# ROTATING COMPONENT MODAL ANALYSIS AND RESONANCE AVOIDANCE RECOMMENDATIONS

by
**Frank Kushner**
**Consulting Engineer**
**Delmont, Pennsylvania**

*Frank Kushner is an independent Consulting Engineer specializing in dynamics and acoustics testing and analysis. He has had 35 years of experience with industrial turbomachinery, mostly all while with The Elliott Company, and four years previous experience with the jet engine combustion section development group at Pratt & Whitney Aircraft. He is a previous author for the Ninth, 25th, 29th, 31st, and 32nd Turbomachinery Symposia, as well as for the ASME.*

*After obtaining a B.S. degree (Mechanical Engineering, 1965) from Indiana Institute of Technology, Mr. Kushner received his M.S. degree (Mechanical Engineering, 1968) from Rensselaer Polytechnic Institute. He is a registered Professional Engineer in the State of Pennsylvania and a member of the ASME and the Vibration Institute. Mr. Kushner holds patents for a blade damping mechanism and a method to prevent one-cell rotating stall in centrifugal compressors.*

## ABSTRACT

Rotating blade and disk fatigue failures cause a fairly small percentage of machine shutdowns, but can require extensive repairs and downtime. This tutorial focuses on steam turbines, centrifugal compressors, and axial compressors primarily for refineries and process plants. For steam turbines, axial compressors, and open-impeller centrifugal compressors, the main concern is resonance giving blade rather than disk vibration. For covered impellers in centrifugal compressors disk modes are the main response for problems, albeit failures are much more rare.

Reviews of methods are to give insight to design evaluation, calculation, and test procedures. Cases will be given where designs were suspect with high potential for cause of fatigue, with other cases due to minor resonance combined with either manufacturing defects, operation, and/or environment such as liquid ingestion and corrodants. For some cases, test data will be correlated with theories provided, while others will use references, some of which will be shown to perhaps require consideration of alternative probable causes. The material will utilize both proven concepts and cases for lessons learned.

## INTRODUCTION

High cycle fatigue (HCF) and its prevention is one of the most serious issues for fluid-handling turbomachinery for the oil, petrochemical, and related industries. Relative to this type of industrial machinery, electric utility turbine/generator sets can have consequences that are often greater; in some cases, blade and/or disk fracture can be catastrophic and a safety issue. In some reports, failures of turbine blades are identified as the leading causes of unplanned outages for these steam turbines. Accidents of low-pressure end turbine blades suffer a high percentage of failures; Bhaduri, et al. (2003), provide some Electric Power Research Institute data as reproduced in Figure 1.

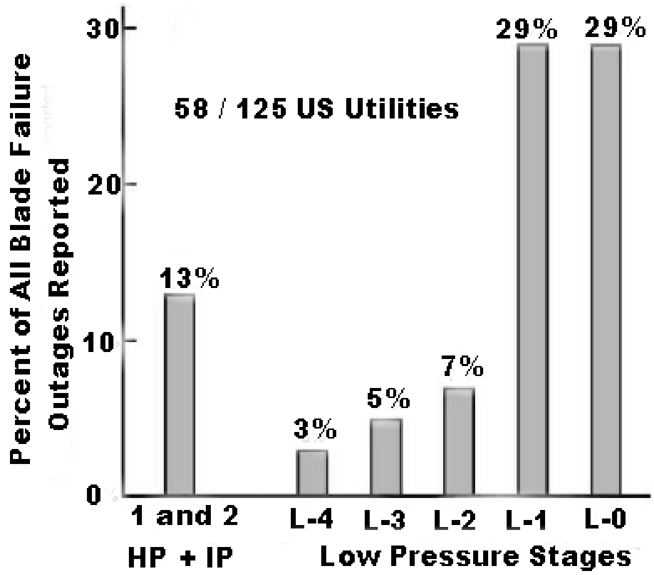


*Figure 1. Location of Rotor Blade Failures—Data for 58 Electric Utilities. (Courtesy EPRI, Palo Alto, CA)*

McCloskey (2002) gives a comprehensive summary of utility turbine blade and disk troubleshooting including many references. There are similar issues with jet engine designs, their land-based derivatives, and other gas turbines including microturbines (refer to Srinivasan, 1997), but these also have many other issues such as low cycle fatigue analysis and interaction of components with much different operational modes and life cycles (Layton and Marra, 2000). Reports give values between 40 and 50 percent of gas turbine outages due to blade fatigue; besides other sources, combustion pulsation can also give rise to alternating pressure forces. Gas expanders such as those for fluid cat cracker (FCC) processes are a special case with the combustion occurring in the process, but there have been limited blade fatigue problems not associated with corrosion, such as described by Dowson, et al. (1995), and catalyst dumps into the turbine inlet casing. Turboexpanders, fans, and some liquid pumps should have similar issues to those that are to be covered, but extensive reviews will only include machinery based on direct experience.

Some of the materials to be presented primarily for fluid-handling turbomachinery for the oil, petrochemical, and related industries are:

- Basic coverage of resonance response.
- Methods to analyze cantilevered and shrouded blade designs.
- Disk modes and disk critical speeds.
- Test procedures and correlation with finite element analysis (FEA).
- Excitation sources with designs for minimizing loads.
- Campbell diagrams—do's and don'ts for both blades and disks.

- Interaction resonance with proper interference diagrams for disks.
- Effects of mistuning of blade natural frequencies on response causing fictitious interference diagrams.

A historical review for the all important interaction resonance effect between blades and vanes for disks will be offered, starting with Kushner (1979). Another objective is to assist with explanations as to why there must be exceptions taken to the extreme specifications for resonance avoidance such as those in API 612 (2003) specifications. There are numerous resonance criteria to consider, including many that give negligible response and thus alternating stress. For many potential resonant speeds, environmental effects that reduce endurance strength can sometimes lead to questions of design adequacy. Other resonant points must always be avoided for all designs, and some have limits that can only be exceeded if there are improper operation and/or environmental factors. Besides forced response for turbine and axial compressor blades instability with flow interaction reducing damping is an issue in component design, whereas instability does occur but is extremely rare for radial flow impellers.

Cases will be given where designs were suspect with high potential for cause of fatigue, with others due to minor resonance combined with either manufacturing defects, operation, and/or environment such as liquid ingestion and corrodants. These will include several cases for steam turbines and multistage centrifugal compressors along with two axial compressors, and a plant air compressor. For some, available test data will be used; some other results will be supplemented with reviews in the literature. In some cases, alternative possible causes will be given for the cited references.

Reliability of turbomachinery components requires that aspects of fitness-for-purpose be properly addressed at various stages of its design, fabrication, testing, and operation. Besides economic considerations, safety considerations influence the inherent quality and reliability of the machine. Safety basically requires proper functioning without presenting hazards to personnel and property. It is possible to build a highly reliable compressor or turbine that ends up being economically competitive, i.e., using the author's personal motto—"there is an optimum to everything." Design specifications are needed to incorporate optimum fitness-for-service attributes. Those specifications must include review of any potential resonance that could cause fatigue failures. This tutorial will hopefully present some important views on how components should be designed so that they meet the service requirements for the required life (20 years per some specifications). Of course components also must be fabricated with specified materials, inspected to show they conform to design concepts, and are operated and maintained properly. Whenever there are failures even from misoperation, it is always wise to use the knowledge gained to achieve optimum designs for "fitness-for-service."

Besides the rotor/bearing system lateral modes of all turbomachinery, and potential severe torsional system resonance for some with motors and/or gears, an equally critical aspect is resonance avoidance for structures on the rotor. If the frequency at which variable loads act on a bladed disk coincides with one of the important natural frequencies of the structure, a very dangerous situation can occur, leading to rapid failure due to high cycle fatigue. High cycle fatigue (HCF) is defined as that which results in cracking or fracture from a large number of stress cycles well below the yield strength of the material, and is associated with fatigue lives greater than about 10,000 cycles. Low cycle fatigue (LCF) such as from thermal effects during startup is normally not an issue for blades and disks in machinery to be covered herein. However, severe forces from improper operation such as continuous surging and/or excessive liquid ingestion can cause failure in much less than a million vibration cycles. For many observers, even 10 million cycles (corresponding to threshold endurance strength for many defect-free, ductile materials) appear to be an unattainable high number of cycles for setting a lower limit. However, consider that structures discussed in this tutorial respond typically in the range or 200 to 10,000 cycles per second. Depending on the severity of excitation and resonant amplification for the structure, a failure can occur in a few minutes to several hours if operating at high loads very close to, let alone exactly at, a resonant operating speed. There have been cases of failures in the first day of operation after startup, as shown by one of the case studies. Resonant response is avoided or minimized by controlling the stimulus or by changing the blade frequency. The stimulus can be controlled by changing the frequency of excitation, changing the strength of the stimulus or its relative phase with respect to the blade or bladed disk. The most common method of modifying blade frequency is to tailor the thickness of the blade or disk along with other variables affecting frequency: aspect ratio, taper, number and thickness of blades, and even radius ratio. More drastic modifications are to use one of many damping methods, e.g., change to a friction damping mechanism at part span or by using a tip shroud incorporating damping and/or phase cancellation.

In some cases, natural frequency variations even for constant speed applications of load may be greater than an acceptable band; thus the blades must be designed with sufficient stiffness, higher strength materials, and/or higher damping to limit stresses whenever blades operate at resonance. HCF failures can otherwise occur when these conditions are not met, or if a forcing function becomes excessive, such as from upstream or downstream blade or vane damage, operation at off-design causing rotating stall, surge, flutter, or other damage, e.g., from blade tip rubs. In some cases there can be simultaneous sources of excitation affecting the same or different modes. Manufacturing tolerances and fabrication variation can influence natural frequencies for a blade assembly; corrosion, wear, and erosion can also reduce resonant frequency margins and/or endurance strength over time. Either or a combination of these conditions can result in high-cycle fatigue initiation and propagation. There also can be a cause of crack initiation, followed by secondary resonant points that can assist in propagation. The cracked structure has lower stress intensity and required number of cycles for crack growth compared to initiation. If inspections do not find cracks to abate further propagation, a structure then could easily eventually reach its fast fracture limit. Many machines have been shut down due to changes in monitored vibration data before a catastrophic wreck. Thankfully ductile materials mitigate this concern. In some cases cracks could lower the resonant frequency such that the structure moves away from the resonant speed. This change could give some relief, especially for constant-speed machines such as turbine/generator sets. In other cases, sometimes after a long period, resonance and/or the same or another speed change or off-design upset giving high loads can reoccur, causing the "beech-mark" patterns on a fractured surface.

The tutorial will include an initial precursory description of resonance, using examples for blades and disks. There will then be a more general description of modal analysis of rotating structures that should apply to other machinery, outlining which vibratory modes are especially important. The risks of resonance for different applications vary; each major type of rotating machine, centrifugal and axial handling gaseous fluids will then be more fully covered in the case studies.

## REVIEW OF BLADE AND DISK RESONANCE

Machinery discussed in this tutorial typically has potential for excitation of either blade or disk modes; however some designs could have coupled disk/blade modes as do many jet engine designs and gas turbine derivatives. As in all aspects of vibration, it is very important to always consider phase, not just amplitude and frequency. As the author likes to repeat, you do not push a child on a swing as he is coming toward you, unless you want him to stop and do his homework.

Natural frequencies with finite element programs are utilized for design, with much less testing as compared to 20 years ago. Some correlation tests as described in APPENDIX A are always beneficial, especially for critical resonant modes such as for blade leading edge mode for impellers to be reviewed later. The equation of motion for a single degree of freedom model is given in Figure 2.

