment and Lagrange payment is minimized in the worst case scenario.

The rest of the chapter is organized as follows. In Section 4.2, a survey of related works is presented. This is followed by a description of the classic VCG framework for the static and dynamic deterministic problem and the corresponding modified SVCG mechanism in Section 4.3.

A layered SVCG payment scheme is introduced for the dynamic stochastic problem in Section 4.4.

4.2 Related Works

In recent years, several papers have been written with the aim of exploring issues arising in dynamic mechanism design. In order to achieve ex post incentive compatibility, Bergemann and Valimaki [62] propose a generalization of the VCG mechanism based on the marginal contribution of each agent and show that ex post participation constraints are satisfied under some conditions. Athey and Segal [63] consider an extension of the d'Aspremont-Gerard-Varet (AGV) mechanism [64] to design a budget balanced dynamic incentive compatible mechanism. Pavan et al. [65] derives first-order conditions under which incentive compatibility is guaranteed by generalizing Mirrlees's [66] envelope formula of static mechanisms. Cavallo et al. [67] considers a dynamic Markovian model and derives a sequence of Groves-like payments which achieves Markov perfect equilibrium. Bapna and Weber [68] solves a sequential allocation problem by formulating it as a multi-armed bandit problem. Parkes and Singh [69] and Friedman and Parkes [70] consider an environment with randomly arriving and departing agents and propose a "delayed" VCG mechanism to guarantee interim incentive compatibility. Besanko et al. [71] and Battaglini et al. [72] characterize the optimal infinite-horizon mechanism for an agent modeled as a Markov process, with Besanko considering a linear AR(1) process over a continuum of states, and Battaglini focusing on a two-state Markov chain. Bergemann and Pavan [73] have an excellent survey on recent research in dynamic mechanism design. A more recent survey paper by Bergemann and Valimaki [74] further discusses the dynamic mechanism design problem with risk-averse agents and the relationship between dynamic mechanism and optimal contracts.

In order to capture strategic interactions between the ISO and market participants, game theory and mechanism design has been proposed in many recent papers. Sessa et al. [75] studies the VCG mechanism for electricity markets and derives conditions to ensure collusion and shill bidding are not profitable. Okajima et al. [76] propose a VCG-based mechanism that guarantees incentive compatibility and individual rationality for day-ahead market with equality and inequality constraints. Xu et al. [77] shows that the VCG mechanism always results in higher per-unit electricity prices than the locational marginal price (LMP) mechanism under any given set of reported supply curves, and that the difference between the per-unit prices resulting from the two mechanisms is negligibly small. Bistarelli et al. [78] derives a VCG-based mechanism to drive users in shifting energy consumption during peak hours. In Samadi et al. [79], it is proposed that utility companies use VCG mechanism to collect private information of electricity users to optimize the energy consumption schedule.

There are also some related works aiming at achieving budget balance for VCG mechanism. Moulin et al. [80] discusses the trade-off between budget balance and efficiency of the mechanism. Cavallo [81] uses domain information regarding agent valuation spaces to achieve redistribution of much of the required transfer payments back among the agents. Similarly, Thirumulanathan et al. [82] propose a mechanism that is efficient and comes close to budget balance by returning much of the payments back to the agents in the form of rebates. In [83], an enhanced (Arrow-dAspremont-Gerard-Varet) AGV mechanism is proposed to tackle the problem of budget balance in demand side management.

To our knowledge, there does not appear to be any result that ensures dominance of dynamic truth-telling for agents comprised of LQG systems, let alone ensuring budget balance of the ISO and individual rationality for all agents.

4.3 The Static and Dynamic Deterministic VCG

Let us begin by considering the simpler static deterministic case. Suppose there are N agents, with each agent having a utility function $F_i(u_i)$, where u_i is the amount of energy produced/consumed by agent i. We will use the convention that $u_i \leq 0$ for a producer and $u_i \geq 0$ for a consumer. $F_i(u_i)$ depends only on its own consumption/generation u_i. However, for convenience of notation, we will occasionally abuse notation and write $F_i(u)$ with the implicit understanding that it only depends on the i-th component u_i of u.

Let $\boldsymbol{u} := (u_1, ..., u_N)^T$, $\boldsymbol{u}_{-i} := (u_1, ..., u_{i-1}, u_{i+1}, ..., u_N)^T$, and let $F := (F_1, \ldots, F_n)$.

In a power system the total energy generated must equal to the total consumed, i.e., $\sum_i u_i = 0$. An independent system operator which does not know the utility functions of the agents, wishes to maximize the social welfare. The social welfare is defined as $\sum_i F_i(u_i)$. Hence it seeks to solve the following optimization problem and assign the resulting generations/consumptions to the agents: $\max_{u_1,u_2,\ldots,u_N} \; F_i(u_i)$, subject to $\sum_i u_i = 0$.

However, as noted above the ISO does not know the individual utility functions of the agents. If it asks them to disclose their utility functions they may lie in order to obtain a better allocation. A solution to this problem of “truth-telling" is provided by the VCG mechanism. In the VCG mechanism, each agent is asked to bid its utility function. Let us denote its bid by $\hat{F}_i$. The agent can lie, so $\hat{F}_i$ may not be equal to F_i. We denote $\hat{F} := (\hat{F}_1, \ldots \hat{F}_n)$. After obtaining the bids, the ISO calculates $\boldsymbol{u}^*(\hat{F})$ as the optimal solution to the following problem:

$$\max_{\boldsymbol{u}} \sum_i \hat{F}_i(u_i)$$

subject to

$$\sum_i u_i = 0.$$

Each agent is then assigned to produce/consume $u_i^*(\hat{F})$, and is obliged to do so, accruing a utility $F_i\left(u_i^*\left(\hat{F}\right)\right)$. Following the rule that it has announced a priori before receiving the bids, the ISO collects a payment $p_i(\hat{F})$ from agent i, defined as follows:

$$p_i(\hat{F}) := \sum_{j \neq i} \hat{F}_j(\boldsymbol{u}^{(i)}) - \sum_{j \neq i} \hat{F}_j(\boldsymbol{u}^*(\hat{F})),$$

where $\boldsymbol{u}^{(i)}$ is defined as the optimal solution to the following problem:

$$\max_{\boldsymbol{u}_{-i}} \sum_{j \neq i} \hat{F}_j(u_j)$$

subject to

$$\sum_{j \neq i} u_j = 0.$$

We can see that p_i is the cost to the rest of the agents due to agent i's presence, which leads agents to internalize the social externality.

In fact, the VCG mechanism is a special case of the Groves mechanism [84], where payment p_i is defined as:

$$p_i(\hat{F}) = h_i(\hat{\boldsymbol{F}}_{-i}) - \sum_{j \neq i} \hat{F}_j(\boldsymbol{u}^*(\hat{F})).$$

where h_i is any arbitrary function of $\hat{F}_{-i} := (\hat{F}_1, .., \hat{F}_{i-1}, \hat{F}_{i+1}, ..., \hat{F}_N)$. Truth-telling is a dominant strategy in the Groves mechanism [84]. That is, regardless of other agents' strategies, an agent cannot do better than truthfully declaring its utility function.

Theorem 3. *[84] Truth-telling ($\hat{F}_i \equiv F_i$) is the dominant strategy equilibrium in Groves mechanism.*

Proof. Suppose agent i announces the true utility function F_i. Let $\bar{F} := (\hat{F}_1, ...\hat{F}_{i-1}, F_i, \hat{F}_{i+1}, ..., \hat{F}_N)$ and $\bar{F}_{-i} := (\hat{F}_1, ...\hat{F}_{i-1}, \hat{F}_{i+1}, ..., \hat{F}_N)$. Let $\bar{F}(u) := \sum_i \bar{F}_i(u_i)$. Let $\bar{u}_i^*$ be what ISO assigns, and $p_i(\bar{F})$ be what ISO charges, when $\bar{F}$ is announced by the agents. Let u_i^* be what ISO assigns and $p_i(\hat{F})$ be what ISO charges when $\hat{F}$ is announced by agents.

Note that $\bar{F}_{-i} = \hat{F}_{-i}$, and so $h_i(\bar{F}_{-i}) = h_i(\hat{F}_{-i})$. Hence for agent i, the difference between the net utilities resulting from announcing F_i and $\hat{F}_i$ is

$$\begin{aligned} &\Big[F_i(\bar{u}_i^*) - p_i(\bar{F})\Big] - \Big[F_i(u_i^*) - p_i(\hat{F})\Big] \\ =&F_i(\bar{u}_i^*) - h_i(\bar{F}_{-i}) + \sum_{j \neq i} \hat{F}_j(\bar{u}_i^*) - F_i(u_i^*) + h_i(\hat{F}_{-i}) \\ &- \sum_{j \neq i} \hat{F}_j(u_i^*) = \bar{F}(\bar{u}^*) - \bar{F}(u^*) \geq 0, \end{aligned}$$

where the last inequality holds since $\bar{u}^*$ is the optimal solution to the social welfare problem with utility functions $\bar{F}$. □

Definition 1. We call a mechanism *incentive compatible (IC)* if truth-telling is a dominant strategy for agents.

One should note that an agent may not necessarily tell the truth even if truth-telling is dominant since there may be another strategy that is also dominant. However, we assume that the agent is "rational," in that if the dominant strategy is unique, then the agent will indeed tell the truth.

Definition 2. We call a mechanism *efficient (EF)* if the resulting allocation $\boldsymbol{u}^*$ maximizes the social welfare $\sum_i F_i(u_i)$.

It is seen that the VCG mechanism is IC, and if all agents declare their utility functions truthfully, then the VCG mechanism is also EF.

In addition to choosing a strategy that maximizes social welfare, there are two more important properties that are sought in a solution.

Definition 3. A mechanism is *individually rational (IR)* if agents actively participate in the mechanism, which they will do if they can gain a nonnegative net utility by participating, that is, $F_i(u_i^*) - p_i \geq 0$.

Definition 4. A mechanism satisfies *budget balance (BB)* if the total payment made by agents is nonnegative: $\sum_i p_i \geq 0$. (In our context this means that the ISO does not have to provide a subsidy).

The VCG mechanism, in general, does not satisfy BB. In fact, more generally, Green and Laffont [61] show that no mechanism can satisfy all the four properties (IC, EF, IR & BB) at the same time.

If ISO knew the true utility functions of all the agents, it could solve the social welfare problem in a centralized manner: calculate the Lagrange multiplier λ^* (price), and collect a payment that equals to $\lambda^* u_i^*$ from agent i. We call this payment naturally defined by the Lagrange multiplier, in the absence of strategic considerations, as the *Lagrange payment* and have the following definition:

Definition 5. If the optimal solution (λ^*, u^*) is unique, we call a mechanism *Lagrange Optimal* if the payment p_i collected from agent i is equal to the Lagrange payment $\lambda^* u_i^*$.

We need to somehow overcome the difficulty recognized by Green and Laffont [61] noted above. We will show in the sequel that while there is no mechanism that satisfies all four properties (IC, EF, IR and BB) *in general*, there does exist such a mechanism under a “market power balance" condition.

