A GENERALIZED AND SIMPLIFIED TRANSIENT TORQUE ANALYSIS FOR SYNCHRONOUS MOTOR DRIVE TRAINS

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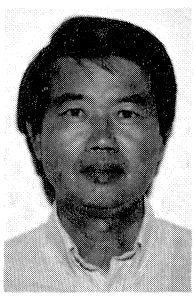

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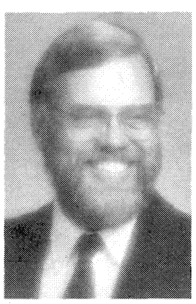

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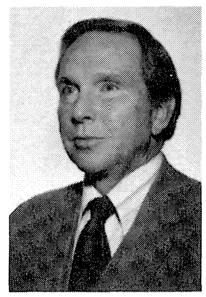

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ABSTRACT

The start-up of synchronous motor drive trains is usually associated with torsionally excited vibrations and low-cycle fatigue problems. Traditional calculation methods used for analysis of such a system involve computerized integrations with very small time steps and many degrees of freedom.

A simple method is presented herein which uses the knowledge of system natural frequencies and mode shapes and a general dimensionless integration data plot. A sample problem is included to demonstrate the application of the method. The results are compared with the more rigorous, traditional method to illustrate the accuracy of the simplified method. A procedure is also included to relate the dynamic torque to the torsional low-cycle fatigue limit, thus establishing a safe number of starts.

INTRODUCTION

Large synchronous motors tend to have a decrease in acceleration just before they reach synchronous speed. At the same time, their vibratory exciting torque increases. This torque is at twice slip frequency, which coincides, for a moment, with the lowest torsional system natural frequency somewhere during the running up process. In other words, the usually large inertia system is excited torsionally during every start-up (Figure 1). The level of vibratory shear stress that is reached depends on how fast the motor can pass the "critical speed," how much damping is available in the shafting, and, of course, the size of the shafting at the weakest link, such as at the bearing journal, shaft end, coupling, etc. It is not uncommon to see shaft torsional low-cycle fatigue problems in these machinery trains, especially those associated with motors of "solid pole" structure [1].

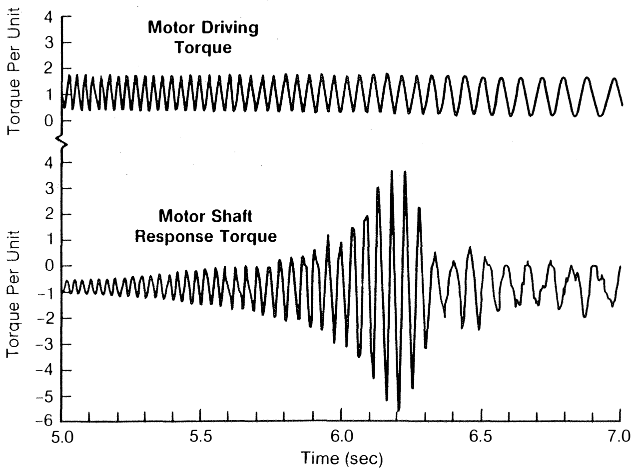

Figure 1. Example of Synchronous Motor Torsional Response.

Traditionally, calculation of the transient torque is a lengthy and costly task, because it involves computerized integrations with very small time steps and many degrees of freedom. Presented herein is a simple method which uses the results of frequency and mode shape from the common Holzer method for a torsional system and a specially generated dimensionless integration data plot. The frequency and mode shape determined by the Holzer method is preliminary to any torsional system analysis, and it always precedes the transient calculation.

