THE CALCULATION AND VERIFICATION OF TORSIONAL NATURAL FREQUENCIES FOR TURBOMACHINERY EQUIPMENT STRINGS

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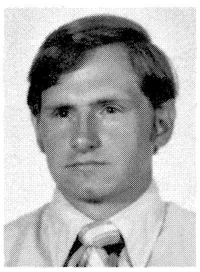
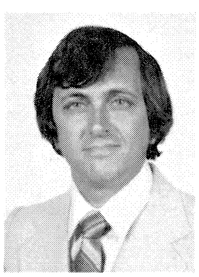


Roy E. Mondy graduated from North Carolina State University honors program in 1973 with a Bachelors Degree in Mechanical Engineering. In 1975 he received his Master of Science Degree in Mechanical Engineering from the University of Virginia. He joined the Westinghouse LRA Division at East Pittsburgh, PA in 1975 as a Mechanical Test Engineer in the Development Engineering Department. There his duties related to the measurement and control of vibrations as related to rotor dynamics and included dynamic balancing of large rotors. In 1978 he joined Ingersoll-Rand's Turbo Products Division as an Analytical Engineer. His primary duties were to conduct analytical studies in the areas of lateral and torsional rotor dynamics as relating to turbomachinery. His duties also included correlation of field data to analytical studies and resolution of test stand problems. He joined the Fossil and Hydro Technical Services group at the Virginia Electric and Power Company (Vepco) in 1982 as a Mechanical Equipment Specialist. There his duties are to provide engineering services to the systems fossil and hydro power stations on an as necessary basis. Roy is a licensed engineer in both New Jersey and Virginia and is an Associate ASME member.



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ABSTRACT

The analytical techniques used to design turbomachinery equipment strings from a torsional dynamics standpoint are discussed. Consideration is also given to both torsional natural frequency placement and potential excitation sources. Three cases are presented wherein the torsional natural frequencies and mode shapes are calculated via Holzer Transfer Matrix computer code and the frequencies then verified by measurement on the actual hardware. The accuracy of both analytical and measuring techniques is discussed from the aspect of the minimum acceptable interference margins for acceptable equipment operation.

ANALYTICAL METHOD

The Ingersoll-Rand proprietary computer code used to calculate undamped torsional natural frequencies and mode shapes of turbomachinery equipment strings employs a Holzer Transfer Matrix routine. Since most linear component equipment strings are considered to have a low level of system torsional damping, the undamped natural frequency calculation sufficiently reflects the actual train natural frequencies without the added complexities and uncertainties of having to estimate levels of torsional damping. The convergence accuracy of this Holzer routine is to within ± 0.01 Hz (0.6 cpm) of the actual analytical value. Instead of using the magnitude of the torque residual at the end of each iteration as the convergence dependent variable, this code searches for the torque residual's crossover points on the frequency axis to within the specified tolerance limit. This is because for all modes above the first several, which are typically coupling torsional stiffness controlled, the slope of the torque residual curves becomes very steep and may result in excessive computer iteration time if a residual torque magnitude convergence routine is employed. Since the frequencies of interest are generally several hundred cpm or larger, the error in the frequency convergence routine is less than $\pm 0.01\%$ regardless of the magnitude of the torque residual.

EQUIPMENT STRING MODELLING

The versatility of this computer code permits both shaft geometry and concentrated mass-elastic data to be used as input in describing the shafting of any equipment string. When shaft geometry is used as input, the program considers the effective flexibility penetration of smaller diameter shaft sections into adjacent larger diameter sections. This allowance for penetration effects more accurately approximates the true flexibility of the physical system than by simply summing the calculated flexibilities of the individual shaft sections.

The entire shafting of systems which include at least one gearbox is modelled using absolute quantities, the same as a single speed system. The program internally accounts for any speed difference between shaft sections caused by the presence of a gearbox and automatically formulates the equivalent single shaft model before calculating the torsional natural frequencies and mode shapes of the system. However, once the equivalent system's mode shapes have been calculated, they are converted back to representing the actual physical system where step changes in both shaft speed and response amplitude occur at each gearmesh.

