AN IMPROVED METHOD TO DETERMINE THE SCORING RESISTANCE OF HIGH POWER HIGH SPEED GEARING

by

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ABSTRACT

The petrochemical and utility industries are experiencing a growing demand for high power turbogear units for use in large gas turbine compressor and generator drives. A growing number of high speed gears with ratings up to 70,000 kW are already in service. For such gears, the failure mode of scoring can become the limiting constraint. The validity of an analytical method to predict scoring resistance is, therefore, becoming increasingly important.

A simplified calculation procedure, that is suitable for high power, high speed gearing, and is based on the Winter and Michaelis integral temperature method, is presented.

INTRODUCTION

The preponderance of operating experience in the U.S. petrochemical and utility industries over the past thirty years has been with soft and medium hard, hobbed and shaved gears. When gears of this type became overloaded for one reason or another, the usual mode of failure was pitting. Occasionally, tooth breakage occurred. Scoring was rare.

To satisfy power requirements in the range of 35,000 hp to 100,000 hp, it is necessary to utilize fully hardened gear teeth. In general terms, gear hardness can be classified as:

Soft	< 300 BHN
Medium Hard	300-400 BHN
Fully Hard	> 56 Rc

The pitting resistance of gear teeth increases by the square of the compressive strength, which is directly proportional to hardness. In a given gear set, a change in hardness from 300 Brinell Hardness Number (BHN) to Rockwell C (Rc) 57 (other things being equal) increases the pitting resistance

over 200 percent. Pitting failures, therefore, are unlikely in Rc 57 gearing.

The resistance to breakage of a gear tooth increases with the first power of the bending strength of the material, which is also directly proportional to hardness. Therefore, a gear set of 300 BHN that has balanced pitting characteristics and bending strength, becomes unbalanced at Rc 57. The size of the teeth must be increased to gain the additional strength required in bending. For this reason, hardened and ground gears characteristically have larger teeth (lower tooth numbers) than their through-hardened predecessors.

The scoring resistance of the gear teeth is not significantly affected by hardness. Sliding velocity, surface finish, oil film characteristics and temperature, profile modification, and anti-weld agents (copper and silver plating, or extreme pressure (EP) additives in the oil) have a much greater influence on this mode of failure.

Tooth size affects the sliding velocity during the tooth engagement cycle. Sliding velocity, in turn, affects the thickness of the elastohydrodynamic oil film and the heat generated therein. Good gear tooth action results when the peripheral velocity is high enough to produce an oil film thick enough to keep the asperities of the tooth surfaces from touching and, at the same time, low enough to avoid the generation of excessive heat and flashing of the oil.

Flashing of the oil film results in metal-to-metal contact and instantaneous welding in small local spots. These small welds are torn apart as fast as they are formed. The sliding action of the teeth causes the torn-out weld to be dragged across the mating surface, creating a gouge or score mark. Hence, the name of the failure mode scoring. The typical appearance of a scored test gear is shown in Figure 1.
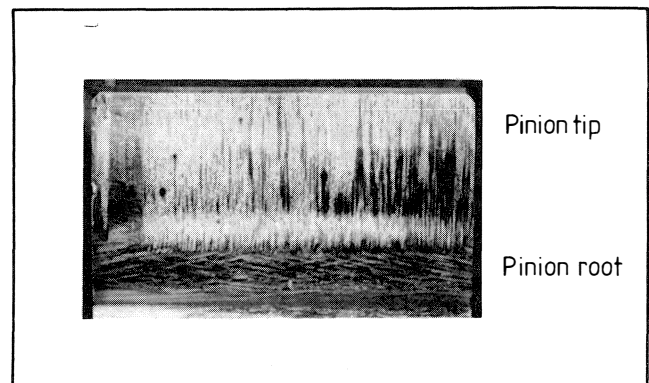

Pinion tip

Pinion root

Figure 1. Typical Scoring Failure of a Test Gear (carburized material).

Continued scoring can ruin the tooth form and create stress risers large enough to cause tooth breakage. Scoring is the most likely failure mode, should overloading occur, in large horsepower, high speed turbogear sets, with hardened teeth.

Unfortunately, neither the American Petroleum Institute (API) nor the American Gear Manufacturers Association (AGMA) offer a means of assessing the risk of scoring on large turbogear units.

BACKGROUND

Scoring failures of gear teeth were first encountered in the small but highly loaded hardened gears used for automotive, truck, tractor, and aircraft transmissions. Two of the early developments in analytical prediction of scoring were the PV and the PVT factors. These indices were developed in the mid-1930s at the General Motors Research Laboratories. J. O. Almen and J. C. Straub were deeply involved in this work. P, V and T represent contact stress, sliding velocity and distance, respectively.

