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SCS-CN METHOD. PART II: ANALYTICAL TREATMENT

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Abstract

This part of the paper, a sequel to Part I, in continuation of the analytical treatment of the Soil Conservation Service curve number (SCS-CN) method, further explores the SCS-CN method for: (a) its functional behaviour, (b) the physical interpretation of its proportional equality and curve number, (c) the derivation of seldom explored potential maximum retention S-CN relation, and (d) the development of CN-antecedent moisture condition (AMC) relations. Finally, an attempt is made to present the SCS-CN concept as a viable alternative to power law.

Key words: antecedent moisture condition, curve number, infiltration, Soil Conservation Service, sorptivity, soil-vegetation-land use complex, hydrologic condition.

1. INTRODUCTION

As discussed in the first part of this two-part paper (Mishra and Singh, 2002a), the Soil Conservation Service curve number (SCS-CN) method is a combination of the following equations:

$$P = I_a + F + Q \,, \tag{1}$$

$$\frac{Q}{P - I_a} = \frac{F}{S} \,, \tag{2}$$

$$I_a = \lambda S\,, \tag{3}$$

where P is the total rainfall, I_a is the initial abstraction, F is the cumulative infiltration excluding I_a, Q is the direct runoff, and S is the potential maximum retention or infiltration. All quantities in eqs. (1) – (3) are in depth or volumetric units. The popular form of the SCS-CN method is obtained by combining eqs. (1) and (2) as

$$Q = \frac{(P-I_a)^2}{P-I_a+S}\,, \tag{4}$$

which is valid for $P \geq I_a$; else $Q = 0$. Parameter S is mapped into curve number (CN) as

$$S = \frac{1000}{CN} - 10\,. \tag{5}$$

The issues related to the SCS-CN method, such as the rational derivation of the method and the rationale of initial abstraction, have been discussed in the first part (Mishra and Singh, 2002a). Others issues, such as the analytical structure of the S-CN mapping relation and the CN-AMC relations are investigated in this part. Thus, the objective of this part of the paper is to revisit the existing SCS-CN method for its functional behaviour and explore its fundamental proportionality concept using the soil porosity. The description of its functional behaviour leads to the development of criteria useful for field applications. The empirical S-CN relationship is investigated for its analytical derivation. The relations linking CN with AMC are also proposed and discussed. Finally, a brief investigation is made for using the SCS-CN concept as an alternative to the power law widely used as a surrogate to the popular Manning equation.

2. FUNCTIONAL BEHAVIOUR OF THE EXISTING AND MODIFIED SCS-CN METHODS

Existing SCS-CN method

The behaviour of the SCS-CN method can be described as follows. Under the state of complete saturation, the basic SCS-CN concept (eq. 2) fails to describe Q, for both F and S are equal to zero; rather Q is described by the water balance (eq. 1). Since $S = 0$ and therefore $F = 0$, $I_a = 0$ for $\lambda = 0.2$ and $Q = P$ from eq. (1). However, for the condition $I_a \neq 0$, $\lambda = \infty$ from eq. (3) and from eq. (1), $Q = P{-}I_a$. Thus, Q or the runoff factor C is significantly governed by I_a, which is consistent with the notion that I_a governs S (McCuen, 1982) that, in turn, governs Q (eq. 4 with $I_a = 0.2\,S$). On the other hand, no runoff (or $Q = 0$) condition can possibly occur if $P = 0$, $I_a \geq P$, or $S = \infty$. If $P = 0$, then each term of eq. (1) should be equal to zero. In such a situation, the left-hand side of eq. (2) equals 0/0 and the right-hand side is equal to zero for $S > 0$; it is indeterminate (= 0/0) if $S = 0$. Under the condition $I_a \geq P$, the entire rainfall is initially abstracted. Therefore, both F and Q are taken as equal to zero to restrict the violation of water balance (eq. 1). These conditions ($I_a \geq P$ and $F = Q = 0$), however, do not limit the validity of eq. (2), for both its left- and right-hand sides are equal to zero for $S > 0$. Finally, if $S = \infty$ (for example, an infinitely deep sandy soil), then, according to the existing SCS-CN method, $F = Q = 0$ because $I_a = \infty$ for $\lambda = 0.2$. However, if $I_a \neq 0$, then for $S = \infty$, $\lambda = 0$ (eq. 3) and $Q = 0$ from eq. (2) and consequently, $F = P - I_a$ from eq. (1). Thus, the condition $\lambda = 0$ does not necessarily describe $I_a = 0$. However, if $I_a = 0$, then $\lambda = 0$, as shown below following Mishra and Singh (1999).