The program has the feature to permit the material shear modulus to be used in modelling each shaft in the equipment string. By specifying the shear modulus, the effects of the different grades of material used in the various shafts are properly incorporated into the natural frequency calculation. The importance of including the proper shear modulus in formulating the shaft models can be seen by considering that the difference caused by the two typical values of 7.50 x 10^4 N/mm^2 (10.8 x 10^6 PSI) and 8.06 x 10^4 N/mm^2 (11.6 x 10^6 PSI) can be up to 3-4% in the frequencies calculated.

Typical items which can easily be modelled by concentrated mass-elastic data exclusively are couplings, gears, impellers, turbine stages, and motor rotor attachments. Experience has shown that couplings are best modelled as one torsional spring with the appropriate half-coupling inertia at each end. Gears lend themselves to lumped mass modelling because the majority of their inertia is in the gear wheels and the shafts closely approximate low inertia torsional springs. Impellers and turbine stages can be frequently considered as lumped inertias since they typically contribute nothing to the torsional stiffness of their respective shafts. Motor rotor attachments such as rotor cores and brushless exciters are not easily modelled since they are shrunk onto the shaft over an extended distance and it is not always obvious how their inertias and stiffnesses enter the motor shaft.

GENERAL DESIGN CONSIDERATIONS

The typical approach to designing an equipment string from a torsional dynamics standpoint is to calculate the system's torsional natural frequencies and attempt to locate them outside of the proposed operating speed range. By setting the train natural frequencies outside of the nominal design speed range on the 1 and 2-per operating lines and away from any other known frequencies of potential torsional excitation, the natural frequencies will not amplify and cause operating problems. In most instances the placement of the natural frequencies of interest can be accomplished by specifying the appropriate torsional stiffness of each coupling. However, occasionally the natural frequency of an individual unit which cannot be moved by a coupling stiffness change will fall into the operating speed range. When this occurs consideration must be given to modifying the basic design of the unit if it is judged that this frequency must be removed from the operating speed zone.

In general, experience has shown it wise to make the spacer of each coupling as torsionally soft as practical while making the shafts of the individual units as torsionally stiff as practical. By following this philosophy, the coupling stiffness controlled natural frequencies are usually placed below the operating speed range while the natural frequencies of the individual units are usually placed above the operating speed range. The major advantage of this technique is to permit the typically more important coupling controlled frequencies to be accurately calculated because the shaft participation of the individual units and their penetration into the coupling hub effects are minimized.

CASE HISTORIES

The following case histories demonstrate the accuracy to which the natural frequencies of complex turbomachinery equipment strings can be routinely calculated. These histories show the need to adhere to the above design techniques for correct frequency calculation. Also discussed are any particular problems associated with either the analytical or experimental techniques used. The test data presented is representative of that typically obtained using a commercially available torsiograph.

Case 1

The subject turbomachinery string consists of an industrial gas turbine driving a four-stage 1067 mm (42 inch) centrifugal load compressor through a speed increasing gearset. Both low speed and high speed continuously lubricated couplings are of the geared type, Figure 1. The design speed range of the train is 80%-105% of the gas turbine rated speed of 77.8 Hz (4670 RPM). The corresponding compressor speed range is 104.2 to 136.8 Hz (6252-8206 RPM).

The in-depth torsional vibration analysis conducted for the turbomachinery user resulted in a required re-design of the high-speed gear-type coupling. The design modification resulted in acceptable operating margins between the fundamental torsional modes of the turborotor train and the first and second harmonics of all operating shaft speeds, Figure 2.