SIMPLIFIED METHOD

During the start-up, the motor will pass a speed band width around the critical speed where the system will be excited. Since the system is usually lightly damped, the speed band is relatively small, compared to the total speed range. The rate of speed increase and the motor exciting torque in the band width can be practically assumed as constants. The vibratory motion of the system at this critical speed is represented by the modal equation:

$$\frac{d^2q_1}{dt^2} + 2\xi\omega_1 \frac{dq_1}{dt} + \omega_1^2 q_1 = \frac{VT}{I_1}\,\text{SIN}\left[2\pi\left(2nN_s\left(1 - \frac{N}{N_s}\right)\right)t\right] \qquad (1)$$

where

q_1 = modal (angular) displacement of first mode
ξ = system damping ratio
V = first mode shape displacement at motor
V_i = first mode shape displacement at ith station of rotor model
T = motor vibratory torque amplitude at 2 × slip frequency
I_1 = first modal inertia* = $\sum_i I_i V_i^2$ (lb-in-sec²)
I_i = inertia at ith station of rotor model
ω_1 = first torsional natural frequency (rad/sec)
N_s = synchronous speed (cps)
n = number of poles = 3 for N_s = 20 cps
= 2 for N_s = 30 cps
N = motor speed (cps)
t = time (sec)

One can solve for q_1 as a function of time, and then the vibratory torque at any shaft location (say between station i and station $(i + 1)$ with stiffness K_i) is calculated as

$$T_i = q_1(V_{i+1} - V_i)\,K_i \qquad (2)$$

To solve Equation 1 by integration in time, one first specifies

$$N = N_o + \frac{1}{2\pi}\frac{T_o}{I_t}\,t \qquad (3)$$

where

N_o = initial speed (cps)
T_o = steady state driving torque minus the load (lb-in)
I_t = total system inertia (lb-in-sec²)

Then substitute Equation 3 into Equation 1 and let

$$\tau = \omega_1 t$$

$$N_1 = \omega_1/2\pi$$

$$B = 2n(N_s - N_o)/N_1$$

Equation 1 becomes

$$\frac{d^2q_1}{d\tau^2} + 2\xi\frac{dq_1}{d\tau} + q_1 = A\,\text{SIN}\left[(B - \alpha\tau)\,\tau\right] \qquad (4)$$

with

$$A = VT/I_1\omega_1^2 \qquad (5)$$

and

$$\alpha = 2nT_o/I_t\omega_1^2 \qquad (6)$$

When the instantaneous forcing [2] frequency is close to 1, the system, represented by Equation 4, will be excited. When solving for q_1, it is sufficient to integrate the equation in the range $0.8 \leq (B - \alpha\tau) \leq 2.0$. One can use the Runge-Kutta method or the simple Euler's scheme for the integration. As shown in the Appendix, a recursive formula derived from the convolution integral method provides another alternative.

The maximum amplitude in Figure 2 is generated by using $A = 1$. Note that, in Figure 2, q_1 approaches $\frac{1}{2\xi}$ as α approaches zero. This is the steady state torsional resonance amplitude. By using Equations 2, 5, and 6 and Figure 2, one can easily calculate the maximum torque in the system with known first mode frequency and mode shape.

*If the torsional frequency analysis applies a consistent mass approach, such as the finite element method, the modal inertias have to be calculated by modal transformation. Frequently they are normalized to the value of 1.0 with respect to the mode shapes.

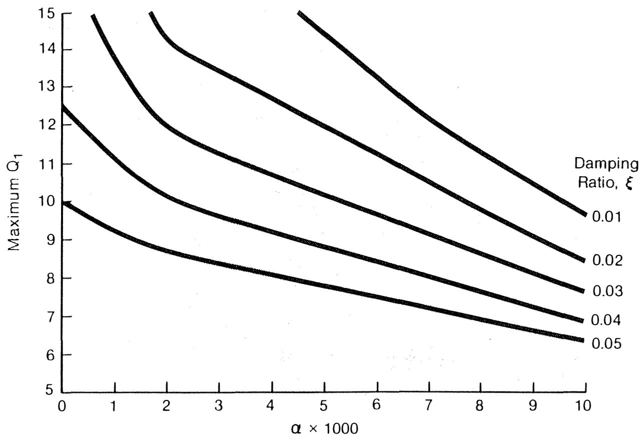

Figure 2. Dimensionless Torque Amplification Chart (Modal Displacement vs. Rotor Acceleration Parameter).