Combining eq. (4) with eq. (3), solving for S, multiplying the resulting equation by λ, and then solving for λ yield (Mishra and Singh, 1999):

$$\lambda = \frac{C\, I_a^*}{(1 - I_a^*)(1 - I_a^* - C)} \tag{6}$$

and $\lambda = 0$. Equation (6) can also be directly derived from eq. (4). In eq. (6), if $I_a^* \to 1$ or $(I_a^* + C) \to 1$, $\lambda \to \infty$. Equation (6) yields prohibitive negative values of λ [$= -1/(1 - I_a^*)$] for C approaching 1. Thus, for λ to be a non-negative value, the following should hold: $Q + I_a \leq P$, which when coupled with eq. (1) yields $F \geq 0$, and its combination with eq. (3) leads to $S \leq (P - Q)/\lambda$. It implies that $S \leq 5(P - Q)$ for $\lambda = 0.2$.

Equation (6) describes the functional behaviour in $C - I_a^* - \lambda$ space, as shown in Fig. 1. This figure shows that λ can assume a value even when I_a is not necessarily equal to zero. In addition, as $C \to 0$, $\lambda \to 0$ and, consequently, $S \to \infty$. Thus, eq.(3) with $\lambda = 0.2$ will yield I_a much larger than zero. Therefore, the existing SCS-CN method performs poorly on very low runoff producing (or low C-values) lands, such as sandy soils and forest lands. Similarly, when λ increases sharply and approaches infinity for a given I_a^* (or $C = 1 - I_a^*$) (Fig. 1), the SCS-CN method may perform poorly.

Figure 1 also shows thick lines indicating the region of SCS-CN applicability for λ-range of (0.1, 0.3) (SCD, 1972). Apparently, as C increases and I_a^* decreases, the applicability region in terms of the runoff-generation potential widens or the range of C-values increases to a certain extent and then decreases as $C \to 1$ and $I_a^* \to 0$. The maximum C-range (0.39, 0.62) can be described mathematically for $I_a^* = 0.113$, implying the existing SCS-CN method to be most applicable to those watersheds exhibiting C-values in the approximate range of (0.4, 0.6) and initial abstraction amount of the order of 10% of the total rainfall.

Since the SCS-CN method with $\lambda = 0.2$ performs well on urban watersheds (SCS, 1986) because of the very low S-values and, in turn, the insignificant amount of

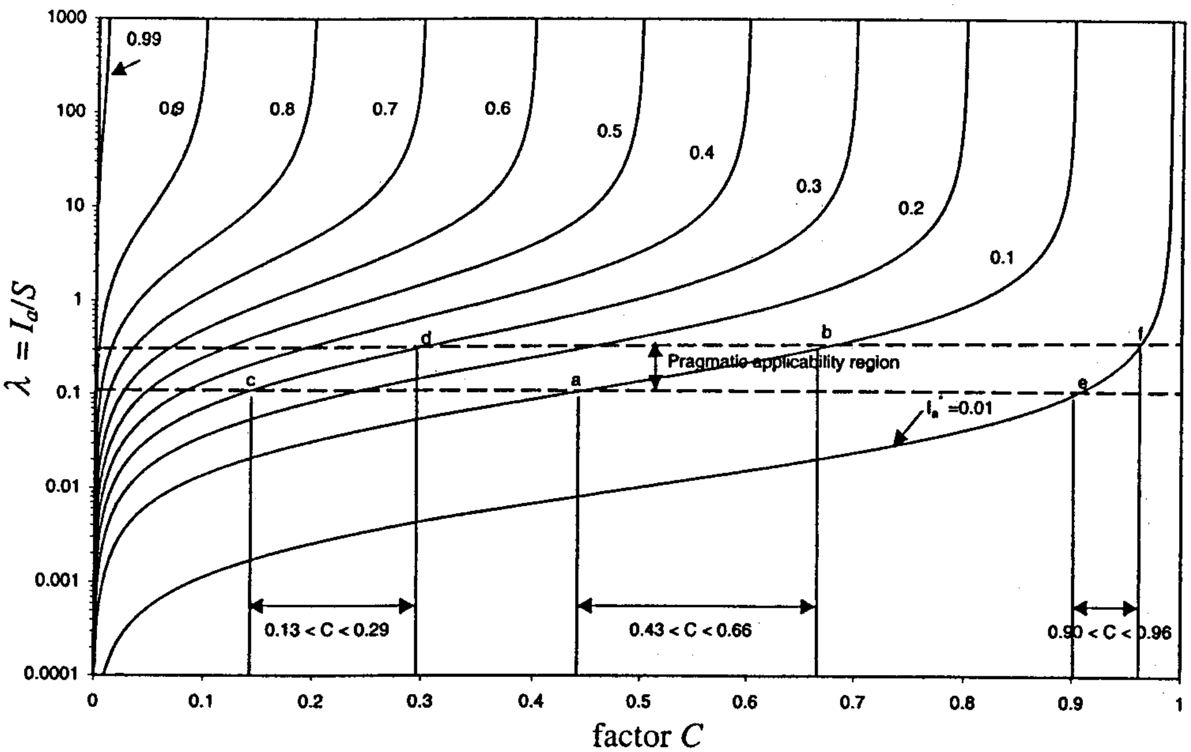

Fig. 1. Variation of initial abstraction coefficient, λ, with runoff factor, C, and nondimensional initial abstraction, I_a^*.

