A MATHEMATICAL FORMULATION FOR FLEXURAL DIAPHRAGM FATIGUE LIFE RELIABILITY EVALUATION

by

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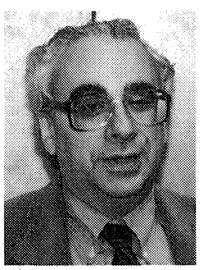

Joseph Stocco was awarded a Ph.D. in theoretical solid state physics by the University of Pittsburgh in 1972. His studies were concentrated in the area of mathematical modelling and simulation of physical representations for the response of solid systems. After leaving the University of Pittsburgh, Dr. Stocco began his career with the Power Transmission Division of Bendix Corporation, where he has remained to the present. In the interim, Bendix Utica has become the Power Transmission Division of Lucas Aerospace. At Lucas, Dr. Stocco specialized in the development of engineering analysis programs to evaluate stress, vibration characteristics, and performance capabilities of rotating power transmission coupling systems. He created a generalized direct integration stress analysis routine for coupling diaphragms. In 1989, Dr. Stocco and Lucas were awarded a patent. This was the first breakthrough in diaphragm technology since 1952. In 1992, Dr. Stocco began a research program to investigate a methodology to evaluate the fatigue life capability of diaphragm couplings. Currently Dr. Stocco's job title is Senior Scientist.

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Mr. Sisson has 24 years experience in the field of rotating equipment, including 10 years with Texaco's Central Engineering Department in Houston and 14 years field experience at the refinery level. He has worked on numerous assignments relating to rotating equipment at various Texaco installations both domestic and overseas.

Mr. Sisson holds a B.S.M.E. degree from the University of Texas at El Paso. He is a member of the Texaco Rotating Equipment Roundtable Advisory Board, and has made technical presentations at many of the sessions.

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Mr Findlay received his B.S. (Mechanical Engineering) from Clarkson College of Technology (1980). He started his career with the Bendix Fluid Power Division over 15 years ago as a Rotating Equipment Engineer for Air Turbine Starters specializing in bearing, gearing, and turbine analysis. After Lucas acquired the Bendix Fluid Division in 1988, Mr. Findlay was transferred into the Power Transmission Coupling group as a Project Engineer. He has worked on both aerospace and industrial products since then, specializing in diaphragm coupling design. He is a member of the task team for the API Standard 671 3rd edition "Special Purpose Couplings."

ABSTRACT

A mathematical formalism is presented for flexural diaphragm fatigue life and reliability evaluation. The use of this formalism enables the diaphragm designer to match a customer's requirements for life and reliability, to the performance characteristics of a diaphragm's geometry. The application requirements for a coupling determine, in conjunction with the geometrical configuration of a diaphragm being evaluated, the stress level that the application loads will induce in the diaphragm profile and ancillary structures. However, as important as a precise knowledge of the induced stress might be, the determination of a design's performance capability is not complete until the induced stresses (including the effects of any stress concentration factors that will be introduced during fabrication) are compared in a physically and

mathematically relevant manner to the strength of the material of which the diaphragm is manufactured. The analysis routine presented herein is an effective and accurate methodology for making this comparison.

These considerations apply to diaphragms which are operated normally within the limits of their normal load ratings. It must also be mentioned that the analysis above applies only to diaphragms that have not sustained incidental damage in the field. A discussion of the life of a diaphragm which has been subjected to incidental damage is also given.

INTRODUCTION

At the beginning phase of every coupling design, a designer will usually list the customer's requirements for torque capability, bend angle to be accommodated, and axial deflection to be accommodated. Following that, the designer can begin to consider various possibilities for the diaphragm profile subject to any size restrictions that may be imposed on the design. As part of this evaluation, the designer must perform a stress analysis of the potential profile candidates. Each design under consideration will generate a specific value of stress. However, when evaluating the suitability of a potential design, the designer must factor in additional stresses that will be generated over and above those of the theoretical design as a result of the effects of defects and flaws and other stress concentration factors. Every experienced designer has a subjective feel for the limit stress, for the material of which a diaphragm is being constructed, that should not be exceeded in order to have a profile that can perform reliably over the required lifetime of the coupling. However, in the past, such evaluations have usually been more qualitative than quantitative. An attempt has been made herein to place the evaluation of diaphragm fatigue life and reliability on a scientific and mathematical basis. This objective has become a reality with the aid of the development of the physics of fatigue failure that is presented.

After having established a method for determining the expected life of a diaphragm for normal operation in the circumstance that it does not sustain any incidental damage, the next consideration becomes the expected life of a diaphragm in the event it sustains damage. It is not generally well know, but the simple fact is that at normal operational stress levels, a diaphragm can sustain considerable damage or can even be cracked or punctured, and yet still be capable of performing reasonably normally. The crucial factor is whether or not the stress at the tip any induced damage is greater or lower than the stress intensity factor at which the crack propagates. If the crack tip stress is less than the critical stress intensity factor, then the damage effects will not propagate, and the remainder of the diaphragm can carry the load normally. In essence, the unaffected regions of the diaphragm are like an intrinsic redundancy load path around the region of damage. However, there are some restrictions to the extent of the stress level that may be tolerated in this mode. This results in some life limitations. Several practical examples of such life considerations and damage tolerance of diaphragms are presented.

A MATHEMATICAL FORMULATION FOR FLEXURAL DIAPHRAGM FATIGUE LIFE RELIABILITY EVALUATION

A methodology for fatigue and reliability analysis that may be applied to mechanical structures that are subjected to loading with cyclic stress is described. The focus is on a particular structure however, namely a variable thickness circular disk with a central hole, which hereafter will be called a diaphragm.

The prime consideration in the evaluation of the fatigue life characteristics of a metallic structure is to determine the amplitude of the difference between the peak stress induced by the structural loading, including the effects of stress concentration factors generated by defects and flaws introduced during the fabrication process, and the fatigue strength of the material of which the structure or component is comprised. Clearly, the lower the peak stress in relation to the strength, the greater will be the number of fatigue cycles that the component will be able to endure, and, consequently, the longer the life of the structure or diaphragm.

