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A Technique for Programming Cuts and Fills when Grading Land for Irrigation

.10'/100'

.15'/100'

											3+	4+	5+	6+	5+	5+	5+	+	2*
										4+	3+	4+	6+	5+	5+	2+	1+	1*	3*
										9+	8+	4+	5+	5+	4+	3+	2+	1*	2*
										7+	8+	4+	5+	3+	3+	1+	1*	1*	2*
									2+	6+	6+	4+	3+	4+	2+	*	+	1*	3*
								3*	1*	*	*	2+	2+	3+	2+	1*	2*	2*	3*
							5*	4*	2*	+	*	1+	2+	2+	+	1*	2*	3*	4*
						9*	7*	5*	5*	3*	1*	+	*	1+	1*	3*	3*	5*	5*
						9*	7*	5*	3*	2*	2*	1*	*	*	1*	3*	5*	5*	5*
					9*	7*	5*	5*	4*	4*	2*	2*	+	+	+	2*	4*	5*	6*
				7*	5*	6*	6*	5*	4*	1*	2*	*	2+	3+	2+	2+	2*	6*	6*
6*	6*	6*	5*	4*	5*	5*	5*	4*	4*	3*	1*	+	3+	4+	4+	2+	+	3*	5*
						3*	3*	3*	3*	1*	1*	1+	3+	4+	5+	4+	2+	1*	4*
							2*	1*	2*	*	2+	3+	4+	4+	3+	4+	3+	1+	5*
								+	+	1+	2+	3+	2+	5+	4+	3+	7+	+	2*
									2+	2+	1+	2+	3+	5+	4+	3+	1+	1+	3+

*Cuts +Fills. Where no number is given, cut or fill is less than .1 foot.

Field layout in 100-foot blocks with cut and fill quantities for leveling in tenths of feet.

TEXAS AGRICULTURAL EXPERIMENT STATION
R. D. LEWIS, DIRECTOR, COLLEGE STATION, TEXAS

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A TECHNIQUE FOR PROGRAMMING CUTS AND FILLS WHEN GRADING LAND FOR IRRIGATION

Ray Billingsley, Associate Professor
Department of Agricultural Economics
Texas Technological College, Lubbock, Texas

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SUMMARY

This report describes and illustrates a technique for selecting a least-cost grading program when leveling land for surface irrigation. The technique is to be used after all of the engineering steps for a land-grading survey have been made. It consists of selecting a program of dirt movement that will move all of the dirt in the cut areas to the fill areas at a minimum cost. The minimum cost dirt movement program is arrived at by a series of trial program changes and an optimum least-cost program has been reached when there exists no change in the program that will result in a decrease in the total cost of making the specified cuts and fills.

The cost referred to in this report consists of cubic-yard-feet. The optimum dirt movement program is the one that moves the dirt measured in cubic yards the least number of feet.

After the optimum dirt moving program is selected, it is possible to compute the total cost of moving the dirt in terms of cubic-yard-feet and these in turn can be converted into a dollar-and-cent figure if the cost of moving a cubic yard of dirt 1 foot is known. This figure is of benefit to both contractors and farmers in that it permits better decision making.

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INTRODUCTION

Linear programming is a mathematical technique that can be used to select a least-cost grading program when moving dirt. This report describes and illustrates how this technique can be used when grading land for surface irrigation.

Land leveling consists of grading and smoothing the land. When irrigating, this practice can result in improved soil and water use. Generally, the land is not made level but usually is graded with gentle uniform slopes. Better irrigation will result from land leveling. The uniform grades allow fields to be organized into irrigation runs of the proper length. This results in a more nearly even stand, improved crop yield and savings in irrigation water, labor and other expenses. Also erosion is prevented and soil fertility is saved. When an uneven field is irrigated, the high spots are watered too little and the low spots too much. This alone is enough to make the crop spotty and reduce the yield.

Before cuts and fills can be programmed, it is necessary to perform all of the engineering steps for a land-grading survey. Acceptable procedures to follow are known by most agricultural and civil engineers and discussions of the procedure are given in many bulletins and books.[1/] [2/] The amount of cut and fill in each area must be determined before the dirt movements can be programmed. The cover page is an example of a topographic map of a field where the cuts and fills have been determined in 100-foot blocks to make a uniform grade to the south and west. The irrigation well is located at the northweat corner of the field.

The dirt programming problem basically is this: "How can the dirt be moved from each cut area and deposited in each fill area so that the least cost is experienced?" Cost may be measured in either travel distance or time per unit of dirt moved. If different costs are involved for different types or depths of cuts and fills, then the cost may be determined in dollars.