initial abstraction with insignificant bearance on the computed Q-values, it follows that the existing SCS-CN method is applicable for $C > 0.6$. Thus, $C \geq 0.4$ and from the relation $S \leq (P-Q)/\lambda$, $S \leq (10/3)(P-Q)$ for $\lambda = 0.3$ forms the bounds for applicability of the method. Coupling of these conditions yields a broad criterion expressed as $S \leq 2P$ or, alternatively, in terms of CN (eq. 5), the criterion is

$$CN \geq \frac{1000}{10+2P} . \tag{7}$$

As a text example, if $P = 1$ inch, $CN \geq 83$, for which $Q \geq 0.23$ inch. Similarly, if $P = 10$ inches, $CN \geq 33$ and, in turn, $Q \geq 2.29$ inches. Thus, the applicability bounds in Fig. 1 are: $\lambda \leq 0.30$, $I_a \leq 0.35P$, and $C \geq 0.23$. It is noted that the condition $S \leq 2P$ allows the SCS-CN method to work even when $S < P$, where the Mockus method (Mishra and Singh, 2002b) fails.

Modified SCS-CN method

The modified version of the existing SCS-CN method is expressed as

$$Q = \frac{(P-I_a)^2}{0.5(P-I_a)+S} , \tag{8}$$

which is applicable for $S \geq 0.5(P-I_a)$, for Q-value to be less than or equal to $(P-I_a)$. Equation (5) is assumed to hold for S-CN conversion.

Following Mishra and Singh (1999), the behaviour of the modified version can be explained from the relation

$$\lambda = \frac{2C I_a^*}{(1-I_a^*)(2-C)}. \tag{9}$$

In eq. (9), if $I_a^* \to 1$, $\lambda \to \infty$ as $C \leq 1$. For $C = 1$, $\lambda = 2I_a^*/(1-I_a^*)$ and if $C = 0$, $\lambda = 0$. Thus, λ can vary between zero and infinity.

3. PHYSICAL SIGNIFICANCE OF THE SCS-CN PROPORTIONAL EQUALITY

From the analytical derivation of the SCS-CN method (Mishra and Singh, 2002a) from the Mockus method, the Zoch method, or the first-order storage hypothesis, it is evident that the proportional equality (eq. 2) is the result of the generalization of the first-order infiltration process coupled with the water balance equation with the assumption that the cumulative rainfall grows linearly with time. Thus, this proportional equality is an improvement over the exponentially decaying infiltration process, for the Mockus method impossibly exhibits runoff (rainfall-excess) rate to exceed the rainfall intensity. It is further noted that the Mockus method assumes the exponential decay of the infiltration rate with time. Since the Horton model is an empirical method, the SCS-CN method can also be construed as an empirical method. Besides, derivation of the SCS-CN method from the second-order storage hypothesis leads to its categorization as a conceptual method. However, the applicability of the SCS-CN method to most hydrological conditions serves as a motivation to explore the physical basis of the method. The following discussion explores this proportional equality using the concept of soil porosity.

Soil porosity

A soil column can be divided into three main parts: volume of solids, water, and air, as shown in Fig. 2. In this figure, V is the total volume, V_w is the volume of water, V_s is the volume of solids, and V_a is the volume of air for a unit surface area. The sum of the volume of air and the volume of water represents the volume of voids V_v. Expressed mathematically,

$$V_v = V_a + V_w. \tag{10}$$

In volumetric terms, the water or moisture content, θ, is defined as $\theta = V_w/V$ and porosity, n, is defined as

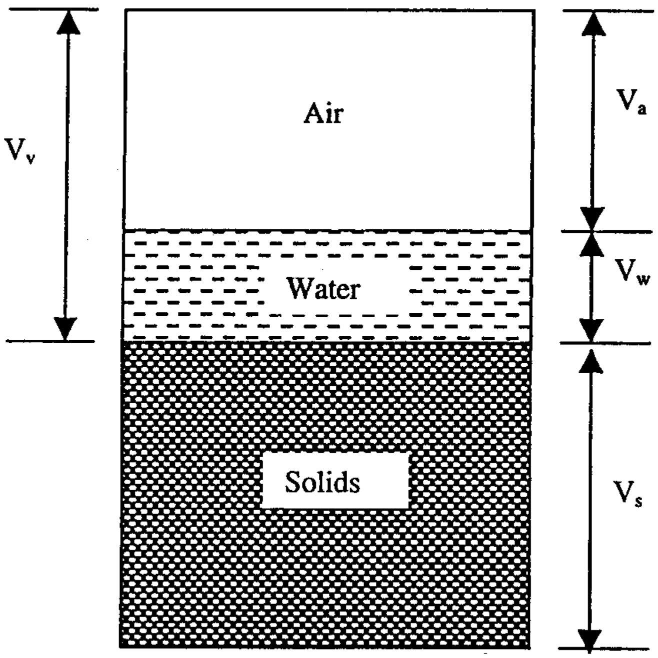

Fig. 2. Schematic diagram showing soil-water-air.

