Case Study, 38th Turbomachinery Symposium

Title:
Torsional – Lateral Coupled Vibration of Centrifugal Compressor System at Interharmonic Frequencies Related to Control Loop Frequencies in Voltage Source PWM Inverter

Abstract:
Centrifugal compressor train in a refinery experienced high vibration problem due to torsional resonance. Sidebands in the VFD output current based on VFD control loop frequencies were identified as the root cause. In this VFD, stator current was used for torque and speed control, hence control loop frequencies had a potential to generate such sidebands. Frequencies of this type of sidebands widely vary with the rotation speed (proportional to harmonics of the fundamental frequency), hence it is difficult to avoid resonance at the train torsional natural frequency. In addition, even if a compressor system is proven to have sufficient safety margin against high cycle fatigue failure due to the torque pulsation by this mechanism, such minute torque pulsation may have a potential to excite high lateral vibration at speed adjusting gear. If unpredicted or overlooked during design stage, such high vibration may disturb plant operation. This case study therefore proposes guidelines to predict such vibration levels by a simplified torsional-lateral coupled vibration analysis.

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High vibration was sometimes observed at gear LS shaft.
Onset of Problem

Lateral Vibration on Low Speed Gear Shaft

50μm (p-p) Vibration Detected on LS Gear Shaft around 1040-1140rpm (17.3-19.0rps), 570kW

Dominant Frequency Component: ca. 16Hz

Close to Calculated 1st Torsional Natural Freq. \( f_{n1} \) (~15.71Hz)

No Significant Vibration on Compressor Shaft or Motor Shaft

Figure 2. Cascade Plot of LS Gear Shaft Vibration
Onset of Problem

Lateral Vibration on High Speed Gear (Pinion) Shaft

Max. 10μm (p-p) Vibration with 16Hz Detected on HS Gear Shaft

Lower Vibration on HS Gear Shaft

Stiffer Pinion Bearings Than Bull Gear Bearings at Part-Load Due to Uploading Bull Gear and Downloading Pinion

Pinion Lighter Than Bull Gear

Concern: What is the Excitation Source?

Figure 3. Cascade Plot of HS Gear Shaft Vibration

\[ f_{n1} = 16 \text{Hz} \]
Torsional Natural Frequencies

Analysis for Torsional Natural Frequencies (FEM)

Figure 4. Campbell Diagram for Torsional Vibration

First natural frequency: 942.6cpm (15.71Hz)

Second natural frequency: 2752cpm (45.87Hz)

Min. operating speed (LS shaft): 1000rpm

Max. operating speed (LS shaft): 1500rpm

Figure 5. Torsional Vibration Mode Shapes

First Natural Frequency: 15.71Hz (943cpm)

Second Natural Frequency: 45.87 Hz (2752cpm)

Third Natural Frequency: 422.1Hz (25324cpm)

Motor

Gear

Compressor

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### Critical Speeds Predicted by Rotor Analyses

#### Lateral & Torsional Vibration Calculation

#### Critical Speeds [rpm]

<table>
<thead>
<tr>
<th></th>
<th>First</th>
<th>Second</th>
<th>Operating Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lateral</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compressor</td>
<td>5220</td>
<td>17700</td>
<td>8588-12882</td>
</tr>
<tr>
<td>Pinion (HS Shaft)</td>
<td>21200-25600</td>
<td>-</td>
<td>8588-12882</td>
</tr>
<tr>
<td>Bull Gear (LS Shaft)</td>
<td>3380-9450</td>
<td>-</td>
<td>1000-1500</td>
</tr>
<tr>
<td>Motor</td>
<td>2078</td>
<td>-</td>
<td>1000-1500</td>
</tr>
<tr>
<td><strong>Torsional</strong></td>
<td>943</td>
<td>2752</td>
<td></td>
</tr>
</tbody>
</table>

Shaft Synchronous Resonance is Excluded. Only Possibility = VFD?

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Estimation of Shaft Torque and Motor Torque

Estimation of Shaft Torque Causing 50μm Shaft Vibration

Shaft Torque Deduced by One-Way Lateral-Torsional Analysis

Force Applied on Teeth Causing 50μm (p-p) Shaft Vibration

Transverse Pressure Angle $\theta$

$\theta = \tan^{-1} \left( \frac{\tan \alpha}{\cos \beta} \right)$

$\alpha =$ Normal Pressure Angle

$\beta =$ Helix Angle

$T_t = \left( \frac{PCD}{2} \right) \times F_t$

$= \left( \frac{PCD}{2} \right) \times F_n \times \cos \theta$

Figure 6. Force Applied on Gear Teeth (Double Helical)
Estimation of Shaft Torque and Motor Torque

Lateral Vibration Analysis of LS Gear Shaft

Unit Excitation on Tooth Surface
\[ F_{n,p.u.} = 9.8 \cdot \sin(2\pi f_{n1}t) \, [N] \]

LS Gear Shaft Vibration Amplitude at Probe @ 16Hz
\[ = 0.101 \mu m \, (p-p) \, (Calculated) \]
\[ F_n = 9.8 \times (50/0.101) = 4851 \, [N] \]
\[ T_t = \frac{(PCD/2)}{F_n} \times \cos \theta \]
\[ = (0.806/2) \times 4851 \times \cos 21.9^\circ \]
\[ = 1815 \, [N \cdot m] \]
\[ \rightarrow 0.213 \, \text{Times Rated Torque} \]

Figure 7. Frequency Response to Unit Excitation Force of Gear Shaft
(Bearing Stiffness and Damping Calculated @ 1140rpm, 570kW)
Estimation of Shaft Torque and Motor Torque

Estimation of Excitation Torque at Motor Air Gap

Amplification Factor for Torsional System

Figure 8. Frequency Response at LS Gear Shaft Assuming $\zeta = 0.02$

$$T_{AG} = \frac{1815}{16.9} = 107 \text{ [N} \cdot \text{m]}$$

1.3\% Rated Torque at Motor Air Gap Suspected.

