TRIANGULAR ARBITRAGE WITH BITCOIN

An Undergraduate Research Scholars Thesis

by

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Submitted to the Undergraduate Research Scholars program at
Texas A&M University
in partial fulfillment of the requirements for the designation as an

UNDERGRADUATE RESEARCH SCHOLAR

Approved by Research Advisor: Dr. Dennis Jansen

May 2017

Major: Economics

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ABSTRACT

Bitcoins: Volatility and Arbitrage

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Bitcoins are bought and sold with most major currencies, and the resulting prices are ‘exchange rates’ of currencies per Bitcoin. The price of Bitcoins in national currencies have been quite volatile over the brief period of Bitcoin’s existence. Among national currencies, the possibility of triangular arbitrage leads to near equality of bilateral exchange rates and exchange rates obtained via triangular trade. We explore the relationship between a bilateral exchange rate of two major national currencies and the exchange rate that can be obtained via triangular trade through Bitcoins. Features of the Bitcoin exchanges make it difficult to meet conditions necessary for strict (risk-free) arbitrage, but market pressures from the threat of arbitrage constrain the national currency prices of Bitcoins to adjust toward the bilateral foreign exchange rate obtained in the traditional foreign exchange market. In this paper, we compare the Bitcoin implied exchange rate between the US Dollar and the Chinese Renminbi to the market US Dollar/Renminbi exchange rate, and show that the Bitcoin implied rate adjusts toward the FOREX market rate due to market pressures from the threat of triangular arbitrage. One implication is that the bilateral rate and the triangular trade rate obtained from trade in Bitcoins are cointegrated, and we study the adjustment process when these exchange rates are misaligned.

INTRODUCTION

Bitcoins are a private electronic currency that exist only on computer hard drives or other storage devices. Bitcoins are created by 'miners' who compete continuously to earn a limited predetermined flow supply of Bitcoins available for solving a difficult mathematical problem involving the prime factorization of a number. Miners profit if the value of the Bitcoins they earn exceed their costs (electricity, capital expenses, labor). Their work on this problem also serves to certify the digital record of Bitcoin transactions including the latest updates called the block chain. See Nadeau and Jansen (2015), or Dwyer (2014), for more information.

Bitcoin use has become more extensive since they were introduced in January 2009. Initially, there were few uses for Bitcoins, but over time, Bitcoins have become accepted for transactions by companies such as WordPress (November 2012), Overstock.com (January 2014), Expedia (June 2014), Dell (July 2014), Amazon (2014), and Apple (2014)(Vessels). Still, it is thought that a major use of Bitcoins is to facilitate underground or illegal activities, and several individuals have been charged with using Bitcoins to facilitate illegal transactions or transactions in support of illegal activities. (For example, the U.S. Department of Homeland Security seized assets from the Bitcoin exchange, Mt. Gox, in May 2013, and, in a related move, the website, Silk Road, was shuttered later in 2013 in connection with illegal transactions facilitated with Bitcoins. The Mt. Gox exchange was eventually shut down in February 2014 due to bankruptcy.)

Bitcoins are useful for transactions involving underground or illegal activities (including money laundering) because they allow anonymity. In this regard, Bitcoins are a substitute for cash in preserving anonymity, and Bitcoins have the advantage of easy portability and easy exchange across large geographic distances as they are electronic in nature. At the same time, Bitcoins have a disadvantage relative to cash in that they have proven to be not completely anonymous in all situations.

The Bitcoin market price has exhibited extreme volatility over the short time of their existence. Bitcoin use is perhaps highest in China and the USA, and Figure 1 plots the USD and RMB price of a Bitcoin. The large increase in Bitcoin prices, in both USD and RMB, are obvious in the graph. I also provide a graph in Figure 2 of the natural log of these two series to more clearly show the size of relative changes over time.

Figure 1 - Chinese RMB per Bitcoin and US Dollar per Bitcoin, August 1 2012 - August 1 2016