ANALYTICAL FRAMEWORK

Basically the linear programming technique consists of selecting a program of dirt movements that will move all of the dirt in the cut areas to the fill areas at a minimum cost.[3] A program, in this case, refers to all the movements necessary to get the dirt from the cut to the fill areas.[4]

To minimize the dirt movement costs, it is necessary to know the cost and the amount of dirt to be moved from each cut area to each fill area. To use linear programming to arrive at an optimum solution, the amount of cut and fill must be adjusted so that they are equal. This may be done by adjusting the fill amounts by a constant ratio so that when totaled, the amount of fill equals the amount of cut shown in Table 1.

TABLE 1. CUTS AND FILLS NECESSARY TO LEVEL FIELD

Cuts		Fill		
			Cubic yards	
Area	Cubic yards	Area	Unadjusted	Adjusted[1]
11	3,404	21	2,072	2,421
12	1,306	22	2,712	3,168
13	2,227	23	744	869
14	1,136	24	981	1,146
15	740	25	618	722
16	973	26	1,369	1,599
17	1,306	27	1,632	1,907
18	1,195	28	389	455
Total	12,287	Total	10,517	12,287

1/ Adjusted cubic yards of fill obtained by $f'_i = \frac{C}{F} f_i$, so that F' = C.

Where: f = planned fill for each area
f'= adjusted fill for each area

$$F = \sum_{i=1}^{n} f_i = \text{total planned fill}$$

C = Total cuts

$$F' = \sum_{i=1}^{n} f'_i = \text{total adjusted fill.}$$

Each area in the topographic map must be given an identifying number. The identifying numbers for the cuts in this example start with one, while the identifying numbers for the fills start with two. The cost (which can be measured in dollars, distance or time) to move a unit of dirt from each cut area to each fill area must be set up in tabular form consistent with the dirt program movement table by listing the cut areas along the side and the fill areas across the top, Table 2. In this way, each cell will contain the cost of moving a unit of dirt from a cut area to a fill area.

TABLE 2. DISTANCE FROM EACH CUT AREA TO EACH FILL AREA IN UNITS OF 10 FEET

Cuts	Fill areas							
areas	21	22	23	24	25	26	27	28
11	99	128	153	107	80	101	108	128
12	47	74	102	57	86	82	102	117
13	68	91	114	66	60	64	80	99
14	96	118	140	91	44	66	71	98
15	87	60	34	63	144	103	129	110
16	86	70	60	53	105	64	88	69
17	92	83	79	59	86	44	67	48
18	117	110	106	84	80	42	53	25

The optimum program is arrived at by a series of trial program changes with each program change representing a decrease in the total cost of the transportation program. An optimum least-cost program has been reached when there exists no change that will result in a decrease in the total cost of satisfying the specified cuts and fills.

COMPUTATIONAL TECHNIQUE AND PROCEDURE

The computational technique is essentially one of starting with any program of dirt movements that satisfies the restriction that the sum of the columns equals the sum of the rows. In some cases, however, simplifying the problem by combining areas will aid computation.

In field leveling problems where many cut and fill areas are involved, the computational task becomes cumbersome. If electronic computer facilities are available, this presents no problem because large problems can be solved quickly and at relatively small cost. Medium-size computers, such as the IBM 650, can handle problems of this type up to 200 rows by 200 columns. No field leveling problem is visualized that would be this large and, if ever encountered, it probably could be broken down easily into manageable parts.

Simplifying by grouping areas. If no computer facilities are available, however, and the area to be leveled is small, close approximation to optimum dirt program movements can be obtained by combining the adjacent cut and fill areas so that no more than 8 to 12 of each occur as in Figure 1. As a general guide, the amount of data that must be manipulated increases in similar fashion to the area of a square and the length of one side.

Step by step procedure. As an example of how the optimum dirt program movement is accomplished, the following steps illustrate the least-cost program dirt movements in cubic-yard-feet to level the field shown on the cover.

