APPLICATION OF A CRACK-CLOSURE MODEL FOR TURBINE LIFE ASSESSMENT

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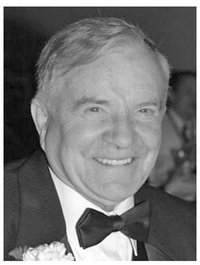

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ABSTRACT

This paper describes the application of a fracture mechanics-based procedure for life estimation that allows complex load cycle effects to be included. The procedure is based on technology developed by the aerospace industry, which includes LCF and HCF loading and incorporates local plastic strain effects at the crack tip.

The procedure assumes that starter cracks are initiated at inherent defects (voids or inclusions) found in the material. Small specimen test data were obtained to calibrate the fracture mechanics characteristics and the early crack development for the blade material. There was a close correlation between the calculated and measured fatigue life with this model.

The procedure was then applied to a steam turbine blade row with documented cracks developed after long-term service (Rieger, et al., 2002). By using known history of the service loads and the fracture properties from the specimen tests, it was possible to develop life predictions for the remaining blades in the row. To date the blades have operated in accordance with the predictions and no failures have occurred since the unit has returned to service.

INTRODUCTION

This paper describes the development of a comprehensive procedure for fatigue life assessment. This procedure was subsequently tested for the prediction of component life under cumulative complex fatigue damage. The components involved in this instance were L-2 steam turbine blades from a nuclear power plant. This unit had a complex history of operation. For the first three years, a blade-disk natural frequency was close to the sixth harmonic of the operating speed. Although the blades had run close to this resonance for three years without failure, these blades were subsequently modified to raise their natural frequency. The particular natural frequency of the blade row was raised from 184 Hz to 192 Hz. In this condition, the detuned blades subsequently operated for an additional 29 years without incident. During a routine inspection it was discovered that some of the blades in this L-2 row of the low-pressure section of the turbine had developed cracks in their attachment regions as seen in Figure 1. A detailed examination showed that 84 out of 820 blades at the site (six rows) had detectable (lengths greater than 0.060 inch) cracks. Cracked blades were replaced with new blades prior to restarting the unit.

It then became important to determine how long would the remaining original (uncracked) blades, which had not been replaced, continue to operate before they too developed cracks?

To resolve this question, a comprehensive fatigue damage investigation was undertaken for this application. A program of small-scale testing was used to determine the fatigue strength characteristics of the AISI 410 blade material, and the manner in which fatigue damage accumulates in 410 material. The various phases of this investigation are described, including the development and application of a calculation procedure that was used to assess the cumulative damage in these blades.

Figure 1. Magnetic Flux Inspection of a Cracked Blade Root.

CUMULATIVE FATIGUE DAMAGE ASSESSMENT METHODS

The subject of cumulative damage has an extensive literature starting with Miner's (1945) restatement of Palmgren's (1924) work in 1945. Many other authors have done additional work. Comprehensive discussions of cumulative damage theories have been presented by authors such as Collins (1981), Dowling, et al. (1975), and by Manson and Halford (1986). For the most part, these theories fall into three categories:

- Simple revisions or extensions to the Palmgren-Miner hypothesis,
- Power law theories that allow for nonlinear damage accumulation at higher strain levels, and
- Elastic-plastic damage with linear or nonlinear summation (Dowling, et al., 1975).

The above fatigue damage studies occur in two recognized categories. The first type is those concerned with so-called fatigue "initiation" life, which is most relevant to high-cycle fatigue (HCF), and associated with small cyclic strain levels. The second type of development concerns the propagation of an established crack. This typically involving a smaller number of larger strain cycles, which may be either elastic or elastic-plastic in nature. Routine low-cycle fracture mechanics deals with relatively large strains and crack growth increments. The use of fracture mechanics as a tool for general-purpose life analysis has been studied extensively by Newman and others (Newman, 1983; Newman and Phillips, 1999; Newman, et al., 1986). This expanded use of fracture mechanics necessarily involves both high-strain and small strain crack growth increments occurring in some sequence that is specific to a given problem. This work has led to more detailed consideration of the plastic strain that occurs in the vicinity of the crack tip during a load cycle, i.e., under crack opening and crack closing conditions.

