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SCS-CN METHOD.
PART I: DERIVATION OF SCS-CN-BASED MODELS

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Abstract

This part of the paper, in a sequence of two, provides an analytical treatment of the Soil Conservation Service Curve Number (SCS-CN) method including its derivation from a) early rainfall-runoff methods, such as the Mockus and Zoch methods, using the Horton method and b) first (linear)- and second (non-linear) -order hypotheses. After a critical review of the available analytical derivations, SCS-CN-based models are proposed for depression, interception storage, and initial abstraction, which form parts of the SCS-CN method. The performance of the existing and modified versions of the SCS-CN method is evaluated using field data.

Key words: soil conservation service, curve number, rainfall-runoff methods, direct runoff, abstractions, maximum retention.

1. INTRODUCTION

The Soil Conservation Service Curve Number (SCS-CN) method (SCS, 1956) is based on the water balance equation and two fundamental concepts. The first concept equates the ratio of the actual amount of direct surface runoff (Q) to the total rainfall (P) (or potential maximum surface runoff) to the ratio of the amount of actual (cumulative) infiltration (F) to the amount of potential maximum retention (S). The second concept relates the initial abstraction (I_a) to the potential maximum retention. Thus, the SCS-CN method consists of: a) water balance equation, b) proportional equality (Fig. 1) concept, and c) I_a–S concept, which, respectively, can be expressed as

$$P = I_a + F + Q\,, \tag{1}$$

$$\frac{Q}{P - I_a} = \frac{F}{S}\,, \tag{2}$$

$$I_a = \lambda S\,, \tag{3}$$

where P is the total rainfall, I_a is the initial abstraction, F is the cumulative infiltration excluding I_a, Q is the direct runoff, and S is the potential maximum retention or infiltration. The cumulative infiltration is the amount of depth of water infiltrated into the soil from the storm.

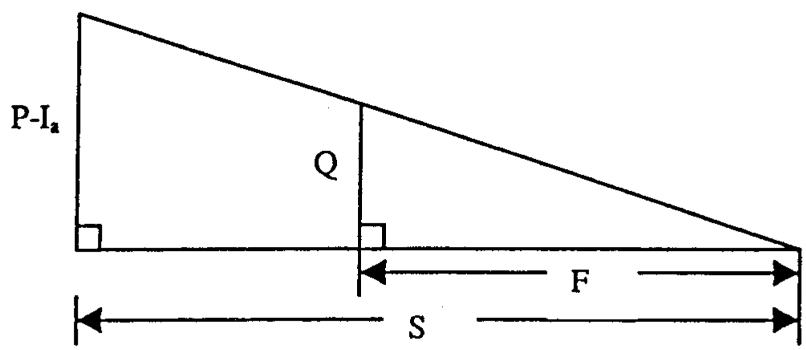

Fig. 1. Proportionality concept.

The fundamental concept (eq. 2) is primarily a proportionality concept (Fig. 1). In this figure, the ratio of vertical ordinates (Q to P–I_a) equals the ratio of corresponding horizontal abscissas (F to S). As $Q \rightarrow (P-I_a)$, $F \rightarrow S$. What this proportionality really says is that the amount of actual runoff as a fraction of the potential runoff will be in proportion to the amount of actual infiltration to the potential infiltration. The two proportions are linearly related. This hypothesis does not take into consideration the time rate of Q, P, or F. The relation of this proportionality to individual rainfall events will become clear from the discussion that follows from eq. (4). This proportionality enables partitioning (or dividing) (P–I_a) into surface water (Q) and subsurface water (F) for given watershed characteristics or S. This partitioning, however, undermines the saturated overland flow or source-area concept that allows runoff generation from only saturated or wet portions of the watershed. Consequently, the statistical theory (Moore and Clarke, 1981; 1982; 1983; Moore, 1983; 1985) based on the runoff production from only saturated (independent or interacting) storage elements is negated.

According to the SCS-CN method, the extent of runoff contribution of a storage element depends on its capacity or, alternatively, the magnitude of S and, therefore, the whole watershed should contribute to runoff, if S is taken to be a definite quantity. Thus, the ratio of the wet and total areas describing the contributing portion should be equal to one.

Parameter S of the SCS-CN method depends on the soil type, land use, hydrologic condition, and antecedent moisture condition (AMC). The initial abstraction accounts for the short-term losses, such as interception, surface storage, and infiltration. Parameter λ is frequently viewed as a regional parameter depending on geologic and climatic factors (Bosznay, 1989; Ramasastri and Seth, 1985). The existing SCS-CN method assumes λ to be equal to 0.2 for practical applications. Many other studies carried out in the United States and other countries (SCD, 1972; Springer *et al.*, 1980; Cazier and Hawkins, 1984; Ramasastri and Seth, 1985; Bosznay, 1989) report λ to vary in the range of (0, 0.3).