In order to satisfy IC, EF, IR and BB at the same time, we inflate the first term in the standard VCG mechanism by a *constant* factor c:

$$p_i(\hat{F}) = c \cdot \sum_{j \neq i} \hat{F}_j(\boldsymbol{u}^{(i)}) - \sum_{j \neq i} \hat{F}_j(\boldsymbol{u}^{*}). \tag{4.3.0.1}$$

We call the VCG mechanism with the above payment structure as a Scaled VCG (SVCG) mechanism, and c as the scaling factor. To achieve BB and IR, one could choose c as a function of the utility bids $\hat{F}_i$, which unfortunately would cease to guarantee incentive compatibility since the first term in (4.3.0.1) is not allowed to be dependent on $\hat{F}_i$ in the Groves mechanism.

We will show below that there is a range of values of c that can ensure BB, and argue that through its long-term operation, the ISO may be able to learn at least a subset of this range of values of c which ensure that all four properties hold. Our presumptive argument rests on the repetitive nature of this problem which is played out every day, allowing the ISO to be able to tune c to avoid a net subsidy. Based on this experience, the ISO could choose a c for which BB and IR hold at the same time. We show in the following theorem that under a certain market power balance condition, it is possible to find such a range of values for c.

As we have shown above, truth-telling is a dominant strategy under the Groves mechanism. In this dominant strategy equilibrium, every agent i will announce its true utility function F_i if that is the unique dominant solution.

Theorem 4. *Let $\boldsymbol{u}^*$ be the optimal solution to the following problem:*

$$\max \sum_i F_i(u_i), \text{ subject to } \sum_i u_i = 0,$$

and suppose that u^* is unique. We will also suppose that $\mathbf{u}^{(i)}$ is the unique optimal solution to the following problem:

$$\max \sum_{j \neq i} F_j(u_j), \textit{ subject to } \sum_{j \neq i} u_j = 0.$$

Let $H_i := \sum_{j \neq i} F_j(\boldsymbol{u}^{(i)})$, and let $H_{max} = \max_i H_i$. Let $F^* = \sum_j F_j(\boldsymbol{u}^*)$. If $F^* > 0$, $H_i > 0$ for all i, and the following Market Power Balance (MPB) condition holds:

$$(N-1)H_{max} \leq \sum_i H_i, \tag{4.3.0.2}$$

then there exists an interval $[\underline{c}, \bar{c}]$ such that for any c chosen in this interval, the SVCG mechanism satisfies IC, EF, BB and IR at the same time.

Proof. With c chosen as a constant, the SVCG mechanism is within the Groves class and thus satisfies IC and EF. To achieve budget balance, we need

$$\sum_i p_i = c \sum_i H_i - (N-1)F^* \geq 0,$$

or equivalently, we need to have,

$$c \geq \frac{(N-1)F^*}{\sum_i H_i}.$$

To achieve individual rationality for agent i, we also need

$$F_i(\boldsymbol{u}^*) - p_i = F_i(\boldsymbol{u}^*) - c \cdot H_i + \sum_{j \neq i} F_i(\boldsymbol{u}^*) \geq 0,$$

or equivalently, we need to have,

$$c \leq \frac{F^*}{H_i}.$$

Combining both the inequalities, we need to be able to choose a c such that

$$\frac{(N-1)F^*}{\sum_i H_i} \leq c \leq \frac{F^*}{H_{max}}. \tag{4.3.0.3}$$

Let $\underline{c} := \frac{(N-1)F^*}{\sum_i H_i}$, and $\bar{c} := \frac{F^*}{H_{max}}$. Such a c exists if

$$(N-1)H_{max} \leq \sum_i H_i,\ F^* > 0,\ H_i > 0 \text{ for all } i.$$

□

The critical condition in the above theorem is (4.3.0.2), which states that no agent that has significantly bigger or smaller market power than others. Individual residential load customers generally have a much smaller scale compared to power plant, and it is thus beneficial to form load aggregators or utility companies at the consumer side as suggested by the SVCG mechanism. This provides an economic justification for the role of load aggregators or load serving entities that guarantee the achievement of social welfare maximization.

In general, a SVCG mechanism is however not Lagrange optimal. Within the feasible range $[\underline{c}, \bar{c}]$, one may like to choose a c that also achieves near Lagrange optimality. This could be formulated as the following MinMax problem:

$$\min_c \max_i |d_i(c)|,\ \text{subject to (4.3.0.3)},$$

where $d_i(c) := \lambda^* u_i^* - p_i = \lambda^* u_i^* - c \cdot H_i + \sum_{j \neq i} F_j(\boldsymbol{u}^*)$. The MinMax problem can be transformed to a linear program:

$$\min\ Z$$

subject to

$$Z \geq d_i(c),\ \text{for all } i,$$

$$Z \geq -d_i(c),\ \text{for all } i,$$

$$\frac{(N-1)F^*}{\sum_i H_i} \leq c \leq \frac{F^*}{H_{max}}.$$

In the absence of knowledge of the utility functions, perhaps the market experience could guide the ISO over time to an appropriate choice. Shortly, we will revisit this problem from a different, asymptotic, point of view. We illustrate the MinMax problem with a numerical example below.

Example 1. All agents have quadratic utility functions: $F_i = r_i u_i^2 + s_i u_i$. $(r_1, r_2, r_3, r_4) = (-1, -1.1, -1.2, -1.1)$ and $(s_1, s_2, s_3, s_4) = (1, 1.2, 4, 5)$. The unique Lagrange optimal solution is $\boldsymbol{u}^* = (-0.86, -0.70, 0.53, 1.03)$, $\lambda^* = 2.73$, and from (4.3.0.3), $1.13 \leq c \leq 1.19$. The optimal solution to the MinMax problem is $(c^*, Z^*) = (1.14, 0.22)$. Thus, by choosing $c = 1.14$, the SVCG mechanism satisfies IC, EF, BB and IR, and the maximum discrepancy between VCG payment and Lagrange payments is 0.22.

In the MinMax problem, one can also replace $d_i(c)$ by the ratio $d_i(c)/\lambda^* u_i^*$ to normalize the nearness to Lagrange payment by the amount of the payment. It also can be written as an LP. Using the above ratio, the optimal solution is $(c^*, Z^*) = (1.18, 0.06)$, showing that all agents pay/receive within 6% of their Lagrange optimal payment.

Now we consider N heterogeneous agents and show that SVCG payments converge to the Lagrange payment as N increases. Let $F_i(u_i) = a_i u_i^2 + b_i u_i$ be the quadratic utility functions for both suppliers and consumers. Denote by $A = diag(a_1, a_2, ..., a_N)$ the diagonal matrix consisting of all the a_i, $B := [b_1; ...; b_N]$ and $U = [u_1; ...; u_N]$. We suppose $A < 0$. The ISO needs to solve the following problem:

$$\max \; U^T A U + B^T U \tag{4.3.0.4}$$

subject to

$$1^T U = 0. \tag{4.3.0.5}$$

where 1 is the all-one vector of proper size. The solution to this problem is:

$$\lambda^{*N} = \gamma 1^T A^{-1} B, \tag{4.3.0.6}$$

$$U^{*N} = \frac{1}{2} A^{-1} \left(\lambda^{*N} \cdot 1 - B \right). \tag{4.3.0.7}$$

where $\gamma = \left(trace \left(A^{-1} \right) \right)^{-1} = 1^T A^{-1} 1$ and index N is used to keep track of the population size. Note also that the optimal social welfare is $\frac{1}{4} \lambda^2 1^T A^{-1} 1 = \frac{1}{4} \frac{(1^T A^{-1} B)^2}{1^T A^{-1} 1}$.

Theorem 5. *For the SVCG mechanism with quadratic utility functions, if* (a_i, b_i) *satisfy the following:*

1. $\underline{a} \le a_i \le \bar{a} < 0$, $0 < \underline{b} \le b_i \le \bar{b}$,

2. $(N-1)H_{max}(N) \le \sum_i H_i(N),\ F^*(N) > 0,\ H_i(N) > 0$, *where the argument* N *denotes that the corresponding quantity refers to the system with agents* $1, 2, ...N$,

then the following holds:

1. *There exists a* c^N *satisfying:*

$$\frac{(N-1)F^*(N)}{\sum_i H_i(N)} \le c \le \frac{F^*(N)}{H_{max}(N)}.$$

   Moreover, any such c^N *satisfies* $\lim_{N\to\infty} c^N = 1$,

2. $\lim_{N\to\infty}(\lambda^{*N} u_i^{*N} - p_i^N) = 0$, *for all* i.

Proof. Without loss of generality, we provide the proof for the first agent. Let $A_{-1} = diag(a_2, ..., a_N)$, $B_{-1} = [b_2; ...; b_N]$, 1_{-1} be the all-one vector of dimension $M + N - 1$ and $\gamma_{-1} = \left(trace\left(A_{-1}^{-1}\right)\right)^{-1}$. We first prove the following Lemma:

Lemma 1. *Let* U^* *and* W^* *be the optimal solutions to the problem consisting of all agents and the problem excluding the first agent, respectively. Then, as the number of agents increases,*

$$\lim_{N\to\infty} \begin{bmatrix} 0_{(N-1)\times 1} & I_{N-1} \end{bmatrix} U^* - W^* = O(1/N), \tag{4.3.0.8}$$

where $0_{(N-1)\times 1}$ *is the* $N-1$ *dimensional column vector of zeroes, and* I_{N-1} *is the* $N-1$ *dimensional identity matrix.*

Proof. According to equations (4.3.0.6) and (4.3.0.7),

$$\begin{bmatrix} 0 & I \end{bmatrix} U^* - W^* =$$
$$= \frac{1}{2} \begin{bmatrix} 0 & I \end{bmatrix} \begin{bmatrix} a_1^{-1} & 0 \\ 0 & A_{-1}^{-1} \end{bmatrix} \left(\gamma \cdot \begin{bmatrix} 1 & 1_{-1}^T \end{bmatrix} \begin{bmatrix} a_1^{-1} & 0 \\ 0 & A_{-1}^{-1} \end{bmatrix} \begin{bmatrix} b_1 \\ B_{-1} \end{bmatrix} \right.$$
$$\left. \begin{bmatrix} 1 \\ 1_{-1} \end{bmatrix} - \begin{bmatrix} b_1 \\ B_{-1} \end{bmatrix} \right) - \frac{1}{2} A_{-1}^{-1} \left(\gamma_{-1} 1_{-1}^T A_{-1}^{-1} B_{-1} 1_{-1} - B_{-1} \right)$$
$$= \frac{1}{2} \left(\gamma a_1^{-1} b_1 + (\gamma - \gamma_{-1}) 1_{-1}^T A_{-1}^{-1} B_{-1} \right) A_{-1}^{-1} 1_{-1}.$$

