THREE-POINT DISC AMPLITUDES AND HIGHER DERIVATIVE COUPLINGS

A Dissertation

by

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ABSTRACT

This dissertation discusses the string disc amplitudes and D-brane higher derivative couplings. In the first part, we investigate all tree-level string theory vacuum to D-brane disc amplitudes involving an arbitrary RR-state and two NS-NS vertex operators. Our method is that, we develop an algorithm to derive the amplitudes in a manifestly gauge invariant and exchange invariant form, then we implement the algorithm on computer to systematically solve the amplitudes. In the second part, we derive the four-derivative brane interactions for one R-R field $C^{(p+5)}$ and two NS-NS B-fields based on the amplitude from the first part. We find the effective coupling actually vanishes, which implies the interaction obeys a non-renormalization theorem.

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1. INTRODUCTION*

D-branes have played many roles in string theory. From the point of view of the string world-sheet they are simply boundary conditions, i.e. strings can end on D-branes. In practice, this means that if we compute string scattering amplitudes in a background with D-branes (including the type I string, which in this language is interpreted to have space-time-filling D9-branes), then we must include contributions from world-sheets with boundaries, in addition to the usual closed world-sheets.

Alternatively, from the point of view of the low-energy effective theory, D-branes host some degrees of freedom that are localized on the D-brane world-volume. In this paper, we will only be considering separated D-branes, in which case the content of the world-volume theory is simply that of maximally supersymmetric super-Yang-Mills with gauge group U(1) for each D-brane. The full effective action is then a sum of a bulk action plus localized actions at each D-brane. These localized actions involve both the world-volume fields and the bulk fields, and can be expanded in derivatives. The details of that expansion are interesting in their own right as an example of an effective theory that admits many different dual perspectives. But even more compellingly, there are examples in which the higher derivative couplings localized on D-branes play an essential role in determining the vacuum structure of string theory, such as in the F-theory duals of M-theory backgrounds on Calabi-Yau four-folds [1, 2]. In the IIB description of these constructions, there are D7-branes which wrap four-cycles of the internal space. These D7-branes host four-derivative

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bulk-field couplings of the schematic form

$$\int_{D7} C^{(4)} \wedge \mathrm{tr}(R \wedge R), \tag{1.1}$$

which lead to the $C^{(4)}$ tadpole equation. This condition is crucial to get consistent solutions. Similarly, there should be more four-derivative couplings which can contribute to charge cancellation in certain other flux backgrounds [3, 4, 5]. For these reasons it is important to systematically compute the entire four-derivative effective action localized on a D-brane.

Of course, the world-sheet and effective theory perspectives are related. The terms in the effective action can be computed by the relevant perturbative string scattering amplitudes. For example, the coupling (1.1) can be obtained by computing a three-point disc amplitude with one R-R vertex operator and two graviton vertex operators [6, 7, 8, 9] (though there are other methods for deducing these particular couplings [10, 11, 12, 13, 14]). As a preliminary step towards computing the full effective action, we need to compute all of the relevant string scattering amplitudes, as we do herein.

We calculate type II superstring scattering amplitudes on world-sheets with the topology of a disk, with closed or open string insertions. We are following references [15], [16], and [3], where the formalism was developed and some simple amplitudes were computed. Similarly as done in these references, the final goal is to extract information about the corresponding Dp-brane effective actions.

The calculation of the two-point function involving one R-R state and one NS-NS state appeared in earlier papers [17, 18, 19, 20, 21] or in the notation and conventions used herein in [15]. The three-point amplitude involving one R-R field of type $C^{(p-3)}$ was calculated in [16, 22], and some pieces for $C^{(p+5)}$ appeared in [22]. Our goal here is to compute the most general tree level string theory vacuum to Dp-brane amplitude with insertion of an arbitrary R-R state and various NS-NS vertex operators. We then restrict to the case of one R-R field and two NS-NS fields. This amplitude is expressed in terms of the R-R potential $C^{(p+1+2k)}$ and two NS-NS fields.

Because the amplitudes are invariant under a certain $\mathbb{Z}_2$ symmetry (combining reflection in the space-time directions that are normal to the brane with worldsheet parity), they are non-vanishing only if

1. k is even and both NS-NS fields are antisymmetric or both are symmetric.
2. k is odd and one of the NS-NS fields is symmetric and the other one is antisymmetric.

The collection of these amplitudes is $C^{(p+5)}BB$, $C^{(p+5)}hh$, $C^{(p+3)}Bh$, $C^{(p+1)}BB$, $C^{(p+1)}hh$, $C^{(p-1)}Bh$, $C^{(p-3)}BB$, $C^{(p-3)}hh$. In principle, we could have $C^{(p+7)}$, $C^{(p-5)}$, $C^{(p-7)}$ as well, but in our calculation we see those amplitudes actually vanish.

In general, the coefficients of the amplitudes cannot be evaluated analytically, so we write them in a complex integral form. Due to conservation of momentum and integration by parts, the integrals can be written in many different ways. We determine a minimum set of variables and express the final result in terms of them. Our choice makes the exchange symmetry of the two NS-NS fields apparent. This is a nontrivial check of our result, since at the level of the vertex operators this symmetry is not manifest.

2. THREE-POINT STRING DISC AMPLITUDES

2.1 Calculational Tools and Strategy

To calculate the expression of the three-point function we are interested in we follow the notation and conventions presented in [15].

2.1.1 Manipulating the region of integration

We construct the n-point correlator as in eqn. 3.1 of [16],

$$\left\langle 0 \left| V_1(z_1,\bar{z}_1)V_2(z_2,\bar{z}_2)\left(\prod_{k=3}^{n}\int_{\mathbb{C}} d^2z_k U_k(z_k,\bar{z}_k)\right)\left(b_0+\tilde{b}_0\right)\right.\right.$$
$$\left.\left. \times \int_{|w|>\max(1/|z_i|)} \frac{d^2w}{|w|^2} w^{-L_0}\bar{w}^{-\tilde{L}_0}\right| B\right\rangle, \quad (2.1)$$

where $U(z,\bar{z})$ is the integrated vertex operator defined as

$$U(z,\bar{z}) = \left\{\tilde{b}_{-1},[b_{-1},V(z,\bar{z})]\right\}, \quad (2.2)$$

and $V(z,\bar{z})$ are closed string vertex operators, whose explicit form is presented later in this section. Also, $|B\rangle$ is a boundary state which encodes the D-brane boundary conditions. We would like to manipulate this expression to eliminate the explicit factor of $w^{-L_0}\bar{w}^{-\tilde{L}_0}$ and convert the integration region to something easier to work with.

In practice, all vertex operators have the form $V(z,\bar{z}) = V(z)\tilde{V}(\bar{z})$ (or a linear combination of such terms) with $V = cU + \eta W$, leading to $U(z,\bar{z}) = U\tilde{U}$. Now we show the general correlator defined in (2.1) can be further simplified. We first use

$$w^{L_0}\bar{w}^{\tilde{L}_0}\mathcal{O}(z,\bar{z})w^{-L_0}\bar{w}^{-\tilde{L}_0} = w^h\bar{w}^{\tilde{h}}\mathcal{O}(zw,\bar{z}\bar{w}), \tag{2.3}$$

for any conformal primary operator of weight $(h,\tilde{h})$, to pull the propagator to the left. Then the correlator can be written as

$$\int_{|w|>\max(1/|z_i|)} d^2w|w|^{2n-6}\left\langle 0\middle| V_1(wz_1,\bar{w}\bar{z}_1)V_2(wz_2,\bar{w}\bar{z}_2)\right.$$
$$\left.\times\left(\prod_{k=3}^{n}\int_{\mathbb{C}} d^2z_kU_k(wz_k,\bar{w}\bar{z}_k)\right)\left(b_0+\tilde{b}_0\right)\middle| B\right\rangle. \tag{2.4}$$

We can use conformal symmetry to set $z_1=\infty$, so it will not affect the $|w|>\max(1/|z_i|)$ condition. Then we can write $\prod_{k=3}^{n}\int_{\mathbb{C}} d^2z_k = \sum_{\alpha=2}^{n}\int_{V_\alpha}\prod_{k=3}^{n} d^2z_k$, where $\int_{V_\alpha}$ denotes the region where $z_\alpha = \min(\{z_i\})$. Also use $\theta_{|w|>\max(1/|z_i|)}$ to denote the Heaviside function $\theta(|w|-\max(1/|z_i|))$. Then we can rewrite the integration $\int_{|w|>\max(1/|z_i|)} d^2w$ as $\int_{\mathbb{C}} d^2w\theta_{|w|>\max(1/|z_i|)}$. The correlator becomes

$$\sum_{\alpha}\int_{\mathbb{C}} d^2w\,\theta_{|w|>1/|z_\alpha|}\,\theta_{|z_\alpha|=\min(|z_i|)}\,|w|^{2n-6}\left\langle 0\middle| V_1(\infty,\infty)V_2(wz_2,\bar{w}\bar{z}_2)\right.$$
$$\left.\times\left(\int_{\mathbb{C}}\prod_{k=3}^{n} d^2z_kU_k(wz_k,w\bar{z}_k)\right)\left(b_0+\tilde{b}_0\right)\middle| B\right\rangle. \tag{2.5}$$

Next we can rescale the coordinates as $z_2'=wz_2$, $z_k'=wz_k$,

$$\sum_{\alpha}\int_{\mathbb{C}}\frac{d^2z_2'}{|z_2'|^2}\,\theta_{|z_\alpha'|>1}\,\theta_{|z_\alpha'|=\min(|z_i'|)}\left\langle 0\middle| V_1(\infty,\infty)V_2(w',\bar{w}')\right.$$
$$\left.\times\left(\int_{\mathbb{C}}\prod_{k=3}^{n} d^2z_k'U_k(z_k',\bar{z}_k')\right)\left(b_0+\tilde{b}_0\right)\middle| B\right\rangle. \tag{2.6}$$

On the other hand, we can conveniently rewrite the Heaviside function as

$$\sum_{\alpha} \theta_{|z'_\alpha|>1} \theta_{|z'_\alpha|=\min(|z'_i|,w')} = \prod_i \theta_{|z'_i|>1}, \tag{2.7}$$

leading to the expression

$$\int_{|z_2|>1} \frac{d^2 z_2}{|z_2|^2} \left\langle 0 \left| V_1(\infty,\infty) V_2(z_2,\bar{z}_2) \left(\prod_{k=3}^{n} \int_{|z_k|>1} d^2 z_k U_k(z_k,\bar{z}_k) \right) \left(b_0 + \tilde{b}_0\right) \right| B \right\rangle, \tag{2.8}$$

where we renamed the dummy variable. This is the formula we use to calculate the three point amplitudes.

We can choose the picture charge for each vertex operator to an arbitrary value, as long as the total picture charge equals -2. The amplitude is independent on how these charges are precisely distributed. See section 4 of [15] for a detailed discussion on these issues. We choose $(-1/2,-1/2), (-1,0)$ for first two vertex operators respectively, then $(0,0)$ for the last.

Let us now evaluate the amplitude with one R-R and two NS-NS vertex operators

$$\int_{|z_2|>1} \int_{|z_3|>1} \frac{d^2 z_2 d^2 z_3}{|z_2|^2} \left\langle 0 \left| V_{-\frac{1}{2},-\frac{1}{2}}(\infty,\infty) V_{-1,0}(z_2,\bar{z}_2) U_{0,0}(z_3,\bar{z}_3) \left(b_0 + \tilde{b}_0\right) \right| B \right\rangle. \tag{2.9}$$

Vertex operators for different picture charges appear in [15]. In particular for this case we use

$$\begin{aligned}
V_{-\frac{1}{2},-\frac{1}{2}} &= f_{AB} : c\tilde{c} e^{-\frac{1}{2}\phi} S^A e^{-\frac{1}{2}\tilde{\phi}} \tilde{S}^B e^{ip_1 X} :, \\
V_{-1,0} &= \epsilon_{2\,\mu\nu} : c\tilde{c} e^{-\phi} \psi^\mu \left(\bar{\partial} X^\nu - i p_2^\rho \tilde{\psi}_\rho \tilde{\psi}^\nu \right) e^{ip_2 X} : \\
&\quad -\frac{1}{2} : c e^{-\phi} e^{\tilde{\phi}} \tilde{\eta} \psi^\mu \tilde{\psi}^\nu e^{ip_2 X} :, \\
U_{0,0} &= \epsilon_{3\,\mu\nu} : (\partial X^\mu - i p_3^\rho \psi_\rho \psi^\mu) \left(\bar{\partial} X^\nu - i p_3^\sigma \tilde{\psi}_\sigma \tilde{\psi}^\nu \right) e^{ip_3 X} : .
\end{aligned} \tag{2.10}$$

Here ϵ_2 and ϵ_3 are the polarizations for the two NS-NS states, while

$$f_{AB} = \left(\mathcal{C} \sum_n \frac{1}{n!} F^{(n)}_{\mu_1 \cdots \mu_n} \Gamma^{\mu_1 \cdots \mu_n} \right)_{AB} , \tag{2.11}$$

with $\mathcal{C}_{AB}$ being the antisymmetric charge conjugation matrix for the Clifford algebra of $(\Gamma^\mu)^A{}_B$. In our amplitude we can also drop the second line in $V_{-1,0}$; it is only included in order to ensure that the vertex operator is BRST-invariant, but it doesn't have the right charges to contribute to this amplitude.

We will also use that

$$\lim_{z_1 \to \infty} \left\langle 0 \right| : c\widetilde{c} e^{-\frac{1}{2}\phi} S^A e^{-\frac{1}{2}\widetilde{\phi}} \widetilde{S}^B(z_1, \bar{z}_1) :: c\widetilde{c} e^{-\phi}(z_2, \bar{z}_2) : \; = \left\langle A, B \right| z_2^{\frac{1}{2}} \bar{z}_2 \;, \tag{2.12}$$

where A and B are spinor indices and $\langle A, B|$ is the corresponding R-R vacuum.

After plugging in the vertex operators, we can separate the correlator into each sector (boson, fermion, bc, and ϕ sectors) and do all possible Wick contractions. Note that we have already dealt with the ghosts by the equation (2.12). This leaves only the matter fields ψ and X. Here we review the evaluation of each sector following [15].

ψ sector:

Each explicit $\widetilde{\psi}(\bar{z})$ is brought to the right and converted into a $\psi(\bar{z}^{-1})$ using the boundary state $|B\rangle$, via

$$\widetilde{\psi}^\mu(\bar{z}) \, |B\rangle = -i \bar{z}^{-1} D^\mu{}_\nu \psi^\nu(\bar{z}^{-1}) \, |B\rangle \, . \tag{2.13}$$

This gives

$$f_{AB}\left\langle A,B\left|\psi^{\mu_1}(z_1)\cdots\psi^{\mu_n}(z_n)\right|B\right\rangle=(-1)^{n+1}\,2^{-\frac{n}{2}}\,(z_1\cdots z_n)^{-\frac{1}{2}}$$
$$\times\left\{T^{\mu_1\cdots\mu_n}+\frac{z_1+z_2}{z_1-z_2}\eta^{\mu_1\mu_2}T^{\mu_3\cdots\mu_n}+\cdots\right.$$
$$\left.+\frac{z_1+z_2}{z_1-z_2}\frac{z_3+z_4}{z_3-z_4}\eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4}T^{\mu_5\cdots\mu_n}+\cdots\right\},\quad(2.14)$$

where $\cdots$ represents all the possible contractions, keeping track of appropriate signs from anticommuting the ψ's or Γ's, and where the objects $T^{\mu_1\cdots\mu_n}$ are given by

$$T^{a_1\cdots a_k i_1\cdots i_\ell}=(-1)^{\frac{1}{2}\left(p^2+p+k^2+k\right)+p\ell+1}\,\frac{32}{(p+1-k)!}\epsilon^{a_1\cdots a_k}{}_{b_1\cdots b_{p+1-k}}F^{b_1\cdots b_{p+1-k}i_1\cdots i_\ell}.\quad(2.15)$$

Recall that we use notation where a, b, etc. represent directions along the D-brane, while i, j, etc. are normal to the D-brane.