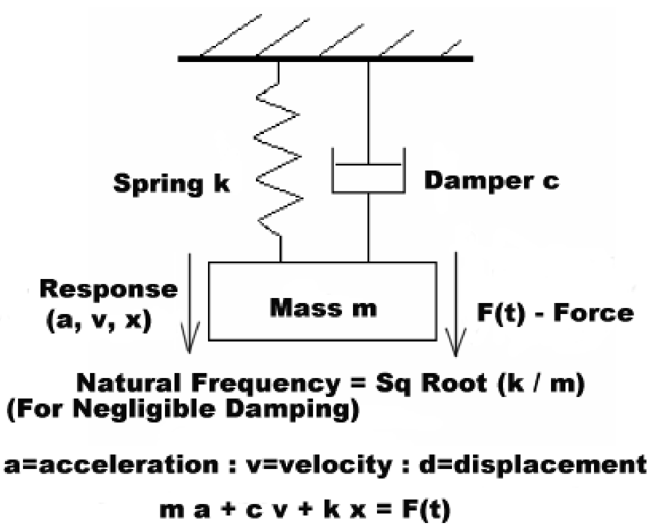


*Figure 2. Equations of Motion for Single Degree of Freedom.*

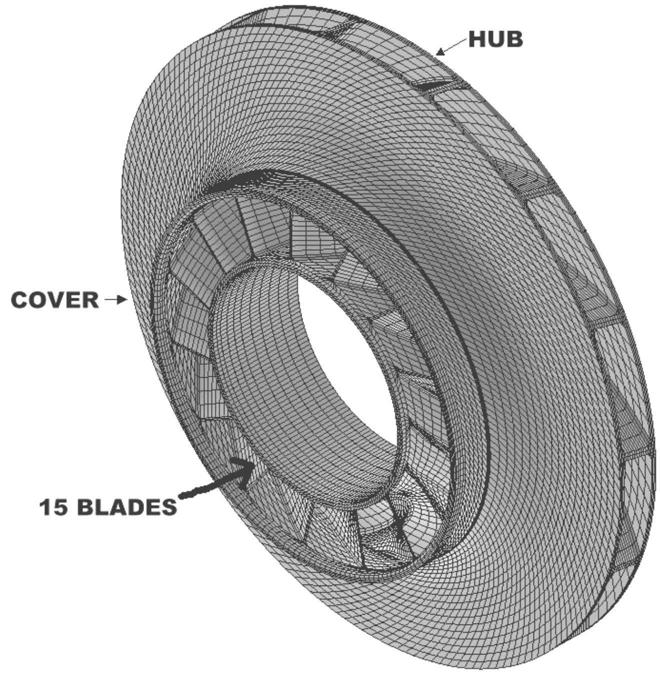


*Figure 3. FEA Model of Covered Impeller with Full Inducer and Curved, 3-D Blades.*

For modal analysis using finite element analysis, the same equations of motion apply; for multiple degrees of freedom they take on a matrix form. For complex structures, the analysis requires careful modeling (Figure 3) and proper techniques including boundary conditions. In order to obtain relative stresses for higher order modes, the entire impeller may require modeling. Modal analysis of a cyclic symmetric structure to determine nodal diameter frequencies can be computed by modeling only one sector per the finite element computer code (Kohnke, 1999). Nodal diameter mode shapes contain lines of zero out-of-plane displacements crossing the entire disk. These lines of zero displacements are commonly called nodal diameters. The number of nodal diameters, n, is an integer that determines the variation in the value of a single degree of freedom at points spaced at a circumferential angle—equal to the basic sector angle (Theta). For a number of nodal diameters equal to n, this variation is described by the function, cos (n [Theta]). This definition allows a varying number of waves to exist around the circumference for a given nodal diameter.

Constraint relationships can be defined to relate the displacements of one edge of the cut boundary to the other edge. This allows for calculation of natural frequencies related to a given number of nodal diameters. The basic sector is used twice to satisfy the required constraint relationships and to obtain nodal displacements. The analysis results will show pairs of frequencies for each nodal diameter solution. Using this technique, it is possible to obtain solutions for up to a number of nodal diameters, n, up to n = B/2. For an even number of blades, B, this would be B/2; for an odd number of blades this would be (B-1)/2. Methods are given to evaluate these frequencies for both static and dynamic (stress stiffened) conditions. More complex modes require a model of the entire impeller as shown in *Case B-6* below.

*Cantilevered Blade Design and Resonance Avoidance*

Rotating blades in axial machinery have the most difficult task of HCF avoidance, as they are most compliant, not only in axial compressors but also in nonshrouded stages of both steam turbines and centrifugal compressors. Freestanding, cantilevered blades come in many forms and sizes. Shorter blades usually do not have much twisting along the profile length and have three fairly independent modes: fundamental tangential, fundamental axial, and fundamental torsional (refer to APPENDIX A). Higher modes have nodal lines and have phase changes along the length and width and thus have inherent phase cancellation for most cases. The fundamental tangential mode is not only the most compliant, but typically experiences the highest excitation forces as the direction of force (a percentage of steady forces) aligns with the predominant steady gas force that "drives" the machine. Many blades are canted so that tangential forces can assist in excitation of the fundamental axial mode, but axial forces are lower and axial modes are less compliant giving lower dynamic stresses. Torsional modes also are of lower concern as there is a node line along the blade, so that the leading edge exciting force is nearly out of phase with that on the trailing side. For short blades, as compared to the tangential bending modes, axial and torsional modes are at least on the order of one tenth of the concern for short blades; there were perhaps one or two field problems in 30 years in the author's former company. There could of course be exceptions, such as from axial forces on a highly loaded reaction-bladed stage.

Medium length blades such as the middle stages of steam turbines typically have much fewer problems (refer to Figure 1). The Campbell diagram in Figure 4 shows that nozzle passing frequency is avoided; resonance of the first modes would only be equal to higher harmonics of speed where severe nonuniformity of nozzle throats is the main issue along with potential excessive corrosion in transition stages (one of the cases below). Higher order modes, not shown in Figure 4, could also present problems with excessive corrosion.

Longer cantilevered blades such as those in the rear of steam turbines (refer to Figure 5) and the front-end of axial compressors can have resonance potential for all three directions. In modern machines, longer blades have significant twist and have larger chord widths; thus the modes for the three directions are coupled. In fact there can be higher axial motion as compared to tangential at the blade tip for the lowest frequency mode; thus higher axial forces for a reaction-type stage of a steam turbine could give relatively more response at resonance. In addition, the fundamental modes also have potential for flow-incidence related flutter. The longer freestanding blades in steam turbine-generator exhaust ends usually must have the first few modes carefully tuned. Gas expanders should have less risk, especially since inlet connections are axial reducing harmonic excitation, other than from support struts; the high number of inlet vanes typically would minimally excite higher order modes.

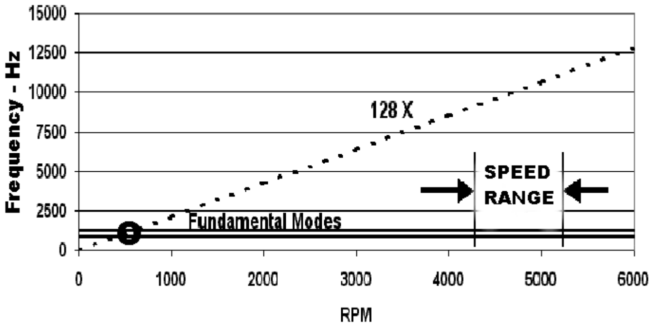


*Figure 4. Campbell Diagram for Turbine Blade, 128 Inlet Nozzles.*

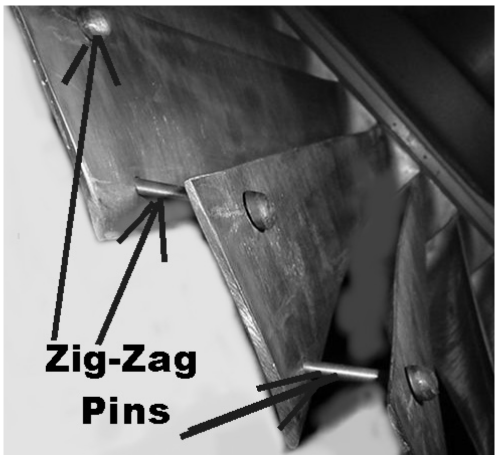


*Figure 5. Looking from Tip of Long Blades with Zigzag Damping Pins.*

Freestanding, cantilevered blades used for open-impellers also have much more risk for HCF than fixed-fixed blades in covered impellers. Inlet guide vane selection must consider the fundamental bending and sometimes the higher modes with nodal lines. Number of inlet guide vanes thus becomes a design variable for integrally geared compressors and radial-inflow turbines. As impeller sizes increase the leading edge flapping mode can become a design issue for covered impellers that can be missed. The fixed-fixed mode is not very compliant, i.e., as compared to a cantilevered blade. Many designs likely could run with having first order resonance as there are thousands running with resonance of the same mode, but excited by two times upstream vanes. Cases such as described by Singh, et al. (2003), and Phillips, et al. (2003), likely were due to other mitigating factors. However, there will always be the issues:

- Was operation within the specified operating map, and
- How much liquid was really being ingested to potentially aggravate the wakes from upstream vanes.

*Shrouded Turbine Blades*

In order to mitigate resonance, phase cancellation has been used for many years in steam turbines and other machines. Weaver and Prohl (1956) give one of the best reviews on this subject. By tying blades into packets, phase cancellation can be optimized for a given mode, so that some blades in a packet are out of phase with the exciting forces. This design feature is likely the most important reason over the years for reducing the number of HCF incidents in steam turbines. Response from partial-admission causing nonuniform loading is also greatly reduced by shrouding. APPENDIX B gives a simplified method to check for optimum blade number per packet for the fundamental mode where all blades vibrate in phase with the same amplitude. Similar procedures, considering amplitude and phase changes, can be done for higher modes. An example is shown in APPENDIX B where the response factor can be forced to give zero response. With the many varieties of designs, it is impossible to obtain complete cancellation for every stage, but typically designs can be utilized with resonant factors down to 0.20, i.e., dynamic stress reduced by a factor of five, as compared to freestanding blades. However shrouding blades into packets also gives some more coupled modes to be concerned with as shown by Weaver and Prohl (1956) and many others. For example, the turbines on the Queen Elizabeth 2 ocean liner had failures due to higher modes for nonoptimized packet configurations. An out-of-phase mode in resonance with upstream vane passing frequency could again have the number of blades per packet optimized giving a different value than for the fundamental mode.

Some manufacturers seem to standardize with a number such as five blades per packet, but it is sometimes very important to optimize a given stage. For example, a high pressure, high temperature Curtis stage with 64 rotating blades and 71 inlet vanes should have eight blades per packet, not four per packet—if a fundamental mode is resonant with 71 times speed. If it is not resonant, it could be acceptable to use four blades per packet. Besides shrouding shorter blades to reduce response at high frequencies from inlet vane passing frequency, "long-arc" blading can be used to minimize response of fundamental modes (refer to Ortolano, et al., 1981; and McCloskey, 2002). In a low-pressure turbine stage, instead of using 24 packets with four blades in each, a six-per-revolution resonance could have six packets with 16 blades per packet around the circumference. APPENDIX B can be used to check that the response factor would be zero when using N equal to 6. If the shroud could not physically fit over the tangs during assembly but could fit with eight packets, the response would still have fairly low response.

*Friction Damping Methods*

Besides shrouding, part-span snubbers and lashing wire designs have greatly reduced failures in steam turbines. The patent on zigzag damping pins for long twisted blades has expired and has been used by many others. An excellent design, especially for shorter blades, and used for decades—is the integral-shroud with rolled-in lashing wire; a more recent design is shown in Figure 6 with improved tip sealing availability. In effect the design has many advantages with the integral shrouds acting as snubbers. They are assembled with zero or negligible gaps at the tips; the small gaps that occur at speed and temperature are usually small enough to limit motion especially in the flexible tangential direction. In addition, rolled-in lashing wire provides a large damping force where there is high relative motion between blade tips. Highly loaded blades can have two rolled-in lashing wires as shown in the figure, with the ends of each wire meeting at blades that are 180 degrees opposite.

Another trend today is to use more z-lock shrouds (Figure 7) for tapered, twisted blades, with friction damping at the tip contact surfaces, eliminating the need for rolled-in wire. The trend likely began with designs in the 1980s such as the power turbines for gas turbine derivatives. With a continuous shroud, the modes to consider are coupled modes where the blades vibrate with mode shapes defined by nodal diameters. Thus instead of individual blade modes, the most responsive disk-type critical speeds must be avoided as described later for disks. For example, the three-diameter mode of a design may have high stress if the frequency was equal to three times operating speed; for either of both sets of modes defined for motion in the predominant tangential and then axial direction. In Figure 8 is shown an interference diagram for a design where the first mode could have resonance for the five- or six-diameter mode for the first family of modes in the speed range. The second family of modes, mostly tangential, could have the nine- through 11-diameter mode in resonance. Thus it would not be wise to have high excitation sources at 5, 6, 9, 10, or 11 times speed. For some designs, the blades must be preloaded to ensure that under centrifugal growths and blade untwisting, there is

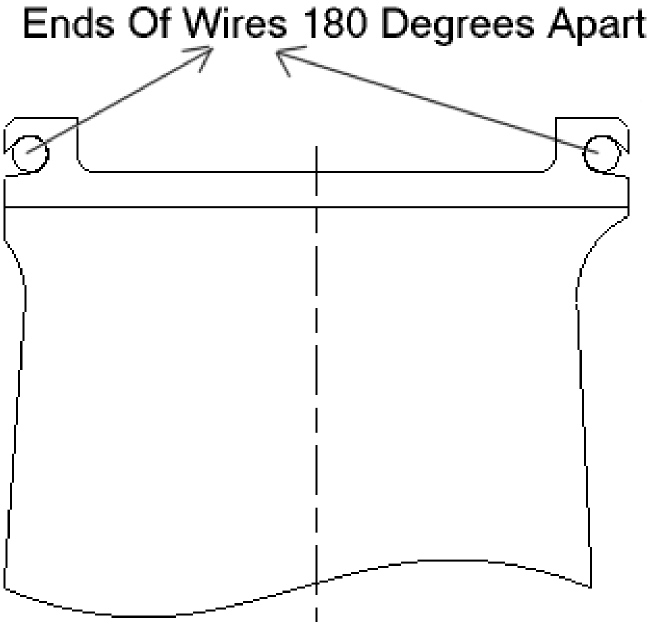


*Figure 6. Blade Integral Tip Shroud with Two Rolled-In Lashing Wires for Damping.*

sufficient contact for the surfaces, which in turn are also properly coated. Other advantages of z-lock shrouds are: permit ease of using tip seals, eliminate inherent aerodynamic losses of lashing wires and snubbers in the steam path, eliminate areas in open shrouds and lashing wire designs where deposits and corrosive medium can accumulate.