PRACTICAL EXAMPLE

Figure 3 shows a synchronous motor driving an axial compressor through a single, step-up gearbox. The motor is rated at 25,000 hp, with synchronous speed at 1200 rpm. The torsional system can be mathematically represented by three inertias: the motor, the gear and the compressor. The first mode frequency and mode shape are calculated by the Holzer method and are shown in Figure 4. The system torques versus speed characteristics during start-up are presented in Figure 5. Figure 6 is the torsional Campbell diagram showing the location of the critical speed where the motor's 2 × slip frequency coincides with the first torsional natural frequency at 972 rpm. From Figure 5, one can read the following at 81% of the synchronous speed (0.81 × 1200 = 972 rpm):

$T = 0.71$ P.U.

$T_m = 1.025$ P.U.

$T = 0.175$ P.U.

$T_o = 0.85$ P.U.

where

$$\text{P.U.} = 63{,}025 \times \text{hp/rpm} = 1.313 \times 10^6 \text{ in-lb}$$

Using Equation 5,

$$A = 1.296 \times 10^{-3} \text{ rad}$$

Using Equation 6 with n = 3, the rotor acceleration parameter is

$$\alpha = 4.23 \times 10^{-3}$$

From Figure 2, the amplification factor is 9.15, assuming

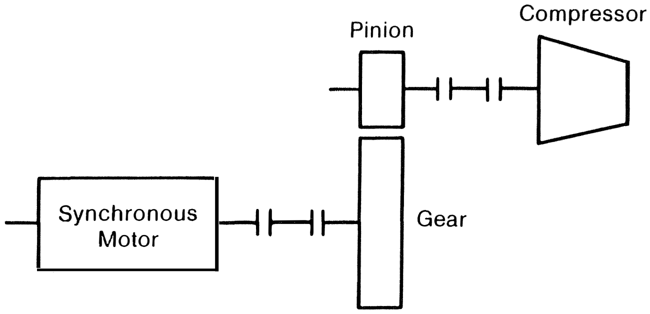

Figure 3. System Diagram.

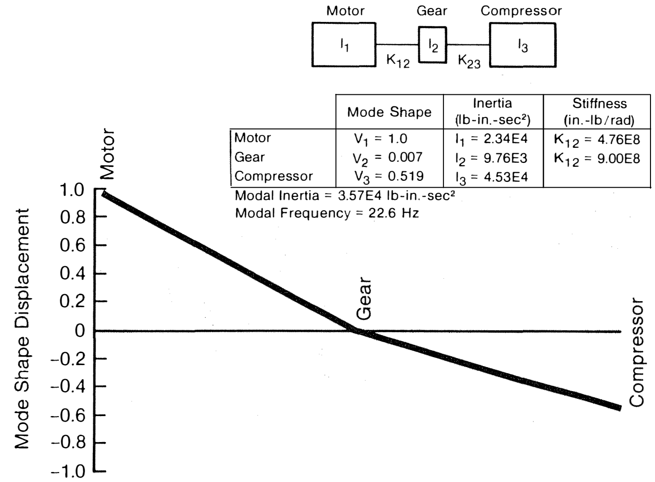

Figure 4. First Mode Shape and System Parameters.

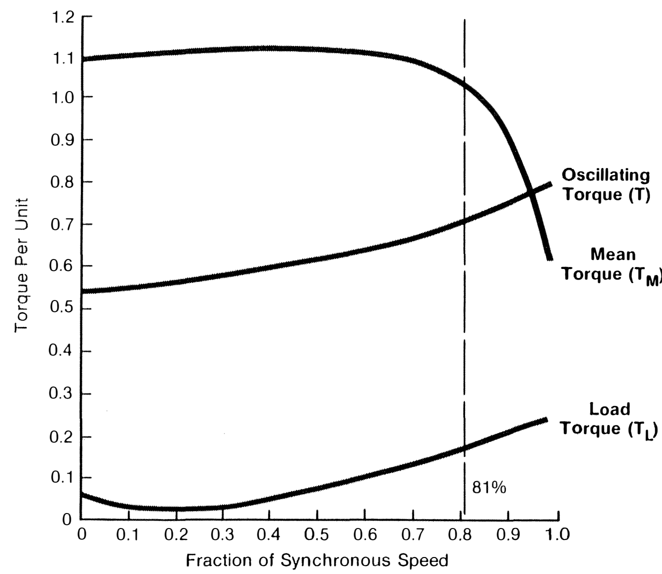

Figure 5. System Torque Characteristics.