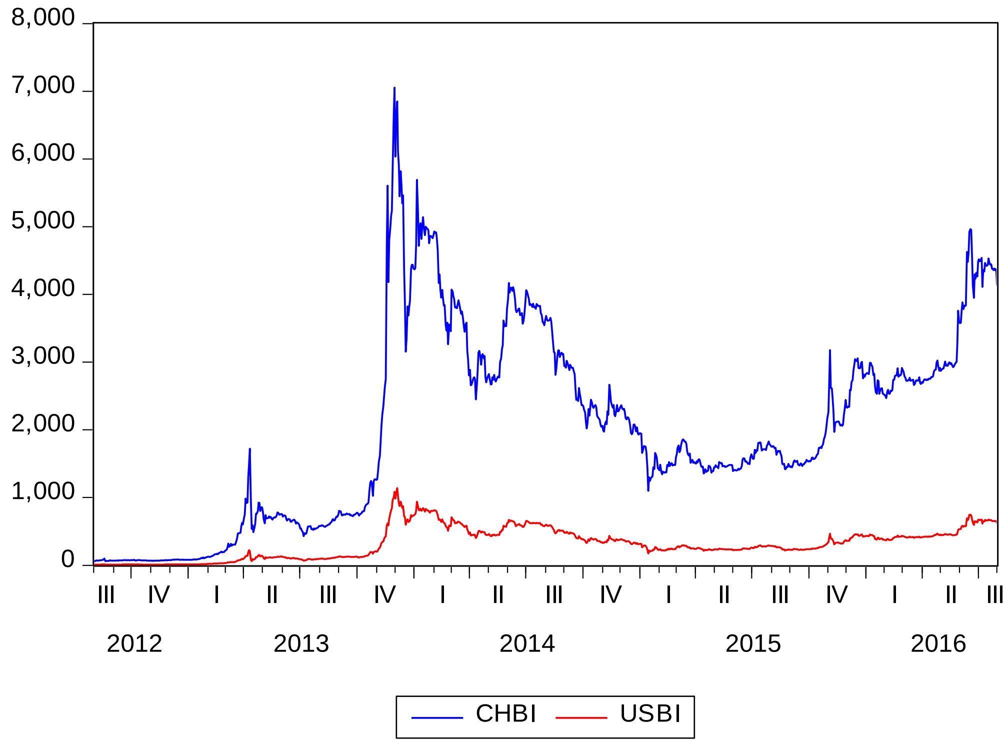
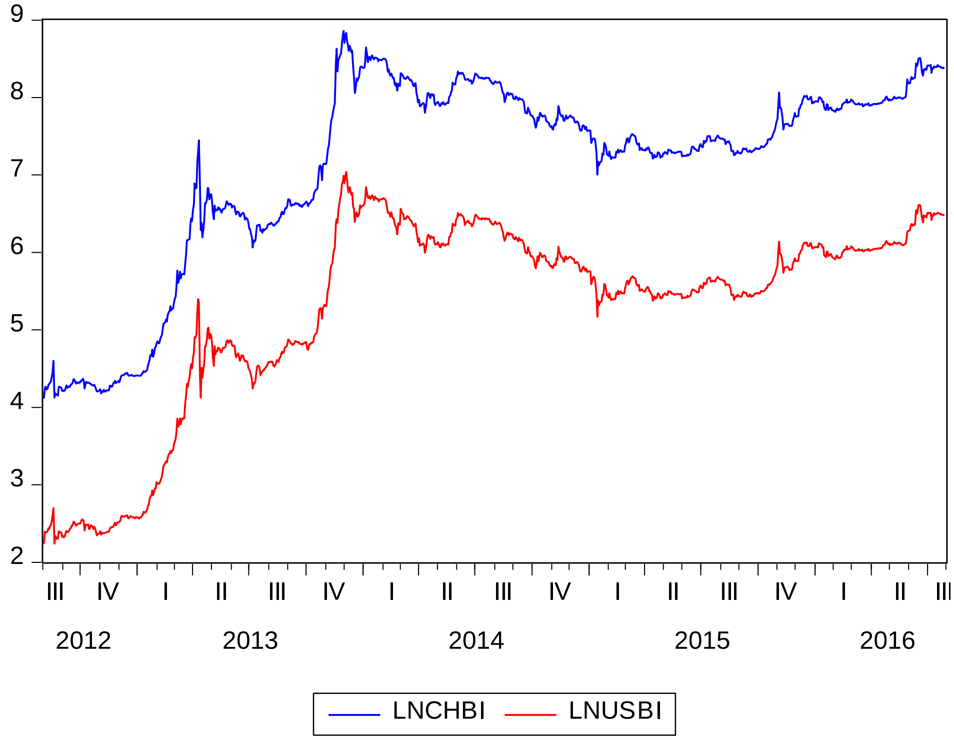


Figure 2 - Log Series for USD/Bitcoin and RMB/Bitcoin



Summary statistics for the USD per Bitcoin and RMB per Bitcoin data series are shown in Table 1. Columns two and three give values for summary statistics for the natural log of the USD per Bitcoin and RMB per Bitcoin rates. A key item to consider is the unit root test, reported near the bottom of the table. The probability values reported are for the Phillips and Perron test and indicate that the log level of the USD/Bitcoin and RMB/Bitcoin rates each have a unit root. Hence, we will not dwell further on the summary statistics for these two series.

Columns four and five of Table 1 give the summary statistics for the differences of the log USD/Bitcoin and log RMB/Bitcoin rates, which I will call the return series. The unit root test clearly rejects a unit root in these rates of return series. A feature to note is the large Kurtosis values, indicated by fat tails. Also, note that the maximum and minimum values of the series are much large in magnitude than the first or third quantile values, as would be expected for series having fat tails. I also calculate the Hill Statistic[1] for the tail index of these two series, in order to provide another measure of the fat tails of these two return series.

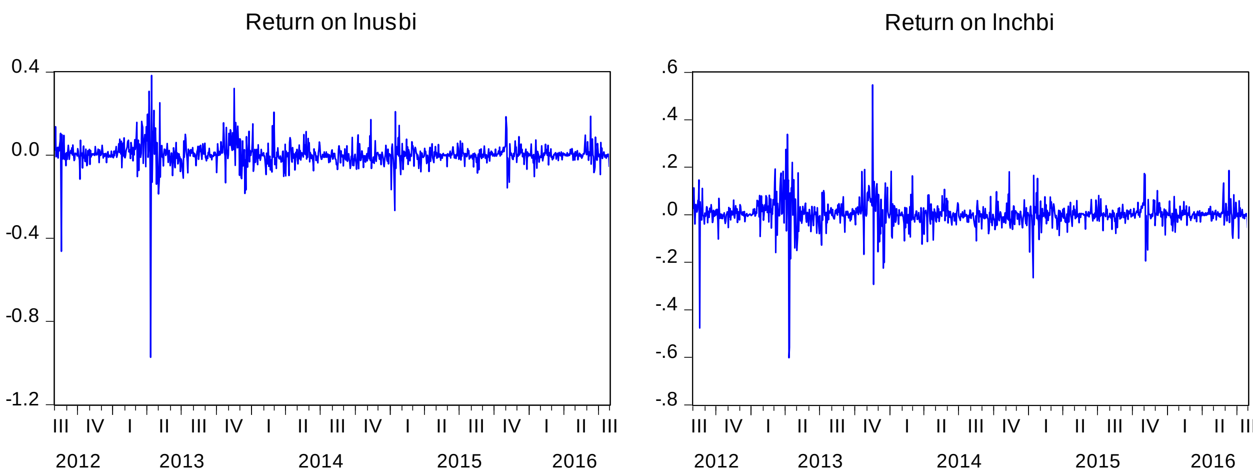
Table 1 - Summary Statistics for the Logs of USD/Bitcoin and RMB/Bitcoin and Return Series