X **sector:**

For the bosons, we again use the boundary state to convert anti-holomorphic operators to holomorphic ones. Indeed, if we split the exponential into left- and right-moving parts[1],

$$:e^{ipX(z,\bar{z})}:=:e^{ipX_L(z)}e^{ipX_R(\bar{z})}:,\quad(2.16)$$

then we can use

$$e^{ipX_R(\bar{z})}\left|B\right\rangle=e^{ipDX_L(\bar{z}^{-1})}\left|B\right\rangle.\quad(2.17)$$

[1]This neglects the zero-mode, but the zero-mode piece is correctly accounted for in the full correlators (2.18) and (2.20).

Then for a correlator with only exponentials, we have

$$\left\langle 0 \left|: e^{ip_1 X(z_1,\bar{z}_1)} : \cdots : e^{ip_n X(z_n,\bar{z}_n)} :\right| B \right\rangle = (2\pi)^{p+1}\, \delta^{p+1}(\frac{1}{2}\,(1+D)\sum_{i=1}^{n} p_i)$$
$$\times \prod_{k=1}^{n} \left(|z_k|^2 - 1\right)^{p_k D p_k} \prod_{1\le \ell < m \le n} |z_\ell - z_m|^{2p_\ell p_m}\, |z_\ell \bar{z}_m - 1|^{2p_\ell D p_m}\,. \quad (2.18)$$

Similarly, if we have explicit factors of $\bar{\partial} X(\bar{z})$, we use

$$\overline{\partial} X^\mu(\bar{z})\, |B\rangle = -\bar{z}^{-2} D^\mu{}_\nu \partial X^\nu(\bar{z}^{-1})\, |B\rangle\,, \quad (2.19)$$

to convert them to holomorphic operators. Then for a correlator that involves these as well, we have for example

$$\left\langle 0 \left|: e^{ip_1 X(z_1,\bar{z}_1)} : \cdots : e^{ip_{n-1} X(z_{n-1},\bar{z}_{n-1})} :: \partial X^\mu(z_n) e^{ip_n X(z_n,\bar{z}_n)} :\right| B \right\rangle$$
$$= \left\langle 0 \left|: e^{ip_1 X(z_1,\bar{z}_1)} : \cdots : e^{ip_n X(z_n,\bar{z}_n)} :\right| B \right\rangle$$
$$\times \left(\frac{ip_1}{z_1 - z_n} + \cdots + \frac{ip_{n-1}}{z_{n-1} - z_n} - \frac{i\bar{z}_1 D p_1}{z_n \bar{z}_1 - 1} - \cdots - \frac{i\bar{z}_n D p_n}{|z_n|^2 - 1} \right)^\mu . \quad (2.20)$$

If there is more than one $\partial X(z)$, then we must also include in the usual way terms where they contract with each other.

2.1.2 The integrand of the amplitudes

After evaluating each sector, we see all the integrands can be spanned by the following set of integrals

$$I_{a,b,c,d,e,f} = \int_{|z_i|\le 1} d^2 z_2 d^2 z_3 \widetilde{\mathcal{K}}\mathcal{K}, \quad (2.21)$$

where

$$\widetilde{\mathcal{K}} = |z_2|^{2a} |z_3|^{2b} \left(1 - |z_2|^2\right)^c \left(1 - |z_3|^2\right)^d |z_2 - z_3|^{2e} |1 - z_2 \bar{z}_3|^{2f} , \tag{2.22}$$

$$\mathcal{K} = |z_2|^{2p_1p_2} |z_3|^{2p_1p_3} \left(1 - |z_2|^2\right)^{p_2Dp_2} \left(1 - |z_3|^2\right)^{p_3Dp_3} \times |z_2 - z_3|^{2p_2p_3} |1 - z_2\bar{z}_3|^{2p_2Dp_3} . \tag{2.23}$$

The matrix $D_{\mu\nu}$ differs for the directions tangent to the brane, denoted with indices a, b, c, etc., and directions transverse to the brane, with indices i, j, k, etc. Explicitly,

$$D_{ab} = \eta_{ab}, \qquad D_{ai} = D_{ia} = 0, \qquad D_{ij} = -\delta_{ij}. \tag{2.24}$$

When writing the result in terms of $I_{a,b,c,d,e,f}$, we still do not see the manifest exchange symmetry under $2 \leftrightarrow 3$. To observe this symmetry, i.e. to show amplitudes with its image under $2 \leftrightarrow 3$ exchanged are the same, we need to go to a minimal set of integrals. For this, we must understand two sets of relations - first the coefficients of the integrals enjoy identities following from conservation of momentum and on-shell conditions, and second there are relations among the $I_{a,b,c,d,e,f}$ themselves that follow from integration by parts.

We now derive the second type of conditions. If we write a polar decomposition $z_i = r_i e^{i\phi_i}$, then we note that the integrand $\widetilde{\mathcal{K}}\mathcal{K}$ depends on r_2, r_3, and the average angle $\frac{1}{2}(\phi_2 + \phi_3)$, but is independent of the relative angle $\phi_2 - \phi_3$. Therefore we expect three relations from integration by parts.

The integration by parts from $\int \frac{\partial}{\partial \bar{z}_2}\frac{\partial}{\partial z_2}\widetilde{\mathcal{K}}\mathcal{K}=0$ (i.e. from the r_2 integration) is

$$\begin{aligned}0 =& 2\left(p_1p_2+a+1\right) I_{a,b,c,d,e,f} - 2\left(p_2Dp_2+c\right) I_{a+1,b,c-1,d,e,f} \\ &+ \left(p_2p_3+e\right)\left(I_{a,b,c,d,e,f}+I_{a+1,b,c,d,e-1,f}-I_{a,b+1,c,d,e-1,f}\right) \\ &+ \left(p_2Dp_3+f\right)\left(I_{a,b,c,d,e,f}-I_{a,b,c,d,e,f-1}+I_{a+1,b+1,c,d,e,f-1}\right).\end{aligned} \tag{2.25}$$

Similarly from $\int \frac{\partial}{\partial \bar{z}_3}\frac{\partial}{\partial z_3}\widetilde{\mathcal{K}}\mathcal{K}=0$ (the r_3 integration) we have

$$\begin{aligned}0 =& 2\left(p_1p_3+b+1\right) I_{a,b,c,d,e,f} - 2\left(p_3Dp_3+d\right) I_{a,b+1,c,d-1,e,f} \\ &+ \left(p_2p_3+e\right)\left(I_{a,b,c,d,e,f}+I_{a,b+1,c,d,e-1,f}-I_{a+1,b,c,d,e-1,f}\right) \\ &+ \left(p_2Dp_3+f\right)\left(I_{a,b,c,d,e,f}-I_{a,b,c,d,e,f-1}+I_{a+1,b+1,c,d,e,f-1}\right).\end{aligned} \tag{2.26}$$

Finally we have a relation from $\int (\frac{\partial}{\partial \bar{z}_3}\frac{\partial}{\partial z_2}+\frac{\partial}{\partial \bar{z}_2}\frac{\partial}{\partial z_3})\widetilde{\mathcal{K}}\mathcal{K}=0$ (which corresponds to the non-trivial angular integration)[2]

$$\begin{aligned}0 =& \left(p_2Dp_3+f\right) I_{a,b,c,d,e+1,f}\left(-\bar{z}_3z_2+\bar{z}_2z_3\right)^2 \\ &+ \left(p_2p_3+e\right) I_{a,b,c,d,e,f+1}\left(-\bar{z}_3z_2+\bar{z}_2z_3\right)^2 + I_{a,b,c,d,e,f}\,\mathcal{Z}_{e,f},\end{aligned} \tag{2.27}$$

where

$$\begin{aligned}\mathcal{Z}_{e,f} =& -\bar{z}_2\bar{z}_3z_2^2+2\bar{z}_3^2z_2^2+2\bar{z}_2\bar{z}_3^2z_2^3-3\bar{z}_3^3z_2^3-\bar{z}_2^2z_2z_3-\bar{z}_3^2z_2z_3-\bar{z}_2\bar{z}_3^2z_2^2z_3+2\bar{z}_3^3z_2^2z_3 \\ &-\bar{z}_2^2\bar{z}_3^2z_2^3z_3+2\bar{z}_2\bar{z}_3^3z_2^3z_3+2\bar{z}_2^2z_3^2-\bar{z}_2\bar{z}_3z_3^2+2\bar{z}_2^3z_2z_3^2-\bar{z}_2^2\bar{z}_3z_2z_3^2-\bar{z}_2^3\bar{z}_3z_2^2z_3^2 \\ &-\bar{z}_2\bar{z}_3^3z_2^2z_3^2-3\bar{z}_2^3z_3^3+2\bar{z}_2^2\bar{z}_3z_3^3+2\bar{z}_2^3\bar{z}_3z_2z_3^3-\bar{z}_2^2\bar{z}_3^2z_2z_3^3.\end{aligned} \tag{2.28}$$

Next we construct a minimal basis for the integrands using the following strategy:

[2] Here we already used relations (2.25) and (2.26) to eliminate p_1p_2 and p_1p_3.

1. We observe that the amplitudes are real. If the integrand is not real, we know that the imaginary part must vanish upon integration, so we can delete the imaginary part without changing the integration. Our integrand can then be written in terms of the integrals $I_{a,b,c,d,e,f}$.

2. Conservation of momentum in the presence of the brane (which comes from evaluating the zero mode part of the boson sector of the correlator) implies

$$0 = p_1 + Dp_1 + p_2 + Dp_2 + p_3 + Dp_3. \tag{2.29}$$

   We use this to eliminate $(p_1)_a$. Whenever p_1 appears, it should be contracted with only normal indices. For example we do allow $p_1 N p_2$ where we define $N = \frac{1-D}{2}$ for contraction of normal indices ($N_{ij} = \delta_{ij}$, all other entries zero), but not $p_1 p_2$.

3. We use the relations (2.25) and (2.26) for $I_{a,b,c,d,e,f}$ to eliminate $p_2 D p_2$ and $p_3 D p_3$, and if any factor of the following form

$$f I_{a,b,c,d,e+1,f} \left(-\bar{z}_3 z_2 + \bar{z}_2 z_3\right)^2 + e I_{a,b,c,d,e,f+1} \left(-\bar{z}_3 z_2 + \bar{z}_2 z_3\right)^2 + I_{a,b,c,d,e,f} \mathcal{Z}_{e,f}, \tag{2.30}$$

   appears, we use relation (2.27) to rewrite it in terms of $p_2 D p_3$, $p_2 p_3$.

2.2 The Amplitudes

Obtaining the concrete expressions for the amplitudes is rather challenging, so we used the aid of a computer, to evaluate the contractions and reducing the integrals to a unique form using the procedure explained above.

For $C^{(p-3)}$ and $C^{(p+5)}$, we have verified that the result agrees with the computations of [16, 22]. For $C^{(p-1)}$, $C^{(p+1)}$, and $C^{(p+3)}$ one can predict pieces of the amplitude using linearized T-duality [3], or linearized T-duality combined with gauge invariance [22, 23, 24]. For $C^{(p-1)}$ and $C^{(p+1)}$, those computations didn't give the full amplitudes, so a direct comparison with our results is difficult. Terms get mixed around as we perform our minimal basis procedure. For $C^{(p+3)}$ our result matches [24].

Finally, for all cases we have written the result in a way that makes the symmetry under exchange of the two NS-NS fields manifest. This is a non-trivial check of the results, since the computation treats the two operators on unequal footing (since they are in different pictures and one operator is in integrated form, while the other one is not).

Since the results are long and elaborate, we list them below without further comments. Each result can be split into pieces according to the number of indices of the R-R polarization $C^{(n)}_{\mu_1\cdots\mu_n}$ which are contracted with the world-volume epsilon tensor $\varepsilon_{a_1\cdots a_{p+1}}$. The remaining indices are contracted with linear combinations of the NS-NS polarizations $\epsilon_{2\,\mu\nu}$ and $\epsilon_{3\,\mu\nu}$ and the three momenta p_1, p_2, and p_3. Finally, each term in this linear combination multiplies a scalar integral I_k, $k = 0, \cdots, 24$ (or I'_k, which is obtained from I_k by interchanging z_2 with z_3 in the integrand) and these combinations are defined in section 2.5.

2.2.1 $C^{(p+5)}$ amplitudes

$$\mathcal{A}_{C^{(p+5)}BB} = \mathcal{A}^{(4)}_{C^{(p+5)}BB} + \mathcal{A}^{(5)}_{C^{(p+5)}BB} + \mathcal{A}^{(6)}_{C^{(p+5)}BB}. \tag{2.31}$$

$$\begin{aligned}
\mathcal{A}^{(4)}_{C^{(p+5)}BB} =& \frac{2i^{p(p+1)}\sqrt{2}}{(p+1)!}\varepsilon^{b_1\ldots b_{p+1}} C^{ijkl}{}_{b_1\ldots b_{p+1}} \big(4p_{2i}p_{3j}(p_1N\epsilon_2)_k(p_2\epsilon_3)_l I_9 \\
& +4p_{2k}p_{3i}(p_1N\epsilon_2)_l(p_1N\epsilon_3)_j I_{10} - 4p_{2k}p_{3i}(p_1N\epsilon_2)_l(p_2D\epsilon_3)_j I_5 \\
& -2\left(p_1N\epsilon_3p_2\right)p_{2i}p_{3j}\epsilon_{2kl}I_9 - 2\left(p_1N\epsilon_3Dp_2\right)p_{2i}p_{3j}\epsilon_{2kl}I_5 \\
& -2\left(p_1Np_2\right)p_{3j}(p_2\epsilon_3)_i\epsilon_{2kl}I_9 - 2\left(p_1Np_3\right)p_{2i}(p_2\epsilon_3)_j\epsilon_{2kl}I_9 \\
& +2\left(p_2p_3\right)p_{3j}(p_1N\epsilon_3)_i\epsilon_{2kl}I_9 + 4\left(p_1Np_2\right)p_{3j}(p_1N\epsilon_3)_i\epsilon_{2kl}I_{10} \\
& -2\left(p_2Dp_3\right)p_{3j}(p_1N\epsilon_3)_i\epsilon_{2kl}I_5 + 2\left(p_2p_3\right)p_{2i}(p_1N\epsilon_3)_j\epsilon_{2kl}I_9 \\
& +2\left(p_2Dp_3\right)p_{2i}(p_1N\epsilon_3)_j\epsilon_{2kl}I_5 - 2\left(p_1Np_2\right)p_{3j}(p_2D\epsilon_3)_i\epsilon_{2kl}I_5 \\
& -2\left(p_1Np_3\right)p_{2i}(p_2D\epsilon_3)_j\epsilon_{2kl}I_5 + \left(p_1Np_2\right)\left(p_2p_3\right)\epsilon_{2jk}\epsilon_{3li}I_9 \\
& -\left(p_1Np_2\right)\left(p_1Np_3\right)\epsilon_{2jk}\epsilon_{3li}I_{10} + \left(p_1Np_2\right)\left(p_2Dp_3\right)\epsilon_{2jk}\epsilon_{3li}I_5\big) \\
& + (2\leftrightarrow 3),
\end{aligned} \tag{2.32}$$

