DOWNHOLE WIRELINE MECHATRONICS AND DRILLSTRING

VIBRATION DYNAMICS

A Thesis

by

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ABSTRACT

The work is divided into two parts: The First part discusses and documents simulation investigations on the interactive effect of the different conveyance accessories tools and their designs on the wireline cable tension force, and how it can influence the wireline cable performance especially in open hole logging operations with highly deviated and rugose zones. A computer simulation model was built to predict the cable tension force applied on conveyed wireline string so as to assess and analyze the effect of the different conveyance accessories, such as centralizers, bottom-nose tools, and wireline coating, on wireline penetration rates. A numerical computing approach was then utilized to represent and analyze the simulation studies output results in a friendly graphical form. Improving the wireline logging performance, especially in highly deviated rugose openhole wells, could increase the percentage of successful logging operations, reducing time, cost and improving data quality with the increased wellbore coverage. The second part discusses the possible violent drillstring vibrations encountered during drilling and its effect on the overall rate of penetration and sustainability. This entails a complete identification and modeling of the drillstring dynamics and the sources of vibrations excitation that include stick-slip, bit-bounce, and whirling with its two forward and backward types to better control its functional operation and improve its performance.

A Matlab numerical simulator model based on Finite-Element-Method of 3D-Timoshenko beam elements is developed for this purpose to predict and simulate the rotordynamic behavior of the bottom-hole-assembly (BHA) and the PDC-Drillbit cutting
dynamics. The model also includes the coupling between the torsional and bending vibrations of drillstrings with the nonlinear effects of drillstring/wellbore friction contacts. The work extends previous models of drillstring vibrations in the literature to include the destructive drillstring vibration backward whirling type with Pure rolling behavior and answers some crucial questions: the operation conditions that possibly causes backward whirl vibrations, possible stabilizers’ configuration to reduce chance of backward whirl, best stabilizers locations in the BHA to minimize the sever vibration effects on the drillstring, and other arising questions.
DEDICATION

To Parents.

To Wife.

..and Baby Murad.
ACKNOWLEDGEMENTS

I would like to thank my committee chairs, Dr. Palazzolo and Dr. Ahmed for their limitless support and guidance throughout the course of this research. I would also like to thank Dr. Kim for serving on my committee, and giving his insights.

Thanks also go to my friends and colleagues and the department faculty and staff for making my time at Texas A&M University a great experience. I also want to extend my gratitude to the Qatar National Research Fund for funding this research and making this work possible.

Finally, thanks to my mother and father for their encouragement and to my wife Yaman for her patience, altruism, and love.
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PDC  Polycrystalline Diamond Compact
R   Rotor radius
\( r \) Radial Displacement of BHA CG
\( R_c \) Radial BHA-Wellbore Clearance
ROP Rate of Penetration
\( s, s_0 \) Formation, Mean Formation Elevation
t time
\( T_b \) Formation friction torque
\( T_c, T_f \) Cutting, Friction Components of TOB
\( T_d \) Top Drive Torque
TOB Torque on Bit
\( t_{pipe} \) Pipe Thickness
\( t_n \) Blade Time Delay
\( V_a \) Axial element velocity
\( V_r \) Radial element velocity
\( V_{rel} \) Relative velocity
W Element weight
\( W_c, W_f \) Cutting, Friction Components of WOB
\( W_a \) Axial component of element weight
\( W_n \) Normal component of element weight
\( W_o \) Axial Applied Load at the Top of the Drill Rig
WOB Weight on Bit
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1.1 Introduction and Problem Identification

During wireline logging there are some challenges that affect the performance of the downhole wireline-string especially when conveying in highly deviated wellbores with rugose openhole zones. These challenges can cause the wireline string to get what called ‘get stuck’. ‘Stuck wireline’ can occur where a combination of wellbore geometry and changes in wellbore direction, together with the wireline bottom-hole assembly stiffness and arrangement of conveyance accessory tools such as centralizers and bottom noses, prevent the wireline string from passing through a section of the wellbore especially in the highly deviated rugose openhole zones. A second major cause that can make the wireline tool-string to get stuck is when the borehole pressure exceeds the formation pressure and a mud cake is formed around the wireline part forcing the wireline to stick to the formation in a condition called differential sticking. Failing to achieve 100% penetration and the tool string got stuck or hanged up somewhere in the borehole can result in costly delays that can directly affect the profit margin for both the operators and service companies that employ the tools. Loss in the measured cable-head tension force of the logged wireline is known to be the main indication of wireline overall logging performance inside the wellbore and whether if it’s getting stuck.

A method to simulate and facilitate a visual investigation of the conveying behavior of a downhole wireline-tool assembly and predict numerically the resultant cablehead-tension could help in better understanding the underlying physics and hence identify the best wireline configuration for highest penetration rates in openhole wellbores.
Having a clearer sense of how different wireline conveyance tools affect the logging performance, could lead in a better realization of the best configuration the wireline should have to suite the different wellbore conditions (e.g. Vertical/inclined, cased/openhole, rugose or flat..etc). This may increase in accordance the percentage of the successful logging operations and hence reduce time, cost and improve data quality and increase wellbore coverage.

1.2 Background and Literature Review

In conventional wireline and slick-line operations, a tool string comprising different tools is lowered in a borehole from a wire or cable spooled from a drum located at the surface of the wellbore. It is often necessary to perform wireline or slick-line operations during for example completion, maintenance and servicing, installation and retrieval of downhole apparatus, intervention and well logging. Tool strings are often comprised of electrical devices that collect data from the wellbore such as temperature, salinity etc. of recovered fluids. In addition to deceinding the tool string, the wire cable acts also as a medium to transfer the electric power to the tools to carry out their functions in the wellbore, and sending the electric signals to convey the data gathered by downhole sensors back to the surface.

Tool strings usually operate better in vertical and near vertical wells, however when they are used in deviated wells problems may arise. Formation evaluation in high-deviation wells still encounters challenges because the logs behave differently in highly deviated wells than in vertical wells. As deviation increases to even moderate angles, the resulting frictional resistance between these heavy tools and the wellbore surface can make
wireline access difficult or even impossible or as said to be stuck. The term of “Stuck wireline” is applied to situations when the movement of the wireline logging tool assembly is restricted due to downhole events or forces, where the wireline tool string is unable anymore to be lowered or pulled inside or outside the borehole. Stuck wireline causes can be broadly referred whether to ‘wellbore geometry’ or to ‘Differential sticking’.

Differential sticking can occur if a portion of the wireline downhole assembly become embedded in the filter/mud cake (an impermeable film of fine solids) opposite a permeable zone (e.g. sand) and is held in place by the difference between hydrostatic and formation pressure, see Figure 1. The resultant force of the overbalance (differential pressure) acting on the contacted surface area of the wireline is the force that sticks the wireline against the wellbore [1]. This mechanism normally occurs:

1- at stationary or very slow moving wireline assembly (during wireline formation sampling)
2- When contact exists between the wireline string and wellbore formation
3- When an overbalance pressure is present
4- Across a permeable formation
5- In a thick filter/mud cake (high water loss/high solids content).

This mechanism can generate extremely large side forces, such that the logging tools cannot be pulled free unless the pressure seal can be broken in some manner.
The sidewall forces increase by increasing the wellbore deviation, which results in a greater risk of differential sticking. The filter cake thickness is critical in differential sticking as the thicker it is the bigger the cross sectional area that the formation pressure acts on and hence a higher differential sticking force. Having a lower formation pressure than the wellbore pressure makes the mud filtrate invade the porous and permeable formation until the solids present in the mud clog enough pores to form a mud cake. The thickness of the mud cake depends on the mud properties and the porosity of the formation. The danger of differential sticking is usually in sand, where the formations have high porosity and permeability and therefore a thick mud cake tends to build up and hence the area of contact can be double by the thickening of the filter cake [2].

This mechanism can generate extremely large side forces, such that the logging tools cannot be pulled free unless the pressure seal can be broken in some manner. The
force holding the wireline logging tools against the borehole wall can be calculated very quickly. The cablehead-tension force has to be higher than this force to prevent the wireline from being stuck in this location, or an extra pulling force should be applied to free up the wireline from this situation. The equation for determining the sticking force is:

\[ F_s = \mu \cdot DP \cdot A_c \]  

(1)

Where,

- \( F_s \): The sticking force (lbs)
- \( DP \): The pressure differential between the borehole and the formation (psi)
- \( A_c \): The area of contact between the wireline tool and the filter cake (in\(^2\))
- \( \mu \): The coefficient of friction between the wireline tool and the filtercake

The effective area of contact is the chord length \( (L_c) \) of the imbedded portion of the wireline tool multiplied by the thickness of the mud cake \( (T_{mc}) \). The friction coefficient factor depends on the formation and the wireline tool material properties. Almost same techniques, for releasing and fishing the differential stuck wireline, are typically used as stuck drill pipe or collar [1, 3]. Other new techniques to reduce or prevent differential sticking are recently proposed [4, 5, and 6].

On the other hand, the second general reason of Wireline sticking due to ‘wellbore geometry’; is attributed to bridges/ledges, borehole caving, severe doglegs, key-seating, casing shoe, or borehole washouts. Wireline logging operation in openhole wellbores can become even worse in highly deviated boreholes. Washout enlargement can be caused by excessive bit jet velocity, soft or unconsolidated formations, chemical attack and swelling or weakening of shale as it contacts fresh water. Generally speaking, washouts become
more severe with time [7]. Another cause of hole rugosity that causes the geometrical irregularities is the Key-seating, which happens when the wireline cuts a groove into the borehole wall.

This can easily happen in deviated or directional wells where the wireline may exert considerable pressure at the contact point with the borehole, usually on the high side of the hole. Key-seating can cause problems since the logging tool diameter is generally much bigger than the groove cut by the wireline [8], and thus can present a serious obstacle to normal ascent out of the hole.

Differential sticking and borehole geometry may raise the risk of the wireline getting stuck. This may cancel the benefits of acquiring log data and lose important well information and incurring the operation cost by running pipe-conveyed logging or some fishing operations. Figure 2 shows a visualization model, based on a caliper data, of a wellbore openhole section with associated rugosity.
One method that is being conventionally used to reduce the likelihood of differential sticking of wirelines during logging, especially in oilfield operations in deviated wells, is the use of wireline stand-offs (i.e. Centralizers). The wireline stand-off device ameliorates the effects of differential sticking and key-seating of the wireline by...
eliminating direct contact of the wireline tool or the logging cable with the borehole wall by lifting the tool string away from the side wall of the bore during the logging operation. This is typically achieved by clamping an array of stand-offs onto the outside of the wireline, resulting in lower contact area per unit length of openhole, lower applied pressure of the wireline against the borehole wall, and lower dragging resistance when conveying the wireline in or out the hole. The conventional stand-offs have fluted fins (straight blades) cut along their external body to allow easy movement through mud cake and other debris which usually build up at the borehole wall during drilling operations [9]. Low-friction Teflon stand-off conveyance accessory, typically used by Schlumberger, reduce the frictional drag forces acting on a tool string in a deviated well by effectively reducing the friction coefficient.

More efficient low-friction stand-off products derived from the Free-roller principle have been developed by oil & gas companies to mitigate differential sticking and increase the net pulling-down force while helping the tool string ride over borehole imperfections and debris. The roller stand-offs provide a rolling resistance, rather than sliding resistance, in the downhole environment, which reduces the effective friction coefficient. In addition, the contact area of the roller assembly with the borehole is reduced in comparison with that of the conventional stand-off, which in accordance reduces the differential sticking force and allow the tool string to roll out of the overbalanced permeable formation with minimum wireline over-pull [10, 11].

In addition to conventional stand-offs and other roller stand-off devices, new conveyance methods have been introduced in the oilfield industry attempting to use a more
reliable conveyance tools that may help the tool string to achieve more wellbore penetration by giving the tool string’s end an additional capability to assist the tools past the breakout zones. These breakouts can form ledges developed at the interface between layers of differing hardness [12], and hence may allow the logging tool to drop out of the path and then lodge into the formation and stop its travel throughout to downhole. These breakout ledges can be approximately described according to the image log of the wellbore as a series of cones spaced along the borehole. When these cones are close together they could support more and guide the logging tool down the hole. The nose of the tool may hence contact the ledges but the contact is at a minimum. In this case the logging tool will hardly fall out the path of the well. However, when these “cones” are spaced far enough apart the logging tool can translate and then impact the ledge. As a result and at the worst case this action can cause the logging tool to hang on the ledge. Therefore, and as an initial idea, a tool with a front rolling nose, low-friction nose surface, or with a free articulating nose could be a good basic solution to let the logging tool roll off the ledges and pass the breakouts with minimum resistance.

A “hole-finder” apparatus incorporating a locally flexible body adapted for attachment to the tool string was introduced commercially to the industry for the purpose to assist the tool string to past the ledges. As the tools string in navigated through the borehole, the flexible body provides local flexibility to lessen the likelihood of jamming the tool string while tripping down the borehole by providing a lateral force at the obstruction [13]. Experience has shown that it only provides a real benefit in deviated holes, as it closely follows the curves of the borehole and guides the tool down [14]. One
drawback was recorded that the flexible rubber nose in certain conditions deflects in an improper manner that causes the rubber extension to deform downward instead of moving upward when hitting an acute ledge.

Another passive tool used for the same purpose has had better success with the use of a spherical Teflon bottom nose at the end of the stool string. The bottom nose tool has proven successful in deflecting off obstructions and practically improved the wireline penetration rates compared to the other passive methods conventionally used before.

Another nose-tool design based on the free-rolling concept has been recently commercially released and experimented in high deviated rugose openholes. A single front self-orienting wheel made of steel provides good stand-off from the wellbore’s surface reducing the overall frictional drag. The big rolling self-orienting wheel ensures minimum loss of momentum while running in borehole by allowing complete flexibility for tool string integration and optimum positioning [15].

The use of these conveyance accessories can significantly reduce the frictional drag forces acting in a tool string in a deviated well by effectively reducing the momentum loss while negotiating the borehole irregularities and avoiding differential sticking trap. For various reasons some logging attempts, in some challenging wellbores, failed to achieve 100% penetration and the tool string got stuck or hanged up somewhere in the borehole. Insufficient downhole tools design may result in costly delays that can directly affect the profit margin for both the operators and service companies that use these tools.

The present research study employs a physics based computational approach to obtain a better understanding of the wireline logging and its behavior negotiating wellbore
geometry, to obtain the highest reliability and operational efficiency, with regards to wireline penetration rate.

Efforts toward achieving the proposed goal involved simulation studies of wireline tool conveyance in a 3D solid wellbore model of 60° degrees from vertical inclined with high rugosity section represents a challenging case study. A wireline model with different conveyance accessories tools were implemented in the wellbore simulator model. The wireline tool model was subjected to descend in the borehole under a constant velocity passing the breakouts interruption in the openhole wellbore model with experimentally measured boundary conditions and gravitational load.

A method to predict the drag forces involved in conveying the tool string into the deviated wellbore model has been presented in order to assess the different factors that directly contributes in the wireline penetration rates, as the cable tension force is the main obvious indication of the wireline behavior inside the wellbore.

Simulation studies results are presented in graphical form, and then discussion and conclusion are provided.

1.3 Objective and Significance

The main objective is focused towards understanding the underlying physics in logging operation, and investigating possible means of improving penetration mainly by studying the behavior of the wireline tool string conveyed inside the wellbore and analyzes the output results of the head tension force graph from the wellbore simulator. This will help identify the major influences on the head-tension force variation and its approximate
contribution rate. Having a clearer sense of how different wireline conveyance tools affect the logging performance, could lead in a better realization of the best configuration the wireline should have to suite the different wellbore conditions (e.g. Vertical/inclined, cased/openhole, rugose or flat..etc).

Using conveyance accessories, or Low-friction coefficient wireline tools coating, can significantly reduce the frictional drag forces acting on a tool string in a deviated well by effectively reducing the momentum loss while negotiating the borehole irregularities and avoiding differential sticking trap.
2. DOWNHOLE SIMULATOR MODEL DESCRIPTION

Until recently, a method to accurately visualize the motion of the wireline string inside a wellbore and to clearly understand the behavior of the logging tools negotiating openhole irregularities was not possible. Usually when there is a loss in head tension force and the wireline is said to be get stuck, the operators up-hole use the output readings from the different gauges as well as they analyze the mud circulation formation and pump pressure to identify the cause of wireline stop. Not like differential sticking, which has specific symptoms and therefore a definite solution to prevent wireline from sticking, there are some other factors, or combination of mixed factors, that can hinder the wireline from complete penetration especially in high deviated openholes. These factors unfortunately cannot be accurately defined under the lack of vision inside the borehole.

The wellbore simulator helps to visualize and quantify what happens downhole to every part of the wireline string as the wireline is conveyed in the wellbore. Caliper data from real open-hole well was used to construct a 3D solid model of the wellbore openhole section in this wellbore simulator tool. After studying the rugosity of various real wellbores, a model for a representative 235 ft section long was constructed from a cloud of points in the simulator. A gauge hole section of another 150 ft flat bore section was added on top of this rugose section to help the tool build the needed momentum and allow for the numerical convergence needs of the simulations. This could also considered as a representative to the cased hole section of the borehole. Since the wellbore model should account the hole deviation and rugosity as an ideal case for better understanding the wireline behavior, the wellbore model was built inclined at 60° from the vertical, and a
gauge hole of 8.5” with a maximum diameter of 24” were applied into the model. A wireline tool string was modelled and mated inside the wellbore computer model having the same weight per unit length and dimensions symmetrically as one of field wireline strings typically used in logging operations with 3-3/8” OD and 110 ft length.

The wellbore simulator also accounted the tool string shape and limberness (taking the material elasticity into consideration). The combination of tool limberness and borehole deviation contributes significantly to the inability of the wireline to penetrate. For example as shown in Fig.3; a sample of 100 feet of a 3-3/8” diameter tool string has approximately a 1 feet drop due to limberness when fixed from one end. The wireline string is connected to a bottom nose tool body via a 3° spherical knuckle joint, and at the end of this tool body is where any nose devices could be attached.

![Figure 3 Illustration of Tool String Limberness](image)

The 110 feet modelled wireline has a steel material with a total mass of 2020 lbs, and connected to a steel bottom nose tool with a total length of 20 feet having a total mass of 600 lbs, which is the maximum mass limit the wireline could handle in real life according to the wireline company specifications.
A model of a stand-off device with fluted straight Teflon blades was used to lift up the wireline assembly from the modelled wellbore internal walls so as to reduce friction and, in real life, to mitigate differential sticking. These stand-offs are typically modelled in the simulator having the same size and dimensions as the commercially used ones with the same material properties and same approximate coefficient of friction between the Teflon blade surface and the borehole internal profile (formation) which is 0.08, while a coefficient of friction between the wireline steel bodies and the openhole formation can be chosen to be of average 0.35, 0.4 dynamic and static respectively according to [16, 17]. The exact value depends on fluids in the well and roughness of the wellbore surface. An array of these Teflon centralizers (9 pieces) have been spaced equally and clamped onto the modelled wireline assembly to cover the openhole section being logged to.

After modelling the wireline components and assembling them together including the Teflon centralizers, the complete wireline assembly is inserted inside the inclined wellbore model. Gravity is then defined in the proper direction, and friction coefficients between the different components and the wellbore are defined. The complete wireline assembly is then driven through the wellbore at a constant descending velocity of 8000 ft/hr (133 fpm). This is the average maximum speed typically used and provided by a logging company to descend on the typically used wireline.

According to an actual logging data and as the wellbore begins to deviate, the friction resistance increase causing the wireline string to slow down and loss its forward momentum until it gets to a hold status for a while till it released again and begin moving after the applied force from the logging cable reaches a value that overcomes the static
resistance friction force. This phenomenon is known as stick-slip which results in wireline intermittent motion rather than smooth continuous motion during logging operations. Depending on actual downhole conditions, the tool string may come to a hold at deviations as low as 40 degrees resulting in failure to reach target depth. This could be even worse in highly deviated openhole sections with severe irregularities that make the tool string stop in even shallower depths.

Accordingly, the wireline motion behavior inside the borehole and the measured cable head tension during the logging operation can be considered the best indication in representing the overall performance of the wireline tool string and its capability of achieving better penetration rates downhole. To simulate the wireline motion behavior inside the 3D borehole model, a model of the conveyance cable should be attached at the end of the wireline assembly where the cable head tension could be measured. A loss of the measured cable tension force therefore indicates that the wireline string begins to slow down and stop, however a continuous tension indicates that the wireline is conveying without problems towards downhole target zone.

Theoretically speaking, the conveyance cable wire should be modelled by having three equivalent stiffnesses:

- Bending stiffness; which represents the bending and buckling in cable.
- Linear stiffness; which represents the elongation and compression in cable.
- Torsional stiffness; which represents the resistance of cable twisting.

Hence, the conveyance cable could be modelled as a series of springs having stiffness, $K$, connected between lumped masses, $M$, whereas each lumped mass represents
a knot that could move freely in the three dimensions equivalent to the real cable entity that moves according to the different forces applied (e.g. buckling, torsional, and tension). For sake of simplicity, and since we are not concerned about the behavior of the conveyance cable itself the presence of bending and torsional stiffness are ignored. Only the linear stiffness will be accounted in our simulation studies as it is the only direct contributor in logging tool performance study. Therefore, the conveyance cable model is modelled with only one spring component to be attached at the end of the wireline assembly. The stiffness value of the linear spring used should be estimated to be equivalent to the stiffness of the conveyance cable used in real logging operations. The linear stiffness, $K$, of the cable-model could be easily computed using the following formula:

$$K = E \cdot A / L \quad (\text{N/m})$$

(2)

Where,

- $E$: Modulus of Elasticity of the cable
- $A$: cross section area of the cable
- $L$: length of the cable

As the logging cables are usually manufactured from different materials (armor, coating, conductor wires, fillings…etc.), it will be hard to directly calculate the overall modulus of elasticity to get the equivalent stiffness. However, the stiffness, $K$, is estimated from the elongation coefficient parameter described from the cable specifications, where:

$$K = F / el \quad (\text{N/m})$$

(3)

Where,

- $F$: is the applied force (1 kN)
el: elongation coefficient (per KN per Km)

K: linear stiffness (N/m)

The elongation coefficient estimation is based on a number of relationships and equations described in details according to reference [18]. Applying the elongation coefficient value of the cable type used in the upper equation, a certain value of $K$ could hence be obtained. Substituting this obtained value of $K$, with the cable cross section area, $A$, and the 1 Km cable length in equation (2) we get the equivalent cable modulus of elasticity, $E$. Also According to the equation, the cable stiffness $K$ will have different values as cable length, $L$, change. Therefore, by reaching more depths, the cable stiffness will decrease accordingly. Reaching extended logging depths the wireline will be subjected more to stop and lose more of its inertia due to impacts when logging in irregular openhole zones. Applying different cable stiffnesses corresponding to different logging depths will be taken into consideration in the presented simulation studies as the wireline behavior will definitely change.

As discussed previously, the efforts and the main objective in this paper study is focused towards understanding the underlying physics in logging operation, and investigating possible means of improving penetration mainly by studying the behavior of the wireline tool string conveyed inside the wellbore and analyze the output results of the head tension force graph from the wellbore simulator. This will help identify the major influences on the head-tension force variation and its approximate contribution rate. Having a clearer sense of how different wireline conveyance tools affect the logging performance, could lead in a better realization of the best configuration the wireline should
have to suite the different wellbore conditions (e.g. Vertical/inclined, cased/openhole, rugose or flat...etc).

Understanding the underlying physics of the wireline logging, requires analyzing and understanding the different forces acts on the system. A computational simulator is developed for this purpose. Some wellbore parameters in the simulator were assumed constant or neglected in order to isolate and focus on the parameters that directly contribute on the wireline logging performance. The neglected parameters include pressure, temperature, fluid flow forces (fluid lift, fluid shear drag, and fluid form drag), cable weight, buoyancy force, mud & drill cuttings existence...etc. Thus; only mechanical forces will be present during wireline logging that is accounted while performing dynamic motion simulation. These mechanical forces can be simplified and classified under the following forces list:

- The wireline and tool string weight
- Friction force between wireline/tool string elements (steel tools and wireline steel segments, Teflon centralizers, and front nose) and the wellbore
- Tension force applied on the wireline resultant from the conveyance cable in order to maintain a constant wireline descending velocity
- Normal forces produced from externally impacts of the wellbore irregularities on the wireline steel tools and centralizers.

According to the Newton’s laws of motion, the calculations are performed by summing the forces along the length of the wireline string at a specific depth in a well, starting from the downhole end of the wireline string and calculating the forces on each
segment of the tool string, progressing up till the surface. The calculation is explained by using a simple wireline segment located in a straight, inclined section of a well without fluids or pressures as shown in Figure 4.

![Diagram of wireline forces in an inclined section of wellbore](image)

**Figure 4** Wireline Mechanical Forces in an Inclined Section of Wellbore

The force-equilibrium analysis of the logging wireline element in a wellbore, assuming steady state conditions, shows the different mechanical forces applied on the wireline element during the logging operation. The element weight, $W$, is divided into two component forces. $W_a$ is the force component in the axial direction along the axis of the wellbore, while, $W_n$, is the force component in the normal direction (normal to the axis of the wellbore). These forces components could be calculated using the following equations:

$$W_a = W \cdot \cos \alpha \quad \text{(N)}$$  \hspace{1cm} (4)

$$W_n = W \cdot \sin \alpha \quad \text{(N)}$$  \hspace{1cm} (5)

The friction force, $F_f$, is calculated by multiplying the normal weight component by the friction coefficient, $\mu$. 
\[ F_f = \mu \cdot W_n \quad \text{(N)} \]  

In our study, the friction coefficient, \( \mu \), will have two values depending on which wireline element is in contact with the wellbore wall, whether it is the Teflon centralizers or the steel tool segments. Also since the wellbore has an irregular profile, then the normal forces, \( W_n \), due to the contact with the different wireline elements (whether the centralizers or the steel tools) will have different directions not necessarily perpendicular to the wellbore axis and therefore a variable friction force, \( F_f \), will generated at each wireline increment. The friction coefficient force, \( F_f \), sign depends on the wireline direction of motion whether it is RIH or POOH.

In addition to these body forces, there is impact force, \( F_i \), which acts on the wireline nose due to impact with the wellbore ledges. These impact forces are applied discontinuously over a short time period on the wireline nose and may have high magnitudes due to the momentum change of the wireline which depends on its overall mass and velocity. This force only appears while conveying the wireline downhole in rugose openhole zones, and is widely believed to be the main cause of the loss of the wireline momentum, ultimately causing the wireline to stop. This view will be discussed and countered in the next section.

The impact forces, \( F_i \), could be calculated according to the following formula according to [19]:

\[ F_i = K \cdot \delta^n + D \cdot \delta' \quad \text{(N)} \]  

Where,

\[ K: \text{ is the approximate stiffness between the part and the wellbore formation} \]
\( \delta \): Relative penetration depth value of one geometry into another

\( n \): A positive real variable that specifies the exponent of the force deformation characteristic.

\( \delta' \): relative impact velocity at contact point

\( D \): hysteresis damping coefficient

The first term of the impact force equation represents the elastic force, and the second term accounts for the energy dissipation. Summing the axial components of these forces applied on all wireline elements results in the equivalent tension force, \( F_T \), applied on the conveyance cable, which will be shown later in the next section how it reflects the overall performance of the wireline during logging operation.

\[
F_T = W_a - F_i \pm F_f \quad \text{(N)}
\]  

(8)

If there is a rotational motion resulted during the wireline logging, a different friction force component will be computed in the axial direction since the resultant velocity vector, \( V_r \), of a rotational wireline element will generated from the sum of the axial element velocity, \( V_a \), and the rotation velocity, \( V_r \), as shown in Figure 5. The actual friction force should be in the direction opposite to \( V_r \), and hence should be break down into two components; one in the axial direction and the other in the rotational direction based on angle between \( V_r \) and \( V_a \), \( \beta \).

\[
F_f = \mu \cdot W_n \cos \beta \quad \text{(N)}
\]  

(9)
The overall performance of the conveyance wireline could be easily predicted and calculated for cased hole logging. Severe irregular profiles in highly deviated wells add considerable complexity to system logging in openhole boreholes and complicate, in accordance, the performance prediction and calculation of different forces acting on the wireline tools. Choosing the right wireline conveyance tools in terms of type and design, according to the intended wellbore condition, will be kind of venture.

As the cable tension force is the main indicator of the wireline behavior inside a wellbore. A computer simulator capability was developed for that purpose to predict and estimate the cable forces applied on the wireline strings conveyed in highly deviated wellbores with rugose openhole sections. This was done to assess different design factors that may directly contribute in the wireline penetration rates and performance.

The wellbore simulator was developed using the COSMOS-Motion computer software tool which is for simulating mechanism motions [20]. The wellbore simulator using COSMOS-Motion employs simulation engine, ADAMS/Solver, to solve the equation of motion of the modelled wireline assembly being conveyed inside the static borehole.
wellbore, and calculates the position, velocity, acceleration, and reaction forces acting on each element in the wireline tool string assembly. The wellbore simulator utilizes a multiple inputs to calculate the total downhole forces acting on each individual wireline element to predict the measured cablehead tension reading acting on the tested wireline.

The wellbore simulator requires several inputs, such as:

- Good 3D structural resolution of the tested wellbore model.
- Accurate data for the tested wireline tool-string assembly (size, dimensions, materials, weight per length).
- Accurate friction coefficient values between wireline assembly elements and wellbore formation.
- Accurate mechanical properties (e.g. elasticity and density) of the wireline assembly elements (steel parts, Teflon parts...etc.), as well as the wellbore model.
- Accounting limberness and flexibility of the wireline parts in the simulator model.
- Applying gravity.
- Good contacts and joints definition between the interacted parts.
- Accurate contact properties defined between the collide parts (contact stiffness, exponent, max. damping coefficient, max. penetration values).
Figure 6  Simulator Snapshot Indicating the Measured Cable Tension Force

Figure 6 shows a snapshot from the simulator built in Solidworks/COSMOS-motion showing the wireline-string logged inside the wellbore model with irregularities, and indicating the wireline measured cable tension force (Black plot), as well as the measured impact forces on the bottom nose (Blue plot).
2.1 Simulation Studies

The wireline assembly model is composed of a set of steel tools having a 3-3/8” OD resulting in a combined tool string length of 110 ft and 2020 lbs of tool weight connected to an additional 20 ft bottom-nose steel tool with a 600 lbs total mass. The simulator calculates the output cablehead tension force by solving the equation of motion of each wireline element in X, Y, and Z coordinates as the wireline descends. The generated cablehead tension output curve from the simulator represents the resultant cumulative drag forces along the length of the wireline for a given depth. This can be compared with measured wireline forces at the surface. The simulation ran until the tool assembly successfully reaches the bottom of the hole, after which results are then obtained and studied. Figure 7 shows a sample of actual cablehead-tension force versus descent depth data obtained from a successful 3-3/8” wireline tool-string run in a calcium carbonate openhole wellbore in the Arabian Gulf area.

Figure 7  Sample Actual Wireline Cablehead-Tension Data versus Logging Depth
The fluctuating of the cablehead-tension force reading, represented in the plot’s dips and peaks, reflects the encountered different resistance forces due to the openhole borehole nature. Similar outputs will be produced by the simulator tool under the different studies. These simulation studies can be listed as follows:

1. Effect of Differential Sticking
2. Effect of Standoffs Design
3. Effect of Wireline Low-Friction Coating
4. Effect of Passive Bottom-Hole Nose Design
3. DOWNHOLE SIMULATOR RESULTS AND DISCUSSIONS

Various configurations were simulated with the wireline tool string assembly – wellbore traversal code to better understand the wireline behavior, and to identify the best wireline configuration for highest penetration rates in openhole wellbores with highly deviated and rugose sections. These simulation studies also revealed the contribution of each wireline component on the overall wireline performance.

Figure 8 Factors Affect Wireline Cablehead Tension Reading

Figure 8 summarizes the different factors that might affect the wireline cable tension reading during the logging operation. (Excluding factors like: Temperature, pressure, mud weight, fluid drag force). These factors have a contribution in the resultant mechanical forces that acts on the wireline assembly body and hence affects the cablehead tension force reading Series of designed simulation studies were configured to achieve the targeted objective. The effects of the following variables on wireline performance were studied:

1. Centralizer Designs
2. Wireline Low-Friction Coefficient Coating

3. Passive Bottom-Hole Nose Design Concepts

The wireline model assembly used in all simulation studies has as prescribed a 3-3/8” OD with a total length of 130 ft steel parts and weighs about 2700 lbs (including the 20ft/600lbs bottom steel tool). The wireline tool string used in simulations is in size, dimensions, and weight as one of field wireline strings typically used in logging operations. Different stand-off designs and bottom-hole noses are used in some simulation studies. A cable-like model with varying stiffness feature is attached to the wireline tool-string upper end. The inclination of the 235ft wellbore model is fixed in all simulation studies to be 60 degrees relative to vertical. The extended upper 150ft flat section of the wellbore model is used to facilitate the validation of the simulation results with the calculated results before running in the irregular section. The applied friction coefficient of any steel parts with the wellbore formation was 0.35 dynamic, and 0.4 static. The friction coefficient between any Teflon model parts and the wellbore formation is selected to be 0.05 dynamic, and 0.08 static. An actuator with a constant descending linear velocity of 8000 fph (133 fpm) is applied on the upper end of the cable-like model in all studies. A total Simulation time of 145 seconds is fixed in all studies.

3.1 Effect of Differential Sticking on Wireline Performance

As a reference guide, the wireline assembly model was first logged in the wellbore simulator without stand-offs or any conveyance accessories attached to its body. This aided in isolating the parameters under investigation from affecting the cablehead tension force readings, and hence to provide a benchmark for comparing results for models which
included the effects. This also considered to be the worst case where the bared wireline tool-string will have a direct contact with the borehole formation with the steel’s high coefficient of friction of 0.4/0.35 static and dynamic respectively. The best case considered when if the whole wireline body is coated by a low-friction material having a COF of 0.08/0.1 dynamic and static respectively without attaching any centralizers or special noses that could affect the head-tension with drops due to impact forces. Figure 9 shows the simulation results of the cablehead-tension force of the bared wireline tool when decent at depth equivalent to 5000 ft (logging Cable stiffness K=4.8 N/mm).

![Figure 9](image)

**Figure 9** Simulator Cablehead Tension of Bared Wireline (Coated Vs. Not-Coated)

The cablehead-tension force readings show values identical to the hand calculated ones according to the body force analysis. According to the simulation resultant plot, the following points could be concluded:

- It is obvious that the coated wireline has a better performance with a higher cable-tension force plot than the bared steel wireline with higher COF.
- The small notches (dips) in the curve reading are due to the wireline nose impacts during logging, since there are no centralizers or any other accessories attached to the wireline body having a projection area or protrusions.

- The Blue curve (Coated wireline) shows better penetration rate than the Black curve (Not-coated one) at the same simulation time period.

- The wireline assembly that will have the best conveyance accessories arrangement (e.g. centralizers, bottom noses, coating material) will considered to have the best configuration, when its cablehead-tension curve is more flattened and have higher average cablehead-tension force reading.

Although the results seem to be good especially with the coated wireline assembly with its flattened cablehead-tension force curve, however this configuration is practically prevented due to the differential sticking concerns. As prescribed previously the resultant sticking force due to differential pressure could be massive. Referring to equation (1), the friction coefficient factor depends on the formation and the wireline tool surface. It can vary from 0.2 and 0.5. As realization how huge the sticking force the wireline might face, consider the friction coefficient to have a value of 0.4 in case of uncoated steel wireline tool, and 0.1 in case of coated wireline.