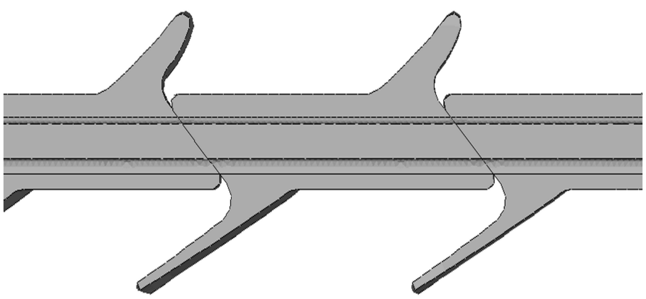

*Figure 7. Schematic of Turbine Blade Z-Lock Shrouds with Friction Damping.*

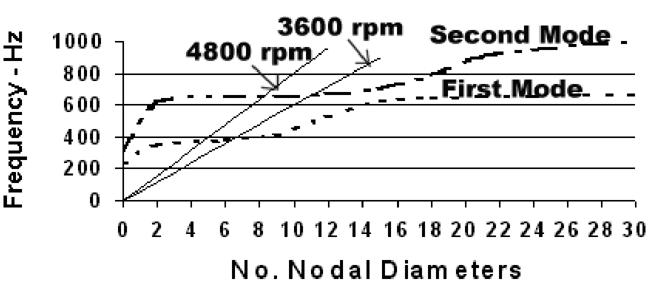


*Figure 8. Interference Diagram for Z-Lock Integral-Shroud Blade Design.*

## DISK MODE RESONANCE

### *Disk Critical Speeds*

The most destructive resonance is that for disk critical speeds as was found in the 1920s by Wilfred Campbell (1924). In fact some of those disks had axial rubs from the high amplitudes involved. Disks had basically become too thin in steam turbines, leading to fatigue failures. The fundamental, two-diameter mode shape is shown in Figure 9. As this disk rotates, at a certain speed each part of the disk will arrive at an area in phase with a stationary exciting force. As shown in the Campbell diagram (Figure 10) the resonant operating speed is equal to the natural frequency divided by the number of diametrical nodal lines. In Figure 10, the "backward wave" line, $|f_r - 2x|$, intersects the x-axis when the disk frequency is equal to two per revolution. Note that $f_r$, frequency at speed, includes effect of temperature, fluid mass loading, and centrifugal loading on stationary frequency, $f_s$. The diagram only shows one of the two frequencies for each "n" diameter mode due to dynamical imperfection (Tobias and Arnold, 1957), with nodal lines shifted by (90/n) degrees. Figure 11 has some data for a more recent impeller with a zoomed fast Fourier transform (FFT) spectrum analysis of a rap test. At speed, the two frequencies will likely get further apart as the cover rotates circumferentially with respect to the hub (refer to plots in Kushner, 1979).

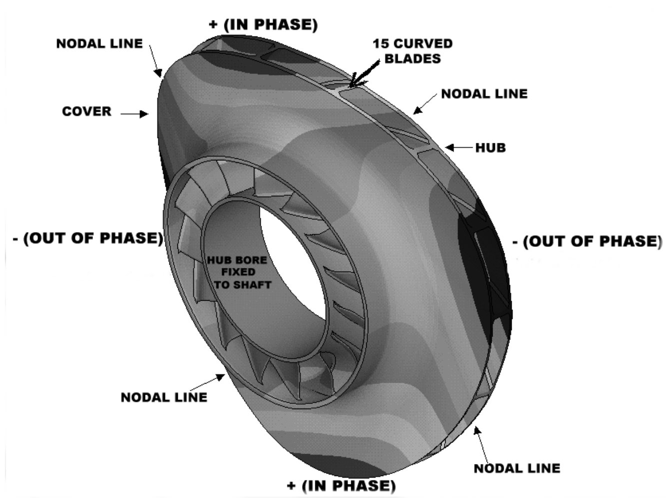


*Figure 9. Mode Shape of Two-Diameter Disk Mode for Covered Impeller.*

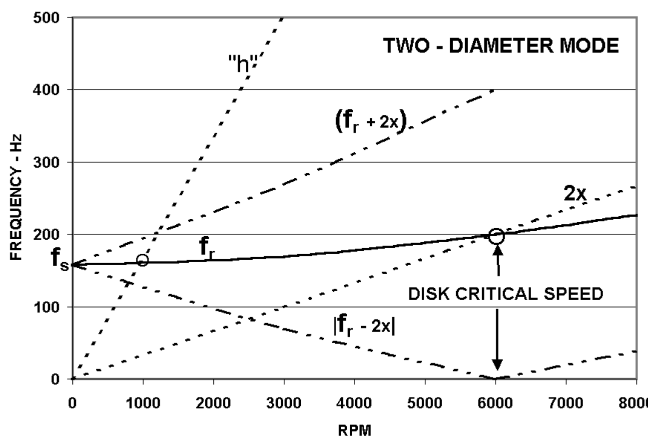


*Figure 10. Campbell Diagram for Disk Critical Speed—Fundamental Two-Diameter Mode.*

Nodal lines remain fixed on the rotating disk because there are two frequencies for each mode, with normal phase changes depending on damping in traversing each resonant peak. With a large enough difference between the two frequencies, resonance at each one of the peaks only has a small component of the other similar mode. A measurement of a rotating, vibrating disk from a particular stationary point would show two frequencies for both of the modes: the frequency measured by an accelerometer or strain gauge attached to the disk, plus and minus the number of nodal

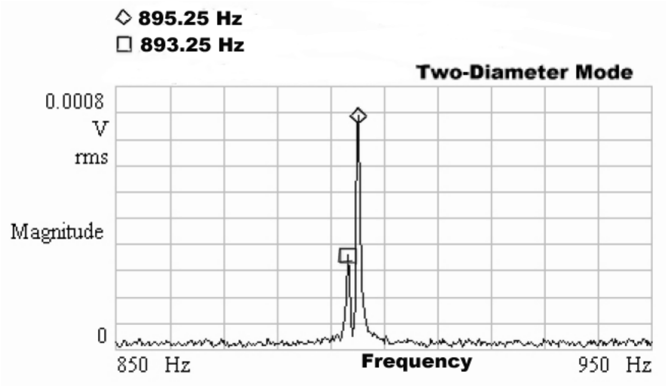


*Figure 11. Zoom Frequency Spectrum from Impact Test Showing Two Frequencies for Same Mode.*

diameters times rotating speed. At a disk critical speed, the lower frequency reaches zero; i.e., the average amplitude of the disk around the circumference in stationary coordinates would form a standing wave in stationary space. Thus even a local high-pressure nonuniformity is able to excite the disk; it is in phase with each point (and thus every blade) on the disk as the disk rotates and vibrates. At all other speeds, there would be phase cancellation over each revolution of the disk. There have not been many disk critical speed problems since the time-honored publication of Campbell (1924); however there must be continuous scrutiny.

*Rotating Blade/Stationary Vane Interaction Resonance*

For other resonant points away from disk critical speeds on Campbell diagrams, to determine whether forces would cancel for the entire disk at this speed, one must review the number of rotating blades interacting with the number of stationary elements. For almost all combinations, there is phase cancellation of forces. For a mode such as a five-diameter mode for a 15-bladed impeller shown in Figure 3, other than for stationary vane numbers of 10 or 20, exciting forces would cancel as the impeller spins and vibrates. Forces do not cancel if the natural frequency is equal to 10 times speed with 10 stationary vanes, or with either 10 or 20 vanes if the frequency is equal to 20 times speed. This is because either 10 or 20 vanes give a difference of five with 15, the number of rotating blades. The parametric equations from Kushner (1979) giving cases where there is not phase cancellation are reiterated as follows:

- Not at disk critical speeds:

$$\begin{aligned} &(a)\ \left|y \cdot S\right| \pm \left|z \cdot B\right| = n \\ &(b)\ y \cdot S = h \\ &(c)\ f_r = y \cdot S \cdot \omega \end{aligned} \qquad (1)$$

- At disk critical speeds:

$$\begin{aligned} &(a)\ For\ B > 1 \\ &(b)\ y \cdot S = h = n \\ &(c)\ f_r = n \cdot \omega \end{aligned} \qquad (2)$$

where:
B = Number of rotating blades
S = Number of stationary elements
$f_r$ = Natural frequency at speed, Hz
h = Harmonic of speed
n = Number of diameter nodal lines
y & z = Integers > 0
ω = Rotating speed, Hz

Note that the natural frequency at speed, $f_r$ is affected by centrifugal loads and in some cases by thermal, pressure, or fluid mass loading changes from ambient. Equation (2) is also given for the extreme excitation case of disk critical speeds, as then it does not matter how many blades are on the disk; each blade is in phase with the force. The extreme case for Equation (2) shows the added danger of having both a disk critical speed and the number of stationary elements equal to the number of nodal diameters. By contrast, interaction resonance has to have a certain combination of numbers besides having operation at a specific resonant speed. The highly beneficial recommended standard of not using equal numbers for blades and vanes is based on Equation (1). That is, when n = 0, circle and torsional modes are not excited unless y × S = z × B. Typically, only values of one and two are used in Equation (1) for integers, "y" and/or "z"; higher factors would give much lower excitation and/or response (refer to calculations in Kushner, 1979). In reviewing these equations, it must be considered that the loading is absorbed by the blade surfaces that transfer the steady-state loads. One of the least understood causes of disk vibration is acoustic interaction with acoustic pressure waves that directly excite the disk. The interaction of pressure waves between disk surfaces and the adjacent stationary wall may also be important, as shown by Eckert (1999) and Ni (1999); the instability of the two-diameter mode from acoustic interaction described could thus also give forced excitation with matching phase angles with the disk modes.

Equation (1b) is actually implied by (1a) and (1c); it is inserted to add emphasis that resonance is at a harmonic related to the number of stationary elements, not the number of rotating blades. This point is sometimes misunderstood, e.g., response for very flexible fan disks can show amplitude peaks at blade passing frequency on bearing housings due to resonance of one-circle and one-diameter modes (Baade, 1998). For impellers with splitter blades, both full or splitter blade numbers and their sums should be evaluated for excitation, as did NASA (1990). To date, the plus sign in Equation (1a) is not important for radial compressors; it results in high values of "n" diameters, difficult to excite, let alone find in testing.

Because of the use of higher values of numbers used and the high differences between numbers of blades and vanes, axial flow turbines typically have very low risk of blade/vane interaction resonance for disk modes. It should also be noted that any design that has a disk critical speed or interaction resonance below operating speed would have a very limited number of cycles while traversing the low speeds. Steady-state stress will be lower and reduced power giving much lower exciting loads will further limit risk.

## VIBRATION EXCITATION AND RESPONSE

*Effects of Mistuning on Blade Modes*

As was concluded by Slater, et al. (1999), "Detailed finite element analysis of tuned bladed assemblies can be prone to large errors." Mistuning, variations of blade natural frequencies in a rotating row, is the reason why there are often "rouge" blade failures (refer to cases below). For forced excitation Bladh, et al. (2002), show that mistuned responses can exceed tuned response levels by nearly 200 percent, if appropriate levels of mistuning and interblade coupling are present. Whitehead (1996) has also provided much research on the subject. There have been cases in the literature where coupled blade/disk modes have been discussed for short blades, using interaction resonance affects similar to those described above. However, for packets of relatively short blades, it is not recommended to use such interference diagrams as prescribed by Dello (1987) and Singh, et al. (1988 and 1994). Mistuning of blades as described in many publications surely prevents the occurrence of such nodal diameters for many but not all cases, in that some blades can be largely decoupled from the disk, especially for the predominant tangential direction. By not considering mistuning, this type of graphing procedure for shrouded packets of rotating blades can sometimes assert "safe" operation at resonant speeds of packeted blades. For short blades in packets, the natural frequencies for a particular mode can easily be a few percent different from packet to packet. With resonance at nozzle passing frequency, a nodal diameter pattern may actually not be important at all. There are inherent variations involved in manufacturing tolerances and assembly differences. Differences can also increase with time of operation due to corrosion, erosion or deposits, and in rare cases from foreign object damage (McCloskey, 2002). Thus each packet can be excited separately; e.g., resonant with nozzle passing frequency; geometry patterns that define response are numbers of blades per packet and ratio of nozzle to rotating blade numbers for phase cancellation (refer to APPENDIX B). In fact, just as for "rouge" cantilevered blades without shrouds there will be increased response of some packets, in effect acting as vibration absorbers for others (refer to analysis for shrouded blades in Bladh, 1999).