The second concept of the SCS-CN method (eq. 3) linearly relates the initial abstraction to the potential maximum retention. It is based on the graphical results of SCS (1956; 1971) depicting the plot between I_a and S. The data for S and I_a were derived from rainfall-runoff records of watersheds less than 10 acres in area. The S-values were derived from rainfall-runoff plots prepared for determining CN for normal AMC. More than 50% of the data points were within the limits $0.095 \le \lambda \le 0.38$. Errors in S were largely attributed to the computation of the average rainfall of the watershed. The I_a-values were computed by accumulating the rainfall amount from the beginning to the time of start of runoff. The large scatter in the data points was attributed to the errors in the estimates of I_a due to (SCS, 1971): a) the difficulty in determining the actual time of the start of rainfall because of storm travel and lack of instrumentation, b) the difficulty in determining the time of the start of runoff largely due to time lag in runoff from the watershed, and c) impossible determination of the amount of interception losses prior to runoff and its delayed contribution to runoff. As originally hypothesized (SCS, 1971), parameter S includes I_a. For this condition, eq. (3) can be re-written as $I_a = \lambda/(1-\lambda)S$ (Chen, 1982). Thus, for $\lambda = 0.2$, $I_a = 0.25\ S$.

Combining eqs. (1) and (2), the popular form of the SCS-CN method is obtained

$$Q = \frac{(P - I_a)^2}{P - I_a + S} \quad . \tag{4}$$

Equation (4) is valid for $P \ge I_a$; $Q = 0$ otherwise. Thus, the existing SCS-CN method with $\lambda = 0.2$ is a one-parameter model for computing surface runoff from daily storm rainfall, for the method was originally developed using daily rainfall-runoff data of annual extreme flows (Rallison and Cronshey, 1979). Mockus (1964; in Rallison, 1980) described the physical significance of parameter S of eq. (4) as the maximum difference of $(P-Q)$ that can occur for the given storm and watershed conditions. He, however, limited S to the rate of infiltration at the soil surface or the amount of water storage available in the soil profile, whichever is smaller. This description, however, compares the magnitude of infiltration rate with the volume of water retention in the soil, which is unwarranted.

Since parameter S can vary in the range of $0 \leq S \leq \infty$, it is mapped into a dimensionless curve number (CN), varying in a more appealing range $0 \leq CN \leq 100$, as follows:

$$S = \frac{1000}{CN} - 10 . \tag{5}$$

The underlying difference between S and CN is that the former is a dimensional quantity $[L]$ whereas the latter is a non-dimensional quantity. Although CN theoretically varies from 0 to 100, the practical design values validated by experience lie in the range (40, 98) (Van Mullem, 1989).

Since the inception of the SCS-CN method, the issues such as the rational derivation of the method, the rationale of initial abstraction, the analytical structure of the S-CN mapping relation, and the CN-AMC relations have been of major concern. These relations appear to be mysterious in a sense that no viable analytical or physical explanation of their development is yet available in the literature. Thus, the objective of this part of the paper is to revisit the existing SCS-CN method from an analytical perspective and to propose alternative analytical means to derive the existing SCS-CN method. To this end, the Mockus, Zoch, and depression and interception storage methods are first derived analytically using the Horton and SCS-CN methods. Then, the SCS-CN method is derived from the first (linear)- and second (non-linear)-order hypotheses. Finally, the implication of the SCS-CN derivation from the generalization of the Mockus method is discussed, and the performance of the existing and modified versions of the SCS-CN method evaluated using field data.

2. EARLY RAINFALL-RUNOFF METHODS

Rainfall-runoff methods developed during the early 1940's utilized infiltration data for computing the runoff amount. For example, Andrews (1954) grouped the infiltration data gathered from Texas, Oklahoma, Arkansas, and Louisiana in the United States using soil texture as a soil characteristic and developed a graphical rainfall-runoff method taking into account the soil texture and type and amount of cover, and conservation practices, combined into what is referred to as soil-cover complex or soil-vegetation-land use (SVL) complex.

Sherman (1949) was probably the first to propose plotting direct runoff versus storm rainfall. Using this concept, Mockus (1949) presented a rainfall-runoff model expressed as

$$Q = P_e (1 - 10^{-bP_e}), \tag{6}$$

where P_e is the storm rainfall amount excluding initial abstraction (inches), Q is the direct runoff (in inches), and b is an index related to the watershed and storm characteristics. Since b is estimated using meteorological data and physical watershed characteristics, the Mockus method can also be used for ungauged watersheds. It is important to note that eq. (6) is also valid for runoff rates and, therefore, parallels the model proposed by Zoch (1934; 1936), as shown below.

The Zoch model for the rising hydrograph can be expressed as

$$q = i_o (1 - \mathrm{e}^{-t/c}), \tag{7}$$

where q is the runoff rate (inch/hr), i_o is the uniform rainfall intensity (inch/hr), t is the time, and c is described as the runoff coefficient. For comparing with the Mockus method, eq. (6) is recast in exponential form as

$$Q = P_e (1 - e^{BP_e}), \tag{8}$$

where $B = b \ln(10)$. A comparison of eq. (7) with eq. (8) yields

$$\frac{Q}{P_e} = \frac{q}{i_0} \tag{9}$$

and

$$P_e = \frac{1}{cB} t. \tag{10}$$

Equation (10) implies that the cumulative rainfall grows linearly with time or uniform rainfall intensity, i_0, is constant during a storm. However, during a typical isolated storm, the rainfall intensity first generally grows with time, attains a peak, and then finally reduces to zero. From eq. (10) a relation between the uniform rainfall intensity, i_0, B, and c can be derived as

$$i_0 = 1/cB. \tag{11}$$

The assumption of constant uniform rainfall intensity is reasonable in the sense that it conforms with the infiltration derived from field measurements. Such an assumption also helps signify parameter b of the Mockus model, as shown later. The following section analytically derives the Mockus and Zoch models from the combination of the Horton model and the SCS-CN proportionality concept (eq. 2).