During the late 1930s, H. Blok, Delft Technical Institute, developed a flash temperature theory which was based on the conversion of friction energy to heat and, in turn, to a local peak temperature.

Bruce W. Kelley, Caterpillar Tractor Company, described certain modifications to the Blok approach required to correlate test and field experience, notably, the surface roughness. This information was presented in AGMA 219.04, 1953.

An allowable specific load intensity expressed in terms of the tangential load per unit of face per unit of diameter was developed by G. Niemann, Technical University, Munich, in 1960. While this criteria had certain usefulness in low speed gears, it was too pessimistic for gears with pitch line velocities over 20 meters per second (m/s).

The *Gear Handbook*, 1962, edited by Darle W. Dudley, presented an equation for a scoring criterion number above which scoring might be encountered and suggested more elaborate analysis be made. AGMA Standard 217.01, 1965, contains an adaptation of the Kelley/Blok work, especially tailored to aerospace spur and helical gears, and also contains the Dudley scoring index as a simplified quick check for aerospace type gears.

More recently, analytical and experimental work at the Technical University of Munich, under the direction of H. Winter, has resulted in further refinements of the Kelley/Blok work. This information was presented in AGMA Technical paper P 219.17, 1983, and is called the integral temperature method. This method is based on a mean integrated tooth flank temperature in contrast to a local peak temperature as used by Blok. Additional refinements include consideration of tooth geometry effects (profile and addendum modification, gear ratio, length of the line of action and tooth size), surface coatings, EP additives, materials effect, surface roughness, etc.

Since the integral temperature method is a generalized approach applicable to many types of gears (spur, bevel, single and double helical), and to a broad range of sizes and speeds, a large number of influencing factors have been included in the calculations. This improves the accuracy of the method, but also makes it relatively complicated to work with.

Calculation According To Blok

In the so-called flash temperature theory by Blok, the scoring resistance is too small when the momentary peak tooth flank temperature reaches a critical value. According to Blok, a local contact temperature is calculated:

$$T_C = T_M + T_{Fl} \qquad (1)$$

where:
$$T_{F1} = 2.52 \, (\mu) \left[\frac{F_{bt}}{b}\right]^{3/4} \left[\frac{n}{60}\right]^{1/2} \frac{\left[R_1 - \frac{R_2}{\mu}\right]^{1/2}}{R^{1/4}} \qquad (2)$$

n = pinion speed, rpm
F_{bt} = tangential tooth load, N
b = face width, mm
μ = local friction coefficient
R, R_1, R_2 = radii of curvature, mm

Utilizing this equation, contact temperature curves, T_C, can be plotted as a function of the line of contact (Figure 2). There are several problems with this procedure. The first problem is the determination of the load distribution along the path of contact. The second problem is the determination of the radii of curvature and the coefficient of friction. Since limiting values are available for only straight mineral oil, the effects of EP additives and special treatments cannot be considered (Figure 3).

The integral temperature method developed by Winter and Michaelis has been adopted as DIN/ISO 3990, Part 4. With this method, a mean integrated tooth flank temperature is determined and compared to an allowable value established from gear tests using the specific lubricant involved. Mean values, rather than peak values, are used for the coefficient of friction and for the load distribution.

The mean flash temperature calculated is multiplied by empirical factors and added to the bulk temperature.

$$T_I = T_M + 1.5 \, T_{Flm} \qquad (3)$$

This mean tooth flank temperature, T_I (also called the integral temperature), corresponds to a measurable tooth flank temperature.

The bulk temperature, T_M, can, in turn, be determined from the oil inlet temperature and is added to a percentage of the mean flash temperature.

$$T_M = 1.2 \, T_{Oil} + 0.84 \, T_{Flm} \qquad (4)$$

The factors 1.5, 1.2 and/or 0.84 result from the adaptation of the measured flank and/or bulk temperature to the calculation method and are to be considered constant parameters. Thus, the mean tooth flank temperature becomes:

$$T_I = 1.2 \, T_{Oil} + 2.34 \, T_{Flm} \qquad (5)$$

The mean flash temperature, T_{Flm}, is dependent on the geometry, the material, the speed, and the load parameters.

$$T_{FLm} = \mu_B \left[\frac{X_M \, X_{BE}}{X_{Ca} \, X_Q}\right] X_\xi \left[\frac{w^{3/4} \, v^{1/2}}{a^{1/4}}\right] \qquad (6)$$

where

μ_B = Coefficient of friction
X_M = Material factor
X_{BE} = Factor for geometry, Hertzian pressure and sliding velocity at the pinion tip
X_{Ca} = Factor for tip relief
X_Q = Rotation factor
X_ξ = Contact ratio factor