The interference/nodal diameter plotting method reviews optimum number of blades per packet primarily based on coupled blade/disk modes. However, the inference is that all blade packets will have the same frequency for the nodal diameter patterns, and does not consider individual packet modes. Reality is that there can be different frequencies for the same mode, including the fundamental, albeit close together; thus in some stages such as those with locking pieces instead of locking buckets, it is necessary to have different numbers of blades per packet that gives added differences. If there ever was a troublesome mode of an entire structure, i.e., blades all coupled through the disk that they are attached to, then the method of nodal diameters may have merit as has been well known. Plotting of frequency versus nodal diameter is a design technique used for decades to avoid disk critical speeds and interaction resonance, so the two techniques are similar for those cases. The method uses the same principles as explained above for Equation (1), except that it uses a graphing method that is applicable to completely bladed disks—in effect having one packet. Both methods are discussed by Wang, et al. (1999). The graphing method is also similar to that offered by Wildheim (1979) for modes dominated by the disk.

With long tapered, twisted blades with relatively flexible disks, the graphing method may be useful for disk-dominating modes. As explained by Wagner and Griffin, (1996a, 1996b), it is only the modes that have out-of-plane motion that are effectively coupled. For designs with very rigid disks or drum construction, individual packets of short, nontwisted blades, phase cancellation analysis is the method to use, especially for the tangential modes. The prime reason of having shrouds for reducing response is phase cancellation for individual packets, again tried and proven for decades, even prior to Weaver and Prohl (1956). Case No. 2 in Dello (1987) will be used for phase cancellation comparisons to minimize resonant response. The case assumes a stage in the turbine having 108 blades, 18 packets, and six blades per packet, with frequency of the fundamental tangential mode equal to 46 upstream nozzles times operating speed. In Figure 12 is the interference diagram for this case. The diagram implies that since the shape of the force with 46 nodal diameters does not match the shape for 18 packets, resonance near 15,000 rpm shown really does not occur at the point encircled. However, per equations in APPENDIX B, the optimum number would be either seven or 12 blades per packet. In Table 1 is a summary of resonant response factors. Since seven blades are not divisible into 108 blades, nine blades could be selected. If this choice gives assembly problems, then a combination of five and seven per packet to total 108 blades would be better than using 18, six-bladed packets. Note that with six blades per packet, response would be about three times higher than for nine per packet. The interference graphing method based on nodal diameters says a true resonance condition does not exist for 18 packets except for up to nine nodal diameters. The blades in a packet may not fail as the resonant response factor is still below 0.20, but resonance will exist and could indeed fail if loads were high enough, or if endurance strength becomes lower with time in operation. The graphing method does not give the result that it is best to use either nine blades per packet or a combination of five and seven blades per packet. A slight change in numbers could result in using an optimum common number of blades per packet for the stage using APPENDIX B. Thus this case shows that response is reduced based on the interference diagram graphing method. But it cannot be said that there will be "safe" operation at resonant speeds of the first in-phase modes with six blades per packet.

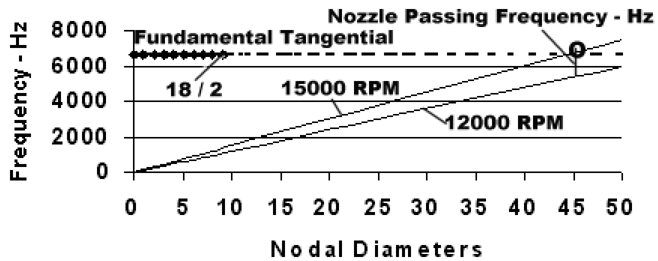


*Figure 12. Interference Diagram for First Mode with Limit of Nine Nodal Diameters.*

*Table 1. Fundamental Mode Resonant Response Factors, 108 Blades, 46 Nozzles.*

| Number of Blades Per Packet | Resonant Response Factor |
|---|---|
| 3 | 0.26 |
| 4 | 0.21 |
| 5 | 0.08 |
| **6** | **0.17** |
| 7 | 0.01 |
| 8 | 0.12 |
| 9 | 0.06 |

A similar example, Case 1 in Singh, et al. (1988), has the first three modes in resonance with nozzle passing frequency. Resonant response factor for six blades is very low, but it is not true that "…a true resonance will not occur." Reviewing that case, one could potentially select three blades per packet and the resonant response factor would be excessive, near 0.33, too high for resonance with nozzle passing frequency (refer to Table 2). For the number used in the example, six blades per packet, the "rocking mode" would also have a fairly high response factor.

*Table 2. Fundamental Mode Resonant Response Factors, 150 Blades, 78 Nozzles.*

| Number of Blades Per Packet | Resonant Response Factor |
|---|---|
| 3 | 0.33 |
| 4 | 0.06 |
| 5 | 0.19 |
| **6** | **0.06** |
| 7 | 0.13 |
| 8 | 0.06 |
| 9 | 0.09 |

Another case described in Singh, et al. (1994), was for higher-order axial modes of six-bladed packets, affected by deposits and corrosion from sodium and/or chlorine. Resonance was with the second harmonic of nozzle passing frequency (exciting force should only be about one-half as strong as those at nozzle passing frequency). The solution, changing the design and reducing governor speed range to remove most (but not all) resonant conditions, was likely only needed because of excessive corrosion, similar to one of the cases to be given below. Corrosion pits such as at the fatigue initiation site are not supposed to occur if contaminants are controlled within specified limits.

The first case in Dello (1987) has the same stage configuration as for the second discussed above, i.e., 108 blades and 46 nozzles, 18 packets, six blades per packet but the fundamental tangential mode is instead resonant with a low harmonic of speed, five per revolution. The interference graphing method based on nodal diameter patterns results in only recommending a change as long as there are less than nine groups of blades. An example is shown in Figure 13 for eight packets, the point shown at four nodal diameters (8 packets/2) is below resonance for five nodal diameters. The resonance still exists however for eight packets at the point encircled. Equations in APPENDIX B can be used with setting value for excitation source number, S = 5. By using combinations of 13 to 16 blades per packet to give either seven or eight groups, response factor would then be between 0.31 and 0.50, much better than freestanding blades value of 1.0. However by using the "long arc" method previously described, if the Goodman factor is very high, it would be optimum to have five groups of blades tied together, two with 21 and three with 22 blades per packet giving a resonant response factor near zero.

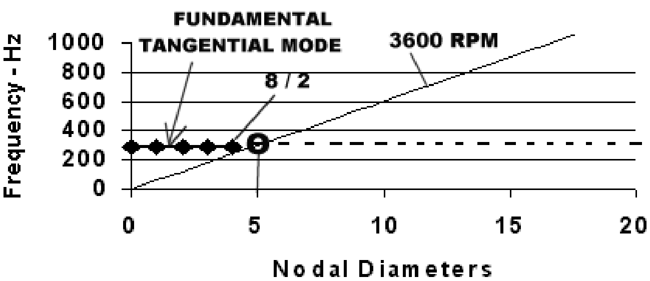


*Figure 13. Interference Diagram for Stage with 108 Blades: Two 15-Bladed Packets, Six 13-Bladed Packets.*

Other machines besides those with open impellers now use blisks, blades machined integrally with the disk. These typically have less damping, especially for higher modes. Mistuning of blades is also much lower in blisks than for fabricated assemblies that have larger differences in geometry; thus flutter could be a larger concern, but "rouge" blade resonance amplitude increase due to mistuning would be of less concern.

For any design, the importance of damping cannot be under emphasized as: forced response vibration amplitude = Σ excitation forces/Σ aerodynamic and mechanical damping. As the resulting amplitude can also be correlated with dynamic stress, the resulting equation that can be used is:

$$\sigma_v = \sigma_s \cdot S \cdot R \cdot (\pi / \delta) \tag{3}$$

where:

$\sigma_v$ = Vibratory stress
$\sigma_s$ = Steady-state gas bending stress
S = Stimulus factor for the harmonic at resonance
R = Response factor
$(\pi/\delta)$ = Amplification factor, where
$\delta$ = Damping log decrement

In effect, in Equation (3), the response is a function of a resonant response factor times a stimulus factor for the gas bending forces in turn multiplied by the amplification factor that is determined by available damping. The response factor, R is both affected by the mode shape and response to the force based on phase angle. Thus for an unshrouded cantilevered blade, R would be close to 1.0, while an optimized packet of blades can usually be reduced to at least 0.2 for the fundamental modes (refer to APPENDIX B). Another way to effectively use this method for vibratory stress values that are acceptable is to use empirical Goodman factors based on experience.

For blade flutter avoidance: Σ aerodynamic and mechanical damping must be > zero. Empirical factors are utilized for reduced frequency factors, in effect limiting how flexible the blades can be, i.e., limiting the fundamental modes to a minimum value for different machinery types. In addition operation must have guidelines, e.g., for surge and choke for axial compressors, and exhaust pressure limits for steam turbine exhaust ends. Precise analytical methods incorporating computational fluid dynamics (CFD) methods are still in development; in fact, especially for transonic stages, there are still issues relative to inclusion of complete viscous and turbulence models even for steady-state conditions. Flutter aeroelastic calculations depend on the solution of unsteady nonlinear fluid mechanics, a Navier-Stokes solver, a solver for eigenvalues with structural equations, as well as a transient structural dynamic solver. Thus both dynamic mesh algorithms for the fluid mechanics and structural interaction are required. Nowinski and Panovsky (2000) and Panovsky and Kielb (2000) provide analysis and experimental data for low-pressure turbine stages for jet engines that once again verify that mistuning of blades greatly assists in preventing flutter. However, mistuning used to assist in guarding against flutter such as in steam turbine fixed-speed units will reduce the separation margins for any tuning for resonance avoidance. The authors also show how movement of the torsional axis from forward to aft position of the profile can greatly increase the propensity of flutter for the torsional mode.

Sources of blade damping are:

- Aerodynamic (viscous, normally positive but can become negative at poor incidence angles).
- Friction (lashing wire, midvane snubbers, platform dampers, root/disk land interface).
- Hysteretic (material)—significant for many of the 12 percent chromium, martensitic steel alloys, small for most other steel alloys, and negligible for titanium.

Mean stress can have a significant effect to lower material damping, but still is often used for added conservatism, especially for mechanical drives where highly responsive modes cannot be tuned as can be done for units only loaded at one rotating speed. Total system damping is one of the most difficult aspects of Equation (3). Values in Table 3 from Kielb (2001) are for materials with very low hysteretic damping and show a wide disparity.

*Table 3. Amplification Factors from Jet Engine Strain Gauge Data.*

| Mode | Total Q |
|---|---|
| Fundamental Bending | 30-130 |
| Second and Third Bending | 80-100 |
| First and Second Torsional | 20-190 |
| Stripe Modes (Plate-type modes) | 60-150 |

All cases in discussions below involved rotating component failures. Stationary vanes only have gas loads and potential thermal effects to give steady-state stresses. In centrifugal compressors inlet and return channel vanes have low excitation forces; in axial machines that use 12 percent chromium steel, material damping is also higher as compared to other machinery with other steel alloys. Without centrifugal loads, there typically is much lower mean stress, thus higher damping values as compared to rotating blades. Many vanes, especially in steam turbines and the high-pressure sections of industrial axial compressors are also shrouded, giving reduced compliance with forced excitation as well as having inherent phase cancellation. Resonance of stationary vanes is much less likely but can occur and must sometimes be scrutinized, such as:

- Avoidance of resonance with rotor blade passing frequency of cantilevered diffuser vanes immediately downstream of rotating blades in a centrifugal design, and
- Rotating stall with multiple stall cells in compressors.

CFD analysis can be utilized to predict magnitudes of excitation forces. As to whether the resulting excitation forces are important for a given design, a Fourier analysis can be performed such as given in APPENDIX C. Excitation can come from non-uniform flow and pressure due to construction geometry besides the inherent wake effects from adjacent vanes. An example would be harmonic excitation at multiples of speed from poorly designed inlets or exhausts. The force distribution can be separated into pulse trains for the analysis. As speeds increase, shock waves become a consideration for normal operation, besides having potential for transient conditions or those inadvertently outside operating performance maps. Differences in manufacturing such as variations in upstream throat areas between vanes can give significant excitation, as can erosion and corrosion differences circumferentially. For compressors, aerodynamic sources also can include rotating stall and surge at one end of the operation map and choke flow on the other. Turbines are inherently more stable, but also more prone to have changes to both excitation forces and endurance strength due to erosion, corrosion, and deposits. In addition, partial admission stages give a rich signature for many harmonics of speed. In effect, as a blade enters the arc it can be in phase, vibrate a number of cycles, and then enter the arc in phase for the next revolution, i.e., in exact harmonic resonance. The higher the number of cycles of vibration per revolution, the lower the response as there are more cycles having a decay of amplitude via damping mechanisms. Partial admission can also aggravate the excitation from the worst offender, wake-passing frequency. The predominant source of excitation for short blades is passing frequency, number of upstream wakes per revolution. Downstream wakes are usually of little concern, along with most cases of difference frequencies between upstream and downstream vanes. Passing frequency is based on the spacing between wakes; Fourier analysis can be utilized for nonuniform spacing of nozzles and vanes.