Since $\underline{a} \leq a_i \leq \bar{a} < 0$, $\gamma = \Theta(1/N)$, and $\gamma < 0$, $\frac{\bar{a}\underline{b}}{\underline{a}N} \leq \gamma a_1^{-1} b_1 \leq \frac{\underline{a}\bar{b}}{\bar{a}N}$, $\frac{\bar{a}^2}{-\underline{a}N(N-1)} \leq \gamma - \gamma_{-1} \leq \frac{\underline{a}^2}{-\bar{a}N(N-1)}$, $\frac{\underline{a}^2 \bar{b}}{-\bar{a}^2 N} \leq (\gamma - \gamma_{-1}) 1_{-1}^T A_{-1}^{-1} B_{-1} \leq \frac{\bar{a}^2 \underline{b}}{-\underline{a}^2 N}$. Therefore,

$$\lim_{N \to \infty} \begin{bmatrix} 0 & I \end{bmatrix} U^* - W^* = 0.$$

□

Let $\begin{bmatrix} 0 & I \end{bmatrix} U^* = V^*$. From Lemma 1, we know that $v_i^* - w_i^* = O(\frac{1}{N})$ where v_i and w_i is the i-th component of V^* and W^*, respectively. Hence,

$$\frac{F^*}{H_1} = \frac{a_1 u_1^{*2} + b_1 u_1^* + \sum_{i=2}^N (a_i v_i^{*2} + b_i v_i^*)}{\sum_{i=2}^N (a_i w_i^{*2} + b_i w_i^*)}$$
$$= \frac{a_1 u_1^{*2} + b_1 u_1^* + \sum_{i=2}^N \left(a_i w_i^{*2} + b_i w_i^* + G_1 \right)}{\sum_{i=2}^N (a_i w_i^{*2} + b_i w_i^*)},$$

where $G_1 = (2a_i w_i^* + b_i) O(\frac{1}{N}) + a_i O(\frac{1}{N^2})$. From equations (4.3.0.6) and (4.3.0.7), we know that $w_i^* = \Theta(1)$. Therefore,

$$\lim_{N \to \infty} \frac{F^*}{H_1} = 1.$$

Similarly, for all other i,

$$\lim_{N \to \infty} \frac{F^*}{H_i} = 1.$$

Therefore,

$$\lim_{N\to\infty} \bar{c}^N = 1.$$

Let $H_{min} = \min_i H_i$. Since $\frac{(N-1)F^*}{NH_{max}} \leq \underline{c}^N \leq \frac{(N-1)F^*}{NH_{min}}$,

$$\lim_{N\to\infty} \underline{c}^N = 1.$$

Consequently,

$$\lim_{N\to\infty} c^N = 1.$$

From Lemma 1, we have $W^* - V^* = -\frac{1}{2}\left(\gamma a_1^{-1} b_1 + (\gamma - \gamma_{-1})\,\xi\right) A_{-1}^{-1} 1_{-1}$, where $\xi = 1_{-1}^T A_{-1}^{-1} B_{-1}$ and $\xi = \Theta(N)$. The payment by Agent 1 is:

$$\begin{aligned} p_1^N &= U_{-1}^{*T} A_{-1} U_{-1}^* + B_{-1}^T U_{-1}^* - V^{*T} A_{-1} V^* - B_{-1}^T V^* \\ &= (U_{-1}^* + V^*)^T A_{-1} (U_{-1}^* - V^*) + B_{-1}^T (U_{-1}^* - V^*). \end{aligned}$$

The difference between Lagrange payment and VCG payment is:

$$\begin{aligned} &\lambda^{*N} u_1^{*N} - p_1^N \\ &= \frac{1}{2a_1}\gamma(a_1^{-1}b_1 + \xi)\left(\gamma\left(a_1^{-1}b_1 + \xi\right) - b_1\right) - p_1^N \\ &= \frac{1}{2a_1}\gamma^2(a_1^{-1}b_1 + \xi)^2 - \frac{b_1}{2a_1}\gamma(a_1^{-1}b_1 + \xi) \\ &- \left[\frac{1}{2}\left[\left(\gamma a_1^{-1}b_1 + (\gamma + \gamma_{-1})\,\xi\right) 1_{-1}^T - 2B_{-1}^T\right] A_{-1}^{-1} A_{-1} + B_{-1}^T\right] \\ &\cdot \frac{-1}{2}\left(\gamma a_1^{-1} b_{-1} + (\gamma - \gamma_{-1})\,\xi\right) A_{-1}^{-1} 1_{-1} \\ &= \frac{1}{2a_1}\gamma^2(a_1^{-1}b_1 + \xi)^2 - \frac{b_1}{2a_1}\gamma(a_1^{-1}b_1 + \xi) \\ &+ \frac{1}{4\gamma_{-1}}\left[\gamma^2 a_1^{-2} b_1^2 + 2a_1^{-1} b_1 \gamma^2 \xi + (\gamma^2 - \gamma_{-1}^{-2})\xi^2\right]. \end{aligned}$$

Since $\gamma = \Theta(\frac{1}{N})$,

$$
\begin{aligned}
&\lim_{N\to\infty} \left(\lambda^{*N} u_1^{*N} - p_1^N\right) \\
&= \lim_{N\to\infty} \left[\frac{\gamma^2\xi^2}{2a_1} - \frac{b_1\gamma\xi}{2a_1} + \frac{b_1\gamma^2\xi}{2a_1\gamma_{-1}} + \frac{(\gamma^2-\gamma_{-1}^2)\xi^2}{4\gamma_{-1}}\right] \\
&= \lim_{N\to\infty} \left[\frac{\xi^2}{4}(\frac{2\gamma^2}{a_1} + \frac{\gamma^2-\gamma_{-1}^2}{\gamma_{-1}}) - \frac{b_1\gamma\xi}{2a_1}(1-\frac{\gamma}{\gamma_{-1}})\right] \\
&= \lim_{N\to\infty} \left[\frac{\xi^2}{4}(\frac{\gamma^2}{a_1} + \gamma - \gamma_{-1})\right].
\end{aligned}
$$

By calculation, we have

$$
\frac{\gamma^2}{a_1} + \gamma - \gamma_{-1} = \frac{-1}{a_1^2}\left[\frac{1}{(\sum_{i=1}^N \frac{1}{a_i})^2(\sum_{j=2}^N \frac{1}{a_j})}\right] = O(\frac{1}{N^3}).
$$

Therefore,

$$
\lim_{N\to\infty} \left(\lambda^{*N} u_1^{*N} - p_1^N\right) = 0.
$$

□

The above VCG scheme can be extended to the important case of deterministic dynamic systems. One can consider the entire sequence of actions taken by an agent as a vector action. That is, one can view the problem as an open-loop control problem, where the entire decision on the sequence of controls to be employed is taken at the initial time, and so treatable as a static problem.

For agent i, let $F_i(x_i(t), u_i(t))$ be the one-step utility function at time t. Suppose that the state of agent i evolves as:

$$
x_i(t+1) = g_i(x_i(t), u_i(t)).
$$

The ISO asks each agent i to bid its one-step utility functions, state equations and initial condition. Denote the one-step utility function bids made by agent i by $\{\hat{F}_i(x_i(t), u_i(t)), t = 0, 1, \ldots, T-1\}$, its state equation bids by $\{\hat{g}_i, t = 0, 1, \ldots, T-1\}$, and its initial condition bid by $\hat{x}_{i,0}$. The ISO then calculates $(x_i^*(t), u_i^*(t))$ as the optimal solution, assumed to be unique, to the following utility maximization problem:

$$\max \sum_{i=1}^{N} \sum_{t=0}^{T-1} \hat{F}_i(x_i(t), u_i(t))$$

subject to

$$x_i(t+1) = \hat{g}_i(x_i(t), u_i(t)), \text{ for } \forall i \text{ and } \forall t,$$

$$\sum_{i=1}^{N} u_i(t) = 0, \text{ for } \forall t,$$

$$x_i(0) = \hat{x}_{i,0}, \text{ for } \forall i.$$

We denote this problem as $(\hat{F}, \hat{g}, \hat{x}_0)$. We can extend the notion of VCG payment p_i to the deterministic dynamic system as follow. Let

$$p_i := \sum_{j \neq i} \sum_{t=0}^{T-1} \hat{F}_j(x_j^{(i)}(t), u_j^{(i)}(t)) - \sum_{j \neq i} \sum_{t=0}^{T-1} \hat{F}_j(x_j^*(t), u_j^*(t)).$$

Here $(x_i^{(i)}(t), u_i^{(i)}(t))$ is the optimal solution to the following problem, which is assumed to be unique:

$$\max \sum_{j \neq i} \sum_{t=0}^{T-1} \hat{F}_j(x_j(t), u_j(t))$$

subject to

$$x_j(t+1) = \hat{g}_j(x_j(t), u_j(t)), \text{ for } j \neq i \text{ and } \forall t,$$

$$\sum_{j \neq i} u_j(t) = 0, \text{ for } \forall t,$$

$$x_j(0) = \hat{x}_{j,0}, \text{ for } j \neq i.$$

More generally, we can consider a Groves payment p_i defined as:

$$p_i := h_{i,t}(\hat{\boldsymbol{F}}_{-i}) - \sum_{j \neq i} \sum_{t=0}^{T-1} \hat{F}_j(x_j^*(t), u_j^*(t)),$$

where $h_{i,t}$ is any arbitrary function. We first show in the following theorem that truth-telling is still the dominant strategy equilibrium in Groves mechanism.

Theorem 6. *Truth-telling of utility function, state dynamics and initial condition ($\hat{F}_i \equiv F_i$, $\hat{g}_i \equiv g_i$ and $\hat{x}_{i,0} = x_{i,0}$) is a dominant strategy equilibrium under the Groves mechanism for a dynamic system.*

Proof. Let $\hat{F} := (\hat{F}_1, ..., \hat{F}_i, ..., \hat{F}_N)$, $\hat{g} := (\hat{g}_1..., \hat{g}_i, ..., \hat{g}_N)$, and $\hat{x}_0 := (\hat{x}_{1,0}, ..., \hat{x}_{i,0}, ..., \hat{x}_{N,0})$. Suppose agent i announces the true one-step utility function F_i, true state dynamics g_i, and true initial condition $x_{i,0}$. Let $\bar{F} := (\hat{F}_1, ...\hat{F}_{i-1}, F_i, \hat{F}_{i+1}, ..., \hat{F}_N)$, $\bar{g} := (\hat{g}_1, ...\hat{g}_{i-1}, g_i, \hat{g}_{i+1}, ..., \hat{g}_N)$, and $\bar{x}_0 := (\hat{x}_{1,0}, ...\hat{x}_{i-1,0}, x_{i,0}, \hat{x}_{i+1,0}, ..., \hat{x}_{N,0})$. Let $(\bar{x}_i^*(t), \bar{u}_i^*(t))$ be what ISO assigns and $p_i(\bar{F}, \bar{g}, \bar{x}_0)$ be what ISO charges when $(\bar{F}, \bar{g}, \bar{x}_0)$ is announced by agents. Let $(x_i^*(t), u_i^*(t))$ be what ISO assigns and $p_i(\hat{F}, \hat{g}, \hat{x}_0)$ be what ISO charges when $(\hat{F}, \hat{g}, \hat{x}_0)$ is announced by agents. Let $\bar{F}(x_i(t), u_i(t)) := \sum_i \bar{F}_i(x_i(t), u_i(t))$.

For agent i, the difference between net utility resulting from announcing $(F_i, g_i, x_{i,0})$ and $(\hat{F}_i, \hat{g}_i, \hat{x}_{i,0})$ is

$$\Big[\sum_t F_i(\bar{x}_i^*(t), \bar{u}_i^*(t)) - p_i(\bar{F}, \bar{g}, \bar{x}_0)\Big] - \Big[\sum_t F_i(x_i^*(t), u_i^*(t)) - p_i(\hat{F}, \hat{g}, \hat{x}_0)\Big]$$

$$\begin{aligned} &= \sum_t F_i(\bar{x}_i^*(t), \bar{u}_i^*(t)) - h_{i,t}(\bar{F}_{-i}) + \sum_{j\neq i} \sum_t \hat{F}_j(\bar{x}_i^*(t), \bar{u}_i^*(t)) \\ &- \sum_t F_i(x_i^*(t), u_i^*(t)) + h_{i,t}(\hat{F}_{-i}) - \sum_{j\neq i} \sum_t \hat{F}_j(x_i^*(t), u_i^*(t)) \\ &= \sum_t \bar{F}(\bar{x}_i^*(t), \bar{u}_i^*(t)) - \sum_t \bar{F}(x_i^*(t), u_i^*(t)) \geq 0, \end{aligned}$$

since $(\bar{x}_i^*(t), \bar{u}_i^*(t))$ is the optimal solution to the problem $(\bar{F}, \bar{g}, \bar{x}_0)$. $\square$

As in the static case, we show below that there exists a range of values c under which the scaled VCG payment achieves IC, EF, BB and IR at the same time. As in the static case, we suppose that from experience, the ISO can choose a value of c in this range, which does not depend on the agents' bids, to achieve BB and IR.