Mean values for these influencing factors, diagrams, and/or calculation formulae are given in DIN/ISO 3990. This method was presented in AGMA 219.17, 1983, by Winter and Michaelis. The factor of safety against scoring, S_{SI}, then results in

$$S_{SI} = \frac{T_{SI}}{T_I} \qquad (7)$$

The allowable mean integral temperature, T_{SI}, has been determined for various types of oil through a large number of gear
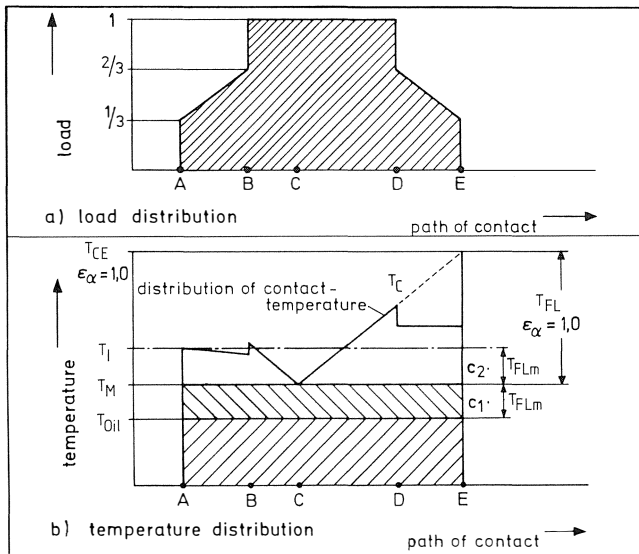
Figure 2. Distribution of Contact Temperature on Tooth Flank According to Blok.

Figure 4. Calculated Scoring Safety Factors of Test Gears According to Winter and Michaelis.
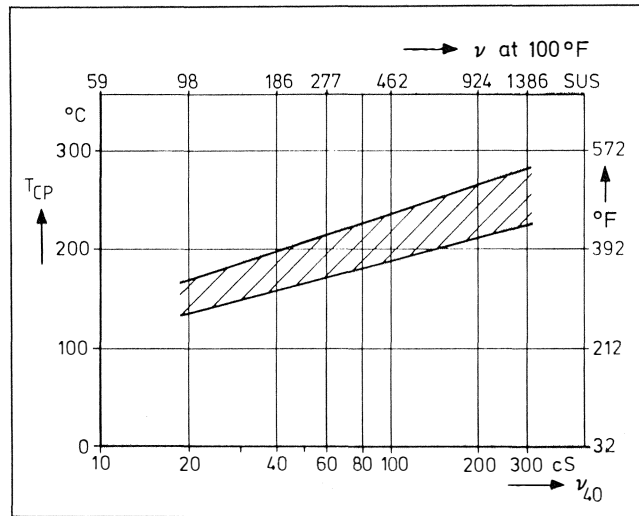
Figure 3. Critical Contact Temperature for Flash Temperature Calculation (straight mineral oils).

tests. The accuracy of this method can be shown from the results of a large number of gear tests in a plot of T_{SI}, as a function of circumferential speed (Figure 4).

The data fall around a mean value of about 1.2 S_{SI}. The teeth are subject to scoring at a temperature approximately 20 percent below the calculated critical temperature, T_{SI}. This may be due to small overloads in the test system not included in the calculation. Additional investigations of gear units of all sizes in actual service yielded similar results.

SIMPLIFIED CALCULATION METHOD

Equation (6), as develoepd by Winter and Michaelis and as adopted by DIN/ISO, is relatively complex and is usually handled in a computer program. One of the attractive aspects of the well known Dudley scoring index or AGMA 217.01, is its simplicity and the ease with which it can be determined with a hand-held calculator.

The range of the variables in equation (6), when applied to high speed turbogearing, was investigated. The more or less
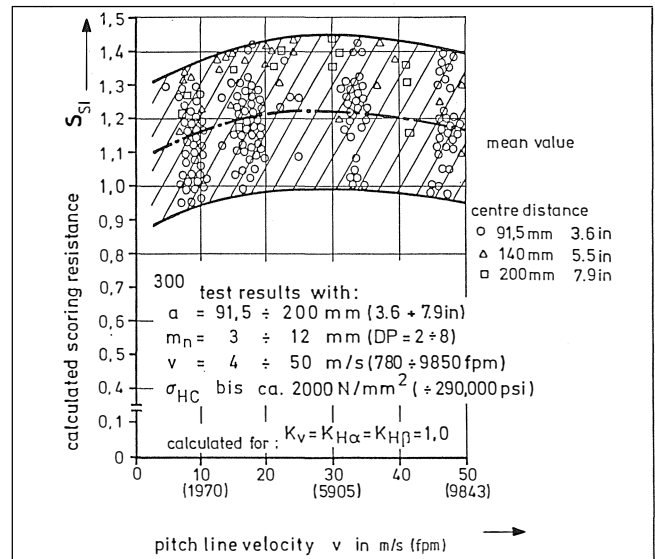
standard practices followed in the design, manufacture and operation of high quality turbogearing with respect to tooth geometry, contact ratio, profile modification, surface finish and lubricant, permit assigning constant values to the following variables:

μ_B Coefficient of friction
X_M Material factor
X_{Ca} Tip relief factor
X_Q Rotation factor

The resulting simplification to equation (6) is an easy to use calculation procedure applicable expressly to high speed gearing. The simplified equations have been verified by recalculating numerous high speed gear units in service.