The justification for employing fracture mechanics procedures to address both the initiation and propagation of small cracks lies in experimental data obtained by a number of small crack investigators. As an example Figure 2 shows the initiation and early growth of a small crack from a SiO_2 inclusion within an AISI 4340 alloy steel matrix. Such an inclusion may be thought of as a stress concentration within the body of the component. The jagged corner (or crack) that exists at the edge of the inclusion then represents a "starter" crack, typically of second-order smallness in size. Recognizing that inclusions and voids are inherent in the material, all parts have a large number of crack initiation points distributed throughout the part. The overall size of such inclusions can be determined by inspection. Both the flaw size and the starter crack are readily modeled for calculation, but the mechanics of early propagation of small cracks under load still remains a very complex problem that has not as yet been fully addressed.

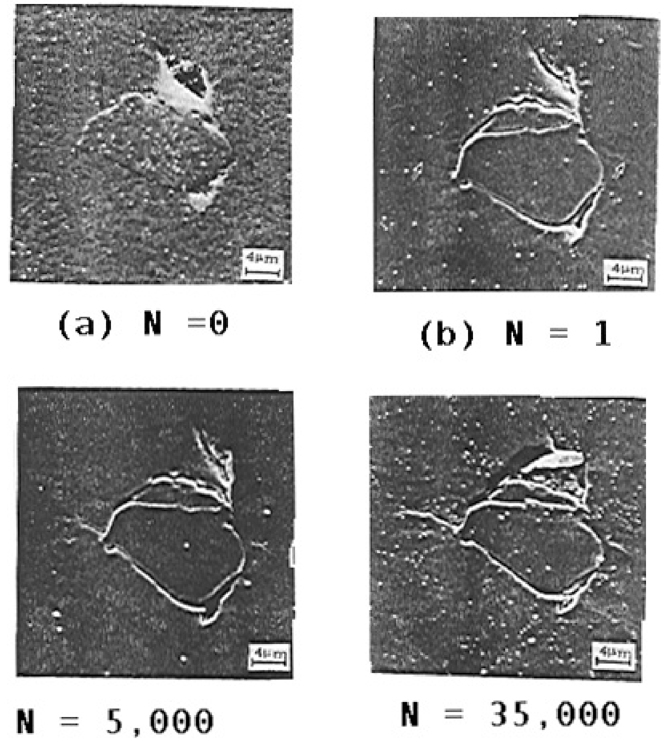

Figure 2. Growth of Crack from Inclusion with Cycles in AISI 4340. (Courtesy Lankford, 1977)

METHOD APPLIED FOR FATIGUE LIFE CALCULATIONS

The initial calculations for blade life were made using the local strain method. The long life calculations involved were found to be extremely sensitive to the values selected for certain parameters. For example a 6 percent decrease in the numerical value of the HCF strength exponent, b, resulted in a fivefold increase in the calculated HCF life. This is probably due to the assumption of a single value for the parameter, b, over a 30-year span. Similarly a 15 percent decrease in the stimulus ratio led to a sixfold increase in the calculated life. In addition these techniques were unsuccessful in predicting cycles to failure for the compact tensile test specimens.

Another method for fatigue life calculation was sought, which had the ability to accurately correlate both the test data and also the plant load cycles. A search of the literature led to the crack-closure fracture mechanics procedure developed by Newman and coworkers in the aerospace industry. Their procedure involved a modified version of Forman's rule and considered the strain cycle in the crack tip region (including residual strain effects) in the crack growth process. The flaw site from which the crack grows is considered as a stress concentration within the strain field, from which a small "starter-crack" has begun to propagate. The rate of crack propagation is described using a modified Paris-Forman expression in which stress intensity ΔK is written as ΔK_{eff}, which is independent of R. The governing equations become:

$$\frac{da}{dN} = B\left[\Delta K_{eff}\right]^m \tag{1}$$

where:

$$\Delta K_{eff} = \left(\sigma_1 - \sigma_0'\right)\sqrt{\pi a} \tag{2}$$

The use of the $\dot{\sigma}_0$ term recognizes that no crack extension can occur without reopening the crack during the load cycle. Newman replotted crack growth rate versus ΔK_{eff} as shown in Figure 3. This resulted in the correlation of numerous tests with a variety of materials. The propagation curve for the AISI 410 blade material tested in this program was found to be close to that of AISI 4340 steel.