Both Ingersoll-Rand and the manufacturer of the gas turbine and gearset performed independent torsional vibration analyses. The results of these studies were presented to the prime contractor and ultimate user. Although all parties agreed to the recommended design modification of the high speed compressor coupling in order to remove the compressor mode from the operational speed range of the train, the actual results of the two analyses were not in good agreement. This disagreement precipitated a customer requirement to accurately determine the torsional critical speed locations through field measurement, torsiograph testing, with the modified coupling in place.

The train second torsional vibration mode was removed from the operating speed range via the coupling modification according to the I-R analysis. The predicted range for Mode 2 of 60.7 Hz (3640 CPM) to 63.3 Hz (3800 CPM) is based on a range of the coupling hub penetration factor for the high speed coupling:

f_2 = 63.3 Hz (3800 CPM) —
Based on ⅓ P.F. assumption

f_2 = 60.7 Hz (3640 CPM) —
Based on 59% P.F., a refinement

I-R took the position that 63.3 Hz (3800 CPM) was a conservative upper limit prediction while 60.7 Hz (3640 CPM) was the most accurate prediction for Mode 2. The user required Mode 2 to be below 62.3 Hz (3736 CPM), which is 80% of power turbine rated speed.

The analysis performed by the gas turbine supplier using the modified coupling, predicted the Mode 2 frequency to be equal to 69.3 Hz (4156 CPM). This frequency was in substantial disagreement with the I-R analysis and did not satisfy the

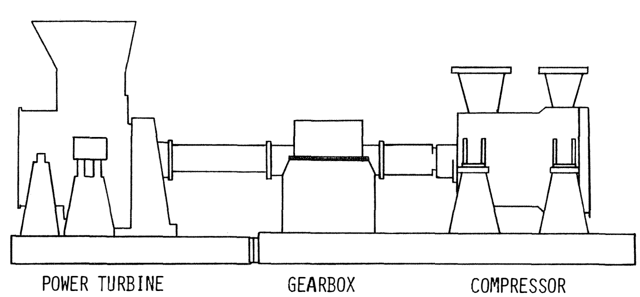

Figure 1. General Arrangement of Equipment String for Case 1.

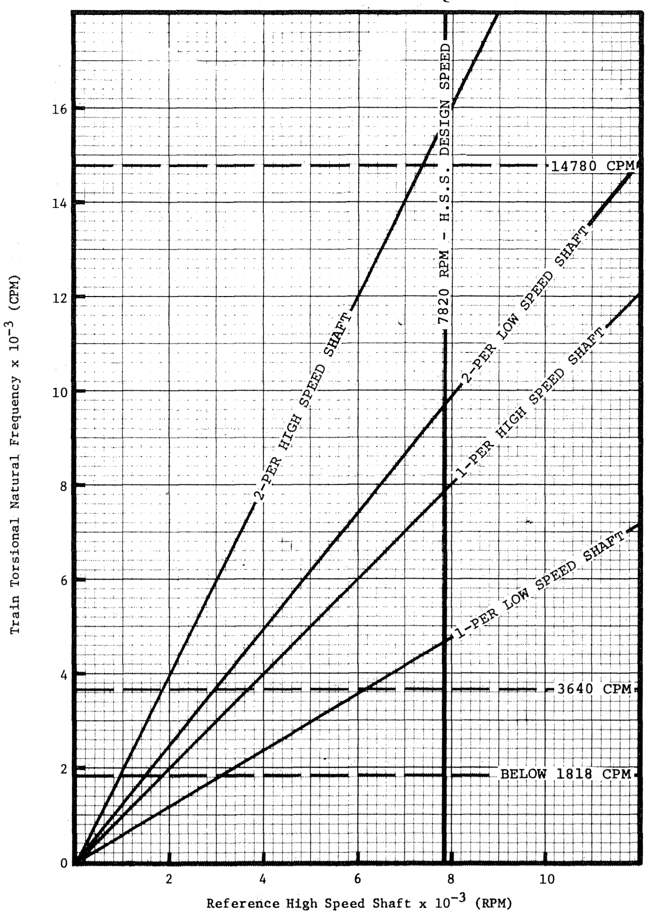

Figure 2. Torsional Interference Diagram for the Turbomachinery Equipment String of Case 1.

required operating speed margins. Attempts to reconcile this disagreement proved futile in that the turbine manufacturer calculated coupling torsional stiffness values by his own in-house computer code irrespective of shaft penetration effects. His calculated stiffnesses were twice as high as those values supplied by the coupling manufacturer and used by I-R.

