

# THE LARGE POWER-OUTPUT RADIAL INFLOW TURBINE

by

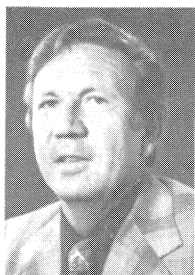
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## ABSTRACT

The performance of advanced radial inflow (RIF) turbines is reviewed and new results of stress and blade vibration analysis are presented. Due to the development of modern manufacturing techniques, large radial inflow turbines are a competitive reality for large power-output cryogenic turboexpanders, hot gas power recovery expanders and industrial gas turbines.

## INTRODUCTION

While the incompressible solution of the large power-output radial inflow turbine (RIF), known as "Francis Turbine" is domineering the hydro-electric industry, the compressible solution for large cryogenic and/or hot gas expander applications appears to be a new and innovative idea. As any new idea, it has to pass Zwicky's test: "First, the new idea is mocked as ridiculous and absurd; then it is admitted to be valid but overrated and of no particular significance; finally it is decided that the idea had been known long ago and that everybody had thought of it for himself." The cryogenic process industry has accepted the basic idea (1)\*\* and numerous RIF turbines are running in gas processing plants up to about 10,000 HP. Beyond this power level, there is some hesitance in accepting custom engineered RIF turbines. The questions are technical and we try to contribute toward their clarification. In case of the hot gas expander application of the RIF turbine, most process and machinery engineers admit that they have thought of it for themselves for a long time.

\*\*Number in parenthesis refers to references summarized on last page of paper.

Why, then, did the major process turbomachinery industry not develop the hot gas RIF expander? It becomes clear that this industry merely followed suit to the aircraft engine and steam turbine industry which, due to their special technical requirements, had to perfect the axial large power output turbine. In addition, the single stage RIF hot gas expander operates at high tip speeds, which requires a high degree of rigorous stress and dynamics analysis together with new forging and machining techniques. Looking, for example, at the new turbine requirements of large coal gasification plants (2) and tail gas turbine requirements of several basic chemical processes such as nitric acid (3), there exists a significant performance and economical potential for the RIF turbine.

The objective of this paper is to indicate that the idea stage has passed and technical solutions are being developed which provide rugged economic and high performance, high power output RIF cryogenic or hot gas expanders. In particular, aerodynamics of RIF turbines are well understood, the method of stress analysis and blade vibration analysis have been perfected and are being verified by tests; and most important, the manufacturing techniques such as forging and machining of large power output RIF turbine wheels have been accomplished. Therefore, new processes based upon the economic and performance potential of the RIF turbine are now seriously being investigated and pursued.

## AERODYNAMICS

Many papers have been written about Aerodynamics of RIF Turbines. The most noteworthy publications are by references (4), (5) and (6). Theory and test results agree reasonably well and 90% efficient machines can be built with confidence. As understood by the Turbomachinery industry, the performance of the radial inflow turbine, like any other compressible turbomachine, is a function of the basic similarity parameters which are: Specific speed  $N_s$ , specific diameter  $D_s$ , relative mach number  $M_w$  and Reynolds number  $Re$ . Detailed discussions about  $N_s$  are contained in (7) and (8). Figure 1 taken from (8) correlates efficiency as a function of specific speed for RIF and axial turbines. In order to achieve efficiencies above 80%, specific speeds larger than 50 should be selected.

For cryogenic applications which are seldom stress or material limited due to the modest enthalpy differential utilized in most air separation, LNG and cryogenic gas processing plants, optimum design values of tip speed to spouting velocity ratios ( $U/C_0$ ), degree of reaction  $R$ , and exit energy ratios  $(c_3/c_0)^2$  can be achieved. The correlation of these design parameters as function of specific speed has been presented in (8).

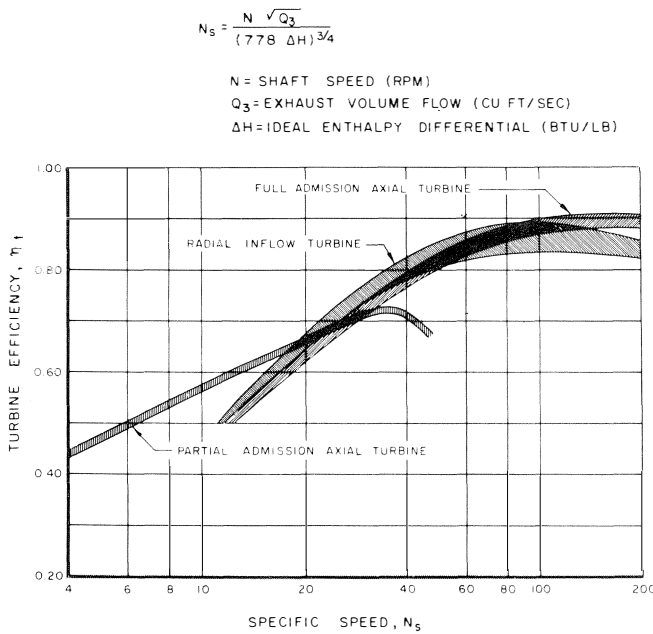


Figure 1. Generalized Turbine Performance.

In case of the hot gas expander, off-design solutions have to be accepted due to stress, dynamics and other systems considerations. For this purpose the general RIF theory is best computerized and applied in accordance to the following basic design rules which have proven to result in high efficiency RIF turbines.

