STUDY OF A ROTORDYNAMIC ANALYSIS METHOD THAT CONSIDERS TORSIONAL AND LATERAL COUPLED VIBRATIONS IN COMPRESSOR TRAINS WITH A GEARBOX

by

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ABSTRACT

Variable speed inverter motors are increasingly being used to drive compressors for petroleum, chemical, and gas industry services. In the past, as this type of application of large inverters, it has been possible to apply variable speed inverter motors to larger compressor trains. The power sources of these inverters have various frequency fluctuations depending on the inverter type. These torque fluctuations may cause torsional vibration on the shafts.

Most centrifugal compressor trains that are driven by variable speed inverter motors use speed-increasing gears. Since the gear meshing force induced by the torsional vibrations also causes lateral vibrations, lateral vibration levels are useful in monitoring torsional vibrations in the compressor trains. This torsional and lateral coupled vibration model shows a difference in the rotordynamic characteristics apart from the results obtained from independent torsional and lateral studies. Therefore, a rotordynamic analysis that considers a coupled torsional and lateral vibration system is effective in evaluating torsional vibrations.

This paper introduces a rotordynamic analysis method that is based on a coupled torsional and lateral vibration system, and its application results of actual machines shown.

INTRODUCTION

A coupled torsional and lateral vibration appears on a centrifugal compressor train with a gearbox, because the torsional vibration is transferred to the lateral vibration at the gear mesh. There are several potential sources of torsional excitation. Under the load condition of continuous operating speed, centrifugal compressors, steam turbines, gas turbines, gearing, and constant speed induction motors are not normally considered significant sources of torsional excitation within a system. However, variable speed inverter motors are a significant potential source of torsional vibration, because variable speed inverter motors have torque fluctuations caused by AC-DC-AC conversion. Since the gear meshing force

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induced by torsional vibrations cause lateral vibrations, lateral vibration levels are useful in monitoring torsional vibrations in the compressor trains.

ANALYSIS OF TORSIONAL AND LATERAL COUPLED VIBRATION

As an example, the authors analyzed a compressor train with a motor and a speed increasing gearbox as shown in Figure 1. The low-speed shaft is composed of a motor and a wheel shaft. The high-speed shaft is composed of a pinion shaft, a low-pressure compressor, and a high-pressure compressor.

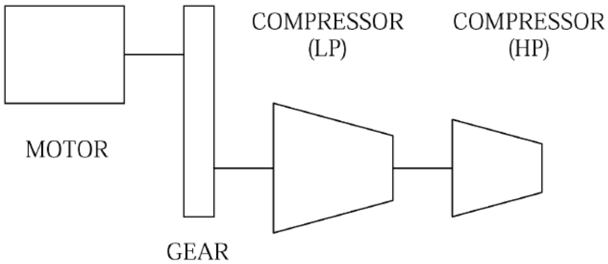


Figure 1. Centrifugal Compressor Train with Gearbox.

Analytical Model of Rotordynamics

Figure 2 shows an analytical model of the train. Here the authors divide the compressor train into two shaft systems at the gear mesh joint. Shaft-1 is the low speed shaft (motor and wheel shaft), and shaft-2 is the high-speed shaft (pinion shaft-low pressure [LP] compressor-high pressure [HP] compressor). First, a lateral natural frequency analysis and a torsional natural frequency analysis were performed for each shaft. The natural frequency, vibration mode, and modal parameters were calculated. Figure 3 depicts the analytical result of the lateral and torsional natural frequency of each shaft.

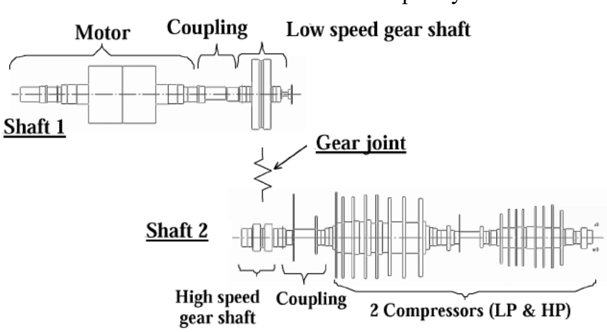


Figure 2. Analytical Model of Rotordynamics.