Simplified Equations

If the product of the terms X_{BE} and X_ξ is equated to a geometry factor, X_{GEO}, accounting for the number of teeth, contact ratio, radii of curvature, and ratio, and if constant values for the terms μ_B, X_M, X_{Ca}, and X_Q are substituted in equations (5) and (6), the result is:

$$T_{IM} = 1.2\, T_{Oil} + X_{GEO}\, SI_M$$

where:

$$SI_M = \text{Modified Scoring Index} \tag{8}$$

$$= \frac{3\, w^{3/4}\, v^{1/2}}{a^{1/4}} \tag{9}$$

The load, w, is the tangential force per unit face width

w = F_{bt}/b, N/mm
v = Circumferential speed, m/s
a = Center distance, mm   (10)

The load, w, should include all overload effects, for example, from non-uniform load distributions and/or from actual operating conditions.

The modified scoring index, SI_M, equation (9), is similar in form to the AGMA 217 scoring index. For example:

$$\text{AGMA Scoring Index} = \frac{W^{3/4} n_p^{1/2}}{P_d^{1/4}} \tag{11}$$

where: W = Tangential force per unit face width

n_p = Pinion speed, rpm
P_d = Diametral pitch

By including the geometry factor, X_{GEO}, in equation (8), the influence of tooth numbers, ratio, and addendum proportions are taken into consideration in addition to tooth size. Geometry effects in equation (11) are limited to tooth size only. Diagrams of X_{GEO} for teeth with zero-addendum modification or zero sum of addendum modification are shown in Figures 5 through 10.
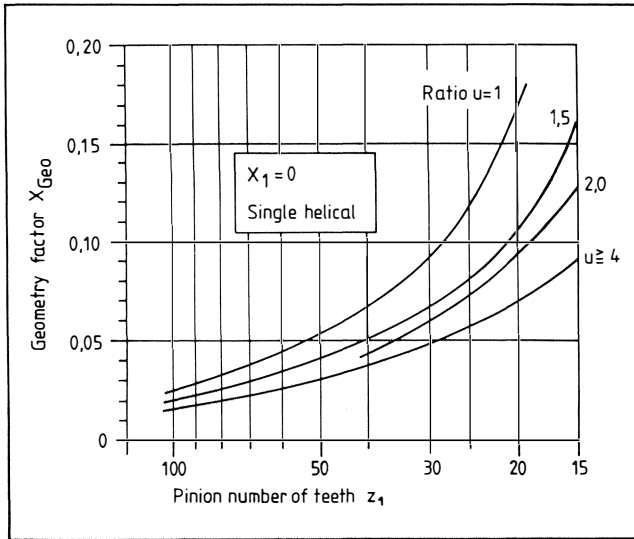