- Prescribe continuous acceleration of velocity through machine including nozzles and rotor.
- Select sufficient large exit energy ratio as to assure elimination of large three-dimensional velocity profile distortions.
- Select sufficient solidity  $1.25 < \sigma < 1.75$  in the exducer section as to avoid separation or undue slip.
- The optimum number of RIF turbine blades ranges between 11 and 15 whereby the lower number is required for high specific speed and the higher blade number for the lower specific speed range.

With the above guidelines high performance RIF Turbines can be custom engineered with a high degree of confidence by utilizing a rigorous mean streamline analysis method. The latter is most conveniently computerized as to study the effect of various prescribed velocity and wrap angle distributions on performance and mechanical integrity. As a matter of fact, aerodynamic analysis and stress analysis have to be conducted concurrently for highly loaded hot gas expanders in order to achieve a sound, rugged and efficient design. Figure 2, for example, presents the manufacturing process of two 35" RIF hot gas expander wheels which have been designed according to the above guidelines. The detailed procedure is briefly described for clarity.

- (1) For the given process input data
- ΔH — Total to total isentropic enthalpy differential across RIF turbine (BTU's per pound)
  - W — Mass flow (pounds per second)

- P<sub>01</sub> — Total inlet pressure (PSIA)
- T<sub>01</sub> — Total inlet temperature (°R)
- M — Average molecular weight based upon gas composition.
- z — Compressibility parameter based on average of the complete expansion process.

The preliminary design analysis determines the optimum specific speed and the resulting efficiency. At the same time, the optimum geometry is determined giving the following basic machine parameters.

- α<sub>2</sub> — Nozzle angle (degrees)
- h/D<sub>2</sub> — Tip blade width to diameter ratio
- D<sub>2</sub> — Tip diameter and therefore resulting tip speed
- L/D<sub>2</sub> — Axial length to tip diameter ratio
- d<sub>m3</sub>/D<sub>2</sub> — Mean discharge to tip diameter ratio
- β<sub>2</sub> — Rotor inlet angle (preferably 90°)
- β<sub>3</sub> — Discharge angle

(2) With the overall dimensions given, which have to satisfy the basic fluid mechanic and thermodynamic laws, the detailed blade geometry is determined as follows:

- Prescribe meridional velocity distribution.
- Prescribe Θ (wrap angle) distribution as function of radius and axial length.

The optimum wrap angle distribution lies between quadratic and cubic or a combination of both resulting in a polynomial solution.

- Prescribe blade thickness distribution throughout wheel based upon input from experience and preliminary stress analysis.
- Prescribe mean streamline shape connecting inlet and discharge determined by preliminary analysis.

With the above data solve continuity and change of momentum equations step by step throughout the wheel whereby nozzle and wheel loss data have to be included. The result of this computerized solution is the determination of hub and shroud shapes and the velocity distribution throughout the turbine wheel.

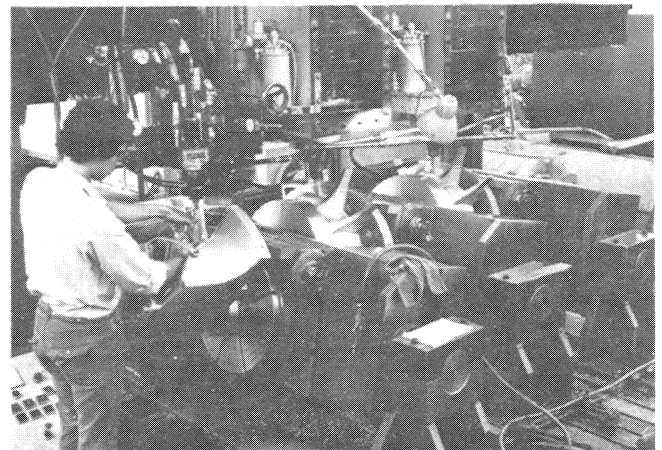


Figure 2. Manufacturing process of two 35-inch RIF hot gas expander wheels.

With the blade coordinates given, a detailed stress analysis can be conducted which will finalize the necessary blade thickness distribution and the overall mechanical design of the turbine wheel.

The final output is preferably an NC-tape which allows cutting of the tooling on an appropriate tape controlled milling machine.

Being specific, the outlined method has been successfully applied to numerous cryogenic expanders which have achieved efficiencies above 84% and has recently been applied to the hot gas expander wheel shown in Figure 2. The preliminary test data substantiated the design efficiency.

The most noteworthy advantages of the RIF turbine over the axial turbine are:

- Expansion of high pressure ratios in single stage.
- Less sensitive to angle of attack.
- Wide operating range with good efficiency when variable two-dimensional nozzles are installed.
- Less sensitive to side and tip clearances.
- Machine runs cooler due to the extraction of more work in the single stage.
- More rugged and economic design when compared with the comparable axial multistage machine.