$$n = \frac{V_v}{V}. \tag{11}$$

The moisture content, θ, can also be expressed as

$$\theta = n\, S_r, \tag{12}$$

where S_r is the degree of saturation, varying from 0 to 1, or alternatively,

$$S_r = \frac{\theta}{n} = \frac{V_w}{V_v}. \tag{13}$$

Proportional equality

In the proportionality concept (eq. 2 for $I_a = 0$), F represents the infiltrated amount of water ($= V_w$) and S is equal to the maximum possible amount of infiltration equal to the maximum ($P-Q$) difference which, in turn, is equal to the volume of voids V_v. Therefore,

$$F/S = S_r \tag{14}$$

The left-hand side of eq. (14) represents the runoff factor C which has a range of (0, 1) as

$$C = \frac{Q}{P}. \tag{15}$$

Thus, substitution of eqs. (14) and (15) into eq. (2) for $I_a = 0$ leads to

$$C = S_r \tag{16}$$

which is the equation that integrates surface flow and subsurface flow linearly on a 1:1 scale, implying that the SCS-CN method is not only a runoff method but also an infiltration method (Aron *et al.*, 1977; Chen, 1982; Gray *et al.*, 1982; Ponce and Hawkins, 1996; Mishra, 1998; Mishra and Garg, 2000; Mishra and Singh, 2002b). Thus, the SCS-CN method can also be categorised as an infiltration model. Using this concept of proportional equality, it is possible to signify the curve number, *CN*.

Significance of CN

Equation (5) of the SCS-CN method defines *CN* and its link with *S* that represents the maximum possible amount of infiltration. *CN* can be signified from the expression for *F* derived from eqs. (4) and (1) for $I_a = 0$ as

$$\frac{F}{S} = \frac{P}{P+S}. \tag{17}$$

Equation (17) describes the variation of the degree of saturation (eq. 14) with rainfall *P*. This equation leads to a derivation of the S-CN mapping relation (eq. 5) as follows.

The ratio *F/S* varies from 0 to 1. To map it on a scale of 0–100, it is necessary to multiply eq. (17) by 100, leading to

$$100\frac{F}{S} = \frac{100P}{P+S}. \tag{18}$$

Defining the left-hand side of eq. (18) as *CN* leads to

$$CN = \frac{100P}{P+S} = \frac{100}{1+S/P} \tag{19}$$

which describes the variation of *CN* with *P* and *S*. Assuming *P* = 10 inches leads to eq. (5). Thus,

$$CN = 100\frac{F}{S}. \tag{20}$$

Equation (20) defines *CN* as the percent degree of saturation of the watershed due to a 10-inch rainfall amount. It is worth noting that the direct use of *CN* in the proportionality concept (eq. 2) for computing *Q* is restricted, because *CN*, by definition, corresponds to the 10-inch base rainfall, not to the actual rainfall. Therefore, eq. (4) with $I_a = 0$ should be resorted to computation of *Q* for a given rainfall amount, as also shown below.

In terms of CN, the runoff factor $C(=Q/P)$ can be defined from eq. (4) (for $I_a = 0$) as

$$C = \frac{1}{1 + \frac{10}{P}\left(\frac{100}{CN} - 1\right)} . \tag{21}$$

To describe C physically it is necessary to explain the bracketed portion in the denominator of eq. (21) in terms of the volumetric elements of the soil (Fig. 2) as

$$\frac{100}{CN} - 1 = \frac{100}{100V_w / V_v} - 1 = \frac{V_v - V_w}{V_w} = \frac{V_a}{V_w}, \tag{22}$$

where V_v is the void space, V_w is the available moisture due to 10-inch rainfall, and V_a is the space available for water retention after 10-inch rainfall. An actual rainfall P greater than 10 inches would result in higher V_w and, consequently, lesser V_a or, in turn, a lesser V_a/V_w ratio and *vice versa*. Therefore, the bracketed term in the denominator of eq. (22) needs to be updated (increased or reduced) in proportion to $10/P$ to describe the actual V_a/V_w ratio that corresponds to P. The actual V_a/V_w ratio computed in the denominator of eq. (22) and the inverse of the resulting sum of the denominator yields the actual degree of saturation that corresponds to P, which equals C to form the proportional equality equivalent to $C = S_r$. Such a description leads to defining the runoff factor C (eq. 21) as the degree of saturation, S_r, of the watershed for the actual rainfall P. It supports the validity of the $C = S_r$ concept described above. The implication of such an assertion is that for CN to represent a watershed characteristic, S/P should form a basic parameter of the SCS-CN model while deriving CN from rainfall-runoff data, rather than S alone.