3. ANALYTICAL DERIVATION OF THE MOCKUS AND OTHER METHODS

The Mockus method and the Andrews method, which lead to the existing SCS-CN method, compute the runoff volume or rate or indirectly compute the infiltration volume or rate using the water balance equation. Therefore, as also pointed out by Ponce and Hawkins (1996) among others, the SCS-CN method can also be construed as an infiltration loss model. Further analysis also shows analytically that the SCS-CN method is an infiltration model. To this end, using the SCS-CN concept of proportionality (eq. 2 for $I_a = 0$), the Mockus method is derived from the Horton model.

Derivation of Mockus method

The Horton infiltration model is expressed as

$$f = f_c + (f_0 - f_c)\,e^{-\alpha t} \tag{12}$$

where f is the infiltration rate $[LT^{-1}]$ at time t, f_0 is the initial infiltration rate $[LT^{-1}]$ at time $t = 0$, α is the decay constant $[T^{-1}]$, and f_c is the final infiltration rate $[LT{-}^{1}]$. It is noted that eq. (12) is valid for the time t past ponding. An integration of eq. (12) leads to the cumulative infiltration F at time t as

$$F = \frac{f_0 - f_c}{\alpha}(1 - e^{-\alpha t}), \tag{13}$$

where F excludes f_c (or static or gravitational) component of infiltration. Thus, as $t \to \infty$, $F \to (f_0 - f_c)/\alpha$. From eq. (2) as $Q \to (P - I_a)$, $F \to S$, which is valid for time t approaching infinity. Therefore,

$$S = \frac{f_0 - f_c}{\alpha}. \tag{14}$$

From common experience on infiltration tests $f_0 = i_0$, where i_0 is the uniform rainfall intensity at time t (time past ponding) equal to zero. Its substitution into eq. (14) yields

$$f_0 - f_c = i_0 - f_c = i_e = \alpha S, \tag{15}$$

where i_e is the effective uniform rainfall intensity. Equation (15) describes the relationship among four parameters f_0, f_c, α, and S and defines the Horton parameter α equal to the ratio of the effective uniform rainfall intensity, i_e, to the potential maximum retention, S. This implies that α increases as i_e increases and decreases as S increases or CN decreases, and *vice versa*. Thus, α depends on the magnitude of the rainfall intensity and soil type, land use, hydrologic condition, antecedent moisture that affect S. Further substitution of eq. (14) into eq. (13) yields

$$\frac{F}{S} = (1 - e^{-\alpha t}). \tag{16}$$

Coupling eq. (16) with eq. (2) (for $I_a = 0$) yields

$$\frac{Q}{P} = (1 - e^{-\alpha t}) . \tag{17}$$

An assumption of rainfall P growing linearly with time t leads to

$$P = i_e t , \tag{18}$$

where P excludes static infiltration. Equation (18) asserts the general notion that P grows unbounded (Ponce and Hawkins, 1996). Substitution of eq. (15) into eq. (18) yields

$$\frac{P}{S} = \alpha t . \tag{19}$$

Substituting eq. (19) into eq. (17) leads to

$$Q = P(1 - e^{-P/S}) , \tag{20}$$

which is equivalent to eq. (8) for $B = 1/S$ and $I_a = 0$. Replacing P by P_e (= $P - I_a$) in eq. (20) yields the Mockus equation (eq. 6) for $S = 1/[b \ln(10)]$. For P to be the total rainfall for derivation of the Mockus method, f_c should be equal to zero. Furthermore, coupling of eq. (19) with eq. (18) yields the same parametric relation as described by eq. (15).

Equation (20) (the Mockus method) can also be derived using a first-order linear hypothesis for the variation of S with time or rainfall as

$$\frac{dS_t}{dt} = -\alpha S_t \qquad \text{or} \qquad \frac{dS_t}{dP} = -B S_t , \tag{21}$$

whose solution is

$$S_t = S_0 e^{-\alpha t} \qquad \text{or} \qquad S_t = S_0 e^{-BP} \tag{22}$$

for $S_t = S_0$ (or S in eq. 20) at $t = 0$ or $P = 0$. It implies that for a given antecedent moisture condition (AMC), $S = S_0$ represents the maximum possible amount of infiltration or the potential maximum space available for moisture retention. Replacing S_t by $(S_0 - F)$ and B by $1/S$, eq. (22) can be recast for F as

$$\frac{F}{S_0} = 1 - e^{-\alpha t} = 1 - e^{-P/S} . \tag{23}$$

Coupling eq. (23) with eq. (2) for $I_a = 0$ leads to eq. (20), which represents the alternative form of the Mockus method. It is worth emphasizing that eq. (22) describes the variation of S with time or rainfall. As time or rainfall increases, S_t decreases exponentially. For antecedent dry, wet, or normal condition, S_o corresponds to dry, wet, or normal condition of the soil before the start of the storm. Thus, $S_o = S$.