However, these advantages can only be realized when blade vibration modes and wheel stresses are well under-

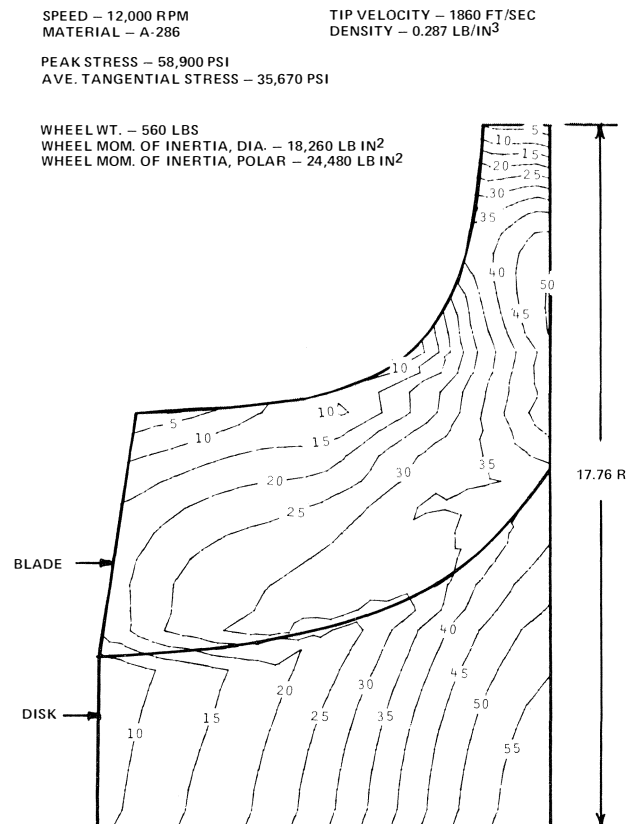


Figure 3. High pressure turbine wheel centrifugal stress lines of constant effective (VON-MISES) stresses.

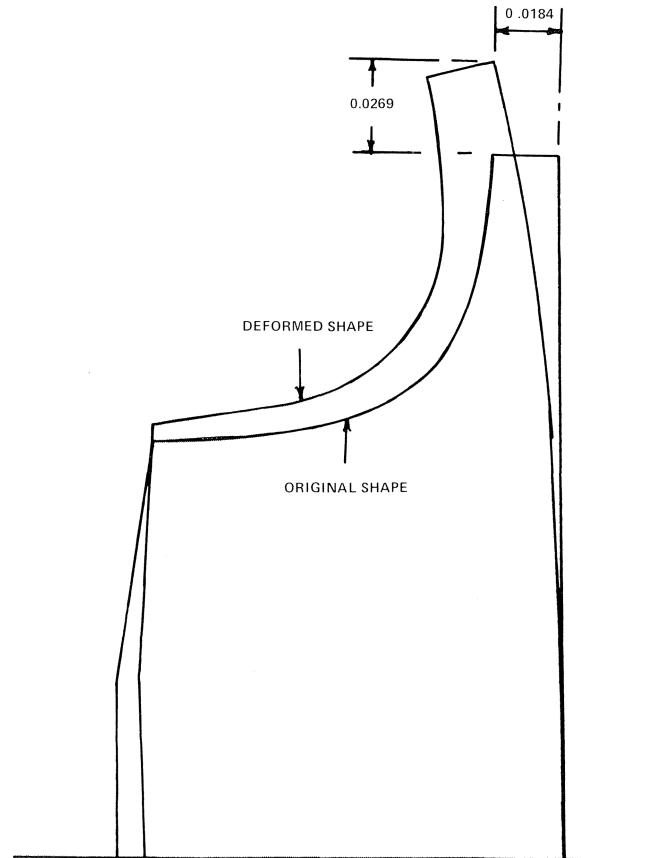


Figure 4. High pressure turbine wheel deformed and undeformed shapes.

stood and designed for the same or higher reliability as demonstrated by multistage axial turbines.

#### TURBINE WHEEL STRESS ANALYSIS

Referring to the 1250°F RIF turbine illustrated in Figure 2, Figures 3 and 4 show the distribution of centrifugal stresses and the boundary displacements, respectively, for the turbine wheel at the design rotating speed of 12,000 RPM. The lines of constant stress were computed for the Von-Mises combined stress criteria,  $\sigma_e$ , defined by the expression:

$$\sigma_e = 1/2 [ (\sigma_r - \sigma_t)^2 + (\sigma_t - \sigma_z)^2 + (\sigma_z - \sigma_r)^2 + 6\tau_{rz}^2 ]^{1/2}$$

where  $\sigma_r$ ,  $\sigma_t$  and  $\sigma_z$  are the radial, tangential, and axial normal stress components while  $\tau_{rz}$  is the shearing stress component.

The stress and deformation analysis was performed with a two-dimensional, finite-element model of the integral disk and blades (9). The disk was modelled with triangular solids of revolution elements while the blades were represented by triangular plates. The complete analytical model was composed of 535 stations and 967 elements. Reference to Figure 3 illustrates the rather moderate stress levels induced in the wheel for a tip speed of 1860 ft/sec. The high margin of safety is also implied from a closer look at the stresses and material strength of the disk and blades. At 12,000 RPM the peak stress throughout the