For simplification purpose, consider the mudcake thickness is (8mm) which makes (1/4) of the wireline tool circumference will be embedded in the mud cake as the tool cross-section dashed lines shows in Figure 10.
As in our simulations we use a 3-3/8th OD wireline tool string, the sticking force could be therefore calculated in terms of the differential pressure (DP) and the subjected wireline contact length (Lc) as follows:

\[ F_s(\text{uncoated}) = 0.4 \cdot \pi \cdot 3.375'' \cdot \frac{1}{4} \cdot (L_c \cdot DP) = 1.06(L_c \cdot DP) \]  
\[ F_s(\text{coated}) = 0.1 \cdot \pi \cdot 3.375'' \cdot \frac{1}{4} \cdot (L_c \cdot DP) = 0.265(L_c \cdot DP) \]  

Figures 11 & 12 show the estimated sticking force in (lbs) of uncoated & coated wireline tool assembly under different differential pressures and contact lengths with the mud cake. Apparently the resultant differential sticking force is HUGE whether the tool is coated or uncoated. By referring to Figure 8 for the simulated cablehead tension forces for bared wireline tool assembly (having no centralizers), almost any condition of DP or Lc will definitely make the wireline string to get stuck. Increased contact area can significantly increase the differential sticking force.
**Figure 11** Sticking Force of Uncoated Steel Wireline Tool Assembly

**Figure 12** Sticking Force of Coated Steel Wireline Tool Assembly
On the other hand and by using centralizers, to left the wireline tool off from the borehole formation and the built-up mudcake in permeable zones, the generated sticking force in this case is dramatically reduced whether the centralizers are having a low-coefficient of friction or not as the contact area comparatively is very small. Figure 10 and Figure 13 show how the contact area between the Mudcake & the Centralizer is very limited comparing when using a bared wireline tool embedded in the same mudcake thickness (8mm). Figure 10 shows how using a fluted centralizers reduces the overall contact area as the circumference contact is reduced. Figure 13 shows how the contact length is reduced as well, according to the centralizer’s length instead of having the full length of the mudcake contacted by the bared wireline’s length.

![Diagram](image)

**Figure 13** Mudcake Contact Area with Centralizer

Figures 14 & 15 show the resultant sticking force of steel & Teflon 0.75” centralizers respectively. The results shows that the sticking force using centralizers could varies between tens and a maximum of several hundred sticking force depending on how many centralizers will be contact simultaneously with the mudcake zone and the mudcake thickness, comparing to tens-to-hundreds of thousands of differential sticking force when using a bared wireline tool in contact.
Figure 14 Sticking Force of Steel 0.75” Centralizer

Figure 15 Sticking Force of Teflon 0.75” Centralizer
As a conclusion; centralizers are required in openhole logging especially when there is a chance of mud cake presence in the borehole.

3.2 Effect of Standoff (centralizer) Design on Wireline Performance

Centralizers are used to mitigate the effect of differential sticking by lifting the wireline assembly from the borehole wall. Wireline logging involves a risk of sticking, in either of two ways. Either the wireline tool will stick and the logging cable in the hole remains free, or the wireline tool remains free while the logging cable itself get stuck above to the wellbore wall [21]. Another purpose for the centralizers is to ensure that the tool string is in centralized position especially in highly deviated wells.

![Reduced Contact Area Teflon Centralizers](image)

**Figure 16** Reduced Contact Area Teflon Centralizers

Certain centralizers designs widely used in oil field offer the opportunity to reduce mechanical friction drag in high angle wells and therefore increase the wireline average cablehead-tension force. For example, roller centralizers provide typical friction factor reductions of between 50% and 70% in cased wells [9]. Use of materials such as Teflon integrated with solid centralizers and also some other composites have shown noticeable reduction in casing running drag. However, their performance in openhole is less certain especially in openholes with severe rugosity. Their shape and size, whether of the roller centralizers or the solid fluted ones, plays an important role in their effectiveness. In order
to understand the effectiveness of such devices, it is important to study different designs and shapes of and their effect on the main performance metric: cablehead-tension force. Figure 16 shows two different centralizers’ shape design used in our simulations studies, one with conventional straight fluted blades, and other with elliptical smooth fluted profile.

The aim of this study is to test the effect of the centralizers design in terms of standoff distance and profile shape on the cablehead tension force reading. Four low-friction Teflon centralizer designs (COF=0.08/0.05 static & dynamic respectively) have been investigated first in the wellbore simulator:

1. Conventional Shape Centralizer with 1.5” Stand-off Distance
2. Conventional Shape Centralizer with 0.75” Stand-off Distance
3. Smooth Shape Profile Centralizer with 1.5” Stand-off Distance
4. Smooth Shape Profile Centralizer with 0.75” Stand-off Distance

Simulation runs were performed using the same prescribed wireline assembly model. The simulation runs were repeated under different logging depths to be sure that the same behavior trend will be obtained for the different centralizer’s designs. Figures 17 to 19 show the simulation results obtained of the four Teflon centralizers’ designs under different logging depths.
Figure 17 Teflon Centralizers Designs @ 5,000 ft Depth

Figure 18 Teflon Centralizers Designs @ 10,000 ft Depth
By analyzing the output results, the following behavior points could be concluded:

- The plots show a significant improve in the wireline string overall performance indicated by the rise of the average cablehead tension readings from 550 lbf (in case of bared steel wireline) to about 1000 lbf when using low friction centralizers onto the wireline string (compare Figure 9 with Figure 17). The percentage of improvement depends on the friction coefficient value of the installed centralizers.

- The upper cablehead-tension limit, and the most optimum reading is considered when the wireline is conveyed and logged inside a flat/-or cased wellbore, where the wireline will always be left on the Teflon centralizers with its low-COF. This

Figure 19 Teflon Centralizers Designs @ 20,000 ft Depth
upper limit appears in the first 10 ft of the simulation results having a value of 1250 lbf when the wireline is still in the flat section of the wellbore model.

- The multiple dips, in the cablehead-tension curves and the wireline velocity curves, appeared as the wireline is being logged in the openhole rugose zone. These dips are mainly due to centralizers impacts, between ledges and stand-offs’ projection, not the nose impacts (compare Figure 9 with Figure 17).

- These impact forces work against the wireline dragging force until the applied force on the conveyance cable builds up and overcomes the resistive force, resulting in a recovery in wireline descent velocity and cable tension force.

- Generally the three graphs show that the shape factor in the centralizer design does not have a significant contribution on the wireline performance, since the difference between the projections areas subjected to impacts of the two centralizers’ shape are very small. From the graphs, the plots of the 1.5” conventional centralizer & the 1.5” smooth one are almost identical, just as the 0.75” conventional & smooth centralizers.

- The stand-off distance factor in centralizer design, on the other hand, appears to have more contribution on the cablehead tension reading. In case of logging in openhole zones, the 1.5” standoff centralizers have higher average tension readings than the 0.75” centralizers. The plots shows an average overall cable-tension drop in the 1.5” centralizers of 150 lbf (i.e. 1250-1100=150 lbf), while for the 0.75” centralizers the overall drop reached (1250-900=350 lbf). That could be explained that by using centralizers with small standoff distance (0.75”), more area of the
wireline body will be subjected in contact and friction with the borehole wall, unlike when using bigger standoffs (1.5”) where the wireline will be more lifted and hence less likely to contact while logging.

- Increasing logging depths (or decreasing cable stiffness K), result in more fluctuations in the cable-tension force and wireline velocity plots. By comparing the three figures (Figure 17 to Figure 19), we will see higher intense of fluctuations with maximum dips values at higher logging depths (or with lower cable stiffness values). For example in the K=1.2 N/mm ~ 20,000 ft depth (Figure 19), there is more dips values lower than 600 lbf compared to at K=4.8 N/mm~5,000 ft depth (Figure 17). Also in Figure 18, the velocities are reaching more to the zero value than in Figure 17 which didn’t exceed the 10 in/sec value. That explains why when reaching higher logging depths the wireline is subjected more to stop and lose of its momentum due to impacts when logging in rugose openhole zones. Also, stiffer conveyance cables are more recommended than cables with low stiffnesses.

As a final conclusion from the previous simulation results, the centralizer shape factor doesn’t have an obvious contribution on the wireline cablehead-tension performance as the results show the mean cablehead-tension force value and the Minimum cablehead-tension force value of the Conventional shape Teflon centralizer versus the smooth elliptical shape of Teflon centralizers when comparing at same stand-off distance, see Figure 27 (Blue section). However, and from the differential sticking prospect, using centralizers with smoother profiles (e.g. elliptical) could help in eliminating differential sticking as the area of contact with mudcake will be minimum comparing to the
conventional straight fluted centralizers or spring centralizers. As previously illustrated, even the very small contact area (AC) could result in high sticking force when multiplied by the pressure differential (PD). Roller steel centralizers is much likely to give better results when conveying in borehole since rolling resistance is lower than dragging resistance (as in the conventional centralizers case), where the contact will be only in a point. In addition, dragging will subject the centralizers’ surface to wear.

Higher stand-off distance, in case of low-friction or roller centralizers, contributes directly to a better wireline logging performance in openhole rugose zones. Refer to the summary simulation results in Figure 27 and compare the mean force values and the minimum tension force values of the obtained simulation results of the Teflon centralizers study in the ‘Blue section’ using the 1.5” and 0.75” as stand-off distances.

The maximum allowable external diameter of the centralizer used is generally governed by the borehole minimum internal diameter, and fishing tools constraints. While the minimum stand-off distance is governed by the minimum allowable distance that could prevents differential sticking caused by wellbore overbalance pressure and mud cake thickness. The spacing between the centralizers depends on the nature for the particular borehole being logged. The higher the borehole severity, the shorter spacing is recommended between installed wireline centralizers.

The previous section discussed the effect using low-friction centralizers, and how the different design factors contributed to the overall wireline performance. As the shape factor appears to not have a direct contribution on the cable-tension force, Figure 20 shows
the simulation results of wireline having steel centralizers (COF=0.4/0.35 static & dynamic respectively) with different stand-offs.

![Graph showing comparison between uncoated wireline performance with and without steel centralizers]

**Figure 20** Uncoated Wireline with Steel Centralizers having Different Standoffs

The results in Figure 20 compared to the bared uncoated wireline plot in Figure 9 shows generally a worse overall wireline performance by using steel centralizers. The effect of centralizers impacts, represented in the plot dips, could lead the wireline string to lose inertia and stop especially when using higher stand-offs. The results could be even worse at higher depths and using logging cables with low stiffness as it appears in the obtained simulation results in Figure 21 where a wireline tool having a 1.5” steel centralizers were logged at depth of 20,000 ft. The plot revealed a coincide trend between
dropping in the cablehead-tension force readings and the deceleration descending of the wireline tool. From the graph the wireline is appeared to be halted at location 181 descent feet until the applied force is build up, and the wireline recovers its inertia.

![Graph showing cable-tension and descent velocity](image)

**Figure 21** 1.5” Steel Centralizer at High Depths

This study is summarized in Figure 27 in the ‘Steel centralizers’ violet section, where using a steel centralizers leads to a worse overall wireline performance compared to the rest of wireline configurations.
3.3 Effect of Wireline Low-Friction Coating on Wireline Performance

As friction is the enemy of conveyance, this study tests the effect of using low-friction coating material with wireline having Teflon centralizers and steel centralizers. The wireline is considered to be coated with a material having COF of 0.08/0.1 dynamic and static respectively. In this case, lower standoff centralizers are recommended, to have the minimal affect due to impacts since the wireline is already coated and no need to lift highly the wireline to prevent contact. Figure 22 shows the simulation results obtained when 0.75” steel and Teflon centralizers are used with the coated wireline.

![Coated Wireline (K 4.8 ~ Depth 5,000 ft)](image)

**Figure 22** Steel Vs. Teflon Centralizers with Coated Wireline

By analyzing the plots, the following points could be concluded:
- Using low-friction (Teflon) small centralizers with coated wireline body leads to the optimum configuration of the wireline when logging in rugose openhole inclined wellbores. The low friction coefficient centralizers significantly decreased the cablehead tension dips due to impacts when comparing it to the steel centralizers. However, this is considered to be the most expensive configuration of a wireline.

- The velocity readings also coincide with the cablehead tension force behavior. A more flattened cable-tension readings result in a more continuous smooth descend velocity plot.

- By comparing the cablehead-tension plot of the steel centralizer with a coated wireline in Figure 22 (blue curve) by the plot of the Steel/Uncoated wireline having low-friction centralizers in Figure 20 (green curve), we will conclude that it is more efficient to use low-friction centralizers with uncoated wireline steel assembly.

Accordingly, the results proved the added improvement on the wireline performance when using a low-friction coating on the wireline tools. The percentage of improvement depends on how efficient and wear resistant the coating is.

In oil field industry downhole tools are typically consist of metallic parts that are often moving and chafing against the borehole wall and passing through mud/hydrocarbon fluids in extreme harsh environment of temperatures and pressures which may also contains chemically aggressive materials, and as a result corrosion occurs. For a solution, an inert coating materials are recently been used for reducing wear and corrosion, and hence improving the downhole tool’s operating life. A further advantage can be added to
the applied wear/corrosion resistance coating to improve the overall performance of the wireline logging string by using coating materials that have a low-friction coefficient. Several papers [22, 23] and patents [24, 25] were recently issued discussing different suggested low-friction coating materials for downhole applications that may be applied, and proven to have positive effects. Examples suggested, using Hardide-T Nano structure material [22], and another one by using Tungsten Disulphide as a coating material [24]. These materials shown to provide friction factors of the order of 0.1 or less as typically applied in this simulation study. Using Low-friction coating on logging tools could also prevent tool sticking when pulling the wireline out of the hole, especially when reaching extended depths.

In addition to the conventional stand-offs and centralizers and using low-friction coating materials, Downhole bottom noses are also believed to have a contribution on improving the penetration rates of the conveyed tools. Different designs of low-friction bottom noses were recently used to reduce the frictional drag forces and enables deeper descents of conventional wireline in deviated wells. The following section will discuss different passive bottom nose designs practically used in logging operations by oil-field companies, and whether they are effective or not.

3.4 Effect of Passive Bottom-Hole Nose Design on Wireline Performance

The aim of this study is to compare two different bottom-hole passive noses, been recently commercially used in logging operations, versus the No-Nose/hemisphere wireline tool end. The first passive nose design (Teflon-Ball nose) is based on the low-friction concept in conveying the wireline downhole, while the other nose design (Self-
Orienting Front-wheel nose) is based on the free-rolling concept in conveying the wireline downhole as shown in Figure 23. The design is to ensure complete flexibility for tool string integration and optimum positioning by having an extra degree of freedom represented in the swivel feature.

Figure 23 Wireline Bottom Noses

In addition, this study should show to which extent the bottom-hole nose contributes in improving the wireline penetration performance. First, a simulation run is conducted to evaluate the effect of the nose size on the cablehead-tension. A followed simulation run is conducted to compare the three passive nose designs of a Teflon-Ball & Front-Wheel having a same size. All simulation runs were conducted at same equivalent depth of 5,000 ft (K=4.86 N/mm) using the same wireline logging tool assembly having a 1.5” Teflon centralizers. Figure 24 shows the output results of Teflon-Ball bottom nose design in three different sizes 5, 6, and 7 inches.
The results show that the size of the bottom nose does not really have a significant effect on the conveying performance. The three plots are almost identical on each other. The fraction square inch of the projection areas differences between the three sizes subjected to impacts does not really have considerable effect on the overall performance comparing to the impacts on the centralizers. Note that the 3-3/8” wireline is lifted on centralizers with a 1.5” standoff making the overall external diameter equals to 6-3/8”. That means the 5” & the 6” Teflon-Ball nose sizes are normally not touching the borehole ground (except during impacts), while the 7” Teflon-Ball nose is normally having contact with the borehole.
Figure 27 in the ‘Cyan’ Section summarizes the simulation output results of the different Teflon ball sizes and shows how the three mean tension-force readings are almost the same as well as their minimum tension force values.

Now, three different nose designs were compared (Teflon-Ball, self-orienting front-wheel, and hemisphere nose) in the wellbore simulator. The Teflon-Ball nose and the Front-wheel nose were chosen to have the same size of 6”. Figure 25 shows the obtained simulation results of the cablehead-tension as well as the wireline descending velocity.

![Passive Noses (K4.8 ~ 5,000 ft)](image)

**Figure 25** Different Bottom-Hole Noses Comparison
By analyzing the output results of the three bottom nose designs as well as observing their behavior in the simulator, the following points could be concluded:

- It is remarked by visual observation, that the Teflon bottom nose and the front-wheel bottom nose have better navigation behavior with ledges than the hemisphere steel nose.

- The cablehead-Tension readings show that the self-orienting Front-wheel concept has a more positive influence on the wireline overall performance than the other two nose designs. The maximum tension dip recorded in the Front-Wheel nose is about 900 (lbf) compared to 700 (lbf) in the Teflon-Ball & hemisphere noses’ case. In conclusion, the self-orienting Front-wheel Nose shows a better navigation to the borehole roughness and ledges, and hence better penetration performance could be achieved. The results can be easily concluded as it appears in the simulations output results summary in Figure 26 in the ‘Orange’ section. This gives more credit to the free rolling self-orienting concept over the low-friction dragging concept. However, using a rolling wheel in openhole environment in presence of cuttings and other debris is still questionable.

- By visually inspecting the simulation runs at the head-tension major dips positions, it has been observed that these dips occur when a multiple centralizer engages with different ledges simultaneously, see Figure 26. Therefore, that confirms that the nose alone does not have the major contribution on the penetration performance. The centralizers do have an obvious effect.
Different bottom nose passive designs were used recently in logging operations especially in the high deviated wells with openhole sections as a trial to prevent the logging tool from stopping by a ledge and losing its momentum. Therefore, different nose concepts based on free-rolling or low-friction dragging has been commercially released and successfully experimented in deviated wellbores. More passive nose devices with extra degree of freedoms are being developed to give the wireline tool-string the flexibility to convey past ledges, washouts, and contractions which may be present in irregular boreholes with highly deviated angles [26, 27]. These nose devices may include swivel, articulated, roller parts to act as a “hole-finder” when negotiating the borehole irregularities. This makes the wireline string experience less resistance forces when navigating the wellbore and hence higher cable-tension force will be achieved as concluded from equation (8). More active downhole tools (Tractors) are being developed to improve penetration especially in extended horizontal wells. These devices are usually powered and controlled that invests friction in order to push the wireline tool-strings out to the end of the wellbore [28- 30].
4. DOWNHOLE SIMULATOR SUMMARY AND CONCLUSION

This work has identified and analyzed some challenges that affect the performance of the downhole wireline-string especially when conveying in highly deviated wellbores with rugose openhole zones. These challenges can cause the wireline string to get what called ‘get stuck’. ‘Stuck wireline’ can occur where a combination of wellbore geometry and changes in wellbore direction, together with the wireline bottom-hole assembly stiffness and arrangement of conveyance accessory tools such as centralizers and bottom noses, prevent the wireline string from passing through a section of the wellbore especially in the highly deviated rugose openhole zones. Figure 26 shows two centralizers are hitting wellbore ledges simultaneously that causes a drop in cable-head tension reading that might reach zero or get ‘stuck’. A major cause also can make the wireline tool-string to get stuck, is when the borehole pressure exceeds the formation pressure and a mud cake is formed around the wireline part forcing the wireline to stick to the formation in a condition called differential sticking.

![Figure 26 Two Centralizers hitting the Ledges Simultaneously](image)

The differential sticking force can be considerably reduced by the use of centralizers. The number of centralizers needed depends on hole deviation, hole condition. The use of centralizers in soft or unconsolidated formations become very difficult, since the
centralizer blades or springs will tend to embed in the formation. The number of centralizers must be sufficient to offset the lateral force and position the wireline string near the middle of the hole. The standoff distance of the used centralizers should be greater than the expected mud cake thickness inside the wellbore. Also using centralizers having small contact area shape design with the wellbore is recommended. This is by using either low-friction drag centralizers with elliptical profile, or by using roller centralizers having the contact in point.

A computer simulation model was built to facilitate a visual investigation of the conveying behavior of a downhole wireline-tool assembly, as well as to predict numerically the resultant cablehead-tension readings as an indication to its penetration performance. Different simulation studies were conducted to assess and analyze the contribution of different wireline components on the wireline overall performance in logging operation.

Figure 27 summarizes the obtained results from all simulation studies conducted by the simulator of the different wireline tool configurations. The figure shows a graphical bar representation of the Mean & Min.-Cablehead tension values of the simulation results recorded from the different studies conducted at same logging depth and same simulation period.
According to the ‘Blue section’ in Figure 27, the obtained results showed that the stand-off distance factor in centralizers design has a more contribution and effect on the total wireline drag force index plot than shape factor assuming no differential sticking. Having a smooth centralizer shape design is highly recommended if overbalance or mudcake is expected.

Comparing the output results in the ‘Blue section’ with the one obtained in the ‘Violet section’ in Figure 27, the results recommend using low-friction centralizers with higher stand-offs when logging in rugose inclined wellbores.

The results obtained from Figure 21 and Figure 27 ‘Violet section’ confirm that using conventional steel centralizers could lead the wireline string to stop in shallower depths. The results showed, as it appears in the ‘Green section’ of Figure 27, that Coating the wireline tools with an efficient low-friction corrosion resistant material will definitely
improve the wireline overall performance during logging and prevents tool sticking especially when pulling the wireline out of the hole after reaching extended depths. The plots showed a more flattened and continuous cablehead-tension curves with smooth wireline velocity motion.

The results also support, as it appears in the ‘Orange section’ of Figure 27, that by using passive bottom nose designs having degrees of freedoms (more flexibility) could considerably reduce the resistance forces acting on the wireline string, and hence improve the penetration performance.

As an overall summary, we can conclude that using wireline tools with efficient low-friction coating in combination with smooth shape ‘low’ centralizers, that comprises rollers or low-friction material and attached to a swivel-articulated-rolling bottom-nose tool could lead to an optimum most efficient wireline tool arrangement when logging in highly deviated rugose wellbores. However this arrangement considers being the most expensive because of the coating feature. Alternatively, uncoated wireline tools is best to be combined with smooth shape ‘high’ centralizers, that comprises rollers or low-friction material, and attached to a swivel-articulated-rolling bottom-nose tool as a second most optimum wireline string arrangement and again that in case of high borehole rugosity. If the borehole openhole profile has less irregularity, then ‘Low’ centralizers should be used.

Using ‘smart’ standoff-adjusting centralizers (maybe by using spring loaded rollers) could lead to a remarkable improvement in the wireline cablehead-tension reading by reducing the dips intense due to impacts. Centralizers may also be clamped to the logging cable itself to prevent key-seating and differential sticking.
The results of this work can be used in conjunction with other factors (e.g. additives for lubrication and reducing mudcake thickness) to improve wireline logging performance especially in highly deviated rugose openhole wells to overcome different challenging wellbore conditions. This may increase in accordance the percentage of the successful logging operations and hence reduce time, cost and improve data quality and increase wellbore coverage.

**Table 1** Suggested Wireline-String Configuration based on Wellbore Condition

<table>
<thead>
<tr>
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<th>CASED</th>
<th>OPEN HOLE</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Vertical</td>
<td>Low inclination</td>
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<tr>
<td></td>
<td>low</td>
<td>severe</td>
</tr>
<tr>
<td>Centralizers</td>
<td>Small</td>
<td>X</td>
</tr>
<tr>
<td></td>
<td>Big</td>
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</tr>
<tr>
<td>Bottom Nose</td>
<td>Hemisphere</td>
<td>X</td>
</tr>
<tr>
<td></td>
<td>Teflon-Ball</td>
<td></td>
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<tr>
<td></td>
<td>Front-Wheel</td>
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<tr>
<td>Coating</td>
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Table 1 provides some suggestions for the recommended conveyance configurations to be used in wireline logging operations based on the investigations done and under the different wellbore conditions to achieve the highest logging penetration rate. In case of cased boreholes, with any inclination, it is suggested to use only centralizers with low friction coefficient having small stand-offs, with a normal steel hemisphere nose at the bottom of the logging tools assembly. However, if the wellbore will have an open-hole zone, the logging performance will be affected significantly on the conveyance accessories.
will be selected. For vertical openholes; centralizers with small stand-offs and a hemisphere steel nose are still recommended, as the small stand-off distance under the gravitational force will be sufficient to levitate the logging tools from contact the formation and prevents any impacts on the bottom nose, therefore a hemisphere steel nose is satisfactory. However, by having more borehole inclination with the vertical and more borehole rugosity, the conveyance tools should have low friction coefficients and more degrees-of-freedom (more flexible) to reduce losses due to friction and drag resistance. For example, in highly deviated openholes with severe rugosity, low friction coated logging tools with swivel front-wheel bottom nose and ‘High’ Teflon centralizers are recommended to achieve the best logging penetration rate. Reducing losses due to friction and drag will cause the wireline string to preserve its momentum to achieve higher penetration depths. Tractors, or more actuated devices, should be used in extended reach horizontal openhole wells, as the tools’ momentum will be totally vanished due to impacts, friction, and gravitational force absence.
5. PART TWO: DRILLSTRING VIBRATION DYNAMICS

In this part we will discuss the dynamic behavior of the drillstring during the drilling operation using different approaches to model and simulate its vibration modes under different operation conditions. First, problem identification will be introduced with quick survey on literatures that tackled the problem previously from one or more sides. The objective will then be defined with a proposed methodology to achieve it step-by-step.

5.1 Problem Identification

Drillstring vibration is one of the major causes of the deterioration of drilling performance by reducing the rate of penetration (ROP) and hence increasing the non-productive time (NPT). This violent downhole vibration can also impact negatively all drillstring subassemblies, including measurement instruments, mud motor, and drillbit, and put them at risk of major failure. Furthermore, these vibrations can cause an interference with the signals from measurement while-drilling (MWD) tools, as well as it can be a source for wellbore instabilities.

Drillstring vibrations can be classified into three modes: axial, torsional, and lateral. Each vibration mode has a different destructive nature, and different excitation sources. Drilling dynamics is a very complex problem, and this necessitates the investigation and analysis of the different types of drillstring vibrations, especially the most sever lateral type of vibration, and finding practical solutions to suppress or mitigate these harmful vibrations in order to increase drilling efficiency and prevent subassemblies from possible operation failure. Figure 28 shows the different vibration modes that could be induced in the
drillstring during the drilling operation. The three vibration type will be discussed in details in the coming section.

![Drillstring Vibration Types](image)

**Figure 28** Drillstring Vibration Types [31]

### 5.2 Background and Literature Review

Drillstrings is typically consists of drillpipe, that transmits drilling fluids, drill-collars, stabilizers, tools, and drillbit. The drillstring is usually made up of hollow steel collars so that drilling fluid can be pumped thoroughly down to the drillbit. The drill collars should be in thick-walled large diameter pipes in order to provide sufficient stiffness to avoid buckling and the chance of bending vibrations. The BHA; is called on the bottom assembly of the drillstring which comprises the drill-collars, stabilizers, and the drillbit in addition to other logging tools and instruments. Stabilizers, known also as centralizers, are tools assembled along the drill collars and above drillbit used to center the BHA, and usually have a lower friction coefficient than the drill-collars. They also have a major contribution
in BHA bending prevention as we will see later in the simulation analysis. The drillstring is resting with the bit on the formation surface and pulled from the top end upward with a hook at the rig. The thin drillpipe section of the drillstring is therefore constantly in tension while the thick-walled lower part is partly in compression. The tension applied on the drillpipes prevents them from buckling or bending. However, the drillpipes are subjected to torsional loads due to their lengths and thin walls. Drilling fluids are pumped down through the drillstring hollow, and then out through the drillbit nozzles to provide lubricating and cooling for the drillbit, as well as to transport the cuttings to the surface for cleaning. Figure 29 shows the main components for the drillstring assembly and how they are assembled.

Figure 29 Drillstring Assembly [32]
While drilling, the axial reaction force on the drillbit, known as weight-on-bit (WOB), can become excessive and results in large fluctuations causing axial vibration in drillstring. This phenomenon is called bit-bounce, which can lead to deterioration in drilling rate-of-penetration (ROP) as well as damaging bit cutters and poor directional control. Torsional vibrations results from bit chatter, stick-slip interaction between drillbit and wellbore formation. Stick-slip is usually occurs while drilling in which the drillbit becomes stationary for a while “sticks” due to buildup of torque-on-bit (TOB) followed by an increasing of rotational acceleration as the bit breaks free “slips”. This causes a severe form of drillstring torsional oscillation. Severe stick-slip motion can cause eventually a reversing of the bit direction. The third and most destructive type of drillstring vibration is due lateral vibrations. Lateral or bending vibration can create large shocks as the BHA impacts the wellbore wall [31].

This type of violent vibration results from the interactions between the BHA components with formation due to bend drillstring, or mass imbalance in the drillstring caused by the embedded Measurement-while-drilling (MWD) tools, drillbit unbalance, and drill collar sag. Drillbit plays an important role in coupling mechanism as it converts axial to lateral vibrations. The interaction between the BHA or drillstring contact points may, under certain circumstances, cause to what called “Backward whirl”. Backward whirl, known also as ‘Rolling’ contact motion, is the most severe type of drillstring vibration that which results into high-frequency large-magnitude bending moment fluctuations that leads to components and connections failure such as fatigue cracking, washouts, and possible twist-offs as shown in figure 30. The increased friction results in
increased torque at the contact point, which causes the BHA collar to rotate in the opposite direction of the rotation of the drillstring.

Forward whirl, known also as sliding contact motion, is another form of drillstring lateral vibration observed when the angular rotation of the BHA component is in the same direction as the whirling direction. This results in one-sided wear “Rubbing” of components in which the BHA rubs the formation along the same part of the collar as the drillstring rotates. If the formation is aggressive excessive wear will occur along the part of the collar that rubs the formation. This wear can be seen as a flat spots on one side of the collar, or as a single worn blade or stabilizer as shown below.

**Figure 30** Drill String Components Failures due to Backward Whirl [31]

Figure 31 shows the possible failure that the drillstring components could account due to forward whirling.
Figure 31 Drillstring Components Failure due to Forward Whirl [31]
Figure 32 describes the different between forward and backward whirling in terms of whirling directions compared to the rotation direction.

The sources of drillstring vibration during drilling can be summarized as follows:

- Bit Bounce
- Stick-Slip
- Mud Motors
- Backward Whirl
- Forward Whirl
- Hydraulic vibration

As described before, there are various potential excitation sources of drillstring vibration such as: mass imbalance, friction factor between drillstring and borehole, operational spin velocity, cutting action of the drillbit, BHA stiffness, clearance between
BHA components and borehole, Bent angle, stabilizers locations, and fluid forces around the drillstring.

Most of drillstring vibration literatures focus on simulation analysis of the torsional and axial vibrations. However there is still a big lack in studies that predicts the lateral chaotic vibrations in drillstring specifically the most severe mode “Backward whirl”.

Mongkolcheep et al [32] presented a methodology to predict lateral vibrations of drillstrings accounting the flexibility of drill collar utilizing a modal coordinate condensed, finite element approach. The nonlinear effects of drillstring/borehole contact, friction and quadratic damping were included. A study that considered the length of time to steady state, the number and duration of linearization sub-intervals, the presence of rigid body modes and the number of finite elements and modal coordinates, was conducted on factors for improving the accuracy of Lyapunov Exponents to predict the presence of the chaotic vibration.

Feng and Zhang [33] discussed a linear system analytical model with only 2-DOF of a simple rotor and fixed stator. The paper discusses the vibration phenomena resultant from rotor rubbing inside a stator by an initial perturbation. The perturbation is an instantaneous change of the radial velocity when the rotor is rotating in its normal steady state. The studies shows the effect of formation coefficient friction and operating speed on the rotor’s dynamic rubbing behavior and the transition from forward whirling to full backward whirling.

When the rotor contacts the stator, the changes of the friction forces acting on the rotor can drastically affect the rotor dynamics. The paper assumed linear system with only
two degrees-of-freedoms. So they ignored the effect of inertia and stiffness non-linearities inherent in rotor systems.

Khulief et al. [34] formulated a finite-element-model (FEM) for the drill-pipes and drill-collars of the drillstring that accounts for gyroscopic and axial-bending coupling via the gravitational force field using a Lagrangian approach in order to study the self-excited nature of stick-slip oscillations and bit-bounce. Explicit expressions of the finite element coefficient matrices were derived using a consistent mass formulation and the developed model is integrated into a computational scheme to calculate time-response of the drillstring system in the presence of stick–slip excitations.

Saeed and Palazzolo [35] proposed a novel concept for a downhole flywheel energy storage module to be embedded in a bottom-hole-assembly. The paper discussed the sizing of the embedded flywheel in the BHA. Magnetic levitation control system was designed and tuned to maintain the continuous suspension of the flywheel under the different drilling vibrations of the BHA excluding the lateral vibration modes.

Lein and Van Campen [36] presented a Stick-slip whirl interaction model as a simplification of an oil well drillstring dynamics confined in a borehole with drilling fluid. Full-scale drilling rig experiment has been conducted to validate the numerical results obtained. The model consists of a sub-model for the whirling motion and a sub-model for the stick-slip motion. The model is a simple 3-DOF model that exposes only the basic phenomena of stick-slip and whirling.

Richard, Germay, and Detournay [37] studied the self-excited stick–slip oscillations of a rotary drilling system with a drag bit, using a discrete model taking into consideration
the torsional and axial vibration modes of the PDC-drillbit. Both axial and torsional vibrations of the bit, as well as the coupling between the two vibration modes through the bit-rock interaction laws are considered as well as the interface laws that account both for cutting of the rock and for frictional contact between the cutter wearflats and the rock. The evolution of the system is governed by two coupled delay differential equations, with the delay being part of the solution, and by discontinuous contact conditions. Detournay et al. [37, 38] presented also a method suggesting that the time-delay in the formation cutting of PDC bits is the vibration cause for stick-slip mode. They divided WOB and TOB into two separate processes, where the drilling action of a drag bit consists of a pure cutting process in front of each blade and a frictional process along wear flats.

Yigit and Christoforou [39] they used a simple model that captures the dynamics and coupling between the axial and torsional vibrations to simulate the effects of varying operating conditions on stick-slip and bit-bounce interactions. The authors demonstrated that the conditions at the bit/formation interface, such as bit speed and formation stiffness, are major factors that can affect the dynamic response of the model. They claim that due to the varying and the uncertain nature of these conditions, simple operational guidelines will not be sufficient to eliminate both stick-slip and bit-bounce. They also suggested parameters that could be used to represent a typical PDC bit on a hard formation. A Non-linear continuous function that represents the relation between drillbit friction torque with bit speed has been provided according experimental results.

Moreover, Yigit and Christoforou [40] have also studied the coupled torsional and bending vibrations of drillstrings subject to impact with friction.
Franca and Mahjoob [41] developed a relationship for tri-cone bits in rotary operations based on experimental results.

Patil and Teodoriu [42] presented a mathematical model of a torsional drillstring based on nonlinear differential equations which were formulated to consider drillpipes and bottom-hole-assembly separately. They represented the bit-rock interaction by a nonlinear friction force. They carried out a parametric study to analyze the influence of drilling parameters such as surface rotations per minute (RPM) and weight-on-bit (WOB) on torsional oscillations. They built the torsional drillstring model using MATLAB/SIMULINK interface.

5.3 Objective and Significance

The objective of the this work is to extend previous works stated in literature of simulating downhole drillstring dynamics, specifically works presented by Saeed and Palazzolo [35], and Mongkolcheep et al [32], and make it more comprehensive by including the lateral vibration dynamics. This will gives a complete identification and modeling of the drillstring dynamics, and in accordance will give a better understanding and control over its functional operation and improve the drilling performance. Knowing the potential excitation sources of the different vibration modes, will lead to better troubleshooting to mitigate vibrations of the BHA subassemblies and measuring tools. That should give more grounded answers to the crucial questions about the operation conditions that possibly causes backward whirl vibrations, possible stabilizers’ configuration to reduce chance of backward whirl, and other arising questions. Vibration
response predictions may assist drilling rig operators in changing a variety of controlled parameters to improve operation procedures and/or equipment.

5.4 Methodology

The objective is being achieved step-by-step as follows:

1) Initiate with a simplified mathematical based model with only two degrees-of-freedom to simulate the rotor dynamics due to friction contact between a simple rotating rotor and a fixed stator. This should give the fundamental understanding of rotor lateral vibrations (in X-Y plane) due to varying the friction coefficient between the rotor and stator as well as stator stiffness, and their effect on forward and backward whirl. Success in getting proper results from this step, by applying the right boundary conditions with logic behind presence or absence of contact, will help in achieving the targeted objective in the final system model. Literature [33] is used as reference in this step.