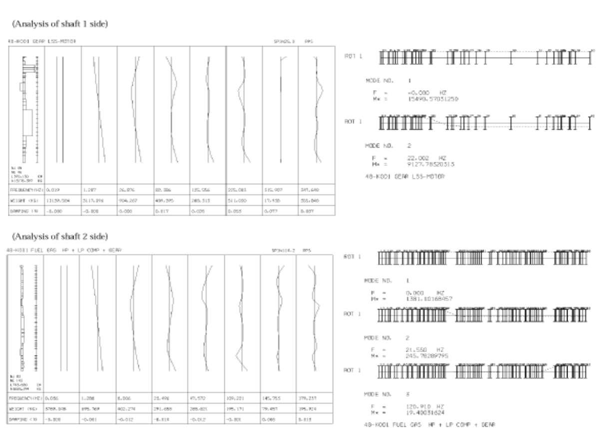


Figure 3. Analytical Result of Lateral and Torsional Frequency.

The following parameters were used for the analytical model of the total system vibration:

- ω_i = i-th natural vibration frequency (= $2\pi f_i$)
- ζ_i = i-th damping ratio
- m_i^* = i-th modal mass
- $k_i^* = m_i^* \omega_i^2$ = i-th modal rigidity
- $c_i^* = 2\zeta_i m_i^* \omega_i$ = i-th modal damping coefficient
- ϕ_{Ii} = i-th external force point mode
- ϕ_{Oi} = i-th response point mode

The coupling of the torsional and lateral vibrations appears at the gear mesh joint. Mathematical computer software is used to examine this coupled vibration analysis. A model of the software analysis was produced using the modal parameters derived from the lateral and torsional natural frequency analysis. The following is the rotordynamic equation:

$$\begin{pmatrix} \dot{x} \\ \ddot{x} \end{pmatrix} = A \begin{pmatrix} x \\ \dot{x} \end{pmatrix} + Bu, \; y = C \begin{pmatrix} x \\ \dot{x} \end{pmatrix} + Du \tag{1}$$

where:
x = State variables (modal coordinate system)
y = Response of each part (actual coordinate system)
u = External force (actual coordinate system)

$$A = \begin{pmatrix} 0 & I \\ -M^{*-1}K^* & -M^{*-1}C^* \end{pmatrix}, \; B = \begin{pmatrix} 0 \\ M^{*-1}\Phi_i^T \end{pmatrix},$$

$$C = \begin{pmatrix} 0 & \Phi_o \end{pmatrix}, \; D = (0)$$

$$M^* = \begin{pmatrix} m_1^* & & 0 \\ & \ddots & \\ 0 & & m_n^* \end{pmatrix} = \text{Modal mass matrix}$$

$$C^* = \begin{pmatrix} c_1^* & & 0 \\ & \ddots & \\ 0 & & c_n^* \end{pmatrix} = \text{Modal damping coefficient matrix}$$

$$K^* = \begin{pmatrix} k_1^* & & 0 \\ & \ddots & \\ 0 & & k_n^* \end{pmatrix} = \text{Modal rigidity matrix}$$

$$\Phi_I = \left(\phi_{I1} \ldots \phi_{In}\right) = \text{Mode matrix of external force point}$$

$$\Phi_O = \left(\phi_{O1} \ldots \phi_{On}\right) = \text{Mode matrix of response point}$$

Figure 4 shows the analytical model of the lateral vibrations for shaft-1. The input to the system is the tangential force of the gears meshing, while the output from the system is the gear radial displacement. The feedback loops in the model are the reactive force of the journal bearings.

Figure 5 shows the analytical model of torsional vibrations for shaft-1. The input into the system is gear torque, while the output from the system is the gear rotation angle.

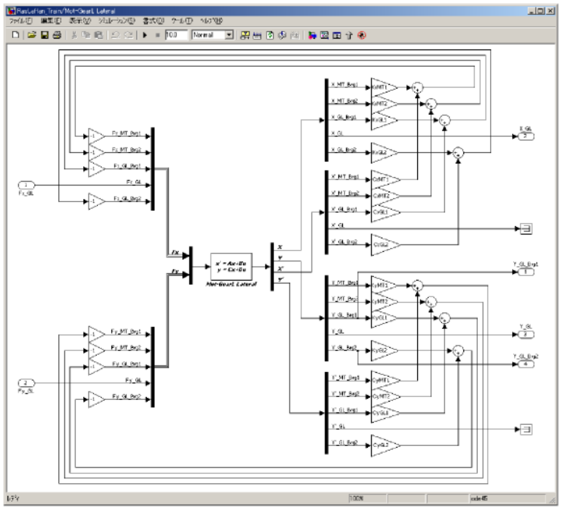


Figure 4. Model of Lateral Vibration Analysis in Shaft-1.