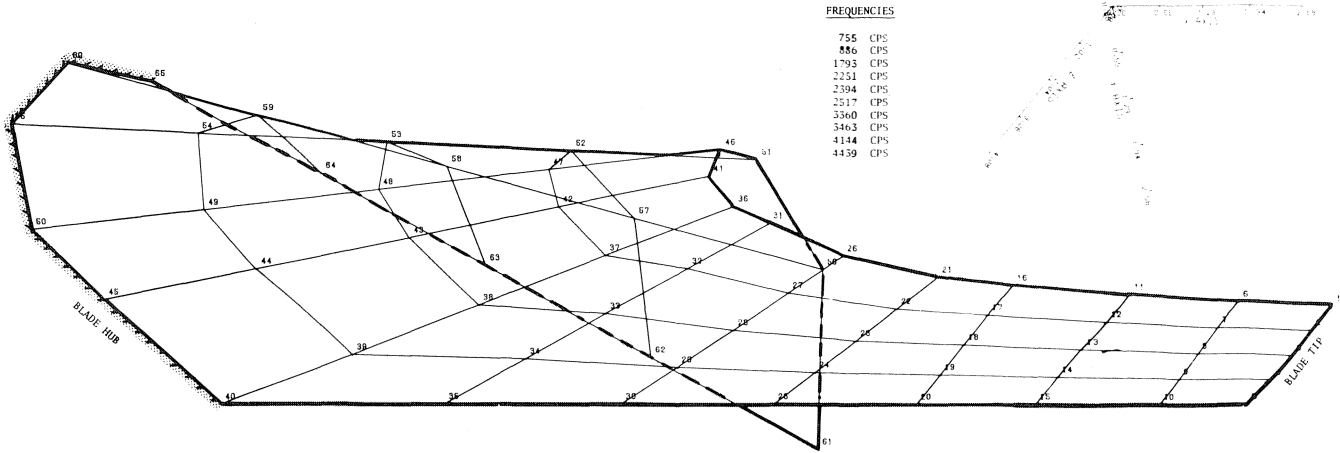


Figure 5. Nodal network of H. P. Turbine wheel blade used in vibration analysis.

wheel is only 58.900 PSI. The peak is at the center of the solid disk where the metal temperature is 525°F. The blade peak stress is 51.740 PSI and occurs approximately 5 inches downward (radially) from the tip. At this location the metal temperature is 730°F. The disk average tangential stress, which usually governs the disk burst failure, is only 35.670 PSI at 12,000 RPM.

Regarding strength and ductility of the wheel material (A-286 Steel) we find the following result: At 525°F the minimum yield stress (0.2% offset) is 74,000 PSI. At this temperature, the material has a minimum ultimate tensile strength of 117,000 PSI and an elongation of 23%. The corresponding yield and ultimate strengths and ductility at 730°F are 72,000 PSI, 115,000 PSI, and 19% respectively.

Based on the induced stresses and material properties discussed one can draw conclusions regarding the overspeed capabilities of the wheel. These are as follows:

- (1) Initial yielding of the wheel will occur at the disk center. This will be at an overspeed of 13,450 RPM.
- (2) Initial yielding of the blade will occur at a speed of 14,150 RPM.
- (3) Blade tip fracture will occur at an overspeed of 19,620 RPM.
- (4) Wheel disk burst will occur at a speed of 21,730 RPM.

Items (1) through (4) listed previously will now be discussed further in an effort to show their relative importance regarding the wheel overspeed capabilities. First, because of the material ductility and stress gradients in the area of initial yielding, items (1) and (2) are less important than items (3) and (4) in accessing the structural reliability of the rotor. Item (3), blade tip tensile fracture, at a speed below the burst speed of the disk is desirable since the blade will be contained by the housing. This failure type can be withstood without the danger of destroying the complete rotor system.

In summary, it is concluded that the overspeed capability of the wheel is quite high, being 163% above the operating speed, and the failure mode far less serious than that for disk burst.

The wheel elastic displacements are presented in Figure 4, and were computed from the following equations:

$$U = \frac{r}{E} [\sigma_t - \mu(\sigma_r + \sigma_t)]$$

$$W = \frac{z}{E} [\sigma_z - \mu(\sigma_r + \sigma_t)]$$

Where r and z are the radial and axial coordinates of the wheel. U and W are the displacements at r and z, and E and  $\mu$  are the Elastic Modulus and Poissons Ratio for the wheel material, respectively. The results of the elastic displacement analysis together with the thermal analysis are being used as a design guide for selecting the assembly and running clearances of the RIF wheel.

### TURBINE WHEEL BLADE NATURAL FREQUENCIES

A three-dimensional, finite-element, model was used to analytically determine the turbine wheel blade natural frequencies and corresponding free vibration modes (10). In the analysis the blade mid-plane geometry was represented by a series of quadrilateral plate elements. The elements are capable of taking into account both in-plane and out-of-plane forces. The complete blade model, shown in Figure 5, consisted of 65 stations and 48 plate elements.

Natural frequencies and modes were computed up to 4500 cycles/second. Ten frequencies, also presented in Figure 5, were identified. Figures 6 through 9 show four of the vibration modes selected for illustration. These particular modes correspond to the natural frequencies computed at 755, 2251, 3360, and 4144 cycles/second, respectively.

Of the ten computed modes eight were, for all practical purposes, uncoupled with regard to simultaneous blade tip and exducer motions. For those modes excitations input near the blade tip will not induce deformations and displacements at the exducer. Similarly, excitations input near the exducer will not excite the tip of the blades.

The two frequencies where significant coupling was involved were at 3463 and 4144 cycles/second. At the lower frequency the blade tip twists simultaneously with the tangential displacement of the exducer. At the upper frequency the exducer moves tangentially while the blade tip both twists and displaces in the tangential direction.