Derivation of Zoch model

The Zoch model (eq. 7) can be analytically derived assuming that the basic SCS-CN proportionality concept also holds for rainfall-runoff rates as below:

$$\frac{Q}{P} = \frac{F}{S} = \frac{q}{i_0}, \tag{24}$$

which is analogous to eq. (9). Combination of eqs. (24) and (2) for I_a and f_c equal to zero leads to a relation among runoff rate (q), rainfall intensity (i_0), and the average storage capacity (s_{av}) as

$$q = \frac{i_0^2}{i_0 + s_{av}}, \tag{25}$$

which is similar to the equation proposed by Yu (1998).

For a given storm duration, eq. (25) computes the storm-averaged runoff rate corresponding to the storm-averaged rainfall intensity, and thus, it does not compute the actual instantaneous runoff rate corresponding to the instantaneous rainfall intensity. The average storage capacity, s_{av}, refers to the ratio of S to the storm duration T_s, and thus, it is the storm-average of the potential maximum storage capacity (rate). There fore, it can be defined as the maximum possible storm-average rate of infiltration. In other words, s_{av} is analogous to the potential maximum phi-index for a storm.

Similar to the derivation of the Mockus method, parametric relations can be derived assuming that

$$\frac{P}{S} = \alpha t = t/c, \tag{26}$$

which also implies that the cumulative rainfall grows linearly with time. Substitution of eq. (26) into eq. (20) yields eq. (7), which is the Zoch model. From eq. (26), α can be derived as

$$\alpha = 1/c. \tag{27}$$

Coupling of eq. (27) with eq. (15) (for $f_c = 0$) leads to

$$S = c\, i_0. \tag{28}$$

Equation (28) misleadingly describes c as the runoff coefficient because it multiplies the rainfall intensity in the equation. However, c in eq. (28) has the dimension of time, unlike the runoff coefficient which is non-dimensional. Furthermore, S does not represent the runoff amount, rather it is the potential maximum retention.

Derivation of depression and interception storage models

The depression storage model is expressed as (Bras, 1990):

$$\frac{V_s}{S_d} = 1 - e^{-\beta P_e} , \tag{29}$$

where V_s is the equivalent depth of depression storage (mm), P_e is the effective rainfall computed as the total rainfall amount minus the sum of interception loss and infiltration (mm), S_d is the depression storage (mm), and β is a constant (mm^{-1}). Linsley *et al.* (1982) recommended S_d to vary in the range 10–50 mm for most of the catchments. The value of β is estimated by assuming that for very small values of rainfall excess (P_e close to 0), essentially all the rainfall goes into depression storage ($dV_s/dP_e = 1$), leading to a definition of β as: $\beta = 1/S_d$.

Similar to the fundamental proportionality concept (eq. 2) of the SCS-CN method, the following proportionality can be assumed to hold:

$$\frac{Q}{P_e} = \frac{V_s}{S_d} , \tag{30}$$

which states that the ratio of the actual runoff (Q) to potential runoff (P or P_e) is equal to the ratio of the actual depth of depression storage (V_s) to the potential maximum depression storage (S_d). Coupling of eq. (30) with eq. (8) yields a relation analogous to eq. (29), which is the depression storage model, for $\beta = 1/S_d$.

The interception loss comprises two distinct elements (Horton, 1919). The first is the interception storage defined as the depth (or volume) of rainfall retained by the foliage against the forces of wind and gravity. The second is the evaporation loss from the foliage surface, which takes place throughout the duration of rain storm and afterwards. A combination of these processes can be expressed mathematically as (Chow, 1964):

$$L_i = S_i + K\,E\,t , \tag{31}$$

where L_i is the interception loss (mm), S_i is the interception storage depth (mm), K is the ratio of evaporating foliage surface to its horizontal projection, E is the evaporation rate in mm/hour, and t is the storm duration in hours. S_i usually varies from 0.25 to 1.25 mm and its variation depends on the amount of rainfall as (Bras, 1990):

$$S_i = S_v(1 - e^{-P/S_v}) , \tag{32}$$

where P is the total precipitation and S_v is the storage capacity of vegetation for the projected area of canopy. Using a similar approach as in the derivation of the depression model, the interception model (eq. 32) can also be derived.

4. DERIVATION OF SCS-CN METHOD

Generalization of the Mockus method

Expanding the terms of the Mockus and SCS-CN methods, Mishra and Singh (1999) analytically approximated the Mockus method to yield the SCS-CN method, which is close to the generalization of the Mockus method (Rallison and Miller, 1982). This generalization was found to hold for $BP < 1$, for which the results of the SCS-CN method deviated from those of the Mockus method by approximately ±20%.