For the purpose of constructing a physical or mathematical representation, a diaphragm may be regarded as a cylindrical beam which accomplishes the function of accommodating misalignment by deflecting in a manner which may be described as bending. This bending deformation accommodates axial and angular misalignment, while at the same time transmitting torque. The applied loadings combine together in a fairly complicated manner to result in a total equivalent cyclic Von Mises stress distribution that extends throughout the diaphragm or structure. However, for purposes of fatigue and reliability analysis, one focuses attention on the specific location of the structure, or the diaphragm, where the equivalent cyclic stress is the highest.

Actual diaphragms that are produced by real world manufacturing processes, simply cannot be loaded as highly as would be predicted by a theoretical design, because the processes involved in manufacturing always introduce small discrepancies and stress concentration factors.

If the effects of stress risers from fabrication are known, they may be included in the analysis as multiplying factors for the stresses developed by the theoretical design. However, since it is usually very difficult to evaluate the effects of defects and flaws and spurious stress concentration factors, one frequently must consider alternative ways of including these effects. Fatigue substantiation testing or acceptance cycle testing is an excellent way to take these factors into consideration.

Empirical considerations often play a significant role in setting up guidelines for designing diaphragms for flexible power transmission couplings. In general, it is good practice to design diaphragms such that the peak stress induced in the theoretical design by the loading should be no more than half of its fatigue strength.

In general, there are three factors that must be taken into account when manufacturing diaphragms. The material must have adequate strength, the theoretical design must have adequate margin from the strength for withstanding the stresses induced by the loading requirements, and the manufacturing operations and processes must produce diaphragms that are in reasonable conformance with the design, and be free from flaws or defects that compromise the integrity of the diaphragm by introducing an excessive stress concentration factor.

The development of a fatigue life and reliability evaluation formalism begins with the physics of fatigue. This can be understood readily by considering a representative phenomena, the physics of the deformation of a spring. It is well known that every spring has a limit to the amount of elastic deformation it can accommodate. If one induces a stress beyond the elastic limit, the spring begins to yield and sustain structural damage. The concept of "damage" is fundamental to an understanding of the nature of failure mechanisms of metallic mechanical structures, such as diaphragms. To answer the question, "What is the mechanism for damage in a metallic structure?," one must define the nature of the structure of a metal. Many volumes would have to be devoted to this if it were necessary to represent a metal diaphragm in anything beyond an extremely simplified model. Fortunately, all the essential physics is contained within a very simple representation that will be given.

From a broad external perspective, a diaphragm is a circular plate arrangement of a metal. But for the purposes of fatigue analysis and reliability evaluation, consider only a small region of the diaphragm where its stress level is a peak value. One zooms in on this region until reaching a magnification where the local zone of metal takes on the appearance of a gigantic lattice of atomic ion cores locked together in the metallic bonding structural configuration.

For purposes of failure evaluations, only this microscopic local zone is involved, and the remainder of the structure is irrelevant. The nature of the local zone, its structural lattice, and its geometrical construction is fundamental to an understanding of fatigue failures, because it is within this zone that cracks, microscopic failures, or other stress induced damage sites initiate. The most important consideration in fatigue is whether the stress, including defect SCF effects, is less than the strength of the material, or in terms of the spring analogy, whether or not the spring is being strained less than its elastic limit.

In order to better visualize a metallic ion core lattice, and how it reacts when it is strained or stressed, imagine placing a golf ball on top of a desk. Next surround it with six more golf balls. If all the golf balls touch, the distance from the center of the central ball to the center of any other golf ball is the diameter of a golf ball. This "lattice" may then be expanded normal to the desktop plane, by adding golf balls on top of the arrangement that already exists. Imagine placing three more golf balls on top of the original group of seven. These three golf balls will be packed in such a way that the distance from the center of any of the three to the ball in the center of the desk plane is also the diameter of one golf ball. Now imagine adding three more golf balls to the total existing lattice by placing them under the ten existing golf balls. The same lattice spacing will apply to the plane below the desktop as the plane above the desktop. This structure is representative of the way metallic ion cores bond into the typical hexagonal close pack arrangement where each ion core has twelve nearest neighbors.

The essential feature of the metallic bonding lattice is that lines drawn to connect ion core centers are all at 60 degree angles with respect to each other. Using this lattice as a model, it is now possible to examine what happens when metallic ion cores are strained or stressed and how the elastic limit comes into the picture. One can simulate strain in a metal by rolling the three golf balls in the top plane away from each other and up against the lower lattice. These balls can only move to a certain limit before they will no longer return to their base position when released. The limit (which is analogous to the elastic limit) is a factor of cos(60 deg)/2 which equals 0.43 or 43 percent of the diameter of a golf ball. For displacements greater than 43 percent of a diameter, the golf balls will have moved beyond their home position and the lattice will have been disrupted as the balls fall into new positions. It seems as if 43 percent displacement of the fundamental dimension is the "elastic" limit for a metallic lattice. This is very interesting because in real life the measurable fatigue strength of a metal is usually close to 43 percent of the yield strength. This illustrates dramatically that a metal can only be deformed by a certain limited amount before you start damaging its structural lattice and fatiguing the part.

As interesting as it is to explore the theoretical considerations for evaluating fatigue strength in terms of yield strength, for the remainder herein only empirical test data will be used to specify fatigue strengths. Some of these data come from manufacturers' metallurgical evaluations, the remainder comes from RR Moore testing for S/N fatigue strength curves. These data are the most demonstrably relevant measure of the damage tolerance limit fatigue strength of the material used to manufacture diaphragms.

It may be seen from the golf ball model that a lattice may be strained reversibly up to a certain point which may be called the zero fatigue damage limit (ZDFL). Up to this level of strain (or stress) the lattice does not suffer any damage. The question now arises, what happens to a lattice if the strain exceeds the zero fatigue damage limit? The extra energy introduced into the lattice causes what is called fatigue damage. There is only just so much extra energy that a lattice may contain beyond the zero fatigue damage without a gross structural change in the lattice. If the golf ball lattice is strained to 100 percent of the diameter of a golf ball, the lattice yields to failure in just that one application of the displacement. If strained between 43 percent and 100 percent, it will take a number of applications of the strain to induce a "fatigue" failure.