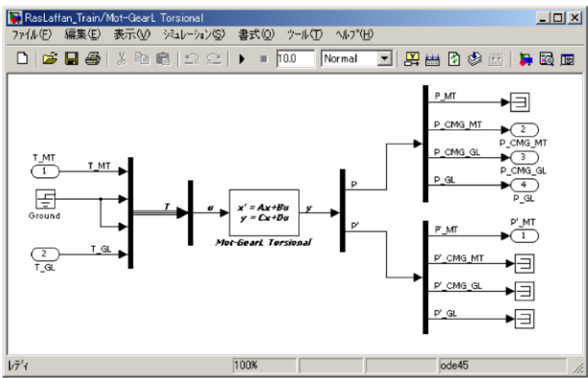


Figure 5. Model of Torsional Vibration Analysis in Shaft-1.

Analytical models of lateral and torsional vibrations for shaft-2 were prepared as same.

Modal Synthesis of Total System

The following shows the relational expressions of relative displacement of both shafts at the gear joint, gear reaction by relative rotation angle and torque:

- $f_{x1} = f_t \sin \alpha$ = X directional compound of gear reaction of shaft 1,
- $f_{y1} = f_t \cos \alpha$ = Y directional compound of gear reaction of shaft 1,
- $T_1 = f_t r_1$ = Torque of shaft 1
- $f_{x2} = f_t \sin \alpha$ = X directional compound of gear reaction of shaft 2,
- $f_{y2} = -f_t \cos \alpha$ = Y directional compound of gear reaction of shaft 2,
- $T_2 = -f_t r_2$ = Torque of shaft 2
- $f_t = k_g \{(z_2 + r_2 \theta_2) - (z_1 + r_1 \theta_1)\}$ = Gear reaction
- $z_1 = x_1 \sin \alpha + y_1 \cos \alpha$ = Gear tangential direction displacement of shaft 1,
- $z_2 = -x_2 \sin \alpha + y_2 \cos \alpha$ = Gear tangential direction displacement of shaft 2,
- x_1 = X directional displacement of shaft i,
- y_i = Y directional displacement of shaft i,
- θ_i = Rotating angle of shaft I, r_i = Gear pitch circle radius of shaft i, k_g = Gear wheel rigidity

Using the above, an analytical model of the coupled lateral and torsional vibrations can be prepared. Figure 6 shows the analytical model of the coupled lateral and torsional vibrations of the total system. The input into the system is motor torque ripple, while the output from the system considers bearing vibrations, motor rotating angle, and motor coupling torque.

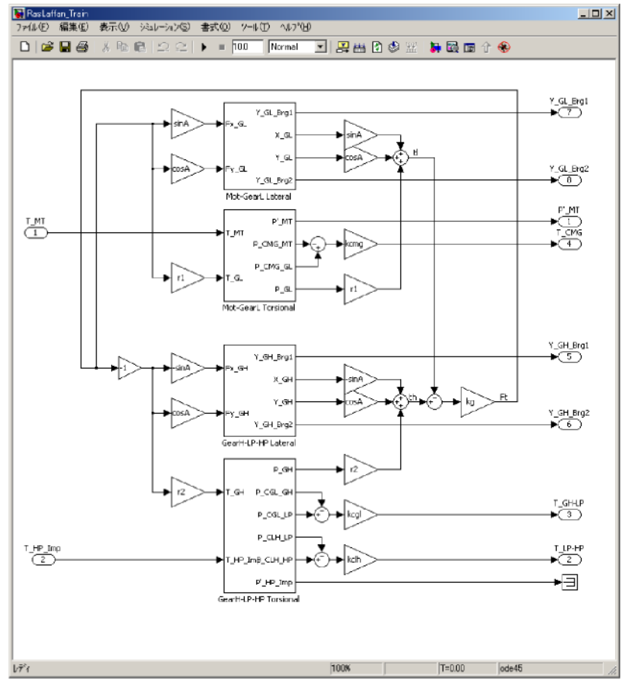


Figure 6. Analytical Model of Vibration of Total System.

In this model, the input is motor torque fluctuation, while the output is bearing vibrations, motor torsional vibrations, and the torque fluctuations of the coupling.

A vibration analysis was performed using the model below.

MEASURING RESULTS OF THE COMPRESSOR ROTOR VIBRATIONS

Occurrences of Torsional Vibration in Onsite Compressors

Large torsional vibrations were observed in the onsite compressors. This compressor train is composed of a variable inverter motor, a speed increasing gearbox, LP compressor, and HP compressors, as show in Figure 7. In order to confirm the magnitude of torsional vibration, actual field measurements were obtained.