2) Build another simplified mathematical based model of two degrees-of-freedom (Z, φz) to capture the dynamics and coupling between axial and torsional vibration of a drillbit model and to simulate the effects of varying operating conditions on stick-slip and bit-bounce interactions taking into consideration the nonlinear effects of drillbit/wellbore friction torque. Literatures [36, 37, and 39] were used as references in this step.

3) Extend the system capabilities of the mathematical based model by combining (1) and (2) together to form an analytical model having a four DOF that captures the lateral vibration in addition to the axial and torsional vibration for a single rotor
mass inside a stator. The rotor here represents the drillbit inside the borehole. Literature [40] is used as a reference in this step.

4) Building a more complex and accurate mathematical model consists of three lumped masses that represents the drillstring assembly in its FULL DOF’s (6-DOF for each lumped mass) based on (3). The upper mass represents the drillpipe model lumped with a stabilizer having a radius bigger that the BHA radius. This mass portion is coupled with the rotary table from top, and with the 2nd mass portion (Intermediate BHA lump mass) with an equivalent torsional & bending stiffness springs, as well as axial damping & stiffness coefficients. The second lumped mass (the intermediate BHA mass portion) is considered to not have a stabilizer component. Therefore the radius of this mass portion will be less than the upper mass portion and the lower mass portion. The lower lumped mass (the third one) is considered to sum the drill-bit model with the second portion of BHA mass. The drillbit is considered to have almost the same radius of the borehole. Literature [42] is used as a reference in this step.

5) Enhance the drillstring dynamics model using a more accurate approach, Finite-Element-Method, to simulate the drillstring vibrations. Instead of treating the BHA as three lumped masses in step (4), the BHA is meshed or discretized into a number 3D- Timoshenko beam elements and the drillpipes can be substituted by a lumped mass, torsional spring, and damper attached to the top node of the BHA. Stabilizers models can be applied and located at any of the BHA’s nodes by adding the corresponding DOF’s and geometry profile. Right boundary conditions and
contact logic can be applied in the same manner as in step (1) for lateral vibration, and step (2) for axial & torsional vibration such that the drillstring experiences axial, torsional, and lateral vibrations. External forces are applied at each BHA nodes including stabilizers’ nodes and the bottom node for the PDC-drillbit. The model accounts the gyroscopic effect, the torsional/bending inertia coupling, and the effect of the gravitational force. This model is a more complex and will give a more accurate prediction to the rotordynamic behavior of the BHA, and the PDC-Drillbit cutting dynamics. Literatures [32] and [35] are used as references in this step.

6) Investigate the behavior of drillstring vibrations from the finite element model to obtain better understanding the effect of system parameters to the system response.

7) Conduct different simulation studies on the final finite element model to fulfill the targeting objective by obtaining the following dynamic behaviors with the corresponding analysis:
   - Forward whirling under self-excited vibration
   - Rolling whirling under self-excited vibration
   - Chaotic whirling under self-excited vibration
   - Stick-slip drilling
   - Bit bounce

8) Verify model by check cases
6. MODELLING OF DRILLSTRING DYNAMICS

In this section theoretical investigations of the drillstring dynamics will be initiated with the simple Jeffcott rotor-stator model with a relatively low degree of freedom approach to simulate the lateral and rotational vibrations. A more accurate but still simplified mathematical model of drillstring is then applied using lumped-system modelling approach to capture more vibration modes, including axial vibrations, with 6-DOF for each lumped mass. A final finite element method is introduced to model a more accurate and reliable model for the drillstring dynamics, including drillbit and centralizers, to investigate the different vibration modes that the drillstring could encounter during the drilling operation in vertical wells by giving a full analysis of the dynamic response and highlighting on the most destructive vibration mode ‘Backward whirling’ and the conditions that leads to it.

6.1 Rotor-Stator Modeling

Objective

As a first step, a simple 3-DOF rotor/stator mathematical based model will be presented to describe the interaction phenomenon between stick-slip and whirl motion as simple as possible. The model will qualitatively simulate the rotor dynamics due to friction contact between a simple rotating rotor and a fixed stator. This should give the fundamental understanding of rotor lateral vibrations in two-DOF model (X-Y plane) due to varying the friction coefficient between the rotor and stator as well as stator stiffness, and their effect on forward and backward whirl. The third-DOF ($\varphi_z$) is to capture the dynamics of the rotor’s torsional vibration with the effects of varying operating conditions.
on stick-slip taking into consideration the nonlinear effects of the Strubeck friction. The rotor mass imbalance will be the self-excitation vibration source and rotor whirling. Fluid forces model will be presented to form the interaction mechanism between torsional and lateral motions.

**Theoretical Background**

The vibration theory for rotor-dynamic systems was first developed by Föppl in 1895 and Jeffcott in 1919 [43]. The presented 3-DOF rotor/stator model is employed based on the simplified Föppl/Jeffcott rotor system, which is often employed to evaluate more complex rotor-dynamic systems in the real world.

To have a good understanding of the various types of vibrations that undergoes the drillstring during drilling and be able to simulate them, we need first to identify the different parameters that contributes to each drillstring vibration phenomenon, and also to understand and analyze the kinematics of a simple rotating rotor inside a fixed stator and the conditions that may enforce the rotor to switch from state to state especially when whirling.

Lateral vibrations in drillstring, also called drillstring whirling, in which the center of rotation of the drillbit, or BHA-stabilizer/BHA-collar, rotates not coincident with the center of the wellbore causing the whirling motion. It is often results from bit/formation interaction, drill collar mass imbalance, and from fluid forces around drillstring. In case of contact with the borehole formation, two different modes can be obtained while whirling: Forward and Backward whirl. Forward whirling is called when either the drill collar, drillbit, or the stabilizer, has a sliding contact with borehole wall and rotates around
the borehole axis in the same direction as around its own axis. However backward whirling is called when rotates clockwise on its axis while traveling counter-clockwise around the inside of the hole. The drillstring hence starts creating traction with the borehole wall and continues to rotate at very high frequency, depending on the clearance between the drillstring component that is in whirl status and the borehole, and reach frequencies 5 to 30 times the rotational speed of the drillstring. This violent vibration can cause a major failure in drillstring components if left unchecked due to a combination of high fatigue & impact loading as well can cause in over-gauged boreholes due to bit whipping & whirling. Figure 9 shows how bit whirling can results in an over-gauged borehole.

![Figure 9: Resultant Over-gauged Boreholes from Whirling Bit](image)

**Figure 33** Resultant Over-gauged Boreholes from Whirling Bit [44]

Backward whirl is more likely to occur at lower wellbore inclinations and as the inclination increases and the drilling trajectory becomes more horizontal, the likelihood of backward whirl decreases, however it can still occur in any wellbore environments. Furthermore, sharp edges on the drillstring can bite into the rock, creating traction that can lead to backward whirl. [45]
The relative speed, $V_{rel}$, between the two contacting surfaces, rotor and stator, is used to determine and differentiate whether there is sliding or rolling contact, and hence forward whirling or backward whirling. When the relative speed between the stator and rotor reaches zero, perfect or pure rolling occurs.

**Figure 34** Rotor Stator Kinematics

**Figure 35** Rotor Rolling on a Flat Surface

To understand the conditions that govern the motion of the rotor inside a stator and predict whether sliding or rolling motion will occur, we need first to analyze the rotor/stator kinematics. From the above figures, $\alpha$ is the angular acceleration of the rotor, $\omega$ is the angular velocity of the rotor, $R$ is the radius of the rotor, $r$ is the radial displacement of the rotor from the stator geometric center, $\Omega$ is the rotor whirling velocity, $V_t$ is the tangential velocity of the geometric center $O$ of the rotor with respect to ground, $a_t$ is the transaltional acceleration of the geometric center $O$ of the rotor with respect to
ground, \( P \) is the contact point between rotor and stator. The rotor rolls without slipping only if there is no translational movement of the rotor at the contact point \( P \). Point \( P \) therefore must also have zero horizontal (or translational) movement, which means not sliding. In this case, ‘Pure rolling’ or ‘rolling without sliding’ occurs and the following conditions must apply:

\[
V_{rel} = 0 \quad (12)
\]

\[
V_t = \omega R \quad (13)
\]

\[
a_t = \alpha R \quad (14)
\]

Otherwise, when the relative velocity is nonzero, and the velocity \( V_t \) will not equal to \( \omega R \) (\( V_t \neq \omega R \)). The relative velocity hence is given by:

\[
V_{rel} = \Omega r + \omega R \quad (15)
\]

Referring to Figure 35, equations (12-14) are also applicable for rolling on curved surfaces. The contact point \( P \) velocity would still be zero for no slipping condition, and equations (13, 14) would still apply. The velocity \( V_t \) and acceleration \( a_t \) are parallel to the tangent to surface at contact point \( P \) and the friction force \( F_t \).

In case of “Rolling with Slipping” or “Transition”: equations (12), (13), and (14) are not applicable, and the relative velocity at contact point \( P \) is not zero and hence there is no relationship between the velocity (and acceleration) of the geometric center \( O \) with the angular velocity (and angular acceleration) of the wheel.

Torsional, or rotational, vibrations are caused by the nonlinear interaction between the drillbit, or the drillstring, with the formation. This type of vibrations caused by stick-slip where the drillstring, specifically the drillbit, characterized by alternating stops then
intervals of large angular velocity after. As the strength of the formation increases, more weight-on-bit (WOB) is required to maintain efficient rate-of-penetration (ROP) and depth-of-cut (DOC). Increased WOB in hard formations with low rotation drilling speeds will often create stick-slip, a violent reaction of building up torsional energy along the drillstring body then releasing it suddenly. During the “stick” phase, the bit stops drilling while WOB and TOB are still applied, the resulting built-up torque in the drillstring will cause the bit eventually give away or “slip” causing a high sudden increase in its rotational speed, which can reach to more than five times the top drive rotational speed. This causes the drillbit to wear out due to friction, and a remarkable decrease in rate-of-penetration (ROP) and increase in the non-productive-time (NPT) in accordance.

Stick-Slip and Whirling Modelling

A dynamic analytical model of a simplified mathematical based model with only three degrees-of-freedom (x, y, φz) is used to simulate the two vibration types of stick-slip and whirl. One-degree of freedom for the torsional vibration type, and the other two degrees of freedom to model the lateral vibration type. The analytical stick-slip whirl model is a simplification of a drillstring confined in a borehole wall with mud. The BHA will be modeled as a rigid disk with mass, m, and inertia, J, and having radius R. The disk is attached at the bottom end of a massless flexible shaft (represents the drill pipe) as indicated in Figure 36. The disk has an eccentricity, e, between its center of mass and its geometric center.
The upper end of the flexible shaft is fixed in two-DOF’s (X, Y) and driven with a constant rotation speed, \( \omega \), which represents the upper constant rotary table speed. The massless shaft is subjected to bending and torsion with bending (or lateral) stiffness, \( K \), in (X, Y) and torsion stiffness, \( K_{\phi} \), about the axial axis (or axis of rotation). The displacement of the geometric center of the rigid disk occurs in the (X, Y) plane of the stationary coordinate system as shown in Figure 36. The disk rotates with an angle, \( \phi \), with angular velocity \( \phi \). The disk has a clearance, \( R_c \), with the stator. A friction torque, \( T_f \), acts on the disk (or the rotor) in the opposite of the rotation direction. This torque parameter will be extended later in the more complete drillstring model as Torque-On-Bit (TOB).

By initial exciting the rotor in the normal steady state will cause the rotor to pass through a transient vibration process. During this process the rotor may or may not contact the stator due to the rotor clearance with the stator. If the centrifugal force, \( F_c \), resultant
from the mass imbalance or eccentricity, $e$, is high enough, the disk will rotates in a spiral motion till it hits the stator wall. The stator will induce normal and tangential forces on the rotor when the radial displacement of the rotor becomes larger than the clearance, $r > R_c$. The normal contact force resultant from formation contact can be modeled simply as spring force having a spring stiffness $k_b$, which represents here the contact stiffness or formation stiffness

$$F_N = \begin{cases} k_b(r - R_c), & r > R_c \\ 0, & r \leq 0 \end{cases}$$

(16)

The normal contact force will induce a tangential friction force due to dry friction between rotor and the wall (stator). Here we will apply two models for the friction coefficient $\mu$. The first assuming a constant value for the friction coefficient, and the other using an approximated smooth Strubeck function for the friction coefficient as adapted from reference [46], where the friction force is simulated as a function of relative velocity and is assumed to be the sum of Strubeck, Coulomb, and viscous components, as shown in figure 37.

$$\mu = -\frac{2}{\pi} \arctan(e_f v_{rel}) \left[ \frac{\mu_s - \mu_d}{1 + \delta_f v_{rel}} + \mu_d \right]$$

(17)
Where \( \varepsilon \), determines the steepness of the approximation function and parameter \( \delta_f \) is a positive number that determines the rate at which the static friction coefficient approaches by the dynamic friction coefficient with respect to relative velocity. If the relative velocity between the rotor and the stator is nonzero, the tangential friction force, \( F_t \), could be then:

\[
F_t = -\mu \cdot \text{sign}(V_{rel})F_N, \, V_{rel} \neq 0
\]  

(18)

From the relative velocity equation (15), the whirling velocity, \( \Omega \), equals:

\[
\Omega = \dot{\alpha} = \frac{d}{dt} \left( \tan^{-1} \frac{y}{x} \right) = \frac{\dot{y} x - \dot{x} y}{r^2}
\]

(19)

Where, \( \dot{\alpha} \) or \( \Omega \) is also known as the angular rate of change of the rotor geometric center position.

In case of pure rolling, as illustrated before, the relative velocity reaches zero (\( V_{rel}=0 \)), and the tangential friction force must be between

**Figure 37** Friction Force Relation with Relative Velocity [46]
\[-\mu F_N \leq F_t \leq \mu F_N \quad (20)\]

The tangential friction force in this case (Pure Rolling) can be determined from the kinematics analysis of the rotor when in contact with the stator. The equations of motion for a whirling rotor in polar coordinates:

\[ma_r + C \dot{r} + Kr = -F_N + F_e \quad (21)\]

\[ma_t + C \Omega r = F_t \quad (22)\]

\[J \ddot{\alpha} + K_\phi (\phi - \omega t) + C_{\phi \dot{\phi}} = -F_t R \quad (23)\]

Where \( r \) is the rotor radial displacement, \( C \) is the lateral damping value, and \( C_{\phi} \) is the rotational damping coefficient due to surrounding drilling fluid (mud).

\[r = \sqrt{x^2 + y^2} \quad (24)\]

From equation (22),

\[a_t = (F_t - C \cdot \Omega \cdot r)/m \quad (25)\]

Then from equation (14): as \( \alpha = a_t/R \), then The tangential friction force can be determined from equation (23):

\[F_t = \frac{C \cdot \Omega \cdot r - \left(\frac{m \cdot R}{J}\right) (K_\phi (\phi - \omega \cdot t) + C_{\phi \dot{\phi}})}{\left(1 + \frac{m \cdot R^2}{J}\right)} \quad (26)\]

Hence,

\[F_t = \begin{cases} \text{eq. (25)} & \text{if } F_t < F_{t_{\text{max}}} \implies \text{Pure Rolling} \\ \text{eq. (18)} & \text{if } F_t > F_{t_{\text{max}}} \implies \text{Sliding/Transition} \end{cases} \quad (27)\]

The contact force can be expressed in stationary coordinate system in x and y as:

\[F_x = \frac{-F_t \cdot y - F_N \cdot x}{r} \quad (28)\]
\[ F_y = \frac{F_t \cdot x - F_N \cdot y}{r} \]  

(29)

The lateral forces due to mass unbalance that causes the lateral motion are given as:

\[ F_{ex} = m \cdot e \cdot \dot{\varphi}^2 \cdot \cos \varphi \]  

(30)

\[ F_{ey} = m \cdot e \cdot \dot{\varphi}^2 \cdot \sin \varphi \]  

(31)

Where, \( \varphi \) is the rotor rotation angle, and \( \dot{\varphi} \) is the rotor angular velocity.

For simplification in this model, we will assume that the rotor disk represents a BHA collar not a drillbit so we will not consider the forces related to WOB and TOB. As the contact between rotor and stator induces the contact forces \( F_N \) and \( F_t \), the tangential contact force \( F_t \) induces a torque on the rotor,

\[ T_b = F_t \cdot R \]  

(32)

**Equation of Motions**

The equations of motion for the rotor’s stick-slip model and whirl model combining the lateral and the torsional vibrations, and taking into account the lateral and rotational damping due to drilling fluids, contact forces, contact torque, and lateral forces due to mass unbalance, gives the following set of equations of motion in its general form:

\[ m \dddot{x} + c \dddot{x} + kx = F_x + F_{ex} \]  

(33)

\[ m \dddot{y} + c \dddot{y} + ky = F_y + F_{ey} \]  

(34)

\[ J \dddot{\varphi} + c_\varphi \dddot{\varphi} + k_\varphi (\varphi - \omega t) = T_b \]  

(35)

Where \((X, Y)\) is the position of the rotor’s geometric center, and \( \varphi \) is the rotor’s rotational angle.
From previous discussions we can summarize the following conditions for Forward and Backward whirling.

**Backward Whirling with Pure Rolling**

The rotor will roll without slipping over the stator only when four conditions are satisfied:

1. There is contact between rotor and stator, \( r \geq R_c \),
2. Relative velocity is zero, \( V_{rel} = \Omega r + \omega R = 0 \),
3. Tangential contact force does not exceed the maximal friction force, \( -\mu F_N \leq F_t \leq \mu F_N \)
4. Whirling direction is opposite to the rotor’s rotational direction about its center.

The mean angular whirling velocity of the rotor in case of backward whirling is found to be close to \( -\left(\frac{R}{r}\right) \omega \), which implies that the contact between the rotor and the stator wall approaches that of (Rolling without slipping) where the rotor tangential velocity \( \Omega r = \omega R \).

Therefore we can calculate theoretically the backward whirling velocity as follows:

\[
\Omega = -\left(\frac{R}{R_c}\right) \omega = -\Gamma \cdot \omega \tag{36}
\]

Where, \( \Gamma \) is the precession frequency ratio (PFR) which represents the ratio of whirl frequency to rotor angular speed that is governed by the measured radius-to-clearance ratio at the contact location. Since the rotor radius, \( R \), is much larger than the radial displacement or the clearance, \( R_c \), the backward whirl velocity can reach to more than 30 times that of the rotor’s angular velocity. Therefore dry-friction instabilities at low speeds can still produce high frequency vibrations, and that makes the backward whirling vibration mode in drillstring is the most severe and violent mode of vibration.
In simulation, as there will be a numerical difficulty for the integrator to reach a whole zero value for the relative velocity, $V_{rel}$, at pure-rolling condition, it is recommended to set minimum velocity value ($V_{min}=0.001$ m/s) so as to consider the rotor in a backward whirling condition if the relative velocity reached below this value ($V_{rel} < V_{min}$).

**Forward Whirling (Pure Sliding)**

The rotor will whip over the stator wall with a slipping friction behavior when three conditions are satisfied:

1. There is contact between rotor and stator, ($r \geq R_c$),
2. Relative velocity is non-zero, ($V_{rel}=\Omega r+\omega R \neq 0$),
3. Whirling direction is the same to the rotor’s rotational direction about its center

Otherwise ‘Pure-rolling’ or ‘pure-sliding’, the rotor can experience a mix of backward-whirling with some sliding (not pure rolling), or a discrete contact with the stator resultant from unstable random impacts with the stator wall.

**Simulation Flowchart Logic**

The flowchart below illustrates the logic behind the rotor/stator simulation program using Matlab, indicating the different modes that could encounter the rotating rotor inside the stator under the different mathematical constraints.
Figure 38 Simulation Flowchart Logic
Simulation Studies

Different studies have been conducted to simulate the different rotor dynamics modes before, during, and after rotor/stator contact under variable friction coefficients between the rotor and the stator, as well as under different operational speeds. Applying the equations of motion (33), (34), and (35), the differential integration solver ODE45 in Matlab is used to solve the differential equations using the 4th order Runge-Kutta method. Stator stiffness is chosen to be $10^4$ times that of the rotor’s to understand the effect of the elastic contact between the rotor and the stator. A uniform clearance, $R_c$, is assumed between the rotor and the stator when the rotor is stationary. The effect of the gravity is ignored. The rotor has an eccentricity, $e$, between its center of mass and its geometric center. The rotor will have a different friction coefficient in each simulation run under a fixed operational speed to study the effect of varying the friction coefficient on the rotor dynamics. The rotor also will have a different operational speed in another patch of simulation runs under a fixed coefficient of friction to study and analyze the resultant dynamics. The code generally follows the same logic illustrated in the flowchart shown above in figure 38. The following parameters have been fixed in all the four simulation studies:
Table 2 Simulation Parameters for Rotor-Stator Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor radius, $R$</td>
<td>0.05 (m)</td>
</tr>
<tr>
<td>Rotor eccentricity, $e$</td>
<td>0.001 (m)</td>
</tr>
<tr>
<td>Rotor clearance, $R_c$</td>
<td>0.0025 (m)</td>
</tr>
<tr>
<td>Shaft bending stiffness, $K$</td>
<td>40 (N/m)</td>
</tr>
<tr>
<td>Shaft torsional stiffness, $K_{\phi}$</td>
<td>0.6 (N.m/rad)</td>
</tr>
<tr>
<td>Stator stiffness, $K_b$</td>
<td>40e4 (N/m)</td>
</tr>
<tr>
<td>Rotor mass, $m$</td>
<td>1 (Kg)</td>
</tr>
<tr>
<td>Rotor inertia, $J$</td>
<td>0.0013 (Kg.m²)</td>
</tr>
<tr>
<td>Mud lateral damping, $C$</td>
<td>1 (N/ms)</td>
</tr>
<tr>
<td>Mud torsional damping, $C_{\phi}$</td>
<td>0.2 (N.s/rad)</td>
</tr>
<tr>
<td>Top drive angular speed, $\omega$</td>
<td>12 – 16 (rad/sec)</td>
</tr>
<tr>
<td>Friction coefficient,</td>
<td>0 - 0.15 (-)</td>
</tr>
</tbody>
</table>

Following are the results obtained from the different studies conducted using the above parameters’ values listed in Table 2.

Simulation Results

*Rotor-Stator Contact with Synchronous Forward Whirling*

For a very small coefficient of friction, the rotor starts with a short transient impact phase, due to initial perturbation from the mass imbalance excitation, followed by full
rubbing with the stator wall. The full rubbing behavior is denoted with a continuous contact with the stator in a forward whirling motion having the same direction and rate as the rotor rotation about its center. Figure 39 shows the dynamic response of the rotor-stator model under self-excitation.

\[ \omega = 12.6491 \text{ (rad/sec)} \]
\[ \mu = 0.001 \]
\[ \frac{k_b}{k} = 10,000 \text{ (N/m)} \]
\[ e = 0.001 \text{ (m)} \]

**Figure 39** Rotor-Stator Contact with Synchronous Forward-Whirl
Figure 39 Continued
Panel (a) shows the orbit of the rotor geometric center in the (X, Y) plane. The dotted red circle indicates the clearance boundary. Panel (b) shows the radial excursion of the rotor as a function of time, where the red line indicates the clearance boundary limit. Panel (c) the angular velocity of the rotor as a function of time. Panel (d) the rotor whirling velocity and the dashed red line indicates the theoretical whirling velocity in case of backward whirling. Panel (e) the relative velocity between the rotor and the stator at the contact point. Panel (f) shows the normal contact force on the rotor. Panel (g) the tangential friction force on the rotor. Panel (h) indicates the tangential friction force at a nonzero relative velocity condition (black line), compared to the friction force calculated at zero relative velocity (red dashed line) using equation (26). Panel (i) indicates the backward status of the rotor throughout the simulation time when all four conditions are satisfied as mentioned previously.

From the results obtained we can conclude that the friction force is not high enough to change the synchronous, or the continuous, forward full rubbing motion of the rotor into “Backward whirling” with pure dry-friction rolling motion.

*Rotor-Stator Dynamic Response with Discrete Impact and Unsteady Whirl*

The increase of friction coefficient changes the response nature of the rotor dynamics. The rotor makes discrete random contact with the stator. The rotor geometric center behaves with a forward whirling motion but not at a constant rate as in the previous study. A discrete rotor impact with chaotic unsteady whirling motion is resultant at an intermediate coefficient of friction.
\[ \omega = 12.6491 \text{ (rad/sec)} \]
\[ \mu = 0.07 \]
\[ kb/k = 10,000 \text{ (N/m)} \]
\[ e = 0.001 \text{ (m)} \]

Figure 40 Rotor-Stator Dynamic Response with Discrete Impact and Unsteady Whirl
From the results we can conclude that the friction coefficient still not high enough to apply sufficient friction force on the rotor contact to resist the sliding motion and switch it in to rolling motion, or to a backward whirling.

*Rotor-Stator Dynamic Response with a Backward Whirl*

By increasing the friction coefficient, the rotor starts with a discrete impacts characterized by unsteady whirling then a backward whirl is fully developed at about 7.3 second when the relative velocity at the contact point between the rotor and stator reaches
zero as shown in figure 41 (e). That leads to a very large whirling velocity, about 26 times the rotor’s angular velocity, where it reaches to about 330 (rad/sec) compared to 12.6 (rad/sec) for the rotor’s angular velocity as shown below in figure 41 (d). A large centrifugal force will result from the very high frequency backward whirl, which can lead to a large stator deformation as shown in the last panels (a) and (b).

\[ \omega = 12.6491 \text{ (rad/sec)} \]
\[ \mu = 0.15 \]
\[ kb/k = 10,000 \text{ (N/m)} \]
\[ e = 0.001 \text{ (m)} \]

**Figure 41** Rotor-Stator Dynamic Response with Backward Whirl
Figure 41 Continued
The increase of friction leads to an increase of the friction torque at the contact point and hence a deceleration and decrease of the rotor’s angular speed occurs. As there is a constant applied torque on the top shaft, the applied torque will build up along the shaft length till it overcomes the friction torque causing a rapid acceleration in the rotor’s rotation speed. This can explain the fluctuations in the rotor’s angular velocity shown in panel (c) reaching double the operating rotation speed. Panel (d) shows how the theoretical backward whirl velocity calculated from equation (36) indicated by the dashed red line, is very close to the whirl velocity indicated by the black line obtained from dynamic response. Panel (h) indicates the tangential friction force at a nonzero relative velocity condition (black line), compared to the friction force calculated at zero relative velocity (red dashed line), where in case of backward whirl the $F_{t-R}$ should be less than $F_t$ as illustrated previously in equations (20) and (26). Panel (i) indicates the backward status of the rotor throughout the simulation time when all four conditions are satisfied as mentioned before. From the plot the pure rolling, or backward whirl, begins to occur at about 7.3 sec when the contact relative velocity reaches to zero, and $F_{t-R}$ is less than the $F_t$.

**Conclusion**

The friction coefficient of the formation and the rotor’s rotational speed can dramatically affects the rotor dynamics. Figure 42 shows how by increasing the friction coefficient between the rotor and stator, the geometric center of the rotor can change from pure sliding forward whirl to backward whirl and even to pure rolling without slipping, indicated by reaching to a zero contact relative velocity. When the rotor rubs the stator,
the friction changes the forces acting on the rotor dynamics which can be drastically different.

Figure 43 shows how by increasing the spinning velocity of the rotor, the backward whirl could occur earlier. The imbalance centrifugal force, due to eccentricity, has a direct proportional relationship with $\omega^2$ as shown in equation (30). Increasing the rotor’s spinning velocity results in higher centrifugal force acting on the rotor providing an early contact with the stator with a higher contact friction force, induced form higher normal contact force, and hence a higher probability of an early backward whirl.

In addition to the coefficient of friction and rotor’s rotational speed parameters, the rotor’s radius to clearance ration ($R/R_c$) affects significantly the rotor dynamics and specifically the backward whirl velocity. The backward whirl velocity will be large if this ratio is large. Therefore, in engineering drillstring design, the clearance between the largest component radius and the borehole is an important parameter, where the ration should be minimal. The eccentricity in the drillbit, or the in the overall drillstring/BHA body length, is another very important parameter. The mass imbalance is the major cause of lateral vibration self-excitation due to the centrifugal force induced.

Rotational vibration is induced mainly from three parameters; rotation speed, coefficient of friction, and axial force or WOB. Operating at low spinning speeds with high friction coefficient will cause in higher friction torque applied on the rotor. This results in rotor’s speed deceleration till the built-up applied torque overcomes it and releases the rotor before reaching the stop or ‘Stick’ status causing the ‘Slip’ behavior.
Figure 42 Effect of Friction Coefficient on the Rotor Dynamics

Figure 43 Effect of Rotor Rotational Angular Velocity on the Rotor Dynamics
In a more advanced system model and by taking into consideration the axial DOF, we will see how the Torque-On-Bit, which is directly proportional to WOB, contributes in Stick-slip vibrations. Increasing the WOB will result in high frictional torque on bit, which will leads eventually to a total ‘Stick’ and ‘Slip’. Stick-slip vibrations are self-excited and caused mainly by the nonlinear frictional torque between the drillstring component and the borehole.

6.2 Drillstring Lumped System Modeling

Objective

The objective here is to build a more accurate, but still simplified, mathematical model to simulate the drillstring dynamics including drillbit consisting of three lumped masses that represents the drillstring assembly with its FULL-DOF’s (6 DOF for each lumped mass). The model allows studying and analyzing the drill-pipes, BHA, stabilizers, and drillbit dynamic behaviors including axial and rotational vibrations with bit/rock interaction.

Theoretical Background

In the previous section we were able to simulate the lateral and rotational vibrations in a simple rotor-stator model with 3-DOF system model representing the drillpipes with lumped mass system. The advantages of using this lumped mass and spring method in modelling drillstring is to keep the simplicity and have a considerably faster analysis compared to the FE-Method. Another advantage is that we can simply adjust the model parameters to reproduce the nonlinear drillstring vibrations for a quick dynamic analysis of the drilling process. To be able to simulate the drillstring dynamics using the
lumped mass approach, we need first to model the drillbit dynamics including the axial and torsional motion and the coupling between them. This requires modeling the nonlinear drillbit-rock interaction, and understanding the relation between the friction Torque-On-Bit \((TOB)\) with the Weight-On-Bit \((WOB)\) and bit diameter.

Drillbit is responsible for rock formation cutting and penetration in drillstring. There are two different kinds of bits exist: fixed cutter and roller cone. A fixed cutter bit is one where there are no moving parts, but drilling occurs due to percussion or rotation of the drill string. A Roller cone bit usually has three cone shape devices with teeth or cutters, whereas the drillstring rotates the cones rotate to drill ahead. The Polycrystalline-Diamond-Compact bit (PDC), which is a fixed cutter bit type, and the tricone Roller-cone bit, are the most generally used in drilling.

For the (PDC) drillbit type, the depth of cut per revolution per blade is given as below:

\[
d_n(t) = x_{db}(t) - x_{db}(t - t_n)
\]  

(37)

Where \(x_{db}\), is the vertical position of the drillbit, \(t_n\)is instantaneous time delay that the bit requires to rotate by \(2\pi/n\) to its current position at time \(t\) obtained by solving the equation:

\[
\varphi_{db}(t) - \varphi_{db}(t - t_n) = 2\pi/n
\]  

(38)

Where \(\varphi_{db}\), is the drillbit angular position, and \(n\) is the number of blades. Hence, the combined depth of cut for the bit is given as:

\[
d_c = n.d_n
\]  

(39)

The bit response depends on two important parameters; Weight-On-Bit \((WOB)\), and Torque-On-Bit \((TOB)\). The drilling action of a bit consists of a pure cutting process,
and a frictional process. Both WOB and TOB thus should be expressed into two components:

\[
WOB = W_c + W_f \tag{40}
\]

\[
TOB = T_c + T_f \tag{41}
\]

The cutting and frictional components for both WOB and TOB are given as [15]:

\[
W_c = a \cdot \xi \cdot \varepsilon \cdot d_c \tag{42}
\]

\[
T_c = \frac{a^2}{2} \cdot \varepsilon \cdot d_c \tag{43}
\]

\[
W_f = a \cdot l \cdot \sigma \tag{44}
\]

\[
T_f = \frac{a^2}{2} \cdot \gamma \cdot l \cdot \sigma \cdot \mu \tag{45}
\]

Where, \(a\) is the bit radius, \(\xi\) characterizes the inclination of the cutting force and typically \(0.6 \leq \xi \leq 0.8\), \(\varepsilon\) is the intrinsic specific energy (the energy required to cut a unit volume of rock), \(\sigma\) is the normal stress acting across the wear flat interface, and \(\gamma\) characterizes the spatial orientation and distribution of the chamfers/wearflats and for a flat-bottom bit is typically \(1 \leq \gamma \leq 1.3\). \(l\) is the equivalent total wear flat for the bit and is given as:

\[
l = n \cdot l_n \tag{46}
\]

Where \(l_n\), is the width of wearflat for a single blade.

The above relations of WOB and TOB are specifically to model a typical PDC drill bit with a time delay function due to the cutting action of the blades. However, in a more general and simple representation form to model the PDC, or roller-cone bits with no time delay term, the instability hence could be represented in terms of the self-excitation with the sinusoidal nature. Therefore, the WOB and TOB can be expressed as follows [39, 40]:
\[ WOB = \begin{cases} 
K_c \cdot (x_{db} - S), & \text{if } x_{db} \geq S \\
0, & \text{if } x_{db} < S 
\end{cases} \quad (47) \]

Where \( K_c \) is the formation contact stiffness, which can be calculated from the following equation:

\[ K_c = a \cdot n \cdot \zeta \cdot \epsilon \quad (48) \]

The formation surface elevation, \( S \), is given as:

\[ S = S_0 \cdot \sin(\varphi_{db}) \quad (49) \]

The depth of cut per revolution, \( d_c \), in this case is given as:

\[ d_c = \frac{2 \cdot \pi \cdot ROP}{\varphi_{db}} \quad (50) \]

The average Rate-Of-Penetration, \( ROP \), is a function of the applied load \( W_0 \), bit speed \( \varphi_{db} \), and the bit/rock characteristics given as [39]:

\[ ROP = C_1 \cdot W_0 \cdot \sqrt{\varphi_{db}} + C_2 \quad (51) \]

The \( TOB \) is function in \( WOB \) and cutting conditions, and is given as:

\[ TOB = WOB \cdot \left( \mu + \sqrt{\frac{d_c}{a}} \right) \cdot a \quad (52) \]

Equations, (47) and (52), describes the coupling between the axial and rotational vibrations induced by the self-excitation axial sinusoidal function from equation (49).

The Strubeck friction model is utilized according to [35, 39] to describe the nonlinear friction behavior between the bit and the formation. This behavior is modeled by a continuous function given as:
\[ \mu = \mu_0 \left( \tan^{-1} \varphi_{db} + \frac{\alpha \dot{\varphi}_{db}}{1 + \beta \dot{\varphi}_{db}^2} + \nu \dot{\varphi}_{db} \right) \] (53)

Where \( \mu_0, \alpha, \beta, \gamma \), and \( \nu \) are the parameters of the friction model, which characterize the friction behavior as shown in figure 20. These parameters depend on the type of formation and of drillbit used. The negative slope at very low speeds causes the instability in the rotational motion and the bit may reaches to speeds up to three times the operation speed (desired speed). This results in self-sustained stick-slip oscillations. Increasing bit speed will result in a more stable system.

**Figure 44** Friction Behavior of the Drillbit and Rock Formation [39]

At normal drilling operation with steady-state condition, the axial penetration velocity of the drillbit is equal to the ROP, and drillbit should have the same rotational velocity as the top drive speed applied at the surface. This implies that the WOB is equal to the applied load, \( W_0 \), on the drillstring at the surface, and the TOB is equal to the driving
torque $T_d$. If the $TOB$ is higher than the top driving torque $T_d$, the drillbit decelerates and could reach to zero spinning which known as ‘sticking’. As the built-up torsional torque potential energy overcomes the applied $TOB$, the drillbit is suddenly released causing it to accelerate to even three times the top drive speed. This behavior is known as ‘slipping’. This Stick-slip cycle will usually repeat again and again as long as the same conditions are applied and the operation left unchecked. Bit-Bounce; is an axial vibration behavior occurs when the normal reaction force from the formation, or $WOB$, is greater than the top applied load, $W_0$, which results in drillbit loss of contact with the bottom formation forcing the BHA upwards with negative axial displacement and velocity, till the top applied load, $W_0$, and the total weight of drillstring and mud weight force the BHA downwards again to contact with the bottom formation.