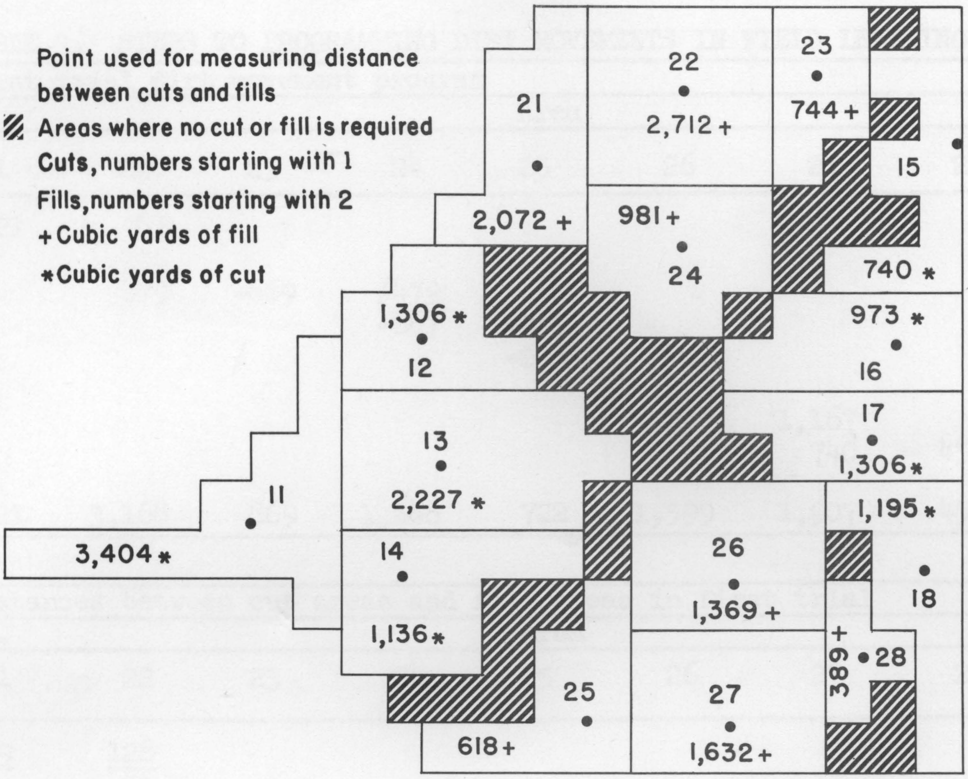

Figure I. Cut and fill areas after adjacent areas are combined to simplify computation.

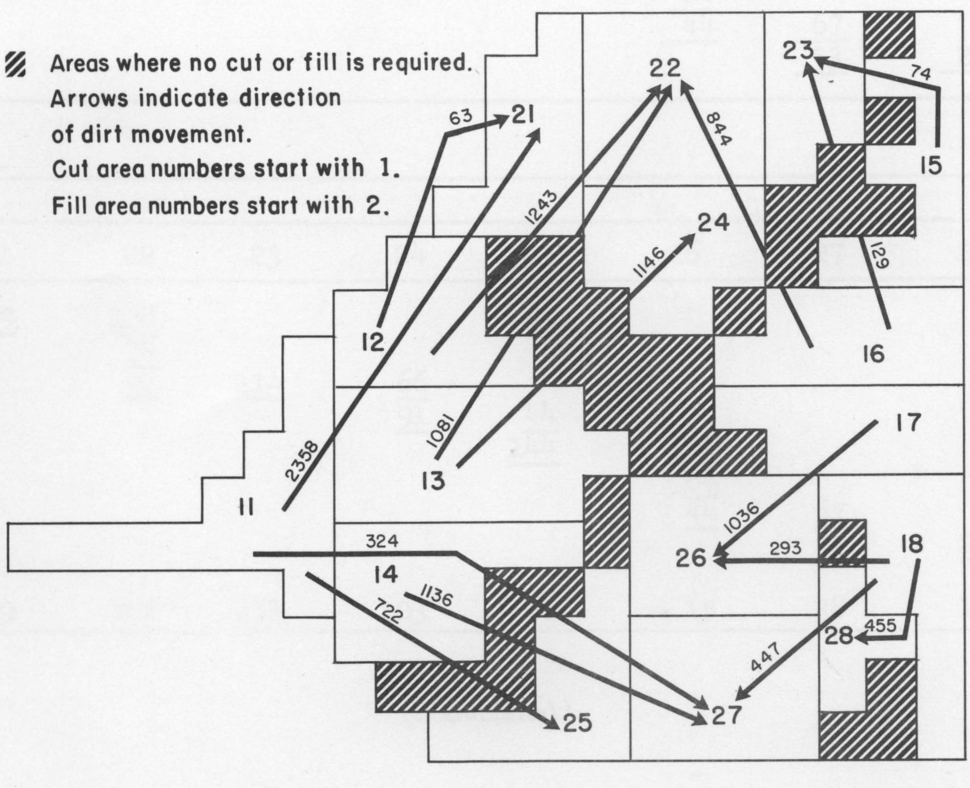

Figure 2. Optimum least-cost program of dirt in cubic yards.