Figure 5. Geometry Factor for Calculation Method with Modified Scoring Index for Addendum Modification Factor $X_1 = 0$ and Helix Angle of Approximately Eight to Twelve Degrees (single helical).
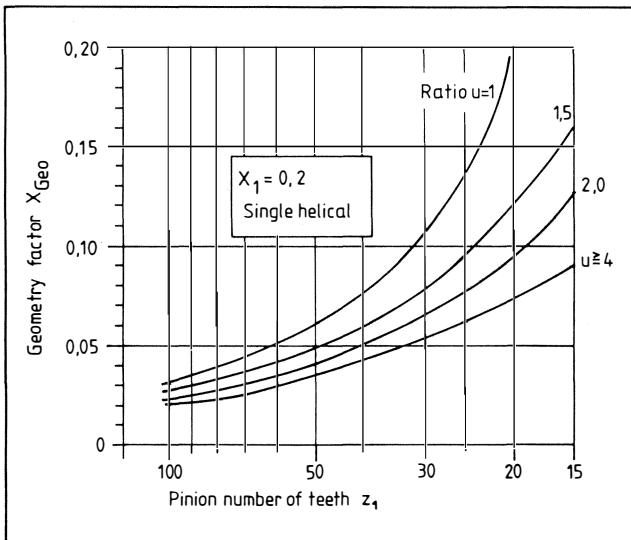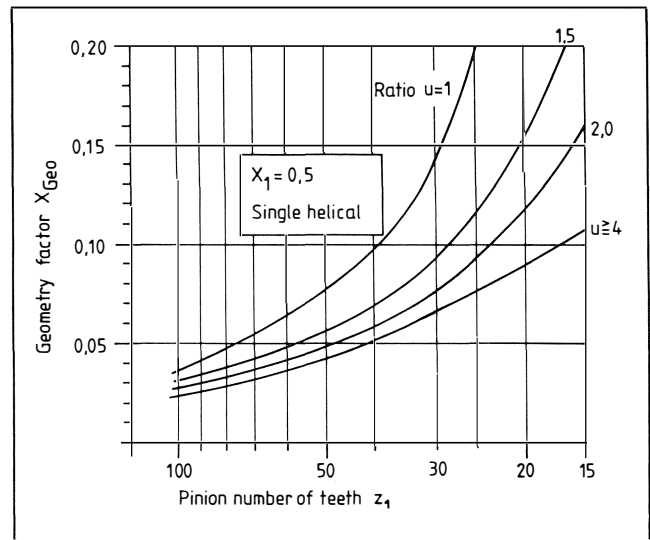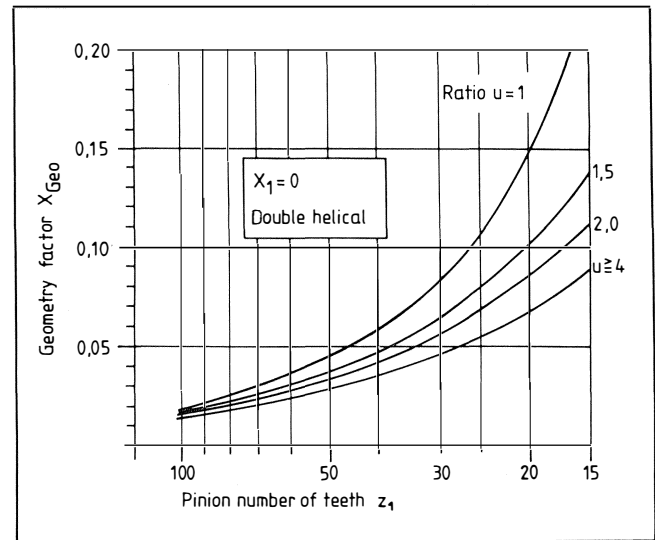

Figure 6. Geometry Factor for Calculation Method with Modified Scoring Index for Addendum Modification Factor $X_1 = 0.2$ and Helix Angle of Approximately Eight to Twelve Degrees (single helical).

Figure 7. Geometry Factor for Calculation Method with Modified Scoring Index for Addendum Modification Factor $X_1 = 0.5$ and Helix Angle of Approximately Eight to Twelve Degrees (single helical).

Figure 8. Geometry Factor for Calculation Method with Modified Scoring Index for Addendum Modification Factor $X_1 = 0$ and Helix Angle of Approximately 25 to 35 Degrees (double helical).

Allowable Values and Margin of Safety

The safety margin in the scoring resistance, S_{SIM}, can be determined from the allowable temperature, T_{IS}.

$$S_{SIM} = \frac{T_{IS}}{T_{IM}} \tag{12}$$

Using equations (8) and (9) and allowable values of T_{IS} from [3], or Table 1, the safety margin for scoring resistance can be determined by equation (12).
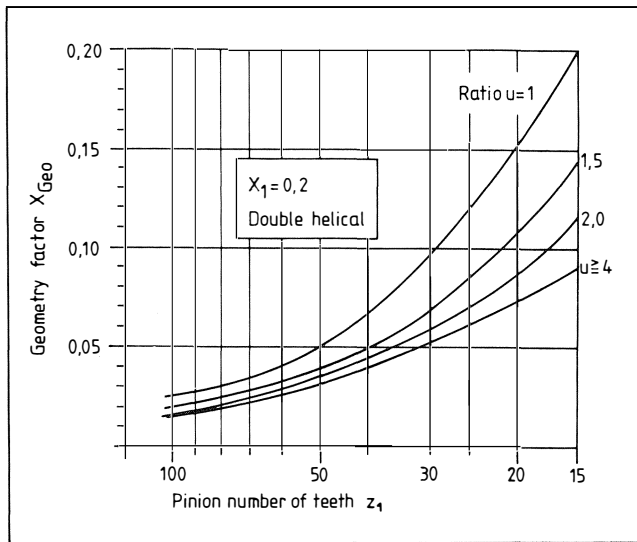
Figure 9. *Geometry Factor for Calculation Method with Modified Scoring Index for Addendum Modification Factor $X_1 = 0.2$ and Helix Angle of Approximately 25 to 35 Degrees (double helical).*
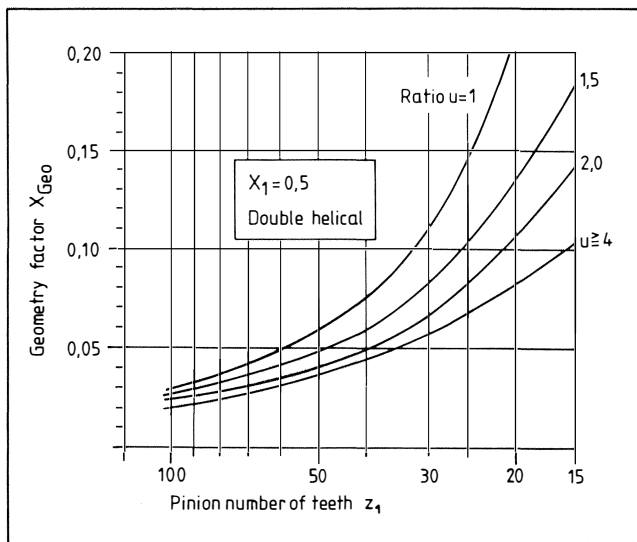