4. ANTECEDENT MOISTURE CONDITIONS

The National Engineering Handbook, Section 4 (NEH-4), (SCS, 1956, 1985) identified three antecedent moisture conditions: AMC I, AMC II, and AMC III for dry, normal, and wet conditions of the watershed, respectively. NEH-4 provides a conversion table from CN for AMC II to the corresponding CNs for AMC I and AMC III. The original values (SCS, 1956) of this table were smoothened in the later versions of NEH-4 (for example, SCS, 1985). Based on the refined values, Sobhani (1975) and Hawkins *et al.* (1985) linked CNs of different AMCs in terms of the potential maximum retention as below

$$S_{\mathrm{I}} = 2.281\, S_{\mathrm{II}}; \qquad r^2 = 0.999 \quad \text{and} \quad SE = 0.206 \text{ inch} \tag{23}$$

and

$$S_{\mathrm{III}} = 0.427\, S_{\mathrm{II}}; \qquad r^2 = 0.994 \quad \text{and} \quad SE = 0.088 \text{ inch}, \tag{24}$$

where SE is the standard error and subscripts I through III correspond to AMC I through III, respectively. Equations (23) and (24) are applicable in the CN range (55, 95). Substitution of eqs. (23) and (24) into eq. (5) leads to

$$CN_{\mathrm{I}} = \frac{CN_{\mathrm{II}}}{2.281 - 0.01281\, CN_{\mathrm{II}}}; \qquad r^2 = 0.996 \quad \text{and} \quad SE = 1.0\ CN \tag{25}$$

and

$$CN_{\mathrm{III}} = \frac{CN_{\mathrm{II}}}{0.427 + 0.00573\, CN_{\mathrm{II}}}; \qquad r^2 = 0.994 \quad \text{and} \quad SE = 0.7\ CN. \tag{26}$$

For practical applications, NEH-4 derives CN based on the amount of the antecedent 5-day rainfall, which forms an index of the initial soil moisture. The term initial stands for the state before the start of the storm. Equations (23) and (25) can be derived from eq. (5), written in general form as

$$CN = \frac{100X}{S + X}. \tag{27}$$

In eq. (5), X = 10 inches of rainfall, which corresponds to the normal condition. Equation (27) is valid for all AMCs according to NEH-4. Alternatively, CN for AMC I and AMC III can be derived from eq. (27), respectively, as

$$CN_{\mathrm{I}} = \frac{100(X - P_{\mathrm{I}})}{S_{\mathrm{II}} + (X - P_{\mathrm{I}})} \tag{28}$$

and

$$CN_{\mathrm{III}} = \frac{100(X + P_{\mathrm{III}})}{S_{\mathrm{II}} + (X + P_{\mathrm{III}})}. \tag{29}$$

These equations imply that the normal rainfall amount X is reduced by P_{I} if CN_{II} converts to CN_{I}. Similarly, an additional rainfall amount of P_{III} over and above the normal X-value is required to raise CN_{II} to CN_{III}. Substitution for S_{II} in eqs. (28) and (29) leads, respectively, to

$$CN_{\mathrm{I}} = \frac{CN_{\mathrm{II}}}{\dfrac{X}{X - P_{\mathrm{I}}} - \dfrac{P_{\mathrm{I}}}{100(X - P_{\mathrm{I}})}} \tag{30}$$

and

$$CN_{\mathrm{III}} = \frac{CN_{\mathrm{II}}}{\dfrac{X}{X + P_{\mathrm{III}}} + \dfrac{P_{\mathrm{III}}}{100(X + P_{\mathrm{III}})}} \tag{31}$$

which are the general expressions for *CN* according to any antecedent moisture condition for a given amount of the normal and the two extreme antecedent rainfalls X, P_{I} and P_{III}, respectively. A comparison of eqs. (30) and (31) with the respective eqs. (25) and (26) leads to

$$X = (2.281/1.281)\, P_{\mathrm{I}} = 1.7806\, P_{\mathrm{I}} \tag{32}$$

and

$$X = (0.427/0.573)\, P_{\mathrm{III}} = 0.7452\, P_{\mathrm{III}}\,. \tag{33}$$

For given P_{I} and P_{III} values in NEH-4 equal, respectively, to 0.5 (1.3 cm) and 1.1 inch (2.8 cm) for a dormant season and 1.4 (3.6 cm) and 2.1 inches (5.3 cm) for a growing season, the normal antecedent rainfall, X, can be computed as

dormant season:

$X = 0.89$ inch for AMC I and $X = 0.82$ inch for AMC III ;

growing season:

$X = 2.49$ inches for AMC I and $X = 1.56$ inches for AMC III .