When a metallic structure is strained beyond its ZDFL it will experience a plastic deformation. The energy that it takes to produce this deformation thereby damaging the lattice may be related to the plastic deformation energy in a spring. Energy stored in a spring is proportional to the displacement squared of the spring. Therefore in a diaphragm, just like in a spring, the energy is proportional to the strain squared or equivalently the stress squared. The excess energy of the strain differential over the zero fatigue damage limit strain (or stress differential above the ZDFL stress) is proportional to the square of the difference between the stress level and the zero fatigue damage limit stress. The total damage sustained by a structure is proportional to the excess energy times the number of times that the material experiences this excess strain (or stress). Since in any given structure there is just so much damage per lattice site that can be tolerated, it is possible to write the following equation for cumulative damage.

$$D = C_1 \times (\sigma_p - \sigma_{ZDFL})^2 \times N \tag{1}$$

where:

D	=	the damage energy density in psi squared
C_1	=	a constant of proportionality
σ_p	=	the peak cyclic stress level in psi
σ_{ZDFL}	=	the zero fatigue damage limit in psi
N	=	the number of applied peak stress cycles

Since the amount of total damage which occurs is proportional to the excess stress squared times cycles, one may make an equivalence between varying stress levels as a function of number of cycles by taking the square root of the damage equation and writing it in the form:

$$(\sigma_A - \sigma_{ZDFL}) \times N_A{}^{1/2} = (\sigma_B - \sigma_{ZDFL}) \times N_B{}^{1/2} \tag{2}$$

where:

σ_A	=	stress level for a damage condition known as "a" in psi
N_A	=	number of cycles at condition "a"
σ_B	=	stress level for a damage condition known as "b" in psi
N_B	=	number of cycles at condition "b"

From this equation one can see that a large number of cycles at a low stress differential can result in the same damage as a small number of cycles at a very high stress differential. When the damage equation is plotted on graph paper using a logarithmic abscissa, the resulting curve is the S/N curve for the material. The S/N curve is a very well known concept in the practice of engineering. Most practitioners are familiar with the generation of these S/N curves through elaborate experimental testing programs. However, one can easily construct virtually identical curves using the simple physics of damage above.

The theory presented will be used to investigate the S/N curves for both 250 grade maraging steel, and also for annealed titanium plate (or forgings). To begin, consider the S/N curve for maraging steel published by a manufacturing company. This curve is reproduced in Figure 1. Certain specific points have been labeled on this curve as noted: for 1E6 cycles (1 million) the fatigue strength for a 50 percent probability of failure is 115,500 psi, for 1E7 cycles (10 million) the fatigue strength is 110,000 psi. If these are substituted into the S/N damage equation (Equation 1), one can solve for the zero damage fatigue limit stress. The ZDFL is 107,456 psi Von Mises, 50 percent probability of failure approaching an infinite number of cycles. S/N curves for titanium appear in many sources including the Governments Material Specification Handbook (MIL-HDBK-5). The 50 percent probability of failure stress for 1E7 cycles is 67,143 psi, for 1E6 cycles it is 74,285 psi, and for 1E5 cycles it is 87,714 psi. Substituting these data into the damage equation gives the ZDFL for titanium as 64,857 psi. From this, the entire S/N curve may be generated. Doing this produces curves that are in reasonably good agreement with the original experimental curves.

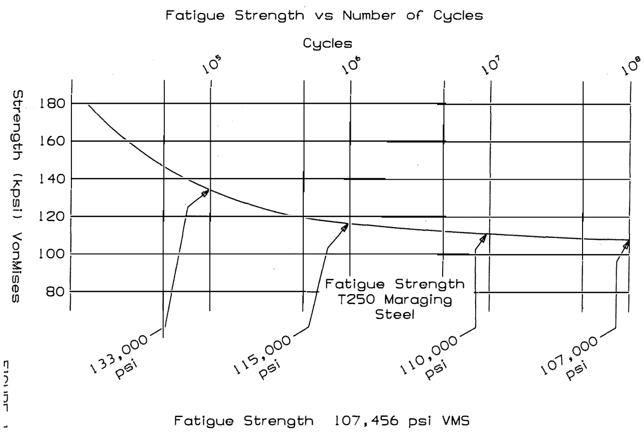

Figure 1. S/N Curve–Fatigue Strength Vs *Number of Cycles.*

These analyses and evaluations confirm the fact that the constant damage equation for stress as a function of the number of cycles is a valid physical and mathematical representation of the S/N curve for a material.

Having shown this, the next step in developing the formalism for fatigue life evaluation is to develop the damage probability density function. The damage probability density function expresses the probability that a sample of material exhibit a particular degree of damage with respect to a mean value of damage. The probability density function for damage, like most naturally occurring phenomena, is a normal or Gaussian statistical distribution function characterizable by a dependent variable (x), an average value (xav), and a variance which is the square of a standard deviation (v). As such, the probability density function must be expressible in the form shown below:

$$P_D = e^{-1/2}\,\frac{(x - xav)^2}{v} \tag{3}$$

The damage probability density function may then be written as:

$$P_D = e^{-1/2}\,\frac{\left[(\sigma_D - \sigma_{ZDFL}) \times N^{1/2} - (\sigma_A - \sigma_{ZDFL}) \times N_A{}^{1/2}\right]^2}{(\Delta\sigma)^2\, N_A} \tag{4}$$

where:

N_A = number of cycles applied in the average case
N_D = number of cycles applied in the case under consideration
$\Delta\sigma$ = variation in stress in psi per standard deviation at the average cycles under consideration

The next phase of the derivation is to consider the failure probability growth function over time (or cycles) from one value on the abscissa to another. The failure probability growth function is the functional relationship between the fatigue failure probability and the number of applied cycles in the case that a constant stress is applied over the range of cycles. In the case of maraging steel the 50 percent probability of failure point is at 110,000 psi for 1E7 cycles. The probability of failure for a different number of applied cycles (N) at 110,000 psi stress is given by:

$$P_F = \left[1 - e^{\,\frac{-.693N}{1E7}}\right] \tag{5}$$

If N equals 1E7 (or ten million cycles) the Pf (probability of failure) is 0.5 as it is supposed to be. However, if N equals 1E6 or one million cycles, the failure probability is 0.067. By using a table of integrated normal probability density, which tabulates failure probability in terms of standard deviations, one will find that the one million cycle data point at 110,000 psi is 1.5 standard deviations below the 50 percent probability of failure point on the S/N curve for one million cycles. Since the 50 percent probability of failure stress level is 115,500 psi at 1E6 cycles, then the 5500 psi stress difference corresponds to 1.5 standard deviations. Therefore, the standard deviation in stress for ten million applied cycles is 5500/1.5 = 3667 psi/standard deviation.