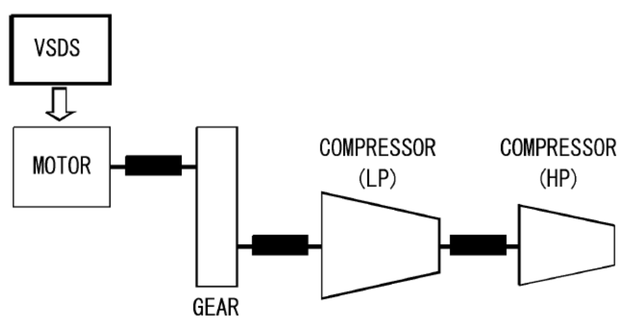
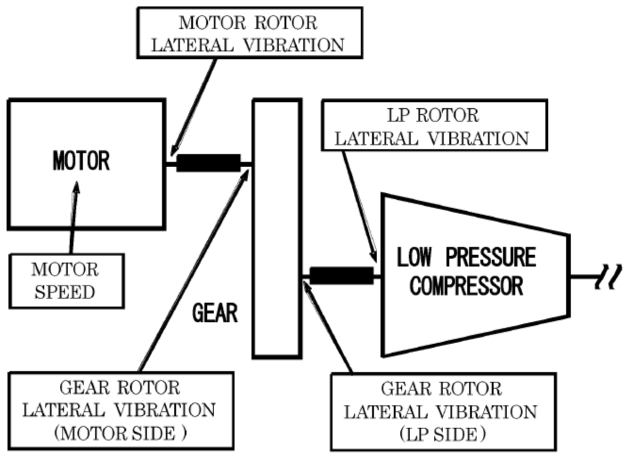


Figure 7. Composition of Fuel Gas Compressors Train.

Measuring Results of Vibrations

A spectral analysis was performed obtaining vibration measurement data on each part of the train during load operation. This was done in order to get to know in detail the conditions of the vibration. The data analysis was derived from the motor speed signal and the lateral vibrations of the motor shaft, the gear shaft, and the low-pressure compressor shaft reacting to torsional vibrations. In this configuration, the motor speed signal was taken from the control signal. For the lateral vibration, relative vibrations of axes were measured by use of an eddy-current type gap sensor. Figure 8 shows the measured items.



Rated Power	11.2 MW
Motor Speed	1470 RPM
Comp. Speed	6712 RPM

Figure 8. Measuring Items.

The results of the spectral analysis of the motor speed signal on startup of the train and the shaft vibration of each part of the compressors are shown in Figures 9, 10, 11 and 12. The rotating speed of the low-speed rotor at startup is 1050 rpm (17.5 Hz), while the high-speed rotor's speed is 4794 rpm (79.9 Hz). The vibration component of the rotating speed and its harmonics were observed at each spectrum. As the other major frequency component, responses were observed near 14 Hz in the gear rotors on the motor side and in the low-pressure compressor side, respectively. Conversely, vibrations of the motor rotor and the low-pressure compressor have hardly any asynchronous component.

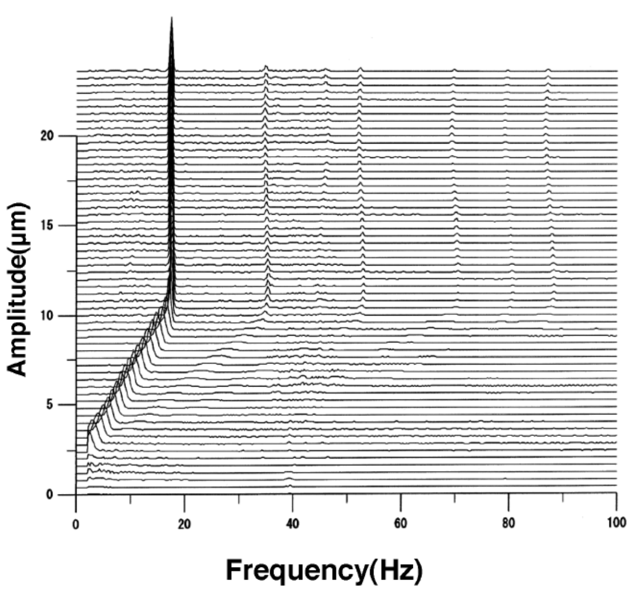


Figure 9. Motor Rotor Lateral Vibration at Startup Period.