Truth-telling is a dominant strategy under the Groves mechanism. Under the dominant strategy equilibrium, every agent i will announce its true utility function F_i, state dynamics g_i and initial condition $x_{i,0}$.

Theorem 7. *Let $u^*(t)$ be the optimal solution to the following problem:*

$$\max \sum_i \sum_t F_i(x_i(t), u_i(t)),$$

subject to

$$x_i(t+1) = g_i(x_i(t), u_i(t)), \textit{ for all } i$$

$$\sum_i u_i(t) = 0, \textit{ for all } t \textit{ and } x_i(0) = x_{i,0}.$$

and let $u^{(i)}(t)$ be the optimal solution to the following problem:

$$\max \sum_{j \neq i} \sum_t F_j(x_j(t), u_j(t)),$$

subject to

$$x_j(t+1) = g_j(x_j(t), u_j(t)), \textit{ for } j \neq i,$$

$$\sum_{j \neq i} u_j(t) = 0, \textit{ for all } t \textit{ and } x_j(0) = x_{j,0} \textit{ for } j \neq i.$$

Let $H_i := \sum_{j \neq i} \sum_t F_j(x_j^{(i)}(t), u_j^{(i)}(t))$, and let $H_{max} = \max_i H_i$. Let $F^* := \sum_t \sum_j F_j(x_j^*(t), u_j^*(t))$. If $F^* > 0$, $H_i > 0$ for all i, and MPB (4.3.0.2) *condition holds, then there exists an $\underline{c}$ and $\bar{c}$, with $\underline{c} \leq \bar{c}$ such that if the constant c is chosen in the range $[\underline{c}, \bar{c}]$, then the Scaled VCG mechanism for the deterministic dynamic system satisfies IC, EF, BB and IR at the same time.*

Proof. Because c is not a function of $(\hat{F}, \hat{g}, \hat{x}_0)$, SVCG is within the Groves class and thus IC and

EF. To achieve budget balance, we need,

$$\sum_i p_i = c \sum_i H_i - (N-1) \sum_t \sum_j F_j(x_j^*(t), u_j^*(t)) \geq 0,$$

or equivalently,

$$c \geq \frac{(N-1)F^*}{\sum_i H_i}.$$

To achieve individual rationality for agent i, we need

$$\begin{aligned}\sum_t F_i(x_i^*(t), u_i^*(t)) - p_i = \sum_t F_i(x_i^*(t), u_i^*(t)) - c \cdot H_i \\ + \sum_{j \neq i} \sum_t F_j(x_j^*(t), u_j^*(t)) \geq 0,\end{aligned}$$

or equivalently,

$$c \leq \frac{F^*}{H_i}.$$

Combining all the inequalities,

$$\frac{(N-1)F^*}{\sum_i H_i} \leq c \leq \frac{F^*}{H_{max}}. \tag{4.3.0.9}$$

Let $\underline{c} := \frac{(N-1)F^*}{\sum_i H_i}$, and $\bar{c} := \frac{F^*}{H_{max}}$. To ensure $\underline{c} \leq \bar{c}$, one sufficient condition is,

$$(N-1)H_{max} \leq \sum_i H_i,\ F^* > 0,\ H_i > 0 \text{ for all } i,$$

□

Similar to the static case, we may want to choose a c that achieves almost Lagrange optimality. This can be formulated as the following MinMax problem:

$$\min_c \max_i |d_i(c)|, \text{ subject to (4.3.0.9)}.$$

where $d_i(c) := \sum_t \lambda^*(t)u_i^*(t) - p_i = \sum_t \lambda^*(t)u_i^*(t) - c \cdot H_i + \sum_{j\neq i}\sum_t F_j(x_j^*(t), u_j^*(t))$. The MinMax problem can in turn be transformed to a linear program:

$$\min\ Z$$

subject to

$$Z \geq d_i(c),\ \text{for all } i,$$

$$Z \geq -d_i(c),\ \text{for all } i,$$

$$\frac{(N-1)F^*}{\sum_i H_i} \leq c \leq \frac{F^*}{H}.$$

One can also replace $d_i(c)$ by the fractional deviation $d_i(c)/\sum_t \lambda^*(t)u_i^*(t)$.

As in the static case, the SVCG mechanism is asymptotically Lagrange optimal as the number of agents goes to infinity. Without loss of generality, we consider the special case where agents have quadratic utility functions and linear state dynamics: $F_i(x_i(t), u_i(t)) = q_i x_i^2(t) + r_i u_i^2(t)$ and $x_i(t+1) = a_i x_i(t) + b_i u_i(t)$. We suppose $q_i \leq 0$, and $r_i < 0$.

Theorem 8. *For SVCG mechanism with quadratic utility functions and linear state dynamics, if* (a_i, b_i, p_i, q_i) *satisfies the following:*

1. $\underline{a} \leq |a_i| \leq \bar{a}$, $\underline{b} \leq |b_i| \leq \bar{b}$, $\underline{q} \leq q_i \leq \bar{q} < 0$ *and* $\underline{r} \leq r_i \leq \bar{r} < 0$,

2. $(N-1)H_{max}(N) \leq \sum_i H_i(N)$, $F^*(N) > 0$ *and* $H_i(N) > 0$ *for all i.*

then the following hold:

1. *There exist* $\underline{c}^N \leq \bar{c}^N$ *such that for any* $c^N \in [\underline{c}^N, \bar{c}^N]$*, BB and IR hold. Moreover,* $\lim_{N\to\infty} c^N = 1$,

2. $\lim_{N\to\infty}\left(\sum_t \left(\lambda^{*N}(t)u_i^N(t)\right) - p_i^N\right) = 0$*, for all i.*

Proof. Let $X(t) = (x_1(t), x_2(t), ..., x_N(t))^T$, $U(t) = (u_1(t), u_2(t), ..., u_N(t))^T$, $A = diag(a_1, a_2, ..., a_N)$, $B = diag(b_1, b_2, ..., b_N)$, $Q = diag(q_1, q_2, ..., q_N)$, and $R = diag(r_1, r_2, ..., r_N)$. The utility maximization problem can be rewritten as the following Linear-Quadratic (LQ) problem:

$$\max \sum_{t=0}^{T-1} X^T(t)QX(t) + U^T(t)RU(t) \tag{4.3.0.10}$$

subject to

$$X(t+1) = AX(t) + BU(t), \tag{4.3.0.11}$$

$$1^T U(t) = 0, \text{ for } \forall t.$$

By substituting (4.3.0.11) into (4.3.0.10), and using the fact that open-loop optimal control is equivalent to the closed-loop optimal solution to LQ problem, we have the following equivalent augmented LQ problem:

$$\max \ \Omega^T(t)W\Omega(t) + V^T\Omega(t) \tag{4.3.0.12}$$

subject to

$$Y^T\Omega(t) = 0. \tag{4.3.0.13}$$

where $\Omega := (U_1; U_2; ...; U_N)$, and $U_i = (u_i(0); u_i(1); ...; u_i(T-1))$, W and V are formed by multiplication and addition of A, B, Q, R and $Y := [I_T; I_T; ...; I_T]$ with N T-dimensional identity matrix I_T. More specifically, W can be partitioned into diagonal blocks: $W = diag(W_1, ..., W_N)$, where each block W_i is a $T \times T$ square matrix consisting of multiplication and addition of a_i, b_i, q_i, r_i.

Noting that the optimization problem (4.3.0.12) and (4.3.0.13) is in the same form as (4.3.0.4) and (4.3.0.5), the unique Lagrange multiplier $\boldsymbol{\lambda}$ is calculated as:

$$\lambda^* = \Gamma Y^T W^{-1} V,$$

where $\Gamma = (Y^T W^{-1} Y)^{-1}$. The key to the proof of Theorem 5 is to show that γ is $\Theta(1/N)$. (Note that $f(N) = \Omega(g(N))$ if $f(N) = \mathcal{O}(g(N))$ as well as $g(N) = \Omega(f(N))$). Similarly, by expanding $\Gamma = (W_1^{-1} + W_2^{-1} + ... + W_N^{-1})^{-1}$ and applying bounded inverse theorem [85], $||\Gamma||$ is also $\Theta(1/N)$

since a_i, b_i, q_i, r_i are all uniformly bounded, respectively.

Let Ω^* be the optimal solution to problem (4.3.0.12) and (4.3.0.13) consisting of all agents and let Ψ^* be the optimal solution to the problem excluding the first agent. By replacing A, B and 1 with W, V and Y respectively, we have

$$\lim_{N\to\infty} \begin{bmatrix} 0_{(N-1)T\times T} & I_{(N-1)T} \end{bmatrix} \Omega^* - \Psi^* = 0.$$

Let $\begin{bmatrix} 0 & I \end{bmatrix} \Omega^* = \Phi^*$. From above, we know that $\Phi_i^* - \Psi_i^* = O(\frac{1}{N})1$ where Φ_i and Ψ_i is the i-th T-length component of Φ^* and Ψ^*, respectively. Hence,

$$\frac{F^*}{H_1} = \frac{U_1^{*T} W_1 U_1^* + V_1^T U_1^* + \sum_{i=2}^N (\Phi_i^{*T} W_i \Phi_i^* + V_i^T \Phi_i^*)}{\sum_{i=2}^N (\Psi_i^{*T} W_i \Psi_i^* + V_i^T \Psi_i^*)} =$$
$$\frac{U_1^{*T} W_1 U_1^* + V_1^T U_1^* + \sum_{i=2}^N (\Psi_i^{*T} W_i \Psi_i^* + V_i^T \Psi_i^* + G_1)}{\sum_{i=2}^N (\Psi_i^{*T} W_i \Psi_i^* + V_i^T \Psi_i^*)}$$

where $G_1 = (2\Psi_i^{*T} W_i 1 + V_i^T 1) O(\frac{1}{N}) + 1^T W_i 1 \cdot O(\frac{1}{N^2})$. Since $\Psi_i^* = \Theta(1)1$, we have

$$\lim_{N\to\infty} \frac{F^{*N}}{H_1^N} = 1.$$

Similarly, for all other i,

$$\lim_{N\to\infty} \frac{F^{*N}}{H_i^N} = 1.$$

Therefore,

$$\lim_{N\to\infty} \bar{c}^N = 1.$$

Let $H_{min} = \min_i H_i$. Since $\frac{(N-1)F^*}{NH_{max}} \leq \underline{c}^N \leq \frac{(N-1)F^*}{NH_{min}}$,