$$\begin{aligned}
\mathcal{A}^{(5)}_{C^{(p+5)}BB} =& \frac{2i^{p(p+1)}\sqrt{2}}{p!} C^{ijklm}{}_{b_1\ldots b_p}\varepsilon^{ab_1\ldots b_p} \big(2p_{2a}p_{2i}p_{3j}(p_2\epsilon_3)_m\epsilon_{2kl}I_9 \\
& +2p_{2i}p_{3a}p_{3j}(p_2\epsilon_3)_m\epsilon_{2kl}I_9 - 4p_{2a}p_{2i}p_{3j}(p_1N\epsilon_3)_m\epsilon_{2kl}I_{10} \\
& +2p_{2a}p_{2i}p_{3j}(p_2D\epsilon_3)_m\epsilon_{2kl}I_5 + 2p_{2i}p_{3a}p_{3j}(p_2D\epsilon_3)_m\epsilon_{2kl}I_5 \\
& -\left(p_2p_3\right)p_{2a}p_{2i}\epsilon_{2jk}\epsilon_{3lm}I_9 + 2\left(p_1Np_3\right)p_{2a}p_{2i}\epsilon_{2jk}\epsilon_{3lm}I_{10} \\
& -\left(p_2Dp_3\right)p_{2a}p_{2i}\epsilon_{2jk}\epsilon_{3lm}I_5 + \left(p_2p_3\right)p_{2a}p_{3i}\epsilon_{2jk}\epsilon_{3lm}I_9 \\
& -\left(p_2Dp_3\right)p_{2a}p_{3i}\epsilon_{2jk}\epsilon_{3lm}I_5\big) + (2\leftrightarrow 3),
\end{aligned} \tag{2.33}$$

$$\mathcal{A}^{(6)}_{C^{(p+5)}BB} = \frac{2i^{p(p+1)}\sqrt{2}}{(p-1)!} C^{ijklmn}{}_{b_1\ldots b_{p-1}} \varepsilon^{abb_1\ldots b_{p-1}} p_{2b}p_{2j}p_{3a}p_{3i}\epsilon_{2kl}\epsilon_{3mn}I_{10} + (2\leftrightarrow 3). \tag{2.34}$$

2.2.2 $C^{(p+3)}$ amplitudes

$$\mathcal{A}_{C^{(p+3)}Bh} = \mathcal{A}^{(2)}_{C^{(p+3)}Bh} + \mathcal{A}^{(3)}_{C^{(p+3)}Bh} + \mathcal{A}^{(4)}_{C^{(p+3)}Bh} + \mathcal{A}^{(5)}_{C^{(p+3)}Bh}. \tag{2.35}$$

$$\begin{aligned}
\mathcal{A}^{(2)}_{C^{(p+3)}Bh} =& \frac{2i^{p(p+1)}\sqrt{2}}{(p+1)!} \varepsilon^{b_1\ldots b_{p+1}} C^{ij}{}_{b_1\ldots b_{p+1}} (\\
& - 2\,(p_1N\epsilon_2\epsilon_3p_2)\, p_{2_j}p_{3_i}I_2 - 2\,(p_1N\epsilon_3\epsilon_2p_3)\, p_{2_j}p_{3_i}I_2 \\
& + 2\,(p_1N\epsilon_2D\epsilon_3p_2)\, p_{2_j}p_{3_i}I_{11} - 2\,(p_1N\epsilon_2\epsilon_3Dp_2)\, p_{2_j}p_{3_i}I_{11} \\
& + 4\,(p_1N\epsilon_2\epsilon_3Np_1)\, p_{2_j}p_{3_i}I_9 - 2\,(p_1N\epsilon_3D\epsilon_2p_3)\, p_{2_j}p_{3_i}I'_{11} \\
& - 2\,(p_1N\epsilon_3\epsilon_2Dp_3)\, p_{2_j}p_{3_i}I'_{11} + 2\,(p_1N\epsilon_2D\epsilon_3Dp_2)\, p_{2_j}p_{3_i}I_1 \\
& - 4\,(p_1N\epsilon_2D\epsilon_3Np_1)\, p_{2_j}p_{3_i}I_5 - 2\,(p_1N\epsilon_3D\epsilon_2Dp_3)\, p_{2_j}p_{3_i}I_1 \\
& - 2\,\mathrm{tr}(D\epsilon_3)\,(p_1N\epsilon_2p_3)\, p_{2_j}p_{3_i}I'_6 - 2\,\mathrm{tr}(D\epsilon_3)\,(p_1N\epsilon_2Dp_3)\, p_{2_j}p_{3_i}I'_7 \\
& + 2\,(p_1N\epsilon_2p_3)\, p_{2_j}(p_2\epsilon_3)_iI_2 - 2\,(p_1N\epsilon_2Dp_3)\, p_{2_j}(p_2\epsilon_3)_iI_{11} \\
& + 2\,(p_1N\epsilon_2p_3)\, p_{3_j}(p_2\epsilon_3)_iI_2 + 2\,(p_1N\epsilon_2Dp_3)\, p_{3_j}(p_2\epsilon_3)_iI'_{11} \\
& + 2\,(p_1N\epsilon_3p_2)\, p_{2_j}(p_3\epsilon_2)_iI_2 + 2\,(p_1N\epsilon_3Dp_2)\, p_{2_j}(p_3\epsilon_2)_iI_{11} \\
& + 4\,(p_1N\epsilon_3Dp_3)\, p_{2_j}(p_3\epsilon_2)_iI'_6 - 4\,(p_1N\epsilon_3Np_1)\, p_{2_j}(p_3\epsilon_2)_iI_9 \\
& - 2\,\mathrm{tr}(D\epsilon_3)\,(p_1Np_3)\, p_{2_j}(p_3\epsilon_2)_iI'_6 + 2\,(p_1N\epsilon_3p_2)\, p_{3_j}(p_3\epsilon_2)_iI_2 \\
& + 2\,\mathrm{tr}(D\epsilon_3)\,(p_1Np_2)\, p_{3_j}(p_3\epsilon_2)_iI'_6 - 2\,(p_1Np_2)\,(p_2\epsilon_3)_j(p_3\epsilon_2)_iI_2 \\
& - 2\,(p_1Np_3)\,(p_2\epsilon_3)_j(p_3\epsilon_2)_iI_2 - 2\,(p_1N\epsilon_3Dp_2)\, p_{3_i}(p_3\epsilon_2)_jI'_{11} \\
& - 2\,(p_1Np_2)\,(p_2p_3)\,(\epsilon_2\epsilon_3)_{ji}I_2 + 2\,(p_1Np_3)\,(p_2p_3)\,(\epsilon_2\epsilon_3)_{ji}I_2 \\
& + 4\,(p_1Np_2)\,(p_1Np_3)\,(\epsilon_2\epsilon_3)_{ji}I_9 - 2\,(p_1Np_2)\,(p_2Dp_3)\,(\epsilon_2\epsilon_3)_{ji}I_{11}
\end{aligned}$$

$$+\,2\,(p_1Np_3)\,(p_2Dp_3)\,(\epsilon_2\epsilon_3)_{ji}I'_{11}+4\,(p_1N\epsilon_3p_2)\,p_{3j}(p_1N\epsilon_2)_iI_9$$

$$+\,4\,(p_2p_3)\,(p_1N\epsilon_2)_i(p_2\epsilon_3)_jI_2-4\,(p_1Np_3)\,(p_1N\epsilon_2)_i(p_2\epsilon_3)_jI_9$$

$$+\,2\,(p_2\epsilon_3p_2)\,p_{2i}(p_1N\epsilon_2)_jI_2-8\,(p_1N\epsilon_3p_2)\,p_{2i}(p_1N\epsilon_2)_jI_9$$

$$+\,4\,(p_2D\epsilon_3p_2)\,p_{2i}(p_1N\epsilon_2)_jI_{11}+4\,(p_2\epsilon_3Dp_3)\,p_{2i}(p_1N\epsilon_2)_jI'_6$$

$$-\,8\,(p_1N\epsilon_3Dp_2)\,p_{2i}(p_1N\epsilon_2)_jI_5-4\,(p_1N\epsilon_3Dp_3)\,p_{2i}(p_1N\epsilon_2)_jI'_4$$

$$+\,8\,(p_1N\epsilon_3Np_1)\,p_{2i}(p_1N\epsilon_2)_jI_{10}+2\,(p_2D\epsilon_3Dp_2)\,p_{2i}(p_1N\epsilon_2)_jI_1$$

$$-\,4\,(p_2D\epsilon_3Dp_3)\,p_{2i}(p_1N\epsilon_2)_jI'_7-2\,\mathrm{tr}(D\epsilon_3)\,(p_2p_3)\,p_{2i}(p_1N\epsilon_2)_jI'_6$$

$$+\,2\,\mathrm{tr}(D\epsilon_3)\,(p_1Np_3)\,p_{2i}(p_1N\epsilon_2)_jI'_4+2\,\mathrm{tr}(D\epsilon_3)\,(p_2Dp_3)\,p_{2i}(p_1N\epsilon_2)_jI'_7$$

$$+\,2\,(p_2\epsilon_3p_2)\,p_{3i}(p_1N\epsilon_2)_jI_2+4\,(p_1N\epsilon_3Dp_2)\,p_{3i}(p_1N\epsilon_2)_jI_5$$

$$-\,2\,(p_2D\epsilon_3Dp_2)\,p_{3i}(p_1N\epsilon_2)_jI_1+2\,\mathrm{tr}(D\epsilon_3)\,(p_2p_3)\,p_{3i}(p_1N\epsilon_2)_jI'_6$$

$$+\,2\,\mathrm{tr}(D\epsilon_3)\,(p_2Dp_3)\,p_{3i}(p_1N\epsilon_2)_jI'_7-4\,(p_1N\epsilon_2p_3)\,p_{2j}(p_1N\epsilon_3)_iI_9$$

$$+\,4\,(p_1N\epsilon_2Dp_3)\,p_{2j}(p_1N\epsilon_3)_iI_5-4\,(p_1Np_2)\,(p_1N\epsilon_3)_i(p_3\epsilon_2)_jI_9$$

$$+\,4\,(p_2p_3)\,(p_1N\epsilon_2)_j(p_1N\epsilon_3)_iI_9-4\,(p_2Dp_3)\,(p_1N\epsilon_2)_j(p_1N\epsilon_3)_iI_5$$

$$+\,2\,(p_1N\epsilon_2p_3)\,p_{2j}(p_2D\epsilon_3)_iI_{11}-2\,(p_1N\epsilon_2Dp_3)\,p_{2j}(p_2D\epsilon_3)_iI_1$$

$$+\,2\,(p_1Np_2)\,(p_2D\epsilon_3)_i(p_3\epsilon_2)_jI_{11}+2\,(p_1Np_3)\,(p_2D\epsilon_3)_i(p_3\epsilon_2)_jI'_{11}$$

$$-\,4\,(p_1Np_3)\,(p_1N\epsilon_2)_j(p_2D\epsilon_3)_iI_5+4\,(p_2Dp_3)\,(p_1N\epsilon_2)_j(p_2D\epsilon_3)_iI_1$$

$$-\,2\,(p_1N\epsilon_2p_3)\,p_{3i}(p_2D\epsilon_3)_jI'_{11}-2\,(p_1N\epsilon_2Dp_3)\,p_{3i}(p_2D\epsilon_3)_jI_1$$

$$-\,2\,(p_1Np_3)\,p_{2j}(p_2\epsilon_3\epsilon_2)_iI_2+2\,(p_1Np_2)\,p_{3j}(p_2\epsilon_3\epsilon_2)_iI_2$$

$$-\,2\,(p_1N\epsilon_3p_2)\,p_{2j}(p_3D\epsilon_2)_iI_{11}-2\,(p_1N\epsilon_3Dp_2)\,p_{2j}(p_3D\epsilon_2)_iI_1$$

$$+\,4\,(p_1N\epsilon_3Dp_3)\,p_{2j}(p_3D\epsilon_2)_iI'_7+4\,(p_1N\epsilon_3Np_1)\,p_{2j}(p_3D\epsilon_2)_iI_5$$

$$-\,2\,\mathrm{tr}(D\epsilon_3)\,(p_1Np_3)\,p_{2j}(p_3D\epsilon_2)_iI'_7+2\,(p_1N\epsilon_3p_2)\,p_{3j}(p_3D\epsilon_2)_iI'_{11}$$

$$+\,2\,\mathrm{tr}(D\epsilon_3)\,(p_1Np_2)\,p_{3j}(p_3D\epsilon_2)_iI'_7+2\,(p_1Np_2)\,(p_2\epsilon_3)_j(p_3D\epsilon_2)_iI_{11}$$

$$-\,2\,(p_1Np_3)\,(p_2\epsilon_3)_j(p_3D\epsilon_2)_iI'_{11}-4\,(p_1Np_2)\,(p_1N\epsilon_3)_j(p_3D\epsilon_2)_iI_5$$

