

Wavelet-Based Hydrological Time Series Forecasting

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Abstract: These days wavelet analysis is becoming popular for hydrological time series simulation and forecasting. There are, however, a set of key issues influencing the wavelet-aided data preprocessing and modeling practice that need further discussion. This article discusses four key issues related to wavelet analysis: discrepant use of continuous and discrete wavelet methods, choice of mother wavelet, choice of temporal scale, and uncertainty evaluation in wavelet-aided forecasting. The article concludes with a personal reflection on solving the four issues for improving and supplementing relevant wavelet studies, especially wavelet-based artificial intelligence modeling. DOI: [10.1061/\(ASCE\)HE.1943-5584.0001347](https://doi.org/10.1061/(ASCE)HE.1943-5584.0001347). © 2016 American Society of Civil Engineers.

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Introduction

Understanding the variability of hydrological processes is an essential and important scientific topic in hydrology studies, but it is also a difficult problem due to the complex stochastic nature of hydrological processes (Shoib et al. 2014). Hydrological time series analysis and forecasting is an effective approach to determine the variability of hydrological processes and predict future values. During recent years wavelet modeling has become popular for hydrological time series forecasting because the wavelet analysis method has the superiority of handling the nonstationary variability of hydrological processes (Nourani et al. 2014; Sang et al. 2015). For wavelet model inputs, the original hydrological series are usually decomposed into a set of subsignals by continuous or discrete wavelet method, called data preprocessing. Each subsignal plays a different role in the original series, and the behavior of each subsignal is distinct. Therefore, a wavelet model is constructed such that wavelet decomposition results of the original input series are the input data vector and the original output series are still

the output data vector (Kisi 2009). Compared with conventional single models, the process of hydrological time series forecasting by wavelet models is more easily guided because it takes the variability of hydrological processes into consideration. However, a set of key issues influences the wavelet-aided data preprocessing and modeling practice, and the choices of proper mother wavelet and temporal scale are two of the most important issues. In fact, these two issues constitute the essential basis of all wavelet analyses. Although there are a multitude of relevant studies (Sang 2012; Nourani et al. 2014; Shoib et al. 2014), these issues have not been completely resolved, and there is no universal method for the choice of mother wavelet and temporal scale yet.

Following the theories of wavelet analysis (Percival and Walden 2000), the suitable mother wavelet and temporal scale should accurately reflect the deterministic characteristics and true components (periodicities, trend, etc.) in a hydrological time series under multitemporal scales. Although we clearly know the mathematical equations and properties of all wavelets, the true components in the raw hydrological data are not known. Thus, there forms a paradox: on one hand, we need to choose a suitable wavelet and temporal scale to identify the true components, but on the other hand, we should first know the true components to choose a suitable wavelet and temporal scale. This is the primary reason for the difficulty in the choice of mother wavelet and temporal scale.

In this article the authors offer personal opinions and suggestions for wavelet modeling, hopefully as an improvement and a supplement for relevant studies. Because a large number of studies describing various types of wavelet models have been reported, they are not repeated here. Four issues of concern in wavelet modeling here are (1) discrepant use of continuous and discrete wavelet method, (2) choice of mother wavelet, (3) choice of temporal scale, and (4) uncertainty evaluation in wavelet-aided forecasting. These issues are now discussed.

Key Issues in Wavelet Modeling

Discrepant Use of Continuous and Discrete Wavelet Method

The basic objective of wavelet analysis is to achieve a complete representation of the localized and transient phenomena occurring at different temporal scales (Percival and Walden 2000; Labat

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2008). Generally, wavelet analysis can be divided into two types: continuous and discrete. Both of them have been widely applied in hydrology. In the specific area of wavelet-aided modeling, the discrete wavelet method has gotten more applications compared with the continuous wavelet method. Many studies discussed the problem and gave the reason that the continuous wavelet method generates large amounts of data and requires more computational time and resources (Adamowski and Sun 2010; Tiwari and Chatterjee 2010). However, they did not clearly explain the problem, and more convincing reasons should be given.