## CASE STUDIES

It is important to acknowledge that there is in an optimum to everything and risk taking is necessary to reach that goal. As part of the risk taking required to compete, for bladed disk resonance, the ramifications of a fracture for a customer can be tremendous. Thus the author at his former company recommended fairly conservative approaches that will be discussed for the various types of machinery to be reviewed. For example, others have taken more risk employing many more freestanding blades. In reviewing field problems and field failures, for original design and/or machinery operation, the optimum may not always be achieved. A large part of the improvement process is to learn from a limited number of mistakes. These case studies and accompanying references hopefully will assist the reader. A reason for achieving success was given in a January 2004 quotation from Lt. Col. Keith "Sully" Sullivan. Per the director of USA Central Command Joint Search and Rescue Center/Saudi Arabia: "…but sometimes unless you do something wrong, it's hard to make changes, and we did a lot of things right."

### *Axial Compressors*

Problems with blade resonance for industrial axial compressors should be less than with counterpart machines, axial turbines. That is, risk is only less if design and operation minimizes rotating stall, and of course a proper surge control system is used to eliminate heavy repetitive surging. Continuous operation at extremely high flow producing stage choking is also taboo (refer to typical performance map in Gresh, 2001). There are several prime design issues:

- Ensure that first stage blades avoid very low harmonic excitation—dictated not only by loading (Goodman factor), but by how well the inlet design minimizes circumferential flow distortion;
- Avoid resonance of fundamental modes with upstream vane passing frequency;
- Aerodynamic design must be free of rotating stall at all operating points including those with variable guide vane settings.

Others in the past have described failures with a poor inlet design without proper straightening vanes, followed by adjustable prewhirl vanes. These vanes assist in minimizing nonuniform flow—the prewhirl vanes also provide flow equalization besides giving adjustable flow angles required by the first rotating row. Proper operation requires that variable vanes be closed at startup to eliminate rotating stall. By retaining subsonic designs, conservatism is ensured; transonic designs are much more demanding as relative tip speed increases. Modern designs have more need to have design procedure to avoid blade flutter of first stage blades, high-pressure blades in choke, and acoustic resonance interaction.

Instead of old National Advisory Committee for Aeronautics (NACA) circular-arc blades, using controlled diffusion airfoils (CDA) blades that are actually designed for transonic conditions greatly assists in not exceeding limits. Improvements by using CDA blades are described by Kilgore (1987). For high flow operation, a design with dewhirl stationary vanes in the discharge casing can be used to guard against cases where process system resistance is such that choking can be within the compressor. For industrial designs, acoustic resonance should be easily avoided with checks as in Parker and Stoneman (1985).

Mazzawy (1980) describes some surge data on gas turbine compressors, still and likely always taboo for jet engines. Issues including measurement of rotating stall are included in Kushner (1996) where there is pressure pulsation data for both surge and stall on a fairly recent axial compressor. In fact the use of dynamic pressure probes:

- Eliminated the need for surging at all in setting the surge controls, and
- Verified a proper design without any rotating stall at much lower flows than the surge-control set points over the complete range of vane settings at full speed.

#### *Case A-1*

Turbocompressors with axial compressors are used to pressurize the boilers of ships, including seven US Navy Wasp-class carriers. The first of many highly successful units had a fatigue crack found on the first stage rotor blades following initial full load testing to confirm performance of the longer blading. Analysis of wake excitation as a percentage of steady-state loads verified that it was severe enough to cause the failure even though the struts supporting the inlet casing were over one chord length removed from the rotor blades. A simple fix was to increase the number of struts in the casing that had previously been used with a rotor with smaller blades. The lesson learned was that using an existing part from an older model must be reviewed for interaction with components in the new design. The change was quickly implemented for the first and 27 subsequent blowers running with no field problems.

#### *Case A-2*

In the 1980s, several rotating blade and variable guide vane failures occurred on a 12-stage axial compressor in FCC service. There was no corrosion issue from contaminated open-inlet atmosphere as has occurred in other plants, requiring coating of some of the front-end stages. Extensive tests were done, including a final test using casing-mounted dynamic pressure probes and strain gauges on several rows of blades. The cause was proven to be excessive surging although the rotating blade roots were not initially optimized and were eventually redesigned, giving an even higher safety factor. The inherent process problem was that the blowoff valve was so undersized that a second blowoff valve had to be added. Initial, continuous, and excessive surging also damaged the linkage for the variable guide vanes, and actually yielded the casing due to transient thermal stress. It was argued that by having skewed roots, adding to steady-state stress, the endurance strength was compromised.

The user and their consultant were insisting on using high mean stress due to the skewed roots as a limiting factor using a modified Goodman or Soderberg diagram. It was argued then and is often exemplified such as by Wang, et al. (2000), that ductile alloys do not always require extreme conservatism. This statement applies to defect-free specimens, unlike the problems with small initial cracks or defects. In addition, there was actual test data for the AISI 403 material showing that actual fatigue limits were higher than the modified Goodman values. The stress concentration was partly due to geometry and partly due to loading effects, with only the latter required to be applied to the mean stress. It was concluded that the blades would have still failed with the redesigned blades if excessive surging had continued. In addition, the blades would not have failed without excessive surging as shown by similar compressors with higher relative loads and identical resonant points, but were not changed and have exceeded many years of operation. The field operations group refused to actually surge the compressor during final tests with strain gauges; but data verified that there was not rotating stall or damaging acoustic pressure pulsation frequencies up to the surge control point. In addition it was verified that excitation at exact resonance points for low speed harmonics gave acceptable stresses—even for the original skewed roots. Speed was changed extremely slowly to obtain maximum peak amplitudes. After surging was eliminated, occasional failures occurred whenever some of the variable linkage failed (also later improved), giving nonuniform flow impulses. The fundamental mode of the first rotating row was close to resonance with four per revolution. As the train was a mechanical drive with a fairly wide speed range, it was impossible to tune all the blades from low harmonic excitation. If the train had been a constant speed unit, proper separation margin could have been feasible for the first four stages but would have required each blade to be accurately tuned. As the harmonic number increases, it becomes exceedingly more difficult to tune even the fundamental mode where there is some speed change with load. Tests were done on all the blades for the first four rows using a fixture. The fixture had mating rotor drum lands such that a contact force at the blade root of approximately 50,000 lb could be exerted using the hydraulic cylinder to equal centrifugal force at speed. The data were used to verify that the first row indeed avoided resonance of the fundamental mode with four per rev, while the next three rows had higher harmonic resonance. Note that variation in fundamental mode frequencies from tolerances was near ±1 percent.

In addition to the concern for surge and low harmonic resonance, dynamic pressure probe data showed there was resonance of acoustic modes of the air stream within the compressor as described by Parker and Stoneman (1985). Description of similar acoustic natural frequencies for analysis of piping noise and interaction with structural modes are explained by Kushner, et al. (2002). In this case the acoustic waves are altered by the presence of the vanes and blades downstream of the inlet. However it was shown that relative frequencies based on the number of nodal diameters of the modes would be well above the blades' fundamental modes as they traversed through the standing acoustic wave. Exciting frequency relative to the rotating blades is the standing wave frequency plus and minus the number of nodal diameters times speed for each of the acoustic modes. Strain gauge tests also confirmed that there were no responsive acoustic resonance peaks in the data. Using one-fifth scale-model testing of the inlet casing, it was found that the inlet struts excited the acoustic modes. Benefits to all the testing were that it verified the excellent steady-state flow characteristics of the inlet. A method was found and then verified to alleviate the high frequency noise by treating the trailing edges of the struts to prevent the vortices from each strut to communicate with each other. The case proved once again a proper surge control system is mandatory for axial compressors; in addition, rotor blade redesign and optimization of root/rim interface resulted in minimal increased cost for the increase in reliability. The strut treatment method greatly reduced ambient noise in subsequent compressors. Many lessons were learned by the vendor and user; in addition, the contractor had a lesson in design of critical surge controls for axial compressors.

*Centrifugal Compressors*

For the axial compressor in the previous case study, the rotor was of drum construction, so disk modes were a moot point. In many other axial compressors that have disks, disk critical speeds can be an issue; some lightweight engines have thin disks and must run up and down through disk resonance points. By contrast, industrial centrifugal compressors almost always avoid disk critical speeds.

*Case B-1*

A unit that did not avoid disk critical conditions was a plant air compressor as detailed by Kushner (2000). In Figure 14 is shown a comparison of the initial and final design. FEA analysis and modal testing showed proper thickening and tapering of the disk hub solved the problem. It requires fairly thin impellers to have disk critical speeds, even for open impellers. Both modal testing and FEA analysis for an unshrouded, plant-air compressor impeller was required to eliminate the four-diameter mode critical speed.

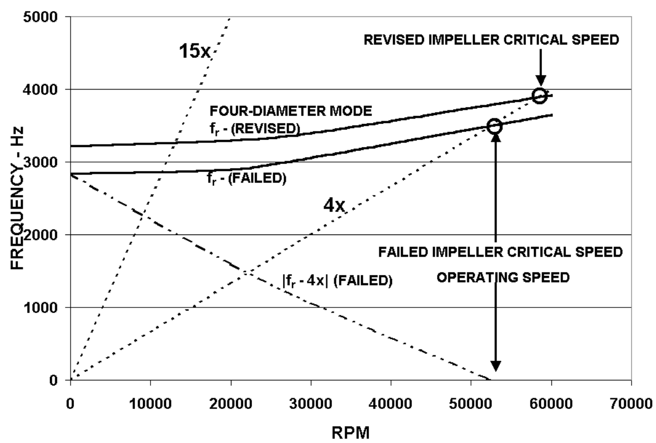


*Figure 14. Campbell Diagram for Open Impeller, Removing Disk Critical Speed.*

Use of four inlet vanes or struts would be especially disconcerting, as would be the second harmonic of two stationary elements equally spaced. It does not matter how many blades would be on the disk; they are the elements that transfer unsteady forces to the disk, just as they absorb steady loads. For this case, four vanes were not used; the number was 15. At this disk critical speed, each blade is in phase with the effective sinusoidal pattern at four per revolution. If the four-diameter mode frequency were resonant with five per revolution, the effective forces would cancel out in each revolution. For the actual impeller, the only stationary vanes were in the diffuser, 15 in number with no critical interaction resonance above 12,000 rpm. It was concluded that nonuniform flow from the downstream volute had sufficient Fourier component for fourth engine order to cause cracking at the periphery of the disk. The solution was to modify disk thickness and to increase the taper, accommodating "flow cuts" both for the disk and blade profiles. The revised impeller operates safely below disk critical speeds. Open impellers could have more risk for disk critical resonance as shown in Figure 15. Other research with blade failures for open impellers such as during the series of laboratory tests by Haupt, et al. (1985), to Jin, et al. (1995), may primarily be due to excessively thin sections for both blades and disks, and use of aluminum as compared to commercial designs.

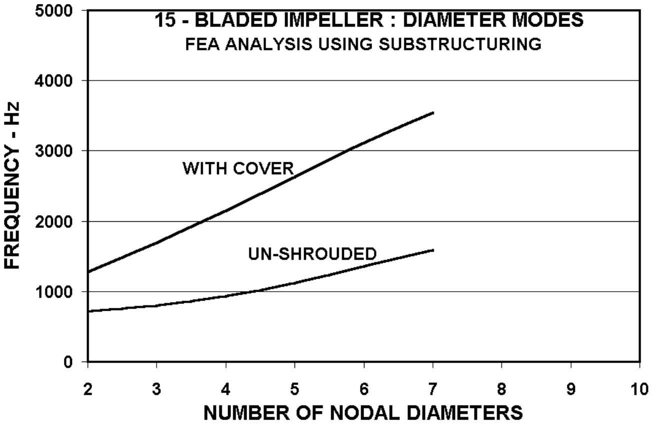


*Figure 15. Comparison of Diametral Mode Frequencies.*

*Case B-2*

Besides more concern for disk critical speed, open impellers are more prone to damage from surging and rotating stall. The case of a failure just after startup given by Kushner (1996) was a definite case of continuous surging, albeit milder than in a multistage compressor, where a measurement is shown in Figure 16. However the air compressor was highly loaded and there was extremely high nonuniform flow excitation from the volute while approaching surge. (Figure 17). Thus the lower harmonics of speed gave resonance at four per rev, adding to the impulse loading from surge itself. Analysis and data showed that the volute was sized properly. A poor discharge volute can give high harmonic excitation from nonuniform circumferential flow. It was fortuitous that rotor vibration data were being tape-recorded and thus verified numerous surge cycles, perhaps for hours. Continuous service has been achieved over many years with the special zigzag damping pins between blades described in the reference for even greater reliability in case of excessive surging. An improved aerodynamic impeller with 3-D curved blades (Aungier, 2000) offset the inherent pressure losses due to use of pins in the air stream.