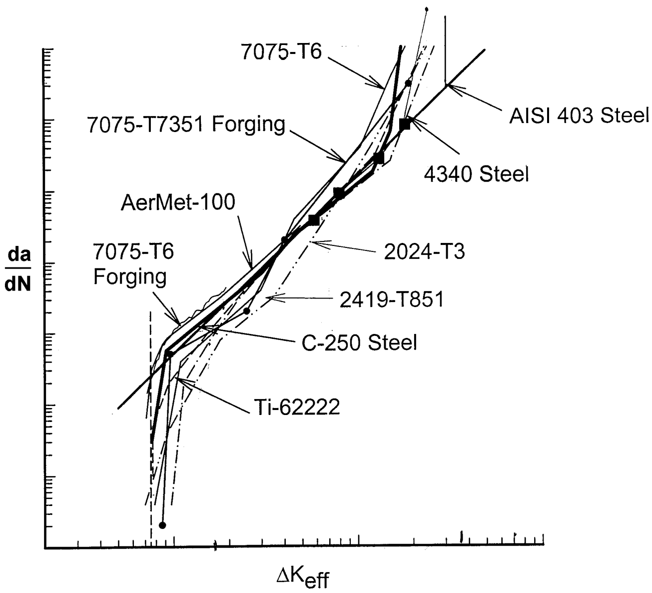

Figure 3. Crack Growth Data for Aerospace Materials. (Courtesy Newman, 2001)

CRACK INITIATION AND PROPAGATION CURVE, BASED ON SPECIMEN TESTS

The da/dN curve for the 410SS material is based on data developed from a crack propagation test program conducted as a part of this investigation. The program used compact tensile test specimens made from discarded blades from the same row that had been in service for over 30 years. Selected test specimens were subjected to multilevel load tests. This application uses a crack growth equation as shown in Equation (1). A curve based on the da/dN data is given as a function of ΔK_{eff}, in Figure 4.

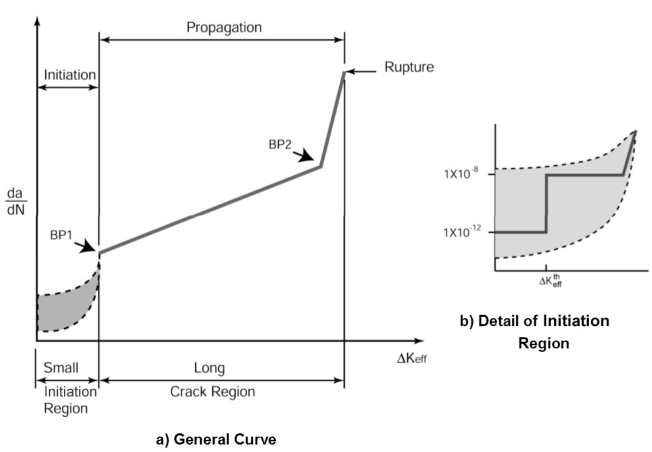

Figure 4. da/dN Curve Used for Life Predictions.

It should also be mentioned that the method described does not rely on the customary "Miner's Rule" procedure to sum the damage cycles that are accumulated, nor does it employ the "rainflow" or "range pair" procedures. Miner's rule does not consider the significance of any sequence in which the damage is accumulated, although in recent years the importance of such sequencing has been widely recognized where high yielding has occurred in the plastic zone at the crack tip. Such yielding is recognized by the local strain method, but the sequence effect is lost where Miner's rule is used for damage summation. The crack-closure model used in the procedure described in this paper allows effects resulting from crack opening and closing (into the yielded zone) to be considered in the sequence in which these events occur.

Initial Crack Size and Initiation Region

The size of a typical inclusion in the surface of the blade attachments was obtained from a low powered microscope examination of several blade surfaces. The maximum size of the inclusions observed had a length of 0.04 mm (0.0016 inch) and a width of 0.02 mm (0.0008 inch). Based on these data, the initial crack size was set to 0.0016 inch × 0.0008 inch. Swain and others (Swain, et al., 1990) also observed inclusions of size 0.008 inch to 0.040 inch in 4340 steel.