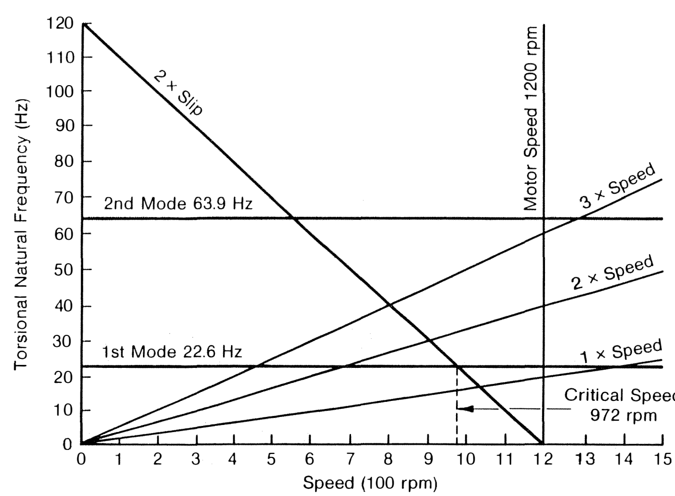

Figure 6. Torsional Campbell (Interference) Diagram.

$\xi = .04$. Therefore, the maximum modal displacement is $(q_1)\max = 9.15\ A = 1.186 \times 10^{-2}$ rad.

The maximum vibratory torque between the motor and the gear is, by Equation 2,

$$T_{1-2} = (q_1)_{max}(V_1 - V_2)\ K_{1-2} = 4.27 \text{ P.U.}$$

Since the steady state torque is 1.025 P.U. at 81% speed, the maximum total torque is

$$T = T_{1-2} + T_m = 5.29 \text{ P.U.}$$

The weakest section between the motor and the gear is at the motor bearing journal, with $d = 9.5$ in. The shear stress due to the maximum total torque is

$$\tau_s = 16\ T/\pi d^3 = 41.3 \times 10^3 \text{ psi}$$

COMPARISON WITH TRADITIONAL METHOD

The above example was treated by the traditional method [3], with three degrees of freedom. The result of the motor shaft torque is presented as the bottom trace of Figure 7. The corresponding simplified one-mode approach resulted in the top trace of Figure 7. The similarity between these two traces at the torsional resonance is obvious, and the accuracy of the simplified method is within 5%. Note that the top trace is plotted in the time scale (t), not the normalized time (τ). Also, it is vertically shifted by the amount of steady state driving torque (T_m).

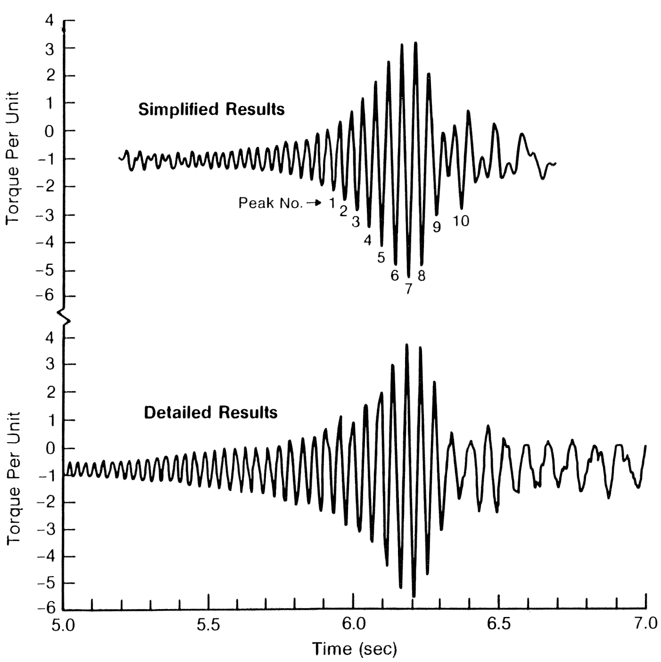

Figure 7. Comparison of Motor Torques.