Close to Maximum Measured Data among Interharmonic Frequencies during Factory Test (1.5\% Rated Torque)

Merely 1.3\% of air gap torque fluctuation can cause 50 \(\mu\)m p-p lateral vibration!
Cause and Effect

**Effect**
- LS Gear Shaft Vibration
  - 50μm
  - 16Hz
- Effective Force on Bull Gear Teeth
  - 4854N
  - 16Hz
- Torque at Bull Gear Teeth
  - 1815Nm
  - 16Hz

**Cause**
- Air Gap Torque on Motor Shaft
  - 107Nm
  - (1.3%) 16Hz

**Actual Cause and Effect**
- Effective Force on Bull Gear Teeth
- Torque at Bull Gear Teeth

**Sequence of Lateral-Torsional Analysis**
- Air Gap Torque on Motor Shaft
- Torsional Frequency Response
  - (ζ=0.02, Geometry)
Strength Evaluation

High Cycle Fatigue Evaluation

Figure 9. Modified Goodman Diagram

Mechanical Strength Verified

No Modification Made on Machinery or VFD

Fatigue Factor of Safety 1.25 for $10^7$ Cycle Life

Stress at the Condition

Mean stress $\sigma_m$ [N/mm²]

Alternating stress $\sigma_a$ [N/mm²]
Investigation of Source of Excitation Torque

Detailed Measurement (LS Gear Shaft Vibration)

Figure 10. Cascade Plot of LS Gear Shaft Vibration
Investigation of Source of Excitation Torque

Detailed Measurement (VFD Output Current)

- $f_c = 1024 \text{Hz}, n=25$, slope: +25
- $f_c = 1024 \text{Hz}, n=23$, slope: -23
- $f_c = 1024 \text{Hz}$, $n=29$, slope: -29
- $f_c = 256 \text{Hz}$, $n=7$, slope: +7

Fundamental Frequency

- $f_{n1} = 16 \text{Hz}$

Figure 11. Cascade Plot of VFD Output Current

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Pattern of Interharmonic Frequency Component

Inclined Streaks of Interharmonic Frequencies in Shaft Vibration Frequencies and VFD Output Frequencies

- LS Shaft Vibration Frequency Content
  Difference of Harmonics of Multiples of 6 and Sampling Frequencies of VFD (1024Hz, 256Hz)

- VFD Output Current Frequency Content
  Difference of Harmonics of Odd Numbers Other Than Multiples of 3 and Sampling Frequencies of VFD
  Inclination Opposite to That of Shaft Vibration

↓

Firm Correlation Between Shaft Vibration and VFD Output Current Suspected
Pattern of Interharmonic Frequency Component

Relation of Between Shaft Vibration Frequencies and VFD Output Frequencies

VFD Output Freq. [Hz]:
\[ f_{bi} = |f_c - nf| \]

Shaft Vibration Freq. [Hz]:
\[ f_{bt} = |f_c - (n \pm 1)f| \]

f : VFD Fundamental Freq. [Hz]
f_c : Arbitrarily Existing Constant Freq. [Hz]
n : Positive Odd Integer Other Than 3

Frequencies of Fluctuating Torque Generated by 3-Phase IM

\[ T_e = pM'I_{s_a}l'_{r} \cdot (9/4) \cdot \sin(2\pi(f_a - f)t + \gamma) \]

\[ f_a : \text{Arbitrarily Existing Current Freq. [Hz]} \]

If \[ f_a = f_{bi} = |f_c - nf| \]
\[ T_e = pM'I_{s_a}l'_{r} \cdot (9/4) \cdot \sin(2\pi(|f_c - (n \pm 1)f|)t + \gamma) \]

Shaft Vibration Caused by Excitation of Motor Torque
Sampling in VFD

VFD Control Loop

Several Sampling Frequencies Used in VFD Control Loop

Figure 12. Block Diagram of VFD Control System
Assumed Cause of Sideband in VFD Output Current

Coarse Pulse of Fundamental Frequency Remained in Current (e.g. Improper Dead Time Compensation)

Harmonics of Odd Numbers Produced by Pulse (Rectangular Wave) of Fundamental Frequency

Harmonics of Multiple of 3 Eliminated in Balanced Three-Wire System

Sideband Frequencies Raised by Modulation between Harmonic Frequencies and Sampling Frequencies Occurred in VFD Control Loop

Harmonics enhance sidebands.
Other Possible Cause of Sideband

**Sideband Frequencies Due to PWM**

Sum and Difference of Frequencies of Harmonics of Triangular Carrier Wave (4.8kHz) and Harmonics of Signal Wave (Fundamental Frequency)

Sideband Frequencies Due to PWM Not Observed in This VFD Output Current

**Figure 13. Mechanism of PWM**
Concluding Remarks

- Measurement of torque & current in factory test is important in case VFD characteristics are unknown.
- Strength evaluation by lateral-torsional analysis is essential to determine mechanical soundness.
- Information of any possible frequencies used in VFD control and the resulting amplitude of torque pulsation should be disclosed in advance by VFD vendor.
- Reduction of amplitude of fundamental frequency harmonics would decrease amplitude of sideband frequencies.