Experimental determination of the train torsional natural frequencies was carried out on site with the modified high speed coupling in place. The results of this torsiograph testing were documented by the turbine supplier. The Mode 2 frequency was pinpointed during all the accel, decel, and load runs to be at 60.8 Hz (3650 CPM).

Table I summarizes the results of the analyses and test for the Mode 2 natural frequency. These results confirm the excellent agreement between the I-R Analysis and the actual test results for the determination of the system's second torsional mode of vibration. No torsional vibration problems existed within the unit's operating speed range.

Case 2

This equipment string consists of a hot gas expander, axial flow compressor, steam turbine, and a motor-generator. The couplings are of the dry diaphragm type. The general arrangement of the train is shown in Figure 3. The requirement was placed on this equipment string that it had to be capable of operating satisfactorily with the expander uncoupled. When the torsiograph tests of these two train configurations, Figures 4 and 5, did not agree with the analytical predictions, Table II,

TABLE I. COMPARISON OF THE TEST AND CALCULATED MODE 2 TORSIONAL NATURAL FREQUENCY OF CASE 1.

Method	Frequency (CPM)	% Deviation From Test Value
Test	3650	—
I-R Analysis	3640	−0.27
Gas Turbine Vendor Analysis	4156	+13.9

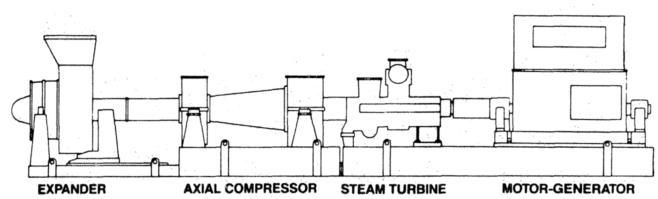

Figure 3. General Arrangement of Equipment String for Case 2.

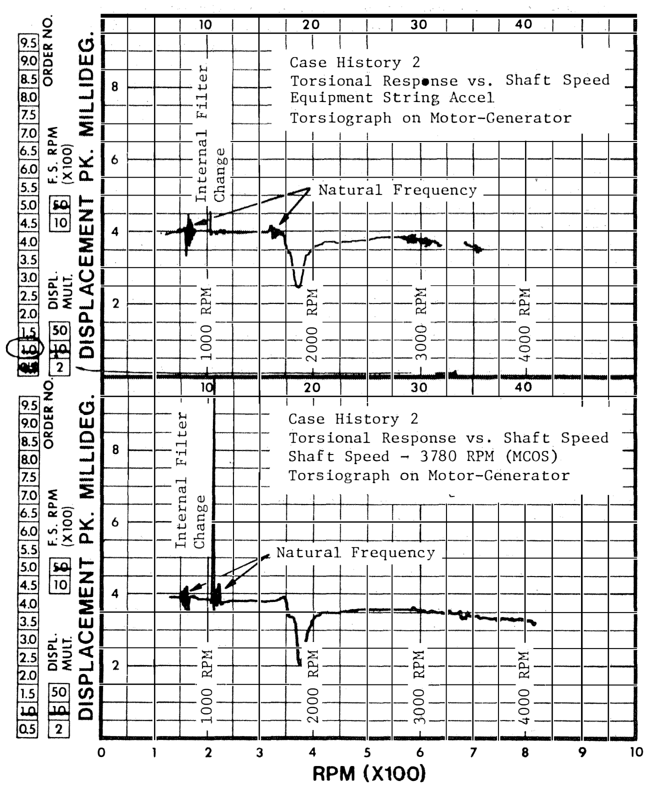

Figure 4. Case 2 Response vs. Frequency Torsiograph Plot for the Full String Configuration.

a review of the torsional natural frequency study was required as well as a calibration check of the torsiograph.