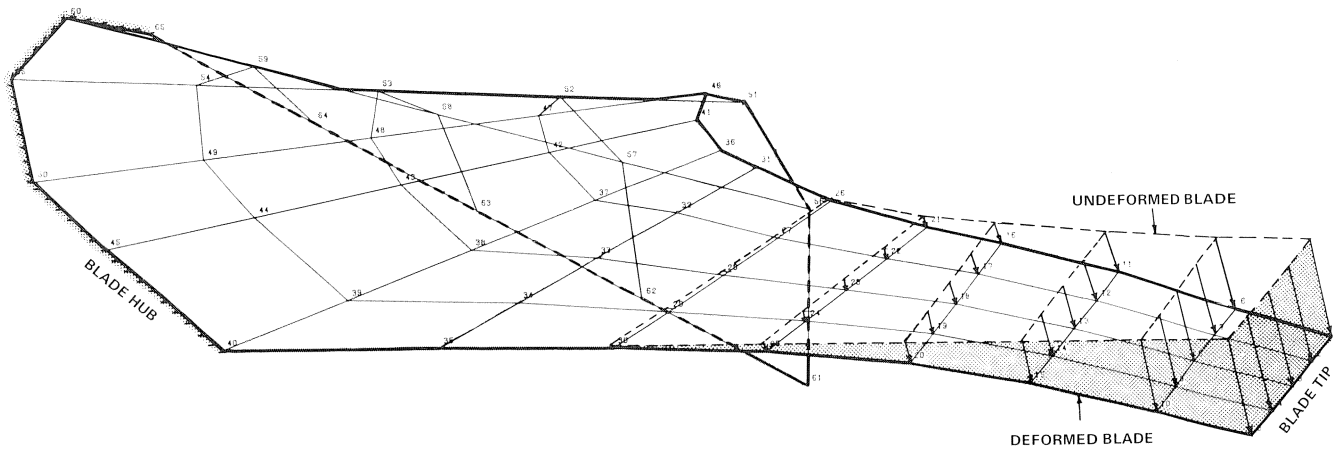


Figure 6. High pressure turbine wheel blade vibration mode; frequency—755 CPS.

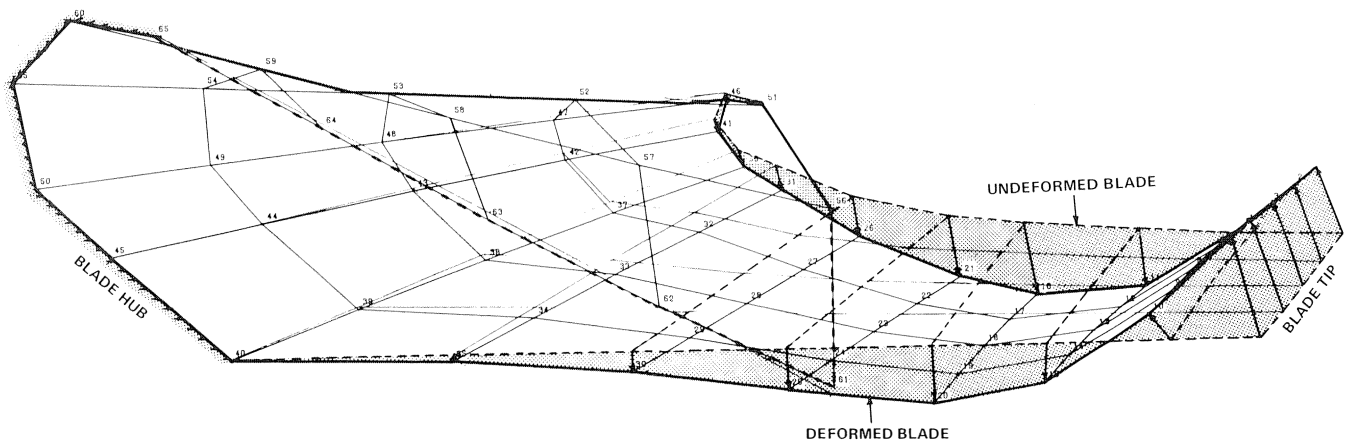


Figure 7. High pressure turbine wheel blade vibration mode; frequency—2251 CPS.

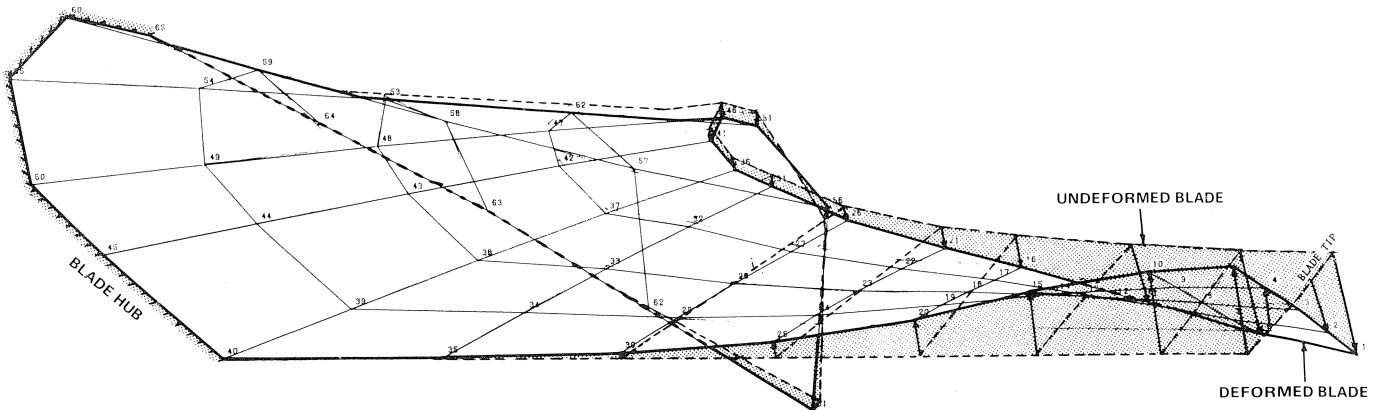


Figure 8. High pressure turbine wheel blade vibration mode; frequency—3360 CPS.