In the authors' opinion, the continuous and discrete wavelet method has different superiority for hydrological time series analysis. The continuous wavelet method is superior for determining both the scale contents of a series and how they vary in time. In their excellent work, Torrence and Compo (1998) placed the continuous wavelet transform method in the framework of statistical analysis by formulating a significance test. This has made the continuous wavelet method become more effective and rapidly develop. The continuous wavelet method of course can be used for wavelet modeling, just as discussed by Shoaib et al. (2014). Generally, more data can give more useful information, which would be favorable for the data-driven modeling practice (Singh 1998). The continuous wavelet method can identify the complex characteristics of a time series under multitemporal scale, based on which the continuous wavelet-based models can perform better than those single models. However, there is much repeated information (called data redundancy) in the continuous wavelet results of a time series, and the results get more impacts from the boundary effects, which would influence the stability of wavelet modeling structure and correspondingly cause more uncertainty. It is the main reason causing the worse performance of the continuous wavelet models compared with the discrete wavelet models, so the former has limited application in wavelet modeling practice.

The superiority of the discrete wavelet method is to decompose a series into subsignals given proper wavelet and temporal scale, and the result can guide wavelet threshold denoising and wavelet decomposition. Because those wavelets used for discrete wavelet transform must meet the orthogonal properties, the results can overcome the problem of data redundancy in the continuous wavelet transform. On the basis of the discrete wavelet results of series, we can avoid the influences of noise and multi components overlapping on the hybrid wavelet-artificial intelligence models. Therefore, the discrete wavelet method is commonly employed for hydrological time series forecasting.

As a result, the continuous and discrete wavelet methods have different advantages for time series analysis. If our purpose is to understand complex localized and nonstationary variability of a time series, we can use the continuous wavelet method. If we want to do denoising, identification of true components, especially wavelet modeling, we should use the discrete wavelet method.

Choice of Mother Wavelet

Differing from other transform techniques whose basis functions are fixed, the choice of proper mother wavelet is the foremost task in all wavelet analyses because results of time series analysis are very sensitive to the wavelets used. There is a large number of wavelets which are available for time series analysis. Different wavelet functions are characterized by their distinctive support region and vanishing moment. The support region of a wavelet reflects its feature of localization ability, and the vanishing moment of a wavelet reflects its ability of representing polynomial behavior or information of the data. Generally, all mother wavelets can be divided into two types, orthogonal or nonorthogonal. Seven

wavelet families, Haar, Daubechies (dbN), Coiflets (coifN), Symlets (symN), BiorSplines (biorM,N), ReverseBior (rbioM,N), and DMeyer (dmey), are orthogonal wavelets. Three wavelet families, Morlet (morl), Mexican hat (Marr), and Gaussian (gaus), are non-orthogonal wavelets. The use of an orthogonal wavelet implies the use of the discrete wavelet transform whereas a nonorthogonal wavelet can be used with either the discrete or the continuous wavelet transform (Torrence and Compo 1998).

The essence of wavelet transform is to discover the similarity between the analyzed series and wavelet (Walker 1999), but it cannot be easily carried out in practice, and an appropriate wavelet cannot be easily chosen. Many studies have discussed the choice of wavelet function. For instance, Torrence and Compo (1998) suggested choosing a nonorthogonal wavelet by comparing "width and shape" similarity between wavelet and series; Schaeffli et al. (2007) suggested that the chosen wavelet should have progressive and linear phases, exhibit good time-frequency localization, and be adapted to the trade-off between time and scale resolutions; and Nourani et al. (2014) suggested that similarity in the shape between wavelet and raw series is often the best guide in choosing a reliable wavelet. These suggestions are mainly qualitative and empirical, and cannot be easily implemented in practice. Besides, many other studies tried to find out one or some suitable wavelets for wavelet modeling through experiments (Nourani et al. 2011; Maheswaran and Khosa 2012; Singh 2011; Shoaib et al. 2014). However, the universality and usability of these selected wavelets are limited by a lack of evidence.