$$\lim_{N\to\infty} \underline{c}^N = 1.$$

Consequently,

$$\lim_{N\to\infty} c^N = 1.$$

From Lemma 1, we have,

$$\Psi^* - \Phi^* = \frac{-1}{2} W_{-1}^{-1} Y_{-1} (\Gamma W_{-1}^{-1} V_{-1} + (\Gamma - \Gamma_{-1})\Xi),$$

where W_{-1}, V_{-1} are formed by removing W_1 and V_1 from W and V, respectively. $Y_{-1} = [I_T; ...; I_T]$ with $(N-1)$ T-dimensional identity matrix. $\Xi = Y_{-1}^{-1} W_{-1}^{-1} V_{-1}$ and $\Xi = O(N)1$. Similarly as in Theorem 5, we have,

$$\begin{aligned}
&\lim_{N\to\infty} \left(\lambda^{*T} U_1^* - p_1^N\right) \\
&= \lim_{N\to\infty} \left[\frac{1}{2}(V_1^T W_1^{-1} + \Xi^T)\Gamma^T [W_1^{-1}\Gamma(W_1^{-1}V_1 + \Xi) - W_1^{-1}V_1]\right. \\
&- \left[\frac{1}{2}\left[\left(\Gamma W_{-1}^{-1} V_{-1} + (\Gamma + \Gamma_{-1})\Xi\right)^T Y_{-1}^T - 2V_{-1}\right]^T W_{-1}^{-1} W_{-1} + V_{-1}^T\right] \\
&\left.\cdot \frac{-1}{2} W_{-1}^{-1} Y_{-1} (\Gamma W_{-1}^{-1} V_{-1} + (\Gamma - \Gamma_{-1})\Xi)\right] \\
&= \lim_{N\to\infty} \left(\frac{1}{4}\Xi^T \left(\Gamma^T W_1^{-1}\Gamma + \Gamma - \Gamma_{-1}\right)\Xi\right)
\end{aligned}$$

It is straightforward to see that,

$$\begin{aligned}
&\Gamma^T W_1^{-1}\Gamma + \Gamma - \Gamma_{-1} \\
=& -\left(\sum_{i=1}^N W_i^{-1}\right)^{-1} W_1^{-1} \left(\sum_{i=1}^N W_i^{-1}\right)^{-1} W_1^{-1} \left(\sum_{i=2}^N W_i^{-1}\right)^{-1} \\
=& O(\frac{1}{N^3}).
\end{aligned}$$

Consequently,

$$\lim_{N\to\infty} \left(\lambda^{*T} U_i^* - p_i^N\right) = 0.$$

□

4.4 The Dynamic Stochastic VCG

In the previous section, we have shown that the VCG mechanism can be naturally extended to deterministic dynamic systems by employing an open-loop solution. However, when agents are stochastic dynamic systems, we need to consider closed-loop control laws for each agent. Such closed-loop control laws depend on the observations of the agents, which are generally private. So the states of the system are private random variables. Hence the problem becomes one of additionally ensuring that each agent reveals its “true" state at all times. However, since an agent's intertemporal payoff depends on the expected future payments and allocations in a dynamic game, the agent's current bid needs not maximize its current payoff. What's more, since dishonest bids distort current and future allocations in different ways, an agent's optimal bid will depend on others' bids. This additional complication precludes a dominant strategy solution for general stochastic dynamic systems. All that one can possibly hope for is a subgame perfect Nash equilibrium where each agent can assume other agents's strategies. Thus it appears one cannot hope to have an incentive compatible and social welfare optimal solution for general stochastic dynamic systems. However, as we will see, in the case of LQG agents one can indeed ensure the dominance of truth telling strategies that reveal the true states.

For agent i, let $w_i(t)$ be the discrete-time noise process affecting state $x_i(t)$ via:

$$x_i(t+1) = g_i(x_i(t), u_i(t), w_i(t)),$$

where $x_i(0)$ is independent of w_i. The uncertainties of all the agents are independent.

The ISO aims to maximize the social welfare:

$$\max \ \mathbb{E} \sum_{i=1}^{N} \sum_{t=0}^{T-1} F_i(x_i(t), u_i(t))$$

subject to

$$\sum_{i=1}^{N} u_i(t) = 0, \ \text{for } \forall t, \tag{4.4.0.1}$$

We first assume that F_i, g_i and the distributions of the uncertainties are known to the ISO, and that agents bid their states $x_i(t)$ as $\hat{x}_i(t)$. A straightforward extension of the static Groves mechanism would be to collect a payment $p_i(t)$ at time t from agent i defined as:

$$
\begin{aligned}
&p_i(t)\\
=&h_i(\hat{X}_{-i}(t)) - \mathbb{E}\sum_{j\neq i}\sum_{\tau=t}^{T-1}\Big[F_j(\hat{x}_j(\tau), u_j^*(\tau)) \mid X(t) = \hat{X}(t)\Big]
\end{aligned}
$$

where $\hat{x}_i(t)$ is what agent i bids for his state at time t, $\hat{X}_{-i}(t) = [\hat{x}_1(t), ..., \hat{x}_{i-1}(t), \hat{x}_{i+1}(t), ..., \hat{x}_N(t)]^T$ and $u_j^*(t)$ is the optimal solution to the following problem:

$$
\max\ \mathbb{E}\sum_{i=1}^{N}\sum_{\tau=t}^{T-1}\Big[F_j(\hat{x}_j(\tau), u_j^*(\tau)) \mid X(t) = \hat{X}(t)\Big]
$$

subject to

$$
\begin{aligned}
x_i(\tau+1) &= g_i(x_i(\tau), u_i(\tau), w_i(\tau)),\\
\sum_{i=1}^{N} u_i(\tau) &= 0,\ \text{for } t \leq \tau \leq T-1, \qquad (4.4.0.2)\\
\hat{X}(\tau) &= [\hat{x}_1(\tau), ..., \hat{x}_N(\tau)]^T.
\end{aligned}
$$

It is easy to verify that truth-telling of states by all agents forms a subgame perfect Nash equilibrium since truth-telling of $x_i(t)$ for agent i is a best response given that all other agents bid truthfully for all $\tau \geq t$. In fact it yields the minimum net cost (payment + one step cost) at time t. In that sense it is “myopically" optimal. However, truth-telling of states does not constitute a dominant strategy because another agent j may bid $\hat{x}_j(t+1)$ at time $t+1$ truthfully, but lies about the state $x_j(t)$ at time t in order to obtain a preferable state at the next time $t+1$. More specifically, if we assume all agents will bid truthfully from $t+1$ onward, then at time t, if agent j bids some untruthful $\hat{x}_j(t)$, truth-telling of state for agent i will be an optimal strategy only if agent j continues to bid “an untruthful but consistent” $\hat{x}_j(t)$ which stems from his untruthful bid $\hat{x}_j(t)$. By “consistent" we mean the state that would result from the untruthful $\hat{x}_j(t)$ but with the truthful state noise $w_j(t)$. In other word, agent i's will bid truthfully only if agent j "consistently" lies about his state, which is not guaranteed using the above payment scheme.

We now show that while an incentive compatible strategy presents fundamental challenges for general stochastic dynamic systems, there is a solution for LQG systems. We need to investigate the structure of LQG system more carefully.

For agent i, let $w_i(t) \sim \mathcal{N}(0, \sigma_i)$ be the discrete-time additive Gaussian white noise process affecting state $x_i(t)$ via:

$$x_i(t+1) = a_i x_i(t) + b_i u_i(t) + w_i(t)),$$

where $x_i(0) \sim \mathcal{N}(0, \zeta_i)$ and is independent of w_i. Each agent has a one-step utility function

$$F_i(x_i(t), u_i(t)) = q_i x_i^2(t) + r_i u_i^2(t).$$

We suppose that $q_i \leq 0$ and $r_i < 0$. Let $X(t) = [x_1(t), ..., x_N(t)]^T$, $U(t) = [u_1(t), ..., u_N(t)]^T$ and $W(t) = [w_1(t), ..., w_N(t)]^T$. Let $Q = diag(q_1, ..., q_N) \leq 0$, $R = diag(r_1, ..., r_N) < 0$, $A = diag(a_1, ..., a_N)$, $B = diag(b_1, ..., b_N)$, $\Sigma = diag(\sigma_1, ..., \sigma_N) > 0$ and $Z = diag(\zeta_1, ..., \zeta_N) > 0$. Let $RSW := \sum_{t=0}^{T-1}[X^T(t)QX(t) + U^T(t)RU(t)]$ be the *random* social welfare, i.e., the variable whose expectation is the social welfare of the agents, and let $SW := \mathbb{E}[RSW]$ denote the (*expected*) social welfare. The random social welfare could also be called the "ex-post social welfare". The ISO aims to maximize the social welfare:

$$\max\ \mathbb{E} \sum_{t=0}^{T-1} \left[X^T(t)QX(t) + U^T(t)RU(t)\right]$$

subject to

$$X(t+1) = AX(t) + BU(t) + W(t),$$

$$1^T U(t) = 0, \text{ for } \forall t, \tag{4.4.0.3}$$

$$X(0) \sim \mathcal{N}(0, Z), W \sim \mathcal{N}(0, \Sigma).$$

We now introduce a "layered" payment structure which ensures incentive compatibility for LQG systems. We begin by rewriting the random social welfare, and thereby also the social welfare, in terms more convenient for us. We will decompose the state $X(t)$ of the entire system comprised of all agents as:

$$X(t) := \sum_{s=0}^{t} X(s,t),\ 0 \le t \le T-1, \tag{4.4.0.4}$$

where $X(s,s) := W(s-1)$ for $s \ge 1$ and $X(0,0) := X(0)$. Let

$$X(s,t) := AX(s,t-1) + BU(s,t-1),\ 0 \le s \le t-1, \tag{4.4.0.5}$$

with $U(s,t)$ yet to be specified. We suppose that $U(t)$ can also be decomposed as:

$$U(t) := \sum_{s=0}^{t} U(s,t),\ 0 \le t \le T-1. \tag{4.4.0.6}$$

Then regardless of how the $U(s,t)$'s are chosen, as long as the $U(s,t)$'s for $0 \le s \le t$ are indeed a decomposition of $U(t)$, i.e., (4.4.0.6) is satisfied, the random social welfare can be written in terms of $X(s,t)$'s and $U(s,t)$'s as:

$$RSW = \sum_{s=0}^{T-1} L_s,$$

where L_s for $s \ge 1$ is defined as:

$$\begin{aligned} L_s :&= \sum_{t=s}^{T-1} \Big[X^T(s,t)QX(s,t) + U^T(s,t)RU(s,t) \\ &+2\left(\sum_{\tau=0}^{s-1} X(\tau,t)\right) QX(s,t) + 2\left(\sum_{\tau=0}^{s-1} U(\tau,t)\right) RU(s,t)\Big], \end{aligned} \tag{4.4.0.7}$$

and L_0 is defined as:

$$L_0 := \sum_{t=0}^{T-1} \Big[X^T(0,t)QX(0,t) + U^T(0,t)RU(0,t)\Big].$$

Hence,

$$SW = \mathbb{E}\sum_{s=0}^{T-1} L_s.$$

In the scheme to follow the ISO will choose all $U(s,t)$'s for future t's at time s, based on the information it has at time s. Hence $X(s,t)$ is completely determined by $W(s-1)$, and $U(s,t)$ for $s \leq t \leq T-1$. Indeed $X(s,t)$ can be regarded as the contribution to $X(t)$ of these variables.