Statistical derivation of SCS-CN method

The probability-based derivations of the SCS-CN method are due to Schaake *et al.* (1996) and Yu (1998; 2000). In their derivation, Schaake *et al.* (1996): a) assumed exponential distributions for the variation of infiltration and rainfall; b) neglected the possibility that instantaneous rainfall rates during the storm might exceed the instantaneous infiltration capacity; and c) assumed that infiltration was independent of rainfall. Yu (1998) derived the SCS-CN method assuming that a) the spatial variation of infiltration capacity has an exponential distribution and b) the temporal variation of rainfall rate also follows an exponential distribution. He was able to derive an expression for the average rainfall-excess rate of the form of eq. (25), leading to the definition of S similar to the phi-index (Istanbulluoglu, 2000). Based on the derivation of a probabilistic mean of two independent variables, Yu (2000) suggested another approach to derive the SCS-CN method. He defined the storage capacity as the maximum amount of water holding capacity of a soil at a given location. However, the basic problem with the statistical derivations is one of infiltration rates assumed to be independent of rainfall intensity, which is in contrast with reality (Cook, 1946; Moldenhauer *et al.*, 1960; Murai and Iwasaki, 1975; Hawkins, 1982; Flanagan *et al.*, 1988; and Yu *et al.*, 1997). Therefore, these derivations lead to an ambiguity in the SCS-CN derivation.

SCS-CN derivation from first-order hypothesis

Similar to the derivation of the SCS-CN method from the generalization of the Mockus method, it is possible to show the variation of S with P from eq. (22) exhibiting the first-order exponential variation with rainfall as: $S_t/S_o = e^{-BP}$. An expansion of the right-hand side of this equation following Mishra and Singh (1999) yields

$$\frac{S_t}{S_0} = \left[1 - BP + \frac{(BP)^2}{2!} - \frac{(BP)^3}{3!} \ldots\ldots + (-1)^N \frac{(BP)^N}{N!}\right], \tag{33}$$

where N is an integer. Equation (33) can be approximated well as (Mishra and Singh, 1999):

$$\frac{S_t}{S_0} \approx \left[1 - BP + (BP)^2 - (BP)^3 ...... + (-1)^N (BP)^N\right], \tag{34}$$

which is valid for $BP < 1$. Equation (34) can also be written as

$$\frac{S_t}{S_0} = \frac{1}{1 + BP}, \tag{35}$$

which is also valid for $BP < 1$. For $B = 1/S_0$, eq. (35) can be re-written as

$$S_t = \frac{S_0^2}{P + S_0}. \tag{36}$$

Equation (36) describes the variation of S_t with P. It is again noted that $S_0 = S$, as above. Substitution of $S_t = S_0 - F$ in eq. (36) yields

$$\frac{F}{S_0} = \frac{P}{S_0 + P}. \tag{37}$$

Coupling eq. (37) with the proportionality concept (eq. 2 for $I_a = 0$) or with the water balance equation with $I_a = 0$ yields eq. (4), which is the SCS-CN method for $I_a = 0$.

Derivation of SCS-CN proportional equality

It is further possible to analytically derive the basic proportionality concept of the existing SCS-CN method using the Horton or the first-order storage hypothesis and water balance equation. Coupling of eq. (16) with eq. (19) with the assumption of the cumulative rainfall growing linearly with time yields $F/S = (1 - e^{-P/S})$, which, upon approximation, leads to eq. (37), for $S_o = S$. Its substitution into the water balance equation (eq. 1) yields $Q/P = P/(S + P)$. A comparison of these equations yields the basic proportional equality expressed by eq. (2) for $I_a = 0$. Thus, the basic SCS-CN concept can be derived from the Horton equation or the first-order storage hypothesis and the water balance equation with the assumption that the cumulative rainfall grows linearly with time.

Non-linear derivation of SCS-CN method

The second-order non-linear hypothesis for storage variation with time can be expressed as

$$\frac{dS_t}{dt} = -\gamma S_t^2, \tag{38}$$

where γ is the proportionality coefficient. Integration of eq. (38) yields

$$-\frac{1}{S_t} = -\gamma t + D\,, \tag{39}$$

where D is an integration constant, which can be derived for the condition that at $t = 0$, $S_t = S_0$ (or S). Therefore, $D = -1/S_0$. Its substitution into eq. (39) leads to

$$\frac{S_t}{S_0} = \frac{1}{1+\gamma S_0 t}\,. \tag{40}$$

Analogous to eq. (26), it is assumed that the following holds:

$$\frac{P}{S_0} = \gamma S_0 t\,, \tag{41}$$

which yields

$$i_e = \gamma S_0^2\,, \tag{42}$$

where i_e is the effective uniform rainfall intensity $[LT^{-1}]$. Equation (42) describes γ as the ratio of i_e to the square of S_0, with dimensions $[L^{-1}T^{-1}]$. Further coupling of eq. (42) with eq. (15) yields $\gamma = \alpha / S_0$, which expresses γ as the ratio of the Horton decay parameter to the potential maximum retention, S_0 (= S). Substitution of eq. (41) into eq. (40) leads to eq. (36), which when coupled with the water balance equation (eq. 1) along with $S_t = (S_0 - F)$ leads to the SCS-CN equation.