	Ln(USBI)	Ln(CHBI)	Return on Ln(USBI)	Return on Ln(CHBI)
Mean	5.276955	7.116290	0.004023	0.004042
Std. Dev.	1.265729	1.267951	0.060746	0.061216
Skew	-1.213169	-1.206261	-3.819886	-1.235289
Kurtosis	3.346330	3.324306	73.43931	30.52082
Max.	7.039660	8.861519	0.383893	0.546911
Q3	6.116432818	7.97863828	0.020117538	0.019441014
Q2	5.649836535	7.475511124	0.001778569	0.001990917
Q1	4.792686495	6.602509864	-0.014038806	-0.013133237
Min.	2.234306	4.110874	-0.972671	-0.602074
Unit Root Test P-Value[2] (Null is a unit root)	-2.350778	-2.251863	-29.92682	-29.25361
Tail Index[3]	0.7465	0.7512	0	0
Number of Observations	1044	1044	1043	1043

[1] The Hill statistic for the tail index is one for one with the number of moments that exist for the distribution, and hence provides information on the 'size' of the fat tails. The number of order statistics used for the Hill statistic is calculated using the method suggest by X&Y (XXXX). I do not calculate the Hill Statistic for the log level series since there is evidence strongly suggesting that these two series are not stationary.
[2] Phillips Perron Test, automatic Newy-West bandwidth selection, Bartlett kernel.
[3] Hill estimate with number of order statistics selected using Drees and Kaufmann (1998).

Given the results of the unit root test, I provide in Figure 3 graphs of the return series calculated as the first difference of the natural log of the USD/Bitcoin and RMB/Bitcoin rates. In this graph, the stationarity of these two series as indicated by the unit root tests is clear to see, as the series show no trend and no evidence of persistent deviation from the mean. This graph shows the extreme realizations of the return series that occurred at various times, including days in the third quarter of 2012, in the second and fourth quarters of 2013, and to a lesser extent during the first and fourth quarter of 2015. Also, apparent in the graph is the nearly identical timing of these extreme realizations in the two series. In fact, the correlation between these two series is extremely high, .9449.

Figure 3 - Differences in Log of USD/Bitcoin and Log of RMB/Bitcoin Series



The graphs in Figures 1-3 certainly suggest a strong relationship between the USD/Bitcoin and RMB/Bitcoin prices, and the relationship between these two price series can be illustrated by using the two series to construct the ratio of USD/RMB. This ration would be the implied USD/RMB exchange rate arrived at via triangular trade through the Bitcoin market. We will analyze this implied exchange rate and compare it to both the direct USD/RMB exchange rate, obtained from the Federal Reserve in St. Louis (FRED), and to an alternative implied

USD/RMB exchange rate arrived at via triangular trade through the direct exchange for USD per Euro and the direct exchange of RMB per Euro.

Our basic approach is to show that the USD/Bitcoin and RMB/Bitcoin, however volatile, exhibit behavior that is consistent with the USD/RMB direct exchange rate, and further that the implied USD/RMB rate from triangular trade via Bitcoins is also consistent with the implied USD/RMB rate from triangular trade via Euros.

Specifically, we will look at the three versions of the RMB/USD exchange rate: the direct market exchange rate, the exchange rate implied by triangular trade via the euro, i.e. the RMB/Euro and Euro/USD exchange rates, and the exchange rate implied by triangular trade via Bitcoins, i.e. the RMB/Bitcoin and Bitcoin/USD prices. Economic intuition tells us that all three of these exchange rates should be approximately equal to the extent that trades can be executed such that trader behavior will enforce a no-arbitrage condition. This type of relationship holds in traditional foreign exchange markets, and we expect to see that trading foreign exchange via Bitcoins will likewise approximately satisfy a no-arbitrage condition. There are some difficulties in making simultaneous trades in Bitcoins, and volume was relatively low in the early years of Bitcoin trading, so the experience in the Bitcoin market may not exactly coincide to behavior in traditional foreign exchange markets.

After examining our data, we will see that innovations to the market exchange rate leads to persistent changes of the Bitcoin price, but innovations to the Bitcoin implied exchange rate does not affect the market exchange rate. Because the Bitcoin implied exchange rate is made of two component Bitcoin prices, either of them could be adjusting. However, we will examine the component parts separately to see that the market with the smallest volume will do most of the adjusting (in this case, the RMB/Bitcoin rate). We will also show that the results we find in the Bitcoin market are mirrored in traditional triangular arbitrage as well.

CHAPTER I

DATA

We use five data series: The USD price of Bitcoins, the RMB price of Bitcoins, the market exchange rate of RMB per USD, the market exchange rate of USD per Euro, and the market exchange rate of RMB per Euro.