The torsiograph calibration check indicated that the torsiograph readout was approximately 1.25 Hz (75 CPM) high, accounting for part of the discrepancy between the analytical and test natural frequency values. The review of the analytical study ultimately led to the discovery that the coupling vendor

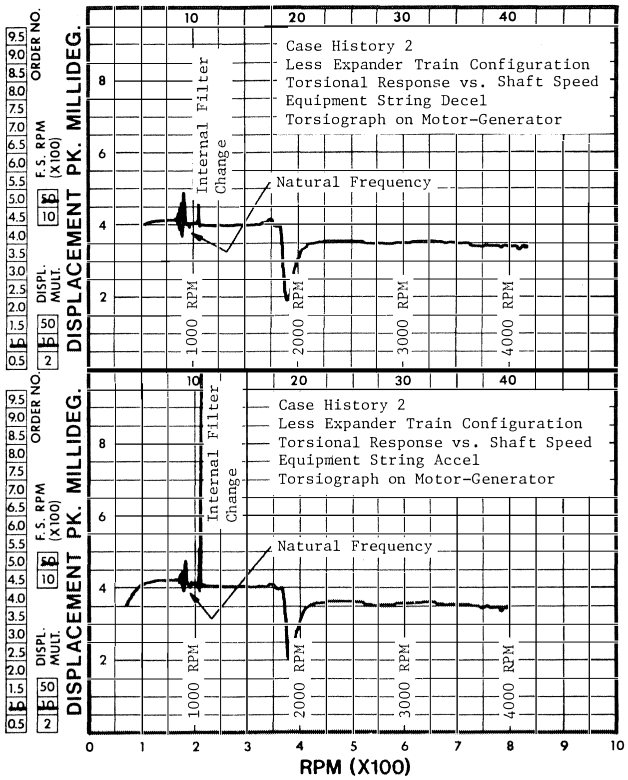

Figure 5. Case 2 Response vs. Frequency Torsiograph Plot for the Less Expander String Configuration.

had made errors in his torsional stiffness calculations and that the torsional stiffness values stated on the drawings were not those of the couplings supplied. When the correct torsional stiffness of each coupling was used to recalculate the natural frequencies of both train configurations, agreement between the test and analysis frequencies was obtained, Table III. Figure 6 presents the torsional interference diagram for both configurations of the subject equipment string.

The torsiograph test data of Figure 4 shows that the three coupling controlled modes of the full train configuration can be identified using the two traces. The torsiograph test data of Figure 5 only clearly identifies the first mode of the less expander equipment configuration. The ripple that occurs in both traces of Figure 5 at approximately 29.2 Hz (1750 RPM) cannot be identified as the second mode of the train. The dip which occurs in all four traces of Figures 4 and 5 at approximately 31.25 Hz (1875 RPM) corresponds to the lateral critical speed of the motor-generator shaft to which the torsiograph was attached. Figure 7 shows the torsional excitation harmonic content, Histogram, for the full equipment string operating at 63.0 Hz (3780 RPM). This figure shows that the majority of the torsional excitation response exists at 1 and 2-per rotational frequency with all other frequency components being of significantly less amplitude.

Case 3

The subject turbomachinery string consists of a hot gas expander, axial flow compressor, steam turbine, speed reducing gearset, and a motor-generator. These units are coupled with dry diaphragm-type couplings. The general arrangement of this equipment string is shown in Figure 8. The torsional interference diagram, Figure 9, shows the analytical placement of the system's lowest 5 torsional natural frequencies.