In the authors' opinion, both the properties of wavelets and the composition of series should be considered for choosing a wavelet, and three key points should be clarified for the issue. The first is that four key properties of wavelets should be concerned: (1) the wavelet should have the progressive and linear phase; (2) the wavelet should exhibit good localization both in time and frequency domains; (3) the wavelet should be adapted to the trade-off between time and scale resolutions (Schaeffli et al. 2007); and (4) the wavelet should also meet the orthogonal condition, because it is required for wavelet decomposition, denoising, multiresolution analyses, and many other wavelet analyses (Sang et al. 2013). The second is that because observed hydrological data usually include noise, which contaminates the true components of the series (Yevjevich 1972), the similarity between wavelet and those true components in raw noisy series, but not raw noisy series, should be the basis for choosing the proper mother wavelet; in other words, the appropriate wavelet should at least meet the need of accurately separating the true components from noise through wavelet denoising practice (Donoho 1995). The third is that because different time series usually show different characteristics and include different true components, it may be more feasible to establish a universal rule or criterion for the choice of wavelet, but not to find a specific method or wavelet for the problem. Sang et al. (2013) discussed the problem, and used the statistical indices to establish the criteria for choosing a proper wavelet (Table 1). The criteria quantify the similarity between the denoised series and raw series, and restrict the pure random characteristics of noise. To be specific, the mean values of the raw and denoised series should be similar; the standard deviation value of the denoised series should be smaller than that of the raw series; the coefficient of skewness of the raw and denoised series should be similar; the lag-1 autocorrelation coefficient of the denoised series should be bigger than that of the raw series; and the lag-1 autocorrelation coefficient of the removed noise should be close to zero. Analyses of various synthetic and observed series have verified its reasonableness, and the results accord well with the essence of wavelet transform. Hence, the criteria can be an approach to choosing a wavelet.

Table 1. Criteria for Assessment of the Reasonability of the Chosen Wavelet and the Denoising Result of Series

Series' type	Statistical indices			
	\bar{X}	σ	C_s	r_1
Original series	\bar{X}_o	σ_o	C_{s_o}	r_{1o}
Denoised series	$\bar{X}_m \approx \bar{X}_o$	$\sigma_m < \sigma_o$	$C_{s_m} \approx C_{s_o}$	$r_{1m} > r_{1o}$
Noise	$\bar{X}_n = \bar{X}_o - \bar{X}_m$	—	—	$r_{1n} \approx 0$

Note: Denoised series = true components in original series; Noise = random components removed from original series; Original series = observed series data. For the original series, denoised series, and noise, \bar{X}_o , \bar{X}_m , and \bar{X}_n are their mean; σ_o , σ_m , and σ_n are their standard deviation; C_{s_o} , C_{s_m} , and C_{s_n} are their coefficient of skewness; and r_{1o} , r_{1m} , and r_{1n} are their lag-1 autocorrelation coefficient, respectively. — = it is unknown about the statistical character of noise. It is cited from Sang et al. (2013).

Choice of Temporal Scale

The choice of proper temporal scale, called decomposition level for the discrete wavelet analysis, is another important issue for wavelet analysis, especially for wavelet modeling, because it directly determines the accuracy of the characteristics identified in a time series under multitemporal scales. In many previous studies, an algorithm of $\log_{10} n$, which is based on historical data length n , is recommended for choosing a temporal scale (Wang and Ding 2003; Belayneh et al. 2014; Nourani et al. 2014). Besides, another algorithm of $\log[n/(2v-1)]/\log(2)$, which is based on both series length n and the number v of vanishing moments of wavelet, is also widely used for choosing a temporal scale (de Artigas et al. 2006; Nalley et al. 2012). These algorithms may be arguable. Hydrological time series usually show dominant characteristics under multitemporal scales, and various time series with the same length show obviously different characteristics. Therefore, the choice of proper temporal scale should be closely based on the composition and characteristics of the analyzed data, but not the series data length, properties of wavelet, or other factors.