TABLE 3. STEPS TO PROGRAMMING DIRT MOVEMENTS IN FIELD LEVELING

a.	First trial dirt movement program								
Fills-cuts area	Area								
	21	22	23	24	25	26	27	28	Total
11	2,421	983							3,404
12		1,306							1,306
13		879	-869	/479					2,227
14				-667	/469				1,136
15			/		-253	487			740
16						973			973
17						139	1,167		1,306
18							740	455	1,195
Total	2,421	3,168	869	1,146	722	1,599	1,907	455	12,287

b.	Distances between cut areas and fill areas in first trial								
Fills-cuts area	b_1. Area								
	21	22	23	24	25	26	27	28	Total
11	99	128							
12		74							
13		91	114	66					
14				91	44				
15					144	103			
16						64			
17						44	67		
18							53	25	

b_2.

Fills-cuts area	21	22	23	24	25	26	27	28	Total
11	99	128							0
12		74							-54
13		91	114	66					-37
14				91	44				-12
15					144	103			88
16						64			49
17						44	67		29
18							53	25	15
	99	128	151	103	56	15	38	10	

(continued)

Table 3. Continued

Fills-cuts area	b_3. Area 21	22	23	24	25	26	27	28	Total
11	99	128	151	103	56	15	38	10	0
12	45	74	97	49	2	-39	-16	-44	-54
13	62	91	114	66	19	-22	1	-27	-37
14	87	116	139	91	44	3	26	-2	-12
15	187(100)	216(156)	239(205)	191(128)	144	103	126	98	88
16	148	177(107)	200(160)	152(99)	105	64	87	59	49
17	128(36)	157(74)	180(101)	132(73)	85	44	67	39	29
18	114	143(33)	166(60)	118(34)	71	0	53	25	15
	99	128	151	103	56	15	38	10	

c. Fills-cuts area	First change in dirt movement program. Area 21	22	23	24	25	26	27	28	Total
11	2,421	983							
12		1,306							
13		879	-616	+732					
14				-414	722		+		
15			+253			-487			
16						973			
17						+139	-1,167		
18							740	455	

d. Fills-cuts area	First change in route distance. Area 21	22	23	24	25	26	27	28	Total
11	99	128	151	103	56	220(101)	243(135)	215(87)	0
12	45	74	97	49	2	166(84)	189(87)	161(44)	-54
13	62	91	114	66	19	183(119)	206(126)	178(79)	-37
14	87	116	139	91	44	208(142)	231(160)	203(105)	-12
15	-18	11	34	-14	-61	103	126	98	-117
16	-57	-28	-5	-53	-100	64	87	59	-156
17	-77	-48	-25	-73	-120	44	67	39	-176
18	-91	-62	-39	-87	-134	30	53	25	-190
	99	128	151	103	56	220	243	215	

The intermediate steps are left out with the final step following.

(continued)

Table 3. Continued

e. : Final route distance

Fills-cuts area	Area 21	22	23	24	25	26	27	28	Total
11	99	126	116	101	80	97	108	80	0
12	47	74	64	49	28	45	56	28	-52
13	64	91	81	66	45	62	73	45	-35
14	62	89	79	64	43	60	71	43	-37
15	17	44	34	19	- 2	15	26	- 2	-82
16	43	70	60	45	24	41	52	24	-56
17	46	73	63	40	27	44	55	27	-53
18	44	71	61	46	25	42	53	25	-55
	99	126	116	101	80	97	108	80	

f. : Final dirt movement program

Fills-cuts area	Area 21	22	23	24	25	26	27	28	Total
11	2,358				722		324		3,404
12	63	1,243							1,306
13		1,081		1,146					2,227
14							1,136		1,136
15			740						740
16		844	129						973
17						1,306			1,306
18						293	447	455	1,195
Total	2,421	3,168	869	1,146	722	1,599	1,907	455	12,287

Step 1. From Table 1, using the adjusted cubic yards of fill, set up the first dirt-movement program, satisfying each of the row and column restriction by starting in the upper left hand corner and progressing to the lower right hand corner. (See Table 3_a.) Notice that the cut areas are listed at the side of the table and the fill areas along the top. Notice also that the totals at the ends of the rows and columns of Table 3_a conform to the cubic yards of cut and fill shown in Table 1.