Here we assume that the ISO knows the true system parameters Q, R, A and B. This may hold if the ISO has previously run the VCG bidding scheme for a dynamic deterministic system, or equivalently, a day-ahead market, and system parameters remain unchanged when agents participate in the real-time stochastic market.

Instead of asking agents to bid their state, we will consider a scheme where agents will be asked to bid their state noise. At each stage, the ISO asks the agents to bid their $x_i(s,s)$ (defined as equal to $w_i(s-1)$) at each time s, for $0 \leq s \leq T-1$. Let $\hat{x}_i(s,s)$ be what the agents actually bid, since they may not tell the truth. Based on their bids $\{\hat{x}_i(s,s)$ for $1 \leq i \leq N\}$, the ISO solves the following problem:

$$\max \ L_s$$

for the system

$$\hat{X}(s,t) = A\hat{X}(s,t-1) + BU(s,t-1), \text{ for } t > s,$$

with

$$\hat{X}(s,s) = [\hat{x}_1(s,s), ..., \hat{x}_N(s,s)]^T,$$

subject to the constraint

$$1^T U(s,t) = 0, \text{ for } s \leq t \leq T-1.$$

Here $\hat{X}(s,t)$ is the zero-noise state variable updates starting from the "initial condition" $\hat{X}(s,s)$. Let $U^*(s,t)$ denote the optimal solution.

The interpretation is the following. Based on the bids, $\hat{X}(s,s)$, which is supposedly a bid of $W(s-1)$, the ISO calculates the trajectory of the linear systems from time s onward, assuming zero noise from that point on. It then allocates consumptions/generations $U(s,t)$ for future periods t for the corresponding deterministic linear system, with balance of consumption and production (4.4.0.3) at each time t. These can be regarded as taking into account the consequences of the disturbance occurring at time s.

Next, the ISO collects a payment $p_i(s)$ from agent i at time s as:

$$p_i(s) := h_i(\hat{X}_{-i}(s,s)) - \sum_{j\neq i}\sum_{t=s}^{T-1}\Bigg[q_j\hat{x}_j^2(s,t) + r_j u_j^{*2}(s,t)$$
$$+2q_j\left(\sum_{\tau=0}^{s-1}\hat{x}_j(\tau,t)\right)\hat{x}_j(s,t) + 2r_j\left(\sum_{\tau=0}^{s-1}u_j(\tau,t)\right)u_j^*(s,t)\Bigg],$$

where $\hat{X}_{-i}(s,s) = [\hat{x}_1(s,s), ..., \hat{x}_{i-1}(s,s), \hat{x}_{i+1}(s,s), ...$
$, \hat{x}_N(s,s)]^T$, and h_i is any arbitrary function (as in the Groves mechanism).

Before we prove incentive compatibility, we need to define the notion of "rational agents".

Definition 6. Rational Agents: We say agent i is rational at time $T-1$, if it adopts a dominant strategy whenever there exists a unique dominant strategy. An agent i is rational at time t if it adopts a dominant strategy at time t under the assumption that all agents including itself are rational at times $t+1, t+2, ..., T-1$, whenever there is a unique such dominant strategy.

Theorem 9. *Truth-telling of state $\hat{x}_i(s,s)$ for $0 \leq s \leq T-1$, i.e., bidding $\hat{x}_i(s,s) = w_i(s-1)$, is the unique dominant strategy for the stochastic ISO mechanism, if system parameters $Q \leq 0$, $R < 0$, A and B are known, and agents are rational.*

Proof. Below, by "net" utility, we mean the utility derived by an agent minus its payment. We show this by backward induction. Let Agent j, $j \neq i$ bid $\hat{x}_j(s,s)$ at time s. Given the bids $\hat{x}_j(s,s)$ of other agents, let $J_i(s)$ be the net utility of agent i from time s onward if it bids truthful $x_i(s,s)$, i.e., $w_i(s-1)$, and let $\hat{J}_i(s)$ be the net utility if it bids possibly untruthful $\hat{x}_i(s,s)$. Let $U^*(s,t)$ be the ISO's assignments if agent i bids truthfully and let $\hat{U}^*(s,t)$ be the ISO's assignments if agent i bids untruthfully.

We will first consider time $T-1$, since we are employing backward induction. Suppose that $x_i(s,T-1)$ for $0 \leq s \leq T-2$ were the past bids, and $u_i(s,T-1)$ for $0 \leq s \leq T-2$, were those portions of the allocations for the future already decided in the past. Our interest is on analyzing what should be the current bid $x_i(T-1,T-1)$, and the consequent additional allocation $u_i(T-1,T-1)$. Now

$$\begin{aligned}
J_i(T-1) &= q_i x_i^2(T-1) + r_i u_i^2(T-1) - p_i(T-1) \\
&= q_i \left[x_i(T-1,T-1) + \sum_{s=0}^{T-2} x_i(s,T-1) \right]^2 \\
&+ r_i \left[u_i^*(T-1,T-1) + \sum_{s=0}^{T-2} u_i(s,T-1) \right]^2 - p_i(T-1).
\end{aligned}$$

Now $x_i(s,T-1)$ for $0 \leq s \leq T-2$ depend only on previous bids $x_i(s,s)$, and thus those terms can be treated as constants. In addition, the h_i term depends only on other agents' bids. As a consequence, when comparing $J_i(T-1)$ with $\hat{J}_i(T-1)$, one can just regard $h_i \equiv 0$. Hence we can simply write $J_i(T-1)$ as:

$$\begin{aligned}
J_i(T-1) &= q_i x_i^2(T-1,T-1) + r_i u_i^{*2}(T-1,T-1) \\
&+ 2q_i \left(\sum_{s=0}^{T-2} x_i(s,T-1) \right) x_i(T-1,T-1) \\
&+ 2r_i \left(\sum_{s=0}^{T-2} u_i(s,T-1) \right) u_i^*(T-1,T-1) \\
&+ \sum_{j \neq i} \Bigg[q_j \hat{x}_j^2(T-1,T-1) + r_j u_j^{*2}(T-1,T-1) \\
&+ 2q_j \left(\sum_{\tau=0}^{T-2} \hat{x}_j(\tau,T-1) \right) \hat{x}_j(T-1,T-1) \\
&+ 2r_j \left(\sum_{\tau=0}^{T-2} u_j(\tau,T-1) \right) u_j^*(T-1,T-1) \Bigg].
\end{aligned}$$

It is seen that $J_i(T-1)$ is in the same form as L_{T-1}. $\hat{J}_i(T-1)$ is obtained by replacing u_i^* with

$\hat{u}_i^*$. We conclude that $J_i(T-1) \geq \hat{J}_i(T-1)$ because u_i^* is the optimal solution to L_{T-1} when $\hat{x}_i(T-1, T-1) = x_i(T-1, T-1)$. Moreover truth telling is the unique optimal strategy since $r_i < 0$.

We next employ induction and so assume that truth-telling of states is the dominant strategy equilibrium at time k. Let $\mathcal{H}_t$ be the history up to time t. If agents are rational, we can take the expectation over future $X(s,s)$, $s \geq k$, which are i.i.d. Gaussian noise vectors, and calculate $J_i(k-1)$ (where, as before, we simply take the first Groves term $h_i \equiv 0$):

$$\begin{aligned}
&J_i(k-1) = \\
&q_i x_i^2(k-1) + r_i u_i^2(k-1) - p_i(k-1) + \mathbb{E}\left[J_i(k)|\mathcal{H}_{k-1}\right] \\
&= q_i \left[x_i(k-1,k-1) + \sum_{s=0}^{k-2} x_i(s,k-1)\right]^2 \\
&+ r_i \left[u_i(k-1,k-1) + \sum_{s=0}^{k-2} u_i(s,k-1)\right]^2 - p_i(k-1) \\
&+ \mathbb{E}\left[\sum_{t=k}^{T-1} \left(q_i x_i^2(t) + r_i u_i^2(t) - p_i(t)\right) \middle| \mathcal{H}_{k-1}\right].
\end{aligned} \tag{4.4.0.8}$$

We first show that $\mathbb{E}[U^*(k,k)|h_{k-1}] = 0$. By completing the square for L_k in (4.4.0.7), we have the following equivalent problem for the ISO to solve for the k-th layer:

$$\begin{aligned}
\max \sum_{t=k}^{T-1} \Bigg[&\left(X(k,t) + \sum_{\tau=0}^{k-1} X(\tau,t)\right)^T Q \cdot \\
&\left(X(k,t) + \sum_{\tau=0}^{k-1} X(\tau,t)\right) + \left(U(k,t) + \sum_{\tau=0}^{k-1} U(\tau,t)\right)^T R \cdot \\
&\left(U(k,t) + \sum_{\tau=0}^{k-1} U(\tau,t)\right)\Bigg]
\end{aligned} \tag{4.4.0.9}$$

Now, for the fixed k of interest, letting $Y(t) := X(k,t) + \sum_{\tau=0}^{k-1} X(\tau,t)$, and $V(t) := U(k,t) + \sum_{\tau=0}^{k-1} U(\tau,t)$, we see that $Y(t) = AY(t-1) + BV(t-1)$ for $t \geq k+1$. The "initial" condition is $Y(k) = X(k)$. For this linear system, the optimal control law for the cost (4.4.0.9) under the balancing constraint for all t is a control law that is linear in the state. Denoting the optimal gain by $K(t)$, we have

$$U^*(k,k) + \sum_{\tau=0}^{k-1} U(\tau,k) = K(k)\left[X(k,k) + \sum_{\tau=0}^{k-1} X(\tau,k)\right].$$

Similarly, at time $k-1$, the ISO chooses the allocation at time k by using the same gain $K(t)$ applied to that portion of the state at time k resulting from disturbances up to time $k-1$:

$$U(k-1,k) + \sum_{\tau=0}^{k-2} U(\tau,k) = \sum_{\tau=0}^{k-1} U(\tau,k) = K(k)\cdot$$
$$\left[X(k-1,k) + \sum_{\tau=0}^{k-2} X(\tau,k)\right] = K(k)\left[\sum_{\tau=0}^{k-1} X(\tau,k)\right].$$

Consequently,

$$\mathbb{E}[U^*(k,k)|h_{k-1}] = K(k)\mathbb{E}[X(k,k)|h_{k-1}] = 0,$$

since all agents are truth-telling at time k, i.e., $\mathbb{E}[X(k,k)|\mathcal{H}_{k-1}] = \mathbb{E}[W(k-1)] = 0$. From (4.4.0.5), by linearity of the system, we consequently also have $\mathbb{E}[X(k,t)|\mathcal{H}_{k-1}] = 0$, $k < t \leq T-1$, and $\mathbb{E}[U(k,t)|\mathcal{H}_{k-1}] = 0$, $k < t \leq T-1$. Therefore, for $k \leq t \leq T-1$,

$$\mathbb{E}[x_i^2(t)|\mathcal{H}_{k-1}] = \mathbb{E}\left[\sum_{\tau=k}^{t} x_i(\tau,t) + \sum_{s=0}^{k-1} x_i(s,t)\right]^2$$
$$= \left[\sum_{s=0}^{k-1} x_i(s,t)\right]^2 + C = x_i^2(k-1,t) + 2x_i(k-1,t)\sum_{s=0}^{k-2} x_i(s,t)$$
$$+ \left[\sum_{s=0}^{k-2} x_i(s,t)\right]^2 + C,$$

where C is a fixed term corresponding to the variance of $\sum_{\tau=k}^{t} x_i(\tau,t)$ and $\left[\sum_{s=0}^{k-2} x_i(s,t)\right]^2$ could be treated as a constant since it depends only previous bids. Similarly, we have, for $t \geq k$,

$$\mathbb{E}[u_i^2(t)|\mathcal{H}_{k-1}] =$$

$$u_i^2(k-1,t) + 2u_i(k-1,t)\sum_{s=0}^{k-2} u_i(s,t) + \left[\sum_{s=0}^{k-2} u_i(s,t)\right]^2 + C,$$