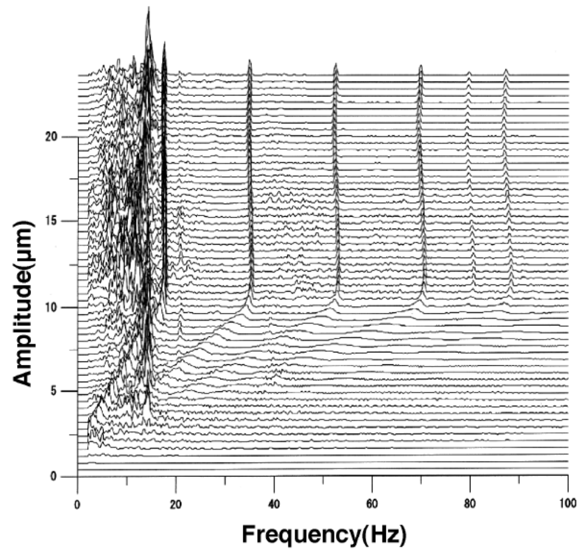


Figure 10. Wheel shaft Lateral Vibration at Motor Side at Startup Period.

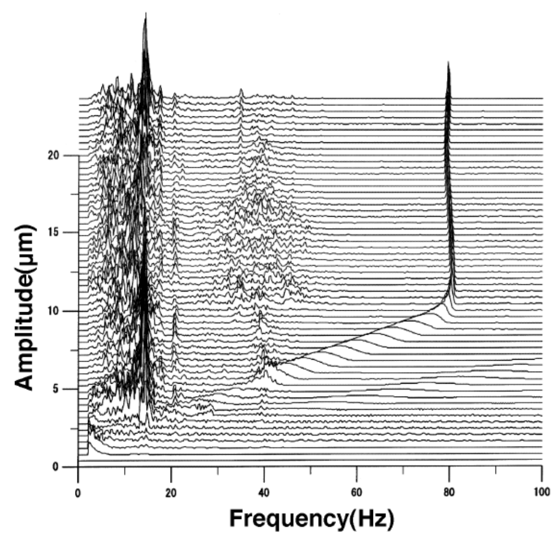


Figure 11. Pinion Shaft Lateral Vibration at LP Side at Startup Period.

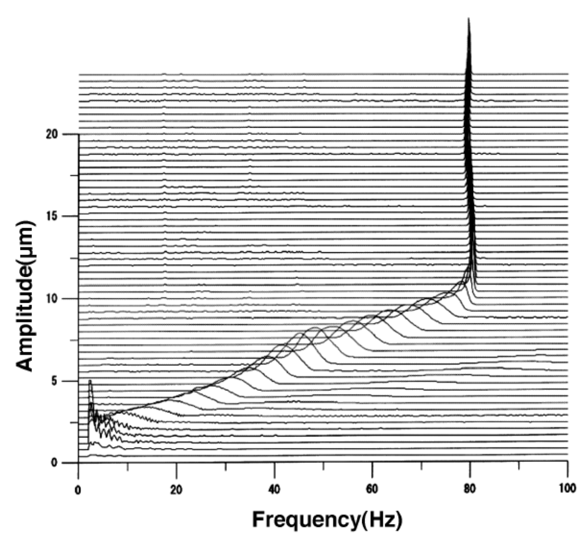


Figure 12. Pinion Lateral Vibration at Startup Period.

This component is excited by the coupled effect of lateral vibration and the first torsional natural frequency as shown in the results of analysis hereinafter described. The vibration mode of the coupled natural frequency shows noticeable responses in torsional vibrations at the end of the motor rotor and lateral vibrations of the gear.

Noticeable appearance of vibrations of the natural frequency of the first torsional mode are assumed to be due to cases where the system damping is lowering for some reason, or where electromagnetic or mechanical fluctuating torque acts on the shaft systems. The compressor rotor is not a factor in causing vibration torque mechanically near 14 Hz, so that the electromagnetic destabilizing effect and the fluctuating torque were presumed to be identified as the cause of asynchronous vibrations. Figures 13 and 14 show the results of spectral analysis of vibrations of the gear shaft at the time of its shutdown.