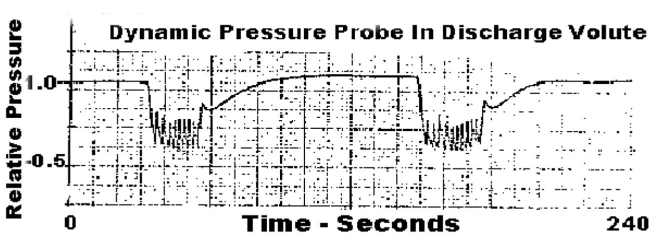


*Figure 16. Multiple Surge Pressure Pulsations in Discharge Volute; 30,000 HP, Four-Stage Centrifugal Compressor.*

*Case B-3*

The damage to many impellers in four identical refrigeration compressors was due to interaction resonance accompanied with liquid ingestion as described by Kushner (2000) and Kushner, et al. (2000). Similar liquid ingestion is likely the reason for many gas compressor failures throughout history. Based on experience with many similar geometries, relative speeds, and loads, it is nearly impossible that the impeller would fail unless liquids aggravated the wakes. First of all, excitation was from upstream vanes, not the much more severe force from diffuser vanes at the impeller tip (the tip of the impeller is also much more compliant to exciting forces than at the eye—especially for higher modes). Secondly, it was the

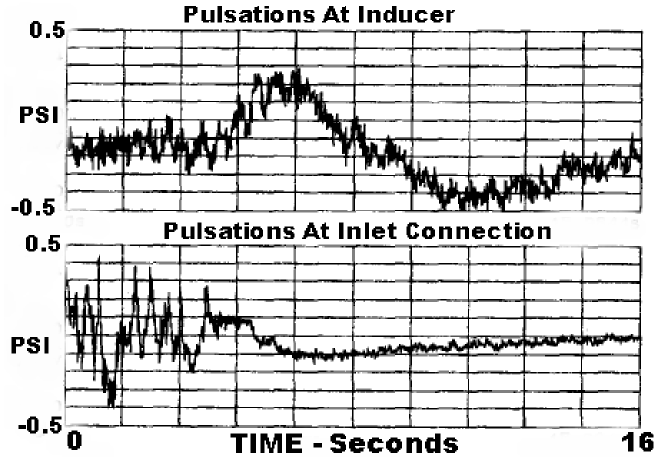


*Figure 17. Pressure Fluctuations Just Before Compressor Surge for Open Impeller.*

second harmonic of vane passing (32 times speed), not the first harmonic of vane passing (16 times speed), interacting with 27 impeller blades, exciting the five-diameter mode. This resonance is shown in Figure 18; similar interaction resonance is a common occurrence with numerous covered impellers with dry gas throughout the world. Even with liquid ingestion, the more responsive three-diameter mode with resonance point shown in Figure 19 did not respond, conclusively verifying the interaction resonance equations given above. Strain gauge data at speed and pressure showed little effect of mass loading on impeller natural frequencies; effects such as the extremely large reductions due to fluid mass for high pressure compressors (Gill, et al., 1999) are likely a function of design variations.

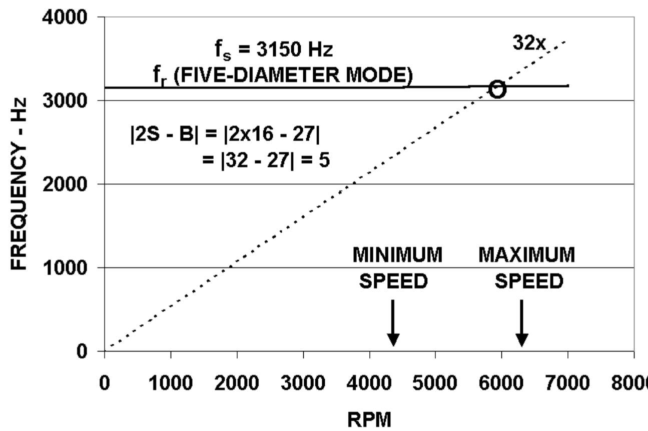


*Figure 18. Campbell Diagram for Mode with Interaction Resonance, Impeller with Excessive Liquid Ingestion.*

For a similar design without liquid ingestion, a two-diameter mode could be questioned for dry gas, but two-diameter mode interaction can be avoided by ensuring an even number for the inlet vanes and an odd number for the rotating blades. Analysis for this case gave the impetus for what is believed the industry's first publication on the subject, Kushner (1979), later backed up by Jay, et al. (1983), followed by many others such as NASA (1990). The same year, Wildheim (1979) published a similar equation to Equation (1). Previous pioneering research into disk modes such as by Ewins (1973) may have included the same discovery in the jet engine industry that had much more proprietary methods prior to engine companies forming and working within consortia. For axial flow jet engines, however, due to much higher blade numbers and differences between numbers, Equation (1) may not be required as is for radial designs. The parametric Equation (1) could have been

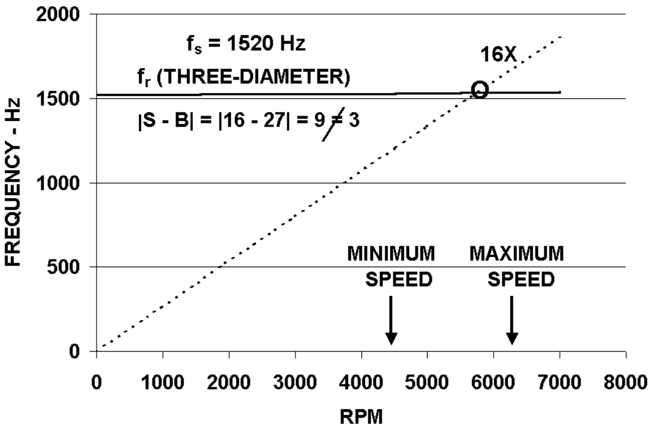


*Figure 19. Campbell Diagram for Mode Without Interaction Resonance, Impeller with Excessive Liquid Ingestion.*

useful for many previous impeller failures such as those unexplained in Bultzo (1975). The equations are sometimes still not applied properly as described by Kushner (2000).

*Case B-4*

Not explained by interaction resonance are some covered impeller failures that occur near the hub and/or at or near the tip. Many cracks initiate at the toe of the blade welds. It is possible for an impeller to actually have an unstable response for the fundamental two-diameter mode as shown by Kushner (2000). Note that in this case operation of the same compressor as Case B-3 had an inherent overload operating condition with an erroneous opening of the recycle valve at high flow. Impeller fatigue cracks initiated at the eye where testing showed maximum values for the two-diameter mode. (FEA analysis was in its infancy in the 1970s.) Liquids that even reached the last stage could also have been associated with heavy gases in the mixture causing a super-condensation effect, such as described for ethylene plant reactors (Parkinson, et al., 1999). Centrifugal compressors are easier to design from the stability standpoint as they do not have steep head rise to surge characteristics as do axial compressors affecting aerodynamic damping. One would look at the structure of an impeller compared to a single long axial compressor blade and never surmise that the entire disk would flutter.

The responding two-diameter mode was shown to respond only with heavy liquid ingestion; transient response reaching the endurance limit was proven by strain gauges. The two-diameter mode is the fundamental mode where vibratory moments are balanced within the disk. The mode had no interaction resonance and the transient response was not at any harmonic of speed. In addition for gas processes, system resistance normally prevents severe impeller overload at high flow. Compressors in parallel and other operations with reduced system resistance, including startup, where an impeller can actually choke should be avoided for significant time periods accumulating high number of cycles. This is especially true if there is likelihood of liquid ingestion, eventually eliminated to solve Cases B-3 and B-4.

*Case B-5*

Following a rerate, a large blast furnace air blower suffered several fatigue failures after many years of service. Instead of the reason given by Phillips, et al. (2003), other data showed that the most likely cause was operation in deep choke when the blowoff valve was opened at high flow. It was much more important for the operators to protect the newly installed furnace than the spared compressor. Granted the impeller had resonance of a leading edge mode with two times inlet vane passing frequency, but this resonance is a common fact in the history of compressors. As this case does not involve liquids, it is perhaps similar to that described by Borer, et al. (1997), for overloading a compressor when a parallel unit tripped offline, driving the running unit into deep choke. The immediate solution that was suggested for the blast furnace blower was to improve controls for the turbine. It was then easier to immediately slow down the string when the "wind" was not needed for the furnaces, i.e., not just open the blowoff valves. A mitigating factor was that a larger blowoff valve also had been installed to protect the larger furnace under emergency shutoff of its air supply. Thus the compressor would have been driven deeper into choke than before the rerate; the stage that choked first may also have moved due to the rerate to a stage that was less resistant. Fatigue cracks likely initiated due to overload condition, then easily propagated at the resonant speed where twice inlet passing frequency was equal to the blade leading-edge frequency. Besides the turbine control change, it was suggested to increase discharge piping system resistance with a longer-range goal of increasing strength and natural frequencies of impellers and blades. After reporting on probable cause using dynamic pressure probes near the impeller, the operators refused to intentionally test off-design curve at full surge or at the overload condition.

*Case B-6*

An impeller that failed in its first day of operation, described by Kushner, et al. (2000), was also due to liquids and operating in overload due to erroneous instrumentation. There was no prime interaction resonance so thus the cracks near the impeller tip were deemed to be due from resonance with inlet vanes of a complex plate mode. A small section of the impeller hub actually tore off, giving high rotor vibration initiating a trip. After scrutiny of the mode, there was a component of a three-diameter pattern shown in Figure 20, and the difference between 18 inlet vanes and 15 rotating blades was equal to three, satisfying Equation (1). This along with likely shock waves and liquids aggravating the vane wakes helps to explain why the mode responded to give high alternating stress.

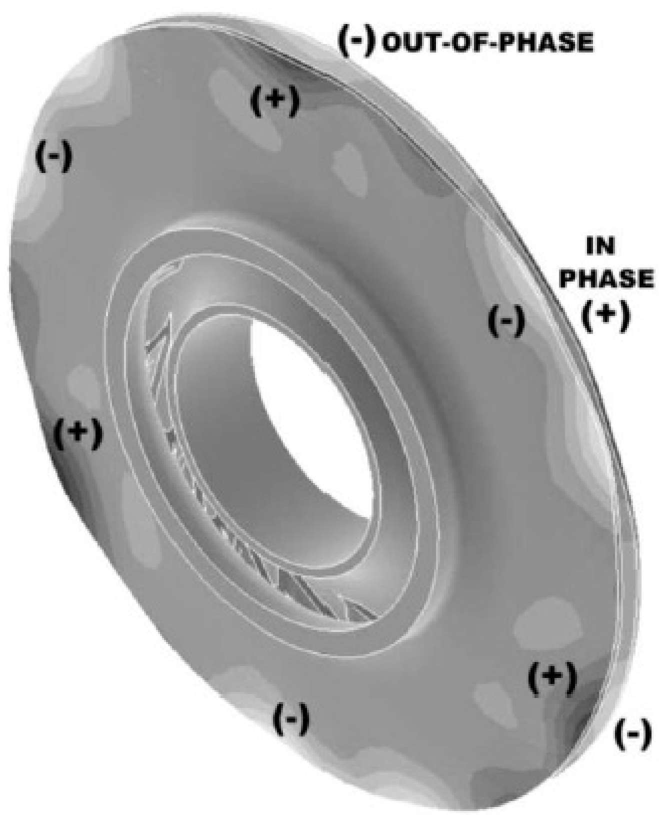


*Figure 20. Mode Shape for Impeller Higher-Order Plate Mode, Having a Three-Diameter Pattern.*

*Case B-7*

Impeller plate modes sometimes are involved in other failures that do not have the same intense analysis as Case B-5; it sometimes is decided it is best to make a change with the best engineering judgement. Experiencing recurring failures with many years between failures is also sometimes a reason to have a quick fix. Some customers just opt to repair the rotor for spared units, as has one customer with two units in parallel. One method to make a change in the past is to scallop out sections of the impeller and cover between blades (aerodynamic effects are small). The method did not work for Case B-3; scalloped impellers still had interaction resonance and liquid wakes were still present. For this case there was no interaction resonance, so the modification shown in Figure 21 was made.

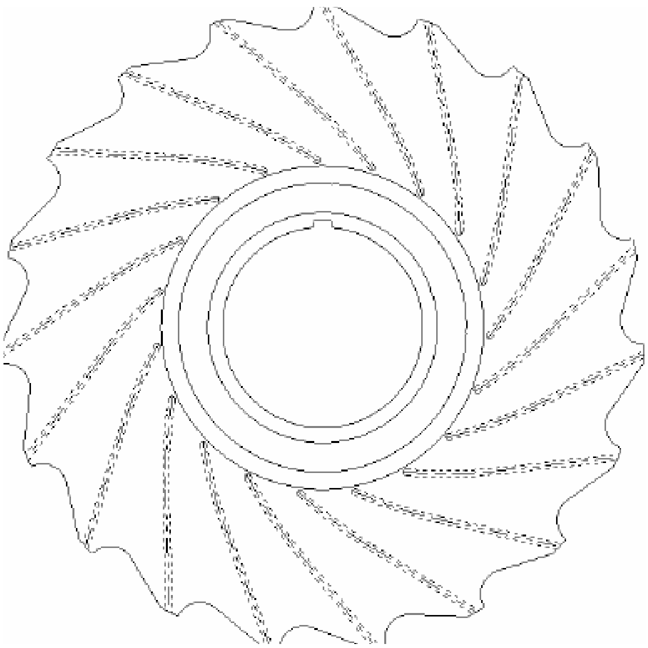

*Figure 21. Covered Impeller Tip Modification, Scalloped Cover and Hub.*

*Case B-8*

High flow compressors employing covered impellers have the most risk of resonance with blade leading edge modes. For the highest flow impellers, potential resonance was found for a new unit with the inlet already designed and manufactured. It was determined that resonance would be avoided as it occurred well below operating speed, so that there would only be occasional ramps up through the resonant point. A check was made that the lower speed would not be used during performance testing, often done with a different gas and running speed, but at equivalent aerodynamic tip speed.