We also have,

$$\mathbb{E}[p_i(t)|\mathcal{H}_{k-1}] = const.,$$

since $\mathbb{E}[x_j(t,\tau)|\mathcal{H}_{k-1}] = 0$ and $\mathbb{E}[u_j(t,\tau)|\mathcal{H}_{k-1}] = 0$, for $\tau \geq t$. By ignoring the constant term, we now have,

$$\begin{aligned}
&J_i(k-1) = \\
&q_i x_i^2(k-1,k-1) + 2q_i x_i(k-1,k-1)\sum_{s=0}^{k-2} x_i(s,k-1) \\
&+r_i u_i^2(k-1,k-1) + 2r_i u_i(k-1,k-1)\sum_{s=0}^{k-2} u_i(s,k-1) \\
&+\sum_{t=k}^{T-1}\left[q_i x_i^2(k-1,t) + 2q_i x_i(k-1,t)\sum_{s=0}^{k-2} x_i(s,t)\right] \\
&+\sum_{t=k}^{T-1}\left[r_i u_i^2(k-1,t) + 2r_i u_i(k-1,t)\sum_{s=0}^{k-2} u_i(s,t)\right] \\
&-p_i(k-1) \\
&= \sum_{t=k-1}^{T-1}\left[q_i x_i^2(k-1,t) + r_i u_i^2(k-1,t)\right. \\
&+ 2q_i\left(\sum_{\tau=0}^{k-2} x_i(\tau,t)\right) x_i(k-1,t) \\
&\left.+ 2r_i\left(\sum_{\tau=0}^{k-2} r_i(\tau,t)\right) r_i(k-1,t)\right] - p_i(k-1).
\end{aligned}$$

It is straightforward to check that $J_i(k-1)$ is in the same form as L_{k-1} and thus we conclude that truth-telling $\hat{x}_i(k-1,k-1) = x_i(k-1,k-1)$ is the unique dominant strategy for agent i at time

$k-1$. □

The above proof yields the following two corollaries.

Corollary 1. In the stochastic VCG mechanism, truth-telling of state constitutes a "subgame perfect dominant strategy equilibrium" in the sense that truth-telling of state is the unique dominant strategy for every subgame of the original game.

Now let us consider systems where the distribution of the noise is not Gaussian, but the noises are independent, mean zero, and have finite variance. Then the optimal centralized solution for social welfare maximization is not necessarily linear in the states. However, one can consider the optimal strategy in the class of linear strategies. Then the ISO can employ the same payment scheme to ensure that the system attains this optimal social welfare in the class of linear strategies:

Corollary 2. Consider the system above, where the noises are independent, mean zero, and have finite variance. Then the ISO can ensure through the mechanism described above that the system attains the optimal social welfare in the class of all linear strategies.

We note that the key to proving incentive compatibility for the layered VCG mechanism lies in the fact that the optimal feedback gain $K(k)$ remains unchanged for each round of bids. This is due to the fact that $K(k)$ is only a function of Q, R, A, and B. Therefore, if bidding of system parameters at the beginning is allowed, then the layered VCG mechanism may not be incentive compatible. We show this by the following counterexample.

Example 2. Let $T=4$. The agents' system equations and cost matrices have the following parameters: $(a_1,a_2,a_3,a_4)=(1,1,1,1)$, $(b_1,b_2,b_3,b_4)=(1,1,1,1)$, $(q_1,q_2,q_3,q_4)=(-1,-1,-1,-1)$, $(r_1,r_2,r_3,r_4)=(-1,-1.1,-1.2,-1.1)$, $(\zeta_1,\zeta_2,\zeta_3,\zeta_4)=(0.3,0.32,0.31,0.3)$ and $(\sigma_1,\sigma_2,\sigma_3,\sigma_4)$ $=(0.1,0.11,0.11,0.12)$. If system operator knows all the parameters of agents, and every agent bid its true state, then the expected net utility of agent 1 (expected total utility minus expected total payment) is 0.629. When agents are also allowed to bid their system parameters at the beginning, truth-telling of state may not be incentive compatible. Suppose that agents 2, 3, 4 remain truthful, namely, bid their true system parameters at the beginning and their true states at all times. Suppose now that agent 1 intentionally bids an untruthful $\hat{q}_1 = -1.3$ while bidding other parameters truthfully at the beginning. Assume also that agent 1 always bids its state as if there is no noise ($w_1(t) \equiv 0$). Now agent 1's net expected utility is 0.631. Therefore, agent 1's optimal strategy is not to bid its true state when it is allowed to bid its system parameters at the beginning.

The assumption that the ISO knows the system parameters of all the agents can perhaps be justified since the ISO can learn these parameters by running the day-ahead market (a dynamic deterministic market) where agents are guaranteed to bid their true system parameters as shown in the previous section.

4.4.1 Budget Balance and Individual Rationality in LQG systems

We extend the notion of the SVCG mechanism to the stochastic dynamic systems as follows:

$$
\begin{aligned}
p_i(s) := c \cdot \sum_{j \neq i} \sum_{t=s}^{T-1} \Bigg[& q_j \hat{x}_j^2(s,t) + r_j u_j^{(i)2}(s,t) \\
& + 2q_j \left(\sum_{\tau=0}^{s-1} \hat{x}_j(\tau,t) \right) \hat{x}_j(s,t) + 2r_j \left(\sum_{\tau=0}^{s-1} u_j^{(i)(\tau,t)} \right) u_j^{(i)}(s,t) \Bigg] \\
- \sum_{j \neq i} \sum_{t=s}^{T-1} \Bigg[& q_j \hat{x}_j^2(s,t) + r_j u_j^{*2}(s,t) \\
& + 2q_j \left(\sum_{\tau=0}^{s-1} \hat{x}_j(\tau,t) \right) \hat{x}_j(s,t) + 2r_j \left(\sum_{\tau=0}^{s-1} u_j(\tau,t) \right) u_j^{*}(s,t) \Bigg],
\end{aligned}
$$

where $u_j^{(i)}(s,t)$ is the optimal solution to the following problem:

$$
\begin{aligned}
\max \sum_{j \neq i} \sum_{t=s}^{T-1} \Bigg[& q_j x_j^2(s,t) + u_j^2(s,t) \\
& + 2q_j \left(\sum_{\tau=0}^{s-1} x_j(\tau,t) \right) x_j(s,t) + 2r_j \left(\sum_{\tau=0}^{s-1} u_j(\tau,t) \right) u_j(s,t) \Bigg]
\end{aligned}
$$

subject to

$$
x_j(s,t) = a_j x_j(s,t-1) + b_j u_j(s,t-1), \text{ for } s < t \leq T-1,
$$

$$\sum_{j\neq i} u_j(s,t) = 0, \text{ for } s \leq t \leq T-1,$$

$$x_j(s,s) = \hat{x}_j(s,s).$$

As in the static case, based on its prior knowledge of a suitable range for c, the ISO can choose a range of c, which does not depend on the agents' bids, to achieve BB and IR.

Truth-telling is a dominant strategy under the SVCG mechanism because it falls under the Groves mechanism. Under the dominant strategy equilibrium, every agent i will bid its true state $x_i(s,s)$, i.e., $w_i(s-1)$.

Theorem 10. *Let $U^*(t)$ be the optimal solution to the following problem:*

$$\max\ \mathbb{E}\sum_{t=0}^{T-1}[X^T(t)QX(t) + U^T(t)RU(t)]$$

subject to

$$X(t+1) = AX(t) + BU(t) + W(t),$$

$$1^T U(t) = 0, \textit{ for } \forall t,$$

$$X(0) \sim \mathcal{N}(0,Z), W \sim \mathcal{N}(0,\Sigma).$$

Let $X^{(i)}(t) := [x_1(t), ..., x_{i-1}(t), x_{i+1}(t), ...x_N(t)]^T$, and similarly let $Q^{(i)}$, $R^{(i)}$, $A^{(i)}$, $B^{(i)}$, $Z^{(i)}$ and $\Sigma^{(i)}$ be the matrix with the i-th component removed. Let $U^{(i)}(t)$ be the optimal solution to the following problem:

$$\max\ \mathbb{E}\sum_{t=0}^{T-1}[X^{(i)T}(t)Q^{(i)}X^{(i)}(t) + U^T(t)R^{(i)}U(t)]$$

subject to

$$X^{(i)}(t+1) = A^{(i)}X^{(i)}(t) + B^{(i)}U(t) + W^{(i)}(t),$$

$$1^T U(t) = 0, \textit{ for } \forall t,$$

$$X^{(i)}(0) \sim \mathcal{N}(0, Z^{(i)}), W^{(i)} \sim \mathcal{N}(0, \Sigma^{(i)}).$$

Let $H_i := \mathbb{E}\sum_{t=0}^{T-1}[X^{(i)T}(t)Q^{(i)}X^{(i)}(t) + U^{(i)T}(t)R^{(i)}U^{(i)}(t)]$ *and let* $H_{max} := \max_i H_i$*. Let* $F^* = \mathbb{E}\sum_{t=0}^{T-1}[X^T(t)QX(t) + U^T(t)RU(t)]$*. If* $F^* > 0$*,* $H_i > 0$ *for all* i*, and MPB* (4.3.0.2) *condition holds, there exists an* $\underline{c}$ *and* $\bar{c}$*, with* $\underline{c} \leq \bar{c}$ *such that if the constant* c *is chosen in the range* $[\underline{c},\ \bar{c}]$*, then the SVCG mechanism for the deterministic dynamic system satisfies IC, EF, BB and IR at the same time.*