Since the torsiograph was attached to the low speed gear, the frequency range of the 1-per torsional response component extended only to just above 25 Hz (1500 CPM). Therefore the fourth and higher frequency modes were not observed. The three modes which were observed are compared to the calculated values in Table IV. This table shows that there is

TABLE II. COMPARISON OF RAW TEST AND INITIALLY CALCULATED TORSIONAL NATURAL FREQUENCIES OF CASE 2.

Configuration	Mode	Frequency Calculated (CPM)	Frequency Measured (CPM)	% Deviation From Measured Value
Full Train	First	657	800	−17.8
	Second	945	1100	−14.1
	Third	1333	1600	−16.7
Less Expander	First	741	900	−21.5
	Second	1321	not excited	—

TABLE III. CORRECTED TEST AND CALCULATED TORSIONAL NATURAL FREQUENCIES OF CASE 2.

Configuration	Mode	Frequency Calculated (CPM)	Frequency Measured (CPM)	% Deviation From Measured Value
Full Train	First	734	725	+1.2
	Second	1000	1025	−2.5
	Third	1510	1525	−1.0
Less Expander	First	834	825	+1.1
	Second	1492	not excited	—

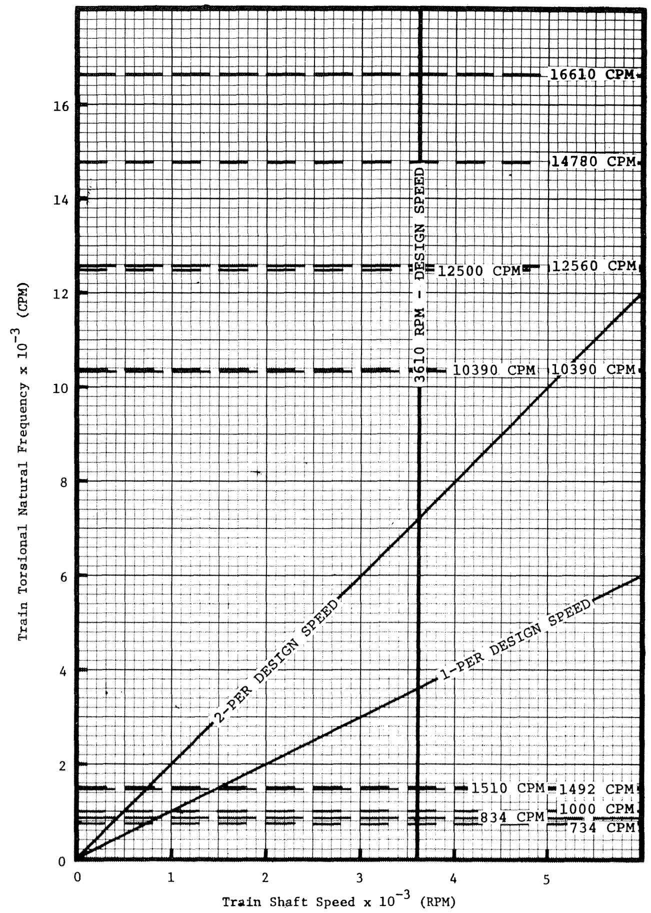

Figure 6. Torsional Interference Diagram for the Turbomachinery Equipment String of Case 2.

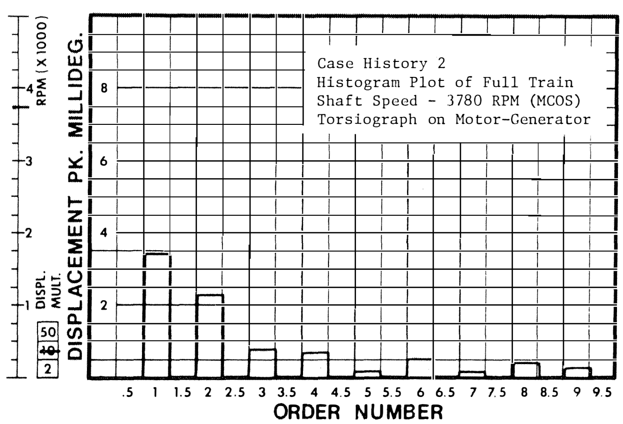

Figure 7. Case 2 Histogram for the Full String Configuration.