Figure 10. *Geometry Factor for Calculation Method with Modified Scoring Index for Addendum Modification Factor $X_1 = 0.5$ and Helix Angle of Approximately 25 to 35 Degrees (double helical).*

An adequate margin of safety exists when the S_{SIM} value of equation (12) is 1.6 or greater. As a quick design check, the permissible modified scoring index, SI_{MP}, can be used from Table 1. The values shown include normal safety factors and are based on average gear geometry.

Qualifications

When using the method previously described, it is necessary that several variables be kept within certain limits in order to maintain the validity of the constants assigned these variables.

Coefficients of Friction

A curve of the coefficient of friction as a function of the pitch line velocity is shown in Figure 11. Surface roughness, Ra, must be 0.5 μm or better (after running-in). The curves shown are believed to be flat beyond 70 m/s. A value of $\mu_B = 0.03$ has been taken for high speed gears. For pitch line velocities below 30 m/s, a slightly larger value should be used.
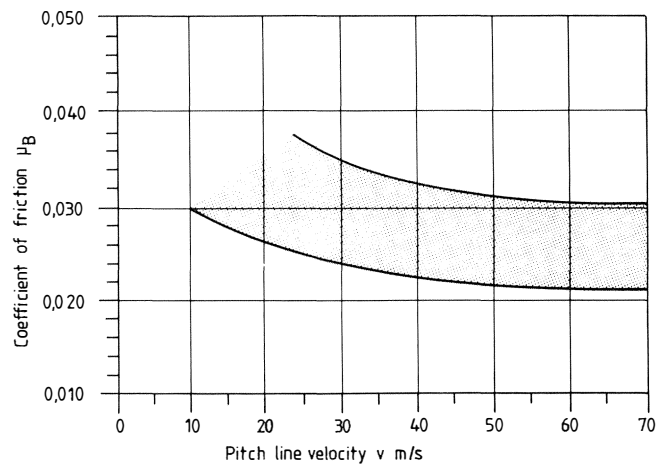

Figure 11. *Mean Coefficient of Friction Versus Circumferential Speed for Typical Lubrication Conditions for High Speed Gearings.*

Involute Modification

Optimum tip relief, C_a, has been assumed. This can be approximated from the expression:

$$C_a = \frac{F_{bt}}{C_\gamma \, b} + F_p$$

where C_γ = tooth stiffness value (approximately 20 N/mm/μm)

F_p = Single tooth spacing error, (8 μm or less).

Table 1. *Allowable Temperatures, T_{IS}, for Several Commonly Used Turbine Oils.*

Oil-Type Viscosity	FZG Test Load-stage	Ryder Gear Test, ppi app.	T_{IS}	Permissable Modified Scoring Index, SI_{MP}
ISO VG 32	5	1300	130°C	550
ISO VG 46	6	1500	160°C	900
ISO VG 32 (EP Oil)	7	2200	180°C	1150
ISO VG 46 (EP Oil)	8	3000	200°C	1400
ISO VG 46 (EP Oil)	10	5000	270°C	2300

With optimal profile modification and normal contact ratios, X_{Ca} can be assigned a value of 1.15.

Pressure Angle

The normal pressure angle is assumed to be 20° or slightly higher.

Addendum Proportions

When addendum proportions are kept within the range of standard to 50 percent long and short, the rotation factor, X_Q, can be taken as unity.

A standard addendum set is defined as having mating pinion and gear with addendums equal to 100 percent × Module, mm. The maximum departure from the standard addendum is 150 percent × Module (driver) mating with 50 percent × Module (driven). The summation addendum lengths shorter and longer than standard should equal 100 percent.

Lubrication

Pressure fed lubrication by spray jet is mandatory.