Proof. It is straightforward to verify that under the earlier dominant strategy,

$$\mathbb{E}\Big[\sum_{s=0}^{T-1} p_i(s)\Big] = c \cdot H_i - \mathbb{E}\left[\sum_{j \neq i}\sum_{t=0}^{T-1}\left(q_j x_j^2(t) + r_j u_j^{*2}(t)\right)\right]$$

since w_i's are i.i.d. and $u_i(t)$ is linear in $x_i(t)$. Hence, to achieve budget balance, we need,

$$\mathbb{E}\Big[\sum_i \sum_{s=0}^{T-1} p_i(s)\Big] = c \cdot \sum_i H_i - (N-1)F^* \geq 0,$$

or equivalently,

$$c \geq \frac{(N-1)F^*}{\sum_i H_i}.$$

To achieve individual rationality for agent i, we need

$$\mathbb{E}\left[\sum_{t=0}^{T-1}\left(q_i x_i^2(t) + r_i u_i^{*2}(t)\right) - \sum_{s=0}^{T-1} p_i(s)\right] = F^* - c \cdot H_i \geq 0,$$

or equivalently,

$$c \leq \frac{F^*}{H_i}.$$

Combining both inequalities, we have

$$\frac{(N-1)F^*}{\sum_i H_i} \leq c \leq \frac{F^*}{H_{max}}.$$

Let $\underline{c} = \frac{(N-1)F^*}{\sum_i H_i}$ and $\bar{c} = \frac{F^*}{H_{max}}$. To ensure $\underline{c} \leq \bar{c}$, one sufficient condition is,

$$(N-1)H_{max} \leq \sum_i H_i,\ F^* > 0,\ H_i > 0 \text{ for all } i.$$

□

4.4.2 Lagrange Optimality in LQG Systems

In general, just as for a static problem, the SVCG mechanism is not Lagrange optimal. Within the feasible range $[\underline{c}, \bar{c}]$, one can choose a c that achieves near Lagrange optimality. This can be formulated as a MinMax problem:

$$\min_c \max_i \left| \frac{d_i(c)}{\mathbb{E}\sum_{t=0}^{T-1}\left[\lambda^*(t)u_i^*(t)\right]} \right|, \text{ subject to (4.3.0.3),}$$

where

$$\begin{aligned} d_i(c) &:= \mathbb{E}\sum_{t=0}^{T-1}\left[\lambda^*(t)u_i^*(t) - p_i(t)\right] \\ &= \mathbb{E}\sum_{t=0}^{T-1}\left[\lambda^*(t)u_i^*(t)\right] - c \cdot H_i + \mathbb{E}\left[\sum_{t=0}^{T-1}\left(q_j x_j^2(t) + r_j u_j^{*2}(t)\right)\right]. \end{aligned}$$

The MinMax problem can be transformed to a linear program:

$$\min\ Z$$

subject to

$$Z \geq \frac{d_i(c)}{\mathbb{E}\sum_{t=0}^{T-1}\left[\lambda^*(t)u_i^*(t)\right]}, \text{ for all } i,$$

$$Z \geq -\frac{d_i(c)}{\mathbb{E}\sum_{t=0}^{T-1}\left[\lambda^*(t)u_i^*(t)\right]}, \text{ for all } i,$$

$$\frac{(N-1)F^*}{\sum_i H_i} \leq c \leq \frac{F^*}{H_{max}}.$$

We illustrate the MinMax problem with a numerical example below.

Example 3. We use the same system parameters as in Example 2. The optimal solution to the MinMax problem is $(c^*, Z^*) = (0.96, 0.21)$. Thus, by choosing $c = 0.96$, the SVCG mechanism satisfies IC, EF, BB and IR, and all agents expect to pay/receive within 21% of their expected Lagrange optimal payments.

However, just as for deterministic systems, as the number of agents increases, the scaled-VCG mechanism does achieve asymptotic Lagrange Optimality.

Theorem 11. *If* $(a_k, b_k, p_k, q_k, \zeta_k, \sigma_k)$ *satisfy the following:*

1. $\underline{a} \leq |a_i| \leq \bar{a}$, $\underline{b} \leq |b_i| \leq \bar{b}$, $\underline{q} \leq q_i \leq \bar{q} < 0$ *and* $\underline{r} \leq r_i \leq \bar{r} < 0$
2. $F^* > 0$, $H_i > 0$ *for all* i*, and MPB condition holds,*

then the following holds:

1. *There is a range of* c^N *that could be chosen to achieve BB and IR, and* $\lim_{N\to\infty} c^N = 1$*,*
2. *Asymptotic Lagrange Optimality:* $\lim_{N\to\infty} \mathbb{E} \sum_{t=0}^{T-1} \left[\lambda^N(t) u_i^N(t) - p_i^N(t)\right] = 0$*, where* $\lambda^N(t)$ *is the stochastic process corresponding to the power balance constraint.*

Proof. At each layer, the ISO is solving a deterministic LQR problem, and from Theorem 8 we have,

$$\lim_{N\to\infty} \frac{L_s^*}{H_{s,1}} = 1, \text{ for } 0 \leq s \leq T-1,$$

where L_s^* is the maximum value of L_s and $H_{s,1}$ is the maximum social welfare when agent 1 is excluded. Moreover, as we have shown in Theorem 9, the sum of $U^*(s,t)$ calculated at each layer is indeed the optimal solution $U^*(t) = \sum_{s=0}^{t} U^*(s,t)$. Consequently,

$$\lim_{N\to\infty} \frac{F^{*N}}{H_1^N} = \lim_{N\to\infty} \frac{\mathbb{E} \sum_{s=0}^{T-1} L_s^*}{\mathbb{E} \sum_{s=0}^{T-1} H_{s,1}} = 1.$$

Similarly we can show that $\lim_{N\to\infty} \frac{F^*}{H_k^N} = 1$ for $k \neq 1$. Therefore,

$$\lim_{N\to\infty} \bar{c}^N = 1.$$

Let $H_{min} = \min H_i$. Since $\frac{(N-1)F^{*N}}{NH_{max}} \leq \underline{c}^N \leq \frac{(N-1)F^{*N}}{NH_{min}}$,

$$\lim_{N\to\infty} \underline{c}^N = 1.$$

Hence,

$$\lim_{N\to\infty} c^N = 1.$$

We next show that the total expected VCG payment converges to the total expected Lagrange payment when N goes to infinity. To calculate $\lambda(t)$, we solve the following one-step problem:

$$\max_{U(t)} X^T(t)QX(t) + U^T(t)RU(t) + \mathbb{E}\left[X^T(t+1)P_{t+1}X(t+1)\right]$$

subject to

$$1^T U(t) = 0.$$

where P_t is the Ricatti matrix of the unconstrained problem where balance constraint $1^T U(t) = 0$, or $u_1(t) = -\sum_{i=2}^N u_i(t)$ is substituted in both the objective and the state equation.

The Lagrangian is,

$$\mathcal{L} = X^T(t)QX(t) + U^T(t)RU(t) + \mathbb{E}\left[X^T(t+1)P_{t+1}X(t+1)\right] - \lambda(t)1^T U(t)$$

Take partial derivative with respect to $U(t)$ and $\lambda(t)$, we have

$$
\begin{aligned}
&\frac{\partial \mathcal{L}}{\partial U(t)} \\
=&2RU(t) + 2B^T P_{t+1} BU(t) + 2B^T P_{t+1} AX(t) - \lambda(t)1 = 0,
\end{aligned}
$$

$$
\frac{\partial \mathcal{L}}{\partial \lambda(t)} = 1^T U(t) = 0.
$$

The Lagrange multiplier $\lambda(t)$ is thus calculated as:

$$
\begin{aligned}
\lambda(t) = 2\left[1^T (R + B^T P_{t+1} B)^{-1} 1\right]^{-1} \cdot \\
1^T (R + B^T P_{t+1} B)^{-1} B^T P_{t+1} AX(t) = \Phi_t X(t). \quad (4.4.2.1)
\end{aligned}
$$

At time s, we denote $\lambda(s,t)$ as the Lagrange multipliers associated with the balance constraint $1^T U(s,t) = 0$. From Theorem 8, we have

$$
\lim_{N\to\infty} \left[\left(\sum_{t=s}^{T-1} \lambda^N(s,t) u_i^{*N}(s,t) \right) - p_i^N(s) \right] = 0.
$$

Summing over s, we have

$$
\begin{aligned}
&\lim_{N\to\infty} \sum_{s=0}^{T-1} \left[\left(\sum_{t=s}^{T-1} \lambda^N(s,t) u_i^{*N}(s,t) \right) - p_i^N(s) \right] \\
=&\lim_{N\to\infty} \sum_{t=0}^{T-1} \left[\left(\sum_{s=0}^{t} \lambda^N(s,t) u_i^{*N}(s,t) \right) - p_i^N(t) \right] = 0.
\end{aligned}
$$

From (4.4.0.9), we have, at time s,

$$
\lambda(s,t) = \Phi_t \sum_{\tau=0}^{s} X(\tau, t),
$$

and at time $s-1$,

$$\lambda(s-1,t) = \Phi_t \sum_{\tau=0}^{s-1} X(\tau,t).$$

Therefore,

$$\lambda(s,t) = \lambda(s-1,t) + \Phi_t X(s,t).$$

The Lagrange multiplier $\lambda(t)$ associated with the balance constraint $1^T U(t) = 0$ can be calculated as:

$$\lambda(t) = \Phi_t X(t) = \Phi_t \sum_{s=0}^{t} X(s,t) = \lambda(t,t).$$

As a result,

$$\begin{aligned}\lambda^N(t) u_i^{*N}(t) &= \lambda^N(t) \sum_{s=0}^{t} u_i^{*N}(s,t)\\ &= \sum_{s=0}^{t} \left[\left(\lambda^N(s,t) + \Phi_t \sum_{\tau=s+1}^{t} X(\tau,t) \right) u_i^{*N}(s,t) \right]\end{aligned}$$

Because $X(0)$ is independent of $W(t)$ and $W(t)$ are i.i.d.,

$$\mathbb{E}[X(\tau,t) u_i^{*N}(s,t)] = 0, \text{ for } \tau \geq s+1,$$

Therefore,

$$\begin{aligned}&\lim_{N\to\infty} \mathbb{E} \sum_{t=0}^{T-1} \left[\lambda^N(t) u_i^{*N}(t) - p_i^N(t) \right]\\ &= \lim_{N\to\infty} \mathbb{E} \sum_{t=0}^{T-1} \left[\left(\sum_{s=0}^{t} \lambda^N(s,t) u_i^{*N}(s,t) \right) - p_i^N(t) \right] = 0.\end{aligned}$$

□

5. CONCLUSIONS

In Chapter 2, we have developed a layered decomposition approach that permits a holistic solution to solving the scheduling, storage and pricing problems of charging stations. The key idea is to decompose problems by time-scale. At the top layer, the long-term pricing is determined with grid power price and renewable energy assumed to be their deterministic long-term averages, and total consumption given by the static price-demand curve. The middle layer determines the amounts of energy to buy from the grid and to use for charging, with the mean rate of the stochastic arrivals depending on the price set at the top-layer. At the bottom layer, the resulting real-time scheduling of EVs becomes trivial and is solved by an earliest deadline first policy.

In Chapter 3, we have formulated the ISO problem of allocating the power supply and demand over heterogeneous energy producing or consuming agents, connected to a smart-grid in a dynamic fashion with network flow constraints, both under a deterministic setting and a stochastic setting when there are uncertainties affecting generation as well as consumption. We have proposed two decentralized iterative algorithms to solve the deterministic version and stochastic version of the problem. We have shown that under concavity assumptions, Algorithm 1 achieves the global maximum of social welfare in the absence of noise. The ISO will play a central role in inducing co-operation among agents by declaring prices, and agents do not need to be aware of each others' states or utilities. The only communication from the ISO is the price policy, and from the agents their energy supply or consumption in response to prices. LMPs arise when constraints on branches for power flow are binding. Compared to the current short-sighted one-step ISO scheme, simulation results shows that there is an increase in social welfare for both day-ahead and real-time market when Algorithms 1 and 2 are applied respectively.

In Chapter 4, we have shown that for the special case of LQG agents, by careful construction of a sequence of layered VCG payments over time, the intertemporal effect of current bids on future payoffs can be decoupled, and truth-telling of dynamic states is guaranteed if system parameters are known and agents are rational. We have also shown that a modification of the VCG payments, called scaled-VCG payments, achieves Budget Balance and Individual Rationality for a range of scaling, under a certain identified Market Power Balance condition. In the asymptotic regime of increasing population of agents, the scaled-VCG payments converge to the Lagrange payments, that is the payments that the agents would make in the absence of strategic consideration.

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