The inference drawn from the above calculations is that the normal amount of rainfall, X, varies with both AMC and season. The variation in X can be interpreted in terms of the initial abstraction amount, I_a, which can be neglected in AMC III. Thus, $I_a = 0.89 - 0.82 = 0.07$ inch for the dormant season and $I_a = 2.49 - 1.56 = 0.93$ inch for the growing season. Similarly, the variation of X with the season can be attributed to evapotranspiration that can be neglected in the dormant season. Thus, the evapotranspiration amount can be computed as equal to 0.74 (= 1.56 – 0.82) inch for the growing season under AMC III, and equal to 1.60 (= 2.49 – 0.89) inches under AMC I.

Variation of *CN* with AMC

Following the above $C = S_r$ concept, eq. (2) (for $I_a = 0$) can be modified for the antecedent moisture, M, as

$$\frac{Q}{P - I_a} = \frac{F + M}{S + M} = \frac{F + M}{S_0}\,, \tag{34}$$

where S_0 represents the volume of air equal to the volume of voids (for complete antecedent dry condition) and M is computed as

$$M = \frac{P_5\, S_0}{P_5 + S_0}\,, \tag{35}$$

where P_5 is the antecedent 5-d rainfall amount and M is derived assuming the completely dry antecedent condition. Thus, *CN* can be described from eqs. (34) and (35) for $P_e = 10$ inch as

$$CN = \frac{100}{S_0}\left[\frac{10S}{10+S} + \frac{P_5 S_0}{P_5 + S_0}\right] = \frac{1000}{10+S} \ . \tag{36}$$

Equation (36) leads to CN for AMC I as

$$CN_{\mathrm{I}} = \frac{100}{S_0}\left[\frac{10S_{\mathrm{I}}}{10+S_{\mathrm{I}}} + \frac{P_{5(\mathrm{I})} S_0}{P_{5(\mathrm{I})} + S_0}\right] = \frac{1000}{10+S_{\mathrm{I}}} \ . \tag{37}$$

It follows that

$$P_{5(\mathrm{I})} = S_0\left[\frac{M_{\mathrm{I}}}{1-M_{\mathrm{I}}}\right], \qquad M_{\mathrm{I}} = \left(1-\frac{S_{\mathrm{I}}}{S_0}\right)\frac{10S_0}{10+S_{\mathrm{I}}}, \tag{38}$$

where subscript I refers to AMC I. Similar expressions can be derived for AMC II and AMC III. It is apparent from eqs. (36) – (38) that S_0 is required *a priori* for computing $P_{5(\mathrm{I})}$ through $P_{5(\mathrm{III})}$. Its derivation using the NEH-4 data is shown below.

As described above by eqs. (23) and (24), S_{I}, S_{II}, and S_{III} are related with each other. However, there does not exist a relationship between S_0 and S_{I}. For this reason, trials were made for S_0 and its relation with S_{I} such that the derived P_5-values approximated the average of AMC II defined by NEH-4 (Table 1). The derived relationship, shown in Fig. 3, which includes I_a (= 0.2 S), yields the limiting P_5-values for various AMCs, as shown in Table 1. The derived S_0 values are equal to 0.11 inch for the dormant season and 0.56 inch for the growing season and S_0/S_{I} = 1.6. For AMC II, the derived limiting P_5-values were equal to 0.8 and 1.75 inch for dormant and growing seasons, respectively. An assumption of soil porosity equal to 0.3 (Singh and Yu,

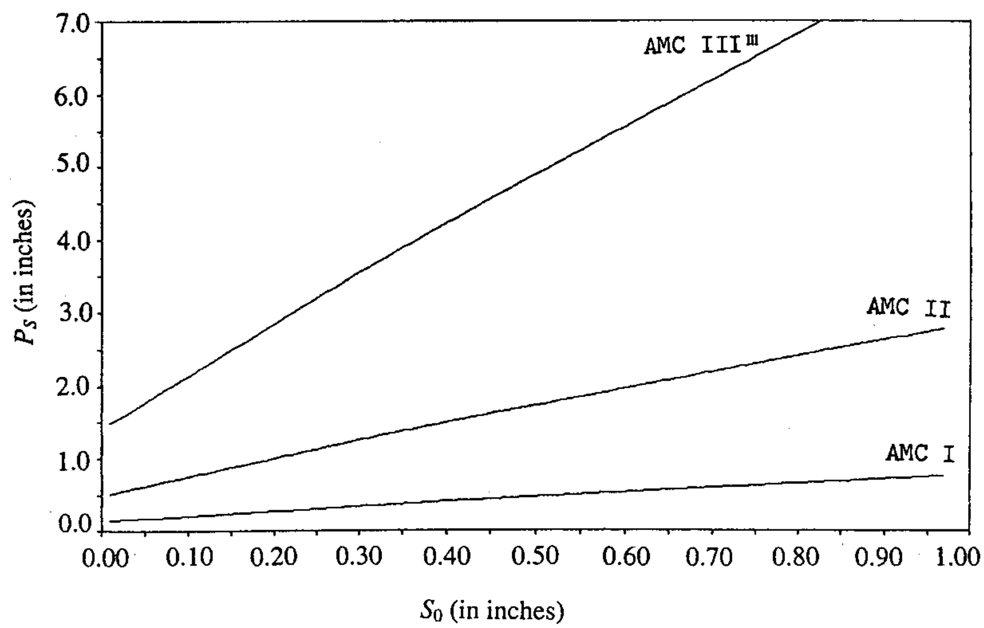

Fig. 3. Variation of P_5 with S_0. P_5 includes I_a = 0.2 S.