CHECK OF UNITS IN SERVICE

Values calculated for a number of gear units in service are presented in Figure 12. These gears have been operating for many years under severe conditions. The units examined fall within a range of safety values, S_{SI}, of 1.6 to 2.0.
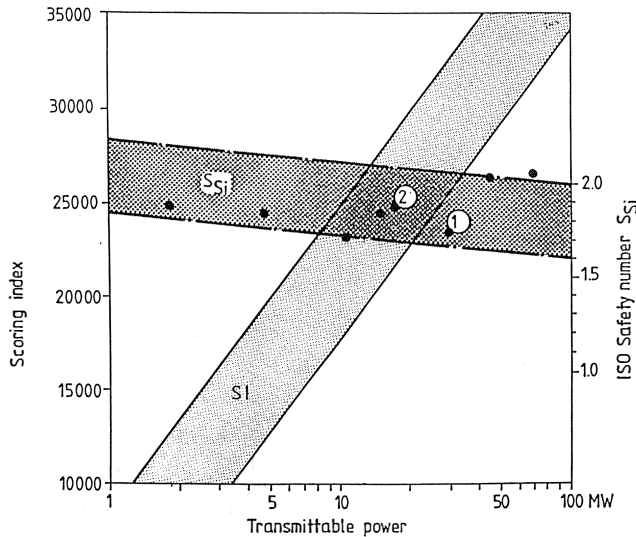

figure 12

Figure 12. Practical Experience Data in Scoring Safety Factors S_{SI} and Scoring Index SI.

A band of scoring index limits using AGMA 217, equation (11), are also reflected in Figure 12. These curves illustrate the limited usefulness of this criteria as applied to large turbogear units. The results are reasonably valid over a small range of transmitted power, but are overly pessimistic or optimistic outside this range.

A single helical gear used in gas turbine generator service is shown in Figure 13. Rated for 30,000 kW, with a turbine speed of 5100 rpm and generator speed of 3000 rpm, it is data point 1 in Figure 12, having a scoring factor of 1.6.

A double helical gear used in a compressor system is shown in Figure 14. Shown in Figure 12 as data point 2, having a scoring safety factor of 1.8, this machine was rated for 18,000
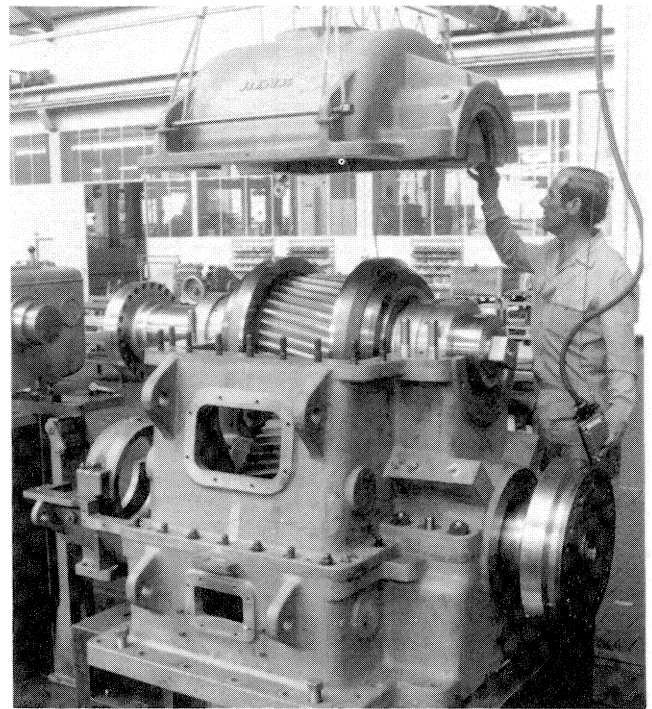

Figure 13. Typical High Speed Gear for Generator Drive of 30,000 kW and Speed Decrease from 5100 rpm to 3000 RPM. Data Point 1 of Figure 12 with a Scoring Safety Factor of 1.6.
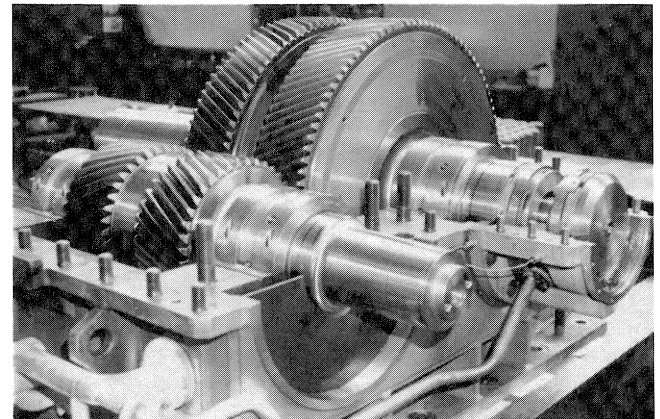

Figure 14. High Speed Gear as Compressor Drive of 18,000 kW and Speeds of 1500/3900 RPM. Data Point 2 of Figure 12 with a Scoring Safety Factor of 1.8.

kW at an input speed of 1500 rpm and output speed of 3900 rpm.