The above non-linear approach also leads to the inclusion of evapotranspiration directly in the SCS-CN method. The derivation is based on the following hypothesis of storage variation with time

$$\mathrm{d}S_t / \mathrm{d}t = -\gamma S_t^2 \left(1 - \bar{E} / E_p\right), \tag{43}$$

where γ is a proportionality coefficient, E_t is the rate of evapotranspiration at time t, and E_p is the potential maximum rate of evapotranspiration. Integration of eq. (43) yields

$$-1/S_t = -\gamma\left(1 - \bar{E}/E_p\right)t + D\,, \tag{44}$$

where $\bar{E}$ is the average rate of evapotranspiration during the storm and D is an integration constant, which can be derived for the condition that at $t = 0$, $S_t = S_0$ (or S). Therefore, $D = -1/S_0$. Its substitution into eq. (44) leads to

$$\frac{S_t}{S_0} = \frac{1}{1+\gamma(1-\bar{E}/E_p)S_0 t}\,. \tag{45}$$

Similar to eq. (41), it is assumed that the following relation holds:

$$\frac{P}{S_0} = \gamma(1 - \bar{E}/E_p)S_0 t. \tag{46}$$

Equation (46) yields

$$i_e = \gamma(1 - \bar{E}/E_p)S_0^2, \tag{47}$$

where i_e is the effective uniform rainfall intensity $[LT^{-1}]$ and γ has the dimension $[L^{-1}T^{-1}]$. Combination of eq. (47) with eq. (15) yields $\gamma = \alpha/[S_0(1 - \bar{E}/E_p)]$, which expresses the dependence of γ on evaporation. Its usual substitution into eq. (46) finally leads to the SCS-CN method. It is worth emphasizing here that since evaporation losses are negligible during a storm, such a derivation is useful for long duration storms, such as a day or longer.

SCS-CN derivation including initial abstraction

The above derivation of the Mockus method or the SCS-CN method from the Horton equation (eq. 12) assumed the time to ponding, t_p, equal to zero. Alternatively, these methods are valid for the time past ponding. It is, however, possible to incorporate t_p in the Horton model as

$$f - f_c = (f_0 - f_c)\,e^{-\alpha(t-t_p)}. \tag{48}$$

Integration of eq. (48) leads to the cumulative infiltration F at time t as

$$F = \frac{f_0 - f_c}{\alpha}\left[1 - e^{-\alpha(t-t_p)}\right], \tag{49}$$

where F excludes the static infiltration. From eq. (15) $i_e = f_0 - f_c = \alpha S$ and its substitution into eq. (49) yields

$$\frac{F}{S} = \left(1 - e^{-P_e/S}\right), \tag{50}$$

where P_e is described as

$$P_e = P - F_c - I_a = \alpha(t - t_p)\,S, \tag{51}$$

where F_c is the static (or gravitational) infiltration ($= f_c\,t$).

Substitution of $i_e = \alpha S$ in eq. (51) leads to an expression for I_a as

$$I_a = i_e\,t_p, \tag{52}$$

which describes I_a as the product of the effective uniform rainfall intensity and the time to ponding. In the context of infiltration, such a computation of I_a is reasonable and consistent with that occurring in the field. It, however, ignores water losses due to surface detention, evaporation from vegetative cover, and so on. An approximation of eq. (50), as for the above SCS-CN derivation from the Mockus method, yields

$$\frac{F}{S} = \frac{P_e}{P_e + S} . \tag{53}$$

Coupling of eq. (53) with the water balance equation (eq. 1) for deriving the Q/P_e ratio and comparing the resulting expression with eq. (53) yields the proportional equality described by eq. (2).

Alternatively, the derivation of Q/P_e ratio yields eq. (4), which is the existing SCS-CN equation for $F_c = 0$. Thus, it is proved that the existing SCS-CN method is the generalization or approximation of the Mockus method, which holds for P excluding both I_a and F_c. The important inference derived from such a derivation is that the formulation of the existing SCS-CN method for I_a has a sound justification within the framework of infiltration phenomenon observed in the field and is not a mere replacement of P by $(P-I_a)$ in eq. (4). Here, it is noted that although I_a is taken to be separate from S in the existing SCS-CN concept, it was assumed as a part of S for deriving $\lambda = 0.2$, on which the NEH-4 curve number tables are based. For the corresponding analytical treatment, the work of Chen (1982) can be referred to.

Development of an initial abstraction model

The initial abstraction includes interception, surface depression, part of infiltration, and evaporation. The interception is described by eq. (31) and the depression storage model is described as:

$$V_s = S_d (1 - e^{-\beta P_e}) , \tag{54}$$

where V_s is the equivalent depth of depression storage (mm), P_e is the effective rainfall computed as the total rainfall amount minus the sum of interception loss and infiltration (mm), S_d is the depression storage (mm), and β is a constant (mm^{-1}). Linsley *et al.* (1982) recommended S_d to vary in the range 10–50 mm for most catchments.

Using either the generalized approach or the second-order non-linear approach, the interception (eq. 31) and surface depression (eq. 54) models can be transformed, respectively, to

$$L_i = \frac{P S_v}{P + S_v} + K E t \tag{55}$$

and

$$V_s = \frac{P_v' S_d}{P_v' + S_d} , \tag{56}$$

where P is the total rainfall and P_v' is the effective rainfall (= $P-L_i$). The initial abstraction due to infiltration can be computed from eq. (52).