The daily Bitcoin prices were obtained from www.bitcoincharts.com. The data covers dates from August 1 2012 through August 1 2016. This period was chosen due to the availability of data at the hourly frequency for the USD price and RMB price of Bitcoins. In earlier periods, volume was low and there were many hours with no recorded trading price for one or both of the prices. The USD price per Bitcoin was obtained from the BitStamp exchange due to its frequently high volume. In years preceding the bankruptcy of the Mt. Gox exchange in Japan[4], BitStamp competed with Mt. Gox as one of the main exchanges for USD and Bitcoins, but beginning January 1 2014 Mt. Gox began experiencing problems and was shut down due to its link to illegal activities. The RMB per Bitcoin data was obtained from the btcnCNY exchange. The daily exchange rates involving the USD were obtained from the St. Louis' Federal Reserve Economic Database (FRED) and the Euro involved exchange rates were obtained from the European Central Bank website, www.ecb.europa.eu.

It is important to gather the exchange rates at the same point in time. The market exchange rates were available at 12:00 noon in New York City, in the USA's Eastern Time

[4]In January 2014, the prices on the Mt. Gox exchange and Bitstamp exchanges were quite similar. Figure XX in the appendix shows the price series in January 2014. Any differences in the prices are less than transaction costs and would not allow arbitrage to occur even if that were possible between the two exchanges.

Zone. Therefore, the Bitcoin prices were taken at the equivalent UTC (GMT) time of 17:00 during Daylight Savings Time, and 18:00 during Standard Time.[5]

Summary statistics for our three price series are presented in Table 2. Note the similarity in the series, especially in the mean and the first, second, and third quantiles. The first two series, the USD per RMB and the USD per RMB obtained via triangular trade through the Euro, are similar in all dimensions. The third series, the USD per RMB rate obtained via triangular trade through Bitcoin, has similar characteristics in the middle of the distribution, but has more extreme values in the tails of the distribution, more outliers, as can be seen in the minimum and maximum values of the series. This also shows up as a larger standard deviation and kurtosis. The Tail Index estimates indicate that these three series do not have fat tails, and this is supported by the Kurtosis values.

Table 2 - Summary Statistics for USD per RMB Exchange Rates

	USD per RMB	USD per RMB via Euro	USD per RMB via Bitcoin
Mean	0.159876	0.159830	0.159066
Std. Dev.	0.003776	0.003825	0.006653
Skew	-0.963253	-0.963307	-2.145455
Kurtosis	3.098107	3.106131	23.77644
Max.	0.165557	0.165670	0.196172
Q3	0.16286645	0.162757092	0.162819771
Q2	0.160913991	0.160805716	0.160505087
Q1	0.157483418	0.1575178	0.154906236
Min.	0.149225	0.148968	0.084926
Tail Index	587.2	437.5	38.0

[5] In the event that no Bitcoin trade took place during the relevant hour, we used the current standing price – the price from the most recent prior transaction – as our measure of the price, or from the next largest exchange if data had not been available for quite some time.

CHAPTER II

ANALYSIS

Despite the extreme volatility in the USD price of Bitcoin and RMB price of Bitcoin, there is a stable relationship between these two price series due to their link to the exchange rate market. The USD per Bitcoin and the RMB per Bitcoin price series can be used to generate an implied value of the USD per RMB exchange rate, one that could be obtained via triangular trade through Bitcoins. This exchange rate also exhibits volatility, but it never wanders too far from the FOREX market's USD per RMB exchange rate. Figure 4 illustrates these two series.

Figure 4 - USD per RMB Exchange Rates: Direct FOREX Rate and Implied Rate from Triangular Trade via Bitcoin