$$
\begin{aligned}
&+2\,(p_1Np_2)\,(p_2D\epsilon_3)_j(p_3D\epsilon_2)_iI_1-2\,(p_1Np_3)\,(p_2D\epsilon_3)_j(p_3D\epsilon_2)_iI_1\\
&-2\,(p_1N\epsilon_3Dp_2)\,p_{3i}(p_3D\epsilon_2)_jI_1+4\,(p_1N\epsilon_2p_3)\,p_{2j}(p_3D\epsilon_3)_iI_6'\\
&+4\,(p_1N\epsilon_2Dp_3)\,p_{2j}(p_3D\epsilon_3)_iI_7'+4\,(p_1Np_2)\,(p_3D\epsilon_3)_i(p_3\epsilon_2)_jI_6'\\
&-4\,(p_2p_3)\,(p_1N\epsilon_2)_j(p_3D\epsilon_3)_iI_6'-4\,(p_2Dp_3)\,(p_1N\epsilon_2)_j(p_3D\epsilon_3)_iI_7'\\
&+4\,(p_1Np_2)\,(p_3D\epsilon_2)_j(p_3D\epsilon_3)_iI_7'-2\,(p_1Np_3)\,p_{2j}(p_3\epsilon_2\epsilon_3)_iI_2\\
&+2\,(p_1Np_2)\,p_{3j}(p_3\epsilon_2\epsilon_3)_iI_2-2\,(p_1Np_2)\,(p_2p_3)\,(\epsilon_2D\epsilon_3)_{ij}I_{11}\\
&-2\,(p_1Np_3)\,(p_2p_3)\,(\epsilon_2D\epsilon_3)_{ij}I_{11}'+4\,(p_1Np_2)\,(p_1Np_3)\,(\epsilon_2D\epsilon_3)_{ij}I_5\\
&-2\,(p_1Np_2)\,(p_2Dp_3)\,(\epsilon_2D\epsilon_3)_{ij}I_1-2\,(p_1Np_3)\,(p_2Dp_3)\,(\epsilon_2D\epsilon_3)_{ij}I_1\\
&-2\,(p_2p_3)\,p_{2i}(p_1N\epsilon_2\epsilon_3)_jI_2+4\,(p_1Np_3)\,p_{2i}(p_1N\epsilon_2\epsilon_3)_jI_9\\
&-2\,(p_2Dp_3)\,p_{2i}(p_1N\epsilon_2\epsilon_3)_jI_{11}+2\,(p_2p_3)\,p_{3i}(p_1N\epsilon_2\epsilon_3)_jI_2\\
&+2\,(p_2Dp_3)\,p_{3i}(p_1N\epsilon_2\epsilon_3)_jI_{11}'+2\,(p_2p_3)\,p_{2j}(p_1N\epsilon_3\epsilon_2)_iI_2\\
&+2\,(p_2Dp_3)\,p_{2j}(p_1N\epsilon_3\epsilon_2)_iI_{11}+2\,(p_2p_3)\,p_{3i}(p_1N\epsilon_3\epsilon_2)_jI_2\\
&+4\,(p_1Np_2)\,p_{3i}(p_1N\epsilon_3\epsilon_2)_jI_9+2\,(p_2Dp_3)\,p_{3i}(p_1N\epsilon_3\epsilon_2)_jI_{11}'\\
&-2\,(p_1Np_3)\,p_{2j}(p_2D\epsilon_3\epsilon_2)_iI_{11}+2\,(p_1Np_2)\,p_{3j}(p_2D\epsilon_3\epsilon_2)_iI_{11}\\
&+2\,(p_1Np_3)\,p_{2j}(p_2\epsilon_3D\epsilon_2)_iI_{11}-2\,(p_1Np_2)\,p_{3j}(p_2\epsilon_3D\epsilon_2)_iI_{11}\\
&-2\,(p_1Np_3)\,p_{2j}(p_3D\epsilon_2\epsilon_3)_iI_{11}'+2\,(p_1Np_2)\,p_{3j}(p_3D\epsilon_2\epsilon_3)_iI_{11}'\\
&-2\,(p_1Np_3)\,p_{2j}(p_3\epsilon_2D\epsilon_3)_iI_{11}'+2\,(p_1Np_2)\,p_{3j}(p_3\epsilon_2D\epsilon_3)_iI_{11}'\\
&+2\,(p_2p_3)\,p_{2i}(p_1N\epsilon_2D\epsilon_3)_jI_{11}-4\,(p_1Np_3)\,p_{2i}(p_1N\epsilon_2D\epsilon_3)_jI_5\\
&+2\,(p_2Dp_3)\,p_{2i}(p_1N\epsilon_2D\epsilon_3)_jI_1+2\,(p_2p_3)\,p_{3i}(p_1N\epsilon_2D\epsilon_3)_jI_{11}'\\
&+2\,(p_2Dp_3)\,p_{3i}(p_1N\epsilon_2D\epsilon_3)_jI_1-2\,(p_2p_3)\,p_{2j}(p_1N\epsilon_3D\epsilon_2)_iI_{11}\\
&-2\,(p_2Dp_3)\,p_{2j}(p_1N\epsilon_3D\epsilon_2)_iI_1+2\,(p_2p_3)\,p_{3i}(p_1N\epsilon_3D\epsilon_2)_jI_{11}'\\
&-4\,(p_1Np_2)\,p_{3i}(p_1N\epsilon_3D\epsilon_2)_jI_5+2\,(p_2Dp_3)\,p_{3i}(p_1N\epsilon_3D\epsilon_2)_jI_1
\end{aligned}
$$

$$+ 2\,(p_1Np_3)\,p_{2j}(p_2D\epsilon_3D\epsilon_2)_iI_1 - 2\,(p_1Np_2)\,p_{3j}(p_2D\epsilon_3D\epsilon_2)_iI_1$$

$$- 2\,(p_1Np_3)\,p_{2j}(p_3D\epsilon_2D\epsilon_3)_iI_1 + 2\,(p_1Np_2)\,p_{3j}(p_3D\epsilon_2D\epsilon_3)_iI_1$$

$$+ 2\,(p_1N\epsilon_3p_2)\,(p_2p_3)\,\epsilon_{2ij}I_2 + 4\,(p_1N\epsilon_3p_2)\,(p_1Np_2)\,\epsilon_{2ij}I_9$$

$$+ (p_1Np_2)\,(p_2\epsilon_3p_2)\,\epsilon_{2ji}I_2 + (p_1Np_3)\,(p_2\epsilon_3p_2)\,\epsilon_{2ji}I_2$$

$$+ 2\,(p_1Np_2)\,(p_2D\epsilon_3p_2)\,\epsilon_{2ji}I_{11} + 2\,(p_1Np_2)\,(p_2\epsilon_3Dp_3)\,\epsilon_{2ji}I'_6$$

$$- 4\,(p_1N\epsilon_3Dp_2)\,(p_1Np_2)\,\epsilon_{2ji}I_5 + 2\,(p_1N\epsilon_3Dp_2)\,(p_2Dp_3)\,\epsilon_{2ji}I_1$$

$$- 2\,(p_1N\epsilon_3Dp_3)\,(p_2p_3)\,\epsilon_{2ji}I'_6 - 2\,(p_1N\epsilon_3Dp_3)\,(p_1Np_2)\,\epsilon_{2ji}I'_4$$

$$- 2\,(p_1N\epsilon_3Dp_3)\,(p_2Dp_3)\,\epsilon_{2ji}I'_7 + 2\,(p_1N\epsilon_3Np_1)\,(p_2p_3)\,\epsilon_{2ji}I_9$$

$$+ 4\,(p_1N\epsilon_3Np_1)\,(p_1Np_2)\,\epsilon_{2ji}I_{10} - 2\,(p_1N\epsilon_3Np_1)\,(p_2Dp_3)\,\epsilon_{2ji}I_5$$

$$+ (p_1Np_2)\,(p_2D\epsilon_3Dp_2)\,\epsilon_{2ji}I_1 - (p_1Np_3)\,(p_2D\epsilon_3Dp_2)\,\epsilon_{2ji}I_1$$

$$- 2\,(p_1Np_2)\,(p_2D\epsilon_3Dp_3)\,\epsilon_{2ji}I'_7 - \mathrm{tr}(D\epsilon_3)\,(p_1Np_2)\,(p_2p_3)\,\epsilon_{2ji}I'_6$$

$$+ \mathrm{tr}(D\epsilon_3)\,(p_1Np_3)\,(p_2p_3)\,\epsilon_{2ji}I'_6 + \mathrm{tr}(D\epsilon_3)\,(p_1Np_2)\,(p_1Np_3)\,\epsilon_{2ji}I'_4$$

$$+ \mathrm{tr}(D\epsilon_3)\,(p_1Np_2)\,(p_2Dp_3)\,\epsilon_{2ji}I'_7 + \mathrm{tr}(D\epsilon_3)\,(p_1Np_3)\,(p_2Dp_3)\,\epsilon_{2ji}I'_7), \tag{2.36}$$

$$\mathcal{A}^{(3)}_{C^{(p+3)}Bh} = \frac{2i^{p(p+1)}\sqrt{2}}{p!}C^{ijk}{}_{b_1\ldots b_p}\varepsilon^{ab_1\ldots b_p}($$

$$2p_{2a}p_{3j}(p_2\epsilon_3)_k(p_3\epsilon_2)_iI_2 + 2p_{3a}p_{3j}(p_2\epsilon_3)_k(p_3\epsilon_2)_iI_2$$

$$- 2p_{2a}p_{2i}(p_2\epsilon_3)_k(p_3\epsilon_2)_jI_2 - 2p_{2i}p_{3a}(p_2\epsilon_3)_k(p_3\epsilon_2)_jI_2$$

$$+ 2\,\mathrm{tr}(D\epsilon_3)p_{2a}p_{2j}p_{3i}(p_3\epsilon_2)_kI'_6 + 2\,\mathrm{tr}(D\epsilon_3)p_{2j}p_{3a}p_{3i}(p_3\epsilon_2)_kI'_6$$

$$+ 2\,(p_2p_3)\,p_{2a}p_{2i}(\epsilon_2\epsilon_3)_{jk}I_2 - 4\,(p_1Np_3)\,p_{2a}p_{2i}(\epsilon_2\epsilon_3)_{jk}I_9$$

$$+ 2\,(p_2Dp_3)\,p_{2a}p_{2i}(\epsilon_2\epsilon_3)_{jk}I_{11} - 2\,(p_2p_3)\,p_{2a}p_{3i}(\epsilon_2\epsilon_3)_{jk}I_2$$

$$- 2\,(p_2Dp_3)\,p_{2a}p_{3i}(\epsilon_2\epsilon_3)_{jk}I'_{11} - 2\,(p_2p_3)\,p_{3a}p_{3i}(\epsilon_2\epsilon_3)_{jk}I_2$$

$$- 4\,(p_1Np_2)\,p_{3a}p_{3i}(\epsilon_2\epsilon_3)_{jk}I_9 - 2\,(p_2Dp_3)\,p_{3a}p_{3i}(\epsilon_2\epsilon_3)_{jk}I'_{11}$$

$$+ 2\,(p_2p_3)\,p_{2_j}p_{3_a}(\epsilon_2\epsilon_3)_{ki}I_2 + 2\,(p_2Dp_3)\,p_{2_j}p_{3_a}(\epsilon_2\epsilon_3)_{ki}I_{11}$$

$$+ 4p_{3_a}p_{3_j}(p_1N\epsilon_2)_i(p_2\epsilon_3)_kI_9 - 4p_{2_i}p_{3_a}(p_1N\epsilon_2)_j(p_2\epsilon_3)_kI_9$$

$$+ 2\,\mathrm{tr}(D\epsilon_3)p_{2_j}p_{3_a}p_{3_i}(p_1N\epsilon_2)_kI'_4 + 4p_{2_j}p_{3_i}(p_1N\epsilon_2)_k(p_2\epsilon_3)_aI_9$$

$$- 4p_{2_j}p_{3_i}(p_1N\epsilon_3)_a(p_3\epsilon_2)_kI_9 - 8p_{2_j}p_{3_i}(p_1N\epsilon_2)_k(p_1N\epsilon_3)_aI_{10}$$

$$+ 4p_{2_a}p_{2_j}(p_1N\epsilon_3)_i(p_3\epsilon_2)_kI_9 + 4p_{2_j}p_{3_a}(p_1N\epsilon_3)_i(p_3\epsilon_2)_kI_9$$

$$+ 8p_{2_j}p_{3_a}(p_1N\epsilon_2)_k(p_1N\epsilon_3)_iI_{10} + 4p_{2_j}p_{3_i}(p_1N\epsilon_2)_k(p_2D\epsilon_3)_aI_5$$

$$- 2p_{2_a}p_{2_j}(p_2D\epsilon_3)_i(p_3\epsilon_2)_kI_{11} - 2p_{2_j}p_{3_a}(p_2D\epsilon_3)_i(p_3\epsilon_2)_kI_{11}$$

$$- 4p_{2_j}p_{3_a}(p_1N\epsilon_2)_k(p_2D\epsilon_3)_iI_5 - 4p_{3_a}p_{3_i}(p_1N\epsilon_2)_k(p_2D\epsilon_3)_jI_5$$

$$- 2p_{2_a}p_{3_i}(p_2D\epsilon_3)_k(p_3\epsilon_2)_jI'_{11} - 2p_{3_a}p_{3_i}(p_2D\epsilon_3)_k(p_3\epsilon_2)_jI'_{11}$$

$$+ 2p_{2_a}p_{2_j}p_{3_i}(p_2\epsilon_3\epsilon_2)_kI_2 + 2p_{2_j}p_{3_a}p_{3_i}(p_2\epsilon_3\epsilon_2)_kI_2$$

$$+ 2p_{2_a}p_{3_j}(p_2\epsilon_3)_k(p_3D\epsilon_2)_iI'_{11} + 2p_{3_a}p_{3_j}(p_2\epsilon_3)_k(p_3D\epsilon_2)_iI'_{11}$$

$$- 4p_{2_i}p_{3_a}(p_1N\epsilon_3)_k(p_3D\epsilon_2)_jI_5 - 2p_{2_a}p_{3_i}(p_2D\epsilon_3)_k(p_3D\epsilon_2)_jI_1$$

$$- 2p_{3_a}p_{3_i}(p_2D\epsilon_3)_k(p_3D\epsilon_2)_jI_1 + 2\,\mathrm{tr}(D\epsilon_3)p_{2_a}p_{2_j}p_{3_i}(p_3D\epsilon_2)_kI'_7$$

$$+ 2\,\mathrm{tr}(D\epsilon_3)p_{2_j}p_{3_a}p_{3_i}(p_3D\epsilon_2)_kI'_7 + 2p_{2_a}p_{2_j}(p_2\epsilon_3)_i(p_3D\epsilon_2)_kI_{11}$$

$$+ 2p_{2_j}p_{3_a}(p_2\epsilon_3)_i(p_3D\epsilon_2)_kI_{11} + 4p_{2_j}p_{3_i}(p_1N\epsilon_3)_a(p_3D\epsilon_2)_kI_5$$

$$- 4p_{2_a}p_{2_j}(p_1N\epsilon_3)_i(p_3D\epsilon_2)_kI_5 + 2p_{2_a}p_{2_j}(p_2D\epsilon_3)_i(p_3D\epsilon_2)_kI_1$$

$$+ 2p_{2_j}p_{3_a}(p_2D\epsilon_3)_i(p_3D\epsilon_2)_kI_1 - 4p_{2_a}p_{2_j}(p_3D\epsilon_3)_i(p_3\epsilon_2)_kI'_6$$

$$- 4p_{2_j}p_{3_a}(p_3D\epsilon_3)_i(p_3\epsilon_2)_kI'_6 - 4p_{2_j}p_{3_a}(p_1N\epsilon_2)_k(p_3D\epsilon_3)_iI'_4$$

$$- 4p_{2_a}p_{2_j}(p_3D\epsilon_2)_k(p_3D\epsilon_3)_iI'_7 - 4p_{2_j}p_{3_a}(p_3D\epsilon_2)_k(p_3D\epsilon_3)_iI'_7$$

$$+ 2p_{2_a}p_{2_j}p_{3_i}(p_3\epsilon_2\epsilon_3)_kI_2 + 2p_{2_j}p_{3_a}p_{3_i}(p_3\epsilon_2\epsilon_3)_kI_2$$

$$+ (p_2p_3)\,p_{2_j}p_{3_a}(\epsilon_2D\epsilon_3)_{ik}I_{11} + 2\,(p_2Dp_3)\,p_{2_j}p_{3_a}(\epsilon_2D\epsilon_3)_{ik}I_1$$

$$- 2\,(p_2p_3)\,p_{2_a}p_{2_i}(\epsilon_2D\epsilon_3)_{jk}I_{11} + 4\,(p_1Np_3)\,p_{2_a}p_{2_i}(\epsilon_2D\epsilon_3)_{jk}I_5$$

$$- 2\,(p_2 D p_3)\, p_{2a} p_{2i} (\epsilon_2 D \epsilon_3)_{jk} I_1 + 2\,(p_2 p_3)\, p_{2a} p_{3i} (\epsilon_2 D \epsilon_3)_{kj} I'_{11}$$