Thus, a complete initial abstraction model based on the SCS-CN concept can be derived as

$$I_a = \frac{P\,S_d}{P+S_d} + K\,E\,t + \frac{(P-L_i-F)S_d}{P-L_i-F+S_d} + i_e\,t_p, \tag{57}$$

where $(P - L_i - F)$ is the rainfall-excess and cumulative infiltration F is given as

$$F = \frac{(P-L_i)\,S}{P-L_i+S}. \tag{58}$$

Let $S_v + S_d = S'$. It is assumed that S' varies with time according to the above second-order law. At time $t = 0$, $S' = S_0'$. Then, at any time t, I_a, excluding evaporation (= $K\,E\,t$) and infiltration (= $i_e\,t_p$), can be given as

$$I_a = S_0' - S_t' = \frac{PS_0'}{P+S_0'}. \tag{59}$$

Coupling of eq. (59) with eq. (4) leads to

$$Q = \frac{P^4}{(P+S_0')[P^2+(P+S_0')S]}, \tag{60}$$

which is another simplified form of the SCS-CN model (eq. 4). It is noted that eq. (60) ignores evaporation and the initial (time to ponding) infiltration losses. S_0' can be derived from the coupling of eq. (59) with eq. (3) as

$$S_0' = \frac{\lambda\,S\,P}{P-\lambda\,S}. \tag{61}$$

If there exists a possibility to determine S_0' by some other means, then λ can alternatively be determined from eq. (61) as $\lambda = PS_0'/[S(P+S_0')]$, which, however, excludes evaporation and initial infiltration losses. Therefore, a general expression for computing λ can be given as

$$\lambda = \frac{PS_0'}{S(P+S_0')} + \frac{K\,E\,t}{S} + \frac{i_e\,t_p}{S}. \tag{62}$$

Equation (62) exhibits the dependence of λ on meteorological factors besides the factors governing the soil-vegetation-land use complex.

5. IMPLICATION OF GENERALIZATION OF THE MOCKUS METHOD

Mishra and Singh (1999) derived a modified version of the SCS-CN method from the Mockus methd which is given as

$$\frac{Q}{P} = \frac{P}{S + 0.5P}. \tag{63}$$

This led to a general form of the SCS-CN model

$$Q = \frac{P^2}{S + aP}, \tag{64}$$

where a is the coefficient replacing 0.5 in eq. (50). These models are explained from an analytical perspective by Mishra and Singh (1999). Upon comparison, they found that, within ±5% deviation, the results due to the existing SCS-CN method were in agreement with those of the Mockus method for $BP < 0.1$ and $BP > 20$ as well as with the results due to the modified method for $BP < 1$. For $1 < BP < 20$, the SCS-CN method deviated most (about −1.69%) from the Mockus method at $BP = 1.788$ whereas the modified method deviated increasingly from the Mockus method with increasing BP; at $BP = 1.0$, there was a +5% deviation from the results of the Mockus method.

Limitations of the SCS-CN and Mockus methods

Since the derivation of the SCS-CN, Mockus, and Zoch methods exclude the static portion of infiltration f_c, they all underestimate infiltration. In practical application, the usually defined condition, $F \rightarrow S$, is possible only when the condition, $(P/S)\, e^{-P/S} = 1$, is satisfied, which is indeterminate implying that F never approaches or equals S. The maximum possible F is equal to P/e and occurs at $P = S$. In addition, the Mockus equation depicts a double curvature at large BP-values indicating $\partial Q/\partial P$ even greater than 1, which is not possible. It can also be shown mathematically as follows.

Differentiation of eq. (8) with respect to P_e for $B = 1/S$ yields

$$\frac{\partial Q}{\partial P} = 1 + (P_e / S - 1)\, e^{-P_e/S}. \tag{65}$$

The condition that the left-hand side of eq. (65) be less than or equal to 1 yields $(P_e/S) \leq 1$. This implies that the Mockus method is applicable only for the rainfall amount less than or equal to S. For P greater than S, the Mockus method will yield runoff rates greater than the rainfall intensity, which is not possible. Alternatively, for application of the Mockus method, it can be shown that the time duration (t) should be less than or equal to the inverse of the Horton parameter α: $t \leq 1/\alpha$. Thus, the Mockus method is restricted in its application. Similarly, the applicability of the SCS-CN method can also be ascertained as follows.

The first derivative of eq. (4) (for $I_a = 0$) with respect to P is

$$\frac{\partial Q}{\partial P} = \frac{(P/S)(2+P/S)}{(1+P/S)^2}. \tag{66}$$

Since the direct runoff rate ($\partial Q/\partial t$) does not exceed the corresponding rainfall intensity ($\partial P/\partial t$), $\partial Q/\partial P$ can range between 0 and 1, as also suggested in the variable -source area concept (for example, Steenhuis *et al.*, 1995), which, however, is not supported by the SCS-CN concept, as described earlier. Evidently, from eq. (66), $\partial Q/\partial P \geq 0$ for all values of $P/S \geq 0$; $\partial Q/\partial P = 0$ at $P = 0$. For the upper limit, it can be shown that as $\partial Q/\partial P \rightarrow 1.0$, $S \rightarrow 0$. Similar inferences can be drawn if P is replaced by P_e in eq. (66); for $\partial Q/\partial P \geq 0$, $P \geq I_a$ since S is a non-negative quantity. Another condition that $\partial Q/\partial P \leq 1$ always holds, for S is a non-negative value. Thus, the existing SCS-CN method is more stable than the Mockus method.