In the authors' opinion, two key points must be considered for the choice of a temporal scale: purpose and uncertainty. Discrete wavelet analysis is usually used for denoising or separating the true components of a series, as the basis of wavelet modeling, so two types of temporal scale need to be chosen for the two purposes, respectively. Besides, decomposition results of the series and the identified true components using the chosen temporal scale have uncertainty; therefore, the evaluation of uncertainty is another important problem. Sang (2012) discussed this problem and proposed the significance testing of discrete wavelet transform (DWT). He first established a stable reference energy function by doing Monte Carlo simulation to diverse noise types. Then, by comparing the energy function of the hydrological time series with the reference energy function, he presented a step-by-step guide for wavelet decomposition of series (Fig. 1), in which a suitable temporal scale can be chosen, and uncertainty can also be quantitatively evaluated using a proper confidence interval. Following the guide, we can easily judge whether a component under a certain temporal scale is noise or a true component, and the result can guide artificial intelligence modeling. Hence, the significance testing of DWT for hydrological time series analysis is recommended here for choosing a proper temporal scale.

Uncertainty Evaluation in Wavelet-Aided Forecasting

The combination of wavelet analysis with artificial intelligence models improves hydrological time series forecasting. Various studies have demonstrated the effectiveness of this practice due

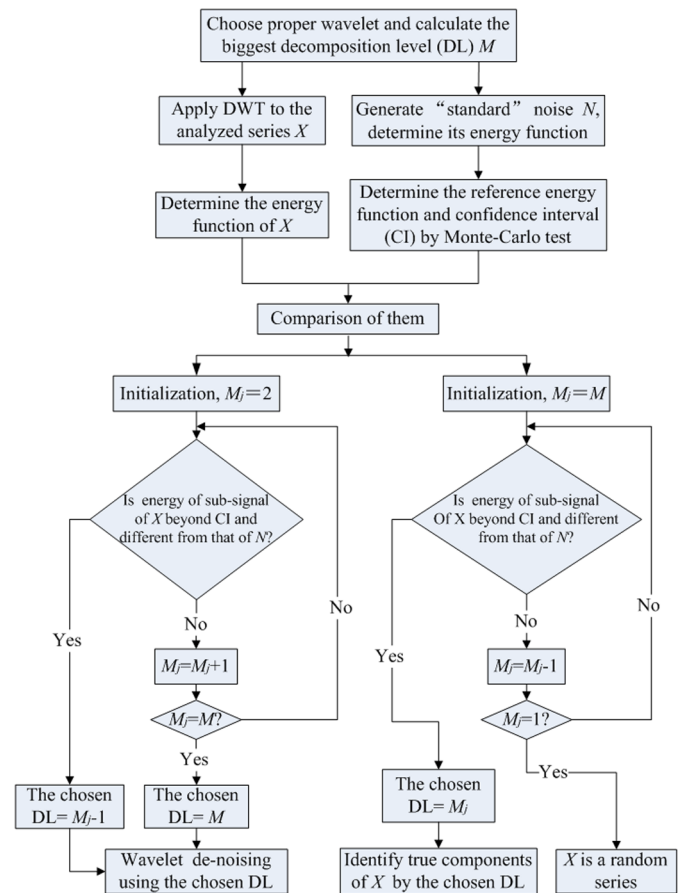


Fig. 1. The steps for comparing energy function of hydrological series with the reference energy function, and the steps for choosing decomposition level. "DWT" is discrete wavelet transform (reprinted from Sang 2012, © ASCE)

to the ability of wavelet analysis (Wang and Ding 2003; Sang 2013; Nourani et al. 2014; Tiwari and Adamowski 2013; Rathinasamy et al. 2013). However, forecasting results of hydrological extremes (including both maximum and minimum) have not been obviously improved in many case studies. These hydrological extremes reflect the complex influence of various random factors on hydrological processes and cause the uncertainty in hydrological time series forecasting. On the whole, present studies about wavelet modeling mainly focused on improving the accuracy of forecasting value through diverse combinations of models, but there is a limited study about uncertainty evaluation.