*Turbines*

Radial-inflow turbines with unshrouded impellers have similar concerns to compressors, but inlet vane excitation can be worse as to excitation of disk modes due to their location - near the compliant impeller tip. For steam turbines, as related in a lecture by the expert Den Hartog (1954), a golden rule for long blades is never to have to have the fundamental mode equal to two per rev, especially for turbine generator sets with severe electrical impulses. There have been catastrophic failures due to transient torsional excitation at twice line frequency; in effect the torsional vibration of the entire rotor is exciting the base of the blades as there were on a shaker table. Otherwise the other engineering standards for long blades (and others) are made by a myriad of individual vendors with many designs, shrouding, and friction damping methods (refer for example to Sohre, 1975). Most designs still use 12 percent chromium steels, as there is some benefit of hysteretic damping as shown in Figure 22, although steady-state (mean) stress reduces values as shown in Figure 23. Some designs are highly scrutinized so that materials such as titanium and other variations of corrosion-resistant steels can be utilized in a specific design or modification for a problem such as those in the transition zone experiencing corrosion fatigue. The risk can be higher for some variables however so that care must be used, such as avoiding flutter conditions where material damping adds to the positive side of the equation. Recently there have been some examples where blade lives have "only" been around 20 years for some long back-end blades employing zigzag damping pins shown in Figure 5. In some cases the most economical solutions would be to add more conservatism by employing z-lock shrouds or integral shrouds with rolled-in lashing wire.

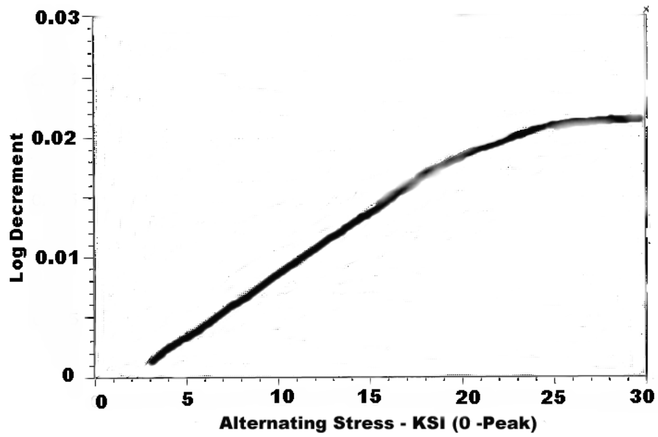


*Figure 22. Typical Hysteretic Damping, AISI 403, Cantilever Beam in Bending.*

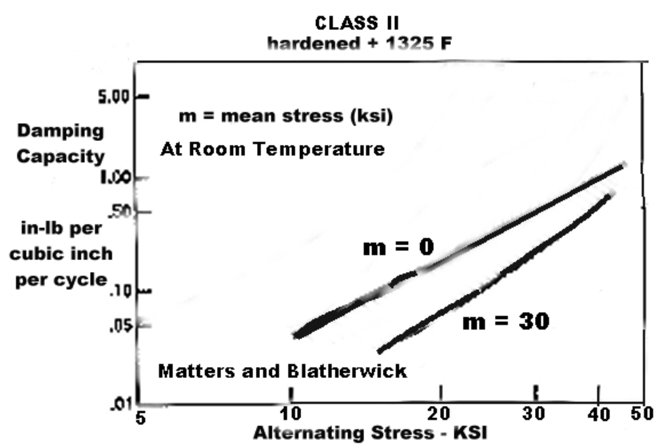


*Figure 23. Damping Capacity of High-Strength 12 Percent Chromium Steel Versus Alternating Stress.*

*Case C-1*

A turbine that had reduced endurance strength in the transition zone suffered a failure with an axial mode having first order resonance with special inlet nozzle vanes used to add rigidity to the diaphragm. Failed blades were at the ends of the packets, a sign that an out-of-phase mode was most likely involved. This is not always true as the ends of the packets have somewhat higher steady-state stresses, so that another mode could have the end blades above the endurance limit. Many years earlier there had been one case for an even shorter Curtis stage blade that was suspected to be an axial mode resonance; it never was proven but the packeted design was successfully changed to the more forgiving design of mating integral shrouds with rolled-in lashing wire. There was not a corrosion issue for the Curtis stage as there was for this case. Both "stress corrosion" and "corrosion fatigue" can become a contentious issue for steam turbines whenever there is a field failure for a stage in the transition zone. Some of these incidents are likely the reason the API 612 (2003) committee used such an impossible hard line for the Fifth Edition with respect to resonance. The higher modes with phase changes along the blade are safe even with resonance as response is typically negligible. It is also impossible to avoid resonance with all harmonics up to 15 times speed (no relief is given in the current edition by showing acceptable stresses as was in the Fourth Edition).

It is known that corrosion fatigue is a special case of stress corrosion caused by the combined effects of cyclic stress and corrosion (refer to McCloskey, 2002). Turbine metals are not immune from some reduction of its resistance to cyclic stressing if the metal is in a corrosive environment. By far the highest propensity is where the water droplets first form, at the transition stage. Water, metal, and the presence of a corrosive medium such as chlorides are the three required elements. The first two are a given, the third—the corrosive medium—is almost always the argument, e.g., makeup water treatment of high quality as specified, or another source of contamination. In this case, the pitting was just starting; however, corrosion fatigue is often encountered not as a visible degradation of the metal but as a premature failure of a component under cyclic conditions.

The excitation and offending vibratory mode still should not have been a problem for the small amount of corrosion; it was finally determined that there was also transient water ingestion into the stage, aggravating the wakes. Damage from corrosion fatigue is greater than the sum of the damage from both cyclic stresses and corrosion. Fatigue corrosion failure occurs in two stages. First the combined action of corrosion and cyclic stresses damages the metal by pitting and possibly small crack formation; then endurance strength and stress risers exist to such a degree that fracture by cyclic stressing ultimately occurs. The second stage is essentially the fatigue stage in which failure proceeds by propagation of the crack and is controlled primarily by stress concentration effects and the physical properties of the metal. Even if the corrosive medium is completely removed, it can be too late to avoid failure with the same excitation. The transient water ingestion likely caused the final fracture. This mechanical drive unit, as for all other likely causes was going in and out of resonance, randomly accumulating many cycles until some blades failed. Even though the blades were shrouded, individual packets could have had much higher response than other packets, from mistuning described earlier. Fracture due to corrosion fatigue occurs at a stress far below the fatigue limit in laboratory air, even though the amount of corrosion is extremely small. The mode for this case was an out-of-phase mode shown to be resonant in Figure 24. Using FEA, the crack initiation site correlated with peak stresses for this out-of-phase mode. As there were operating issues, it was best to provide a more rugged design employing integral shrouds with rolled-in lashing wire. A more recent type of coating was also considered, but in this case a rerate due to plant changes had already been necessary so that improvements in design and operation fit into the plans.

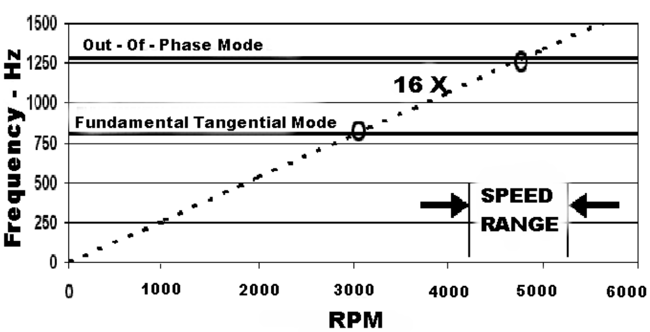


*Figure 24. Campbell Diagram for Medium-Length Steam Turbine Blade.*

*Case C-2*

A variable-speed steam turbine last-stage blade using 12 percent chromium steel, with zigzag lashing pins experienced two "rouge" failures; only one blade cracked on two rotors with long lives between failures, i.e., years apart. Failure was at a small stress riser as the surface had limited liquid impingement damage near the stellite overlay. FEA analysis confirmed that the failure site showed a high dynamic stress for a higher mode with a nodal line near the zigzag pins. The excitation was from nozzle passing frequency, normally not causing problems for high frequency modes of long blades due to phase cancellation. However, every mode needs some damping; in this case records were found that the exhaust pressure was often well below limits, which can greatly reduce aerodynamic damping of last stage blades, in fact likely causing it to go negative. The zigzag pins and some inherent material damping would prevent flutter for the fundamental modes; however as the pins were located near a nodal line for the mode shape, forced excitation and response were compromised by the high aerodynamic incidence angles during low exhaust pressure conditions. The options were to take some risk and just have operations monitor that maintaining that exhaust limits are adhered to, or also get a more robust design. The user opted for an improved design. In this case, the loss of damping occurred at low exhaust pressure; it can also be greatly reduced with abnormally high pressure while at or near design speed.

*Case C-3*

A rerated and repaired steam turbine had the next-to-last stage assembled where due to large number of blades, tolerances gave a condition that all packets would not have the same number of blades. However it was only the nonresonant fundamental mode that would have a low response factor for the different packet based on APPENDIX B. The one packet with a different number of blades would definitely have somewhat different frequencies from mistuning; however, higher modes are much less likely to cause problems. Similar calculations for the first out-of-phase modes showed that response factor would actually be less for the one different packet, compensating for the potential increase in amplitude from mistuning effects. Many designs have slightly different numbers per packet in a stage and thus should be compared for critical cases, adding to other reasons for mistuning. This could explain why in some cases only one or perhaps a few packets experience fatigue cracks.

*Case C-4*

A radial turbine stage had two different upstream sources of excitation for a fundamental mode of a cantilevered blade. Calculations were made to show that the resultant excitation was as shown in Figure 25. This rare cause of resonant excitation was identified as being due to use of 15 stationary inlet vanes interacting with 12 other equally spaced elements upstream of the vanes. Besides giving harmonic excitation at 15 times and 12 times speed, the sources will cause interaction depending on spacing between them and the strength of each source. Interaction affects the local gas loads on the turbine blades downstream, accentuating various harmonics of speed. As the difference between 15 and 12 was three, three times speed could be a prime source of excitation; and as verified by Fourier analysis, multiples of three could also produce significant excitation. Using the variation shown in Figure 25, the harmonic peak-to-peak excitations from the predominant sources are 5.5 percent for the 15 per rev component, and 2.2 percent for the 12 per rev component. The interaction between the two sources of nonuniformity results in a value equal to 1.9 percent excitation for the three times speed, and 1.2 percent excitation for the six per rev component. Natural frequencies of the blades avoided harmonics of 3, 12, and 15, but the second mode was resonant with six per revolution.

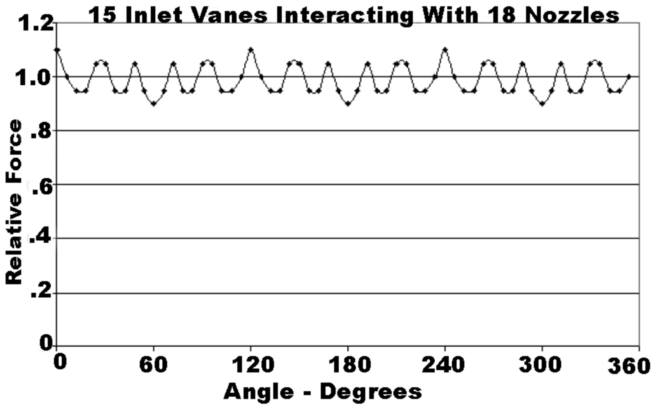


*Figure 25. Excitation Force Around Circumference for Radial Inflow Turbine.*

Finite element analysis was used to show that maximum dynamic stress at the crack initiation point (identified through fracture mechanics analysis) correlated for the second mode. A small modification that also alleviated a stress concentration factor was utilized to both avoid the resonance problem and the need for major design revisions to the structural components. Rather than a complete redesign, the solution was successful for the constant-speed application.