LOW CYCLE FATIGUE ANALYSIS

In the lifetime of a synchronous motor drive machine, the number of starts is usually estimated to be 1,000 or more. During every start-up, the weak links in the rotor train will experience several high peak stress cycles. Here, in the example, the motor bearing journal, being a weak link, will have several instantaneous peak stresses higher than its yield when the stress concentration factor is considered. One must be sure that the "accumulated" damage at the torsional resonance will not break the journal in the designed life. Material fatigue data are available in the form shown in Figure 8, where

$\beta = 1 + \eta\,(K-1)$
K = stress concentration factor
η = notch sensitivity factor
τ_w = shear fatigue limit (psi)
τ_s = calculated peak shear stress (psi)

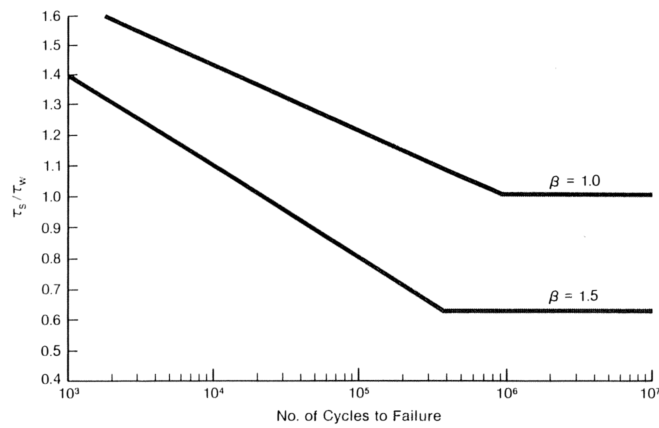

Figure 8. Low-Cycle Fatigue Data.

For every peak at resonance, there corresponds a peak shear stress. The occurrence of each peak consumes a certain amount of the designed life. Table 1 shows how the peak stresses are related to the life in the conventional way. It indicates that the sum of all the consumed life fractions is 94%. Although it is less than 100%, and the 1,000 starts may be achievable, it does not leave much safety margin. Therefore, to increase the journal diameter from 9.5 in. to 10 in. is necessary. In order to avoid the detailed calculations of Table 1, which also need the time integration of the resonance peaks, it is proposed herein that the sum of the consumed life fraction be $\Sigma f_n = 5 \times$ maximum peak life fraction, and also that it be less than 1.0. Experience suggests that this simple but conservative rule provides enough safety margin.

Table 1. Low Cycle Fatigue Calculation.

Peak No.	Calculated Shear Stress (1000 psi)	Stress Ratio	Cycles to Failure	1000 Starts Life Fraction f_n
1	17.9	0.557	645142	0.002
2	21.1	0.654	306690	0.003
3	23.4	0.727	175583	0.006
4	28.9	0.896	47787	0.021
5	33.1	1.029	17189	0.058
6	39.0	1.211	4263	0.235
7	41.3	1.284	2440	0.410
8	38.2	1.187	5134	0.195
9	24.6	0.763	132853	0.008
10	23.0	0.715	192686	0.005
			Σf_n =	0.941

NOTES:
$\beta = 1.5$, $\tau_w = 32{,}000$ psi
$\Sigma f_n = 5 \times .410 = 2.05$ (by simple rule)
Refer to Figure 7 for peak number.