**Lumped System Modeling**

The model describing the drillstring is obtained by assuming that the drillstring is divided into three lumped masses (with inertias) connected one to each other by lateral, axial, and torsional stiffness, as well as axial, and lateral damping. Viscous torsional damping is applied on the whole drillstring assembly that represents the drilling fluid damping effect.
Figure 45 describes the simplified drillstring assembly in its full DOFs. The model consists of four elements:

1) A massless top-rotary system with applied axial load from top $W_0$, and constant rotational speed $\omega$. The massless rotary system is fixed in 4-DOFs except the axial and rotational direction about the Z-axis.
2) The drillpipes with lumped mass $m_{dp}$ and inertias $J_{dp}$ and $I_{dp}$. The lumped mass model has 6-DOFs (X, Y, Z, $\phi_x$, $\phi_y$, $\phi_z$) and coupled with the rotary table from the top with axial, lateral, and torsional stiffness, as well as with axial, and lateral torsional damping. A stabilizer, with a radius bigger than the BHA radius, is attached to the drillpipes lumped mass.

3) The intermediate lumped mass represents the first half of the BHA-drill collars. This model has mass $m_{BHA}$ and inertias $J_{BHA}$ and $I_{BHA}$. The lumped mass model has 6-DOFs (X, Y, Z, $\phi_x$, $\phi_y$, $\phi_z$) and coupled with the drillpipes lumped mass model from top with axial, lateral, and torsional stiffness, as well as with axial, and lateral torsional damping.

4) The bottom lumped mass is considered to sum the drillbit model with the second half of the BHA-drill collars. A second stabilizer is attached to the lumped mass model having a radius equal to the drillbit. A very small clearance is considered between the stabilizers and the borehole. This mass portion will have 6-DOFs as well.

The Stribeck nonlinear friction model described in equation (17) is applied on both stabilizers in case of contact, as well as to the intermediate BHA lumped mass model. The Stribeck friction model that describes the nonlinear friction behavior between the bit and the formation (equation 53) is considered at the bit. $WOB$, and $TOB$ are applied at the bit when in contact with formation. A ground viscous torsional damping, $C_f$, is applied on the whole drillstring assembly that represents the rotational damping due drilling fluid (mud).

The equations below describe the equation of motion of the drillstring assembly in its complete DOFs.
EOM-Drillpipes lumped mass

\begin{align}
    m_{dp} \ddot{X}_{dp} + C_{dp}(\dot{X}_{dp}) + C_{BHA}(\dot{X}_{BHA} - \dot{X}_{dp}) + K_{dp}(X_{dp} - X_{BHA}) + K_{BHA}(X_{dp} - X_{BHA}) &= F_{x,dp} \\
    m_{dp} \ddot{Y}_{dp} + C_{dp}(\dot{Y}_{dp}) + C_{BHA}(\dot{Y}_{BHA} - \dot{Y}_{dp}) + K_{dp}(Y_{st}) + K_{BHA}(Y_{dp} - Y_{BHA}) &= F_{y,dp} \\
    m_{dp} \ddot{Z}_{dp} + C_{z,dp}(\dot{Z}_{dp}) + C_{BHA}(\dot{Z}_{BHA} - \dot{Z}_{dp}) + K_{z,dp}(Z_{dp}) + K_{z,BHA}(Z_{dp} - Z_{BHA}) &= W_0 + F_{z,dp} \\
    J_{dp} \ddot{\phi}_{z,dp} + C_f(\phi_{z,dp}) + K_{\phi_z,dp}(\phi_{z,dp} - \omega t) + K_{\phi_z,BHA}(\phi_{z,dp} - \phi_{z,BHA}) &= -T_{z,dp} \\
    I_{dp} \ddot{\phi}_x + C_f(\phi_x) + K_{\phi_x,dp}(\phi_{x,dp}) + K_{\phi_x,BHA}(\phi_{x,dp} - \phi_{x,BHA}) &= -T_{x,dp} \\
    I_{dp} \ddot{\phi}_y + C_f(\phi_y) + K_{\phi_y,dp}(\phi_{y,dp}) + K_{\phi_y,BHA}(\phi_{y,dp} - \phi_{y,BHA}) &= -T_{y,dp} \end{align}

\begin{align}
    m_{BHA} \ddot{X}_{BHA} + C_{BHA}(\dot{X}_{BHA} - \dot{X}_{dp}) + C_{BHA}(\dot{X}_{BHA} - \dot{X}_{db}) + K_{BHA}(X_{BHA} - X_{dp}) + K_{BHA}(X_{BHA} - X_{db}) &= F_{x,BHA} + F_{ex} \\
    m_{BHA} \ddot{Y}_{BHA} + C_{BHA}(\dot{Y}_{BHA} - \dot{Y}_{dp}) + C_{BHA}(\dot{Y}_{BHA} - \dot{Y}_{db}) + K_{BHA}(Y_{BHA} - Y_{dp}) + K_{BHA}(Y_{BHA} - Y_{db}) &= F_{y,BHA} + F_{ey} \\
    m_{BHA} \ddot{Z}_{BHA} + C_{z,BHA}(\dot{Z}_{BHA} - \dot{Z}_{dp}) + C_{z,BHA}(\dot{Z}_{BHA} - \dot{Z}_{db}) + K_{z,BHA}(Z_{BHA} - Z_{dp}) + K_{z,BHA}(Z_{BHA} - Z_{db}) &= +F_z \\
    (J_{BHA} + J_e) \ddot{\phi}_z + C_f(\phi_{z,BHA}) + K_{\phi_z,BHA}(\phi_{z,BHA} - \phi_{z,dp}) + K_{\phi_z,BHA}(\phi_{z,BHA} - \phi_{z,db}) &= -T_{z,BHA} \end{align}
\[ I_{BHA}\dot{\phi}_{X_{BHA}} + C_f(\dot{\phi}_{X_{BHA}}) + K_{\phi x-BHA}(\phi_{X_{BHA}} - \phi_{X_{db}}) = -T_{X_{BHA}} \]  
(64)

\[ I_{BHA}\dot{\phi}_{Y_{BHA}} + C_f(\dot{\phi}_{Y_{BHA}}) + K_{\phi y-BHA}(\phi_{Y_{BHA}} - \phi_{Y_{db}}) = -T_{Y_{BHA}} \]  
(65)

**EOM-Drillbit Lumped Mass**

\[ m_{BHA}\ddot{x}_{db} + C_{BHA}(\dot{x}_{db} - \dot{x}_{BHA}) + K_{BHA}(x_{db} - x_{BHA}) = F_{x_{db}} - WOB_x \]  
(66)

\[ m_{BHA}\ddot{y}_{db} + C_{BHA}(\dot{y}_{db} - \dot{y}_{BHA}) + K_{BHA}(y_{db} - y_{BHA}) = F_{y_{db}} - WOB_y \]  
(67)

\[ m_{BHA}\ddot{z}_{db} + C_{z_{BHA}}(\dot{z}_{db} - \dot{z}_{BHA}) + K_{z_{BHA}}(z_{db} - z_{BHA}) = F_{z_{db}} - WOB_z \]  
(68)

\[ I_{db}\ddot{\phi}_{z_{db}} + C_f(\dot{\phi}_{z_{db}}) + K_{\phi z_{BHA}}(\phi_{z_{db}} - \phi_{z_{BHA}}) = -TOB_T_{z_{db}} \]  
(69)

\[ I_{db}\ddot{\phi}_{x_{db}} + C_f(\dot{\phi}_{x_{db}}) + K_{\phi x_{BHA}}(\phi_{x_{db}} - \phi_{x_{BHA}}) = -T_{x_{db}} \]  
(70)

\[ I_{db}\ddot{\phi}_{y_{db}} + C_f(\dot{\phi}_{y_{db}}) + K_{\phi y_{BHA}}(\phi_{y_{db}} - \phi_{y_{BHA}}) = -T_{y_{db}} \]  
(71)

For the upper lumped mass that represents the Drillpipes mass with stabilizer, the contact forces can be expressed or analyzed in the Cartesian stationary system (X, Y, Z) as:

\[ F_{x_{dp}} = [-F_{N_{dp}} \cdot (X_{dp}) - F_{t_{dp}} \cdot (Y_{dp})]/r_{dp} \]  
(72)

\[ F_{y_{dp}} = [-F_{N_{dp}} \cdot (Y_{dp}) + F_{t_{dp}} \cdot (X_{dp})]/r_{dp} \]  
(73)

\[ F_{z_{dp}} = -\text{sign}(\dot{Z}_{dp}) \cdot \mu_0 \cdot F_{N_{dp}} \]  
(74)

Hence, the toques applied due to contact forces in X, Y, Z axes:

\[ T_{x_{dp}} = \text{sgn}(\dot{\phi}_{x_{dp}}) \cdot \mu_0 \cdot F_{N_{dp}} \cdot R_s \cdot \frac{|Y_{dp}|}{r_{dp}} \]  
(75)
\[T_{y,dp} = \text{sign}(\dot{\varphi}_{y,dp}) \cdot \mu_0 \cdot F_{N,dp} \cdot R_{st} \cdot \frac{|X_{dp}|}{r_{dp}} \quad (76)\]

\[T_{z,dp} = \text{sign}(\dot{\varphi}_{z,dp}) \cdot F_{t,dp} \cdot R_{st} \quad (77)\]

For the intermediate lumped mass that represents the first portion of the BHA-Drill collars, the contact forces can be expressed or analyzed in the Cartesian stationary system (X, Y, Z) as:

\[F_{x,BHA} = \left[-F_{N,BHA} \cdot (X_{BHA}) - F_{t,BHA} \cdot (Y_{BHA})\right]/r_{BHA} \quad (78)\]

\[F_{y,BHA} = \left[-F_{N,BHA} \cdot (Y_{BHA}) + F_{t,BHA} \cdot (X_{BHA})\right]/r_{BHA} \quad (79)\]

\[F_{z,BHA} = -\text{sign}(\dot{Z}_{BHA}) \cdot \mu_0 \cdot F_{N,BHA} \quad (80)\]

Hence, the toques applied due to contact forces in X,Y,Z axes:

\[T_{x,BHA} = \text{sign}(\dot{\varphi}_{x,BHA}) \cdot \mu_0 \cdot F_{N,BHA} \cdot R_{BHA} \cdot \frac{|Y_{BHA}|}{r_{BHA}} \quad (81)\]

\[T_{y,BHA} = \text{sign}(\dot{\varphi}_{y,BHA}) \cdot \mu_0 \cdot F_{N,BHA} \cdot R_{BHA} \cdot \frac{|X_{BHA}|}{r_{BHA}} \quad (82)\]

\[T_{z,BHA} = \text{sign}(\dot{\varphi}_{z,BHA}) \cdot F_{t,BHA} \cdot R_{BHA} \quad (83)\]

The eccentricity force induced due to mass imbalance can be expressed in X, Y coordinates as:

\[F_{ex} = F_e \cdot \cos \varphi_{z,BHA} \quad (84)\]

\[F_{ey} = F_e \cdot \sin \varphi_{z,BHA} \quad (85)\]

Where;

\[F_e = e \cdot m_{BHA} \cdot \varphi_{z,BHA}^2 \quad (86)\]
For lower lumped mass that represents the second portion of the BHA-Drill collars in addition to the drillbit and stabilizer, the contact forces can be expressed or analyzed in the Cartesian stationary system (X, Y, Z) as:

\[
F_{x,db} = \left[-F_{N,db} \cdot (X_{db}) - F_{t,db} \cdot (Y_{db})\right] / r_{BHA}
\]

(87)

\[
F_{y,db} = \left[-F_{N,db} \cdot (Y_{db}) + F_{t,db} \cdot (X_{db})\right] / r_{db}
\]

(88)

\[
F_{z,db} = -\text{sign}(\dot{Z}_{db}) \cdot \mu_0 \cdot F_{N,db}
\]

(89)

Hence, the toques applied due to contact forces in X,Y,Z axes:

\[
T_{x,db} = \text{sign}(\dot{\varphi}_{x,db}) \cdot \mu_0 \cdot F_{N,db} \cdot a \cdot \frac{|Y_{db}|}{r_{db}}
\]

(90)

\[
T_{y,db} = \text{sign}(\dot{\varphi}_{y,db}) \cdot \mu_0 \cdot F_{N,db} \cdot a \cdot \frac{|X_{db}|}{r_{db}}
\]

(91)

\[
T_{z,db} = \text{sign}(\dot{\varphi}_{z,db}) \cdot F_{t,db} \cdot a
\]

(92)

Where, \(a\) is the drillbit radius.

Most of the drill collar weight is supported by the drillpipe and a fraction of it is applied on the bit (applied WOB). Under static conditions this applied force \(W_0\) is the difference between the total weight and the hook load. \(WOB\) is the weight on bit, which is the axial force applied at the bit under dynamic conditions, and in our case here is given as:

\[
WOB_z = \begin{cases} 
K_c (z - s) & \text{if } z \geq s \\ 
0 & \text{if } z < s 
\end{cases}
\]

(93)

Where \(K_c\) is the formation contact stiffness and \(S\) is the formation surface elevation given as:
\[ S = S_0 \cdot \sin(\varphi_{z,db}) \]  

(94)

If there is a drillbit tilt angle, \( \alpha \), there will be lateral forces components for the \( WOB \) in X,Y direction acts on the drillbit as well:

\[ WOB_x = WOB_z \cdot \tan \alpha \cdot \cos \varphi_{z,db} \]  

(95)

\[ WOB_y = WOB_z \cdot \tan \alpha \cdot \sin \varphi_{z,db} \]  

(96)

The toque on bit, \( TOB \), is related to the \( WOB \) and cutting conditions and is given as:

\[ TOB = WOB_z \left( \mu(\varphi_{z,db}) + \sqrt{\frac{d_c}{a}} \right) \alpha \]  

(97)

Where \( d_c \) is the depth of cut per revolution, and \( \mu(\varphi_{z,db}) \) is the Striebeck nonlinear friction function of the drillbit rotation speed and are given as:

\[ \mu(\varphi_{z,db}) = \mu_0 \left( \tan^{-1} \varphi_{z,db} + \frac{2\dot{\varphi}_{z,db}}{1 + \varphi_{z,db}^2} + 0.01\varphi_{z,db} \right) \]  

(98)

\[ d_c = \frac{2\pi \cdot ROP}{\varphi_{z,db}} \]  

(99)

The rate of penetration, \( ROP \), here will be a function of applied weight on bit \( W_0 \), and the bit speed, and rock/bit characteristics given as:

\[ ROP = C_1 \cdot W_0 \cdot \sqrt{\varphi_{z,db}} + C_2 \]  

(100)

Table 3 contains the simulation parameters for the different drillstring components that correspond to a real system design.
Table 3 Simulation Parameters for Drillstring Lumped-System Model

<table>
<thead>
<tr>
<th>Drillpipe Parameters</th>
<th>Value</th>
<th>BHA Collar Parameters (One Segment)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length, $L_{dp}$</td>
<td>5700 m</td>
<td>Length</td>
<td>180/2 m</td>
</tr>
<tr>
<td>OD</td>
<td>5 in</td>
<td>OD</td>
<td>6.75 in</td>
</tr>
<tr>
<td>ID</td>
<td>3/8 in</td>
<td>ID</td>
<td>4 in</td>
</tr>
<tr>
<td>Cross section area, $A_c$</td>
<td>0.0126 m$^2$</td>
<td>Cross section area, $A_c$</td>
<td>0.015 m$^2$</td>
</tr>
<tr>
<td>Nominal wt., $M_{dp}$</td>
<td>5.6363x10$^5$ Kg</td>
<td>Nominal wt., $M_{BHA}$</td>
<td>1.30583x10$^4$ Kg</td>
</tr>
<tr>
<td>Inertia (z), $J_{dp}$</td>
<td>1.1427x10$^3$ Kg.m$^2$</td>
<td>Inertia (z), $J_{BHA}$</td>
<td>52.5467 Kg.m$^2$</td>
</tr>
<tr>
<td>Inertia (x,y), $I_{dp}$</td>
<td>1.9075x10$^{11}$ Kg.m$^2$</td>
<td>Inertia (x,y), $I_{BHA}$</td>
<td>8.9396x10$^5$ Kg.m$^2$</td>
</tr>
<tr>
<td>Radial stiffness, $K_x/K_y$</td>
<td>2.4720x10$^8$ N/m</td>
<td>Radial stiffness, $K_x/K_y$</td>
<td>1.4699x10$^9$ N/m</td>
</tr>
<tr>
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<td>Axial stiffness, $K_z$</td>
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<table>
<thead>
<tr>
<th>Stabilizer Parameters</th>
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<th>Eccentricity, $e$</th>
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<td>OD</td>
<td>7 in</td>
<td>Eccentricity Inertia, $J_e$</td>
<td>$M_{BHA}.e^2$</td>
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<table>
<thead>
<tr>
<th>Drillbit Parameters</th>
<th>Value</th>
<th>Material Parameters</th>
<th>Value</th>
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<td>7 in</td>
<td>Young modulus, $E$</td>
<td>200x10$^9$ N/m$^2$</td>
</tr>
<tr>
<td>Radius, $a$</td>
<td>3.5 in</td>
<td>Steel density, $\rho$</td>
<td>7850 Kg/m$^3$</td>
</tr>
<tr>
<td>Bit bent angle, $\alpha$</td>
<td>0*Pi/180</td>
<td>Shear Modulus, $G$</td>
<td>$(3/8)*E$</td>
</tr>
</tbody>
</table>
The stiffnesses of the drillpipe as well as the BHA collar are modeled as beam elements having 6-DOF. All stiffnesses are calculated accordingly and using the below stiffness matrix for 3D beam element:

\[
K_p = \begin{pmatrix}
\frac{EA}{l} & 0 & 0 & 0 & 0 & -\frac{EA}{l} & 0 & 0 & 0 & 0 \\
\frac{12EI_x}{l^2} & 0 & 0 & \frac{6EI_y}{l^2} & 0 & -\frac{12EI_x}{l^2} & 0 & 0 & 0 & \frac{6EI_y}{l^2} \\
\frac{12EI_z}{l^2} & 0 & 0 & 0 & 0 & -\frac{12EI_z}{l^2} & 0 & 0 & \frac{6EI_z}{l^2} & 0 \\
0 & \frac{GJ}{l} & 0 & 0 & 0 & 0 & -\frac{GJ}{l} & 0 & 0 & 0 \\
0 & 0 & \frac{4EI_y}{l} & 0 & 0 & 0 & 0 & \frac{2EI_y}{l} & 0 & 0 \\
0 & 0 & 0 & \frac{6EI_z}{l^2} & 0 & 0 & 0 & 0 & -\frac{6EI_z}{l^2} & 0 \\
0 & 0 & 0 & 0 & \frac{12EI_z}{l^2} & 0 & 0 & 0 & 0 & \frac{2EI_z}{l} \\
0 & 0 & 0 & 0 & 0 & \frac{GJ}{l} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{4EI_y}{l} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{4EI_z}{l} \\
\end{pmatrix}
\]

(101)
Simulation Results

In this section we will present the results obtained from one of the simulation studies done on the drillstring lumped system model. The vibrations are initially self-induced due to imbalance mass eccentricity in the intermediate lumped mass model. Boundary conditions, represented in the top drive constant speed $\omega$, dynamic friction coefficient $\mu$, eccentricity value $e$, and applied static load $W_0$, were applied. Stiffness and damping parameters for the three lumped masses were slightly tuned by try and error to provide the required dynamic response.

Figure 46 shows simulation results obtained from the lumped system model of the drillstring. Panel (a) shows the 2D plane C.G. orbit of the upper stabilizer inside the borehole. Panel (b) shows the 2D orbit of the BHA C.G. inside the borehole. As the BHA collar has a smaller diameter than the stabilizer and drill bit, the clearance hence is bigger. Panel (c) shows the 3D pathline of the drill bit inside the borehole indicating the drilling direction, while panel (d) shows the projection 2D plane orbit of the drill bit. The centrifugal force due to mass imbalance is induced in the intermediate BHA C.G. lump mas, which in accordance affects the dynamic behavior of the upper and lower lump masses. The whirling motion induced in the three lump masses appears to be unstable causing an unstable drilling operation and hence reduction in the rate of penetration. Panel (e) shows the rotation velocities of the three lumped masses. Once there is lateral contact with the borehole formation and friction torque is applied, more oscillations begin to appear which leads to higher torsional vibrations. Panel (f) shows the relative velocities between the lump masses and formation. The relative velocities measurement gives the
first indication if pure rolling is going to happen once a zero value is reached. Panel (g) shows the whirling velocities of the mass bodies. Panels (h) and (i) show $WOB$ and $TOB$ respectively. The plots show bit-bounce results from axial drillstring vibrations. Axial vibrations reduce the rate of penetration of drilling due to fluctuations in $WOB$. Bit-Bounce are caused when there is a temporary lift-off of the drillbit from the formation and the $WOB$ and $TOB$ readings reach zero. Panels (j) and (k) show the normal contact forces on the three masses, and the friction contact force respectively. When there is more continuous contact with the formation, in case of forward whirling or backward whirling, continuous readings are obtained with higher magnitudes.

Figure 46 Simulation Results of Lumped-System Drillstring
Figure 46 Continued
Conclusion

Lumped system modelling is a very useful method in order to have a better understanding of the equation of motions and better sense of how applied forces and boundary conditions could affect the system dynamics, as well as how each mass dynamic behavior affects others due to the interaction of stiffnesses. Another advantage in using this method is to have a relatively fast numerical calculation. However this method is not accurate as the system is discrete and lumped into a limited number of masses instead of one continuous mass. The stiffness parameters that couple the lump masses together might need extensive tuning by try and error so as to give a close system behavior as in real.

6.3 Drillstring Finite Element Modeling

In this section, Finite-Element-Method is used to enhance the drillstring dynamics model using a more accurate approach. Instead of treating the BHA as three lumped masses as discussed in the previous section, the BHA will be meshed or discretized into a number of 3D-Cylindrical Timoshenko beam elements, where each node will have 6-
DOF. The drillpipes can be substituted by a lumped mass, axial & torsional springs, and
damper attached to the top node of the BHA, as they are generally much lighter than the
BHA and have a negligible contribution on the drillstring dynamics. Stabilizers models
will be applied and located at any of the BHA’s nodes by adding the corresponding DOF’s
and geometry profile. Right boundary conditions and contact logic can be applied in the
same manner as in the previous two sections for lateral vibration, and axial & torsional
vibrations such that the drillstring experiences axial, torsional, and lateral vibrations.
External forces are applied at each BHA nodes including stabilizers’ nodes and the bottom
node for the PDC-drillbit. The model accounts the gyroscopic effect, the torsional/bending
inertia coupling, and the effect of the gravitational force. This model is a more complex
and will give a more accurate prediction to the rotordynamic behavior of the BHA, and
the PDC-Drillbit cutting dynamics. Reference [35] is used as a major reference in this
step, as the work will be extended to account the Backward whirl.

Different simulation studies will be conducted to investigate the drillstring different
vibration modes from the finite element model to obtain better understanding of the effect
of system parameters on system response.

Objective

To simulate more accurately the dynamics of the drillstring while drilling by
building a 3D Finite-Element-Model using cylindrical Timoshenko beam element to
describe the axial, lateral, and rotational vibration modes with the nonlinear bit/rock
interaction. Gyroscopic effects will also be included. Following are the different dynamic
vibration behaviors that will be investigated with detailed analysis:
- Forward whirling, under self-excited vibration
- Backward whirling with pure-rolling, under self-excited vibration
- Chaotic and random whirling vibration
- Stick-slip drilling
- Bit bounce

**Theoretical Background**

The Timoshenko finite element model takes into account the shear deformation and rotational inertia effects, unlike Euler-Bernoulli beam theory that assumes the plane across sections remain plane and normal to the longitudinal axis after bending which results in zero transverse shear strain. Timoshenko beam theory assumes that the rotation of a transverse normal plane about the longitudinal axis, $\Theta_3$, is not equal to $-dv/dx_1$, therefore the transverse shear strain is not zero [47]. Figure 47 shows the kinematics of the Timoshenko beam theory in comparison to the Euler- Bernoulli beam theory.
The BHA- drill collars in the drillstring will be discretized using a number of hollow cylindrical Timoshenko beam elements. Each element consists of two nodes at its ends. Each node has six DOF consisting three translations \((u, v, w)\) in the \((x, y, z)\) and three rotations about the three axis \((\Theta_1, \Theta_2, \Theta_3)\). Figure 48 describes the translation and rotation in a 3D beam element.

**Figure 47** Deformation in Timoshenko Beam versus Euler-Bernoulli Beam [47]

**Figure 48** Translation and Rotation in 3D Beam Element

Mongkolcheep [48] discussed in details the generation of the shape function matrix \([N]\) of the 3D finite element, and the correspondent translation deformations, rotations and
torsional deformations based on the shape function matrix. Calculating the elastic strain energy of the axial, torsional, shear and bending deformations is used to formulate the element stiffness matrices. Equation (102) shows the formulated element stiffness matrix of a 3D 2-node Timoshenko beam element based on the total elastic body strain energy

\[
K_e = \begin{bmatrix}
R^a & 0 & 0 & 0 & 0 & -R^a & 0 & 0 & 0 & 0 \\
0 & 12R_y^b & 0 & 0 & 0 & 6R_y^b & 0 & -12R_y^b & 0 & 0 \\
0 & 0 & 12R_y^b & 0 & -6R_y^b & 0 & 0 & -12R_y^b & 0 & 0 \\
0 & 0 & 0 & R^t & 0 & 0 & 0 & 0 & -R^t & 0 \\
0 & 0 & 0 & 0 & R^t & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -6R_y^b & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -12R_y^b & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -6R_y^b & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -R^t & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\] (102)

Where \([K_e] = [K_A] + [K_T] + [K_S] + [K_B] + [K_G]\) is the element stiffness matrix given by the axial stiffness matrix \([K_A]\), the torsional stiffness matrix \([K_T]\), the shear stiffness matrix \([K_S]\), the bending stiffness matrix \([K_B]\), and the stress stiffening matrix \([K_G]\). The entries of the element stiffness matrix in equation (102) are presented in details in reference [49] and in the appendix.

**Finite-Element-Modeling**

The model consists of the BHA, drillbit, two stabilizers, and drillpipes model. A typical 200m BHA is meshed into six 3D-Timoshenko hollow cylindrical beam elements to represent the heavy drill collars in the drillstring. Each element consists of 2-nodes with added axial and torsional stiffness. The assembled Global Mass, Stiffness, Damping, as well as Gyroscopic Matrices have been successfully generated and verified. The drillpipes are substituted by a single lumped mass, torsional spring, as well as torsional and axial
damper, and attached to the top node of the BHA. Each to its correspondent node’s DOF. The static load, $W_0$, is applied axially on the top node. The top drive torque (motor torque) with a constant driving rotation speed is applied as well on the BHA top node. The PDC-Drillbit model and its characteristics have been applied on the BHA bottom node with the associate $WOB$, and $TOB$ when the drillbit is penetrating the rock formation and by taking into consideration the time delay term. Two rigid massless stabilizers are considered at two location along the BHA; one at the top node, and the other at the last node before the bottom. The stabilizers have the same radius as the drillbit. The borehole is modeled as formation with spring stiffness $K_f$ from the radial direction, and $K_c$ from the bottom. A clearance $R_c$ is left between the borehole and the stabilizers. An imbalance centrifugal force is induced at the second stabilizer node at the bottom. The contact forces between the stabilizers and wellbore occur when the lateral displacement of the stabilizer becomes larger than the clearance $R_c$. Lateral and torsional damping are applied on the BHA due to drilling fluid (mud) viscosity. Two choices are given in the code to implement any mass body (e.g. flywheel) under dynamics investigation inside the BHA collars. Choices are either the node above the drillbit location at the bottom of the BHA-Assembly, or at the BHA’s C.G. location. This will give the advantage for any future work to extend the code capability to study the effect of the induced drillstring vibrations on a possible implemented flywheel inside the BHA collars.
Figure 49 describes the FEM of the BHA indicating the position of the drillbit, drillpipes, and stabilizers correspondent to the BHA elements nodes as well as the applied loads. The table below (Table 4) shows the parameters with the values used in the FEM-Drillstring simulation studies.
### Table 4 Simulation parameters for Drillstring FEM

<table>
<thead>
<tr>
<th>Drillpipe Parameters</th>
<th>Value</th>
<th>BHA Collar Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>110 m</td>
<td>Length</td>
<td>200 m</td>
</tr>
<tr>
<td>OD</td>
<td>5 in</td>
<td>OD</td>
<td>6.75 in</td>
</tr>
<tr>
<td>ID</td>
<td>3/8 in</td>
<td>ID</td>
<td>variable in</td>
</tr>
<tr>
<td>Cross section area, $A_c$</td>
<td>0.0126 m$^2$</td>
<td>Eccentricity, $e$</td>
<td>0.003 m</td>
</tr>
<tr>
<td>Nominal wt., $M_{dp}$</td>
<td>1.0877x10$^4$ Kg</td>
<td><strong>Stabilizers Parameters</strong></td>
<td><strong>Value</strong></td>
</tr>
<tr>
<td>Inertia about Z, $J_{dp}$</td>
<td>22.0528 Kg.m$^2$</td>
<td>OD</td>
<td>8.5 in</td>
</tr>
<tr>
<td>Axial Damping, $C_z$</td>
<td>15x10$^3$ Nms/rad</td>
<td><strong>Formation Parameters</strong></td>
<td><strong>Value</strong></td>
</tr>
<tr>
<td>Torsional Damping, $C_{\phi z}$</td>
<td>500 Nm/rad</td>
<td>Borehole diameter</td>
<td>9.5 in</td>
</tr>
<tr>
<td>Torsional stiffness, $K_{\phi z}$</td>
<td>600 Nm/rad</td>
<td>Formation Stiffness, $K_f$</td>
<td>variable N/m</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Drillbit Parameters</th>
<th>Value</th>
<th><strong>Other</strong></th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>OD</td>
<td>8.5 in</td>
<td>Mud axial damping, $C_{rf}$</td>
<td>10 Nms/rad</td>
</tr>
<tr>
<td>Radius, $a$</td>
<td>4.25 in</td>
<td>Mud torsional damping, $C_{\phi f}$</td>
<td>2000 Nm/rad</td>
</tr>
<tr>
<td>Bit tilting angle, $\alpha$</td>
<td>0*Pi/180</td>
<td>Friction coefficient, $\mu_0$</td>
<td>variable</td>
</tr>
<tr>
<td>Number of blades, $n$</td>
<td>8</td>
<td><strong>Other</strong></td>
<td>Value</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Material Parameters</th>
<th>Value</th>
<th>Applied Static load, $W_0$</th>
<th>variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young modulus, $E$</td>
<td>200x10$^9$ N/m$^2$</td>
<td>Rotary table RPM, $\omega$</td>
<td>variable</td>
</tr>
<tr>
<td>Steel density, $\rho$</td>
<td>7850 Kg/m$^3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shear Modulus, $G$</td>
<td>(3/8)*$E$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Poisson ratio, $\nu$</td>
<td>0.3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Simulation Results

The main objective of the simulation studies conducted is to examine the drillstring dynamic response under influence of the operation parameters and examine the interaction or coupling between torsional and axial vibrations, rather than predicting quantitatively a real drilling dynamics using a particular drillbit type.

Although the drillstring might be perfectly symmetric, assuming perfect symmetric drillbit and any other implemented instruments including mud motor, the drillstring can still encounter the different types of vibrations. Formation stiffness, as well as the stiffness of the drill-collars itself, play a major role in initiating one mode of vibration that could lead to the other two modes of vibrations. For example, drilling in hard formations (high WOB) with low rotation speeds could initiate first the bit stick-slip vibration mode due to the non-linear Strubeck friction function that causes instability in torsional motion at low speeds. Accordingly, and due to the direct proportional between the TOB and the WOB expressed in equation (97), axial vibrations will be then imitated. Another example, in case of hard formations and weak (non-stiff) drill-collars and high applied static load, that could cause drillstring to bend or to buckle which will lead in accordance to lateral or whirling motion.

The studies below summarize the different vibration modes that could encounter the drillstring during the drilling operation.

Chaotic lateral vibration with Bit-bounce

Drilling under low upper static applied load will lead to low WOB, which under normal to high rotational speed will eliminate the stick-slip behavior but can cause on the other hand
bit-bounce. Mass Imbalance (or eccentricity) under normal to high rotational speeds will induce whirling motion in the drillstring. If the formation has a low coefficient of friction, forward whirling will occur. However, if the formation has a slightly higher coefficient of friction, a chaotic unstable whirling behavior will occur till the friction force at the point of contact is high enough that encourages the body to roll.

Table 5 shows the parameters values used in this simulation study that induced the chaotic lateral drillstring vibrations with bit-bounce behavior.

**Table 5** Simulation Parameters for Chaotic Drillstring Vibrations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotary Table Speed, $\omega$</td>
<td>100 (rpm)</td>
</tr>
<tr>
<td>Upper static applied load, $W_0$</td>
<td>50e3(N)</td>
</tr>
<tr>
<td>Coulomb dry friction coefficient, $\mu_0$</td>
<td>0.07</td>
</tr>
<tr>
<td>Radial formation contact stiffness, $K_f$</td>
<td>2e8 (N/m)</td>
</tr>
<tr>
<td>Eccentricity, $e$</td>
<td>0.003 (m)</td>
</tr>
<tr>
<td>Applied eccentricity node</td>
<td>Node #6</td>
</tr>
<tr>
<td>BHA inner diameter, $ID$</td>
<td>4(in)</td>
</tr>
</tbody>
</table>

Panel (a) shows the axial penetration displacement of the drillbit by time. The plot shows increasing axial fluctuating (vibration) as the rotation speed of the drillbit increases to catch the upper applied rotary table speed. That causes decrease in $ROP$ and depth of cut. Panel (b) shows the 3D path-line orbit of the drillbit function of time. As the drillbit
is increasing its rotation speed from zero to the operating top drive speed, higher \( ROP \) and DOC are achieved. As axial vibration increases ROP and the achieved DOC is decreased accordingly. Panel (c), (d), (e), and (f) show the 2D lateral vibration of the upper stabilizer, BHA-CG, bottom stabilizer, and drillbit respectively. The red dotted circle indicates the clearance boundary between the component CG and the borehole. The BHA collar has the biggest clearance with the borehole, as it has the smallest diameter compared to the stabilizers and the drillbit. The bottom stabilizer has the most severe lateral vibrations induced, as the eccentricity is applied at its node. The upper stabilizer is about 167m far from the bottom stabilizer, and that is why it has the least whirling motion induced. A chaotic lateral vibration is induced that causes the bottom stabilizer to have discrete impacts with unsteady whirl at intermediate coefficient of friction with the formation \( (\mu_0=0.07) \). Panel (g) shows the rotational velocities of the BHA components, where the drillbit has the most severe rotation speed variation due to friction with the bottom formation while cutting, as well as the torsional stiffness of the BHA and drill pipes. Panel (h) shows the relative velocities of the different BHA components. The relative velocity might reach to zero value, but still other conditions are still not satisfied to achieve the pure rolling status. That appears in panel (i) as the whirling velocity doesn’t indicate high whirling frequency with a negative value. Panels (j), and (k) show the \( WOB \) and \( TOB \) respectively. The zero values of the \( WOB \) and \( TOB \) indicate bit-bounce as the drillbit lose contact with the bottom formation and hence now weight is applied on the bit as well torque. Panel (l) shows the normal contact force applied on the bottom stabilizer due to radial impacts with the borehole formation.
Figure 50 Simulation Results of Chaotic Vibration Behavior with Bit-Bounce
Figure 50 Continued
Figure 50 Continued
Figure 50 Continued
Backward Whirling with Pure Rolling Motion

Five major parameters can lead to backward whirling with pure rolling; to have a weak (or non-stiff) drillstring, assembly imbalance, low WOB, high rotational speed, and high friction coefficient with the formation. Table 6 shows the input parameters values that were used to achieve the backward whirling with pure rolling. The stiffness of the BHA has been reduced by decreasing the drill-collars wall thickness by 0.25” compared to the previous study. Rotary table drive speed has been increased from 100 rpm to 120 rpm. Formation friction coefficient increased from 0.07 to 0.2. Low WOB can be achieved, according to equations, either by having weak formation stiffness or be reducing the applied top static load W0.