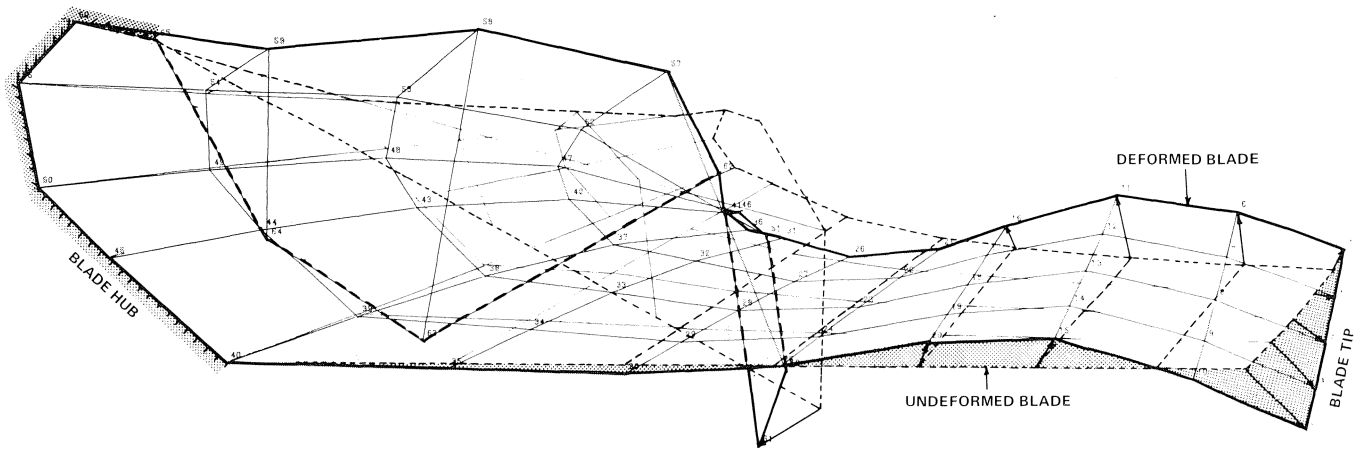


Figure 9. High pressure turbine wheel blade vibration mode; frequency—4144 CPS.

The modal results were used, together with vibration and damping results obtained from sweep tests on the blades, as the basis for selection of the number of nozzles for the turbine. In this selection a frequency vs. rotating speed, Campbell type vibration graph or interference diagram, was constructed. The natural frequencies and excitation lines, corresponding to number of nozzles, were superimposed in the diagram. For very lightly damped systems, such as integral turbine or compressor blades, the intersection of the lines of nozzle excitation with the natural frequency curves, locates the rotating speeds where the dynamic response will peak.

In order to verify the analysis and to determine the amplification factors of the natural frequencies of interest, a detailed test program was conducted. The test program and results are discussed briefly in the following.

The blade vibration analysis tests were performed at the Aeronutronic Division of the Philco-Ford Corporation located in Newport Beach, California. The test apparatus and instrumentation consisted of the following:

- Vibration Exciter: Ling Model 335 Shaker capable of 15,000 force pounds.
- Power Amplifier: Ling 120/150 capable of an output of 120 KVA.
- Accelerometers: Endevco Model 2226.
- Force Transducer: Endevco Model 2106E.
- Charge Amplifiers: Unholtz Dickie 8PMCVA.
- Tracking Filters: Spectral Dynamics SD101A.
- Phase Angle Meter: Acton Model 3298.
- Log Converters: Houston Model 150.
- XY Plotter: Moseley Model 4B.
- Co-Quad Analyzer: Spectral Dynamics SD109.
- Sweep Oscillator: Spectral Dynamics SD104.
- Sine Servo Control: Spectral Dynamics SD105.
- Oscilloscope: Tektronics Model RM502A dual trace.

From the aerodynamic design of the blade, a mathematical computer model of the blade was constructed. Re-

sults from the computer predicted resonant frequencies and accompanying mode shapes. Based on the acceptability of the computer results, a full sized vane was then machined. This full sized vane model served two purposes: (1) it became the tooling for cutting the final wheel and (2) it is also the test specimen used to determine actual blade resonant frequencies and mode shapes.

The wheel model was mounted on top of a Team hydrostatic bearing and driven with the vibration exciter from a point source through the center of gravity of the test package as indicated in Figure 10. The blade was instrumented with piezoelectric accelerometers in several locations over the areas of interest to acquire mode shapes of different resonant frequencies. Sine sweeps were conducted over the entire frequency range which the machine would experience while in actual operation. Figure 11 shows a view of the instrumentation for control and data acquisition utilized during the test program.

The first step in the testing was to identify the resonant frequencies of the vanes. This was accomplished by sine sweeps through the operating frequency range and plotting vane input vibration levels and levels at different points on the vane versus frequency. From these plots, resonant frequencies were identified. Once this was accomplished the mode shape of each resonant frequency was defined by use of mechanical impedance methods. By measuring the phase angle between the input forcing function to the blade and the acceleration of various points on the blade, the relative position of each point on the vane to the base of the vane can be determined by graphically depicting the relative position of each point on the vane, the mode shape at a resonant frequency may be drawn.