$$+ 2\,(p_2 D p_3)\, p_{2a} p_{3i} (\epsilon_2 D \epsilon_3)_{kj} I_1 + 2\,(p_2 p_3)\, p_{3a} p_{3i} (\epsilon_2 D \epsilon_3)_{kj} I'_{11}$$

$$- 4\,(p_1 N p_2)\, p_{3a} p_{3i} (\epsilon_2 D \epsilon_3)_{kj} I_5 + 2\,(p_2 D p_3)\, p_{3a} p_{3i} (\epsilon_2 D \epsilon_3)_{kj} I_1$$

$$+ 4 p_{2j} p_{3a} p_{3i} (p_1 N \epsilon_2 \epsilon_3)_k I_9 - 4 p_{2a} p_{2j} p_{3i} (p_1 N \epsilon_3 \epsilon_2)_k I_9$$

$$+ 2 p_{2a} p_{2j} p_{3i} (p_2 D \epsilon_3 \epsilon_2)_k I_{11} + 2 p_{2j} p_{3a} p_{3i} (p_2 D \epsilon_3 \epsilon_2)_k I_{11}$$

$$- 2 p_{2a} p_{2j} p_{3i} (p_2 \epsilon_3 D \epsilon_2)_k I_{11} - 2 p_{2j} p_{3a} p_{3i} (p_2 \epsilon_3 D \epsilon_2)_k I_{11}$$

$$+ 2 p_{2a} p_{2j} p_{3i} (p_3 D \epsilon_2 \epsilon_3)_k I'_{11} + 2 p_{2j} p_{3a} p_{3i} (p_3 D \epsilon_2 \epsilon_3)_k I'_{11}$$

$$+ 2 p_{2a} p_{2j} p_{3i} (p_3 \epsilon_2 D \epsilon_3)_k I'_{11} + 2 p_{2j} p_{3a} p_{3i} (p_3 \epsilon_2 D \epsilon_3)_k I'_{11}$$

$$- 4 p_{2j} p_{3a} p_{3i} (p_1 N \epsilon_2 D \epsilon_3)_k I_5 + 4 p_{2a} p_{2j} p_{3i} (p_1 N \epsilon_3 D \epsilon_2)_k I_5$$

$$- 2 p_{2a} p_{2j} p_{3i} (p_2 D \epsilon_3 D \epsilon_2)_k I_1 - 2 p_{2j} p_{3a} p_{3i} (p_2 D \epsilon_3 D \epsilon_2)_k I_1$$

$$+ 2 p_{2a} p_{2j} p_{3i} (p_3 D \epsilon_2 D \epsilon_3)_k I_1 + 2 p_{2j} p_{3a} p_{3i} (p_3 D \epsilon_2 D \epsilon_3)_k I_1$$

$$+ 2\,(p_2 p_3)\, p_{2a} (p_2 \epsilon_3)_k \epsilon_{2ij} I_2 - 2\,(p_1 N p_3)\, p_{2a} (p_2 \epsilon_3)_k \epsilon_{2ij} I_9$$

$$+ 2\,(p_2 p_3)\, p_{3a} (p_2 \epsilon_3)_k \epsilon_{2ij} I_2 + 2\,(p_1 N p_2)\, p_{3a} (p_2 \epsilon_3)_k \epsilon_{2ij} I_9$$

$$+ 2\,(p_2 p_3)\, p_{2a} (p_3 D \epsilon_3)_k \epsilon_{2ij} I'_6 + 2\,(p_2 D p_3)\, p_{2a} (p_3 D \epsilon_3)_k \epsilon_{2ij} I'_7$$

$$- (p_2 \epsilon_3 p_2)\, p_{2a} p_{2i} \epsilon_{2jk} I_2 + 4\,(p_1 N \epsilon_3 p_2)\, p_{2a} p_{2i} \epsilon_{2jk} I_9$$

$$- 2\,(p_2 D \epsilon_3 p_2)\, p_{2a} p_{2i} \epsilon_{2jk} I_{11} - 2\,(p_2 \epsilon_3 D p_3)\, p_{2a} p_{2i} \epsilon_{2jk} I'_6$$

$$+ 4\,(p_1 N \epsilon_3 D p_2)\, p_{2a} p_{2i} \epsilon_{2jk} I_5 + 2\,(p_1 N \epsilon_3 D p_3)\, p_{2a} p_{2i} \epsilon_{2jk} I'_4$$

$$- 4\,(p_1 N \epsilon_3 N p_1)\, p_{2a} p_{2i} \epsilon_{2jk} I_{10} - (p_2 D \epsilon_3 D p_2)\, p_{2a} p_{2i} \epsilon_{2jk} I_1$$

$$+ 2\,(p_2 D \epsilon_3 D p_3)\, p_{2a} p_{2i} \epsilon_{2jk} I'_7 + \mathrm{tr}(D \epsilon_3)\,(p_2 p_3)\, p_{2a} p_{2i} \epsilon_{2jk} I'_6$$

$$- \mathrm{tr}(D \epsilon_3)\,(p_1 N p_3)\, p_{2a} p_{2i} \epsilon_{2jk} I'_4 - \mathrm{tr}(D \epsilon_3)\,(p_2 D p_3)\, p_{2a} p_{2i} \epsilon_{2jk} I'_7$$

$$- (p_2 \epsilon_3 p_2)\, p_{2i} p_{3a} \epsilon_{2jk} I_2 + 2\,(p_1 N \epsilon_3 p_2)\, p_{2i} p_{3a} \epsilon_{2jk} I_9$$

$$- 2\,(p_2 D \epsilon_3 p_2)\, p_{2i} p_{3a} \epsilon_{2jk} I_{11} - 2\,(p_2 \epsilon_3 D p_3)\, p_{2i} p_{3a} \epsilon_{2jk} I'_6$$

$$+ 2\,(p_1 N \epsilon_3 D p_2)\, p_{2i} p_{3a} \epsilon_{2jk} I_5 - (p_2 D \epsilon_3 D p_2)\, p_{2i} p_{3a} \epsilon_{2jk} I_1$$

$$+\,2\,(p_2D\epsilon_3Dp_3)\,p_{2i}p_{3a}\epsilon_{2jk}I_7' + \mathrm{tr}(D\epsilon_3)\,(p_2p_3)\,p_{2i}p_{3a}\epsilon_{2jk}I_6'$$

$$-\,\mathrm{tr}(D\epsilon_3)\,(p_2Dp_3)\,p_{2i}p_{3a}\epsilon_{2jk}I_7' - (p_2\epsilon_3p_2)\,p_{2a}p_{3i}\epsilon_{2jk}I_2$$

$$+\,(p_1N\epsilon_3p_2)\,p_{2a}p_{3i}\epsilon_{2jk}I_9 - 2\,(p_1N\epsilon_3Dp_2)\,p_{2a}p_{3i}\epsilon_{2jk}I_5$$

$$+\,(p_2D\epsilon_3Dp_2)\,p_{2a}p_{3i}\epsilon_{2jk}I_1 - \mathrm{tr}(D\epsilon_3)\,(p_2p_3)\,p_{2a}p_{3i}\epsilon_{2jk}I_6'$$

$$-\,\mathrm{tr}(D\epsilon_3)\,(p_2Dp_3)\,p_{2a}p_{3i}\epsilon_{2jk}I_7' - (p_2\epsilon_3p_2)\,p_{3a}p_{3i}\epsilon_{2jk}I_2$$

$$+\,(p_2D\epsilon_3Dp_2)\,p_{3a}p_{3i}\epsilon_{2jk}I_1 - 2\,(p_2p_3)\,p_{2a}(p_1N\epsilon_3)_i\epsilon_{2jk}I_9$$

$$+\,2\,(p_2Dp_3)\,p_{2a}(p_1N\epsilon_3)_i\epsilon_{2jk}I_5 - 2\,(p_2p_3)\,p_{3a}(p_1N\epsilon_3)_i\epsilon_{2jk}I_9$$

$$-\,4\,(p_1Np_2)\,p_{3a}(p_1N\epsilon_3)_i\epsilon_{2jk}I_{10} + 2\,(p_2Dp_3)\,p_{3a}(p_1N\epsilon_3)_i\epsilon_{2jk}I_5$$

$$+\,2\,(p_1Np_3)\,p_{2a}(p_2D\epsilon_3)_i\epsilon_{2jk}I_5 - 2\,(p_2Dp_3)\,p_{2a}(p_2D\epsilon_3)_i\epsilon_{2jk}I_1$$

$$+\,2\,(p_1Np_2)\,p_{3a}(p_2D\epsilon_3)_i\epsilon_{2jk}I_5 - 2\,(p_2Dp_3)\,p_{3a}(p_2D\epsilon_3)_i\epsilon_{2jk}I_1$$

$$+\,2\,(p_2p_3)\,p_{3a}(p_3D\epsilon_3)_i\epsilon_{2jk}I_6' + 2\,(p_1Np_2)\,p_{3a}(p_3D\epsilon_3)_i\epsilon_{2jk}I_4'$$

$$+\,2\,(p_2Dp_3)\,p_{3a}(p_3D\epsilon_3)_i\epsilon_{2jk}I_7' + 2\,(p_1Np_3)\,p_{2j}(p_2\epsilon_3)_a\epsilon_{2ki}I_9$$

$$-\,2\,(p_1Np_2)\,p_{3j}(p_2\epsilon_3)_a\epsilon_{2ki}I_9 - 2\,(p_2p_3)\,p_{2j}(p_1N\epsilon_3)_a\epsilon_{2ki}I_9$$

$$-\,2\,(p_2Dp_3)\,p_{2j}(p_1N\epsilon_3)_a\epsilon_{2ki}I_5 + 2\,(p_2p_3)\,p_{3j}(p_1N\epsilon_3)_a\epsilon_{2ki}I_9$$

$$+\,4\,(p_1Np_2)\,p_{3j}(p_1N\epsilon_3)_a\epsilon_{2ki}I_{10} - 2\,(p_2Dp_3)\,p_{3j}(p_1N\epsilon_3)_a\epsilon_{2ki}I_5$$

$$+\,2\,(p_1Np_3)\,p_{2j}(p_2D\epsilon_3)_a\epsilon_{2ki}I_5 - 2\,(p_1Np_2)\,p_{3j}(p_2D\epsilon_3)_a\epsilon_{2ki}I_5$$

$$+\,\mathrm{tr}(D\epsilon_3)\,(p_2p_3)\,p_{3a}p_{3i}\epsilon_{2kj}I_6' + \mathrm{tr}(D\epsilon_3)\,(p_1Np_2)\,p_{3a}p_{3i}\epsilon_{2kj}I_4'$$

$$+\,\mathrm{tr}(D\epsilon_3)\,(p_2Dp_3)\,p_{3a}p_{3i}\epsilon_{2kj}I_7' + 4\,(p_1Np_3)\,p_{2j}(p_3\epsilon_2)_k\epsilon_{3ai}I_9$$

$$+\,2\,(p_1Np_2)\,(p_2p_3)\,\epsilon_{2jk}\epsilon_{3ai}I_9 - 2\,(p_1Np_3)\,(p_2p_3)\,\epsilon_{2jk}\epsilon_{3ai}I_9$$

$$-\,4\,(p_1Np_2)\,(p_1Np_3)\,\epsilon_{2jk}\epsilon_{3ai}I_{10} + 2\,(p_1Np_2)\,(p_2Dp_3)\,\epsilon_{2jk}\epsilon_{3ai}I_5$$

$$+\,2\,(p_1Np_3)\,(p_2Dp_3)\,\epsilon_{2jk}\epsilon_{3ai}I_5 + 4\,(p_1Np_2)\,p_{3i}(p_3\epsilon_2)_k\epsilon_{3aj}I_9$$

$$+\,4\,(p_2p_3)\,p_{2i}(p_1N\epsilon_2)_k\epsilon_{3aj}I_9 - 8\,(p_1Np_3)\,p_{2i}(p_1N\epsilon_2)_k\epsilon_{3aj}I_{10}$$

$$+ 4\,(p_2Dp_3)\,p_{2i}(p_1N\epsilon_2)_k\epsilon_{3aj}I_5 - 4\,(p_2p_3)\,p_{3i}(p_1N\epsilon_2)_k\epsilon_{3aj}I_9$$
$$+ 4\,(p_2Dp_3)\,p_{3i}(p_1N\epsilon_2)_k\epsilon_{3aj}I_5 - 4\,(p_1N\epsilon_2p_3)\,p_{2j}p_{3i}\epsilon_{3ak}I_9$$
$$+ 4\,(p_1N\epsilon_2Dp_3)\,p_{2j}p_{3i}\epsilon_{3ak}I_5 + 4\,(p_1Np_3)\,p_{2j}(p_3D\epsilon_2)_i\epsilon_{3ak}I_5$$
$$- 4\,(p_1Np_2)\,p_{3j}(p_3D\epsilon_2)_i\epsilon_{3ak}I_5), \tag{2.37}$$

$$\begin{aligned}
\mathcal{A}^{(4)}_{C^{(p+3)}Bh} =&\frac{2i^{p(p+1)}\sqrt{2}}{(p-1)!}C^{ijkl}{}_{b_1\ldots b_{p-1}}\varepsilon^{abb_1\ldots b_{p-1}}\left(-4p_{2b}p_{2j}p_{3a}p_{3i}(\epsilon_2\epsilon_3)_{kl}I_9\right.\\
&+4p_{2b}p_{2j}p_{3a}p_{3i}(\epsilon_2D\epsilon_3)_{kl}I_5 - 2p_{2b}p_{2i}p_{3a}(p_2\epsilon_3)_l\epsilon_{2jk}I_9\\
&+2p_{2a}p_{3b}p_{3i}(p_2\epsilon_3)_l\epsilon_{2jk}I_9 + 4p_{2b}p_{2i}p_{3a}(p_1N\epsilon_3)_l\epsilon_{2jk}I_{10}\\
&-2p_{2b}p_{2i}p_{3a}(p_2D\epsilon_3)_l\epsilon_{2jk}I_5 + 2p_{2b}p_{3a}p_{3i}(p_2D\epsilon_3)_l\epsilon_{2jk}I_5\\
&-2p_{2b}p_{2i}p_{3a}(p_3D\epsilon_3)_l\epsilon_{2jk}I_4' - tr\,(D\epsilon_3)\,p_{2b}p_{2j}p_{3a}p_{3i}\epsilon_{2kl}I_4'\\
&-2p_{2b}p_{2j}p_{3i}(p_2\epsilon_3)_a\epsilon_{2kl}I_9 - 2p_{2j}p_{3b}p_{3i}(p_2\epsilon_3)_a\epsilon_{2kl}I_9\\
&+4p_{2b}p_{2j}p_{3i}(p_1N\epsilon_3)_a\epsilon_{2kl}I_{10} - 2p_{2b}p_{2j}p_{3i}(p_2D\epsilon_3)_a\epsilon_{2kl}I_5\\
&-2p_{2j}p_{3b}p_{3i}(p_2D\epsilon_3)_a\epsilon_{2kl}I_5 - 4p_{2b}p_{2i}p_{3j}(p_3\epsilon_2)_l\epsilon_{3ak}I_9\\
&-4p_{2i}p_{3b}p_{3j}(p_3\epsilon_2)_l\epsilon_{3ak}I_9 - 4p_{2b}p_{2j}p_{3i}(p_3D\epsilon_2)_l\epsilon_{3ak}I_5\\
&-4p_{2j}p_{3b}p_{3i}(p_3D\epsilon_2)_l\epsilon_{3ak}I_5 - 2\,(p_2p_3)\,p_{2b}p_{2i}\epsilon_{2jk}\epsilon_{3al}I_9\\
&+4\,(p_1Np_3)\,p_{2b}p_{2i}\epsilon_{2jk}\epsilon_{3al}I_{10} - 2\,(p_2Dp_3)\,p_{2b}p_{2i}\epsilon_{2jk}\epsilon_{3al}I_5\\
&+2\,(p_2p_3)\,p_{2b}p_{3i}\epsilon_{2jk}\epsilon_{3al}I_9 - 2\,(p_2Dp_3)\,p_{2b}p_{3i}\epsilon_{2jk}\epsilon_{3al}I_5\\
&-2\,(p_2p_3)\,p_{2j}p_{3a}\epsilon_{2kl}\epsilon_{3bi}I_9 - 2\,(p_2Dp_3)\,p_{2j}p_{3a}\epsilon_{2kl}\epsilon_{3bi}I_5\\
&+2\,(p_2p_3)\,p_{3a}p_{3j}\epsilon_{2kl}\epsilon_{3bi}I_9 + 4\,(p_1Np_2)\,p_{3a}p_{3j}\epsilon_{2kl}\epsilon_{3bi}I_{10}\\
&\left.-2\,(p_2Dp_3)\,p_{3a}p_{3j}\epsilon_{2kl}\epsilon_{3bi}I_5 - 8p_{2j}p_{3a}p_{3i}(p_1N\epsilon_2)_l\epsilon_{3bk}I_{10}\right),
\end{aligned} \tag{2.38}$$

$$\mathcal{A}^{(5)}_{C^{(p+3)}Bh} = \frac{8i^{p(p+1)}\sqrt{2}}{(p-2)!}C^{ijklm}{}_{b_1\ldots b_{p-2}}\varepsilon^{abcb_1\ldots b_{p-2}}p_{2c}p_{2j}p_{3a}p_{3i}\epsilon_{2kl}\epsilon_{3bm}I_{10}. \tag{2.39}$$