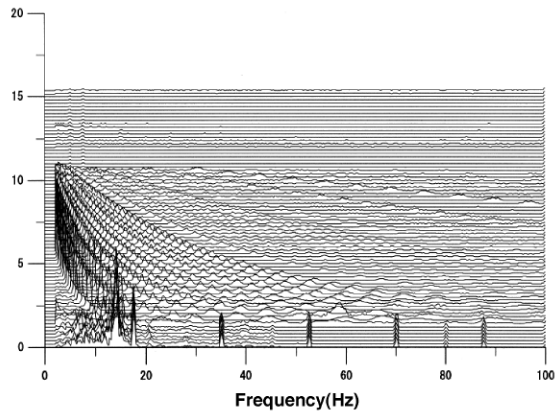


Figure 13. Wheel Shaft Lateral Vibration at Motor Side at Shutdown Period.

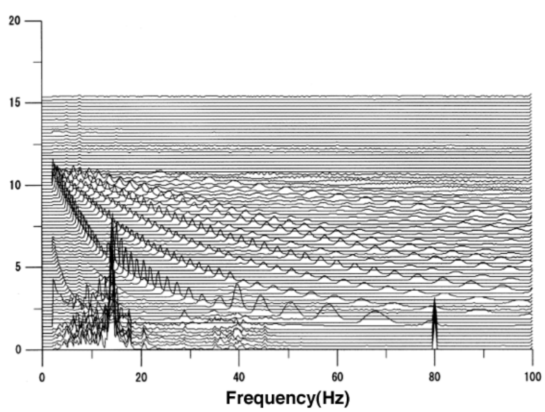


Figure 14. Pinion Shaft Lateral Vibration at LP Side at Shutdown Period.

Concurrently with the shutdown the power supply to the motor was cut off, and so the electromagnetic force on the motor rotor disappeared. The asynchronous component of vibration had vanished at the same moment with the shutdown as shown in the figure, and it was confirmed that asynchronous vibrations were caused by electromagnetic effect.

Analytical Results of Coupled Lateral and Torsional Vibrations

With the aim of a quantitative grasp of vibrations, the analytical method of coupled lateral and torsional vibrations was applied to the train. Table 1 shows the analytical results of the characteristic values with and without consideration of the coupled vibrations. Figure 15 shows the natural mode of torsion and lateral vibrations.

Table 1. Analytical Results of Characteristic Values.

		without consideration of the coupled vibrations	with consideration of the coupled vibrations	
stiffness of bearings of the gear	N/m		10e+8	10e+9
1st	Hz	13.4	12.4	13.4
2nd	Hz	21.7	21.8	21.8
3rd	Hz	37.9	30.6	37.4

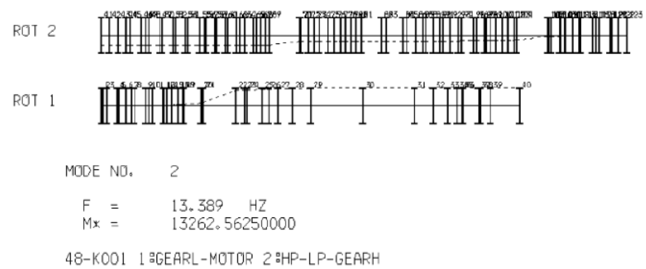


Figure 15. Vibration Mode.

The first torsional mode shown in Figure 15 has a large amplitude in the speed increasing gearbox, so it is assumed that the effects of coupled vibrations of torsion and lateral are observed noticeably. The frequency of asynchronous element vibrations shown in the results of measured vibrations of actual equipment is about 14 Hz. Clear-cut responses were confirmed in the lateral vibrations of the motor speed and gear shaft. It is also assumed that the vibrations in this mode were energized under the aforesaid conditions. The amplitude in this mode is small at the vibration measuring point of the motor shaft and low-pressure compressor shaft, and this feature corresponds with the results of vibration measurement of the actual equipment.

The coupled torsional and lateral vibrations were found to occur at the gear part; hence, the effect of the coupling is small for the mode where the gear part forms the node. Conversely, for the mode where the gear part forms the under part, the lateral vibration is energized by the torsional vibration, and so, it is necessary that vibration measurement shall be carried out according to the standpoint of coupled vibrations.

Comparison of Observation and Analytical Results

Figure 16 shows the analytical results of responses in cases where the torque fluctuation is added at the motor. It turns out that the torque fluctuation on the torsional primary mode energizes the lateral vibrations.

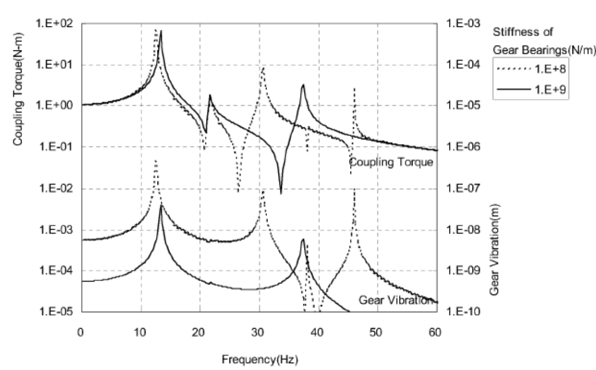


Figure 16. Analytical Results of Coupled Responses.