Table 1

Antecedent soil moisture conditions (AMC)

AMC	Total 5-day antecedent rainfall, P_5 (in inches)			
	NEH-4 (McCuen, 1982)		derived from Fig. 3	
	Dormant season	Growing season	Dormant season $S_0 = 0.11$ inch	Growing season $S_0 = 0.51$
I	less than 0.5	less than 1.4	less than 0.22	less than 0.49
II	0.5 to 1.1	1.4 to 2.1	0.22 to 2.28	0.49 to 5.02
III	more than 1.1	more than 2.1	more than 2.28	more than 5.02

1990) leads to the soil depth above the impeding layer (Huggins and Monke, 1966) equal to 0.36 and 1.87 inches for dormant and growing seasons, respectively. Since the existing SCS-CN method accounts for only dynamic (or capillary) portion of infiltration and excludes its static (or gravitational) portion, these values of S_0 represent the soil depths responsible for the dynamic infiltration process. It is consistent with the notion that only the top-soil-zone significantly affects the dynamic infiltration process (Fok and Chiang, 1984). In other words, the antecedent moisture model considers only the uppermost soil layer as a reservoir (Schaake *et al.*, 1996), rather than the complete root zone depth. Figure 3 can be used for deriving the AMC criteria for any other S_0-value that depends on regional soil, vegetation, and land use characteristics.

5. SCS-CN CONCEPT AS AN ALTERNATIVE TO POWER LAW

With the above analytical, conceptual, and physical background of the SCS-CN method, it is now appropriate to explore if the SCS-CN concept could be a replacement of the widely used power law. In hydrology, the widely used steady-state stage -discharge (or velocity) relation generally expressed as a power function represents another form of the popular Manning equation

$$\upsilon = \frac{1}{n} R^{2/3} s_0^{1/2}, \tag{39}$$

where υ is the mean flow velocity (in m/s), R is the hydraulic radius (in m), n is Manning's roughness, and s_0 is the channel bed slope (in m/m). Here, it is necessary to describe first the basis for making such an attempt and to adapt an SCS-CN-based function that exhibits the growth or decay of a dependent variable with its independent variable.

Representing the spatial variation of interacting storage elements over a watershed by a power distribution function, Moore (1985) gave a relationship for computing the surface runoff volume (or rainfall-excess) from a given effective storm rainfall volume. This relationship when plotted is quite close to that produced by the SCS-CN-generated runoff with rainfall. It implies that the power distribution may represent the SCS-CN concept and *vice versa.*

From eq. (17) the variation of the cumulative infiltration F with rainfall for a given S can be described as

$$F = \frac{SP}{S+P}. \tag{40}$$

Replacing P by $i_e t$, (eq. 18 of Part I; Mishra and Singh, 2002) leads to

$$F = \frac{at}{b+t}, \tag{41}$$

where $a = S$ and $b = S/i_e$, which is equal to the Horton decay parameter α. Thus, parameter a describes the system's potential maximum capacity, and b the decay of the phenomenon. Equation (41), however, describes the growth of the cumulative infiltration with time t. Thus, parameters a and b have a physical significance. A division of F by time t describes the decay of the average infiltration rate, f_{av}, with t as

$$f_{av} = \frac{a}{b+t}. \tag{42}$$

Equations (41) and (42) describing, respectively, the growth and decay functions can, in general, be expressed as

$$y = \frac{ax}{b+x}; \qquad y = \frac{a}{b+x}, \tag{43}$$

where y and x are dependent and independent variables, respectively, and a and b are the parameters. Equation (43) is analogous to that given by Ponce (1989), among others, for describing the rainfall depth-duration relationship:

$$P = \frac{at}{b+t} \tag{44}$$

which describes the growth of rainfall with the rainstorm duration. In eq. (44), parameter a represents the maximum possible amount of P that can occur on a watershed in time t, similar to S representing the upper bound of F in eq. (40). Thus, eq. (43) is based on the capacity of the considered system. Parameter b bears the dimension of time and describes the growth pattern of P with t.