2.2.3 $C^{(p+1)}$ amplitudes

$$\mathcal{A}_{C^{(p+1)}BB} = \mathcal{A}^{(0)}_{C^{(p+1)}BB} + \mathcal{A}^{(1)}_{C^{(p+1)}BB} + \mathcal{A}^{(2)}_{C^{(p+1)}BB} + \mathcal{A}^{(3)}_{C^{(p+1)}BB} + \mathcal{A}^{(4)}_{C^{(p+1)}BB}. \quad (2.40)$$

$$\begin{aligned}
\mathcal{A}^{(0)}_{C^{(p+1)}BB} =& \frac{2i^{p(p+1)}\sqrt{2}}{(p+1)!} C_{b_1\ldots b_{p+1}} \varepsilon^{b_1\ldots b_{p+1}} \left(2\left(p_1 N\epsilon_2 p_3\right)\left(p_1 N\epsilon_3 p_2\right) I_2\right. \\
& -2\left(p_1 N\epsilon_3 p_2\right)\left(p_2 D\epsilon_2 p_3\right) I_{20} - 2\left(p_1 N\epsilon_2 p_3\right)\left(p_2 D\epsilon_3 p_2\right) I_{14} \\
& -16\left(p_1 N\epsilon_2 D p_3\right)\left(p_1 N\epsilon_3 p_2\right) I_0 + 2\left(p_1 N\epsilon_2 D p_3\right)\left(p_2 D\epsilon_3 p_2\right) I_{15} \\
& -2\left(p_1 N\epsilon_2 D p_3\right)\left(p_2 \epsilon_3 D p_3\right) I_{22} + 4\left(p_1 N\epsilon_2 \epsilon_3 p_2\right) I_{23} \\
& -2\left(p_1 N\epsilon_2 \epsilon_3 p_2\right)\left(p_2 p_3\right) I_{16} + 2\left(p_1 N\epsilon_2 D p_3\right)\left(p_1 N\epsilon_3 D p_2\right) I_1 \\
& -2\left(p_1 N\epsilon_3 D p_2\right)\left(p_2 D\epsilon_2 D p_3\right) I_{21} + 2\left(p_1 N p_3\right)\left(p_2 D\epsilon_2 \epsilon_3 p_2\right) I_{20} \\
& -2\left(p_1 N\epsilon_2 p_3\right)\left(p_2 D\epsilon_3 D p_3\right) I_{22} - 2\left(p_1 N p_3\right)\left(p_2 D\epsilon_3 \epsilon_2 p_3\right) I_{14} \\
& -2\left(p_1 N p_2\right)\left(p_2 \epsilon_3 D\epsilon_2 p_3\right) I_{14} + 2\left(p_1 N\epsilon_2 D\epsilon_3 p_2\right)\left(p_2 p_3\right) I_{14} \\
& +2\left(p_1 N\epsilon_2 \epsilon_3 D p_2\right)\left(p_2 D p_3\right) I_{15} - 2\left(p_1 N\epsilon_2 \epsilon_3 D p_3\right)\left(p_2 p_3\right) I'_{20} \\
& +2\left(p_1 N\epsilon_2 \epsilon_3 D p_3\right)\left(p_2 D p_3\right) I_{22} + 2\left(p_1 N\epsilon_2 \epsilon_3 N p_1\right)\left(p_2 p_3\right) I_2 \\
& -8\left(p_1 N\epsilon_2 \epsilon_3 N p_1\right)\left(p_2 D p_3\right) I_0 - 4\left(p_1 N\epsilon_3 D\epsilon_2 p_3\right)\left(p_1 N p_2\right) I'_{11} \\
& -4\left(p_1 N\epsilon_3 \epsilon_2 D p_3\right)\left(p_1 N p_2\right) I'_{11} - 2\left(p_1 N p_2\right)\left(p_2 D\epsilon_3 D\epsilon_2 p_3\right) I_{15} \\
& -2\left(p_1 N p_2\right)\left(p_2 D\epsilon_3 \epsilon_2 D p_3\right) I_{15} - 2\left(p_1 N p_2\right)\left(p_3 D\epsilon_2 \epsilon_3 D p_3\right) I_{22} \\
& -2\left(p_1 N p_2\right)\left(p_3 D\epsilon_3 D\epsilon_2 p_3\right) I_{22} + 4\left(p_1 N\epsilon_2 D\epsilon_3 D p_2\right) I_{24} \\
& -2\left(p_1 N\epsilon_2 D\epsilon_3 D p_2\right)\left(p_2 D p_3\right) I_{17} + 2\left(p_1 N\epsilon_2 D\epsilon_3 D p_3\right)\left(p_2 p_3\right) I_{22} \\
& -2\left(p_1 N\epsilon_2 D\epsilon_3 D p_3\right)\left(p_2 D p_3\right) I'_{21} - 8\left(p_1 N\epsilon_2 D\epsilon_3 N p_1\right)\left(p_2 p_3\right) I_0 \\
& +2\left(p_1 N\epsilon_2 D\epsilon_3 N p_1\right)\left(p_2 D p_3\right) I_1 + 2\left(p_1 N p_3\right)\left(p_2 D\epsilon_2 D\epsilon_3 D p_2\right) I_{21}
\end{aligned}$$

$$
\begin{aligned}
&+2\,\mathrm{tr}(-\epsilon_2\epsilon_3)\,(p_1Np_2)\,I_{23}-\mathrm{tr}(-\epsilon_2\epsilon_3)\,(p_1Np_2)\,(p_2p_3)\,I_{16}\\
&+\mathrm{tr}(-\epsilon_2\epsilon_3)\,(p_1Np_2)\,(p_1Np_3)\,I_2-\mathrm{tr}(-\epsilon_2\epsilon_3)\,(p_1Np_2)\,(p_2Dp_3)\,I_{14}\\
&-2\,\mathrm{tr}(D\epsilon_2D\epsilon_3)\,(p_1Np_2)\,I_{24}+\mathrm{tr}(D\epsilon_2D\epsilon_3)\,(p_1Np_2)\,(p_2p_3)\,I_{15}\\
&-\mathrm{tr}(D\epsilon_2D\epsilon_3)\,(p_1Np_2)\,(p_1Np_3)\,I_1\\
&+\mathrm{tr}(D\epsilon_2D\epsilon_3)\,(p_1Np_2)\,(p_2Dp_3)\,I_{17})+(2\leftrightarrow 3),
\end{aligned}
\tag{2.41}
$$

$$
\begin{aligned}
\mathcal{A}^{(1)}_{C^{(p+1)}BB}=&\frac{2i^{p(p+1)}\sqrt{2}}{p!}C^{i}{}_{b_1\dots b_p}\varepsilon^{ab_1\dots b_p}\,(-2\,(p_2\epsilon_3D\epsilon_2p_3)\,p_{2a}p_{2i}I_{14}\\
&-2\,(p_2\epsilon_3\epsilon_2Dp_3)\,p_{2a}p_{2i}I_{14}+2\,(p_3D\epsilon_3\epsilon_2p_3)\,p_{2a}p_{2i}I'_{20}\\
&-4\,(p_1N\epsilon_3D\epsilon_2p_3)\,p_{2a}p_{2i}I'_{11}-4\,(p_1N\epsilon_3\epsilon_2Dp_3)\,p_{2a}p_{2i}I'_{11}\\
&-2\,(p_2D\epsilon_3D\epsilon_2p_3)\,p_{2a}p_{2i}I_{15}-2\,(p_2D\epsilon_3\epsilon_2Dp_3)\,p_{2a}p_{2i}I_{15}\\
&-2\,(p_3D\epsilon_2\epsilon_3Dp_3)\,p_{2a}p_{2i}I_{22}-2\,(p_3D\epsilon_3D\epsilon_2p_3)\,p_{2a}p_{2i}I_{22}\\
&+2\,(p_3D\epsilon_2D\epsilon_3Dp_3)\,p_{2a}p_{2i}I'_{21}+2\,\mathrm{tr}(-\epsilon_2\epsilon_3)p_{2a}p_{2i}I_{23}\\
&-\mathrm{tr}(-\epsilon_2\epsilon_3)\,(p_2p_3)\,p_{2a}p_{2i}I_{16}+2\,\mathrm{tr}(-\epsilon_2\epsilon_3)\,(p_1Np_3)\,p_{2a}p_{2i}I_2\\
&-\mathrm{tr}(-\epsilon_2\epsilon_3)\,(p_2Dp_3)\,p_{2a}p_{2i}I_{14}-2\,\mathrm{tr}(D\epsilon_2D\epsilon_3)p_{2a}p_{2i}I_{24}\\
&+\mathrm{tr}(D\epsilon_2D\epsilon_3)\,(p_2p_3)\,p_{2a}p_{2i}I_{15}-2\,\mathrm{tr}(D\epsilon_2D\epsilon_3)\,(p_1Np_3)\,p_{2a}p_{2i}I_1\\
&+\mathrm{tr}(D\epsilon_2D\epsilon_3)\,(p_2Dp_3)\,p_{2a}p_{2i}I_{17}+2\,(p_1N\epsilon_2\epsilon_3p_2)\,p_{2i}p_{3a}I_2\\
&-2\,(p_1N\epsilon_3\epsilon_2p_3)\,p_{2i}p_{3a}I_2-2\,(p_2\epsilon_3D\epsilon_2p_3)\,p_{2i}p_{3a}I_{14}\\
&-2\,(p_2\epsilon_3\epsilon_2Dp_3)\,p_{2i}p_{3a}I_{14}+2\,(p_3D\epsilon_3\epsilon_2p_3)\,p_{2i}p_{3a}I'_{20}\\
&+4\,(p_1N\epsilon_2\epsilon_3Dp_3)\,p_{2i}p_{3a}I'_6-4\,(p_1N\epsilon_2\epsilon_3Np_1)\,p_{2i}p_{3a}I_9\\
&-2\,(p_1N\epsilon_3D\epsilon_2p_3)\,p_{2i}p_{3a}I'_{11}-2\,(p_1N\epsilon_3\epsilon_2Dp_3)\,p_{2i}p_{3a}I'_{11}\\
&-2\,(p_2D\epsilon_3D\epsilon_2p_3)\,p_{2i}p_{3a}I_{15}
\end{aligned}
$$

$$
\begin{aligned}
&-2\,(p_2D\epsilon_3\epsilon_2Dp_3)\,p_{2i}p_{3a}I_{15}-2\,(p_3D\epsilon_2\epsilon_3Dp_3)\,p_{2i}p_{3a}I_{22}\\
&-2\,(p_3D\epsilon_3D\epsilon_2p_3)\,p_{2i}p_{3a}I_{22}-2\,(p_1N\epsilon_2D\epsilon_3Dp_2)\,p_{2i}p_{3a}I_1\\
&+4\,(p_1N\epsilon_2D\epsilon_3Dp_3)\,p_{2i}p_{3a}I'_7+4\,(p_1N\epsilon_2D\epsilon_3Np_1)\,p_{2i}p_{3a}I_5\\
&-2\,(p_1N\epsilon_3D\epsilon_2Dp_3)\,p_{2i}p_{3a}I_1+2\,(p_3D\epsilon_2D\epsilon_3Dp_3)\,p_{2i}p_{3a}I'_{21}\\
&+2\,\mathrm{tr}(-\epsilon_2\epsilon_3)p_{2i}p_{3a}I_{23}-\mathrm{tr}(-\epsilon_2\epsilon_3)\,(p_2p_3)\,p_{2i}p_{3a}I_{16}\\
&-\mathrm{tr}(-\epsilon_2\epsilon_3)\,(p_2Dp_3)\,p_{2i}p_{3a}I_{14}-2\,\mathrm{tr}(D\epsilon_2D\epsilon_3)p_{2i}p_{3a}I_{24}\\
&+\mathrm{tr}(D\epsilon_2D\epsilon_3)\,(p_2p_3)\,p_{2i}p_{3a}I_{15}+\mathrm{tr}(D\epsilon_2D\epsilon_3)\,(p_2Dp_3)\,p_{2i}p_{3a}I_{17}\\
&-2\,(p_1N\epsilon_3D\epsilon_2p_3)\,p_{2a}p_{3i}I'_{11}+2\,(p_1N\epsilon_3\epsilon_2Dp_3)\,p_{2a}p_{3i}I'_{11}\\
&+2\,(p_1N\epsilon_2p_3)\,p_{2i}(p_2\epsilon_3)_aI_2+2\,(p_1N\epsilon_2p_3)\,p_{3i}(p_2\epsilon_3)_aI_2\\
&+2\,(p_1N\epsilon_2Dp_3)\,p_{3i}(p_2\epsilon_3)_aI'_{11}-2\,(p_1N\epsilon_2p_3)\,p_{2a}(p_2\epsilon_3)_iI_2\\
&+2\,(p_2D\epsilon_2p_3)\,p_{2a}(p_2\epsilon_3)_iI_{20}+2\,(p_3D\epsilon_2p_3)\,p_{2a}(p_2\epsilon_3)_iI_{14}\\
&-2\,(p_1N\epsilon_2p_3)\,p_{3a}(p_2\epsilon_3)_iI_2+2\,(p_2D\epsilon_2p_3)\,p_{3a}(p_2\epsilon_3)_iI_{20}\\
&+2\,(p_3D\epsilon_2p_3)\,p_{3a}(p_2\epsilon_3)_iI_{14}-2\,(p_1N\epsilon_3Dp_2)\,p_{3i}(p_3\epsilon_2)_aI'_{11}\\
&+2\,(p_1Np_2)\,(p_2\epsilon_3)_i(p_3\epsilon_2)_aI_2+2\,(p_1Np_3)\,(p_2\epsilon_3)_i(p_3\epsilon_2)_aI_2\\
&-2\,(p_1N\epsilon_3Dp_2)\,p_{2a}(p_3\epsilon_2)_iI'_{12}+2\,(p_2D\epsilon_3Dp_3)\,p_{2a}(p_3\epsilon_2)_iI_{22}\\
&-2\,(p_1N\epsilon_3Dp_2)\,p_{3a}(p_3\epsilon_2)_iI'_{11}+2\,(p_2D\epsilon_3Dp_3)\,p_{3a}(p_3\epsilon_2)_iI_{22}\\
&+2\,(p_1Np_2)\,(p_2p_3)\,(\epsilon_2\epsilon_3)_{ai}I_2-2\,(p_1Np_3)\,(p_2p_3)\,(\epsilon_2\epsilon_3)_{ai}I_2\\
&-4\,(p_1Np_2)\,(p_1Np_3)\,(\epsilon_2\epsilon_3)_{ai}I_9-2\,(p_1Np_3)\,(p_2Dp_3)\,(\epsilon_2\epsilon_3)_{ai}I'_{11}\\
&+2\,(p_1Np_3)\,(p_2Dp_3)\,(\epsilon_2\epsilon_3)_{ia}I'_{11}+4\,(p_1N\epsilon_3p_2)\,p_{3i}(p_1N\epsilon_2)_aI_9\\
&+4\,(p_1N\epsilon_3Dp_2)\,p_{3i}(p_1N\epsilon_2)_aI_5+4\,(p_1Np_3)\,(p_1N\epsilon_2)_a(p_2\epsilon_3)_iI_9\\
&-4\,(p_1N\epsilon_3p_2)\,p_{3a}(p_1N\epsilon_2)_iI_9-4\,(p_2\epsilon_3Dp_3)\,p_{3a}(p_1N\epsilon_2)_iI'_6\\
&+4\,(p_1N\epsilon_3Dp_2)\,p_{3a}(p_1N\epsilon_2)_iI_5-4\,(p_2D\epsilon_3Dp_3)\,p_{3a}(p_1N\epsilon_2)_iI'_7
\end{aligned}
$$