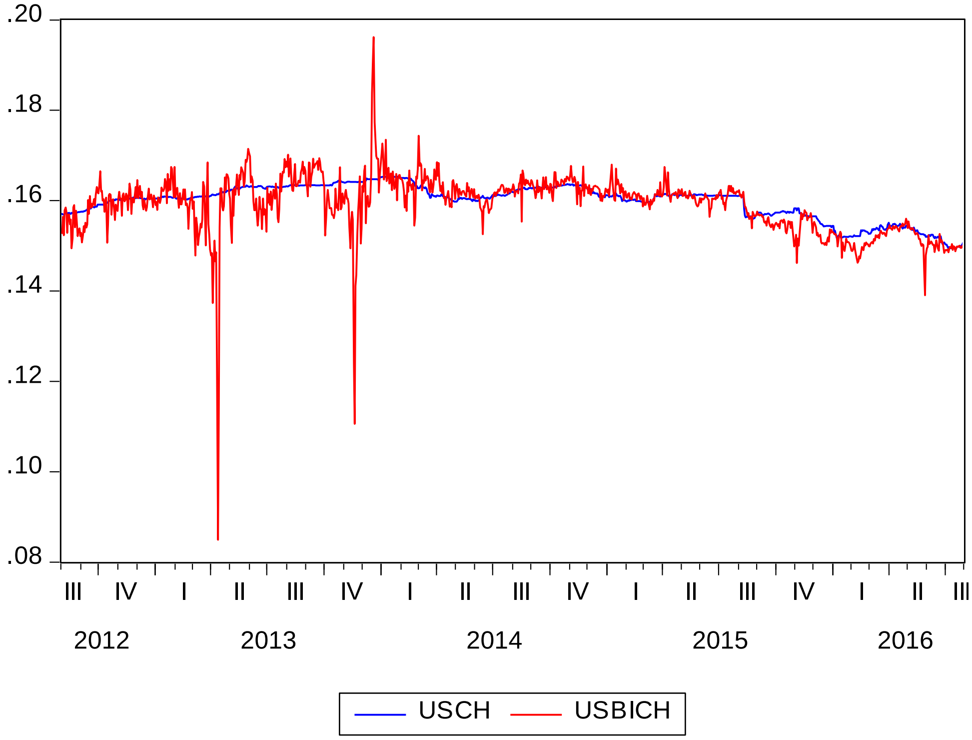
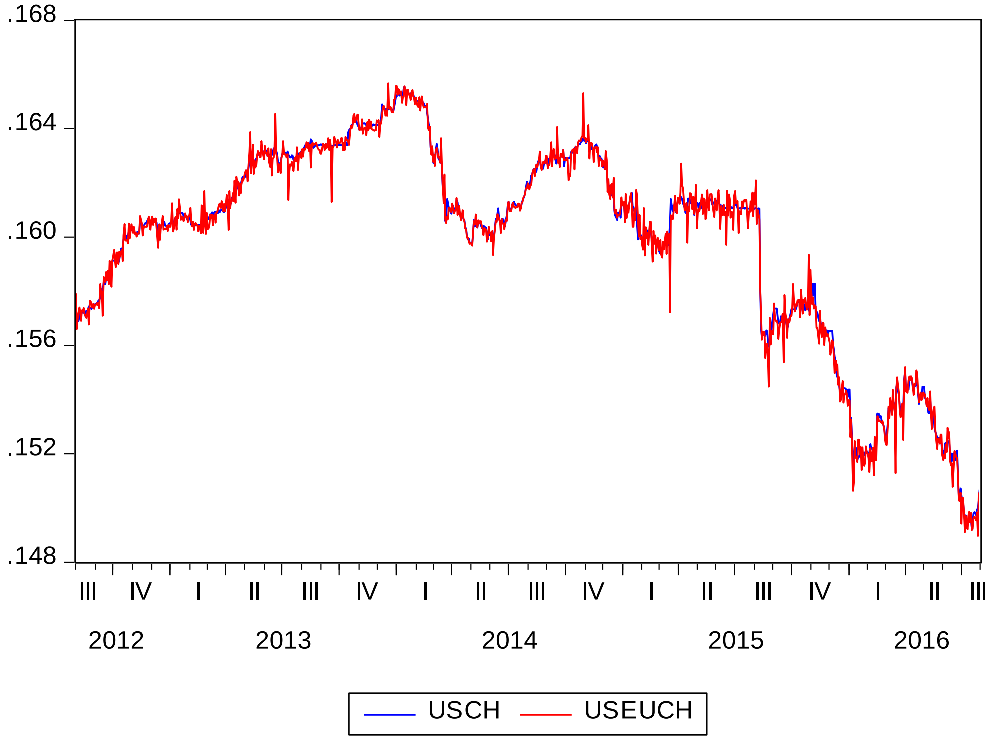


Figure 4 shows several important features of these two series. First, the series implied by triangular trade through Bitcoin is more volatile than the direct FOREX rate. Second, the series implied by trade through Bitcoin can move away from the direct FOREX rate and persistently stay some distance above or below the direct FOREX rate, but then eventually returns to the FOREX rate. This is the type of price movement that would be expected if there are transaction costs and difficulties with arbitraging price discrepancies, but still there are market forces poised to take advantage of pricing discrepancies whenever the two exchange rates get too far out of line. In fact, in the absence of transaction costs and in the presence of arbitrage possibilities, these two series would be expected to be in perfect agreement. Figure 4 suggests that arbitrage pressures work, but not immediately, and that there are significant transaction costs that allow prices to diverge before market forces act to drive them back together. Finally, the figure shows larger volatility and large discrepancies in the first part of the graph, which we will argue is due to lower volume in the Bitcoin markets during that more volatile period. Thinner, lower volume markets make it harder to arbitrage price discrepancies.

We also show a graph of the direct FOREX rate for USD per RMB against the implied rate from triangular trade through the EURO. This implied exchange rate is obtained from direct FOREX rate for USD per Euro and the direct FOREX rate for RMB per Euro. Since FOREX markets for all these currencies are characterized as thick markets, we would expect arbitrage pressures to keep the direct and implied rates almost equal. Figure 5 shows that this is indeed the case. The USD per RMB rate calculated from trade via the Euro is more volatile than the direct FOREX rate, but much less volatile than that shown in Figure 4.

Figure 5 - USD per RMB Exchange Rates: Direct FOREX Rate and Implied Rate from Triangular Trade via the Euro



Figures 4 and 5 suggest that our three USD per RMB exchange rates are cointegrated. We test for cointegration and estimate VECM models for two bivariate models corresponding to Figures 4 and 5, respectively, and for a trivariate model for all three series.

Cointegration tests are reported in Figure 3A, and cointegrating vectors along with adjustment coefficients (on the error correction terms) in Figure 3B. The first cointegration test is for the variables USD/RMB (direct) and USD/RMB (Bitcoin). Here, we strongly reject the hypothesis of zero cointegrating vectors, and we fail to reject the hypothesis of one cointegrating vector. Estimates for that one cointegrating vector are <1, -.56>, whereas we might expect the cointegrating vector to be <1, -1> in the presence of perfect arbitrage. The adjustment coefficients indicate that, when the two variables are out of equilibrium in the sense that the cointegrating vector differs from zero, then the adjustment occurs in the USD/RMB (Bitcoin) rate. The adjustment coefficient in the equation for USD/RMB (direct) is small in magnitude, indicating economic insignificance, and it is statistically insignificant as well.