The line between BP1 and BP2 shows rate of growth of an established crack plotted as a function of ΔK_{eff}. This region is well defined and the current test results are consistent with previously published results for growth in this region. The shape of the curve in the "initiation region" of Figure 4 was obtained from additional observations with a trial and error procedure. In this study it was found that the reported data could best be represented using a two step model as shown in Figure 4b. The parameters needed for this are the values of ΔK_{eff} at BP1 and at ΔK^{th}_{eff} plus the values for da/dN at the two steps. After several preliminary calculations, the bottom step was set at da/dN = 1 3 10212 in/cycle for ΔK_{eff} less than ΔK^{th}_{eff}, which was fixed at 3 ksi-in$^{1/2}$ for all cases.

Tests 5a and 5b were made using four compact tensile specimens with a mean stress of 150 ksi and a dynamic stress range, $\Delta\sigma$, of 130 ksi (R = 0.395). Sample 5a had no prestress (prestrain), while sample 5b had a single cycle of 0 to 240 ksi (pseudo-stress). The results in the top part of Table 1 show the calculated cycles to failure as a function of BP1 and da/dN for the second step in the initiation region. The closest agreement was found in the case BP1 = 8.0 ksi-in$^{1/2}$, and with $da/dN_{start} = 1.0 \times 10^{-8}$ in/cycle (highlighted in Table 1). Table 1b shows the comparison of the individual data points, the average values for cycles to failure, and the corresponding calculated fatigue life using a fatigue life prediction computer code.

Table 1. Comparison of Calculated Life Results with AISI 410 Fatigue Test Data.

(a) Parametric Values of LifeCycle Control

BP2 Rupture Point	BP1 First Break Point	da/dN Start Value	Calculated Cycles to Failure Test 5a	Test 5b
		1.00E-09	3.24E+06	3.55E+06
	ΔK$_{eff}$ =10	1.00E-08	439,448	480,040
		1.00E-07	74,826	79,402
		1.00E-09	1,364,424	1,883,757
	ΔK$_{eff}$ =8	1.00E-08	215,832	265,850
ΔK$_{eff}$=50ksi-√in		1.00E-07	49,831	56,574
		1.00E-09	42,532	47,618
	ΔK$_{eff}$ =5	1.00E-08	42,538	48,022
		1.00E-07	41,732	47,894
		1.00E-10	41,881	46,634
	ΔK$_{eff}$ =3	1.00E-09	41,888	46,641
		1.00E-08	41,866	46,593

NOTE: Highlighted cells of Table 1(a) match the test data most closely

(b) Comparison of Test Cycles to Failure and LifeCycle Calculations (based on the Row Highlighted Above)

Test No.:	Test Sample 1	Test Sample 2	Average	Calculation
5a	215,754	189,621	202,688	215,832
5b	332,144	329,819	330,982	265,850

Tests 6a, 6b, 6d, and 6e had load cycles as described in Table 2. When multiple values were tested for BP1 and da/dN, once again the closest agreement was found for the case BP1 = 8.0 ksi-in$^{1/2}$ with $da/dN_{start} = 1.0 \times 10^{-8}$ in/cycle, as highlighted in Table 1.

Table 3 shows the individual sample points and the average cycles to failure compared with the calculated values of cycles to failure.

Table 2. Cyclic Loads for Test 6.

Test No.:	Initial	Intermediate		Final
6a	2 cycles 160 ksi, R=0.05	10^5 Cycles 140 ksi, R=0.05		To Failure 180 ksi, R=0.05
6b	2 cycles 160 ksi, R=0.05	20 cycles 140 ksi, R=0.05	10^6 cycles 110 ksi, R=0.05	To Failure 180 ksi, R=0.05
		Repeat To Failure		
6d & 6e	1 cycle 0-260 ksi	1 cycle 0-260 ksi	5000 Cycles 150 ksi, R=0.395	

The specimens used in 6e are exactly the same as in tests 5a, 5b, 6a, 6b & 6d, except for a 0.002" notch.

Table 3. Comparison of Calculated Cycles with AISI 410 Compact Specimen Test Results.