$$-4\,(p_2p_3)\,(p_1N\epsilon_2)_i(p_2\epsilon_3)_aI_2+4\,(p_1Np_3)\,(p_1N\epsilon_2)_i(p_2\epsilon_3)_aI_9$$

$$+4\,(p_2p_3)\,(p_1N\epsilon_2)_i(p_1N\epsilon_3)_aI_9-4\,(p_2Dp_3)\,(p_1N\epsilon_2)_i(p_1N\epsilon_3)_aI_5$$

$$+4\,(p_3D\epsilon_2p_3)\,p_{2_a}(p_1N\epsilon_3)_iI'_{11}+4\,(p_1N\epsilon_3p_2)\,p_{3_i}(p_2D\epsilon_2)_aI_6$$

$$+4\,(p_1N\epsilon_3Dp_2)\,p_{3_i}(p_2D\epsilon_2)_aI_7+4\,(p_1Np_3)\,(p_2D\epsilon_2)_a(p_2\epsilon_3)_iI_6$$

$$(p_2p_3)\,(p_1N\epsilon_3)_i(p_2D\epsilon_2)_aI_6-4\,(p_2Dp_3)\,(p_1N\epsilon_3)_i(p_2D\epsilon_2)_aI_7$$

$$-2\,(p_1N\epsilon_2Dp_3)\,p_{2_i}(p_2D\epsilon_3)_aI_1+2\,(p_1N\epsilon_2p_3)\,p_{3_i}(p_2D\epsilon_3)_aI'_{11}$$

$$+2\,(p_1N\epsilon_2Dp_3)\,p_{3_i}(p_2D\epsilon_3)_aI_1+2\,(p_1Np_3)\,(p_2D\epsilon_3)_a(p_3\epsilon_2)_iI'_{11}$$

$$-4\,(p_1Np_3)\,(p_1N\epsilon_2)_i(p_2D\epsilon_3)_aI_5+4\,(p_2Dp_3)\,(p_1N\epsilon_2)_i(p_2D\epsilon_3)_aI_1$$

$$+2\,(p_3D\epsilon_2p_3)\,p_{2_a}(p_2D\epsilon_3)_iI_{15}-2\,(p_1N\epsilon_2Dp_3)\,p_{2_a}(p_2D\epsilon_3)_iI_1$$

$$+2\,(p_2D\epsilon_2Dp_3)\,p_{2_a}(p_2D\epsilon_3)_iI_{21}+2\,(p_3D\epsilon_2p_3)\,p_{3_a}(p_2D\epsilon_3)_iI_{15}$$

$$-2\,(p_1N\epsilon_2Dp_3)\,p_{3_a}(p_2D\epsilon_3)_iI_1+2\,(p_2D\epsilon_2Dp_3)\,p_{3_a}(p_2D\epsilon_3)_iI_{21}$$

$$-2\,(p_1Np_3)\,(p_2D\epsilon_3)_i(p_3\epsilon_2)_aI'_{11}+4\,(p_1Np_3)\,(p_1N\epsilon_2)_a(p_2D\epsilon_3)_iI_5$$

$$+4\,(p_1Np_3)\,(p_2D\epsilon_2)_a(p_2D\epsilon_3)_iI_7+2\,(p_1Np_3)\,p_{2_i}(p_2\epsilon_3\epsilon_2)_aI_2$$

$$-2\,(p_1Np_2)\,p_{3_i}(p_2\epsilon_3\epsilon_2)_aI_2+4p_{2_a}(p_2\epsilon_3\epsilon_2)_iI_{23}$$

$$-2\,(p_2p_3)\,p_{2_a}(p_2\epsilon_3\epsilon_2)_iI_{16}+2\,(p_1Np_3)\,p_{2_a}(p_2\epsilon_3\epsilon_2)_iI_2$$

$$+4p_{3_a}(p_2\epsilon_3\epsilon_2)_iI_{23}-2\,(p_2p_3)\,p_{3_a}(p_2\epsilon_3\epsilon_2)_iI_{16}$$

$$-2\,(p_1Np_2)\,p_{3_a}(p_2\epsilon_3\epsilon_2)_iI_2+2\,(p_1N\epsilon_3p_2)\,p_{3_i}(p_3D\epsilon_2)_aI'_{11}$$

$$+2\,(p_1Np_3)\,(p_2\epsilon_3)_i(p_3D\epsilon_2)_aI'_{11}+2\,(p_1Np_2)\,(p_2D\epsilon_3)_i(p_3D\epsilon_2)_aI_1$$

$$-2\,(p_1Np_3)\,(p_2D\epsilon_3)_i(p_3D\epsilon_2)_aI_1-2\,(p_1N\epsilon_3p_2)\,p_{2_a}(p_3D\epsilon_2)_iI'_{12}$$

$$+2\,(p_2\epsilon_3Dp_3)\,p_{2_a}(p_3D\epsilon_2)_iI_{22}-2\,(p_1N\epsilon_3p_2)\,p_{3_a}(p_3D\epsilon_2)_iI'_{11}$$

$$+2\,(p_2\epsilon_3Dp_3)\,p_{3_a}(p_3D\epsilon_2)_iI_{22}+2\,(p_1Np_3)\,(p_2\epsilon_3)_a(p_3D\epsilon_2)_iI'_{11}$$

$$+2\,(p_1Np_3)\,(p_2p_3)\,(\epsilon_2D\epsilon_3)_{ai}I'_{11}-4\,(p_1Np_2)\,(p_1Np_3)\,(\epsilon_2D\epsilon_3)_{ai}I_5$$

$$+2\,(p_1Np_2)\,(p_2Dp_3)\,(\epsilon_2D\epsilon_3)_{ai}I_1+2\,(p_1Np_3)\,(p_2Dp_3)\,(\epsilon_2D\epsilon_3)_{ai}I_1$$

$$+2\,(p_1Np_3)\,(p_2p_3)\,(\epsilon_2D\epsilon_3)_{ia}I'_{11}+2\,(p_2p_3)\,p_{2i}(p_1N\epsilon_2\epsilon_3)_aI_2$$

$$-4\,(p_1Np_3)\,p_{2i}(p_1N\epsilon_2\epsilon_3)_aI_9-2\,(p_2p_3)\,p_{3i}(p_1N\epsilon_2\epsilon_3)_aI_2$$

$$-2\,(p_2Dp_3)\,p_{3i}(p_1N\epsilon_2\epsilon_3)_aI'_{11}+2\,(p_2p_3)\,p_{2a}(p_1N\epsilon_2\epsilon_3)_iI_2$$

$$-4\,(p_1Np_3)\,p_{2a}(p_1N\epsilon_2\epsilon_3)_iI_9+2\,(p_2p_3)\,p_{3a}(p_1N\epsilon_2\epsilon_3)_iI_2$$

$$+2\,(p_2Dp_3)\,p_{3i}(p_1N\epsilon_3\epsilon_2)_aI'_{11}+2\,(p_2Dp_3)\,p_{2a}(p_1N\epsilon_3\epsilon_2)_iI'_{12}$$

$$+2\,(p_2Dp_3)\,p_{3a}(p_1N\epsilon_3\epsilon_2)_iI'_{11}-2\,(p_2p_3)\,p_{2a}(p_2D\epsilon_2\epsilon_3)_iI_{20}$$

$$-4\,(p_1Np_3)\,p_{2a}(p_2D\epsilon_2\epsilon_3)_iI_6-2\,(p_2p_3)\,p_{3a}(p_2D\epsilon_2\epsilon_3)_iI_{20}$$

$$+2\,(p_2Dp_3)\,p_{2a}(p_2D\epsilon_3\epsilon_2)_iI_{15}+2\,(p_2Dp_3)\,p_{3a}(p_2D\epsilon_3\epsilon_2)_iI_{15}$$

$$+2\,(p_2p_3)\,p_{2a}(p_2\epsilon_3D\epsilon_2)_iI_{14}+2\,(p_2p_3)\,p_{3a}(p_2\epsilon_3D\epsilon_2)_iI_{14}$$

$$-2\,(p_1Np_3)\,p_{2i}(p_3D\epsilon_2\epsilon_3)_aI'_{11}+2\,(p_1Np_2)\,p_{3i}(p_3D\epsilon_2\epsilon_3)_aI'_{11}$$

$$-2\,(p_1Np_3)\,p_{2a}(p_3D\epsilon_2\epsilon_3)_iI'_{11}-2\,(p_1Np_2)\,p_{3a}(p_3D\epsilon_2\epsilon_3)_iI'_{11}$$

$$+2\,(p_2Dp_3)\,p_{2a}(p_3D\epsilon_3\epsilon_2)_iI_{22}+2\,(p_2Dp_3)\,p_{3a}(p_3D\epsilon_3\epsilon_2)_iI_{22}$$

$$-2\,(p_1Np_3)\,p_{2i}(p_3\epsilon_2D\epsilon_3)_aI'_{11}+2\,(p_1Np_2)\,p_{3i}(p_3\epsilon_2D\epsilon_3)_aI'_{11}$$

$$+2\,(p_1Np_3)\,p_{2a}(p_3\epsilon_2D\epsilon_3)_iI'_{11}-2\,(p_1Np_2)\,p_{3a}(p_3\epsilon_2D\epsilon_3)_iI'_{11}$$

$$+4\,(p_1Np_3)\,p_{2i}(p_1N\epsilon_2D\epsilon_3)_aI_5-2\,(p_2Dp_3)\,p_{2i}(p_1N\epsilon_2D\epsilon_3)_aI_1$$

$$-2\,(p_2p_3)\,p_{3i}(p_1N\epsilon_2D\epsilon_3)_aI'_{11}-2\,(p_2Dp_3)\,p_{3i}(p_1N\epsilon_2D\epsilon_3)_aI_1$$

$$-4\,(p_1Np_3)\,p_{2a}(p_1N\epsilon_2D\epsilon_3)_iI_5+2\,(p_2Dp_3)\,p_{2a}(p_1N\epsilon_2D\epsilon_3)_iI_1$$

$$+2\,(p_2Dp_3)\,p_{3a}(p_1N\epsilon_2D\epsilon_3)_iI_1-2\,(p_2p_3)\,p_{3i}(p_1N\epsilon_3D\epsilon_2)_aI'_{11}$$

$$+2\,(p_2p_3)\,p_{2a}(p_1N\epsilon_3D\epsilon_2)_iI'_{12}+2\,(p_2p_3)\,p_{3a}(p_1N\epsilon_3D\epsilon_2)_iI'_{11}$$

$$-4\,(p_1Np_3)\,p_{2a}(p_2D\epsilon_2D\epsilon_3)_iI_7-2\,(p_2Dp_3)\,p_{2a}(p_2D\epsilon_2D\epsilon_3)_iI_{21}$$

$$-2\,(p_2Dp_3)\,p_{3a}(p_2D\epsilon_2D\epsilon_3)_iI_{21}+2\,(p_1Np_3)\,p_{2i}(p_2D\epsilon_3D\epsilon_2)_aI_1$$

$$-2\,(p_1Np_2)\,p_{3i}(p_2D\epsilon_3D\epsilon_2)_aI_1+4p_{2a}(p_2D\epsilon_3D\epsilon_2)_iI_{24}$$

$$+2\,(p_1Np_3)\,p_{2a}(p_2D\epsilon_3D\epsilon_2)_iI_1-2\,(p_2Dp_3)\,p_{2a}(p_2D\epsilon_3D\epsilon_2)_iI_{17}$$

$$
\begin{aligned}
&+4p_{3a}(p_2D\epsilon_3D\epsilon_2)_iI_{24}+2\,(p_1Np_2)\,p_{3a}(p_2D\epsilon_3D\epsilon_2)_iI_1\\
&-2\,(p_2Dp_3)\,p_{3a}(p_2D\epsilon_3D\epsilon_2)_iI_{17}+2\,(p_2p_3)\,p_{2a}(p_3D\epsilon_3D\epsilon_2)_iI_{22}\\
&+2\,(p_2p_3)\,p_{3a}(p_3D\epsilon_3D\epsilon_2)_iI_{22})\,,
\end{aligned}
\tag{2.42}
$$