Evaluation of performance of models using field data

The performance of models was evaluated using storm rainfall-runoff data of the sub-watersheds (Table 1) of the Luni basin which is located in an arid region of India. These watersheds vary significantly in size, ranging from 83.1 to 3050 km^2, and characteristics. Using physical characteristics of the watersheds, viz., soil type and land use, and NEH-4 tables, areally-weighted curve numbers were computed for each watershed and these are shown in Table 1. For this purpose, remote sensing data of IRS-1A LISS II were used to prepare land use, and soil maps were prepared from irrigation atlas. After converting to digital form using a geographical information system (GIS), these two maps were superimposed to classify different land use and soil types. Each resulting map was re-classified using the CN values obtained from the NEH-4 tables and a CN map was prepared. From this CN map, the average CN values for each sub-watershed were derived and these are presented in Table 1.

Employing the root mean square error (RMSE) criterion, model parameters were computed using the Marquardt algorithm (Lawson and Hanson, 1974). In all applications, the initial estimate of parameter *CN* of both the models was taken as 70 and the additional parameter of the modified SCS-CN model, λ, was taken as 0.2 (a standard value for the existing version of the SCS-CN method). *CN* ranged between 0 and 100 and λ was assumed to vary in the range (0, 1). The computed parameter-values are shown in Table 1. From the table it is apparent that *CN* of the existing SCS-CN method varies from 36.21 to 47.27 in all applications. It is noted that the computed curve numbers that largely depend on the antecedent moisture condition correspond to the values fitting the above events. Since the basic proportionality concept of the

Table 1

Computation of *CN* and *RMSE* for existing and modified SCS-CN methods

Seria l. No.	River	Gauging site	Catchment area [km^2]	No. of rainfall -runoff events	Parameter estimation					*RMSE* [mm]	
					Existing SCS-CN method		Modified SCS-CN method			Existing SCS-CN method	Modified SCS-CN method
					CN^a	CN^b	λ	CN^a	CN^c		
1	Sukri	Sojat Road	358.6	39	42.23	89.13	0.0000	17.86	80.4	53.678	42.199
2	Sukri	Sojat	626.0	37	40.01	85.66	0.0021	16.33	82.2	24.527	14.339
3	Sukri	Sarangwan	316.9	37	43.25	79.75	0.0000	17.09	81.3	63.714	49.030
4	Sukri	Sheopura	1285.0	36	36.21	81.46	0.0000	12.08	87.1	22.581	12.788
5	Guhia	Jhupelav	349.2	32	47.27	87.58	0.0000	20.56	77.1	67.405	59.523
6	Modiya	Hariamali	177.5	38	43.69	61.25	0.0000	15.58	83.1	67.325	56.862
7	Lilri	Dhaneri	463.3	42	38.96	86.03	0.0000	16.10	82.5	36.151	23.646
8	Phupheria	Dataredan	373.3	35	39.73	80.80	0.0000	15.88	82.8	26.713	11.420
9	Radia	Rohat	251.0	37	37.92	84.79	0.0000	13.42	85.6	29.100	14.388
10	Guria	Sandia	264.0	40	43.87	78.31	0.0000	19.37	78.6	34.796	26.692
11	Guhia Bala	Artia	209.0	36	39.92	78.81	0.0000	14.12	84.8	23.177	12.452
12	Guhia	Karniyali	3050.0	37	33.96	80.78	0.0000	7.58	92.1	34.883	24.568
13	Guhia	Singari	2837.6	37	41.04	80.59	0.0007	16.69	81.8	19.455	8.417
14	Guhia	Sabalpura	83.1	41	42.67	78.31	0.0000	13.87	85.1	44.006	34.725

Notations: Superscripts *a* and *c* stand for *CN* computed from rainfall-runoff data for the existing and modified version, respectively, and superscript *b* stands for *CN* computed from NEH-4 tables.

modified SCS-CN model differs from the existing SCS-CN method for which $\lambda = 0.2$, the computed CN values of both the models in an application will be different from each other. The modified model yields *CN* values ranging from 7.58 to 20.56 in these applications. Except for two watersheds, watersheds with Serial Nos. 2 and 13 in Table 1, for which λ is equal to 0.0021 and 0.0007, respectively, the value of λ of the modified model is equal to zero. Since the RMSE values from the modified model in all these applications are lower than those from the existing SCS-CN method, the former performs better than the latter. It is consistent with the results of Mishra and Singh (1999).

6. CONCLUSIONS

The following conclusions are derived from the study:

1. The SCS-CN method is a proportionality concept that partitions the total rainfall into direct surface runoff and infiltration.

2. Both the Zoch and Mockus methods can be derived from the Horton and SCS-CN methods.

3. The SCS-CN method is derived from (a) the generalization of the Mockus method, and (b) the linear first-order or second order non-linear hypothesis. Thus, the SCS-CN method can be construed as a conceptual model.

4. The SCS-CN method can be extended for evapotranspiration, enabling its applicability to long-duration storms.

5. The initial abstraction can be analytically incorporated in the existing SCS-CN method.

6. The development of initial abstraction model leads to another formulation of the SCS-CN method (eq. 60).

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