Results of the actual vibration test when compared to the computer predicted mode shape and resonant frequencies were in general agreement. Based on the test results, the number of nozzles for the expander were selected in such a way that no resonant frequency would be excited in the operating speed range of the machine. Another interesting result from the vibration testing of this particular large, three-dimensional, RIF turbine revealed that resonant frequencies in the tip and exducer portions of the wheels were for the most part decoupled from the rest of the blade. This is significant in that it suggests

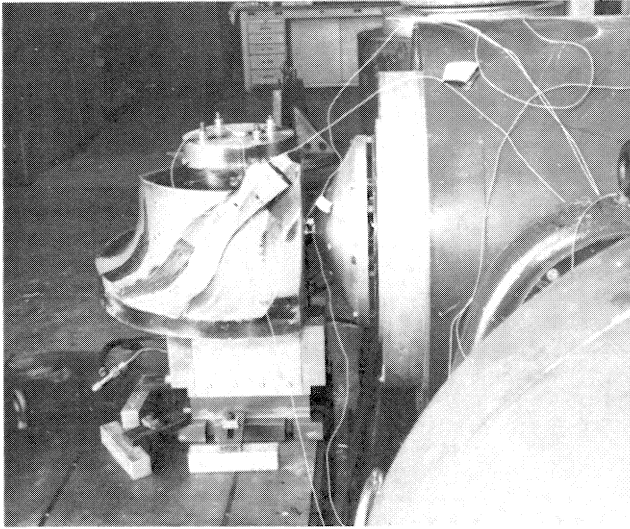


Figure 10. The wheel model mounted on top of a Team hydrostatic.

that some forcing functions which could lead to blade resonance can be tolerated as long as the forcing function is confined to selected portions of the wheel.

#### MANUFACTURING ADVANCES

Significant achievements in manufacturing techniques of custom engineered turbo-machinery are the application of NC controlled milling machines and the development of new forging methods and techniques which result in reliable and rugged large diameter, large power-output RIF turbines.

While the discussion of the NC Controlled machining and the required computer application is beyond the scope of the present paper, and many discussions and papers will be published in the near future during the forthcoming gas turbine meeting of the ASME, we want to concentrate on the achievements of forging large RIF turbine discs.



Figure 11. Instrumentation for control and data acquisition.

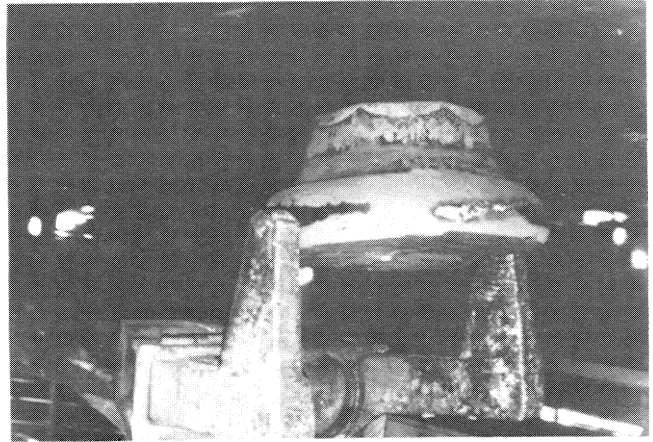


Figure 12. Removal from Die.

One of the reasons that the large power output RIF turbine has not been developed is the lack of industry experience and effort to develop reliable forgings in a 30-50 inch diameter class. As a matter of fact, the largest gas turbine using an RIF turbine is only 2000 HP shaft output and uses a turbine wheel of 20 inches diameter. Significant recent accomplishments have led to the development of forging techniques which provide rugged and reliable 40 inch dia, and 20 inch thick forgings for hot gas expander applications. For example, figure 12 presents a large forging in the intermediate stage of the forging process, and figure 13 presents the finished forging shown in a tool at the forging facility. The forging is manufactured from Allegheny Ludlum's Altemp A-286 high temperature (1,000 to 1,300°F) alloy. Design criteria for the huge hot gas expander require a forging of uniform properties, high strength and ductility especially in the radial direction. To achieve the high quality forging necessary and to ascertain that properties specified by the design were present, a full size forging was sectioned and completely tested. During the processing of the initial forging, a careful documentation of conditions and procedures was maintained, so the manufacturing process would exactly duplicate the processing step in making of subsequent forgings. Starting point was an ingot carefully melted and carefully bloomed, tested, cropped and edged

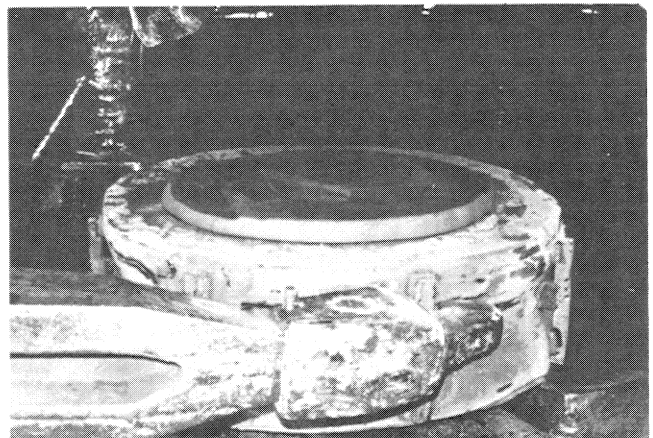


Figure 13. Last upset.