Table 6 Simulation Parameters for Drillstring with Pure Rolling Motion

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotary Table Speed, $\omega$</td>
<td>120 (rpm)</td>
</tr>
<tr>
<td>Upper static applied load, $W_0$</td>
<td>50e3(N)</td>
</tr>
<tr>
<td>Coulomb dry friction coefficient, $\mu_0$</td>
<td>0.2</td>
</tr>
<tr>
<td>Radial formation contact stiffness, $K_f$</td>
<td>2e8 (N/m)</td>
</tr>
<tr>
<td>Eccentricity, $e$</td>
<td>0.003 (m)</td>
</tr>
<tr>
<td>Applied eccentricity node</td>
<td>Node #6</td>
</tr>
<tr>
<td>BHA inner diameter, $ID$</td>
<td>4.5(in)</td>
</tr>
</tbody>
</table>
Panel (a) in figure 51 shows the axial penetration achieved by the drillbit as a function of time. The high frequency oscillation in the plot at certain time periods with magnitudes below the mean value indicates bit-bounce, and as indicated in panel (c) where the drillbit has a negative axial penetration velocity. Plot (a) also indicates reduction in rate of penetration at certain time periods due to lateral motion instead of axial or vertical motion. This could be concluded more clearly from panel (b), where at certain depths the drillbit motion is switched to lateral motion rather than vertical motion. This could be due to forward or backward whirling. Three measurements could indicate whether the body is in forward whirling mode or in backward, and whether it is mixed or fully (purely) developed. These measurements are the relative velocity, the whirling velocity or whirling frequency, and whirling direction. As the eccentricity is applied at node #6 at the bottom stabilizer location, we should suspect first that the whirling is occurring at this location. If the whirling is too severe with a really weak BHA stiffness, whirling behavior could be extended to other BHA component that has lower clearance with the borehole (e.g. drillbit or upper stabilizer). To affirm what components exactly that might experience whirling behavior we should inspect the lateral vibration of each component body or the plane orbit of the components’ C.G. Panels (d), (e), (f), and (g) show the 2D C.G. orbit of the upper stabilizer, BHA, bottom stabilizer, and drillbit. The results show only a whirling behavior in the bottom stabilizer as suspected, while other components barely have contact with borehole formation.

Panel (h) indicates high oscillations in the drillbit and bottom stabilizer rotation velocities resultant from formation contact friction that causes applied friction torques on
the rotating body. The plot shows that the drillstring is taking almost 50 seconds to reach the full operation set drive speed 120 rpm. Panel (i) indicates the relative velocity measurement of each BHA component. The plot indicates a sudden drop in the relative velocity of the bottom stabilizer reaching zero value with high oscillations around it. This is a first indication that backward whirling is happening here, but probably mixed with sliding motion as the tangential velocity vector \( \Omega_{\text{whirl,r}} \) is bigger than \( \phi R \) (refer to figures 34 & 35).

**Figure 51** Simulation Results of Backward Whirling with Pure Rolling Motion
Figure 51 Continued
**Figure 51** Continued
Figure 51 Continued
Panel (j) shows the whirling velocities, $\Omega_{whirl}$, of the BHA components. A second indication for backward whirling is the negative jump in the whirling velocity reading of the bottom stabilizer reaching high frequency values as the rotation velocity increase. Other whirling velocity readings for the drillbit, upper stabilizer, and BHA-C.G indicate forward whirling with positive sign direction and low stable magnitudes. The red dashed lines indicates the theoretical, or the calculated, whirling velocity of the bottom stabilizer according to equation (36), as well the conditions described in simulation flowchart (figure 38). From equation, the value of the backward whirl velocity depends on the component rotation velocity, and the radial displacement of the component C.G. (or simply the clearance between the body and borehole). The smaller clearance we have, the higher whirling frequency is obtained. This value could reaches more than 30 to 60 times that of rotation velocity $\varphi$, which is totally destructive vibration behavior to the drillstring assembly. Panel (k) shows the normal contact force applied on the bottom stabilizer due
to radial impacts with the borehole formation. Panel (l) shows the tangential friction force applied on the bottom stabilizer due to contact with formation, and the corresponding calculated tangential friction force if pure rolling occurs (red dashed line). The calculated value is used to indicate whether the bottom stabilizer will switch from mixed or forward whirling to pure rolling status (rolling without slipping) or not, where the tangential contact force should not exceed the maximal friction force and be bounded between $-\mu F_N \leq F_t \leq \mu F_N$. Panel (m) indicates the pure rolling status of the bottom stabilizer if all mentioned conditions are satisfied and according to the simulation logic presented in figure 38. The figure shows a discontinuous pure rolling behavior of the bottom stabilizer associated with interval sliding motion in-between. Panels (n) and (o) show the WOB and TOB respectively. The zero values of the WOB and TOB indicate bit-bounce as the drillbit lose contact with the bottom formation and hence now weight is applied on the bit as well torque.

**Conclusion**

Modeling using finite element method, and more specifically using Timoshenko beam element theory, gives a more accurate and trustful results than using the lumped system method. This method doesn’t require intervention to tune the element stiffness parameters as needed when using lumped system method.

Drillbit cutting dynamics has a coupling between axial and torsional modes expressed in the relationship between the WOB and TOB. The non-linear Striebeck friction function between the drillbit and formation can initiate a self-excited torsional vibration mode on the drillbit at lower values of top drive spin speeds and in accordance to an axial
vibration mode which can be characterized by bit bouncing and a total stick-slip. Increasing the top drive spin speed should mitigate the instability effect of the non-linear Strubeck friction and leads to a more stable cutting process and rate of penetration. Reducing the applied WOB could help alleviate torsional vibrations by reducing the TOB, however lower WOB will negatively affect the drilling efficiency by having a lower rate of penetration as well as can lead to bit bounce.

Drillstring stiffness is one of the major factors that lead to lateral vibrations even if the drillstring is perfectly symmetric and has no eccentricity or mass imbalance. Drilling in hard formation under high static applied load could cause the BHA to buckle and bend which under the rotation velocity leads to BHA whirling.

Formation friction coefficient as well as rotation speed are responsible in characterizing the type of whirling motion induced, whether forward or backward whirling. Five major parameters can lead to backward whirling with pure rolling; to have a weak (or non-stiff) drillstring, assembly imbalance, low WOB, high rotational speed, and high friction coefficient with the formation. The calculated tangential friction force, at zero contact relative velocity, in case of pure rolling should not exceed the maximal friction force \( \mu F_N \) otherwise the body will switch to a non-pure rolling motion status.

The clearance between the borehole and the BHA components, especially the installed stabilizers, plays a major role in identifying the severity of the whirling frequency in case of pure rolling.
Axial, torsional, and lateral vibrations lead to reduction in rate of penetration and in accordance increasing the non-productive time, despite the destructive nature on the drillstring assembly.

Checking and determining drillstring mass unbalance magnitude and angular position before the drilling operation could help in determining the optimal positions to install the stabilizers and ways to suppress the vibrations.
7. DRILLSTRING VIBRATION DYNAMICS CONCLUSIONS

The following conclusions are based upon the studies conducted to simulate the drillstring dynamics using different methods, and from the results obtained.

- There are various potential excitation sources of drillstring vibration such as: mass imbalance, friction factor between drillstring and borehole, operational spin velocity, cutting action of the drillbit, BHA stiffness, clearance between BHA components and borehole, bent angle, stabilizers locations, and fluid forces around the drillstring.

- The friction coefficient of the formation and the top drive angular speed can drastically affect the rotor dynamics. Increasing the friction coefficient can change rotation behavior from forward whirl with pure sliding to backward whirl and even to pure rolling without slipping. The non-linear Stribeck friction function causes instability in the rotation motion at low angular velocities which can leads to drillbit stick-slip torsional vibration.

- Stick-slip vibration behavior is induced generally at low drilling operational speeds and high applied $WOB$ (high formation stiffness).

- There is a coupling between the torsional and axial vibrations of the drillstring dynamics. Drillbit is the major cause of this coupling expressed in the direct relationship between the $TOB$ and the $WOB$.

- Drillbit with bent angle is a major source in self-exciting the three types of vibration in the drillstring while drilling, as the lateral component of the $WOB$ causes a coupling between the axial and the lateral modes of vibration and hence
torsional. The coupling between the three vibration modes is first addressed by Saeed and Palazzolo [35], and is implemented in the codes presented here as a feature used in the simulation investigations.

- Lateral vibrations are the most destructive type of vibration and are generally induced due to drillstring bending from the imbalance of the assembly or weak BHA stiffness, and at high angular velocities. The interaction between a BHA component (e.g. stabilizer, drillbit) and the borehole formation determined by the friction coefficient value leads either to forward whirl, chaotic discrete lateral vibration, or in certain circumstances to backward whirl.

- Backward whirl is an undesired vibration mode since it can lead to a pure rolling contact motion behavior that causes destructive reverse bending vibration mode with high frequency whirling motion. This results in high rates of BHA component and connection fatigue, as well as in an over gauged borehole due to high frequency rubbing.

- Five major parameters can lead to backward whirling with pure rolling; to have a weak (or non-stiff) drill-collars, assembly imbalance, low WOB, high rotational speed, and high side friction coefficient with the formation.

- The clearance between the borehole and the BHA components, especially the installed stabilizers, plays a major role in identifying the severity of the whirling frequency in case of pure rolling.

- A Novel model has been described for investigating the drillstring dynamics while drilling, using Timoshenko finite-element-modelling including the most
destructive mode of vibration “Backward whirling with pure rolling”. This gives a more accurate examination of the drillstring dynamic response under the influence of the operation parameters, more than any other models addressed before in literature.

- Some features are added to the code to give a wider range of input conditions such as; non-linear Stribeck friction, drilling mud damping effect, drillbit bent angle, locations for implemented mass bodies inside the BHA collars (e.g. flywheel), node selection for applied eccentricity, and node selection for attached stabilizers.
REFERENCES


Entries of the element stiffness matrix of a 3D Timoshenko beam element with 2-nodes according to reference [26]. The other not listed entries are zeroes.

\[
\begin{align*}
    k_{1,1} &= k_{7,7} = -k_{1,7} = \frac{EA}{L} \\
    k_{2,2} &= k_{2,8} = \frac{12kGAEl_y(12El_y + kGA)^2}{L(12El_y - kGA)^2} \\
    k_{2,6} &= k_{2,12} = \frac{6kGAEl_y(12El_y + kGA)}{(9kEl_y - kGA)^2} \\
    k_{3,3} &= k_{3,9} = \frac{12kGAEl_z(12El_z + kGA)}{L(12El_z - kGA)^2} \\
    k_{3,5} &= k_{3,11} = \frac{6kGAEl_z(12El_z + kGA)}{(12El_z - kGA)^2} \\
    k_{4,4} &= k_{7,7} = -k_{4,10} = \frac{G(I_y+I_z)}{L} \\
    k_{5,5} &= \frac{4EI_y[(kGA)^2L^4 + 3kGA^2 EI_y + 36(EI_y)^2]}{L(12El_y - kGA)^2} \\
    k_{5,9} &= \frac{6kGAEl_y(12El_y + kGA)^2}{(12El_y - kGA)^2} \\
    k_{5,11} &= -\frac{2EI_y[72(El_y)^2 - (kGA)^2 L^4 - 30kGA^2 EI_y]}{L(12El_y - kGA)^2} \\
    k_{6,6} &= \frac{4EI_y[(kGA)^2L^4 + 3kGA^2 EI_y + 30(EI_y)^2]}{L(12El_y - kGA)^2} \\
    k_{6,8} &= -\frac{6kGAEl_y(12El_y + kGA)}{(12El_y - kGA)^2} \\
    k_{6,12} &= -\frac{2EI_y[(kGA)^2L^4 + 30kGA^2 EI_y] + 72(EI_y)^2]}{L(12El_y - kGA)^2} \\
    k_{8,8} &= \frac{12kGAEl_y(12El_y + kGA)}{L(12El_y - kGA)^2} \\
    k_{9,11} &= \frac{6kGAEl_y(12El_y + kGA)}{(12El_y - kGA)^2} \\
    k_{11,11} &= \frac{4EI_z[(kGA)^2L^4 + 3kGA^2 EI_z + 36(EI_z)^2]}{L(12El_z - kGA)^2} \\
    k_{12,12} &= \frac{4EI_z[(kGA)^2L^4 + 3kGA^2 EI_z + 36(EI_z)^2]}{L(12El_z - kGA)^2}
\end{align*}
\]
APPENDIX B

MATLAB CODES

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vmin</td>
<td>Minimum relative velocity</td>
</tr>
<tr>
<td>Kb</td>
<td>Borehole Contact Stiffness</td>
</tr>
<tr>
<td>mu0</td>
<td>Borehole Contact Friction</td>
</tr>
<tr>
<td>mud</td>
<td>Borehole dynamic Contact Friction</td>
</tr>
<tr>
<td>mus</td>
<td>Borehole static Contact Friction</td>
</tr>
<tr>
<td>ma</td>
<td>Rotor mass</td>
</tr>
<tr>
<td>e</td>
<td>Eccentricity of rotor's center mass</td>
</tr>
<tr>
<td>R</td>
<td>Rotor radius</td>
</tr>
<tr>
<td>Rh</td>
<td>Borehole radius</td>
</tr>
<tr>
<td>Rc</td>
<td>Rotor-Borehole Clearance according to (Rh)</td>
</tr>
<tr>
<td>Kc</td>
<td>Rotor-Borehole Clearance</td>
</tr>
<tr>
<td>w</td>
<td>Top Drive Spin Speed (rad/s)</td>
</tr>
<tr>
<td>Jc</td>
<td>Rotor's moment inertia at center</td>
</tr>
<tr>
<td>Je</td>
<td>Moment Inertia at unbalance mass</td>
</tr>
<tr>
<td>Jt</td>
<td>Total rotation moment of inertia</td>
</tr>
<tr>
<td>K_phi</td>
<td>Torsional stiffness of rotor</td>
</tr>
</tbody>
</table>

% % [1] % INPUT SYSTEM PARAMETERS

% Set the Minimum relative velocity
Vmin=0.001;
% Borehole Contact Stiffness
Kb=400000;
% Borehole Contact Friction
mu0=0.12;
% Borehole dynamic Contact Friction
mud=0.07;
% Borehole static Contact Friction
mus=1.1*mud;
% Rotor mass
ma=1;
% Eccentricity of rotor's center mass
e=0.001;
% Rotor radius
R=0.05;
% Borehole radius
Rh=0.503;
% Rotor-Borehole Clearance according to (Rh)
Rc=0.0025;
% Rotor-Borehole Clearance
Kc=40;
% Rotor radial stiffness
zeta=0.3;
% Rotor damping coefficient calculated
C=1;
% Rotor lateral damping coefficient
Cphi=0.2;
% Rotor natural freq
wn=sqrt(Kb/ma);
% Top Drive Spin Speed (rad/s)
w=12.6491;
% Rotor's moment inertia at center
Jc=0.5*ma*R^2;
% Moment Inertia at unbalance mass
Je=ma*e^2;
% Total rotation moment of inertia
Jt=Jc+Je;
% Torsional stiffness of rotor
K_phi=0.6;
% % Input Fluid Mud Parameters
% If presence of Mud
D=0.1;
% fluid friction coefficient
B1=0.1;
% constant 1
B2=0.0;
% constant 2
mf=0.5;
% fluid mass
% No Mud
D=0;
% fluid friction coefficient
B1=0;
% constant 1
B2=0;
% constant 2
mf=0;
% fluid mass
% % Calculate total system mass
m=ma+mf;
% Total sys mass
% % [2] % SIMULATION TIME
tstart=0; % Start Simulation Time
tend=10; % End Simulation Time
n=300000; % Number of steps
maxstep=0.01; % Maximum Simulation Step (Output Data)

tspan=linspace(tstart,tend,n); % (fixed step size=tend/n)
\% tspan= [tstart tend]; % (variable step size)
\% Define the initial conditions making sure to use the right ordering
\% X(0)=0 , X'(0)=0
\% Y(0)=0 , Y'(0)=0
\% Phi(0)=0 , Phi'(0)=0
\% initials= [initial_x initial_y initial_phi ];
\% \% ODE - INTEGRATION
options = odeset('AbsTol',1e-6,'RelTol',1e-5,'initialStep',0.01,'MaxStep',maxstep);
tic
\% Ode- Integrator Type Selection
[t,x]=ode45(@Lumped_RotorStator_sub,tspan,initials,options); % more STIFF Integrator
\% \% Initializing Flags
NC= zeros (size(X)); % NO-CONTACT
SL= zeros (size(X)); % SLIDING (Forward Whirling)
RL= zeros (size(X)); % Pure Rolling with No Sliding (Backward Whirling) !
TR= zeros (size(X)); % TRANSITION (Mixed)
for i= 1 : size(X)
\% Initialize Theoritical whirling velocity (Omega)
\% \% Applying Stribeck Friction between Rotor and Formation
\% \% Check whirling Status - FLAGS !
\% Initializeing Flags
NC= zeros (size(X)); % NO-CONTACT
SL= zeros (size(X)); % SLIDING (Forward Whirling)
RL= zeros (size(X)); % Pure Rolling with No Sliding (Backward Whirling) !
TR= zeros (size(X)); % TRANSITION (Mixed)
\% \% \% \%
NC(i)=1; % There is Rotor/Formation Contact
elseif Fbn(i)==0
if abs(V_rel(i))>Vmin && sign(Phidot(i))== sign(omega(i))
  SL(i)=1; % SLIDING (Forward Whirling)
elseif abs(V_rel(i))<=Vmin && abs(Fbt_R(i))<=abs(Fbt(i))
  RL(i)=1; % Pure Rolling with No Sliding (Backward Whirling) !
  omega_theo(i)= -(R./r(i)).*Phidot(i);
else TR(i)=1; % TRANSITION (Mixed)
end
end

Total_Fc= sqrt(Fbn.^2 + Fbt.^2); % Magnitude of total contact force

% [4] % Plotting Output Data
figure (1)
plot (X,Y);
hold on
Radius=Rc;
[cx,cy,z] = cylinder(Radius,100);
axis equal
plot(cx(:,1,:),cy(:,1,:),'r.');
grid
title('Rotor Center Orbit');
xlabel('Rotor Xg (m)');
ylabel('Rotor Yg (m)');

% Plotting- Rotor whirl velocity
figure (2)
plot (t,omega,'k');
hold on
grid
plot (t, omega_theo,'--r');
title('Rotor whirl velocity');
xlabel('Time (S)');
ylabel('Omega (rad/s)');
legend ('omega','omega_tho');

% Plotting- Rotor contact force
figure (3)
plot (t,Fbn);
grid
title('Contact Force ');
xlabel('Time (S)');
ylabel('Fn (N)');

% Plotting- Rotor friction force
figure (4)
plot (t,Fbt);
grid
title('Stator Friction force');
xlabel('Time (S)');
ylabel('Ft (N)');

% Plotting- Rotor relative velocity
figure (5)
plot (t,V_rel);
grid
title('Relative velocity');
xlabel('Time (S)');
ylabel('V_rel (rad/s) ');

% Plotting- Rotor rotational velocity
figure (6)
Lumped_RotorStator_sub.m

%% Omar Abdelzaher
%% Rotor-Stator 3-DOF lateral/rotational model using lumped mass method
%% Created September 10, 2013
%% Updated April 10, 2014

%% call integration function
funtion xdot= Lumped_RotorStator_sub(t,x)

plot (t,Phidot);
grid
title('Rotor rotational velocity');
xlabel('Time (S)');
ylabel('Phidot (rad/s) ');

figure (7)
plot (t,r);
grid
title('Rotor radial displacement');
xlabel('Time (S)');
ylabel('r (m) ');
hold on
plot (t,Rc,'--r');
hold off

figure (8)
plot (t,Total_Fc);
grid
title('Total Contact force (magnitude) ');
xlabel('Time (S)');
ylabel('F contact (N) ');

figure (9)
plot (t,Fbt,'k');
grid
hold on
plot (t,Fbt_r,':r');
title('Tangential force ');
xlabel('Time (S)');
ylabel('Ft (N) ');
legend ('Fbt','Fbt_R');

figure (10)
plot (t,NC,'-k');
hold on
grid on
plot (t,SL,'-B');
plot (t,RL,'-R');
plot (t,TR,'-G');
title ('Backward Whirling Condition');
xlabel('Time (S)');
ylabel('0=NOT active    1= ACTIVE ');
legend ('No Contact','Sliding', 'ROLLING !!','Transition');

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% Calculating variables
r = sqrt(x(1)^2 + x(3)^2); % Rotor radial displacement
teta1 = B1 * r^2; % Fluid nonlinear function 1
teta2 = B2 * r^2; % Fluid nonlinear function 2
omega = (x(1) * x(4) - x(3) * x(2)) / r^2; % Rotor whirl velocity
V_rel = x(6) * r + omega * r; % Rotor Relative velocity
Fe = e * m_a * x(6)^2; % Centrifugal force
Fe_x = Fe * cos(x(5)); % Eccentricity- Centrifugal force in X-dir
Fe_y = Fe * sin(x(5)); % Eccentricity- Centrifugal force in Y-dir

%% Logic for rotor motion status
if r <= R_c % If (NO CONTACT)
    Fbn = 0; % No Contact force
    Fbt = 0; % No friction force
    disp('---------- NO CONTACT ---------------------');
else % If (THERE IS CONTACT)
    Fbn = K_b * (r - R_c); % Calculate Normal Contact force
    Fbt = -m_a * sign(V_rel) * Fbn; % Calculate friction force
    % Calculate theoretical friction force Incase of Pure-Rolling (Vrel=0)
    Fbt_R = ((m * K_phi * R / J_t) * (w * t - x(5)) + C * omega * r - Cphi * m * R * x(6) / J_t) / (1 + (m * R^2) / J_t); % Theory friction force for Pure Rolling
    if abs(V_rel) > V_min && sign(x(6)) == sign(omega) %     disp('--------- SLiding ---------------');
        elseif abs(V_rel) <= V_min && -m_a * Fbn <= Fbt_R <= (m_a * Fbn)
            Fbt = Fbt_R;
        %     disp('--------------- Pure Rolling ------');
        else
            disp('------------------- TRANSITION ------');
        end
end

%% Calculate applied forces on Rotor
Fb_x = -(Fbt * x(3) - Fbn * x(1)) / r; % Contact force in X-dir
Fb_y = (Fbt * x(1) - Fbn * x(3)) / r; % Contact force in Y-dir
Tb = Fbt * R; % Friction Torque on Rotor

%% EOM's
% % (X-axis)
xdot_1 = x(2);
xdot_2 = (-C * x(2) - K * x(1) + Fe_x + Fb_x) / m; % Incase of No Mud fluid
% xdot_2 = -(C + D + eta2) * x(2) - (x(6)*mf/m) * x(3) - ((K - (x(6)^2)/4)*mf*eta1)/m * x(1) - (((x(6)/2)*D + (w/2)*eta2)/m)*x(3) + (Fe_x)/m + Fb_x/m; % Incase of No Mud fluid
% % (Y-axis)
xdot_3 = x(4);
xdot_4 = -(C * x(4) - K * x(3)) / m; % Incase of No Mud fluid
% xdot_4 = -(Fe_y/m + Fb_y/m - (C + D + eta2)/m)*x(4) - (x(6)*mf)*x(2) - ((K + eta1 = (x(6)^2)/4)*mf/m)*x(3) - (((-x(6)/2)*D - (x(6)/2)*eta2)/m)*x(1); % Incase of No Mud fluid
% % (Z-axis)
xdot_5 = x(6);
xdot_6 = (-K_phi * x(5) - w * t) - Cphi * x(6) - Tb / J_t;

xdot = [ xdot_1; xdot_2; xdot_3; xdot_4; xdot_5; xdot_6];
t end
Lumped_Drillstring.m

% % Omar Abdelzaher
% Drillstring Lumped-System-Model using Three lumped masses with 6-DOF
% Created December 15, 2014
% Updated April 24, 2014
%
clc
clear all
close all
%dbsstop if naninf

difine Global Variables
global Tx_db Ty_db Tz_db X_0 mu_s WOB landa f Rh w Vmin w_f1 W0 M_st J_st
globalJe I_st M_BHA M_fl J_fl M_d BHA J_d e q S0 b C1 C2 alpha
global r_st r_f1 R_st Rc_st omega_st Vrel_st mu_st mu Kf Kc Fn_st Ft_st
global Kxy_BHA Kxy_dp Kxy_f1 Kz_dp Kz_BHA Kz_f1 KphiZ_dp KphiZ_BHA KphiXY_BHA
global KphiXY_dp Cxy_dp Cxy_BHA Cxy_f1 Cz_dp Cz_BHA Cz_f1 Cf J_m Rm Km Lm
global r_BHA R_BHA Rc_BHA omega_BHA Vrel_BHA mu_BHA Fn_BHA Ft_BHA n J_rt
global r_db a Rc_db omega_db Vrel_db mu_db S Fn_db Ft_db Cf_z WOB_z TOB

% [1]% INPUT SYSTEM PARAMETERS

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% A)STEEL material properties
% Modulus of Elasticity Pa
E= 200e9;
% Poisson's Ratio
nu= 0.4;
% Density kg/m^3
mass_rho = 7850;
% Calculate Element Shear Moduli  . (lb/in^2)
G= (3/8)*E;
% Mass=mass_rho*Ac*L;                   % Mass of element

% B)Rotary Table parameters
% rotary speed of rotary table (RPM)
rpm= 140;
% rotary speed of rotary table (rad/s)
w= rpm*2*pi/60;
% armature resistance (Ohm)
Rm=0.01 ;
% armature inductance (H)
Lm=0.005;
% Rotary table inertia (Kg.m2)
J_rt=930;
% Motor inertia (Kg.m2)
J_m=23;
% Motor constant
Km= 6;

% C)loads parameters
% upper applied Static load 100 KN
W0=50e3;
Vmin=0.001;

% D)Formation parameters
% Borehole radius
Rh=(7.4/2)*0.0254;
% lateral formation Contact Stiffness // for bit bounce result try softer formation stiffness let Kb=50000000
Kf=2e10;
% axial formation Contact Stiffness
Kc=67e5;
% formation Contact Friction
mu=0.3;
% Formation surface function constant
b=1;
% Formation elevation amplitude = 1mm
C1= 1.35e-8;
% Penetration Constant
C2= -1.9e-4;

% E)Drilling fluids parameters
% Drilling fluid torsional Damping
Cf= 2e2;
% Drilling fluid Axial Damping
Cf_z=10e0;
% Drillpipe parameters

L_dp=5700; % Drill-pipe length
OD_dp=(5)*0.0254; % Drill-pipe OD
ID_dp=(3/8)*0.0254; % Drill-pipe ID
Ac_dp = pi/4*(OD_dp.^2 - ID_dp.^2); % Drill-pipe cross section area
M_dp=mass_rho*Ac_dp*L_dp; % Drill-pipe total mass
J_dp=(M_dp/8)*(OD_dp^2+ID_dp^2); % Drill-pipe moment of Inertia about Z
I_dp=(17(12*8))*(OD_dp^2+ID_dp^2)+L_dp^2); % Equivalent drillpipe Bending stiffness in x,y
KphiXY_dp=4*E*I_dp/L_dp; % Drillpipe Torsional stiffness about X,Y
KphiZ_dp=G*J_dp/L_dp; % Drillpipe Torsional stiffness about Z
Kz_dp=(5)*(Ac_dp*E)/L_dp; % Axial drillpipe stiffness in Z
Kxy_dp=(0.1)*(12*E*I_dp/L_dp^3)/1000; % Equivalent drillpipe Torsional stiffness in x,y
KphiXY_dp=10e5; % Axial drillpipe damping
KphiZ_dp=1e5; % Equivalent lateral drillpipe damping
Cxy_dp=10e1; % Equivalent lateral drillpipe damping
Cz_dp=1e2; % Equivalent axial drillpipe damping

% Drillcollar parameters

L_BHA=180/2; % BHA length/segment ((we have TWO segments))
e= 0.135; % eccentricity of rotor's center mass
OD_BHA = (6.75)*0.0254; % Outer Diameter (m)
R_BHA= OD_BHA/2; % BHA drill-collar radius
RC_BHA=Rh-R_BHA; % BHA drill-collar/Borehole Clearance
ID_BHA = (4)*0.0254 ; % BHA drill-collar Inner Diameter (m)
Ac_BHA = pi/4*(OD_BHA.^2 - ID_BHA.^2); % BHA drill-collar Cross Sectional Area (m^2)
M_BHA=mass_rho*Ac_BHA*L_BHA; % Mass BHA/segment
J_BHA=((M_BHA+M_fl)/8)*(OD_BHA^2+ID_BHA^2)+L_BHA^2); % Total rotation moment of inertia
Je=M_BHA*e^2/2; % BHA drill-collar moment of inertia about Z
Jz_BHA=(1/(12*8))*(OD_BHA^2+ID_BHA^2)+L_BHA^2); % Equivalent BHA axial stiffness
Kxy_BHA=(0.5)*(12*E*I_BHA/L_BHA^3)/1000; % BHA radial stiffness in x,y
KphiZ_BHA= 5e5; % Equivalent BHA Torsional stiffness about Z
KphiXY_BHA= KphiXY_dp*2 ; % Equivalent BHA Bending stiffness about X,Y
Cxy_BHA= Cxy_dp*2; % Equivalent lateral BHA damping coefficient in x,y
Cz_BHA= Cz_dp*10; % Axial BHA drill-collar damping

% J) Drillbit

OD_db= (7)*0.0254; % DRILLBIT Diameter
a=OD_db/2; % Tricone Drillbit - radius
Rc_db=M_BHA; % Rotor-Borehole Clearance
M_db=M_BHA; % Drillbit assembly total mass
J_db=J_BHA; % Total rotation moment of inertia about X,Y
I_db= I_BHA; % Total rotation moment of inertia about X,Y
alpha=0*pi/180; % Drillbit bent/tilt angle
X_0=0.001; % Drillbit parameter 1
f=0.005; % Drillbit parameter 2
landa= 0.9; % Drillbit parameter 3

% [2] % SIMULATION TIME

to=0; % Start Simulation Time
tfinal=20; % Final Simulation Time
Nsamples=5001; % Number of steps
maxstep=0.01; % Max. step size
dt=tfinal/(Nsamples-1);
fixed step
tsim=[to:dt:tfinal];

% define the initial conditions making sure to use the right ordering
X0= 0.00001; % Initial X-location
Y0= 0.00001; % Initial Y-location
Z0= -0.001; % Initial Z-location

% Stabilizer (1)
X0_st= [X0 0 ]; % X(0)=X0 , X'(0)=0
Y0_st= [Y0 0 ]; % Y(0)=Y0 , Y'(0)=0
Z0_st= [Z0 0 ]; % z(0)=Z0 , z'(0)=0
PhiZ0_st= [0 w ]; % Phi_z(0)=0, Phi'(0)=w
PhiX0_st= [0 0 ]; % Phi_x(0)=0, Phi_x'(0)=0
PhiY0_st= [0 0 ]; % Phi_y(0)=0, Phi_y'(0)=0
initialize_st= [ X0_st Y0_st Z0_st PhiZ0_st PhiX0_st PhiY0_st];

% BHA Drill-collars
X0_BHA= [X0 0 ]; % X(0)=X0 , X'(0)=0
Y0_BHA= [Y0 0 ]; % Y(0)=Y0 , Y'(0)=0
Z0_BHA= [Z0 0 ]; % z(0)=Z0 , z'(0)=0
PhiZ0_BHA= [0 w ]; % Phi_z(0)=0, Phi'(0)=w
PhiX0_BHA= [0 0 ]; % Phi_x(0)=0, Phi_x'(0)=0
PhiY0_BHA= [0 0 ]; % Phi_y(0)=0, Phi_y'(0)=0
initialize_BHA= [ X0_BHA Y0_BHA Z0_BHA PhiZ0_BHA PhiX0_BHA PhiY0_BHA];

% Flywheel
X0_fl= [X0 0 ]; % X(0)=X0 , X'(0)=0
Y0_fl= [Y0 0 ]; % Y(0)=Y0 , Y'(0)=0
Z0_fl= [Z0 0 ]; % z(0)=Z0 , z'(0)=0
initialize_fl= [ X0_fl Y0_fl Z0_fl];

% Drillbit + Stabilizer (2)
X0_db= [X0 0 ]; % X(0)=X0 , X'(0)=0
Y0_db= [Y0 0 ]; % Y(0)=Y0 , Y'(0)=0
Z0_db= [Z0 0 ]; % z(0)=Z0 , z'(0)=0

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Φz0_db = [0 w]; % Φ_z(0)=0, Φ_z'(0)=w
Φx0_db = [0 0 ]; % Φ_x(0)=0, Φ_x'(0)=0
Φy0_db = [0 0 ]; % Φ_y(0)=0, Φ_y'(0)=0
initialize_db= [X0_db Y0_db Z0_db Φz0_db Φx0_db Φy0_db];

% Motor + Rotary Table
I0_m= 0.001; % Drive motor Inertia
Φ0_rt= [0 w]; % Rotary table fixed angular speed
initialize_rt= [I0_m Φ0_rt];

% Initialization Matrix
initials= [initialize_st initialize_BHA initialize_fl initialize_db initialize_rt];

% % ODE - INTEGRATION
options = odeset('AbsTol',1e-6,'RelTol',1e-5,'initialStep',0.0001,'MaxStep',maxstep);
tic
[t,x]=ode45(@Lumped_Drillstring_sub,tsim,initials, options);
% [t,x]=ode23s (@Lumped_Drillstring_sub,tsim,initials, options);
% [t,x]=ode15s (@Lumped_Drillstring_sub,tsim,initials, options);
% [t,x]=ode113 (@Lumped_Drillstring_sub,tsim,initials, options);
toc

% [3]% Defining Output variables:
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Stabilizer (1)
X_st=x(:,1);
Xdot_st=x(:,2);
Y_st=x(:,3);
Ydot_st=x(:,4);
Z_st=x(:,5);
Zdot_st=x(:,6);
Φz_st=x(:,7);
Φzdot_st=x(:,8);
Φx_st=x(:,9);
Φxdot_st=x(:,10);
Φy_st=x(:,11);
Φydot_st=x(:,12);

% BHA
X_BHA=x(:,13);
Xdot_BHA=x(:,14);
Y_BHA=x(:,15);
Ydot_BHA=x(:,16);
Z_BHA=x(:,17);
Zdot_BHA=x(:,18);
Φz_BHA=x(:,19);
Φzdot_BHA=x(:,20);
Φx_BHA=x(:,21);
Φxdot_BHA=x(:,22);
Φy_BHA=x(:,23);
Φydot_BHA=x(:,24);

% Flywheel
X_fl=x(:,25);
Xdot_fl=x(:,26);
Y_fl=x(:,27);
Ydot_fl=x(:,28);
Z_fl=x(:,29);
Zdot_fl=x(:,30);

% Drillbit
X_db=x(:,31);
Xdot_db = x(:, 32);
Y_db = x(:, 33);
Ydot_db = x(:, 34);
Z_db = x(:, 35);
Zdot_db = x(:, 36);
PhiZ_db = x(:, 37);
PhiZdot_db = x(:, 38);
PhiX_db = x(:, 39);
PhiXdot_db = x(:, 40);
PhiY_db = x(:, 41);
PhiYdot_db = x(:, 42);

% Motor dynamics + Rotary table
I_m = x(:, 43);
Phi_rt = x(:, 44);
Phidot_rt = x(:, 45);

%% [4]% Output variables calculation

% Radial displacements
r_st = sqrt(X_st.^2 + Y_st.^2);
r_BHA = sqrt(X_BHA.^2 + Y_BHA.^2);
r_db = sqrt(X_db.^2 + Y_db.^2);
r_fl = sqrt(X_fl.^2 + Y_fl.^2);