DISCUSSION

A number of important points related to the forcing frequency, the damping ratio, gear backlash, critical torque, and extension of the present procedure are discussed in the following:

"Instantaneous" Forcing Frequency

In Equation 4, the forcing frequency may appear to be ($B-\alpha\tau$). But, one will find out in integration that at peak resonance, the value of ($B-\alpha\tau$) is far from 1.0. Also, it is different for a different value of B assigned, while the resonance peak amplitudes stay the same. The truth is that we should be dealing with the "instantaneous" forcing frequency, i.e.,

$$\frac{d}{d\tau}\left[(B-\alpha\tau)\tau\right] = B - 2\alpha\tau$$

and that this frequency will not change with the value of B. It is, however, a function of the rotor acceleration parameter α.

Damping Ratio

It is evident from Figure 2 that the damping ratio, ξ, of the first system torsional mode is one of two dominant parameters for evaluating the peak resonance torque. Damping ratios of 0.03 to 0.05 are the common values used for a torsional system without a large damping element, such as a Holset coupling. In practice, there are different physical interpretations of the damping ratio. For example, some engineers specify different Q (which equals $\frac{1}{2}\xi$) factors for different sections of the shafting. Strictly speaking, when one assigns a damping ratio to a lightly damped torsional mode, it means that every stiffness element in the model is in parallel with an equivalent, viscous damping of the value

$$C = 2\,\xi/\omega$$

where ω = the modal frequency (rad/sec). The damping is proportional to the stiffness [4] only.

For the first torsional mode, only one section of the shafting is twisting the most. Therefore, it is reasonable to take the system damping ratio as the same as that section.

Gear Backlash

The authors' experiences suggest that the amount of the backlash in the synchronous motor gear system do not have significant effects on the first torsional mode.

Critical Torque

Also, the most serious transient torque problem is not at the instance of switch-on, nor in cases of short-circuits, but at the first mode resonance speed.

Procedure Extension

The one-mode approach presented herein may be extended to systems with large damping and non-linear stiffness elements, as long as the first system model damping is not larger than 0.20 [4]. Further study is needed in the areas of equivalent damping ratio and linearized stiffness.

CONCLUSION

The simplified torsional transient method presented herein provides a fast alternative for evaluating the low-cycle fatigue problem frequently encountered in synchronous motor machinery. It is ideal for decision-making at the early design stage, and for field trouble-shooting.

APPENDIX

The impulse response of the system represented by Equation 4 is

$$h(\tau) = e^{-\xi\tau}\sin t$$

for small ξ and A = 1.

The forced response can be calculated by the convolution integral:

$$q(\tau) = \int_0^{\tau} \sin\left[(B-\alpha\lambda)\lambda\right] e^{-\xi(\tau-\lambda)}\sin(\tau-\lambda)\,d\lambda$$

or

$$q(\tau) = e^{-\xi\tau}\left[F_1(\tau)\sin\tau + F_2(\tau)\cos\tau\right] \quad (A1)$$

where

$$F_1(\tau) = \int_0^{\tau} \sin\left[(B-\alpha\lambda)\lambda\right] e^{\xi\lambda}\cos\lambda\,d\lambda \quad (A2)$$

$$F_2(\tau) = -\int_0^{\tau} \sin\left[(B-\alpha\lambda)\lambda\right] e^{\xi\lambda}\sin\lambda\,d\lambda \quad (A3)$$

Let $\triangle\tau$ be a finite increment of τ. Using equation A1, the following recursive equation is derived:

$$\begin{aligned} q(\tau+\triangle\tau) = {} & \bar{e}^{\xi\triangle\tau}\cos\triangle\tau\, q(\tau) \\ & + e^{-\xi(\tau+\triangle\tau)}\left[(F_1\cos\tau - F_2\sin\tau)\sin\triangle\tau + \triangle F_1\sin(\tau+\triangle\tau) + F_2\cos(\tau+\triangle\tau)\right] \end{aligned} \quad (A4)$$

where

$$\triangle F_1 = F_1(\tau+\triangle\tau) - F_1(\tau) \quad (A5)$$

$$\triangle F_2 = F_2(\tau+\triangle\tau) - F_2(\tau) \quad (A6)$$

Equations A1 to A6 can be implemented into a simple computer routine for calculating the transient modal response.

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