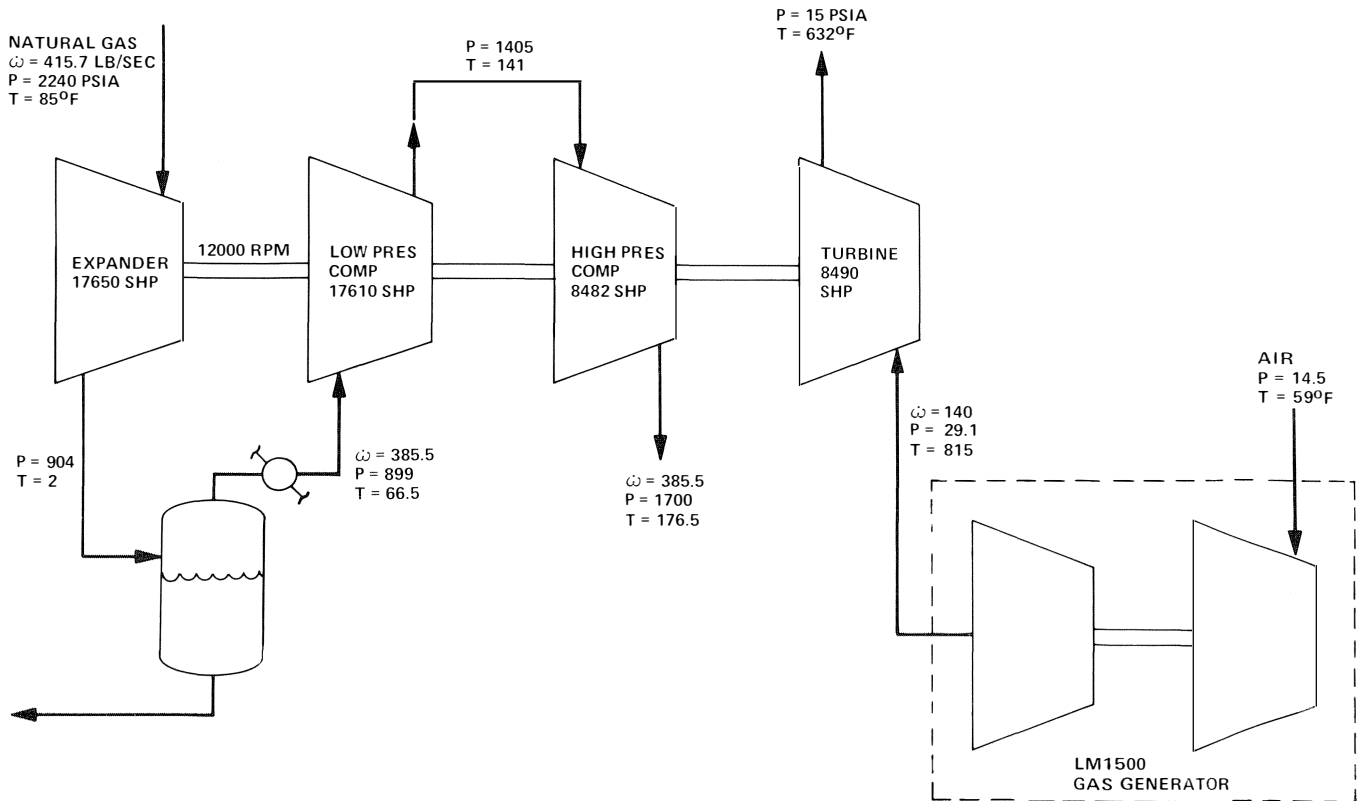


Figure 14. Turboexpander—Compression System.

sampled. The resulting cut billet was subjected to a series of forging operations calculated to give the maximum physical properties in the direction of future high stress areas. These high stress areas were identified by the stress analysis discussed previously. After forging, the part was subjected to a controlled heat treatment to develop the uniform properties needed. Comparing the specification of AMS5737E for A286, it was very encouraging to find that the actual tensile strength was 152,000 instead of 140,000 PSIA. The yield strength was 113,000 PSIA instead of 90,000 PSIA. In summary, utilizing new forging techniques such as the one discussed in (11), the development of large power-output RIF turbines has become a reality and will lead to the development of many new applications in the process as well as power RIF turbine industry.

#### APPLICATIONS

The sound development of the large power-output RIF Turbine opens the door for a wealth of new and unique turbomachinery applications. These applications range from large cryogenic RIF expanders for base load plants to large power-output RIF gas turbines, and certainly include the numerous applications for power recovery turbines in several new petrochemical and power systems processes. For the purpose of this discussion, we will limit ourselves to one specific example which combines cryogenic and hot gas RIF technology in one package. Figure 14, for example, presents the flow schematic of a combined cryogenic expander and recompression package for a large gas processing plant. As indicated, a

17,650 shaft horsepower cryogenic RIF gas expander is combined on one shaft with two stages of radial compressors for the purpose of recompression and the additional hot gas turbine for covering the power differential between the requirement of the recompressor and the power available from the cryogenic expander. The hot gas power turbine provides 8,490 shaft horsepower and consists of a single stage RIF turbine. The hot gas source is an industrial gas generator. The basic simplicity of the single frame design not only assures the utmost in reliability, but also reduces the machinery installation task to the bare fundamentals. The setting in place and alignment of a multiplicity of coupling connected shaft segments belonging to individual casings are eliminated, thus simplifying installation and improving reliability. The latter results from fewer components and the elimination of misalignment of several coupled casings due to thermal transients.

The makeup of additional power is achieved by the aerodynamic coupling between the industrial gasifier and the hot gas turbine mounted on the same shaft with the recompressor and the large cryogenic expander.

#### SUMMARY

The large power output RIF turbine has a great potential within the process and power turbomachinery industry. We hope the present discussion contributes toward better understanding and clarification of the RIF technology available and being applied toward large power output requirements.



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