% Whirling velocities
omega_st = (X_st.*Ydot_st - Y_st.*Xdot_st) ./ r_st.^2;
omega_BHA = (X_BHA.*Ydot_BHA - Y_BHA.*Xdot_BHA) ./ r_BHA.^2;
omega_db = (X_db.*Ydot_db - Y_db.*Xdot_db) ./ r_db.^2;

% Relative velocities
Vrel_st = PhiZdot_st.*R_st+omega_st.*r_st;
Vrel_BHA = PhiZdot_BHA.*R_BHA+omega_BHA.*r_BHA;
Vrel_db = PhiZdot_db.*a+omega_db.*r_db;

% friction coefficients
mu_st = mu;
mu_BHA = mu;
mu_db = mu.*(tanh(PhiZdot_db)+(2.*PhiZdot_db)./(1+PhiZdot_db.^2)+(0.01).*PhiZdot_db);

% Contact forces
Fn_st = Kf.*(r_st-Rc_st);
Fn_BHA = Kf.*(r_BHA-Rc_BHA);

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$F_{t_{BHA}} = \mu_{BHA} \cdot \text{sign}(V_{rel_{BHA}}) \cdot F_{n_{BHA}}$;  
\text{intermediate BHA mass} 

$F_{n_{db}} = Kf \cdot (r_{db} - R_{c_{db}})$;  
\text{contact force on drillbit mass} 

$F_{n_{db}}(F_{n_{db}} < 0) = 0$;  
\text{set zero for any negative value in} 

$F_{t_{db}} = -\mu \cdot \text{sign}(V_{rel_{db}}) \cdot F_{n_{db}}$;  
\text{friction force on drillbit mass} 

\% WOB 
\% Approach 1 
$S = S_0 \cdot \sin(b \cdot \Phi_{Z_{db}})$;  
formation surface elevation 

$WOB = Kc \cdot (Z_{db} - S)$;  
Weight on bit 

$WOB(WOB < 0) = 0$;  
formation contact 

$WOB_{z} = \cos(\alpha) \cdot WOB$;  
Axial WOB in Z-dirc dueto bit angle 

$WOB_{x} = \sin(\alpha) \cdot WOB \cdot \cos(\Phi_{Z_{db}})$;  
Lateral WOB in X-dirc dueto bit angle 

$WOB_{y} = \sin(\alpha) \cdot WOB \cdot \sin(\Phi_{Z_{db}})$;  
Lateral WOB in Y-dirc dueto bit angle 

$WOB = Kc \cdot X_0 \cdot (1 - \sin(2 \cdot \pi \cdot f \cdot t))$;  
Vertical WOB in Z-dirc dueto bit inclination 

$T_{OB} = WOB \cdot a \cdot (\mu_{db} + \sqrt{dc/a})$;  
Torque on bit 

\% Check Drillbit pure-rolling status 
\text{theoretical applied friction force incase of pure-rolling (Vrel=0)} 

$F_{R_{db}} = \text{zeros (size(Vrel_{db}))}$;  
\text{Initialize Drillbit} 

$Rolling_{db} = \text{zeros (size(Vrel_{db}))}$;  
\text{Initialize Pure-Rolling Flag} 

\% Approach 2 Ref.Parimal Patil (Model Development of Torsional drillstring and Investigating..) 
$S = S_0 \cdot \sin(b \cdot \Phi_{Z_{db}})$;  
\text{Vertical WOB in Z-dirc dueto} 

$WOB = Kc \cdot X_0 \cdot (1 - \sin(2 \cdot \pi \cdot f \cdot t))$;  
\text{Vertical WOB in Z-dirc dueto} 

$TOB = 50 \cdot \Phi_{Z_{db}} + a \cdot WOB \cdot (\mu + (\mu_s - \mu) \cdot e^{-\lambda \cdot \text{abs(\Phi_{Z_{db}})}})$;  
\text{Calculate applied torques} 

$T_d = K_{phiZ_{dp}} \cdot (\Phi_{rt} - \Phi_{Z_{st}})$;  

\[Tx_{db} = \mu \cdot F_{n_{db}} \cdot a \cdot (\text{abs}(Y_{db})/r_{db}); \quad \text{\% Applied torque on drillbit about X-axis}\]
\[Ty_{db} = \mu \cdot F_{n_{db}} \cdot a \cdot (\text{abs}(X_{db})/r_{db}); \quad \text{\% Applied torque on drillbit about Y-axis}\]
\[Tz_{db} = F_{t_{db}} \cdot a; \quad \text{\% Applied torque on drillbit about Z-axis}\]

% Convert obtained Angular velocities from (Rad/sec) to (RPM)
PhiZdot_st_RPM = (PhiZdot_st.*60)./(2.*pi);
PhiZdot_BHA_RPM = (PhiZdot_BHA.*60)./(2.*pi);
PhiZdot_db_RPM = (PhiZdot_db.*60)./(2.*pi);

%% [5] % Plotting Output Data
figure (1)
plot (X_st,Y_st,'b');
hold on
grid
Radius_st=Rc_st;
.cx_st,cy_st,z_st = cylinder(Radius_st,100);
axis equal
plot(cx_st(1,:),cy_st(1,:),,'r.');
title('Stabilizer C.G. Orbit');
xlabel(' Xg (m)');
ylabel(' Yg (m) ');

% Plotting- intermediate BHA mass x,y position
figure (2)
plot (X_BHA,Y_BHA,'b');
hold on
% plot (X_fl,Y_fl,'k--');  % plot the inside flywheel x,y position
grid
Radius_BHA=Rc_BHA;
.cx_BHA,cy_BHA,z_BHA = cylinder(Radius_BHA,100);
plot(cx_BHA(1,:),cy_BHA(1,:),,'r.');
axis equal
title('BHA C.G. Orbit');
xlabel(' Xg (m)');
ylabel(' Yg (m) '); legend('Borehole','BHA Orbit');

% Plotting- Drillbit x,y position
figure (3)
% subplot(1,3,3);
plot (X_db,Y_db,'c');
hold on
grid
Radius_DB=Rc_db;
.cx_DB,cy_DB,z_DB = cylinder(Radius_DB,100);
axis equal
plot(cx_DB(1,:),cy_DB(1,:),,'r.');
title('Drill-Bit C.G. Orbit');
xlabel(' Xg (m)');
ylabel(' Yg (m) ');

% Plotting- Drillbit 3D pathline inside the borehole
figure (4)
hold on
grid
NN=100;
.X,Y,Z = cylinder(Rc_db,NN);
Z(2,:) = min(Z_db);
Z(2,:) = max(Z_db);
h2 = surf(X,Y,Z);
set(h2,'FaceAlpha',0.3)
% plot3(Xdb,Ydb,Z_db,'c');
% plot3(Xst,Yst,Z_st,'b');
% plot3(Xbha,Ybha,Z_BHA,'g');
plot3(X_db,Y_db,Z_db,'c');
% plot3(X_st,Y_st,Z_st,'b');
% plot3(X_BHA,Y_BHA,Z_BHA,'g');
xlabel(' Xg (m)');
ylabel(' Yg (m)');
zlabel(' Zg (m)');
title('3D Pathline Orbit');
legend('Borehole','Drillbit Pathline');
view([-37.5 30])
hold off

% Plotting- the Three masses axial displacements
figure (5)
hold on
grid
plot(t,Z_st,'b');
plot(t,Z_BHA,'g');
plot(t,Z_db,'c');
title('Axial displacement');
xlabel(' Time (S)');
ylabel(' Z (m)');
legend('Stabilizer (1)', 'BHA ', 'Drillbit ');

% Plotting- the upper stabilizer mass radial displacement
figure (6)
plot(t,r_st,'b');
hold on
grid
plot(t,Rc_st,'r');
title('Upper Stabilizer Radial Displacement');
xlabel(' Time (S)');
ylabel(' r (m)');
legend('Borehole', 'Stabilizer (1)');
axis square

% Plotting- the upper stabilizer mass radial displacement
figure (7)
plot(t,r_BHA,'g');
hold on
grid
% plot (t,r_fl,'k--'); % Plot flywheel radial displacement
plot(t,Rc_BHA,'r');
title('BHA Radial Displacement');
xlabel(' Time (S)');
ylabel(' r (m)');
% legend('Borehole', 'BHA ', 'Flywheel');
legend('Borehole', 'BHA ');
axis square

% Plotting- the drillbit radial displacement
figure (8)
plot(t,r_db,'c');
hold on
grid
plot(t,Rc_db,'r');
title('Drillbit Radial Displacement');
xlabel(' Time (S)');
ylabel(' r (m)');
legend('Borehole', 'Drillbit');
axis square
% Plotting- the drillstring components rotational angles in degrees
figure (9)
hold on
grid
plot (t,PhiZ_st*180/pi, 'b');
plot (t,PhiZ_BHA*180/pi,'g');
plot (t,PhiZ_db*180/pi, 'c');
title('Rotation angles around Z-dir ');
xlabel('Time (S)');
ylabel(' Phi-Z (Degrees)');
legend('Stabilizer (1)','BHA','Drillbit');
hold off

% Plotting- Drillbit WOB and Applied static load
figure (10)
hold on
grid
plot (t,WOB_z);
plot (t,W0,'k-');
title('Weight On Bit ');
xlabel('Time (S)');
ylabel(' WOB (N)');

% Plotting- Drillbit TOB and Applied top drive torque
figure (11)
hold on
grid
plot (t,TOB);
plot (t,Td,'r--');
title('Torques ');
xlabel('Time (S)');
ylabel(' TOB (Nm) ');
legend('TOB','Top torque');

% Plotting- Applied contact forces on drillstring components
figure (12)
hold on
grid
plot (t,Fn_st,'b');
plot (t,Fx_st,'g-');
plot (t,Fn_db,'c');
plot (t,Fx_db,'g');
plot (t,Fn_BHA,'g');
title('Contact Forces ');
xlabel('Time (S)');
ylabel(' Contact forces (N)');
legend('Normal force on Stabilizer (Fn_st)', 'Normal force on Drillbit (Fn_db)');

% Plotting- drillstring components angular velocities in RPM
figure (13)
hold on
grid
plot (t,PhiZdot_st_RPM,'b');
plot (t,PhiZdot_db_RPM,'c');
plot (t,PhiZdot_BHA_RPM,'g');
title('Rotational velocities');
xlabel('Time (S)');
ylabel(' Phidot-Z(RPM)');
legend('Upper Stabilizer','DrillBit','BHA');

% Plotting- drillstring components relative velocities
figure (14)
hold on
grid
plot (t,Vrel_st,'b');
plot (t,Vrel_db,'c');
plot (t,Vrel_BHA,'g');
% Omar Abdelzaher
% Drillstring Lumped-System-Model using Three lumped masses with 6-DOF
% Created December 15, 2014
% Updated April 24, 2014

%% call integration function

function xdot= Lumped_Drillstring_sub(t,x)

Lumped_Drillstring_sub.m
%% % State Variables

%%%%%%%% Stabilizer 1
% X_st= x(1);
% Xdot_st= x(2);
% Y_st= x(3);
% Ydot_st= x(4);
% Z_st=x(5);
% Zdot_st=x(6)
% PhiZ_st=x(7);
% PhiZdot_st=x(8);
% PhiX_st=x(9);
% PhiXdot_st=x(10);
% PhiY_st=x(11);
% PhiYdot_st=x(12);

%%%%%%%% BHA DOF's
% X_BHA=x(13);
% Xdot_BHA=x(14);
% Y_BHA=x(15);
% Ydot_BHA=x(16);
% Z_BHA=x(17);
% Zdot_BHA=x(18)
% PhiZ_BHA=x(19);
% PhiZdot_BHA=x(20);
% PhiX_BHA=x(21);
% PhiXdot_BHA=x(22);
% PhiY_BHA=x(23);
% PhiYdot_BHA=x(24);

%%%% Flywheel
% X_fl= x(25)
% Xdot_fl= x(26);
% Y_fl= x(27);
% Ydot_fl=x(28);
% Z_fl=x(29);
% Zdot_fl=x(30);

%%%% DrillBit + Stabilizer 2
% X_bit= x(31);
% Xdot_bit= x(32);
% Y_bit= x(33);
% Ydot_bit=x(34);
% Z_bit=x(35);
% Zdot_bit=x(36);
% PhiZ_bit=x(37);
% PhiZdot_bit=x(38);
% PhiX_bit=x(39);
% PhiXdot_bit=x(40);
% PhiY_bit=x(41);
% PhiYdot_bit=x(42);

%% Calculating variables %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%% A) Rotary Table + Motor

Tm=Km*x(43);
Vc=Km*n*w;

%%% B) STABILIZER (1)

r_st= sqrt(x(1)^2+x(3)^2);
rho= sqrt(x(1)^2+x(3)^2);

omega_st= (x(4)*x(1)-x(2)*x(3))/r_st^2;
% upper stabilizer mass radial displacement
omega_st= (x(4)*x(1)-x(2)*x(3))/r_st^2;
% upper stabilizer mass whirl velocity
\[ V_{rel_st} = x(8) \cdot R_{st} + \omega_{st} \cdot r_{st}; \]  
% upper stabilizer mass Relative velocity

\[ \mu_{st} = \mu; \]  
% applied friction coefficient

% Logic for upper stabilizer mass motion status
if \( r_{st} < Rc_{st} \) % If No contact
  \[ F_{n_{st}} = 0; \]  
  \[ F_{t_{st}} = 0; \]  
  \[ \text{disp}('-- NO CONTACT -------------------'); \]
else
  \[ F_{t_{st}} = -\mu_{st} \cdot \text{sign}(V_{rel_st}) \cdot F_{n_{st}}; \]  
  \[ \text{Calculate theoretical friction contact force in case of pure-rolling (Vrel=0)} \]
  \[ F_{tR_{st}} = (C_{xy} \cdot \omega_{st} \cdot R_{st} - (M_{st} \cdot R_{st} / J_{st}) \cdot (K_{phiZ} \cdot x(4)) \cdot K_{phiZ} \cdot x(19)) + C_f \cdot M_{st} \cdot R_{st} \cdot x(8)) / (1 + (M_{st} \cdot R_{st}^2) / J_{st}); \]
  \[ \text{if abs(Vrel_st) > Vmin && sign(x(8)) == sign(\omega_{st})} \]  
  \[ \text{disp('---------- SLiding ----------------');} \]
  elseif \( \text{abs(Vrel_st) <= Vmin && -\mu_{st} \cdot F_{n_{st}} <= F_{tR_{st}} <= \mu_{st} \cdot F_{n_{st}}} \]  
  \[ \text{Check for pure-rolling (Backward whirling)} \]
  \[ F_{t_{st}} = F_{tR_{st}}; \]  
  \[ \text{if YES !} \]
  \[ \text{disp('--------------- Pure Rolling ----');} \]
else
  \[ \text{disp('---------- TRANSITION ----');} \]
end

% Calculate applied loads
\[ F_{x_{st}} = (-F_{t_{st}} \cdot x(3) - F_{n_{st}} \cdot x(1)) / r_{st}; \]  
% Applied force in X-dir
\[ F_{y_{st}} = (F_{t_{st}} \cdot x(1) - F_{n_{st}} \cdot x(3)) / r_{st}; \]  
% Applied force in Y-dir
\[ F_{z_{st}} = -\text{sign}(x(6)) \cdot \mu_{st} \cdot F_{n_{st}}; \]  
% Applied force in Z-dir
\[ T_{x_{st}} = -\text{sign}(x(10)) \cdot \mu_{st} \cdot F_{n_{st}} \cdot R_{st} \cdot (\text{abs}(x(3)) / r_{st}); \]  
% Applied friction Torque about X-dir
\[ T_{y_{st}} = -\text{sign}(x(12)) \cdot \mu_{st} \cdot F_{n_{st}} \cdot R_{st} \cdot (\text{abs}(x(1)) / r_{st}); \]  
% Applied friction Torque about Y-dir
\[ T_{z_{st}} = -\text{sign}(x(8)) \cdot F_{t_{st}} \cdot R_{st}; \]  
% Applied friction Torque about Z-dir

%%% C) BHA intermediate mass
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
\[ r_{BHA} = \sqrt{x(13)^2 + x(15)^2}; \]  
% Intermediate BHA mass radial displacement
\[ \omega_{BHA} = (x(16) \cdot x(13) - x(14) \cdot x(15)) / r_{BHA}^2; \]  
% Intermediate BHA mass whirl velocity
\[ V_{rel_{BHA}} = x(20) \cdot R_{BHA} + \omega_{BHA} \cdot r_{BHA}; \]  
% Intermediate BHA mass Relative velocity
\[ \mu_{BHA} = \mu; \]  % applied friction coefficient

% Logic for Intermediate BHA mass motion status
if \( r_{BHA} < Rc_{BHA} \) % If No contact
  \[ F_{n_{BHA}} = 0; \]  
  \[ F_{BHA} = 0; \]  
  \[ \text{disp('-- NO CONTACT -------------------');} \]
else
  \[ F_{BHA} = K_f \cdot (r_{BHA} - R_{BCA}); \]  
  \[ \text{Normal Contact force} \]
  \[ F_{t_{BHA}} = -\mu_{BHA} \cdot \text{sign}(V_{rel_{BHA}}) \cdot F_{n_{BHA}}; \]  
  \[ \text{Calculate theoretical friction contact force in case of pure-rolling (Vrel=0)} \]
  \[ F_{tR_{BHA}} = (C_{xy} \cdot \omega_{BHA} \cdot r_{BHA} - (M_{BHA} \cdot R_{BHA} / (J_{BHA} + J_{e})) \cdot (K_{phiZ} \cdot x(19) - (x(19) - x(37)) + C_f \cdot M_{BHA} \cdot R_{BHA} \cdot x(20)) / (1 + (M_{BHA} \cdot R_{BHA}^2) / (J_{BHA} + J_{e})); \]
  \[ \text{if abs(Vrel_{BHA}) > Vmin && sign(x(20)) == sign(\omega_{BHA})} \]  
  \[ \text{Check for Forward whirling} \]
  \[ \text{disp('---------- SLiding ----------------');} \]
  elseif \( \text{abs(Vrel_{BHA}) <= Vmin && -\mu_{BHA} \cdot F_{n_{BHA}} <= F_{tR_{BHA}} <= \mu_{BHA} \cdot F_{n_{BHA}}} \]  
  \[ \text{Check for pure-rolling (Backward whirling)} \]
  \[ F_{t_{BHA}} = F_{tR_{BHA}}; \]  
  \[ \text{if YES !} \]
  \[ \text{disp('--------------- Pure Rolling ----');} \]
else
  \[ \text{disp('---------- TRANSITION ----');} \]
end

%%%
% Calculate applied loads
Fx_BHA= (-Ft_BHA*x(15) - Fn_BHA*x(13))/r_BHA; % Applied force in X-dir
Fy_BHA= (Ft_BHA*x(13) - Fn_BHA*x(15))/r_BHA; % Applied force in Y-dir
Fz_BHA= -sign(x(18))*mu*Fn_BHA; % Applied force in Z-dir
TxBHA= -sign(x(22))*mu*Fn_BHA*R_BHA*(abs(x(15))/r_BHA); % Applied friction Torque about X-dir
Ty_BHA= -sign(x(24))*mu*Fn_BHA*R_BHA*(abs(x(13))/r_BHA); % Applied friction Torque about Y-dir
Tz_BHA= -sign(x(20))*Ft_BHA*R_BHA; % Applied friction Torque about Z-dir

% Calculate centrifugal force due to eccentricity (Mass imbalance)
Fe= e*M_BHA*x(20)^2; % Total centrifugal force
Fe_x= Fe*cos(x(19)); % centrifugal force in X-dir
Fe_y= Fe*sin(x(19)); % centrifugal force in Y-dir

%%% D)FLYWHEEL
r_fl= sqrt(x(25)^2+x(27)^2); % Flywheel mass radial displacement
Fq=q*M_fl*w_fl^2; % Flywheel mass Total centrifugal force due to mass imbalance
Fq_x= Fq*cos(w_fl*t); % Flywheel mass centrifugal force in X-dir due to mass imbalance
Fq_y= Fq*sin(w_fl*t); % Flywheel mass centrifugal force in Y-dir due to mass imbalance

%%% E)DRILL-BIT + STABILIZER (2)
r_db= sqrt(x(31)^2+x(33)^2); % drillbit mass radial displacement
omega_db= (x(34)*x(31)-x(32)*x(33))/r_db^2; % drillbit mass whirl velocity
Vrel_db= x(38)*a+omega_db*r_db; % drillbit mass Relative velocity
mu_db= mu*(tanh(x(38))+(2*x(38))/(1+(x(38)^2))+0.01*x(38)); % Drillbit/formation Stribeck friction

% Logic for Drillbit mass motion status
if r_db<Rc_db % If No contact
    Fn_db=0; % No Normal Contact force
    Ft_db=0; % No friction Contact force
    disp('--------------- NO CONTACT -----------------');
else
    Fn_db= Kf*(r_db-Rc_db); % Normal Contact force
    Ft_db= -mu*sign(Vrel_db)*Fn_db; % tangential friction Contact force
end

% Calculate theoretical friction contact force incase of pure-rolling(Vrel=0)
Cf=Mu*(x(38))/(1+ (M_db*a^2)/J_db);% Friction coefficient
FTR_db= Cx_BHA*omega_db*r_db -(M_db*a/J_db)*(KphiZ_BHA*(x(37)-x(19)))+
if abs(Vrel_db)<=Vmin && sign(x(38)) == sign(omega_db) % Check for Forward whirling
    disp('------------- SLiding --------------');
elseif abs(Vrel_db)<Vmin && -(mu*Fn_db)<=FTR_db <=(mu*Fn_db) % Check for Backward whirling with Pure-Rolling
    disp('--------------- Pure Rolling -------');
else
    disp('----------------- TRANSITION -----');
end