Similarly, eq. (43) is analogous to that relating the uniform rainfall intensity with the rainstorm duration as (Ponce, 1989)

$$i_0 = \frac{a}{b+t}. \tag{45}$$

Parameters a and b of eq. (45) can also be described, respectively, as the potential maximum rainfall intensity that can occur in time t on a watershed and decay factor of i_0. From eq. (43) an expression for the steady-state rating curve for a unit-width rectangular channel can be given as

$$\upsilon = \frac{a\,h}{b+h}, \tag{46}$$

where υ is the flow velocity, h is the corresponding stage of flow, a represents the potential maximum velocity, and b is the decay factor of velocity. For a unit-width rectangular channel, discharge q can be given as

$$q = \frac{a\,h^2}{b+h}. \tag{47}$$

On the other hand, the steady-state rating curve in terms of the power law can be expressed as

$$q = a_1\, h^{1+m}, \tag{48}$$

where q is the discharge, h is the depth of flow, and a_1 and m are the coefficient and the exponent, respectively. Parameters of eq. (48) can be described by equating the wave celerity derived from the Seddon formula (Mishra and Singh, 2001) as below.

From eqs. (47) and (48) the wave celerity, c, can be derived, respectively, as

$$c = \left[\frac{2b+\ h}{b+h}\right]\upsilon, \qquad c = (1+m)\,\upsilon. \tag{49}$$

It follows that

$$m = \left(\frac{b}{a}\right)\frac{\upsilon}{h}. \tag{50}$$

This implies that parameter m is a function of υ and h, which is consistent with the wave derivations for m (Mishra and Singh, 2001) describing its dependence on the Froude number and the kinematic wave number which, in turn, depend on υ and h.

Furthermore, an assumption of υ and h varying exponentially (Mishra and Singh, 2001) leads to describing the υ/h ratio for the kinematic wave as

$$\frac{\upsilon}{h} = \frac{V_0}{H_0}, \tag{51}$$

where V_0 is the peak flow velocity and H_0 is the corresponding depth of flow. Since $m = 2/3$ for kinematic wave (Mishra and Singh, 2001), parameter a relates to parameter b as $a = (3V_0/2H_0)\,b$. V_0 can be related to H_0 using eq. (46) as

$$V_0 = \frac{aH_0}{b+H_0}. \tag{52}$$

It follows that $a = 3V_0$ and $b = 2H_0$.

A replacement of h by the hydraulic radius R in eq. (46) yields a general expression for velocity and, in turn, discharge as

$$q = \frac{aRA}{b+R}, \tag{53}$$

where A is the area of cross-section. The advantage of such a relationship is that its parameters are fully describable in terms of the SCS-CN concept. Including the Manning roughness and bed slope, a more general SCS-CN-based expression for the computation of discharge can be given as

$$q = \frac{aRs_0A}{n(b+R)(c+s_0)}, \tag{54}$$

where n is Manning's roughness, s_0 is the bed slope, and a, b and c are the parameters.

To demonstrate the applicability of eq. (54), a set of data was generated using Manning's equation with n varying from 0.01 to 0.1, s_0 from 0.001 to 0.1, and R from 0.5 to 5 m. The fitting (Fig. 4) of eq. (54) to this data set exhibited the Nash and Sutcliffe (1970) efficiency equal to 98.91% with a = 2.075, b = 3.358, and c = 0.0433. The high efficiency value indicates a good performance of the SCS-CN -based eq. (54).

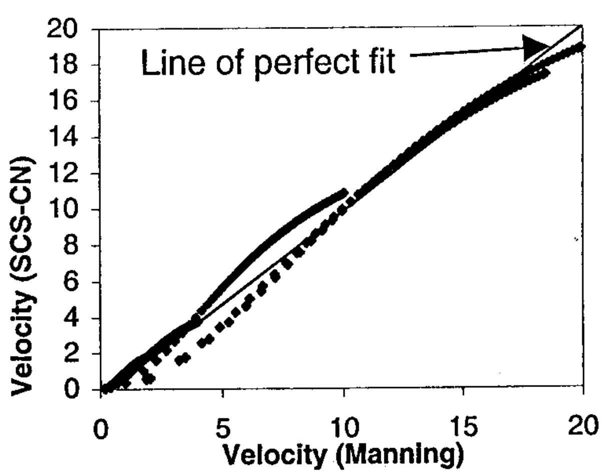

Fig. 4. Velocities computed from Manning's and SCS-CN-based equations.

6. CONCLUSIONS

The following conclusions are derived from the study:

1. The proportionality concept, eq. (2), of the SCS-CN method represents the $C = S_r$ concept that is based on the volumetric concept of soil physics. Thus, the SCS-CN method is a conceptual model founded on physical considerations.

2. The applicability of the SCS-CN method is restricted to the CN values given by eq. (7), and the initial abstraction coefficient varies from zero to infinity.

3. The curve number CN can be defined as percent degree of saturation of watershed due to the 10-inch base rainfall amount.

4. It is possible to describe the variation of CN with any AMC, even other than the existing ones.

5. The SCS-CN method can be taken as a viable alternative to power law.

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