Hydrological processes generally have uncertainty, and the forecasting result with a single optimal value is not convincing and "honest." The optimal results do not take uncertainty into account effectively, so they cannot meet the practical needs adequately (Krzysztofowicz 2001). Sang (2013) tentatively proposed a wavelet modeling framework for hydrological time series forecasting (Fig. 2). The framework first separates different true components and removes noise in the original series through the discrete wavelet method. It then forecasts the former and quantitatively describes the random characteristics of noise. Finally, it adds them up and obtains the final forecasting results. Forecasting of true components is for obtaining the deterministic forecasting results, and noise analysis is for estimating the occurrence possibility of extremes. In the future, more efforts should be made to employ proper methods of uncertainty evaluation into the wavelet modeling

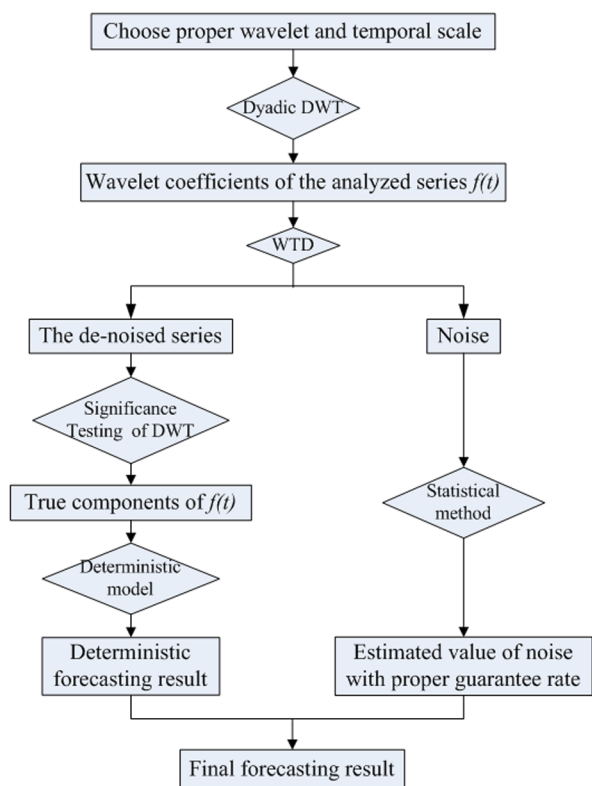


Fig. 2. Steps of hydrological time series forecasting by the wavelet modeling framework. “DWT” is discrete wavelet transform, and “WTD” is wavelet threshold de-noising (reprinted from Sang 2013, © ASCE)

processes and further to make the results of hydrological time series forecasting more reliable.

Conclusions

In this paper four key issues related to wavelet modeling are discussed, and the methods and suggestions for solving them are given. On the whole, the continuous and discrete wavelet methods have different advantages for time series analysis. We can use the continuous wavelet method to identify complex localized and non-stationary variability of a time series and use the discrete wavelet method to do denoising, identification of true components, especially wavelet modeling. In the process of wavelet analysis, both the properties of wavelets and the composition of series should be considered for choosing a proper mother wavelet; the choice of proper temporal scale should be closely based on the composition and characteristics of the analyzed data, and both the purpose and uncertainty should be considered in the choice of a proper temporal scale.

In conclusion, the wavelet models can improve hydrological time series forecasting. Because accurate wavelet analysis of time series is the kernel of wavelet modeling, more studies should focus on it in future studies. Especially, uncertainty evaluation is still an open issue in wavelet-aided modeling, and more studies should be given to the problem.

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