The second cointegration test is for the USD/RMB (direct) and the USD/RMB (Euro). Again, we find evidence of one cointegrating vector, and we estimate it to be <1, -.99>, consistent with strong arbitrage pressures between these two series. The adjustment coefficient indicates that it is the USD/RMB (Euro) that adjusts to deviations of the cointegrating from zero. The adjustment coefficient in the equation for USD/RMB (direct) is small in magnitude and is statistically insignificant.

The third cointegration test is for all three variables, the USD/RMB (direct), USD/RMB (Bitcoin), and USD/RMB (Euro). We reject the hypothesis of zero cointegrating vectors, and we also reject the hypothesis of one cointegrating vector, but we fail to reject the hypothesis of two cointegrating vectors. Our estimates of these two cointegrating vectors are reported in Figure 3B. For our three variables, the first cointegrating vector (normalized) is <1, 0, -.99>, and the second is <0, 1,-1.77>. The first cointegrating vector is consistent with our results for the bivariate model for USD/RMB (direct) and USD/RMB (Euro), and the adjustment coefficients indicate that it is the USD/RMB (Euro) variable that adjusts to restore equilibrium when this cointegrating vector differs from zero. The second cointegrating vector is consistent with our results for the USD/RMB (direct) and the USD/RMB (Bitcoin). This can be seen by substituting the first cointegrating vector into the second cointegrating vector to eliminate the USD/RMB (Euro) variable, resulting in the cointegrating vector USD/RMB (direct) -.559 USD/RMB (Bitcoin), almost identical to the bivariate result for these two variables. For this second cointegrating vector, the adjustment coefficients indicate that it is the USD/RMB (Bitcoin) rate that adjusts when the cointegrating vector differs from zero.

Table 3A - Johansen Cointegration Tests

Variables	Number of Cointegrating Vectors	Trace Statistics	Critical Value (5%)	Marginal Probability
USD/RMB (Direct) USD/RMB (Bitcoin)	None	66.83734	15.495	0.0000
	One	0.105577	3.841	.7452
USD/RMB (Direct) USD/RMB (Euro)	None	158.7246	15.495	0.000
	One	0.111069	3.841	0.7389
USD/RMB (Direct) USD/RMB (Bitcoin) USD/RMB (Euro)	None	227.0887	29.797	0.0001
	One	66.74888	15.495	0.0000
	Two	0.104056	3.841	0.7470

Table 3B - Estimated Cointegrating Vectors and Adjustment Coefficients

Variables	Cointegrating Vectors	Adjustment Coefficients		
		First Equation	Second Equation	Third Equation
USD/RMB (Direct), USD/RMB (Bitcoin	<1, -0.8964>	0.001262 (0.00180)	0.259528** (0.03154)	NA
USD/RMB (Direct), USD/RMB (Euro)	<1, -0.990757>	-0.011929 (0.04471)	0.916685** (0.08411)	NA
USD/RMB (Direct), USD/RMB (Bitcoin), USD/RMB (Euro)	<1, 0, -0.99072>	-0.013418 (0.04482)	-0.199402 (0.79481)	0.915908** (0.08417)
	<0, 1, -1.10484>	-0.001175 (0.00160)	-0.233840** (0.02836)	0.002059 (0.00300)

Table 4 - Lag Length Selection

Variables	AIC	SIC	HQ
USD/RMB (Direct), USD/RMB (Bitcoin)	3	1	2
USD/RMB (Direct), USD/RMB (Euro)	2	1	2
USD/RMB (Direct), USD/RMB (Bitcoin), USD/RMB (Euro)	3	1	1

The cointegration tests were run with models that included five lags. Table 4 provides results from estimated VAR models with up to twenty lags and choosing the lag length that minimizes the AIC, SIC, and HQ statistics. All three criteria indicate VARs with a small number of lags, and based on these results, we estimate models with 2 lags.

Estimates of VECM models consistent with level VARs with 2 lags are presented in the following tables.

Table 5 VECM for USD/RMB (Direct) and USD/RMB (Euro)