Now returning to the S/N curve, if N = 1E6 cycles and the stress is 115,500 psi, then the probability of failure is 0.5. However, for 1E5 cycles at 115,500 psi, the failure probability is once again 0.067, or 1.5 standard deviations down from the 50 percent probability of failure datum. Since the 50 percent probability of failure stress level is 133,000 psi, then the 17500 psi stress difference corresponds to 1.5 standard deviations. Therefore, the standard deviation in stress for 1E6 applied cycles is 17500/1.5 = 11666 psi/standard deviation.

Having established this feature of the failure probabilities, the next step is to confirm this feature utilizing the statistics of the distribution function itself. Because of the nature of the constant damage equation, the standard deviations themselves should follow the same relationship. This means that the distribution's standard deviation at a particular value for the number of cycles times the square root of the number of those cycles is constant, in the same manner that the damage is constant. In mathematical form one writes:

$$(\Delta\sigma_E) \times N_E{}^{1/2} = (\Delta\sigma_F) \times N_F{}^{1/2} \tag{6}$$

where:

N_E = number of applied cycles in case E
N_F = number of applied cycles in case F
$\Delta\sigma_E$ = stress variation in psi per standard deviation in case E
$\Delta\sigma_F$ = stress variation in psi per standard deviation in case F

At this point, one may verify the theory as follows. The square root of 1E6 cycles is 1000, and the square root of 1E7 cycles is 3162. The products 1000*11666 and 3667*3162 are within one percent of each other. This agreement is very reasonable, which confirms and cross validates our determination of the stress variation to be used in the reliability evaluations to follow.

As of this point, it has been shown that one may use either experimental test S/N data curves or the general theory of the physics of damage to establish variance or standard deviation for the material strength for a particular number of base cycles.

Using the formulation given above, and applying it to titanium, one arrives at a value of variance of 4743 psi per standard deviation for a 1E7 cycle reference base. This variance is greater than the value for maraging steel, which is 3667 psi per standard deviation.

The theoretical S/N curve (which results from the theory of fatigue damage) and the variance of the S/N strength curve can now be used in a mathematical formalism for flexural diaphragm fatigue life and reliability evaluation. However, before proceeding, it is necessary to perform a stress analysis on the proposed diaphragm design to establish the location and value of the peak theoretical stress for the application loading requirements. Subsequent to this, one must take into consideration the fact that the design as manufactured may develop greater stresses than the theoretical design, as a result of stress concentration factors due to fabrication defects or flaws.

There are two methods for incorporating these effects into the analysis. In the world of large power industrial power transmission couplings, where acceptance or fatigue substantiation testing is not practical, one has to include the effects of stress concentration factors empirically. Defects which produce a stress concentration factor of 1.3333, or greater, are easily observable. Therefore, it is reasonable to assume that the actual stress in the diaphragm will not be any higher than 1.333 times the design theoretical peak value.

The peak stress including the effects of stress concentration factors may then be compared to the strength taking into consideration the number of cycles of operation.

In the realm of aerospace applications where couplings and diaphragms are generally smaller, it is often possible to conduct an acceptance cycle test, or fatigue substantiation test, at stress levels known to exceed the worst case operating stresses. This establishes a measurable bound on the worst case stress that the component could ever see in service by, in effect, "measuring" the worst case stress concentration factor. The worst case theoretical operating stress may then be multiplied by that stress concentration factor, and the resulting absolute peak stress may be compared with the strength.

Regardless of the method, however, that one arrives at an absolute peak stress including effects of stress concentration factors for defects, the next step in the formalism is to input the peak stress values, the material strength, the variance of the strength (in psi per standard deviation), and the number of cycles of operation for which the evaluation is being conducted, into a computer analysis program. An example program is written out below:

```
rem flre–fatigue life reliability evaluation program
rem 18,2-96
print "fatigue life reliability evaluation DQRLA"
print "flrea001: a version: analysis with SCF compared to base material strength"
print "based on damage theory of fatigue failure"
print "18,2-96"
rem
rpm    = 5850
hrs    = 28.7*1
rem for 1E7 cycles
stds   = 4750
fact   = 1.45
rem fact   =     fact is an empirical SCF–no test
str    = 28000
fl     = 61,000
rem
cycles = rpm*hrs*60
print "for a run of hrs ————————-,"hrs
print "at a speed of rpm ————————,"rpm
print "the number of cycles is —————,"cycles
print "if the nominal operating stress is ,"str
print "using an assumed SCF worst case of ,"fact
str    = str*fact
print "then accounting for defects, and SCFs"
print "after ATP, the worst stress could be ,"str
print "the derated fatigue strength is —,"fl
print "the stress variance std is ———-,"stds
cys    = 10000000.
cyc    = rpm*60*hrs
qq     = cyc/cys
astds  = (str-fl)/stds
print "the stress relative to strength is stds,"astds
astdc  = log(qq)
s2     = astdc/(sqr(astdc*astdc))

print "the run is at relative cycle stds,"astdc
cstds  = astdc+astds
print "the combined cycle and stress stds is ,"cstds
s      = -12.5
ds     = 0.125
x      = 0.
for j  = 1 to 200
s      = s+ds
a      = exp(-.5*s*s)/sqr(2.*3.14159)
x      = x+a*ds
if s>cstds then goto 25
rem print j,s,x
next j
25 print "the probability of failure is,"x
cl     = (1.-x)*100
print "the confidence level is — percent ——,"cl
mtbf   = 0.5/x*hrs
print "estimated MTBF is (hrs) —————-,"mtbf
poh    = 1./mtbf
print "the FMECA prob of failure per op hr is ,"poh
print "this last value applies for flight safety considerations"
end
```

Four different versions of this fatigue life reliability evaluation program were written to account for the various ways one could deal with stress concentration factors and test results. The four versions are described as follows:

1. FLREA uses an empirical SCF as a multiplier for the theoretical nominal peak stress of a design subjected to loading conditions. The absolute peak stress is compared to the fatigue strength.