## CONCLUSIONS

- The methods described using parametric Equations (1) and (2) should be utilized whenever possible, not only for failure analysis but also in the design phase such as in selecting number of vanes for vaned diffusers and for all new bladed-disk designs.
- Equal numbers of vanes and blades should be always avoided; often while also reviewing aerodynamic effects, a more optimum design can utilize a difference of one between stationary vane and rotating blade numbers. Even and odd combinations are usually a better choice.
- Typically, an odd blade number is best used for the impeller for better mode splitting of diameter modes; also a prime number appears to be a better choice for mistuning effects for open impellers.
- Although covered centrifugal compressor impellers are definitely much more rugged than open impellers, first order resonance of the first blade leading edge mode requires as much scrutiny as the first few modes for freestanding cantilevered blades in other machinery.
- There is always a need to scrutinize operating and environmental parameters and the design, not only in design specifications, but also in problem resolution.
- Extensive finite element analysis can be used to correlate vibratory modes and location of peak stresses with fracture analysis documenting crack initiation sites for a failed specimen.
- It is sometimes necessary to opt to use a much more rugged design when investigation of a field failure is inconclusive. Past unresolved problems should be addressed as more knowledge is gained.
- Operation of machinery at too low or too high flow cause many failures of compressors and for the longer steam turbine blade designs. These conditions also could present added interaction resonance concerns, especially for high gas density and loads in radial impellers. Running in deep stonewall should be avoided; if the high flow condition occurs, limits should be used for transients with limited number of cycles, just as is recommended for surge.
- Liquid ingestion must also be avoided since it intensifies engine order resonance, and has caused at least one verified case of loss of overall damping. Better online liquid monitoring and prevention procedures should be developed for both compressors and turbines.
- For steam turbines, API 612, Fifth Edition (2003), specification is overly restrictive as to handling limits for blade resonance. It should be revised to add consideration of what modes really need scrutiny, with verification that stresses will be within Goodman factor limits.
- Additional research and implementation of new techniques replacing glass bead shot peening such as laser shock-peening, and low plasticity burnishing as well as new coatings will aid in precluding fatigue failures.
- Case studies such as those given will hopefully lead to more prevention of inherent design limitations, as well as ensuring proper operation of all fluid-handling turbomachinery for the oil, petrochemical, and related industries.
- Steam turbine transition stages require more attention in both design and achievement of minimal corrosion.
- Because mistuning of blades is unavoidable, it is recommended to use care in replacing phase analyses as those in APPENDIX B with the graphing method suggested by others in conjunction with nodal diameters, especially for modes of short blades.
- Interference diagrams using nodal diameters should be employed for cases of blades all coupled together with a continuous shroud or locked-up with z-shrouds, or if conditions are such that there is a disk-dominating mode of concern.
- There definitely is an optimum to everything; increased research and development will get turbomachinery manufacturers closer to that achievement for high cycle fatigue (HCF) analysis.

## APPENDIX A

### *Modal Analysis Test Procedures*

Avitabile (2001) provides a good review of experimental modal analysis. For machinery components, use of a two-channel fast Fourier analyzer is recommended. Modal frequency tests are to be described. A small accelerometer is attached to the test object. Frequency response functions, FRFs, are obtained while striking the object at various points, using a small hammer that has a force gauge in the head. The FRFs provide ratios of response to applied dynamic force; at the peaks of response/force ratios in the frequency spectrum, mode shapes are then obtained from extracted data for impacts at all required points. The size of the hammer as well as the hardness of the tip determines whether proper amplitude and frequency window of the impulse force is generated.

To measure tangential, torsional, and axial natural frequencies of a single blade mounted in a vise, at the top land of the root, a small accelerometer is attached with glue or thin wax to the blade (near the root to minimize effect of accelerometer mass). Both the hammer and accelerometer must have proper frequency response capabilities; data are verified using coherence checks between the input and response for each test. In Figure A-1 is a model used showing the points that were utilized for striking the blade to obtain the tangential and torsional modes; hammer blows are approximately perpendicular to the blade neutral axis at each position along the blade height. Typically, only the points along the leading edge would be used for obtaining axial mode shapes, striking the blade parallel to the neutral axis at each position along the blade height.

“Rap tests” can also be utilized with microphone response without the accelerometer attached; data verify negligible effect of accelerometer mass. “Rap tests” can also be utilized for expeditious checks to correlate with values obtained with FEA analysis. Modal frequencies are listed in Table A-1; refer to modal plots in Figures A-2, A-3, and A-4 (fundamental axial mode shape would have entire length of blade in phase).

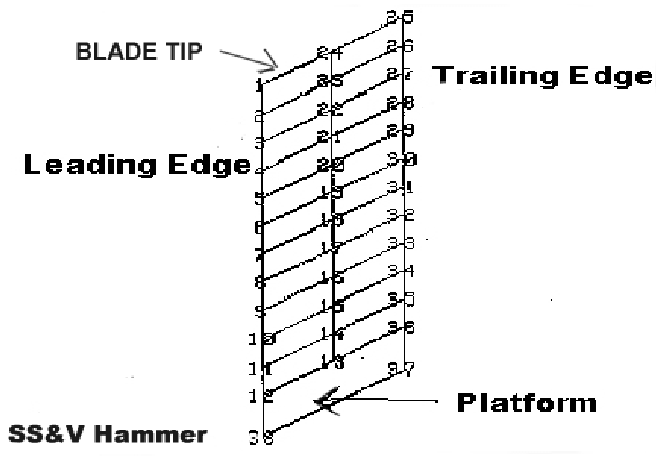


*Figure A-1. Modal Test Point Definition for Tangential and Torsional Modes.*

*Table A-1. Modal Frequencies Obtained for Example Medium-Length, Nontwisted Blade.*

| MODE | FREQUENCY - Hz |
|---|---|
| Fundamental Tangential | 1580 |
| Fundamental Torsional | 3210 |
| Fundamental Axial | 4950 |
| Second Tangential | 5770 |

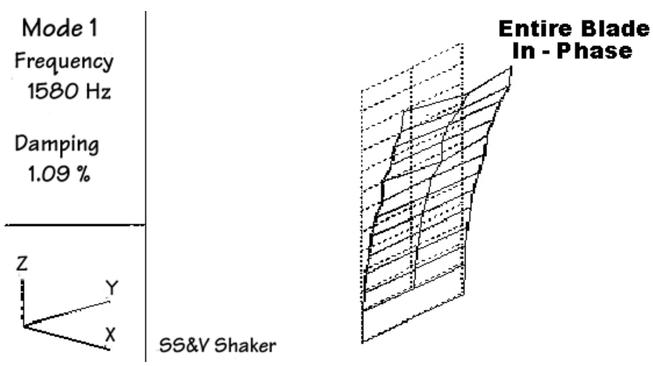


*Figure A-2. Modal Test of Single Blade—Fundamental Tangential Mode.*

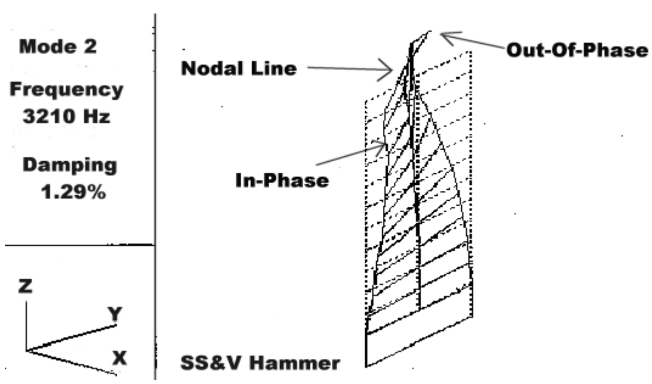


*Figure A-3. Modal Test of Single Blade—Torsional Mode.*

For an impeller, diametral modes, two-diameter through B/2 diameters are usually obtained, where B is the number of impeller blades, but many other modes can also be found. An accelerometer would be placed in the axial direction near the tip of the impeller. The impeller would be impacted with the instrumented hammer at a number of points, both in line with blades and in between blades.

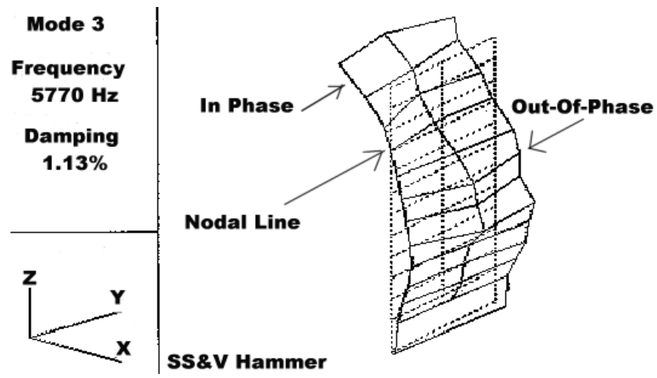


*Figure A-4. Modal Test of Single Blade—Second Bending Mode.*

Frequencies are then obtained from extracted data from amplitude Vs frequency spectrums. The amplitudes are those for ratio of amplitude to force (FRFs), so the peaks are the natural frequencies. The frequency values are used to verify predicted values. For the pure diametral modes without nodal circles, only points near the tip are usually needed for impacts. The extracted data would show the mode shapes for the frequency peaks.

## APPENDIX B

### *Blade Packet Optimization for the Fundamental Mode*

The equation to calculate response factor, R, for a packet of blades where all blades are in phase with each other and have the same amplitude is:

$$R = \left(X^2 + Y^2\right)^{1/2} / p \qquad \text{(B-1)}$$

where:

X = Σ (from i = 1, to i = p) cos [(i−1) • α]
Y = Σ (from i = 1, to i = p) sin [(i−1) • α]
α = 360 • (S/B)
α = Relative phase angle, degrees
S = Number of stationary vanes or nozzles
B = Number of rotating blades per row
p = Number of blades per packet

These equations can be consolidated for a single equation to more readily find the optimum number of blades per packet, p:

$$j = k \cdot \left|S - B\right| / B \qquad \text{(B-2)}$$

Set k = 1, k = 2, k = 3, etc., and calculate values of j. Then optimum value of blades per packet, p, is the value used for k whenever j is equal to, or is closest to, a whole integer. Multiples of the optimum value of p can also be used, as long as the shroud can be assembled with higher number of blades per packet. An example is given in Table B-1. In this case, using k = 4 or 8 results in an integer (not always the case—the closer to an integer the better). The resonant response factor can be verified using Equation (B-1), the response factor for p = 4 or p = 8 would be zero, i.e., perfect phase cancellation. The row of 64 blades could thus be assembled with either 16 packets of four blades, or eight packets of eight blades. Assume that this was a fairly long blade at the transition zone of a steam turbine, and there was no chance of resonance with an in-phase mode. If an out-of-phase mode was a cause of an unusual field problem, such as with excessive corrosion, then it could be much better to choose two packets with five blades and nine packets having six blades.

## APPENDIX C

### *Harmonic Excitation—Fourier Analysis*

For a pulse train made up of a number of rectangular pulses, m, over 360 degrees (one revolution), to calculate $S_h$, peak-to-peak exciting force at harmonic, h:

*Table B-1. Values of "J" for Selected Values of Integer "K" for S = 80, B = 64.*

| Integer "k" | "j" |
|---|---|
| 1 | 0.25 |
| 2 | 0.50 |
| 3 | 0.75 |
| **4** | **1.00** |
| 5 | 1.25 |
| 6 | 1.50 |
| 7 | 1.75 |
| 8 | 2.00 |

$$S_h = \left(C^2 + D^2\right)^{1/2} \qquad \text{(C-1))}$$

where:
C = Σ from 1 to m of $A_m$ [cos(h • $\theta_m$)]
D = Σ from 1 to m of $A_m$ [sin(h • $\theta_m$)]

where:
h = Harmonic of speed
$A_m$ = Peak-to-peak force of each pulse
$\theta_m$ = Angle from zero degrees to center of each pulse

and:

$$A_m = a_m\left[2 / (h \cdot \pi)\right] \cdot \sin\left(h \cdot \beta_m / 2\right) \qquad \text{(C-2)}$$

where:
$a_m$ = Fraction of maximum force of any pulse
$\beta_m$ = Rectangular width of pulse, degrees

For a sine wave shown in Figure C-1, at resonance, the exciting force is in-phase with the response over the entire vibration cycle. As the force is out-of-phase over part of the cycle, a square wave for example would only have 0.637 times the exciting force of a sine wave at the first harmonic. For the square wave, a rectangular pulse with β = 180 degrees, Equation (C-1) simplifies to:

$$S_h = \left|\left[2 / (h \cdot \pi)\right] \cdot \sin\left(h \cdot 180 / 2\right)\right| \qquad \text{(C-3)}$$

First harmonic, $S_1$ = |2 / π • sin (90)| = 0.637
Second, $S_2$ = |2 / 2π • sin (180)| = 0.0
Third, $S_3$ = |2 / 3π • sin (270)| = 0.212
Fourth, etc.…

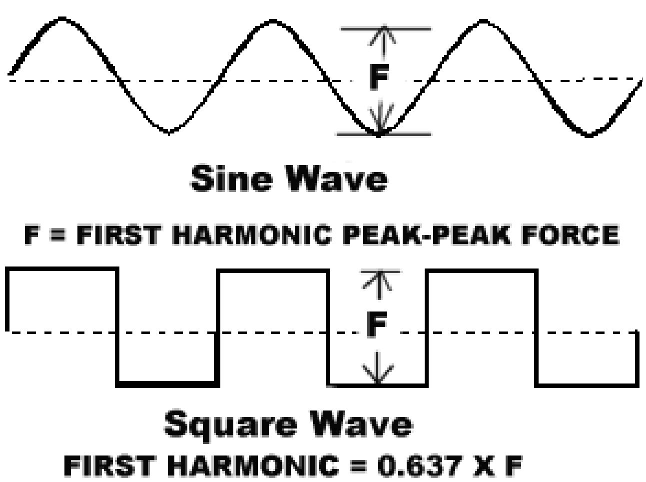


*Figure C-1. First Harmonic Excitation Factor, Square Wave Compared to Sine Wave.*

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