% Calculate applied loads
Fx_db= (-Ft_db*x(33) - Fn_db*x(31))/r_db; % Applied force in X-dir
Fy_db= (Ft_db*x(31) - Fn_db*x(33))/r_db; % Applied force in Y-dir
Fz_db= -sign(x(36))*mu*Fn_db; % Applied force in Z-dir
Tdb= -sign(x(40))*mu*Fn_db*a*(abs(x(33))/r_db); % Applied friction Torque about X-dir


```matlab
Ty_db = sign(x(42)) * mu * Fn_db * a * (abs(x(31)) / r_db);  % Applied friction Torque about Y-dir
Tz_db = sign(x(38)) * Ft_db * a;  % Applied friction Torque about Z-dir

% % Check if Bit-Bounce or No drilling
S = S0 * sin(b * x(37));
if x(35) < S  % if No drillbit/formation contact
    WOB = 0;  % set No WOB
    disp('-- No Drilling ---------------------');
else
    WOB = Kc * (x(35) - S);  % axial WOB in Z-dir due to bit inclination
    disp('--------------- Drilling -----------');
end

% Calculate applied loads
WOB_z = cos(alpha) * WOB;  % Axial WOB in Z-dir due to bit angle
WOB_x = sin(alpha) * WOB * cos(x(37));  % Lateral WOB in X-dir due to bit angle
WOB_y = sin(alpha) * WOB * sin(x(37));  % Lateral WOB in Y-dir due to bit angle

ROP = C1 * W0 * sqrt(x(38)) + C2;  % rate of penetration
dc = (2 * pi * ROP) / x(38);  % Depth of cut
TOB = sign(x(38)) * WOB * a * (mu_db + sqrt(dc / a));  % Torque ON BIT

%% EOM's %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% EOM's
%% UPPER STABILIZER MASS %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%X
xdot_1 = x(2);
xdot_2 = (-Kxy_dp * x(1) - Kxy_BHA * (x(1) - x(13)) - Cxy_dp * x(2) - Cxy_BHA * (x(2) - x(14))) + Fx_st) / M_st;
%Y
xdot_3 = x(4);
xdot_4 = (-Kxy_dp * x(3) - Kxy_BHA * (x(3) - x(15)) - Cxy_dp * x(4) - Cxy_BHA * (x(4) - x(16))) + Fy_st) / M_st;
%Z
xdot_5 = x(6);
xdot_6 = (-Kz_dp * x(5) - Kz_BHA * (x(5) - x(17)) - Cz_dp * x(6) - Cz_BHA * (x(6) - x(18))) + W0 + Fz_st) / M_st;
%PhiZ
xdot_7 = x(8);
xdot_8 = (-KphiZ_dp * x(7) - KphiZ_BHA * (x(7) - x(19))) - Cf * x(8) + Tz_st) / J_z;
%PhiX
xdot_9 = x(10);
xdot_10 = (-KphiXY_dp * x(9) - KphiXY_BHA * (x(9) - x(21))) - Cz_z * x(10) + Tx_st) / I_z;
%PhiY
xdot_11 = x(12);
xdot_12 = (-KphiXY_dp * x(11) - KphiXY_BHA * (x(11) - x(23))) - Cf_z * x(12) + Ty_st) / I_z;

%%% EOM's INTERMEDIATE MASS %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%X
xdot_13 = x(14);
xdot_14 = (-Kxy_BHA * (x(13) - x(1)) - Kxy_BHA * (x(13) - x(31)) - Kxy_fl * (x(13) - x(25)) - Cxy_BHA * (x(14) - x(2)) - Cxy_BHA * (x(14) - x(32)) - Cxy_fl * (x(14) - x(26)) + Fx_BHA + Fe_x) / M_BHA;
%Y
xdot_15 = x(16);
xdot_16 = (-Kxy_BHA * (x(15) - x(3)) - Kxy_BHA * (x(15) - x(33)) - Kxy_fl * (x(15) - x(27)) - Cxy_BHA * (x(16) - x(4)) - Cxy_BHA * (x(16) - x(34)) - Cxy_fl * (x(16) - x(28)) + Fy_BHA + Fe_y) / M_BHA;
%Z
xdot_17 = x(18);
xdot_18 = (-Kz_BHA * (x(17) - x(5)) - Kz_BHA * (x(17) - x(35)) - Kz_fl * (x(17) - x(29)) - Cz_BHA * (x(18) - x(6)) - Cz_BHA * (x(18) - x(36)) - Cz_fl * (x(18) - x(30)) + Fz_BHA) / M_BHA;
```
```
% PhiZ
xdot_19 = x(20);
xdot_20 = -KphiZ_BHA*(x(19) - x(7)) - KphiZ_BHA*(x(19) - x(37)) - Cf*x(20) + Tz_BHA / (J_BHA+J_e);

% PhiX
xdot_21 = x(22);
xdot_22 = -KphiXY_BHA*(x(21) - x(9)) - KphiXY_BHA*(x(21) - x(39)) - Cf_z*x(22) + Tz_BHA / I_BHA;

% PhiY
xdot_23 = x(24);
xdot_24 = -KphiXY_BHA*(x(23) - x(11)) - KphiXY_BHA*(x(23) - x(41)) - Cf_z*x(24) + Ty_BHA / I_BHA;

%%% EOM's FLYWHEEL MASS %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% X
xdot_25 = x(26);
xdot_26 = -Kxy_fl*(x(25) - x(13)) - Cxy_fl*(x(26) - x(14)) + Fq_x / M_fl;
% Y
xdot_27 = x(28);
xdot_28 = -Kxy_fl*(x(27) - x(15)) - Cxy_fl*(x(28) - x(16)) + Fq_y / M_fl;
% Z
xdot_29 = x(30);
xdot_30 = -Kz_fl*(x(29) - x(17)) - Cz_fl*(x(30) - x(18)) / M_fl;

%%% EOM's DRILLBIT MASS %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% X
xdot_31 = x(32);
xdot_32 = -Kxy_BHA*(x(31) - x(13)) - Cxy_BHA*(x(32) - x(14)) + Fx_db + WOB_x / M_db;
% Y
xdot_33 = x(34);
xdot_34 = -Kxy_BHA*(x(33) - x(15)) - Cxy_BHA*(x(34) - x(16)) + Fy_db + WOB_y / M_db;
% Z
xdot_35 = x(36);
xdot_36 = -Kz_BHA*(x(35) - x(17)) - Cz_BHA*(x(36) - x(18)) + Fz_db + WOB_z / M_db;

%%% EOM MOTOR DYNAMICS %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
xdot_43 = -Rm*x(43) - Km*n*x(45) + Vc / Lm;

%%% EOM's ROTARY TABLE %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
xdot_44 = x(45);
xdot_45 = (-KphiZ_dp*(x(44) - x(7)) + n*Tm) / (J_rt + (n^2)*J_m);

end

FEM_BHA.m

% % Omar Abdelzaher
% Finite Element Method for BHA (Discretizing )
% creating BHA global mass Matrix [M], stiffness matrix [K], and Dmaping MATRIX [C].
% Created December 15, 2014
clear all; close all; clc;

% [1] % SECTION 1: INPUT SYSTEM PARAMETERS

BHA_L=200; % Total BHA length in m
noe=6; % # of elements desired %m: 4 to 6
E= 210e9*ones(1,noe); % Modulus of Elasticity Pa
nu= 0.3*ones(1,noe); % Poisson's Ratio
d1 = 8e-6; d2 = 8; % d1, d2: dampening ratio (D = d1*K + d2*M)
rho = 7850*ONES(1,noe); % Density kg/m^3 %m: 7859 to 8500
BHA_OD = (6.75)*0.0254*ONES(1,noe); % Outer Diameters (m) (inch to 0.0254 meter)
BHA_ID = (4.5) *0.0254*ones(1,noe) ; % Inner Diameters (m) --> (ID=3")
default

Node_z_coord = [0: BHA_L/noe :BHA_L]; % Get The Z coordinate for each element
NumberNodes = size(Node_z_coord,2); % Get Total Number of Nodes

for i=1:1:NumberNodes-1
    element_length(1,i) =   Node_z_coord(1,i+1) - Node_z_coord(1,i); % Get element length
end

% Axial and Torsional Restraint Stiffnesses Attached Between Node 1 and Ground
KZ = 1.0;
KTHETA = 1.0;

% %%%%%%%%%%%%%%%%%%%%%%%%% (OPTIONAL SECTION) %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% % If added mass Inerta @ nodes // unbalance mass // lumber mass
% % Added Inertias. Enter any disc, etc. inertias not accounted for with the
% % stiffness or mass diameter distributions
% % (Define the following row vectors. Leave vectors blank [] if no added inertias exist)
% node_added_inertia = [0 0];             % List all node numbers where added
% inertia will be attached %m: [1 2] to [2 4]
% mass_added= [0 0];                       % kg
% IP_added= [0 0];                        % kg.m^2
% IT_added= [0 0];                        % kg.m^2
% mass_rho = 7850*ONES(1,noe);            % Density kg/m^3
% mass_ID = 0*0.0254*ones(1,noe);        % kg
% mass_OD = 0*0.0254*ones(1,noe);


LenTot=0 ;

% Nodes coordinate in X
x1(1,1)=0;
NEL = size(element_length,2);
for i=1:1:NEL
    LenTot = LenTot+element_length(1,i);
    x1(1,i+1) = LenTot;
end

Number_Nodes = size(x1,2); % Number of Nodes
Number_Elem = Number_Nodes-1 ; % Number of Elements

% Integer parameters for arrays
Nnode = Number_Nodes; % No. of Nodes
Nd = 6*Nnode; % No. of Global-Free system degrees of freedom
% Nnpd = Nd; % No. of non-prescribed degrees of freedom:
Ne = Number_Elem; % No. of elements

% Nodes coordinates in Y, Z
x2 = zeros(1,Number_Nodes);
x3 = zeros(1,Number_Nodes);

% Nodal Connectivities (Ref.E12.14.1(a)), Element Types and Orientation Angle
% BEAMS
for ee = 1:1:Ne
    ICON(ee,1) = ee;
    ICON(ee,2) = ee+1;
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%% (OPTIONAL SECTION) %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Plotting BHA Finite-Element geometry
% irotor_geom=0; % 1--> Plot, 0--> Don't Plot
if irotor_geom==1
    % MODEL CHECK PLOT
    for i =1:1:Ne
        x1plt = [x1(ICON(i,1)), x1(ICON(i,2))];
x2plt = [x2(ICON(i,1)), x2(ICON(i,2))];
x3plt = [x3(ICON(i,1)), x3(ICON(i,2))];
        figure(1)
        plot3(x1plt,x2plt,x3plt,'k*
        if i==1
            xlabel('z position (m)');
ylabel('x position (m)');
zlabel('y position (m)');
title('Shaft Model Geometry Verification Plot');
        end
        view(45,45);
grid on
zoom on
hold on
end
end % if irotor_geom==1

% Plotting BHA Finite-Element profile (OPTIONAL)
irotor_profile=0; % 1--> Plot, 0--> Don't Plot
if irotor_profile==1
    for i=1:1:NEL
        subplot(2,1,1)
        for j=1:1:4
            if j==1
                xplot=[x1(1,i+1), x1(1,i)];
yplot=[BHA_OD(1,i)/2,BHA_OD(1,i)/2];
    elseif j==2
        xplot=[x1(1,i), x1(1,i)];
yplot=[BHA_OD(1,i)/2,-BHA_OD(1,i)/2];
    elseif j==3
        xplot=[x1(1,i), x1(1,i+1)];
yplot=[-BHA_OD(1,i)/2,-BHA_OD(1,i)/2];
    elseif j==4
        xplot=[x1(1,i), x1(1,i+1)];
yplot=[-BHA_OD(1,i)/2,BHA_OD(1,i)/2];
end

end
plot(xplot,yplot,'k-')
hold on
end

for j=1:4
    if j==1
        xplot=[x1(1,i+1), x1(1,i)];
        yplot=[BHA_ID(1,i)/2,BHA_ID(1,i)/2];
    elseif j==2
        xplot=[x1(1,i), x1(1,i)];
        yplot=[BHA_ID(1,i)/2,-BHA_ID(1,i)/2];
    elseif j==3
        xplot=[x1(1,i), x1(1,i+1)];
        yplot=[-BHA_ID(1,i)/2,-BHA_ID(1,i)/2];
    elseif j==4
        xplot=[x1(1,i+1), x1(1,i+1)];
        yplot=[-BHA_ID(1,i)/2,BHA_ID(1,i)/2];
    end
    if BHA_ID(1,i)~=0
        plot(xplot,yplot,'k--')
        hold on
    end
end
if i==1
    title('Stiffness Inner and Outer Diameter Profile Plot of Rotor Model')
    xlabel('X-axis');
    ylabel('Y-axis');
end
end % if irotor_profile==1

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% BHA Cross Sectional Area according to element type. (in^2)
A_BHA = pi/4*(BHA_OD.*BHA_OD - BHA_ID.*BHA_ID);
% Area Moment of Inertia Ix3tilde according to element type.(in^4),(Ref.12.3.24 )
Ix3_BHA= pi/64*(BHA_OD.*BHA_OD.*BHA_OD.*BHA_OD - BHA_ID.*BHA_ID.*BHA_ID.*BHA_ID);
% BHA Area Moment of Inertia Ix2tilde according to element type.(in^4),(Ref.12.3.24 )
Ix2_BHA=Ix3_BHA;
% BHA Torsion Constant according to element type. (in^4) (Ref.12.3.24 )
Jp_BHA= Ix2_BHA*Ix3_BHA;
% Calculate Element Shear Moduli . (lb/in^2)
G= 1/2*E./( ones(1,Ne) + nu) ;
% Calculate Shear Form Factors .
ksh = 6*ones(1,Ne) + nu)./(7*ones(1,Ne) + 6*nu);
% Form the Element Inertias
L_element = x1(1,2:Nnode) - x1(1,1:Nnode-1);
M_element_stiff = pi/4*rho.*L_element.*{ BHA_OD.*BHA_OD - BHA_ID.*BHA_ID};
N_element = M_element_stiff ;
Ip_element = 1/8*M_element_stiff.*{ BHA_OD.*BHA_OD + BHA_ID.*BHA_ID };
It_element = 1/12*M_element_stiff.*( 3/4*( BHA_OD.*BHA_OD + BHA_ID.*BHA_ID )+... 
+ L_element.*L_element );
% Form the Nodal Lumped Inertias
M_node =zeros(1,Nnode);
Ip_node =zeros(1,Nnode);
It_node =zeros(1,Nnode);
for e=1:1:Ne
    M_node(1,e) = M_node(1,e) + M_element(1,e)/2;
    M_node(1,e+1) = M_node(1,e+1) + M_element(1,e)/2;
    Ip_node(1,e) = Ip_node(1,e) + Ip_element(1,e)/2;


% Form the Global Mass Matrix \([Mc]\)
% ---------------------------------

\[ Mc = \text{zeros}(Nd,Nd); \]

\[
\text{for } i=1:1:Nnode
\]

\[
Mc(6*(i-1)+1,6*(i-1)+1) = Mc(6*(i-1)+1,6*(i-1)+1) + M_node(1,i);
\]

\[
Mc(6*(i-1)+2,6*(i-1)+2) = Mc(6*(i-1)+2,6*(i-1)+2) + M_node(1,i);
\]

\[
Mc(6*(i-1)+3,6*(i-1)+3) = Mc(6*(i-1)+3,6*(i-1)+3) + M_node(1,i);
\]

\[
Mc(6*(i-1)+4,6*(i-1)+4) = Mc(6*(i-1)+4,6*(i-1)+4) + Ip_node(1,i);
\]

\[
Mc(6*(i-1)+5,6*(i-1)+5) = Mc(6*(i-1)+5,6*(i-1)+5) + It_node(1,i);
\]

\[
Mc(6*(i-1)+6,6*(i-1)+6) = Mc(6*(i-1)+6,6*(i-1)+6) + It_node(1,i);
\]

end

% Determine the DOF connectivity array ICONDOF

\[
\text{for } e=1:1:Ne
\]

\[
\text{for } j =1:1:2
\]

\[
\text{for } k=1:1:6
\]

\[
l = 6*(j-1) + k ;
\]

\[
\text{ICONDOF}(e,l) = 6*(ICON(e,j)-1) + k ;
\]

end

end

% Form the Global Stiffness Matrix \([Kc]\)
% ----------------------------------------

\[ Kc=\text{zeros}(Nd,Nd) ; \]

% First form the beam element contributions

\[
\text{for } e=1:1:Ne
\]

\[
del1 = x1(ICON(e,2)) - x1(ICON(e,1)) ;
del2 = x2(ICON(e,2)) - x2(ICON(e,1)) ;
del3 = x3(ICON(e,2)) - x3(ICON(e,1)) ;
L = \sqrt{del1^2 + del2^2 + del3^2} ;
\]

% Form the element stiffness matrix in element coordinates

\[
\text{Ref. (12.9.6) for the following variables}
\]

\[
PHI12 = 12*E(1,e)*Ix3_BHA(1,e)/ksh(1,e)/A_BHA(1,e)/G(1,e)/L^2 ;
\]

\[
PHI13 = 12*E(1,e)*Ix2_BHA(1,e)/ksh(1,e)/A_BHA(1,e)/G(1,e)/L^2 ;
\]

\[
BETA_a_{12} = E(1,e)*Ix3_BHA(1,e)/(1+PHI12) ;
\]

\[
BETA_a_{13} = E(1,e)*Ix2_BHA(1,e)/(1+PHI13) ;
\]

\[
BETA_b_{12} = (4+PHI12)*BETA_a_{12} ;
\]

\[
BETA_b_{13} = (4+PHI13)*BETA_a_{13} ;
\]

\[
BETA_c_{12} = (2-PHI12)*BETA_a_{12} ;
\]

\[
BETA_c_{13} = (2-PHI13)*BETA_a_{13} ;
\]

% REF. TABLE 12.9.1 for all Ketilda definitions

\[
Ketilda = \text{zeros}(12,12) ;
\]

\[
Ketilda(1,1) = E(1,e)*A_BHA(1,e)/L ;
\]

\[
Ketilda(7,7) = Ketilda(1,1) ;
\]

\[
Ketilda(7,1) = -Ketilda(1,1) ;
\]

\[
Ketilda(4,4) = G(1,e)*Jp_BHA(1,e)/L ;
\]
Ketilda(10,10) = Ketilda(4,4) ;
Ketilda(10,4) = Ketilda(4,4) ;

Ketilda(2,2) = 12*BETA_a_12/L^3 ;
Ketilda(8,8) = Ketilda(2,2) ;
Ketilda(8,2) = Ketilda(2,2) ;

Ketilda(6,2) = 6*BETA_a_12/L^2 ;
Ketilda(8,6) = Ketilda(6,2) ;
Ketilda(12,2) = Ketilda(6,2) ;
Ketilda(12,8) = Ketilda(6,2) ;

Ketilda(6,6) = BETA_b_12/L ;
Ketilda(12,12) = Ketilda(6,6) ;

Ketilda(12,6) = BETA_c_12/L ;
Ketilda(3,3) = 12*BETA_a_13/L^3 ;
Ketilda(9,9) = Ketilda(3,3) ;
Ketilda(9,3) = Ketilda(3,3) ;

Ketilda(5,5) = BETA_b_13/L ;
Ketilda(9,12) = BETA_c_13/L ;

for i = 1:1:12 % Symmetry
    for j = i:1:12
        Ketilda(i,j) = Ketilda(j,i) ;
    end
end

% Form the element stiffness matrix in global coordinates (Ref.12.3.10 and 12.12.4)
Ke = Ketilda ;

% Assemble Global Condensed Matrices
for m=1:1:12
    for n =1:1:12
        g_e_m = ICONDOF(e,m) ;
        g_e_n = ICONDOF(e,n) ;
        Kc(g_e_m,g_e_n) = Kc(g_e_m,g_e_n) + Ke(m,n) ;
    end
end

end % "e" loop

% Assemble the axial and torsional restraint stiffnesses at Node 1 into the Kc matrix
Kc(1,1) = Kc(1,1) + KZ ;
Kc(4,4) = Kc(4,4) + KTHETA;

% Form the Global Damping Matrix [Cc]
Cc = zeros(Nd,Nd);

% SECTION 4: Form System State variables matrices
N = 2*Nd;
AS = zeros(N,N);
AS(1:Nd, Nd+1:2*Nd) = eye(Nd);
AS(Nd+1:2*Nd,1:Nd) = -inv(Mc)*Kc;

% Form Input system Matrix
BS = zeros(2*Nd, Nd);
BS(Nd+1:2*Nd,1:Nd) = inv(Mc);

% gForce = zeros(Nd,1);

% Form the Gyrosopic Matrix [Gyro]

rpm= 110; % Typically should be range from 80 rpm to 130 rpm
omega= rpm*pi/30;
Gyro= zeros(Nd,Nd);
for i=1:1:Nnode
  Gyro(6*(i-1)+5,6*(i-1)+6) = omega*Mc(6*(i-1)+4,6*(i-1)+4);
  Gyro(6*(i-1)+6,6*(i-1)+5) = -omega*Mc(6*(i-1)+4,6*(i-1)+4);
end

% % updating the State/system Matrix due to Geroscopic
AS(Nd+1:2*Nd,Nd+1:2*Nd) = -inv(Mc)* (Gyro+Cc) ;

%%
% irpm=1;
% System_eigenvalues(irpm,1:N)=eig(AS); % un-time scale the eigenvalues
% real_eigen= real(System_eigenvalues(irpm,1:N));
% imag_eigen= imag(System_eigenvalues(irpm,1:N));
%%
% [sort_imag_eigen(irpm,1:N),Iorder(irpm,1:N)= sort(imag_eigen);
% for i=1:1:N
%    sort_real_eigen(irpm,i) = real_eigen(1,Iorder(irpm,i));
% end

%% [5]% SECTION 5: Eigenvalues & Mode Shapes
[eigvector_mat,Eigen] = eig(AS);
damp(AS);
EV = diag(Eigen);

% % boundary condition
% Kc(:,1:6)=[ ]; Kc(:,1:6)=[];
% Mc(:,1:6)=[ ]; Mc(:,1:6)=[];
% dampening matrix
DS = d1*Kc + d2*Mc ;
% global output vectors
CS=zeros(1,6*N);
CS(1,N*6-3)=1; % output: beam tip, +z
% ================ second order --> first order ===============
% FS = eye(6*N);
% ES = [F zeros(6*N,6*N); zeros(6*N,6*N) Mc];
% sys = dss(AS,BS,CS,ES);
% Determining Natural frequencies with ordering
omega_nf=sqrt(EV);
natfreq=omega_nf/2/pi;
[natfreq_ordered,Iorder] = sort(natfreq);

% % printeig(1:N,2)=real(EV);
% printeig(1:N,3)=imag(EV);
% printeig(1:N,1)= (1:1:N)';
% % Sorting Eigenvalues
% sorted_eigvectors= eigvector_mat;
% sorted_eig-printeig;
% sorted_eig(:,3)=abs(sorted_eig(:,3))*30/pi;


% for i=1:N
%     if abs(sorted_eig(i,2))<=1e-4
%         sorted_eig(i,2)=0;
%     end
% end
% for i=1:N-1
%     for j=i+1:N
%         if sorted_eig(j,3)<=sorted_eig(i,3)
%             zwap3=sorted_eig(i,3);
%             sorted_eig(i,3)=sorted_eig(j,3);
%             sorted_eig(j,3)=zwap3;
%             zwap2=sorted_eig(i,2);
%             sorted_eig(i,2)=sorted_eig(j,2);
%             sorted_eig(j,2)=zwap2;
%             zwap1=sorted_eig(i,1);
%             sorted_eig(i,1)=sorted_eig(j,1);
%             sorted_eig(j,1)=zwap1;
%             zwap0=EV(i,1);
%             EV(i,1)=EV(j,1);
%             EV(j,1)=zwap0;
%             zwap=sorted_eigvectors(:,i);
%             sorted_eigvectors(:,i)=sorted_eigvectors(:,j);
%             sorted_eigvectors(:,j)=zwap;
%         end
%     end
% end
% disp('-----------------------------------------------')
% disp('------Sorted DNF & Damping Ratios--------------')
% disp('---------Mode #-------Real------Imag (CPM)-------Zeta (%)')
% damp_ratio=real(EV)./abs(EV)*100;
% [sorted_eig, damp_ratio];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
MBHA=Mc;
KBHA=Kc;
CBHA=Cc;
save ('MBHA.mat', 'MBHA');
save ('KBHA.mat', 'KBHA');
save ('CBHA.mat', 'CBHA');

FEM_Drillstring.m

% Omar Abdelzaher
% Drillstring Finite-Element-Model using Timoshenko Beam Element method
% Drillstring Dynamics Analysis ( Forward & Backward whirling, Bit-Bounce, Stick-Slip)
% Stick Slip Via Time Delay (Detournay et al.) - PDC Drillbit
% Created January 15, 2014
% Updated April 20, 2014
%
clear all; close all; clc;
% Define Global Variables

global Wo A0 M_BHA B0 Vmin Ctf R_BHA cDB e psi cBH cST R_st kT_dp w

global Kf eta gma a eps sgma l mu0 n xi phii xtni phitni tni tn0 bounce x

global omega_st1 Vrel_st1 Fn_st1 Ft_st1 r_CG omega_BHA Vrel_BHA Fn_CG Ft_CG

%% [1] % INPUT SYSTEM PARAMETERS %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% 
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% A) Formations parameters
% --------------------------------------------------------------
Vmin=0.001; % Mean Friction Coefficient ****************************
mu0=0.2; % Mean Friction Coefficient ****************************
mus=1.2*mu0; % Mean Friction Coefficient ****************************
Kf=2e9; % Formation Contact Stiffness
Cf=1000;  % Formation Contact damping
Ctf=2000; % adding tortional damping constant ofr drilling
Rh=(9.5/2)*0.0254; % Borehole radius
eta=0.7; % Bit-formation interaction factor
gma=1.2; % Bit-formation interaction factor
eps=13.3e6*3*3; % Rock Intrinsic Specific Energy
sgma=9.33e6*3*3; % Rock Cutting Stress
Cr=10; % Adding fluid radial damping to BHA

% B) FEM-BHA parameters
% --------------------------------------------------------------
load 'MBHA'; % Load [M],[C],[K] for BHA-FEM ((From FEM_BHA.m))
load 'KBHA';
load 'CBHA';
noe=6; % No of elements used in ((FEM_BHA.m))
e=0.003; % eccentricity of BHA's center mass ****************************
OD_BHA= (6.75)*0.0254;  % BHA Drill-collars Outer Diameters (m) Default
ID_BHA= (4.5)*0.0254;  % BHA Drill-collars Inner Diameters (m)  Default
L_BHA=200;  % BHA total length used in ((FEM_BHA.m))
rho=7850; % BHA steel density
M_BHA=pi/4*rho*L_BHA*( OD_BHA^2-ID_BHA^2);  % BHA Effective Mass (use for mass unbalance) SHOULD CORRESPOND TO KBHA CBHA MBHA OF THE FEM_Drillstring.m
R_BHA= OD_BHA/2; % BHA Drill-collars radius
cBH= Rh-R_BHA; % Borehole-BHA Clearance

% C) Applied static load at the Top of the Rig
% --------------------------------------------------------------
Wo=50e3; % Applied Load (Nfft) ****************************

% D) Drillpipe parameters
% --------------------------------------------------------------
mass_rho = 7850; % Density kg/M_BHA^3
L_dp=110; % Drill-pipe length (M_BHA)
OD_dp=(5)*0.0254; % Drill-pipe OD
ID_dp=(3/8)*0.0254; % Drill-pipe ID
Ac_dp = pi/4*(OD_dp.^2 - ID_dp.^2); % Drill-pipe cross section area
M_dp=mass_rho*Ac_dp*L_dp; % Drill-pipe mass on top of BHA
J_dp=(M_dp/8)*(OD_dp^2+ID_dp^2); % Drill-pipe Moment of Inertia
kT_dp=600;  % Equivalent Torsional Stiffness-to-ground
cT_dp=500;  % Equivalent Torsional damping-to-ground
cb_dp=(0.5)*30e3;  % Equivalent Axial damping-to-ground

% E) DrillBit Parameters
% --------------------------------------------------------------
n=8; % Number of PDC Blades
OD_db = (8.5)*0.0254; % DRILLBIT OD
a = OD_db/2; % PDC Drillbit radius
cDB = Rh-a; % Drillbit/borehole clearance
l = 5e-3; % Wearflat length
psi = 0*180; % WOB Vertical Inclination wrt to the BHA

% F) Rotary Table parameters
% ---------------------------------------------
rpm = 120; % Top Drive Spin (RPM)
w = rpm*2*pi/60; % (wd) Top Drive Spin rotary table Speed (rad/s)
tn0 = 1*pi/t/w; % time required for the bit to rotate by 2Pi/n to its current position at time t

% G) ADDING FLYWHEEL
% -------------------------------------------------------------------------------------
N_fl = 3000; % flywheel (rpm)
w_f = N_fl*2*pi/60; % flywheel angular speed (rad/sec)
q = 5e-6; % Flywheel eccentricity
E_fw = 30456846*6894.76; % Flywheel Young Modulus of Elasticity in Pa
rho_f = 7850; % Flywheel Density in kg/m^3
nu_f = 0.3; % Flywheel Poisson's Ratio
EW_desired = 1.0e3*3600; % Desired Energy to be stored in Flywheel in W.s OR J
ID_fw = 124.3662e-3; % Flywheel Inner Diameter (M_BHA)
OD_fw = 177.1357e-3; % Flywheel Outer Diameter (M_BHA)
max_rpm = 2.9146e+04; % Flywheel Maximum Speed (RPM)
E_Whr = 7.5761; % Flywheel Energy Density (W.hr/kg)
L_fw = 1.3456; % Flywheel Length (M_BHA)
M_fw = 131.9935; % Flywheel Mass (kg)
Jp_fw = (0.5*M_fw*(ID_fw^2+(OD_fw^2))/2)+M_fw*q^2; % Flywheel Inertia (kgm^2)
zeta = 0.3; % damping ration

% FW Axial (X-axis)
kb_fw = M_fw*(w_f)^2; % FW Axial Stiffness
c_fw = 2*(zeta)*w_f*(M_fw); % FW Axial Damping

% FW Torsional (X-axis)
kTb_fw = 1*60; % FW Torsional Stiffness
cTb_fw = 1*500; % FW Torsional Damping

% FW Lateral/Radial (Y/Z-axis)
k_r_fw = 1*M_fw*(zeta)^2; % FW Lateral Stiffness
c_r_fw = 0.1*2*(w_f)*(zeta)*M_fw; % FW Lateral Damping

% H) ADDING STABILIZERS
% -------------------------------------------------------------
% Stabilizers
R_st = (8.5/2)*0.0254; % Stabilizer radius
cST = Rh-R_st; % Stabilizer/Borehole clearance
klat_st = 0*5000; % Stabilizer Stiffness
clat_st = 0*600; % Stabilizer Damping

% [2]% AASEMBLING MATICES ...........................................................................
% % Get Total number of DOF's for the loaded BHA only
NNODE = size(MBHA,2);

% Get the CG node index for the BHA
node_un = ceil(NNODE/2);
loc_unx = 6*(node_un-1)+1;
loc_uny = 6*(node_un-1)+2;
loc_unz = 6*(node_un-1)+3;
loc_unphi = 6*(node_un-1)+4;

% Constructing SYSTEM MATRICES
MC = zeros(NNODE+4,NNODE+4);
CC = zeros(NNODE+4,NNODE+4);
 KC=zeros(NNODE+4,NNODE+4);
%
% Adding the BHA matrices to the global
 MC(1:NNODE,1:NNODE)=MBHA;
 CC(1:NNODE,1:NNODE)=CBHA;
 KC(1:NNODE,1:NNODE)=KBHA;
%
% Adding equivalent top drillpipe weight, Tortional stiffness, damping
 MC(1,1)=MC(1,1)+M_dp;
 KC(4,4)=KC(4,4)+kT_dp;
 CC(4,4)=CC(4,4)+cT_dp;
 CC(1,1)=CC(1,1)+cb_dp;
%
% Adding drilling fluid radial damping constant to BHA in (Y)
 for i=1:NNODE/6
   CC(i+(1+5*(i-1)),i+(1+5*(i-1)))=CC(i+(1+5*(i-1)),i+(1+5*(i-1)))+Cr;
 end
%
% Adding drilling fluid radial damping constant to BHA in (Z)
 for i=1:NNODE/6
   CC(i+(2+5*(i-1)),i+(2+5*(i-1)))=CC(i+(2+5*(i-1)),i+(2+5*(i-1)))+Cr;
 end
%
% Adding drilling fluid radial damping constant to BHA (phiX)
 for i=1:NNODE/6
   CC(i+(3+5*(i-1)),i+(3+5*(i-1)))=CC(i+(3+5*(i-1)),i+(3+5*(i-1)))+Ctf;
 end
%
% % Adding Flywheel inside the BHA- Drill Collars
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% % OPTION (i) Either placing the FW Near Drillbit
% MC(NNODE+1,NNODE+1)=M-fw;
% MC(NNODE+2,NNODE+2)=Jp-fw; % ??! --------- CHECK HERE PLZ from the original file !!
% MC(NNODE+3,NNODE+3)=M-fw;
% MC(NNODE+4,NNODE+4)=M-fw;
% % Add FW axial stiffness (X-dir)
% KC(NNODE-5,NNODE-5)=KC(NNODE-5,NNODE-5)+kb-fw;
% KC(NNODE-5,NNODE+1)=-kb-fw;
% KC(NNODE-5,NNODE+1)=-kb-fw;
% KC(NNODE+1,NNODE-5)=-kb-fw;
% % Add FW Tortional stiffness (phiX-dir)
% KC(NNODE-2,NNODE-2)=-KC(NNODE-2,NNODE-2)+kTb-fw;
% KC(NNODE-2,NNODE+2)=-kTb-fw;
% KC(NNODE-2,NNODE+2)=-kTb-fw;
% % Add FW Radial/Lateral stiffnesses (Y-dir)
% KC(NNODE-4,NNODE-4)=KC(NNODE-4,NNODE-4)+kr-fw;
% KC(NNODE+3,NNODE+3)=kr-fw;
% KC(NNODE+4,NNODE+4)=kr-fw;
% % Add FW axial damping (X-dir)
% CC(NNODE-5,NNODE-5)=CC(NNODE-5,NNODE-5)+cf-fw+cb_dp;
% CC(NNODE+1,NNODE+1)=cf-fw;
% CC(NNODE-5,NNODE+1)=cf-fw;
% CC(NNODE+1,NNODE-5)=cf-fw;
% % Add FW axial Tortional damping (phiX-dir)
% CC(NNODE-2,NNODE-2)=CC(NNODE-2,NNODE-2)+cTb-fw;
% CC(NNODE+2,NNODE+2)=cTb-fw;
% CC(NNODE-2,NNODE+2)=cTb-fw;
% CC(NNODE+2,NNODE-2)=cTb_fw;
% Add FW Radial/Lateral damping (Y-dir)
% CC(NNODE-4,NNODE-4)=CC(NNODE-4,NNODE-4)+cr_fw;
% CC(NNODE+3,NNODE+3)=cc_fw;
% CC(NNODE+4,NNODE+4)=cc_fw;
% Add FW axial damping (Z-dir)
% CC(NNODE-3,NNODE-3)=CC(NNODE-3,NNODE-3)+cr_fw;
% CC(NNODE+4,NNODE+4)=cr_fw;
% CC(NNODE-3,NNODE+4)=cr_fw;
% CC(NNODE+3,NNODE-3)=cr_fw;
% % Adding Stabilizers to the BHA model
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% OPTION (1) Adding first stabilizer at BHA's CG location
% Lateral (Y-dir)
% KC(loc_uny,loc_uny)=KC(loc_uny,loc_uny)+klat_st;
% CC(loc_uny,loc_uny) = CC(loc_uny,loc_uny) + clat_st;
% Lateral (Z-Dir)
% KC(loc_unz,loc_unz) = KC(loc_unz,loc_unz) + klat_st;
% CC(loc_unz,loc_unz) = CC(loc_unz,loc_unz) + clat_st;

% OPTION (2) Adding Second stabilizer at Top of the BHA
% Lateral (Y-Dir)
KC(2,2) = KC(2,2) + klat_st;
CC(2,2) = CC(2,2) + clat_st;
% Lateral (Z-Dir)
KC(3,3) = KC(3,3) + klat_st;
CC(3,3) = CC(3,3) + clat_st;

% OPTION (3) Adding Third stabilizer at the bottom
% Lateral (Y-Dir)
KC(NNODE-4,NNODE-4) = KC(NNODE-4,NNODE-4) + klat_st;
CC(NNODE-4,NNODE-4) = CC(NNODE-4,NNODE-4) + clat_st;
% Lateral (Z-Dir)
KC(NNODE-3,NNODE-3) = KC(NNODE-3,NNODE-3) + klat_st;
CC(NNODE-3,NNODE-3) = CC(NNODE-3,NNODE-3) + clat_st;

%% [3] % FORMING STATE-SPACE %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
A0 = zeros(2*NNODE+8, 2*NNODE+8);
A0(1:NNODE+4, NNODE+4+1:2*NNODE+8) = eye(NNODE+4);
A0(NNODE+4+1:2*NNODE+8, 1:NNODE+4) = -(inv(MC)*KC);
A0(NNODE+4+1:2*NNODE+8, NNODE+4+1:2*NNODE+8) = -(inv(MC)*CC);
B0 = zeros(2*NNODE+8, NNODE+4);
B0(NNODE+4+1:2*NNODE+8, :) = inv(MC);

%% [4] % SIMULATION TIME %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

to = 0; % Start Simulation Time
tfinal = 50; % Final Simulation Time
maxstep = 0.0025; % Max. step size
Nsamples = (tfinal / maxstep) + 1; % Get Number of steps
dt = tfinal / (Nsamples - 1); % Integration with fixed step

% Initializing system state space...
% Initializing DRILLBIT Outputs variables (Node #7)
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
x = zeros(Nsamples, 1); % Drillbit axial penetration in X-dir
xDot_db = zeros(Nsamples, 1); % Drillbit axial velocity in X-dir
y_db = zeros(Nsamples, 1); % Drillbit Lateral position in Y-dir
z_db = zeros(Nsamples, 1); % Drillbit Lateral position in Z-dir
yDot_db = zeros(Nsamples, 1); % Drillbit Lateral velocity in Y-dir
zDot_db = zeros(Nsamples, 1); % Drillbit Lateral velocity in Z-dir
phi = zeros(Nsamples, 1); % Drillbit axial rotation angle X-dir
PhiXDot_db = zeros(Nsamples, 1); % Drillbit axial rotation velocity Phi-X
rDB = zeros(Nsamples, 1); % Drillbit radial displacement
omegaDB = zeros(Nsamples, 1); % Drillbit whirling velocity
VrelDB = zeros(Nsamples, 1); % Drillbit relative velocity
wob = zeros(Nsamples, 1); % Drillbit WOB
tob = zeros(Nsamples, 1); % Drillbit TOB

% Initializing Flywheel Outputs variables (Node #8)
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
x fw = zeros(Nsamples, 1); % FW axial motion X-dir
xDot_fw=zeros(Nsamples,1);  % FW axial velocity X-dir
PhiX_fw=zeros(Nsamples,1);  % FW axial rotation angle X-dir
PhiXDot_fw=zeros(Nsamples,1);  % FW axial rotation velocity X-dir
y_fw=zeros(Nsamples,1);  % FW lateral motion Y-dir
yDot_fw=zeros(Nsamples,1);  % FW lateral velocity Y-dir
z_fw=zeros(Nsamples,1);  % FW lateral motion Z-dir
zDot_fw=zeros(Nsamples,1);  % FW lateral velocity Z-dir

% initializing STABILIZER 1 Outputs variables (Node #1)
% -----------------------------------------------
y_st1=zeros(Nsamples,1);  % ST1 lateral motion Y-dir
z_st1=zeros(Nsamples,1);  % ST1 lateral motion Z-dir
r_st1=zeros(Nsamples,1);  % ST1 radial displacement
omegast1=zeros(Nsamples,1);  % ST1 whirling velocity
Vrelst1=zeros(Nsamples,1);  % ST1 relative velocity
Fnst1=zeros(Nsamples,1);  % ST1 normal contact force
Ftst1=zeros(Nsamples,1);  % ST1 friction force

% initializing BHA CG outputs variables (Node #4)
% -----------------------------------------------
yG=zeros(Nsamples,1);  % C.G. Lateral position in Y-dir
zG=zeros(Nsamples,1);  % C.G. Lateral position in Z-dir
yDot_G=zeros(Nsamples,1);  % C.G. Lateral velocity Y-dir
zDot_G=zeros(Nsamples,1);  % C.G. Lateral velocity Z-dir
PhiX_G=zeros(Nsamples,1);  % C.G. rotation angle about X-dir
PhiXDot_G=zeros(Nsamples,1);  % C.G. rotation velocity about X-dir
rCG=zeros(Nsamples,1);  % C.G. radial displacement
omegABHA=zeros(Nsamples,1);  % C.G. whirling velocity
VrelBHA=zeros(Nsamples,1);  % C.G. relative velocity

% initializing BHA outputs variables at (Node #5)
% -----------------------------------------------
y_5=zeros(Nsamples,1);  % BHA-N5 Lateral position in Y-dir
z_5=zeros(Nsamples,1);  % BHA-N5 Lateral position in Z-dir
r_5=zeros(Nsamples,1);  % BHA-N5 radial displacement

% initializing STABILIZER 2 Outputs at (Node #6)
% -----------------------------------------------
y_st2=zeros(Nsamples,1);  % ST2 lateral motion Y-dir
z_st2=zeros(Nsamples,1);  % ST2 lateral motion Z-dir
rst2=zeros(Nsamples,1);  % ST2 radial displacement
omegast2=zeros(Nsamples,1);  % ST2 whirling velocity