Figure 17 shows the first torsional natural frequency and the ratio of gear shaft vibration to coupling torque with stiffness of the gear bearings parameterized. In this case, the gear bearings stiffness was designed to be about 5e+8 N/m, so the first torsional natural frequency was calculated to be 13.2 Hz and the ratio between shaft vibration and coupling torque to be about 1.1e-9 m/N-m.

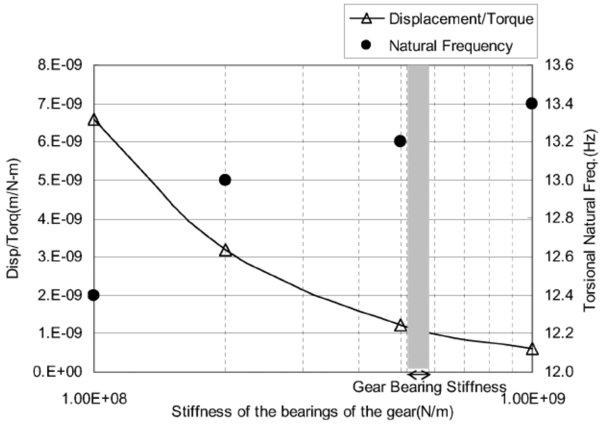


Figure 17. First Torsional Natural Frequency and the Ratio of Gear Shaft Vibration to Coupling Torque Versus Gear Bearings Stiffness.

Figure 18 shows the fast Fourier transform (FFT) analysis results of the shaft vibration and coupling torque measured at site. The dominant frequency is 14.0 Hz.

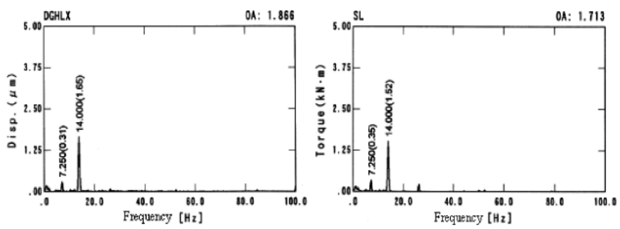


Figure 18. FFT Analysis Results of Shaft Vibration and Coupling Torque.

Table 2 shows a comparison of analytical and measured results of natural frequency and the ratio of coupling torque and lateral vibrations between the motor and the gear. As shown in the table, the analytical first torsional natural frequency is a little lower than the measured one, but both of the ratios almost coincide and it was confirmed that coupled responses are predictable with accuracy.

Table 2. Comparison of Response Levels.

		Analytical	Measured
First torsional natural frequency	Hz	13.2	14.0
Ratio of gear shaft vibration to coupling torque	m/N-m	1.1e-9	1.0e-9

Consequently, when using the results of a coupled analysis, the torsional vibration level, not usually measured, can be estimated with accuracy by the lateral vibration level measured by the shaft vibration sensor.

Results of Vibration Measurement after Countermeasures

After an evaluation of the vibration measurements and a parameter study of the electric system conducted by the motor manufacturer, it was decided that the control logic of the inverter needed to be changed as a countermeasure.

Figures 19 and 20 show the results of vibration measurements made after implementation of the countermeasures. As the natural frequency stays almost unchanged, the vibration component can be observed near 14 Hz. The authors therefore confirmed that both levels lowered significantly.

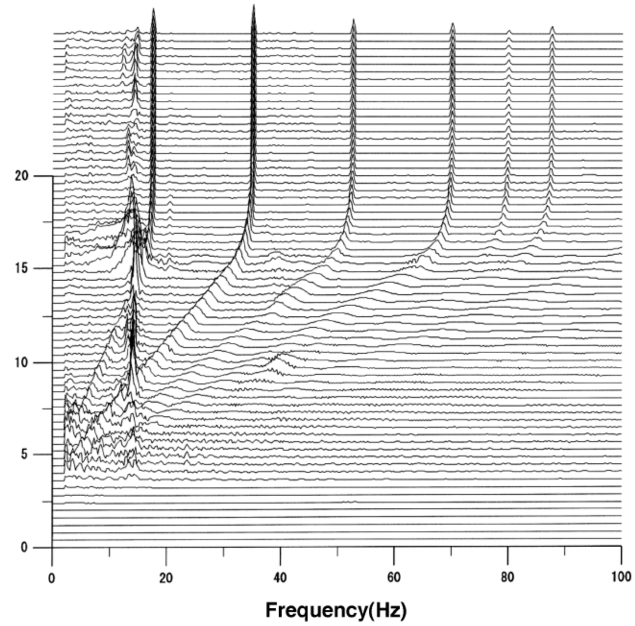


Figure 19. Wheel Shaft Lateral Vibration at Motor Side at Startup Period after Countermeasures.