Test No.	Sample 1	Sample 2	Average	Calculation
Test 6a	169,846	147,565	158,705	132,914
Test 6b	1,086,282	1,078,009	1,082,145	1,036,451
Test 6d	142,418	141,042	141,730	157,210
Test 6e	121,247	115,135	118,191	129,656

Data from Tests 5 and 6 give consistent results. For 12 test specimens at six different conditions, the test results obtained with BP1 at $\Delta K_{eff} = 8$ ksi-in$^{1/2}$ and $(da/dN)_{start} = 1.0 \times 10^{-8}$ in/cycle gave calculated results whose values agreed most closely with the test data. On the average, the predicted life using the calculation procedure was within 4 percent of the mean data. In addition the standard deviation was only 15 percent. Both the test data and the crack-closure model show the benefits of prestress, as shown by the results of Tests 5a versus 5b. The test results and the crack-closure model also have similar sensitivity to a sharp notch (Test 6d versus Test 6e). In addition, because the local strain method did not fail parts, many of the specimens were subjected to several levels of fatigue loading prior to failure of the specimens. In total this is an indication of excellent agreement between the calculated and actual life of these test specimens. The compact tensile test specimen data became the basis for defining the material properties for crack growth calculations for the steam turbine blades.

CALCULATIONS OF BLADE OPERATING LIFE

Operation History

The plant operators have created their estimation of the blade load histories, which are shown in Figure 5 and Figure 6. During the first three years of operation there was a single factory 120 percent overspeed event, which is identified as Load 1 on Figure 5, and four 112 percent overspeed events identified as Load 2. The normal operating condition is identified as Load 3a on Figure 5. The unit operated for three years with the L-2 blade resonance frequency at 184 Hz. This is very close to 6× the operating frequency (180 Hz) for the 30 Hz operating speed. In this condition, blade dynamic response due to diaphragm excitation was amplified, resulting in a higher dynamic stress. This condition persisted during the first three years of operation until the manufacturer identified the problem and detuned the rotor row. The dynamic stress range is defined as the difference between the maximum and minimum stress during steady-state operation. Looking at Figure 5, Load 3a is at a nominal 111 ksi with a HCF variation. The difference between the maximum and minimum values is defined as $\Delta\sigma$, the dynamic stress.

The next 29 years of operation are shown in Figure 6. During this period the operating dynamic stresses were decreased because the blade frequency was intentionally detuned by the original equipment manufacturer (OEM) from 184 Hz to 192 Hz. Based on

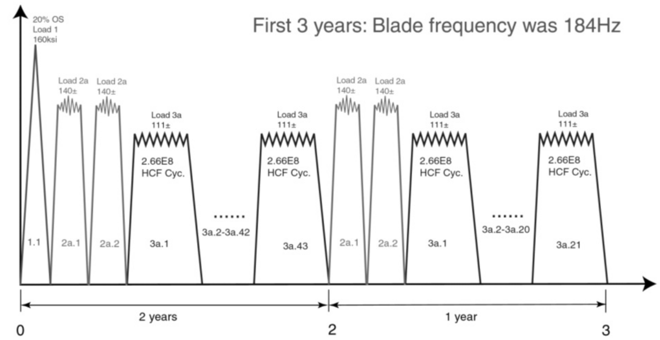

Figure 5. Load History During the First Three Years.

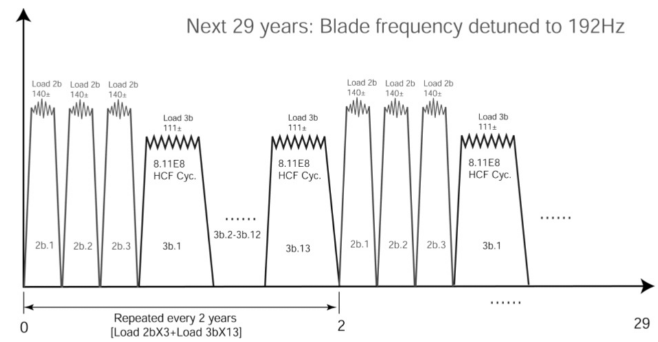

Figure 6. Load History, 1972 to 2001.

harmonic response analysis, the amplification factor for the first three years of operation was calculated to be 2.8 times the operating dynamic stresses during the succeeding 29 years. During this same operating period, it was determined (or estimated) that there were on average three 112 percent overspeed events for trip setting, and 13 offline events for each two year refueling cycle. This load history has been used for all life calculations.