Sample Calculation

A sample calculation (data point two, Figure 12) is shown below:

Step 1—Determine the SI_M value from gear geometry, power and speed:

$$SI_M = \frac{3\,W^{3/4}\,V^{1/2}}{a^{1/4}}$$

where: W = 940 N/mm
         V = 81 m/s
         a = 710 mm

$$SI_M = \frac{3\,(940^{3/4})\,(81^{1/2})}{710^{1/4}} = 887$$

Step 2—Determine the X_{GEO} value from Figure 9, (double helical, Ratio $= 2.64 \times 120$ percent addendum):
$X_{GEO} = 0.04$

Step 3—Determine the mean tooth flank temperature:

$T_I = 1.2\,T_{Oil} + X_{GEO}\,SI_M$
$T_I = 1.2 \times 45 + 0.04 \times 887 = 89°C$

Step 4—Determine the safety margin in scoring:

The allowable temperature for ISO VG 46 from Table 1 is:
$T_{IS} = 160°C$

Therefore: $S_{SIM} = \dfrac{T_{IS}}{T_I} = \dfrac{160}{89} = 1.80$

A graphic illustration of the sample calculation data point 2 in Figure 12 is presented in Figure 15. The flash temperature curve, according to Blok, is shown as line T_C. The drop of this curve at the beginning and the end of the mesh can only be achieved with optimum tip relief. Less than optimum relief will result in a very high peak temperature at the end points. The mean tooth flank temperature is shown as line T_I.
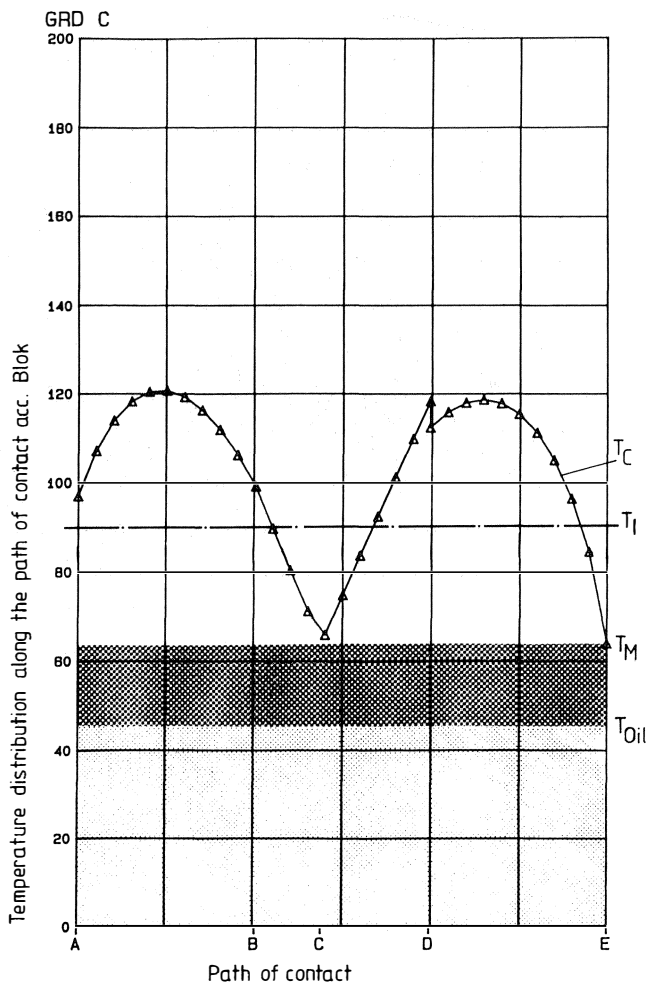
These curves illustrate one of the advantages of the integral temperature method, namely, the greater accuracy with which a mean temperature can be predicted as opposed to a peak temperature.

CONCLUSION

Improved knowledge of the scoring resistance of high speed gears is essential if high power gear units are to be applied successfully. This is particularly true in the power range of 35,000 hp to 100,000 hp, where fully hardened, carburized and ground gears are required.

The calculation method presented herein, a simplified version of the Winter and Michaelis integral temperature method, provides the gear engineer with an analytical tool that will permit him to properly balance his design in terms of scoring and bending strength margins.

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Figure 15. Flash Temperature Distribution of Data Point According to Blok or AGMA 217.01 and Mean Tooth Flank Temperature T_I.