Step 2. In Table 3_{b1} set up the distance for moving a unit of dirt from each cut area to each fill area. These are called route costs. This is done by recopying the elements from the distance table, Table 2, that correspond with the dirt movements of the first feasible program, Table 3_a. These are shown in Table 3_{b1} as the elements of the table that are underlined.

In practice it is suggested that a different colored pencil be used for these values because all of the steps shown in b_1, b_2 and b_3 can be combined into one table as they are done later in Table 3_{b3}.

Step 3. Determine row and column border elements for the route distance table by starting with 0 at the end of the first row and adding it to the intersecting element or elements in the first row and placing the sum at the bottom of the corresponding column or columns. Following in Table 3_{b2}:

0 + ? = 99, 99 goes at bottom of first column:
0 + ? = 128, 128 goes at bottom of second column;
128 + ? = 74, -54 goes at end of second row.

Continue in this manner until all row and column border elements are filled as in the row and border elements of Table 3_{b2}.

Step 4. The next step is to finish filling in the route cost table by adding the corresponding row and column border elements as in Table 3_{b3}. This is done by placing the sum of the border elements in the intersecting cells.

Step 5. The next step is to substract each element in Table 2 from the corresponding elements in Table 3b3 and select the largest non-negative value. All negative values may be ignored. In Table 3b3, these are the values in parentheses.

Step 6. Next, place in the program table, Table 3_a, a + in the cell corresponding with the cell in the route cost table with the largest non-negative value, the values in parentheses. In Table 3_{b3}, this is (205) and so a + is placed in the corresponding cell in Table 3_a.

Step 7. Consider the + as a number and locate all cells affected by placing a - or a + in each non-zero cell so that the basic row and column restriction is not violated.

Determine the amount the program is to be adjusted by selecting the smallest positive value that has a - in its cell.

Step 8. Repeat steps 3 through 7 until no positive values show up when the cells in Table 2 are subtracted from the corresponding cells in the route distance table.

INTERPRETING RESULTS

The dirt program movements shown in Table 3_f represents the program with the minimum cubic-yard-feet. No other program is possible with this set of data that will have fewer cubic-yard-feet; but there may be other programs that equal or are very close to this figure. A schematic presentation of the optimum program of dirt movement is shown in Figure 2. The arrows and numbers in the figure show the direction and amount of dirt to be moved from each cut area to a fill area. That would result in the lowest total cost in this particular cut and fill problem.

To obtain the total cost of moving this amount of cut and fill for this problem, simply multiply the non-zero elements in Table 3 by the corresponding elements in Table 3_e and sum. The least cost dirt movement program in this problem is obtained by multiplying and summing (99)(2,358) + (47) (63)+...+(25) (455) = 8,726,160 cubic-yard-feet. If the cost of moving a cubic yard of dirt 1 foot is known, the total cost can be determined by multiplying the unit cost times the total cubic-yard-feet.

Following the relatively simple procedure outlined in this report can result in benefits both to contractors and people having land leveled. By having a better estimate of the actual cost, contractors can bid closer on a particular job and farmers can better evaluate whether the benefits derived from leveling will be worth the cost.

ACKNOWLEDGMENTS

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REFERENCES

1/ James C. Marr, "Grading Land for Surface Irrigation," California Agricultural Experiment Station, Circular 438, July, 1957.

2/ John G. Bamesberger, "Land Leveling for Irrigation," U. S. Department of Agriculture Leaflet No. 371.

3/ For a mathematical formulation of the analytical framework, see T. C. Koopman's (ed.), "Activity Analysis of Production and Allocation," Cowles Commission for Research and Economics, Monograph No. 13, (New York; John Wylie & Son), 1951.

4/ The dirt programming problem can be illustrated by the following notation:

i Fills $_j$ -: cuts :	(1)	(2)	...	(n)	: Total
(1)	x_{11}	x_{12}	...	x_{1n}	a_1
(2)	x_{21}	x_{22}	...	x_{2n}	a_2
...	...	...	...	...	...
(m)	x_{m1}	x_{m2}	...	x_{mn}	a_m
Total	b_1	b_2	...	b_n	$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_i$

The x_{ij} values which are quantities of dirt that are to be moved, must be chosen so that the rows and columns sum to the marginal totals a_i and b_j. The basic relations to be satisfied are:

(1) $\sum_{j=1}^{n} x_{ij} = a_i \quad (i = 1, 2, \ldots, n)$

(2) $\sum_{i=1}^{m} x_{ij} = b_j \quad (j = 1, 2, \ldots, m)$

(3) $x_{ij} = 0$

(4) $\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} = \min.$