2. FLREB uses a fatigue substantiation test result. The theoretical nominal peak stress is calculated for the application loading, then the component is fabricated and tested at an elevated stress level (induced by elevated loading conditions). The life is evaluated by comparing the nominal stress to the substantiated "strength."

3. FLREC uses an acceptance test result to specify a worst case SCF. This SCF is applied to the nominal theoretical peak stress, and that stress is then compared to the strength.

4. FLRED is a routine that is used in the case one assumes that a design will be configured such that the theoretical nominal peak stress may not exceed a certain percentage of the material strength. This program is most frequently used in the proposal stages of a program.

In the world of industrial power transmission couplings, the FLREA program is the most appropriate, since one rarely has an opportunity to acceptance test those couplings or fatigue substantiate the design concept. The output of program FLREA is given below:

```
fatigue life reliability evaluation          DQRLA
flrea001: a version: analysis with SCF compared to base material
strength based on damage theory of fatigue failure
18,2-96
for a run of hrs                             28.7
at a speed of rpm                            5850
the number of cycles is                      1.00737E+07
if the nominal operating stress is           28000
using an assumed SCF worst case of           1.45
```

then accounting for defects, and SCFs	
after ATP, the worst stress could be	40600
the derated fatigue strength is	61000
the stress variance std is	4750
the stress relative to strength is stds	4.294737
the run is at relative cycle stds	1.689166E-05
the combined cycle and stress stds is	4.29472
the probability of failure is	1.393398E-05
the confidence level is — percent	99.9986
estimated MTBF is (hrs)	1029856
the FMECA prob of failure per op hr is	9.710093E-07
this last value applies for flight safety considerations	

The output itself illustrates how the evaluation is performed. First, there is a determination of the number of cycles of operation being considered. This number of cycles is then expressed in terms of the variance in standard deviations of this value from ten million cycles. Next the absolute peak stress including the empirical SCF is expressed in terms of its variance in standard deviations from the strength. The variance in stress in essence is a means of extending the S/N curve in the vertical direction while taking into account the variance in the probability of failure due to the stress. Similarly, the variance in the number of cycles in essence is a means of extending the S/N curve in the horizontal direction while taking into account the variance in the probability of failure due to the number of cycles. These two variances are combined and then integrated over the probability density distribution function to give the total probability of failure for a run of the specified number of cycles at the specified stress level. This probability of failure is then converted into a life for the diaphragm, and a mean time between failure value (MTBF). The MTBF may then be used for reliability analysis and in the construction of a failure modes effects and criticality analysis (FMECA).

As a final note, it should be mentioned that an extra measure of conservatism may be introduced into the formalism by using a derated (or worst case) fatigue strength instead of the nominal 50 percent probability of failure strength.

DIAPHRAGM COUPLING DAMAGE TOLERANCE

In the preceding section, the life and the reliability of diaphragm couplings were considered in the presence of normal inherent manufacturing defects or fabrication flaws. These inherent fabrication defects contribute to the effective peak stress levels that are generated as a result of the loads and deflections imposed on the coupling during normal and peak transient modes of operation. It was shown that diaphragm couplings have rather considerable tolerance to inherent manufacturing damage or defects.

In the following section, the ability will be considered empirically of a diaphragm coupling to withstand or tolerate incidentally induced damage. Experience has shown that diaphragm couplings are far more tolerant to incidental damage or operational overloading than is generally realized in the engineering community. Even in the presence of incidentally sustained damage, diaphragm couplings have a considerable life expectancy and a surprising degree of reliability. Various incidents will be reported that show that at normal operational stress levels, a diaphragm coupling can sustain damage to the point that it is cracked or even has a hole in the diaphragm, yet the diaphragm can nonetheless still function for a considerable period of time before experiencing a failure.

The crucial factor is whether or not the stress at the tip of any induced damage is greater or lower than the stress intensity factor at which the crack propagates. If the crack tip stress is less than the critical stress intensity factor, then the damage effects will not propagate, and the remainder of the diaphragm can carry the load normally. The unaffected regions of the diaphragm are like an intrinsic redundancy load path around the region of damage. However, there are some restrictions to the extent of the stress level that may be tolerated in this mode. This results in some life limitations. Several practical examples of such life considerations and damage tolerance of diaphragms will be presented herein.

Single contoured diaphragm couplings shown in Figure 2 have been perceived by some of the turbomachinery community to have an unacceptable failure mode. It has been assumed that the only failure of this coupling is instantaneous with no advanced warning to the operator or vibration monitoring devices. The perception seems valid, since most contoured diaphragm failures are from over torque. This typical failure mode will cause the diaphragm to shear from buckling and has a rippled shape as shown in Figure 3 that is customarily found when the diaphragm fails from torque alone.

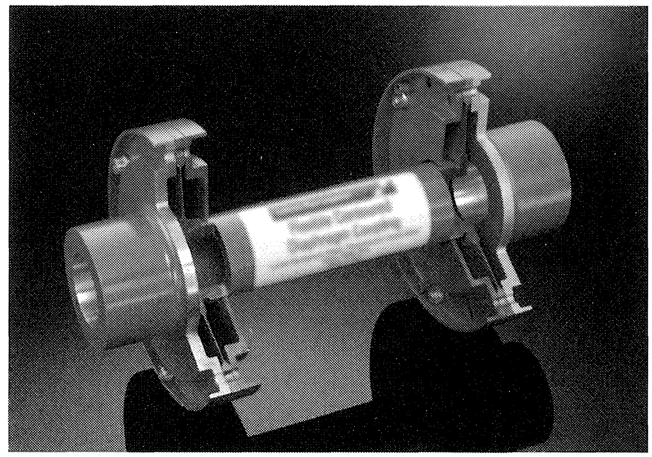

Figure 2. Single Contoured Diaphragm Coupling.