Vrelst2=zeros(Nsamples,1);  % ST2 relative velocity
Fnst2=zeros(Nsamples,1);  % ST2 normal contact force
Ftst2=zeros(Nsamples,1);  % ST2 friction force
Ftst2R=zeros(Nsamples,1);  % ST2 calculated friction force incase of pure-rolling
ROLLING_st2=zeros(Nsamples,1);  % ST2 Pure-Rolling FLAG!
SLIDING_st2=zeros(Nsamples,1);  % ST2 Sliding FLAG!

Wf=a*1*sigma;  % fractional component of WOB
v0=(Wo-Wf)*w/2/pi/a/eta/eps;  % Penetration rate
d0=2*pi*v0/w;

% Initializing the State matricies for ode
% -----------------------------------------------
z0=zeros(NNODE+4,1);  % initialization of the state
% dy= zeros(size(x));  % initialization of the state derivative
% % apply initial axial velocity of V0 to all nodes
% for i=1:1:NNODE/6
% xtn(6*(i-1)+1,1)=v0;
% end

xtn=zeros(Nsamples,1);
phitn=zeros(Nsamples,1);
dd=zeros(Nsamples,1);
tn=zeros(Nsamples,1);
tni=tn0;
tn(1,1)=tni;

% initializing condition flags
% -----------------------------------------------------------

bounce=0;
casex=zeros(Nsamples,1);

% [5]% ODE-Integrating ... %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
tic
for i=1:Nsamples-1
    tii=tsim(1,i)
    tfi=tsim(1,i+1);
    loci=i;
    xi=x(i,1);
    phii=phi(i,1);
    options = odeset('AbsTol',1e-4,'RelTol',1e-3);
    [tr zi]=ode45('FEM_Drillstring_sub', [tii tfi], [z0;dz0], options);
    % %
    % [tr zi]=ode23s('FEM_Drillstring_sub', [tii tfi], [z0;dz0]);
    % [tr zi]=ode15s('FEM_Drillstring_sub', [tii tfi], [z0;dz0]);
    % [tr zi]=ode113('FEM_Drillstring_sub', [tii tfi], [z0;dz0]);
    % ni=size(zi,1);
    % z0=zi(ni,NNODE+4+1:2*NNODE+8)';
    xtn(i+1,1)=xtni;
    phitn(i+1,1)=phitni;
    tni(i+1,1)=tni;
    dd(i+1,1)=d;

% Outputs for DB (Node# 7)
% ----------------------------------------------------------

x(i+1,1)=z0(NNODE-5,1);  % axial penetration X
y_db(i+1,1)=z0(NNODE-4,1);  % lateral position Y
z_db(i+1,1)=z0(NNODE-3,1);  % lateral position Z
phi(i+1,1)=z0(NNODE-2,1);  % axial rotation angle Phi-x
xDot_db(i+1,1)=dz0(NNODE-5,1);  % axial velocity Xdot
yDot_db(i+1,1)=dz0(NNODE-4,1);  % Lateral velocity Ydot
zDot_db(i+1,1)=dz0(NNODE-3,1);  % Lateral velocity Zdot
PhiXDot_db(i+1,1)=dz0(NNODE-2,1);  % axial angular velocity PhiDot-X
rDB(i+1,1)=rdb;  % radial displacement
omega_DB(i+1,1)=omega_db;  % Whirling velocity
Vrel_DB(i+1,1)=Vrel_db;  % Relative velocity
wob(i+1,1)=WOB;  % WOB
tob(i+1,1)=TOB;  % TOB

% Outputs for FW at it's second Node (Node# 8)
% ----------------------------------------------------------

x_fw(i+1,1)=z0(NNODE+1,1);  % Axial displacement X
xDot_fw(i+1,1)=dz0(NNODE+1,1);  % Axial velocity X
Phix_fw(i+1,1)=z0(NNODE+2,1);  % rotation angle about X
PhixDot_fw(i+1,1)=dz0(NNODE+2,1);  % rotation velocity about X
y_fw(i+1,1)=z0(NNODE+3,1);  % Lateral displacement Y
yDot_fw(i+1,1)=dz0(NNODE+3,1);  % Lateral velocity Y
z_fw(i+1,1)=z0(NNODE+4,1);  % Lateral displacement z
zDot_fw(i+1,1)=dz0(NNODE+4,1);  % Lateral velocity z
% Outputs for BHA C.G. (Node# 4)
% ---------------------------------------------------------------------------
yG(i+1,1)=z0(loc_uny,1); % Lateral displacement Y
zG(i+1,1)=z0(loc_unz,1); % Lateral displacement z
yDot_G(i+1,1)=dz0(loc_uny,1); % Lateral velocity Y
zDot_G(i+1,1)=dz0(loc_unz,1); % Lateral velocity z
PhiX_G(i+1,1)=z0(6*(node_un-1)+4,1); % rotation angle about X
PhiXDot_G(i+1,1)=dz0(6*(node_un-1)+4,1); % rotation velocity about X
rCG(i+1,1)=r_CG; % radial displacement
omegaBHA(i+1,1)=omega_BHA; % Whirling velocity
VrelBHA(i+1,1)=Vrel_BHA; % Relative velocity
FnBHA(i+1,1)=Fn_CG; % Contact force
FrBHA(i+1,1)=Fr_CG; % friction force

% Outputs for STABILIZER 1 (Node# 1)
% ---------------------------------------------------------------------------
y_st1(i+1,1)=z0(6*(1-1)+2,1); % Lateral displacement Y
z_st1(i+1,1)=z0(6*(1-1)+3,1); % Lateral displacement z
yDot_st1(i+1,1)=dz0(6*(1-1)+2,1); % Lateral velocity Y
zDot_st1(i+1,1)=dz0(6*(1-1)+3,1); % Lateral velocity z
PhiX_st1(i+1,1)=z0(6*(1-1)+4,1); % rotation angle about X
PhiXDot_st1(i+1,1)=dz0(6*(1-1)+4,1); % rotation velocity about X
rst1(i+1,1)=r_st1; % radial displacement
omegast1(i+1,1)=omega_st1; % Whirling velocity
Vrelst1(i+1,1)=Vrel_st1; % Relative velocity
Fnst1(i+1,1)=Fn_st1; % Contact force
Frst1(i+1,1)=Fr_st1; % friction force

% Outputs for (Node# 5)
% ---------------------------------------------------------------------------
y_5(i+1,1)=z0(6*(5-1)+2,1); % Lateral displacement Y
z_5(i+1,1)=z0(6*(5-1)+3,1); % Lateral displacement z
yDot_5(i+1,1)=dz0(6*(5-1)+2,1); % Lateral velocity Y
zDot_5(i+1,1)=dz0(6*(5-1)+3,1); % Lateral velocity z
PhiX_5(i+1,1)=z0(6*(5-1)+4,1); % rotation angle about X
PhiXDot_5(i+1,1)=dz0(6*(5-1)+4,1); % rotation velocity about X

% Outputs for STABILIZER 2 (Node# 6)
% ---------------------------------------------------------------------------
y_st2(i+1,1)=z0(6*(6-1)+2,1); % Lateral displacement Y
z_st2(i+1,1)=z0(6*(6-1)+3,1); % Lateral displacement z
yDot_st2(i+1,1)=dz0(6*(6-1)+2,1); % Lateral velocity Y
zDot_st2(i+1,1)=dz0(6*(6-1)+3,1); % Lateral velocity z
PhiX_st2(i+1,1)=z0(6*(6-1)+4,1); % rotation angle about X
PhiXDot_st2(i+1,1)=dz0(6*(6-1)+4,1); % rotation velocity about X
rst2(i+1,1)=r_st2; % radial displacement
omegast2(i+1,1)=omega_st2; % Whirling velocity
Vrelst2(i+1,1)=Vrel_st2; % Relative velocity
Fnst2(i+1,1)=Fn_st2; % Contact force
Frst2(i+1,1)=Fr_st2; % friction force
Ftst2R(i+1,1)=Frst2_R; % calculated friction force incase of Pure-rolling
ROLLING_st2(i+1,1)=ROLLINGst2; % Pure-Rolling FLAG !
SLIDING_st2(i+1,1)=SLIDINGst2; % Sliding FLAG !
end

toc

% Theoretical whirling velocity at Stabilizer-2 when Backward whirling
omega_Theo_ST2= zeros(size(Fnst2));
for j= 1: size(Fnst2)
    if Fnst2(j)~=0 && (-mus*Fnst2(j))<= Ftst2R(j)<= (mus*Fnst2(j)) &&
        abs(Vrelst2(j))<Vmin
        omega_Theo_ST2(j)= -(R_st/cST)*PhiXDot_st2(j);
    else
end
end
% % FFT
% -----------------------------------
m=length(omegast2);
Nfft= pow2(nextpow2(m));
Fs= Nsamples/tfinal; % sample frequency (Hz)
% f= (0:Nfft/2-1) * (Fs/Nfft); % Frequency vector
% f= (0:Nfft-1) * (Fs/Nfft); % Frequency range % Fs/Nfft=frequency increment

% ST1
FFT_Omega_ST1= fft(omegast1./(2*pi),Nfft);
FFT_Omega_ST1= abs(FFT_Omega_ST1);
FFT_Omega_ST1 = FFT_Omega_ST1(1:Nfft/2); % FFT is symmetric, throw away second half
FFT_Y_ST1= abs(fft(y_st1,Nfft));
FFT_Z_ST1= abs(fft(z_st1,Nfft));
FFT_Y_ST1 = FFT_Y_ST1(1:Nfft/2);
FFT_Z_ST1 = FFT_Z_ST1(1:Nfft/2);

% ST2
FFT_Omega_ST2= fft(omegast2./(2*pi),Nfft);
FFT_Omega_ST2= abs(FFT_Omega_ST2);
FFT_Omega_ST2 = FFT_Omega_ST2(1:Nfft/2); % FFT is symmetric, throw away second half
FFT_Y_ST2= abs(fft(y_st2,Nfft));
FFT_Z_ST2= abs(fft(z_st2,Nfft));
FFT_Y_ST2 = FFT_Y_ST2(1:Nfft/2);
FFT_Z_ST2 = FFT_Z_ST2(1:Nfft/2);

% BHA
FFT_Omega_BHA= fft(omegaBHA./(2*pi),Nfft);
FFT_Omega_BHA= abs(FFT_Omega_BHA);
FFT_Omega_BHA = FFT_Omega_BHA(1:Nfft/2);

% db
FFT_Omega_db= fft(omegaDB./(2*pi),Nfft);
FFT_Omega_db= abs(FFT_Omega_db);
FFT_Omega_db = FFT_Omega_db(1:Nfft/2);
FFT_X_db= abs(fft((x+v0*tsim'),Nfft));
FFT_X_db = FFT_X_db(1:Nfft/2);

%% [6] % Plotting Output Data %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Plotting- Drillbit axial penetration
figure(1)
plot(tsim,(x+v0*tsim')*1000)
xlabel('time (s)')
ylabel('x (mm)')
title('Drillbit Axial Penetration');
grid on

% Plotting- Drillbit 3D-pathline inside Borehole
figure (2)
hold on
grid
NN=100;
[X,Y,Z] = cylinder(cDB*1000,NN);
Z(Z,:) = min((x+v0*tsim')*1000);
Z(Z,:) = max((x+v0*tsim')*1000);
h2 = surf(X,Y,Z);
set(h2,'FaceAlpha',0.3)
pplot3(y_db*1000,z_db*1000,(x+v0*tsim')*1000,'b');
xlabel(' Yg (mm)');
ylabel(' Zg (mm)');
zlabel(' Xg (mm)');
title('3D Pathline Orbit');
% legend('Borehole','Drillbit Pathline');
view([-37.5 30])

% Plotting - Drillbit Axial Penetration Velocity
figure(3)
plot(tsim,(v0+xDot_db)*1000)
xlabel('time (s)')
ylabel('xDot db (mm/s)')
title('Drillbit Axial Penetration Velocity');
grid on

% Plotting - Drillbit Rotation Angle in Degrees
figure(4)
plot(tsim,phi*180/pi)
xlabel('time (s)')
ylabel('phi (deg)')
title('Drillbit Rotation Angle');
grid on

% Plotting - Spinning angular Velocities
figure(5)
plot(tsim,PhiXDot_db*30/pi,'b')
hold on
plot(tsim,PhiXDot_st1*30/pi,'g-')
plot(tsim,PhiXDot_G*30/pi,'c-')
plot(tsim,PhiXDot_st2*30/pi,'r-')
xlabel('time (s)')
ylabel('PhiDot X (rpm)')
title('Spinning angular Velocities')
legend ('DB', 'ST1', 'BHA C.G', 'ST2');
grid on

% Plotting - FW Axial Displacement
% figure(6)
% plot(tsim,x_fw*1000)
xlabel('time (s)')
ylabel('x_f_w (mm)')
title('FW Axial Displacement')
grid on

% Plotting - FW-BHA Axial Clearance
% figure(7)
% plot(tsim,abs(x_fw-x)*1000)
xlabel('time (s)')
ylabel('dx (mm)')
title('FW-BHA Axial Clearance')
grid on

% Plotting - WOB
figure(8)
plot(tsim,wob/1000)
xlabel('time (s)')
ylabel('Force (kN)')
title('WOB')
grid on

% Plotting - Depth Of Cut
figure(9)
plot(tsim,dd*1000)
xlabel('time (s)')
ylabel('Depth of Cut (mm)')
grid on

% Plotting - TOB
figure(10)
plot(tsim,tob/1000)
xlabel('time (s)')
ylabel('Torque (kN.m_BHA)')
title('TOB')
grid on

% Plotting - Drillbit 2D-Orbit
figure(11)
plot(y_db*1000,z_db*1000);
hold on
Radius_db=cDB*1000;
[cx_db,cy_db,z_db] = cylinder(Radius_db,100);
plot(cx_db(1,:),cy_db(1,:),’r.’);
xlabel(’y_D_B (mm)’)
ylabel(’z_D_B (mm)’)
title(’DrillBit Orbit’);
axis square
axis equal
grid on;

% Plotting - BHA CG 2D-Orbit
figure(12)
plot(yG*1000,zG*1000);
hold on
Radius_cg=cBH*1000;
[cx_cg,cy_cg,z_cg] = cylinder(Radius_cg,100);
plot(cx_cg(1,:),cy_cg(1,:),’r.’);
xlabel(’y_C_G (mm)’)
ylabel(’z_C_G (mm)’)
title(’BHA CG Orbit’);
axis square
axis equal
grid on;

% Plotting - Upper Stabilizer 2D-Orbit
figure(13)
plot(y_st1*1000,z_st1*1000);
hold on
Radius_st=cST*1000;
[cx_st,cy_st,z_st] = cylinder(Radius_st,100);
plot(cx_st(1,:),cy_st(1,:),’r.’);
xlabel(’y_st1 (mm)’)
ylabel(’z_st1(mm)’)
title(’Upper Stabilizer Orbit’);
axis square
axis equal
grid on;

% Plotting - Bottom Stabilizer 2D-Orbit
figure(14)
plot(y_st2*1000,z_st2*1000);
hold on
grid on;
Radius_st=cST*1000;
[cx_st,cy_st,z_st] = cylinder(Radius_st,100);
plot(cx_st(1,:),cy_st(1,:),’r.’);
xlabel(’y_st2 (mm)’)
ylabel(’z_st2(mm)’)
title(’Bottom Stabilizer Orbit’);
axis square
axis equal

% Plotting - Relative velocities
figure (15)
plot(tsim,Vrelst1,’b-’);
hold on
plot (tsim,VrelDB,’c’);
plot (tsim,VrelBHA,'g-');
plot (tsim,Vrelst2,'r');
title('Relative velocities');
xlabel('Time (S)');
ylabel(' Vrel(rad/sec)');
legend('Upper Stabilizer','DrillBit','BHA CG','Bottom Stabilizer');
grid on

% Plotting- Whirling velocities
figure (16)
hold on
plot (tsim,omegaBHA,'g-');
plot (tsim,omegaDB,'c');
plot (tsim,omegast1,'b--');
plot (tsim,omegast2,'k');
plot (tsim,omega_Theo_ST2,'r--');
title('Whirling velocity');
xlabel('Time (S)');
ylabel(' Omega(rad/sec)');
legend(' BHA','Drillbit','Upper stabilizer','Bottom stabilizer','Theoretical Bottom stabilizer');
grid on

% Plotting- BHA CG. Radial Displacement
figure (17)
plot (tsim,rCG,'b');
hold on
grid on
plot(tsim,cBH,'r');
title('BHA CG. Radial Displacement');
xlabel(' Time (S)');
ylabel(' r (mm)');
legend('Borehole','BHA CG radial');
axis square

% Plotting- Bottom Stabilizer Tangential Friction Force
figure (18)
plot (tsim,Ftst2/1000,'k');
hold on
grid on
plot(tsim,Ftst2R/1000,'r--');
title('Sliding friction Vs. Rolling friction');
xlabel(' Time (S)');
ylabel(' Tangential Friction Force (KN) ');
legend('Ft','Ft_R');

% Plotting- Bottom Stabilizer Normal contact Force
figure (19)
plot (tsim,Fnst2/1000,'k');
hold on
grid on
title('Normal Friction Force');
xlabel(' Time (S)');
ylabel(' Fn (KN) ');
legend('Ft','Ft_R');

% Plotting- Bottom Stabilizer Pure-Rolling Status
figure (20)
plot (tsim,ROLLING_st2,'r');
title('Bottom stabilizer Rolling status');
xlabel(' Time (S)');
ylabel(' 0=> No Rolling 1=> Rolling is ACTIVE ! ');

% Plotting- FFT whirling frequencies
figure (21)
plot(f,FFT_Omega_ST2,'r');
plot(f,power,'r');
hold on
grid on
plot(f,FFT_Omega_ST1,'g');
plot(f,FFT_Omega_BHA,'b');
plot(f,FFT_Omega_db,'k');
title ('Omega FFT');
xlabel('Frequency (HZ)');
ylabel('Amplitude');
legend('Bottom Stabilizer','Upper Stabilizer','BHA C.G','DrillBit');

% Plotting- FFT Bottom Stabilizer radial frequencies
figure (22)
plot(f,FFT_Y_ST2,'b');
hold on
grid on
plot(f,FFT_Z_ST2,'k');
% plot(f,FFT_Y_ST1,'--g');
% plot(f,FFT_Z_ST1,'--c');
title ('Radial displacement(Y/Z FFT)-Bottom Stabilizer');
xlabel('Frequency');
ylabel('Amplitude');
legend('FFT_Y_ST2', 'FFT_Z_ST2');

% Plotting- FFT Drillbit axial frequencies
figure (23)
plot(f,FFT_X_db,'r');
grd on
title ('axial penetration FFT - Drillbit');
xlabel('Frequency');
ylabel('Amplitude');
legend('FFT_X_DB');

% Plotting- FFT Upper Stabilizer radial frequencies
figure (24)
plot(f,FFT_Y_ST1,'b');
hold on
grid on
plot(f,FFT_Z_ST1,'k');
title ('Radial displacement(Y/Z FFT)-Upper Stabilizer');
xlabel('Frequency');
ylabel('Amplitude');
legend('FFT_Y_ST1', 'FFT_Z_ST1');
%% PDC Cutting Dynamics & Obtaining Time History

% Determining tni, xtni, phitni

phi(loci+1,1)=z(NNODE-2);  % Phi-X
phitni=z(NNODE-2)-2*pi/n;

if phitni<0  % First step time delay
  phitni=0;
  tni=tn0;
  xtni=0;
  casex(loci,1)=-1;
else  % In case not first step after getting previous time history
  j=loci;
  for jj=j:-1:2
    if phitni==phi(jj,1)
      tni=t-sim(1,jj);
      xtni=x(jj,1);
      casex(loci,1)=2;
      break;
    elseif phitni==phi(jj-1,1)
      tni=t-sim(1,jj-1);
      xtni=x(jj-1,1);
      casex(loci,1)=3;
      break;
    elseif phitni>phi(jj-1,1) && phitni<phi(jj,1)
      tni=t-(sim(1,jj)+sim(1,jj-1))/2;
      xtni=(x(jj,1)+x(jj-1,1))/2;
      casex(loci,1)=4;
      break;
    elseif jj<=1
      tni=tn0;
      xtni=0;
      casex(loci,1)=5;
    end
  end
end

%% Calculating variables

% Node 2

node_2=2;
xDot_2= x_2+NNODE+4;
y_2=6*(node_2-1)+2;
yDot_2= y_2+NNODE+4;
z_2=6*(node_2-1)+3;
zDot_2= z_2+NNODE+4;
PhiX_2= 6*(node_2-1)+4;
PhiXDot_2= PhiX_2+NNODE+4;
PhiY_2= 6*(node_2-1)+5;
PhiYDot_2= PhiY_2+NNODE+4;
PhiZ_2= 6*(node_2-1)+6;
PhiZDot_2= PhiZ_2+NNODE+4;

% STABILIZER ONE (Node# 1)

node_st1=1;  % First Node at Top
x_st1=6*(node_st1-1)+1;
xDot_st1=x_st1+NNODE+4;
y_st1=6*(node_st1-1)+2;
yDot_st1=y_st1+NNODE+4;
z_st1=6*(node_st1-1)+3;
\[ \text{zDot}_{\text{st1}} = \text{z}_{\text{st1}} + \text{NNODE} + 4; \]
\[ \Phi_{\text{X}}_{\text{st1}} = 6 \times (\text{node}_{\text{st1}} - 1) + 4; \]
\[ \Phi_{\text{X}}_{\text{Dot}}_{\text{st1}} = \Phi_{\text{X}}_{\text{st1}} + \text{NNODE} + 4; \]
\[ \Phi_{\text{Y}}_{\text{st1}} = 6 \times (\text{node}_{\text{st1}} - 1) + 5; \]
\[ \Phi_{\text{Y}}_{\text{Dot}}_{\text{st1}} = \Phi_{\text{Y}}_{\text{st1}} + \text{NNODE} + 4; \]
\[ \Phi_{\text{Z}}_{\text{st1}} = 6 \times (\text{node}_{\text{st1}} - 1) + 6; \]
\[ \Phi_{\text{Z}}_{\text{Dot}}_{\text{st1}} = \Phi_{\text{Z}}_{\text{st1}} + \text{NNODE} + 4; \]
\[ r_{\text{st1}} = \sqrt{((z(y_{\text{st1}})^2) + 1e-7) + z(z_{\text{st1}})^2}; \]
\[ r_{\text{Dot}}_{\text{st1}} = (z(y_{\text{st1}}) \times z(y_{\text{Dot}}_{\text{st1}}) + z(z_{\text{st1}}) \times z_{\text{Dot}}_{\text{st1}}) / r_{\text{st1}}; \]
\[ \text{theta}_{\text{st1}} = \text{atan2}(z(z_{\text{st1}}), z(y_{\text{st1}})); \]
\[ \text{omega}_{\text{st1}} = (z(z_{\text{Dot}}_{\text{st1}}) \times z(y_{\text{st1}}) - z(y_{\text{Dot}}_{\text{st1}}) \times z(z_{\text{st1}})) / r_{\text{st1}}^2; \]
\[ \text{Vrel}_{\text{st1}} = z(\Phi_{\text{X}}_{\text{Dot}}_{\text{st1}}) \times R_{\text{st}} + \text{omega}_{\text{st1}} \times r_{\text{st1}}; \]
\[ \text{Fe}_{\text{st1}} = (0) \times M_{\text{BH}} \times e \times z(\Phi_{\text{X}}_{\text{Dot}}_{\text{st1}})^2; \]
\[ \text{Fe}_{\text{y}}_{\text{st1}} = 1 \times \text{Fe}_{\text{st1}} \times \sin(z(\Phi_{\text{X}}_{\text{st1}})); \]
\[ \text{Fe}_{\text{z}}_{\text{st1}} = 1 \times \text{Fe}_{\text{st1}} \times \cos(z(\Phi_{\text{X}}_{\text{st1}})); \]
\[ \text{F}_{\text{tst1}}_{\text{R}} = \frac{(((M_{\text{st1}} \times R_{\text{st}}) / J_{\text{st1}}) \times (K_{\phi_{\text{st1}}}) \times (z(\Phi_{\text{X}}_{\text{st1}}) - z(\Phi_{\text{X}}_{\text{2}}) - C_{\text{tf}} \times z(\Phi_{\text{X}}_{\text{Dot}}_{\text{st1}})) + C_{\text{r}} \times \omega_{\text{st1}} \times R_{\text{st}})}{1 + ((M_{\text{st1}} \times R_{\text{st}} \times R_{\text{st}}) / J_{\text{st1}})} - \text{Fe}_{\text{st1}}_{\text{t}}; \]
\[ \text{fn}_{\text{st1}} = 1 \times K_{\text{f}} \times (r_{\text{st1}} - c_{\text{ST}}) + C_{\text{f}} \times (r_{\text{Dot}}_{\text{st1}}); \]
\[ \text{mu}_{\text{st1}} = (-2 / \pi) \times \text{atan}(10^4 \times \text{Vrel}_{\text{st1}}) \times (((\mu_{\text{s}} - \mu_{0}) / (1 + 10^2 \times \text{abs(\text{Vrel}_{\text{st1}})))) + \mu_{0}); \]
\[ \text{F}_{\text{t}}_{\text{st1}} = \mu_{\text{st1}} \times \text{fn}_{\text{st1}}; \]
\[ \text{F}_{\text{t}}_{\text{st1}}_{\text{R}} = \frac{(((M_{\text{st1}} \times R_{\text{st}}) / J_{\text{st1}}) \times (K_{\phi_{\text{st1}}}) \times (z(\Phi_{\text{X}}_{\text{st1}}) - z(\Phi_{\text{X}}_{\text{2}}) - C_{\text{tf}} \times z(\Phi_{\text{X}}_{\text{Dot}}_{\text{st1}})) + C_{\text{r}} \times \omega_{\text{st1}} \times R_{\text{st}})}{1 + ((M_{\text{st1}} \times R_{\text{st}} \times R_{\text{st}}) / J_{\text{st1}})} - \text{Fe}_{\text{st1}}_{\text{t}}; \]
\[ \text{Fy}_{\text{st1}} = -\text{Ft}_{\text{st1}} \times \sin(\text{theta}_{\text{st1}}) - \text{Fn}_{\text{st1}} \times \cos(\text{theta}_{\text{st1}}) + \text{Fey}_{\text{st1}}; \]
\[ \text{Fz}_{\text{st1}} = \text{Ft}_{\text{st1}} \times \cos(\text{theta}_{\text{st1}}) - \text{Fn}_{\text{st1}} \times \sin(\text{theta}_{\text{st1}}) + \text{Fez}_{\text{st1}}; \]
\[ \text{Fx}_{\text{st1}} = -\mu_{0} \times \text{sign}((\text{Vrel}_{\text{st1}})) \times (\text{Fn}_{\text{st1}}) + \text{Fz}_{\text{st1}}; \]
\[ \text{Ty}_{\text{st1}} = -\mu_{0} \times \text{sign}((\text{Vrel}_{\text{st1}})) \times \text{Fz}_{\text{st1}} \times \text{abs((\text{theta}_{\text{st1}}))}; \]
\[ \text{Tz}_{\text{st1}} = -\mu_{0} \times \text{sign}((\text{Vrel}_{\text{st1}})) \times \text{Fy}_{\text{st1}} \times \text{abs((\text{theta}_{\text{st1}}))}; \]
\( Tx_{st1} = -\text{sign}(z(\Phi X_{dot \_st1})) \cdot Ft_{st1} \cdot R_{st}; \)

\[\begin{align*}
\text{node} \_5 &= 5; \quad \% \text{The node before the last one at the bottom} \\
x_5 &= 6^* (\text{node} \_5 - 1) + 1; \\
x_{\text{dot}}_5 &= x_5 + N\text{NODE}; \\
y_5 &= 6^* (\text{node} \_5 - 1) + 2; \\
y_{\text{dot}}_5 &= y_5 + N\text{NODE}; \\
z_5 &= 6^* (\text{node} \_5 - 1) + 3; \\
z_{\text{dot}}_5 &= z_5 + N\text{NODE}; \\
\Phi X_5 &= 6^* (\text{node} \_5 - 1) + 4; \\
\Phi X_{\text{dot}}_5 &= \Phi X_5 + N\text{NODE}; \\
\Phi Y_5 &= 6^* (\text{node} \_5 - 1) + 5; \\
\Phi Y_{\text{dot}}_5 &= \Phi Y_5 + N\text{NODE}; \\
\Phi Z_5 &= 6^* (\text{node} \_5 - 1) + 6; \\
\Phi Z_{\text{dot}}_5 &= \Phi Z_5 + N\text{NODE}; \\
\end{align*}\]

\[\begin{align*}
\text{node} \_6 &= 6; \quad \% \text{The node before the last one at the bottom} \\
x_{\text{st2}} &= 6^* (\text{node} \_2 - 1) + 1; \\
x_{\text{dot}}_{\text{st2}} &= x_{\text{st2}} + N\text{NODE}; \\
y_{\text{st2}} &= 6^* (\text{node} \_2 - 1) + 2; \\
y_{\text{dot}}_{\text{st2}} &= y_{\text{st2}} + N\text{NODE}; \\
z_{\text{st2}} &= 6^* (\text{node} \_2 - 1) + 3; \\
z_{\text{dot}}_{\text{st2}} &= z_{\text{st2}} + N\text{NODE}; \\
\Phi X_{\text{st2}} &= 6^* (\text{node} \_2 - 1) + 4; \\
\Phi X_{\text{dot}}_{\text{st2}} &= \Phi X_{\text{st2}} + N\text{NODE}; \\
\Phi Y_{\text{st2}} &= 6^* (\text{node} \_2 - 1) + 5; \\
\Phi Y_{\text{dot}}_{\text{st2}} &= \Phi Y_{\text{st2}} + N\text{NODE}; \\
\Phi Z_{\text{st2}} &= 6^* (\text{node} \_2 - 1) + 6; \\
\Phi Z_{\text{dot}}_{\text{st2}} &= \Phi Z_{\text{st2}} + N\text{NODE}; \\
r_{\text{st2}} &= \sqrt{(z(y_{\text{st2}})^2 + 1e^{-7} + z(z_{\text{st2}})^2)}; \\
r_{\text{dot}}_{\text{st2}} &= (z(y_{\text{st2}}) \cdot z(y_{\text{dot}}_{\text{st2}}) + z(z_{\text{st2}}) \cdot z(z_{\text{dot}}_{\text{st2}}))/r_{\text{st2}}; \\
\text{theta}_{\text{st2}} &= \text{atan2}(z(z_{\text{st2}}), z(y_{\text{st2}})); \\
\omega_{\text{st2}} &= (z(z_{\text{dot}}_{\text{st2}}) \cdot z(y_{\text{st2}}) - z(y_{\text{dot}}_{\text{st2}}) \cdot z(z_{\text{st2}}))/r_{\text{st2}}^2; \\
\end{align*}\]

\[\begin{align*}
\text{Mst2 \_NODE} &= 3.3561e3; \quad \% \text{at Node} \_6 \ Y, Z (32, 33) \\
\text{Jst2 \_NODE} &= 17.8126; \quad \% \text{at Node} \_6 \ \text{PhiX} (34) \\
\text{Kphist2 \_NODE} &= 3.289e5; \quad \% \text{at Node} \_6 (34) \\
\text{ROLLINGst2} &= 0; \\
\text{SLIDINGst2} &= 0; \\
\text{Ftst2} &= 0; \\
\text{Fe}_{\text{st2}} &= 1 \cdot \text{Fe}_{\text{st}} \cdot \cos(z(\Phi X_{\text{st2}})); \quad \% \text{Phi-X \& C.G.} \\
\text{Fe}_{\text{st2}} &= 1 \cdot \text{Fe}_{\text{st}} \cdot \sin(z(\Phi X_{\text{st2}})); \quad \% \text{Phi-X \& C.G.} \\
\text{Fe}_{\text{st2}} &= \text{Fe}_{\text{st}} \cdot \text{cos(}\text{theta}_{\text{st2}}\text{)} - \text{Fe}_{\text{st}} \cdot \sin(\text{theta}_{\text{st2}}); \\
\text{node} \_db &= 7; \\
\Phi X_{\text{db}} &= 6^* (\text{node} \_db - 1) + 4; \quad \% \text{CG-PhiX} \\
\text{There is stabilizer if} \\
\text{if} \ r_{\text{st2}} > cST \quad \% \text{THERE IS CONTACT} \\
\text{Ftst2} &= \text{Kf} \cdot (x_{\text{st2}} - cST) + \text{Cf} \cdot (\text{dot} \_x_{\text{st2}}); \quad \% \text{Normal Contact force} \\
\text{mu}_{\text{st2}} &= -2/pi \cdot \text{atan}(10^4 \cdot \text{Vrel}_{\text{st2}}) \cdot (((\text{mus} \cdot \text{mu0}) \div (1 + 10^2 \cdot \text{abs(\text{Vrel}_{\text{st2}})})) \cdot \text{mu0}); \\
\text{Fe}_{\text{st2}} &= \text{mu}_{\text{st2}} \cdot \text{Ftst2}; \quad \% \text{tangential friction Contact force} \\
\text{Ftst2} &= ((\text{Mst2 \_NODE} \cdot R_{\text{st}}) \cdot \text{Jst2 \_NODE}) \cdot (-(\text{Kphist2 \_NODE}) \cdot (z(\Phi X_{\text{st2}}) - z(\Phi X_{\text{db}})) + \text{Cf} \cdot \omega_{\text{st2}} \cdot r_{\text{st2}}) / (1 + ((\text{Mst2 \_NODE} \cdot R_{\text{st}}) \cdot \text{Jst2 \_NODE})); \\
\text{if} \ \text{abs} \cdot (\text{Vrel}_{\text{st2}}) >= \text{Vmin} && \text{sign} \cdot (z(\Phi X_{\text{Dot \_st2}})) \cdot \text{sign} \cdot (\text{omega}_{\text{st2}})
SLIDINGst2=1;
% disp('------- ST2 Pure Sliding ---------------');
elseif (-mus*Fn_st2) <= Ftst2_R <= (mus*Fn_st2) && abs(Vrel_st2)<Vmin
    Ft_st2=Ftst2_R;
    ROLLINGst2=1;
    disp('----------- ST2 Pure Rolling -------');
else
    disp('---------- ST2 TRANSITION ----------');
end
else
    disp('------ ST2 nO side CONTACT -----------');
end

% NOO is stabilizer

% if r_st2>=cBH   % THERE IS CONTACT
%    Fn_st2= Kf*(r_st2 - cBH);                          % Normal Contact force
%    disp('-- Stabilizer (2) side CONTACT --------------');
%    else Fn_st2=0;
%        disp('------ NO Stabilizer (2) side CONTACT ------------');
%    end

% z/r=sin(theta_st2)
% y/r=cos(theta_st2)

Fx_st2= -Ft_st2*sin(theta_st2) - Fn_st2*cos(theta_st2) + Fey_st2; % Contact force in X-dir
Fz_st2= Ft_st2*cos(theta_st2) - Fn_st2*sin(theta_st2) + Fez_st2; % Contact force in Y-dir
Fx_st2=-sign(z(xDot_st2))*mu0*Fn_st2; % axial friction force

% There is stabilizer
Ty_st2= -sign(z(PhiYDot_st2))*mu0*Fn_st2*R_st*abs(sin(theta_st2));
Tz_st2=-sign(z(PhiZDot_st2))*mu0*Fn_st2*R_st*abs(cos(theta_st2));
Tx_st2=-sign(z(PhiXDot_st2))*Ft_st2*R_st;

% No stabilizer
Ty_st2=-sign(z(PhiYDot_st2))*mu0*Fn_st2*R_BHA*abs(sin(theta_st2));
Tz_st2=-sign(z(PhiZDot_st2))*mu0*Fn_st2*R_BHA*abs(cos(theta_st2));
Tx_st2=-sign(z(PhiXDot_st2))*Ft_st2*R_BHA;

% Adding Mass Unbalance Forces @ C.G.

node_CG=ceil(NNODE/6/2);

G C.G. (Node# 4)

x_CG=6*(node_CG-1)+1; % CG-Y
xDot_CG=x_CG+NNODE+4;
y_CG=6*(node_CG-1)+2; % CG-Y
yDot_CG=y_CG+NNODE+4;
z_CG=6*(node_CG-1)+3; % CG-Z
zDot_CG=z_CG+NNODE+4;
PhiX_CG=6*(node_CG-1)+4; % CG-PhiX
PhiXDot_CG=PhiX_CG+NNODE+4;
PhiY_CG=6*(node_CG-1)+5; % CG-PhiY
PhiYDot_CG=PhiY_CG+NNODE+4;
PhiZ_CG=6*(node_CG-1)+6; % CG-PhiZ
PhiZDot_CG=PhiZ_CG+NNODE+4;

z_CG= sqrt(((z(y_CG)^2)+1e^-7)+z(z_CG)^2);
xDot_CG= (z(y_CG)*z(yDot_CG)+z(z_CG)*z(zDot_CG))/r_CG;
theta_BHA= atan2(z(z_CG),z(y_CG));
omega_BHA= z(PhiXDot_CG)R_BHA + omega_BHA*R_CG;
Vrel_BHA= z(PhiXDot_CG)R_BHA + omega_BHA*z_CG^2;

% Adding Mass Unbalance Forces @ C.G.
% change to PhiDot @ C.G.
Fe=1*Fe*cos(PhiX_CG); % Phi-x @ C.g
Fez=1*Fe*sin(PhiX_CG); % Phi-x @ C.g
Fe_CG_t=Fez*cos(theta_BHA)-Fey*sin(theta_BHA);

Mcg_NODE= Mst2_NODE; % at Node 4 Y,Z (20,21)
Jcg_NODE= Jst2_NODE; % at Node 4 PhiX (22)
Kphicg_NODE= Kphist2_NODE; % at Node 4 (22)
ROLLINGcg=0;
SLIDINGcg=0;
FtCG_R=0;

if r_CG>=cBH % THERE IS CONTACT
   Fn_CG= 1*Kf*(r_CG-cBH)+Cf*(rDot_CG); % Normal contact force
   mu_CG= -((mus-mu0)/(1+10^2*abs(Vrel_BHA)))+mu0; % tangential friction Contact force
   Ft_CG= FtCG_R;
   ROLLINGcg=1;
   disp('---- BHA Center side CONTACT ----------');
   if abs(Vrel_BHA)>=Vmin && sign(omega_BHA) == sign(Vrel_BHA)
      SLIDINGcg=1;
      disp('----- BHA Center Pure SLiding --------------');
   elseif (-mus*Fn_CG)<= FtCG_R <=(mus*Fn_CG) && abs(Vrel_BHA)<Vmin
      Ft_CG=FtCG_R;
      disp('---------- BHA Center Pure Rolling ----');
   else
      disp('----------------- BHA Center TRANSITION ------');
   end
else % NO CONTACT
   Fn_CG=0;
   Ft_CG=0;
   FtCG_R=0;
   disp('------- BHA Center NO side CONTACT ----------');
end
% z/r=sin(theta_BHA)
y/r=cos(theta_BHA)

% Ft_CG= -mu0*Fn_CG*sign(Vrel_BHA);

Fy_BHA= Fey + (Ft_CG)*sin(theta_BHA) - Fn_CG*cos(theta_BHA)); % Contact force in X-dir
Fz_BHA= Fey + (Ft_CG)*cos(theta_BHA) - Fn_CG*sin(theta_BHA)); % Contact force in Y-dir
Fx_BHA= -sign(z(xDot_CG))*mu0*Fn_CG; % axial friction force

% DRILLBIT (Node # 7)
node_db=7;
xDot_db=x_db+NNODE+4;
yDot_db=y_db+NNODE+4;
zDot_db=z_db+NNODE+4;
Phix_db=6*(node_db-1)+4; % CG-PhiX

rdb=sqrt(((z(y_db)^2)+1e-7)+z(z_db)^2);
rDot_db=(z(y_db)*yDot_db+z(z_db)*zDot_db)/rdb;
theta_db=atan2(z(z_db),z(y_db));
omega_db=(z(y_db)*zDot_db-z(z_db)*z(yDot_db))/rdb^2;
\begin{verbatim}
Vrel_db = z(\Phi XDot_db)\cdot a + \omega_db \cdot rdb;

% Stribeck friction for DB
muf = tanh(z(\Phi XDot_db)) + 2 \cdot ((1 + 1 \cdot z(\Phi XDot_db)) \cdot z(\Phi XDot_db)) + 0.01 \cdot z(\Phi XDot_db);
mu_db = muf \cdot mu0;

dn = (z(x_db) - xtni);  % Depth of cut per revolution per blade
d = n \cdot dn;          % Depth of cut for all blades

if d <= 0
    bounce = 1;
else if z(x_db) < 0 && xtni < 0
    bounce = 1;
else
    bounce = 0;
end

if bounce == 1  % No DB cutting or friction torques or frictions
    Tf = 0;
    Tc = 0;
    Wf = 0;
    Wc = 0;
else
    Tf = a \cdot a/2 \cdot gma \cdot mu_db \cdot l \cdot sgma;
    Tc = a \cdot a/2 \cdot eps \cdot d;
    Wf = a \cdot l \cdot sgma;
    Wc = a \cdot eta \cdot eps \cdot d;
end

% disp('---------------- DRILLING :))--');

% Inclined Reaction Force from WOB
WOB_x = Wc + Wf;
WOB_y = Wc + Wf;

% disp('--------------- DRILLING :))--');

% % Lateral and radial vibrations
if rdb >= cDB  % There is contact
    Fn_db = 1 \cdot Kf \cdot (rdb - cBH) + Cf \cdot (rDot_db);
    mu = -(2/pi) \cdot atan(10^4 \cdot Vrel_db) \cdot ((\mu_s - mu0)/(1 + 10^2 \cdot abs(Vrel_db))) + mu0;
    Ft_db = mu \cdot Fn_db;
    Ft_R = ((Mdb_NODE \cdot a)/Jdb_NODE) \cdot (-Kphi_NODE \cdot (z(\Phi X_db) - z(\Phi X_st2)) - TOB -
            Ctf \cdot (\Phi XDot_db) - WOB_t + Cr \cdot omega_db \cdot rdb)/(1 + (Mdb_NODE \cdot a/Jdb_NODE) - Fe_t;
    if abs(Vrel_db) >= Vmin && sign(z(\Phi XDot_db)) == sign(omega_db)
        disp('-------- Pure SLiding --------------');
        disp('-------- Pure SLiding --------------');
    elseif (\mu_s \cdot Fn_db) <= Ft_R <= (\mu_s \cdot Fn_db) && abs(Vrel_db) < Vmin
        Ft_db = Ft_R;
        disp('-------- Pure Rolling -------');
    else
        disp('-------- Pure ROLLING TRANSITION ------');
    end
else
    disp('-------- Pure ROLLING TRANSITION ------');
end

end
\end{verbatim}
\begin{verbatim}
Ft_db=0;
\% disp('-- NO CONTACT ------------------');
end

FnY_db= -Fn_db*cos(theta_db); \% Norma(radial) force from polar to cartisian
FnZ_db= -Fn_db*sin(theta_db); \% Tangential force from polar to cartisian
\% Ft_db= mu0*Fn_db*sign(Vrel_db);
FtY_db= Ft_db*sin(theta_db); \% Radial force
FtZ_db= -Ft_db*cos(theta_db); \% Tangential force

Fy_db= WOB_y + FnY_db + FtY_db +Fey_db;
Fz_db= WOB_z + FnZ_db + FtZ_db +Fez_db;

Tb=Ft_db*a; \% Friction Torque on Rotor
\% Tb=0;

\% Nodal Assignment of Forces Vector
F=zeros(NNODE+4,1);
\% @ BHA Top node @ ST1
F(1,1)= Wo + Fx_st1;
F(2,1)= Fy_st1;
F(3,1)= Fz_st1;
F(4,1)= kT_dp*y*t + Tx_st1;
F(5,1)= Ty_st1;
F(6,1)= Tz_st1;
\% @ BHA C.G.
F(x_CG,1)= Fx_BHA;
F(y_CG,1)= Fy_BHA;
F(z_CG,1)= Fz_BHA;
F(PhiX_CG,1)= Tx_BHA;
F(PhiY_CG,1)= Ty_BHA;
F(PhiZ_CG,1)= Tz_BHA;
\% @ Node before last one @ ST2
F(x_st2,1)= Fx_st2;
F(y_st2,1)= Fy_st2;
F(z_st2,1)= Fz_st2;
F(PhiX_st2,1)= Tx_st2;
F(PhiY_st2,1)= Ty_st2;
F(PhiZ_st2,1)= Tz_st2;
\% @ BHA Bottom Node @ DB
F(NNODE-2,1)= -TOB +Tb;
F(NNODE-3,1)= Fz_db;
F(NNODE-4,1)= Fy_db;
F(NNODE-5,1)= -WOB;

zdot=A0*z+B0*F;
\end{verbatim}