Calculated Results—1969 to 2001

Calculations for blade root crack growth during this time period were made based on the operating history and the initial flaw size of 0.0016 inch, based on the microscopic determined inclusion size. Since dynamic stress levels are not known accurately under such circumstances, initial dynamic stress values of 3, 5, 7, and 9 ksi were selected based on experience with similar turbines. A dynamic stress range of less than 5 ksi is considered normal, depending on the given operating conditions. Dynamic stress values during the first three years were calculated to be higher by a factor of 2.8 because the 6× harmonic of operating speed during that period was known to be close to resonance. The material properties for this analysis are based on the laboratory crack growth data. Crack growth was calculated based on the four assumed values for dynamic stress levels. The results are given in Table 4.

Table 4. Crack Growth Between 1969 and 2001.

Assumption for Dynamic stress $\Delta\sigma$ (ksi)		LifeCycle Crack Growth
After 1972	First 3 years	
3	8.4	0.255" after 32 years
5	14.0	0.434" after 32 years
7	19.6	**Fails within** 19.7 years
9	25.2	**Fails within** 1.87 years

Table 4 shows that 32 years of operation under conditions of low-cycle fatigue (LCF) and HCF at modest dynamic stress levels can produce a crack of significant size. Based on a reasonably low, 3 ksi, value for dynamic stress, such a crack is predicted to reach a length of 0.26 inch after 32 years. If in practice some blades had actually been working at a somewhat higher (5 ksi) dynamic stress, the crack length could be over 0.45 inch. Both of these values are well within the range of crack sizes observed during the 2001 inspection. While it is reasonable to expect a range of values for crack length, values of dynamic stress that equal or exceed 7 ksi are likely to have produced cracks that would have been significantly larger than those observed during the 2001 inspection.

One such blade (refer to Figure 1) showed a crack that was about 1.0 inch in length. About 10 percent of blades had one or more cracks that ranged from 0.060 inch to over 0.50 inch in length. For completeness, it should be noted that no measurable cracks were found in around 90 percent of the blades examined. The crack lengths that were calculated to occur with applied dynamic stresses of 3 to 5 ksi (shown in Table 4) are within the range of crack lengths observed in the blades examined. Dynamic stress levels of 7 to 9 ksi would therefore produce cracks much larger than those found in the turbine blade row.

Prediction of Remaining Life for Continued Operation

The remaining life of the blade row was also calculated using the same parameters that successfully modeled the test specimen failures and blade cracking after 32 years. The critical item in this prediction is the simulation of the microdamage condition, which exists at the hook radius for the remaining (nominally uncracked) blades, which were reinstalled in 2001. Most (736 of 820) of these blades have already been in operation for 32 years. After 32 years of operation, the initial crack lengths, even if still microscopic, must be at least marginally larger than those that existed in the virgin material, having a maximum inclusion size of 0.0016 inch. The maximum crack size is therefore the *principal parametric variable* for the determination of blade survival in the coming years. The anticipated operating conditions for the next four years (2001 to 2005) are given in Figure 7. The corresponding calculated values for maximum crack growth are given in Table 5. Dynamic stress, $\Delta\sigma$, values of 5 and 7 ksi were used for this forecast. While prior analysis of operating history had indicated that 5 ksi was the most likely value, there was a potential for performance degradation, and so consideration of the 7 ksi assumption should afford a margin of safety.

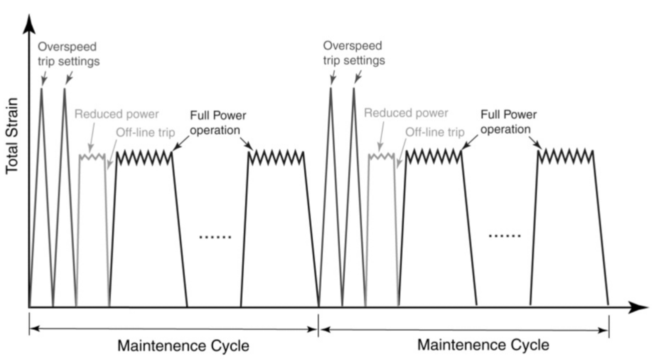

Figure 7. Recommended Operation, 2001 to 2005.