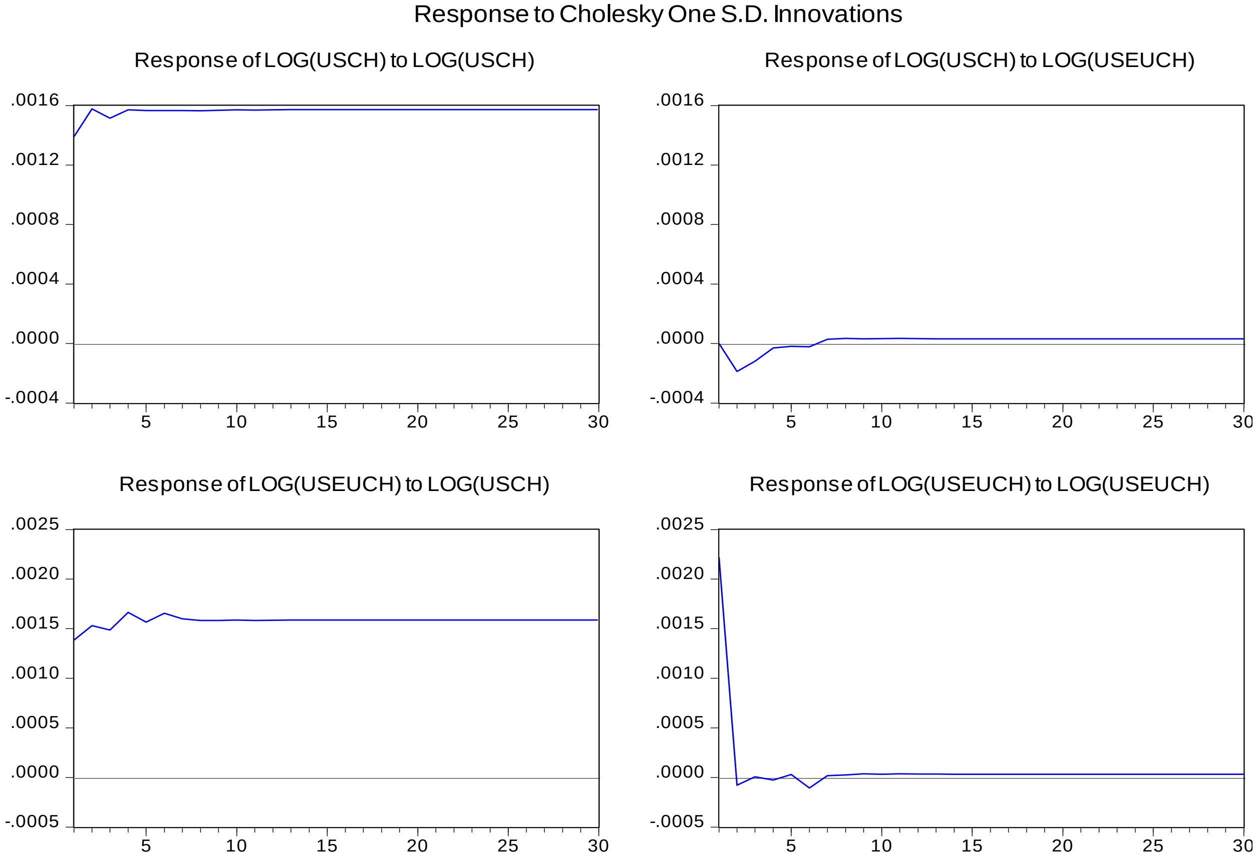


Table 6 VECM for USD/RMB (Direct) and USD/RMB (Bitcoin)