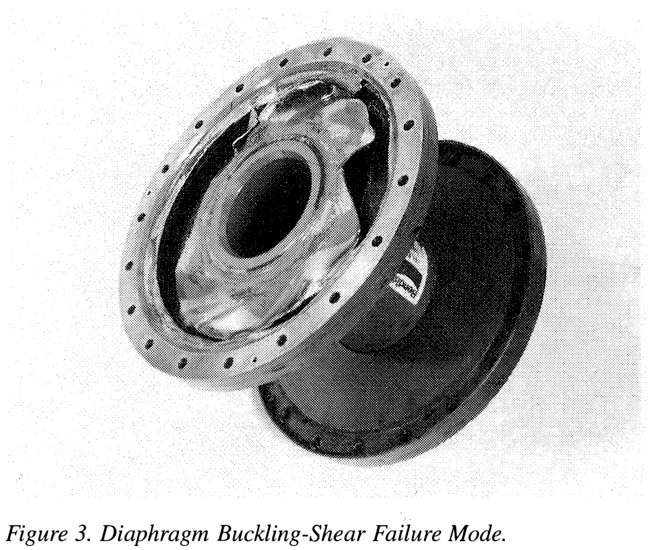

Figure 3. Diaphragm Buckling-Shear Failure Mode.

This failure mode is caused by a surge/overload condition in the turbomachinery equipment. The coupling is the ideal "fuse" and all types of couplings will fail instantaneously, thereby reducing the damage of downstream equipment from the overload condition.

The other type of failure mode common to dry type couplings is fatigue. Fatigue overstress could be caused by excessive misalignment, corrosion, and cyclic (torsional, axial, etc.). The MTBF of a single contoured diaphragm coupling is over 10 million hr. Registered failure of one brand of single contoured diaphragm couplings show that less than 20 percent are from a fatigue failure mode.

A portion of the turbomachinery community have requested a redundancy feature for this type failure mode and both multi disc and multi convoluted diaphragm couplings have been readily accepted as having the redundant feature via the "multi" flex elements. The intent of this redundant feature is typically for short term operation such that a controlled shutdown can be implemented.

A single contoured diaphragm coupling has not been readily accepted as have a redundant feature since there is only one flex element. Seven incidents will be presented involving five field coupling and two in house test couplings. The results presented will show that a single contoured diaphragm coupling in essence has an intrinsic redundancy feature giving the coupling far more tolerance to mechanical damage then previously thought.

FATIGUE TYPE FAILURES

Case 1: A Hydrogen Recycle Compressor, El Dorado, Kansas

• The 1500 hp steam turbine hydrogen recycle compressor is fitted with a model 67E406 flexible diaphragm coupling and operates at a speed of 8,000 to 13,600 rpm. The original coupling guard was constructed of fiberglass.

• The train is equipped with noncontact radial vibration and axial position continuous monitoring systems on both turbine and compressor.

• The unit had been down for turnaround in October 1995 and was running satisfactorily. About two weeks after startup, the turbine inboard bearing oil seal began leaking excessively. This resulted in a fire on the exhaust end of the turbine from oil spraying onto the hot turbine casing. The fire burned away the fiberglass coupling guard and damaged some of the instrument wiring, before it was extinguished by operators. The train was shut down for inspection.

• A piece of heavy expanded-wire material was fitted over the coupling to provide personnel protection until a proper coupling guard could be installed. After making (external) inspection and wiring repairs, the unit was put back online and was running smoothly with acceptable vibration levels.

• The leaking oil seal was not changed out at this time because no spare was available in a reasonable period of time. A steam hose was trained on the oil leak area to prevent another fire.

• Because of the steam hose, insufficient steam leak off to the gland condenser, and additional moisture ingested into the oil reservoir, the unit began to experience problems of water in the lube oil. High differential pressure was frequently experienced on the oil filter and on at least one occasion the filter collapsed due to the water contamination, causing a short shutdown. A project had been initiated to replace the oil seal at the next opportunity, and to add a nitrogen positive pressure purge on the bearing housing seal areas. Additionally, methods were being evaluated to remove the water from the oil by clarifying/reclaiming the oil during operation.

• On February 15, 1996, an emergency ticket was issued to change the oil filters due to excessive differential pressure due to water contamination.

• On February 19, 1996, there was a sudden "step-change" increase in vibration levels on both outboard and inboard turbine radial bearings. The amplitude went from 0.5 mil to over 1.0 mil. There was also an increase in radial bearing metal temperature. Vibration levels on the compressor were still in the normal range. The unit was operating at about 12,000 rpm at the time.

• The unit speed was reduced to about 11,000 rpm and the vibration levels dropped back to about 0.5 mil. Bearing temperatures also dropped.

• On February 20, 1996, the oil filters again required changing due to fouling caused by excess water in the system.

• On February 23, 1996, the vibration on both outboard and inboard turbine radial bearings continued an upward trend (1.2 mil and 1.0 mil). Compressor vibrations continued to remain normal.

• At this point, there was significant concern due to the sudden increase in vibration levels. There was also a noticeable increase in structural vibration around the machine.

• It was suspected that the turbine bearings had sustained damage from the water in the oil system. Plans were initiated to have a controlled unit shutdown and change the bearings.

• It was discovered that no spare bearings were in stock yet. The train speed was reduced again, which reduced the vibration levels slightly and the unit was kept online pending a rush delivery of the spare bearings.

• On February 25, 1996, the unit was shutdown. The bearing housings were opened and the bearings inspected. Both outboard and inboard bearings were found to be in good condition, with no evidence of damage.

• It was decided to uncouple the turbine and run it solo to make a further evaluation on the vibration problem.

• During removal of the coupling spacer, it was noted that the coupling bolts appeared to be out of alignment and the coupling was discolored (black instead of blue). Upon removal, it was discovered that the coupling bolts were in fact bent, and the bolt holes were distorted. The diaphragm portion of the coupling spacer was found to have an open crack at the outside diameter of the web, just under the flange as shown in Figure 4. The crack was about 90 percent of the way completely around the periphery of the diaphragm. There was approximately 2.0 in of unbroken material remaining.