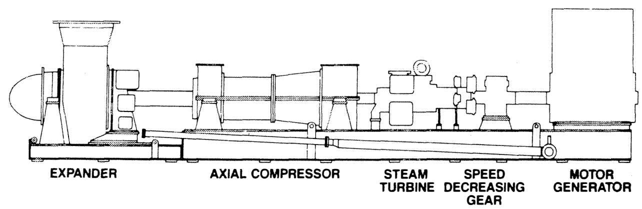

Figure 8. General Arrangement of Equipment String for Case 3.

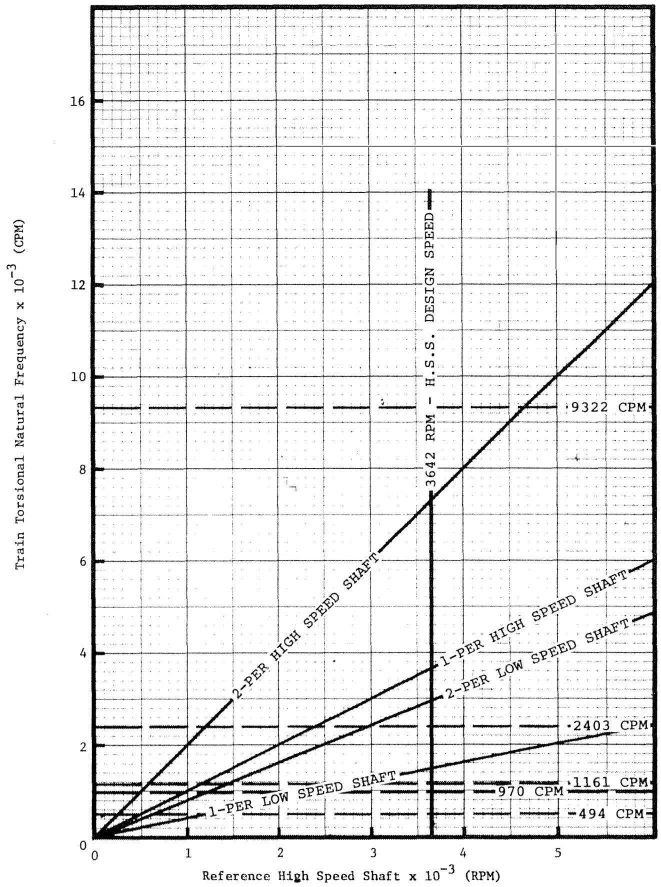

Figure 9. Torsional Interference Diagram for the Turbomachinery Equipment String of Case 3.

TABLE IV. COMPARISON OF TEST AND CALCULATED TORSIONAL NATURAL FREQUENCIES FOR CASE 3.

Mode	Frequency Calculated	Frequency Measured	% Deviation From Measured Value
First	494	500	−1.2
Second	970	1012	−4.2
Third	1161	1187	−2.2

acceptable agreement between the calculated and measured natural frequencies.

Figure 10 shows the Displacement vs. Speed data for the torsiograph test. This data reflects the torsional response of the system to 1-per, fundamental, torsional excitation. Figure 11 presents two histogram plots. These plots show that the fundamental as well as the first three harmonics (1, 2, 3 and 4-per) are major potential sources of excitation.