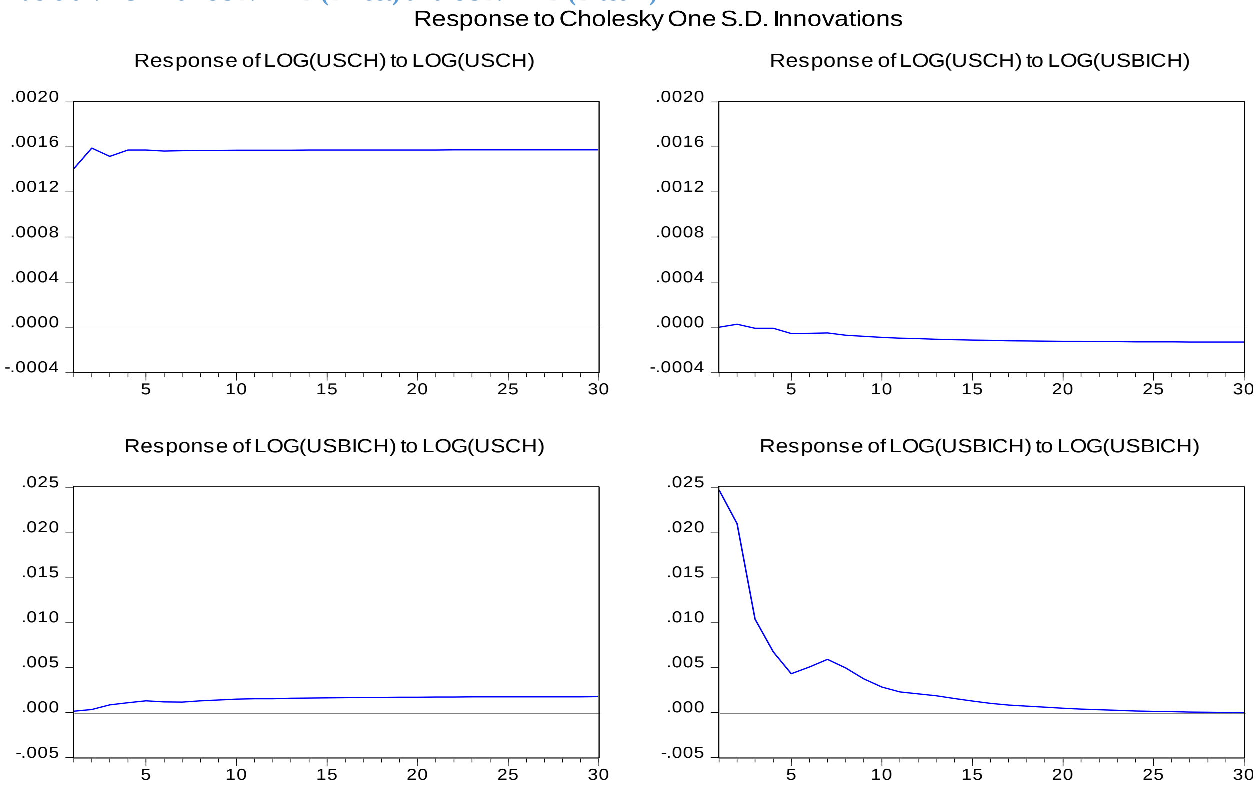
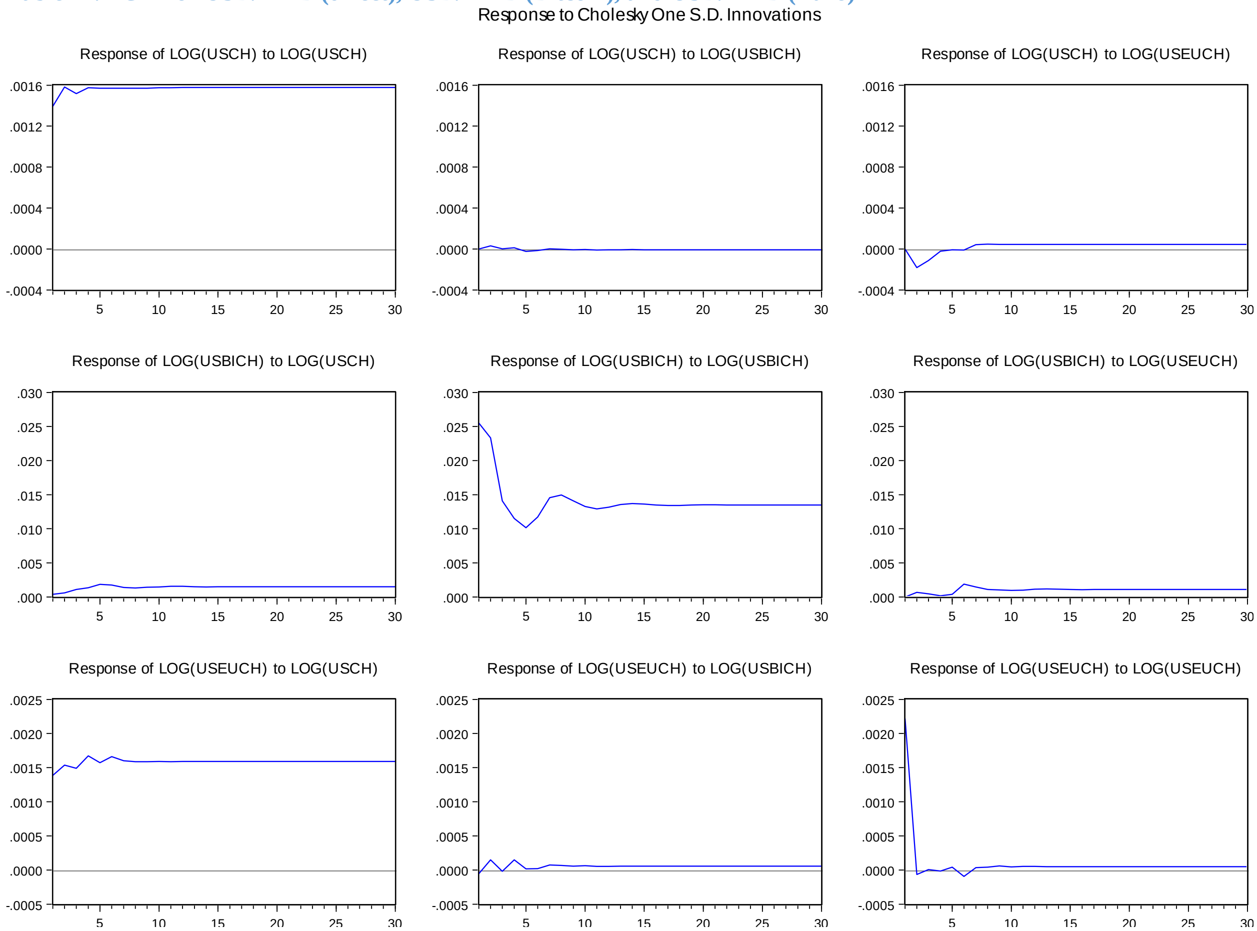


Table 7 VECM for USD/RMB (direct), USD/RMB (Bitcoin), and USD/RMB (Euro)



CONCLUSION

Since its introduction, Bitcoin has strived to establish itself as a national currency on par with the U.S. Dollar and the Chinese Renminbi, but has struggled with issues such as volatility and a reputation related to illegal activities. Above, we examined whether Bitcoin's volatility affected its response to arbitrage pressures by comparing the direct exchange rate of USD to RMB with the implied rates via Bitcoin and Euro. While the implied rate via the Euro revealed an extremely similar history to the direct rate, the Bitcoin exhibited greater innovations, especially in its early periods in 2012. We attributed this to the low volume and trading done in its early years. As Bitcoin became more popular and was traded with a higher volume, the implied rate grew closer to the direct rate and the implied rate via the Euro. Next, we found that it was the USD to RMB implied rate via the Bitcoin that adjusted to market pressures, as would be expected due to the heavy regulations that the other national currencies experience. We used cointegration tests between the direct rate and the implied rate via the Bitcoin, the direct rate and the implied rate via the Euro, and the direct rate and both implied rates. The first and last tests showed that it was the implied rate via the Bitcoin that adjusted.

The implied rate via the Bitcoin having a similarity to the direct rate and the implied rate via the Euro is important because it shows that even a currency outside government control and with as much volatility as Bitcoin is still subject to arbitrage pressures like other national currencies even if it doesn't have the same regulations. Though the innovations were greater in the beginning, the implied rate via the Bitcoin always adjusted back to the direct rate.

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