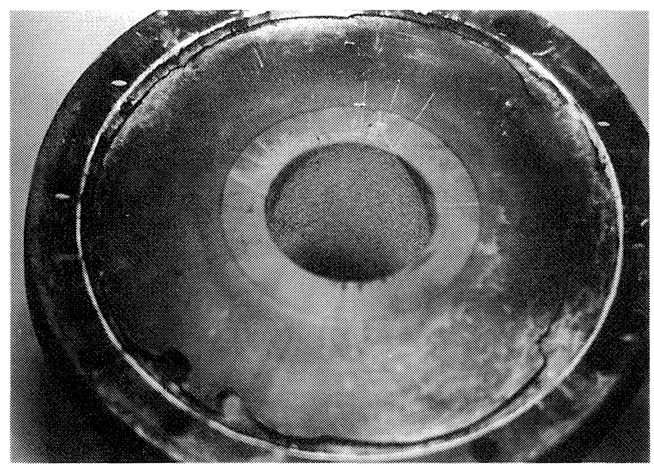

Figure 4. Diaphragm Failure Crack 90 percent of Periphery.

• The coupling apparently had sustained excessive heat damage during the October 1995 fire. The heat (and water quench) had caused a heat affected zone from which the crack had initiated and propagated around the periphery during subsequent operation. At some point a shift in alignment had caused the increased vibration response, which continued to deteriorate as operation continued. It obviously would not have lasted much longer without failing.

• The turbine was reassembled with the same bearings (the leaking oil seal was replaced) and run up solo to check the overspeed trip, which was set at 14,900 rpm. The o/s trip *did not operate.* After numerous attempts at setting and repairing the knockout bolt and o/s linkage assembly, no reliable repeatable response could be obtained.

- It was conceded that the o/s bolt assembly was faulty and a new spring was installed.
- Although numerous attempts were still required, the new spring assembly responded properly and a repeatable trip was obtained at 14,900 rpm. The spare coupling was installed and the unit put back online the morning of March 2, 1996.
- The failed coupling was returned to the manufacturer for inspection, and their analysis confirmed the conclusions drawn about the heat causing the crack in the diaphragm.

RECOMMENDATIONS

- Any time a coupling is exposed to excessive heat, it should be considered suspect until properly examined by the manufacturer. If a spare coupling is available, there is minimal down time to change out the suspect coupling. This is one unexpected incident that supports the stocking of a spare coupling.
- Install a properly designed metal coupling guard with vents and drains. This will help reduce oil leakage and provide protection for the coupling from physical or fire damage.

This unit now has a completely enclosed metal coupling guard with proper vents, internal windage baffles, and drain connected to the oil return header to contain leakage.

- Make sure the o/s trip system is in proper working order by proper inspection, testing, and maintenance.

This unit now has been retrofitted with an electronic governor and overspeed trip with 2/3 voting logic.

CONCLUSIONS

- The coupling failure was caused by excessive heat damage experienced in a brief fire during operation which engulfed the coupling. The excessive heat effected a metallurgical change in the coupling diaphragm material from which a crack initiated. The crack propagated during continued operation until it affected the alignment/balance of the rotating system sufficiently to cause an increase in vibration response.
- The inoperable overspeed trip protection was caused by an unreliable overspeed trip spring and excessively loose overspeed trip linkage that activates the oil pressure dump valve and causes the trip and throttle valve to close, shutting down the train. The system had been in continuous operation for a number of years and appeared to be excessively worn.
- The coupling was dangerously close to complete failure. Had the coupling run to failure, the instantaneous loss of load would have allowed the turbine speed to increase drastically in a very short period of time. Without overspeed protection, this could have resulted in catastrophic damage to the turbine with disastrous results.
- The impending failure was detected by the permanently installed vibration monitoring system. The change in vibration levels resulted in an early diagnosis and prevented the machine from running to destruction.
- This incident provides tremendous support for the application of continuous vibration monitoring and high-vibration shutdown, especially on critical equipment.

Case 2: A Model 67E412 that coupled a steam turbine to a compressor failed on July 7, 1984, with over 10 years of service. The customer noticed a increase in vibration and decreased speed. Three days later torque transmission ceased.

Case 3: A Model 67E310 that coupled a gear to a compressor failed after two years of service. Excessive vibration noted before failure.

Case 4: A Model 67E408 that coupled a steam turbine to a compressor shutdown in 1989 with over 10 years of service. Excessive vibration noted and the train was shutdown. A crack had propagated circumferentially over a 300 degree arc as shown in Figure 5. The unit was still transmitting torque prior to shutdown.

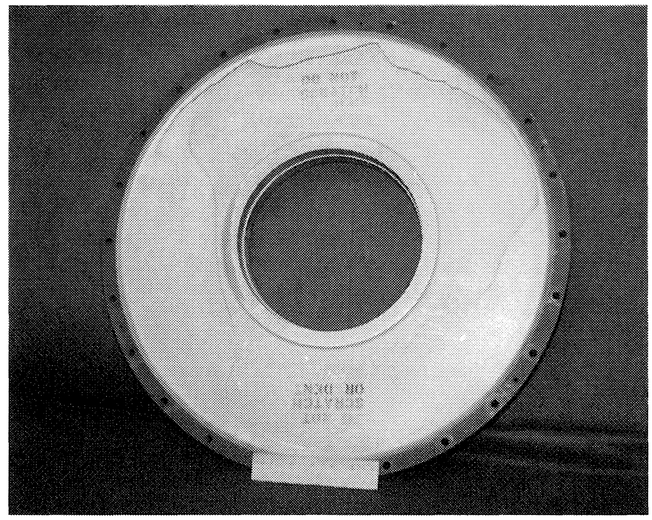

Figure 5. Cirumferential Diaphragm Failure 300 Deg Arc.