SUMMARY

This paper has presented 3 case histories of turbomachinery strings which clearly demonstrate the need for rigorous analysis in the design phases for proper natural frequency placement, as well as the benefit of torsiograph testing for frequency placement verification. The major conclusions which can be drawn from this paper are:

1. Analytical prediction of actual torsional natural fre-

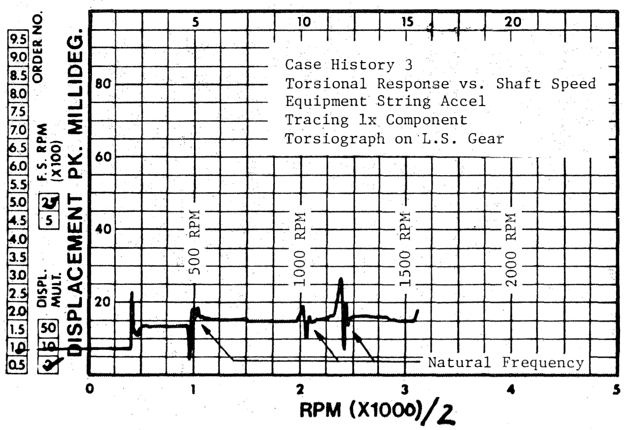

Figure 10. Case 3 Response vs. Frequency Torsiograph Plot.

quencies of rotating systems, turbomachinery, can be accomplished with a great degree of accuracy given the proper mathematical modelling techniques as well as a thorough understanding of the engineering principles involved in rotating equipment.

2. The error of using an undamped model to calculate the torsional natural frequencies of turbomachinery strings can be limited to less than ±5% of the actual frequency.
3. The largest possible torsional interference margins should be designed into turbomachinery strings to minimize the possible impact of manufacturing and analysis errors.
4. Coupling shaft end penetration effects must be considered when shaft end flexibility and coupling total flexibility are of the same order of magnitude.
5. In general couplings with the largest possible spacer flexibility should be selected for large turbomachinery equipment string applications.
6. The presence of a gearset in an equipment string requires both fundamental and harmonic displacement vs. frequency plots in torsiograph testing.
7. When practical torsiograph tests should be used to verify that turbomachinery strings are manufactured as designed, torsionally.

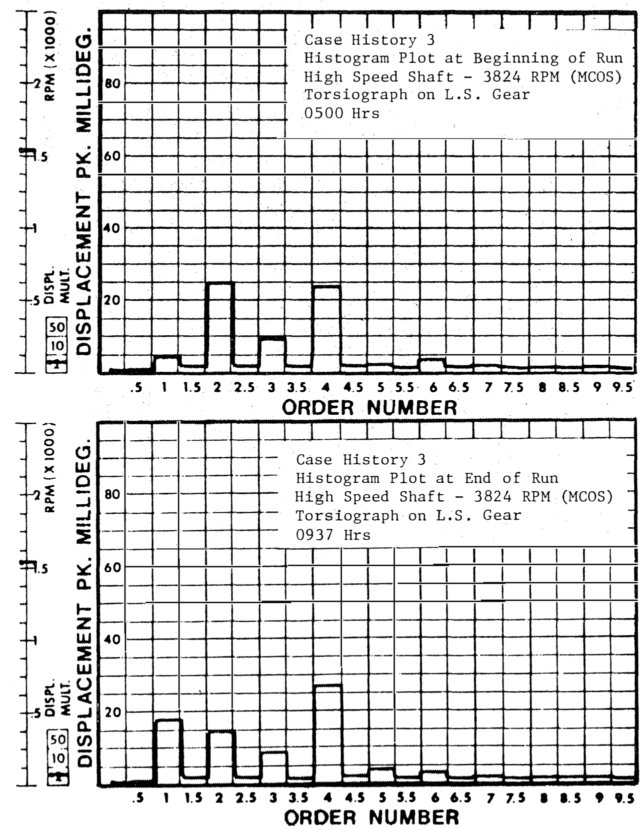

Figure 11. Case 3 Histograms at Beginning and End of Four Hour Mechanical Test Run.