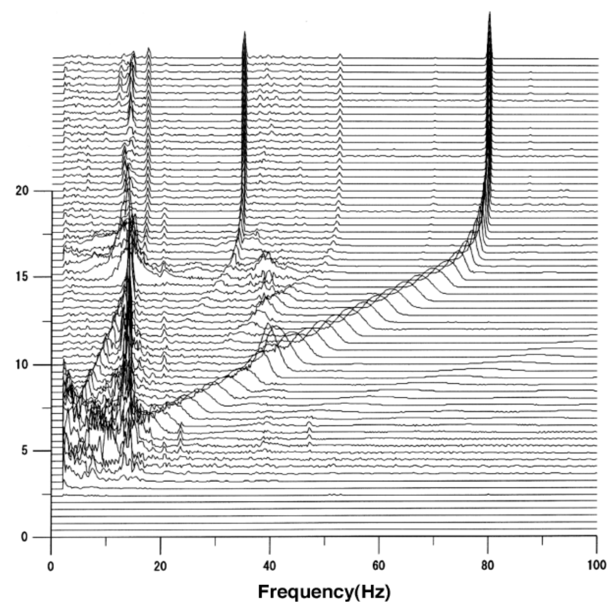


Figure 20. Pinion Shaft Lateral Vibration at LP Side at Startup Period after Countermeasures.

CONCLUSIONS

The analysis method for rotor vibrations of a compressor train with a gearbox was shown from a perspective of coupled lateral and torsional vibrations, and this method was applied to evaluate an actual rotor vibration. The measured results of the compressor rotor vibrations were examined based on the analytical results.
A summary of the results is as follows:

- The torsional and lateral vibrations interact with each other depending on the vibration mode at the gear part. Therefore, lateral vibration might be excited by coupled vibration modes that do not appear by lateral vibration analysis alone.

• In the mode where lateral and torsional vibrations are significantly coupled, the response ratio of torsional and lateral vibrations almost corresponds with the analytical results in actual measurement data, which included various measured errors. Therefore, it is possible to estimate the torsional vibration level with data from the shaft vibration probe.

BIBLIOGRAPHY

API Standard 617, 2002, "Axial and Centrifugal Compressors and Expander-Compressors for Petroleum, Chemical and Gas Industry Services," Seventh Edition, American Petroleum Institute, Washington, D.C.

API Standard 684, 2005, "Tutorial on Rotordynamics: Lateral Critical, Unbalance Response, Stability, Train Torsional and Rotor Balancing," Second Edition, American Petroleum Institute, Washington, D.C.

Iannuzzelli, R. J. and Elward, R. M., 1985, "Torsional/Lateral Coupling in Geared Rotors," International Journal of Turbo Jet-Engines JST, *2,* (1), pp. 11-18.

Schwibinger, P. and Nordmann, R., 1986, "Influence of Torsional-Lateral Coupling on Stability Behavior of Geared Rotor Systems," NASA Conference Publication, pp. 531-553.

Simmons, P. E., 1982, "The Response of a Geared Compressor Set to Torsional Excitation Accounting for Damping and Flexibility in the Bearings and Damping and Backlash in the Gear," ASME Paper, p. 9P.

Shiraki, K. and Kanki, H., 1979, "A New Vibration Criteria for High Speed Large Capacity Turbomachinery," *Proceedings of the Eighth Turbomachinery Symposium,* Turbomachinery Laboratory, Texas A&M University, College Station, Texas, pp. 59-70.

Viggiano, F. and Wattinger, W., 1994, "A Torsional Damping Model for Geared Compressor Shaft Trains," Vibration Fluid Machinery, pp. 1-6.

Wachel, J. C. and Szenasi, F. R., 1980, "Field Verification of Lateral-Torsional Coupling Effects on Rotor Instabilities in Centrifugal Compressors," Texas A&M University Workshop, NASA CP2133, pp. 15-34.