A test was completed inhouse on a model 67E224 diaphragm coupling. The first phase of this test was a 10 million cycle test at a stress level more severe than the actual operating environment. The parallel offset was set at 0.263 in and the axial deflection at 0.100 in stretch. The operating speed was 870 rpm and the 10 million cycles was completed in 200 hr. Nondestructive testing was performed on the coupling after successful completion of the 10 million cycle test to verify that there was no damage to the profile.

A 10 million cycle test is completed on all aerospace couplings before shipment with the intent to verify acceptability and design substantiation. Ten million cycles is approximately the point where the stress level on the S/N curve starts to flatten out *vs* number of cycles as can be seen in Figure 1. Therefore, if a 10 million cycle acceptance test is completed successfully that assures that the diaphragm is free of defects that would compromise the integrity or performance capability of the part.

The coupling was then installed with 0.330 in axial stretch and 0.304 in parallel offset (1.133 degrees misalignment). This condition overstressed the coupling and should have caused a fatigue type failure at about one million cycles. Testing was initiated and after 1.472 million cycles a 3.0 in crack occurred in the drive side diaphragm at the thinnest section of the profile near the outer edge. Testing continued and after an additional hour of testing the vibration levels doubled. The crack propagated from 3.0 in to 12 in in 1.5 hr and a second crack initiated 180R from the first crack. Testing was continued until the 4.6 hr mark when the test was stopped. It was determined that the crack propagated 80 percent of the circumference of the diaphragm.

Vibration levels exceeded acceptable levels within the first 1.5 hr of testing. Torque and misalignment could still have been transmitted even though 80 percent of the diaphragm had cracked.

SUMMARY

The four field failures and one inhouse test are all cases of fatigue type failures caused by either material degradation or operational over stress. In all cases, after the initial failure of the material a crack was formed and propagated over time. At some point this "crack" caused vibration levels exceeding acceptable limits. However, the crack does not impede transmission of torque during this mode.

In conclusion, this type of failure is not instantaneous and fails such that monitoring systems and/or personnel have advance warning to complete an orderly shutdown.

MECHANICAL INDUCED DAMAGE TOLERANT DESIGN

The US Army's OH-6A Helicopter was extensively used as an active combat vehicle in Vietnam. There were two field reports in the late 1960s of aircraft being exposed to VietCong small arms fire. In both cases, the diaphragm couplings that were used in the main drive shaft were damaged by bullet holes through the profile. After the hit, the pilots noticed increased vibration levels, reduced their power by 25 percent and completed their flight mission. In one case the mission lasted an additional 2.5 hr before the damaged aircraft returned safely to base.

It is significant that the type of damage described in these incidents resulted in edge tears and cracks, as shown in Figure 6, which are generally very detrimental to the fatigue life of cyclically stressed parts. However, an examination of the couplings did not locate any evidence of propagation of the ballistic damage.

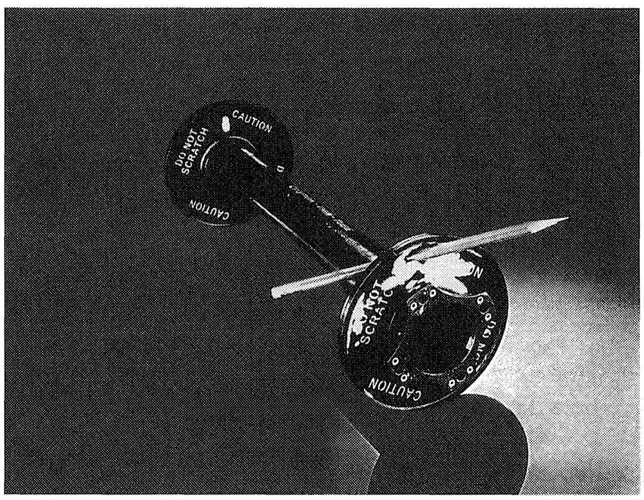

Figure 6. Coupling Survives Incidental Combat Damage.

During the development program of the US Army's Apache (YAH-64) Helicopter, ballistic testing was completed to verify the tail rotor drive shaft was capable to survive impacts from 12.7 mm projectiles. The diaphragm profile was penetrated with these projectiles and then endurance tested for 2.6 hr after impact. No abnormal operating conditions were observed during these tests. The survivability trait of the diaphragm was verified and demonstrated in full-scale ballistic tests.

Inhouse research has also been completed where diaphragms were subjected to external damage. In experimentation, the damage was induced by tearing a cut into the diaphragm with a chisel. Cycle testing was completed and the "tear" propagated, but did not induce a complete failure. In the second experiment, diaphragm damage was induced with the penetration of two 50 caliber rounds, one impact was through the thin profile area. Cycle testing was completed and propagation only occurred from the impact through the profile area. The testing was stopped after 11 hr. The damage had spread to a 90 degree arc on the diaphragm. (See Figure 7).

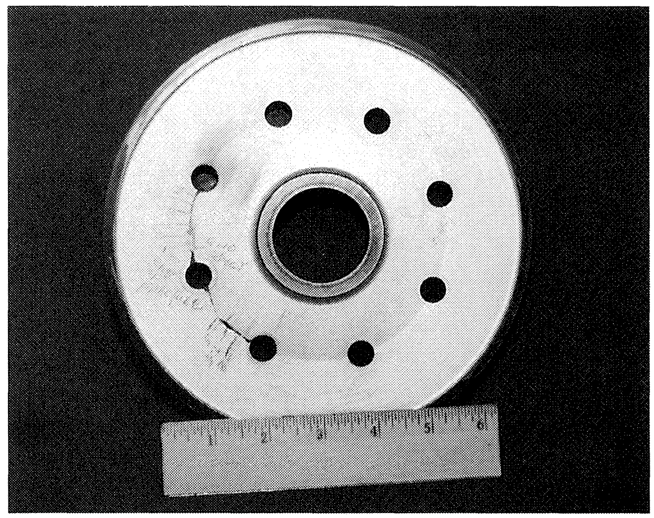

Figure 7. Crack Propagation Test Results.

In conclusion, a diaphragm is more tolerant to external damage then the "Do Not Scratch" note implies. Ideally, no flex element surface should be damaged, but the survivability of a diaphragm coupling has been proven time and again over the last 40 years.