The results in Table 5 show that based on the fatigue life prediction computer code procedure if:

- None of these replaced blades have a crack length that is larger than 0.06 inch, and
- The unit is operated within the conditions shown in Figure 7,

then the blade can be expected to survive more than 10 years. The greatest risk factor appears to be the quality of the preceding blade inspection. If the inspection procedure had successfully identified and removed all blades with defects greater than 0.06 inch, then it appears unlikely that a crack would be able to grow to critical size in only 10 years. If the inspection of any blade has missed a defect in the hook region with the highest dynamic stresses, and if that defect is on the order of 0.30 inch in size, then there is a good chance that a blade would fail within a six-year time. The plant personnel were confident about the quality of the inspection. Based on the calculated predictions for crack growth, the power plant was able to safely get the unit back online and schedule the replacement of the complete blade row.

Table 5. Projected Crack Growth for 2001 to 2005.

Initial Crack Length (in)	Dynamic Stress $\Delta\sigma$ (ksi)	LifeCycle Prediction
0.06	5	0.121" in ten years
	7	0.139" in ten years
0.10	5	0.170" in ten years
	7	0.228" in ten years
0.20	5	0.298" in ten years
	7	**Fails within in SIX years**
0.30	5	**Fails within in SIX years**
	7	**Fails within in 27 hours**

CONCLUSIONS

- This paper presents the application of a practical procedure for the prediction of crack growth and component life for parts subjected to a complex load history. It has been successfully applied to both compact tensile test specimens with a short life (10^5 to 10^6 cycles) and to steam turbine blades with a very long (over 160×10^9 cycles) life to first observed failure.
- A calculation procedure has been developed based on a well-established crack growth algorithm from the aerospace industry. The analysis described is believed to be the first application of this technology to the analysis of steam turbine components.
- The material properties related to crack initiation and propagation (da/dN versus ΔK_{eff} curve) were developed for the specific blade material obtained from blades that had been in service for over 30 years.
- The material properties related to crack initiation and propagation were based on failure data obtained from 12 specimens, which had been subjected to complex loading. The part life for the specimen test conditions ranged from about 100,000 cycles to 1,000,000 cycles. The results showed that the specimen life ranged from 12 percent less to 28 percent greater than the calculated life. Such agreement represents close correlation where the calculations are based on test data having a well-documented range of statistical variability.
- Using material properties developed from test specimens the crack growth model predicted that cracks could reach lengths of 0.26 inch to 0.43 inch. This is consistent with the crack lengths observed for the 84 failed blades.
- There is little doubt that had the row not been detuned in 1972, the blades would have failed much earlier than 2001.
- The projected life for reinstalled blades with undetected cracks has been estimated using the crack growth model. This life is strongly dependent on the presence of any hidden damage associated with undetected flaws. Based on the model and the specified operating conditions for the case studied:
  - For undetected crack sizes smaller than 0.06 inches, and
  - For turbine operation within the recommended conditions,

the blade can be expected to operate safely until the scheduled blade replacement.

- The final proof will be successful operation of the blades until the reblading operation occurs (scheduled for 2007).

NOMENCLATURE

a	=	Crack length
N	=	Number of cycles at a given strain level
B	=	Material coefficient, found from specimen test data
m	=	Exponent found from specimen test data
ΔK_{eff}	=	Effective stress intensity defined by Equation (1)
ΔK^{th}_{eff}	=	Threshold value for the initiation region shown in Figure 4
R	=	Stress ratio, = (σ_2/σ_1) or $(\sigma_{min}/\sigma_{max})$
σ_1	=	Maximum elastic equivalent stress at the location of interest
σ_2	=	Minimum elastic equivalent stress
$\dot{\sigma}_0$	=	Crack-opening stress developed during the previous cyclic load
$\Delta\sigma$	=	Dynamic stress range associated with HCF during operation

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