$$
\begin{aligned}
\mathcal{A}^{(2)}_{C^{(p+1)}BB}=&\frac{2i^{p(p+1)}\sqrt{2}}{(p-1)!}C^{ij}{}_{b_1\ldots b_{p-1}}\varepsilon^{abb_1\ldots b_{p-1}}\,\big(\mathrm{tr}(-\epsilon_2\epsilon_3)p_{2b}p_{2j}p_{3a}p_{3i}I_2\\
&-\mathrm{tr}(D\epsilon_2D\epsilon_3)p_{2b}p_{2j}p_{3a}p_{3i}I_1+2p_{2b}p_{2j}(p_2\epsilon_3)_i(p_3\epsilon_2)_aI_2\\
&+2p_{2j}p_{3b}(p_2\epsilon_3)_i(p_3\epsilon_2)_aI_2+2p_{2b}p_{3j}(p_2\epsilon_3)_i(p_3\epsilon_2)_aI_2\\
&+2p_{3b}p_{3j}(p_2\epsilon_3)_i(p_3\epsilon_2)_aI_2-2\,(p_2p_3)\,p_{2b}p_{2i}(\epsilon_2\epsilon_3)_{aj}I_2\\
&+4\,(p_1Np_3)\,p_{2b}p_{2i}(\epsilon_2\epsilon_3)_{aj}I_9+2\,(p_2p_3)\,p_{2b}p_{3i}(\epsilon_2\epsilon_3)_{aj}I_2\\
&+2\,(p_2Dp_3)\,p_{2b}p_{3i}(\epsilon_2\epsilon_3)_{aj}I'_{11}-2\,(p_2p_3)\,p_{2j}p_{3a}(\epsilon_2\epsilon_3)_{bi}I_2\\
&-2\,(p_2p_3)\,p_{3a}p_{3i}(\epsilon_2\epsilon_3)_{bj}I_2-4\,(p_1Np_2)\,p_{3a}p_{3i}(\epsilon_2\epsilon_3)_{bj}I_9\\
&-2\,(p_2Dp_3)\,p_{3a}p_{3i}(\epsilon_2\epsilon_3)_{bj}I'_{11}+4\,(p_2Dp_3)\,p_{2b}p_{3a}(\epsilon_2\epsilon_3)_{ij}I_0\\
&-2\,(p_2Dp_3)\,p_{2b}p_{3i}(\epsilon_2\epsilon_3)_{ja}I'_{11}+2\,(p_2Dp_3)\,p_{3a}p_{3i}(\epsilon_2\epsilon_3)_{jb}I'_{11}\\
&-4p_{2b}p_{3i}(p_1N\epsilon_2)_a(p_2\epsilon_3)_jI_9-4p_{3b}p_{3i}(p_1N\epsilon_2)_a(p_2\epsilon_3)_jI_9\\
&+4p_{3b}p_{3j}(p_1N\epsilon_2)_i(p_2\epsilon_3)_aI_9+4p_{2i}p_{3a}(p_1N\epsilon_2)_j(p_2\epsilon_3)_bI_9\\
&-8p_{2i}p_{3a}(p_1N\epsilon_2)_j(p_1N\epsilon_3)_bI_{10}-4p_{2b}p_{3i}(p_2D\epsilon_2)_a(p_2\epsilon_3)_jI_6\\
&-4p_{3b}p_{3i}(p_2D\epsilon_2)_a(p_2\epsilon_3)_jI_6-4p_{2b}p_{3i}(p_1N\epsilon_3)_j(p_2D\epsilon_2)_aI_4\\
&-2p_{2b}p_{3i}(p_2D\epsilon_3)_a(p_3\epsilon_2)_jI'_{11}-2p_{3b}p_{3i}(p_2D\epsilon_3)_a(p_3\epsilon_2)_jI'_{11}\\
&+4p_{3b}p_{3i}(p_1N\epsilon_2)_j(p_2D\epsilon_3)_aI_5+4p_{2i}p_{3a}(p_1N\epsilon_2)_j(p_2D\epsilon_3)_bI_5\\
&+2p_{2b}p_{3i}(p_2D\epsilon_3)_j(p_3\epsilon_2)_aI'_{11}+2p_{3b}p_{3i}(p_2D\epsilon_3)_j(p_3\epsilon_2)_aI'_{11}\\
&+8p_{2b}p_{3a}(p_2D\epsilon_3)_j(p_3\epsilon_2)_iI_0-4p_{2b}p_{3i}(p_1N\epsilon_2)_a(p_2D\epsilon_3)_jI_5\\
&-4p_{3b}p_{3i}(p_1N\epsilon_2)_a(p_2D\epsilon_3)_jI_5-4p_{2b}p_{3i}(p_2D\epsilon_2)_a(p_2D\epsilon_3)_jI_7
\end{aligned}
$$

$$-4p_{3b}p_{3i}(p_2D\epsilon_2)_a(p_2D\epsilon_3)_jI_7-2p_{2b}p_{2j}p_{3i}(p_2\epsilon_3\epsilon_2)_aI_2$$

$$-2p_{2j}p_{3b}p_{3i}(p_2\epsilon_3\epsilon_2)_aI_2+2p_{2b}p_{2i}p_{3a}(p_2\epsilon_3\epsilon_2)_jI_2$$

$$+2p_{2b}p_{3a}p_{3i}(p_2\epsilon_3\epsilon_2)_jI_2-2p_{2b}p_{3i}(p_2\epsilon_3)_j(p_3D\epsilon_2)_aI'_{11}$$

$$-2p_{3b}p_{3i}(p_2\epsilon_3)_j(p_3D\epsilon_2)_aI'_{11}+2p_{2b}p_{2j}(p_2D\epsilon_3)_i(p_3D\epsilon_2)_aI_1$$

$$+2p_{2j}p_{3b}(p_2D\epsilon_3)_i(p_3D\epsilon_2)_aI_1+2p_{2b}p_{3i}(p_2D\epsilon_3)_j(p_3D\epsilon_2)_aI_1$$

$$+2p_{3b}p_{3i}(p_2D\epsilon_3)_j(p_3D\epsilon_2)_aI_1+2p_{2b}p_{3j}(p_2\epsilon_3)_a(p_3D\epsilon_2)_iI'_{11}$$

$$+2p_{3b}p_{3j}(p_2\epsilon_3)_a(p_3D\epsilon_2)_iI'_{11}+4\left(p_1Np_3\right)p_{2b}p_{2i}(\epsilon_2D\epsilon_3)_{aj}I_5$$

$$-2\left(p_2Dp_3\right)p_{2b}p_{2i}(\epsilon_2D\epsilon_3)_{aj}I_1-2\left(p_2p_3\right)p_{2b}p_{3i}(\epsilon_2D\epsilon_3)_{aj}I'_{11}$$

$$-2\left(p_2Dp_3\right)p_{2b}p_{3i}(\epsilon_2D\epsilon_3)_{aj}I_1-2\left(p_2Dp_3\right)p_{2j}p_{3a}(\epsilon_2D\epsilon_3)_{bi}I_1$$

$$+2\left(p_2p_3\right)p_{3a}p_{3i}(\epsilon_2D\epsilon_3)_{bj}I'_{11}-4\left(p_1Np_2\right)p_{3a}p_{3i}(\epsilon_2D\epsilon_3)_{bj}I_5$$

$$+2\left(p_2Dp_3\right)p_{3a}p_{3i}(\epsilon_2D\epsilon_3)_{bj}I_1-2\left(p_2p_3\right)p_{2b}p_{3i}(\epsilon_2D\epsilon_3)_{ja}I'_{11}$$

$$+2\left(p_2p_3\right)p_{3a}p_{3i}(\epsilon_2D\epsilon_3)_{jb}I'_{11}-4\left(p_2p_3\right)p_{2b}p_{3a}(\epsilon_2D\epsilon_3)_{ji}I_0$$

$$-4p_{2j}p_{3a}p_{3i}(p_1N\epsilon_2\epsilon_3)_bI_9-4p_{2b}p_{3a}p_{3i}(p_1N\epsilon_2\epsilon_3)_jI_9$$

$$-4p_{2b}p_{3a}p_{3i}(p_2D\epsilon_2\epsilon_3)_jI_6+2p_{2b}p_{2j}p_{3i}(p_3D\epsilon_2\epsilon_3)_aI'_{11}+2p_{2j}p_{3b}p_{3i}(p_3D\epsilon_2\epsilon_3)_aI'_{11}$$

$$+2p_{2b}p_{2i}p_{3a}(p_3D\epsilon_2\epsilon_3)_jI'_{11}-2p_{2b}p_{3a}p_{3i}(p_3D\epsilon_2\epsilon_3)_jI'_{11}+2p_{2b}p_{2j}p_{3i}(p_3\epsilon_2D\epsilon_3)_aI'_{11}$$

$$+2p_{2j}p_{3b}p_{3i}(p_3\epsilon_2D\epsilon_3)_aI'_{11}+2p_{2b}p_{2i}p_{3a}(p_3\epsilon_2D\epsilon_3)_jI'_{11}+2p_{2b}p_{3a}p_{3i}(p_3\epsilon_2D\epsilon_3)_jI'_{11}$$

$$+4p_{2j}p_{3a}p_{3i}(p_1N\epsilon_2D\epsilon_3)_bI_5-4p_{2b}p_{3a}p_{3i}(p_1N\epsilon_2D\epsilon_3)_jI_5$$

$$-4p_{2b}p_{3a}p_{3i}(p_2D\epsilon_2D\epsilon_3)_jI_7-2p_{2b}p_{2j}p_{3i}(p_2D\epsilon_3D\epsilon_2)_aI_1$$

$$-2p_{2j}p_{3b}p_{3i}(p_2D\epsilon_3D\epsilon_2)_aI_1-2p_{2b}p_{2i}p_{3a}(p_2D\epsilon_3D\epsilon_2)_jI_1$$

$$+2p_{2b}p_{3a}p_{3i}(p_2D\epsilon_3D\epsilon_2)_jI_1+2\left(p_1N\epsilon_3p_2\right)p_{2j}p_{3i}\epsilon_{2ab}I_9$$

$$+2\left(p_1N\epsilon_3Dp_2\right)p_{2j}p_{3i}\epsilon_{2ab}I_5-2\left(p_1Np_2\right)p_{3j}(p_2\epsilon_3)_i\epsilon_{2ab}I_9$$

$$-2\left(p_1Np_3\right)p_{2i}(p_2\epsilon_3)_j\epsilon_{2ab}I_9+2\left(p_2p_3\right)p_{3j}(p_1N\epsilon_3)_i\epsilon_{2ab}I_9$$

$$
\begin{aligned}
&+4\,(p_1Np_2)\,p_{3j}(p_1N\epsilon_3)_i\epsilon_{2ab}I_{10}-2\,(p_2Dp_3)\,p_{3j}(p_1N\epsilon_3)_i\epsilon_{2ab}I_5\\
&+2\,(p_2p_3)\,p_{2i}(p_1N\epsilon_3)_j\epsilon_{2ab}I_9+2\,(p_2Dp_3)\,p_{2i}(p_1N\epsilon_3)_j\epsilon_{2ab}I_5\\
&-2\,(p_1Np_2)\,p_{3j}(p_2D\epsilon_3)_i\epsilon_{2ab}I_5-2\,(p_1Np_3)\,p_{2i}(p_2D\epsilon_3)_j\epsilon_{2ab}I_5\\
&+2\,(p_2\epsilon_3Dp_3)\,p_{2b}p_{3a}\epsilon_{2ij}I'_6+2\,(p_2D\epsilon_3Dp_3)\,p_{2b}p_{3a}\epsilon_{2ij}I'_7\\
&-2\,(p_1N\epsilon_3p_2)\,p_{2a}p_{3b}\epsilon_{2ij}I_9+2\,(p_1N\epsilon_3Dp_2)\,p_{2a}p_{3b}\epsilon_{2ij}I_5\\
&+2\,(p_2p_3)\,p_{2b}(p_2\epsilon_3)_a\epsilon_{2ij}I_2-2\,(p_1Np_3)\,p_{2b}(p_2\epsilon_3)_a\epsilon_{2ij}I_9\\
&+2\,(p_2p_3)\,p_{3b}(p_2\epsilon_3)_a\epsilon_{2ij}I_2+2\,(p_1Np_2)\,p_{3b}(p_2\epsilon_3)_a\epsilon_{2ij}I_9\\
&-2\,(p_2p_3)\,p_{2b}(p_1N\epsilon_3)_a\epsilon_{2ij}I_9+2\,(p_2Dp_3)\,p_{2b}(p_1N\epsilon_3)_a\epsilon_{2ij}I_5\\
&+2\,(p_1Np_3)\,p_{2b}(p_2D\epsilon_3)_a\epsilon_{2ij}I_5-2\,(p_2Dp_3)\,p_{2b}(p_2D\epsilon_3)_a\epsilon_{2ij}I_1\\
&+2\,(p_1Np_2)\,p_{3b}(p_2D\epsilon_3)_a\epsilon_{2ij}I_5-2\,(p_2Dp_3)\,p_{3b}(p_2D\epsilon_3)_a\epsilon_{2ij}I_1\\
&+2\,(p_2p_3)\,p_{2b}(p_3D\epsilon_3)_a\epsilon_{2ij}I'_6+2\,(p_2Dp_3)\,p_{2b}(p_3D\epsilon_3)_a\epsilon_{2ij}I'_7\\
&-2\,(p_2p_3)\,p_{3a}(p_1N\epsilon_3)_b\epsilon_{2ji}I_9-4\,(p_1Np_2)\,p_{3a}(p_1N\epsilon_3)_b\epsilon_{2ji}I_{10}\\
&+2\,(p_2Dp_3)\,p_{3a}(p_1N\epsilon_3)_b\epsilon_{2ji}I_5+2\,(p_2p_3)\,p_{3a}(p_3D\epsilon_3)_b\epsilon_{2ji}I'_6\\
&+2\,(p_1Np_2)\,p_{3a}(p_3D\epsilon_3)_b\epsilon_{2ji}I'_4+2\,(p_2Dp_3)\,p_{3a}(p_3D\epsilon_3)_b\epsilon_{2ji}I'_7\\
&+(p_1Np_2)\,(p_2p_3)\,\epsilon_{2ji}\epsilon_{3ab}I_9-(p_1Np_3)\,(p_2p_3)\,\epsilon_{2ji}\epsilon_{3ab}I_9\\
&-2\,(p_1Np_2)\,(p_1Np_3)\,\epsilon_{2ji}\epsilon_{3ab}I_{10}+(p_1Np_2)\,(p_2Dp_3)\,\epsilon_{2ji}\epsilon_{3ab}I_5\\
&+(p_1Np_3)\,(p_2Dp_3)\,\epsilon_{2ji}\epsilon_{3ab}I_5+2\,(p_3D\epsilon_2p_3)\,p_{2b}p_{3a}\epsilon_{3ij}I'_{11}\Big)+(2\leftrightarrow3)\,,
\end{aligned}
\tag{2.43}
$$

$$
\begin{aligned}
\mathcal{A}^{(3)}_{C^{(p+1)}BB}=&\frac{2i^{p(p+1)}\sqrt{2}}{(p-2)!}C^{ijk}{}_{b_1\ldots b_{p-2}}\varepsilon^{abcb_1\ldots b_{p-2}}\,\Big(4p_{2c}p_{2j}p_{3a}p_{3i}(\epsilon_2\epsilon_3)_{bk}I_9\\
&+4p_{2c}p_{2j}p_{3a}p_{3i}(\epsilon_2D\epsilon_3)_{bk}I_5+2p_{2c}p_{2k}p_{3i}(p_2\epsilon_3)_j\epsilon_{2ab}I_9\\
&+2p_{2k}p_{3c}p_{3i}(p_2\epsilon_3)_j\epsilon_{2ab}I_9-4p_{2c}p_{2k}p_{3i}(p_1N\epsilon_3)_j\epsilon_{2ab}I_{10}\\
&+2p_{2c}p_{2k}p_{3i}(p_2D\epsilon_3)_j\epsilon_{2ab}I_5+2p_{2k}p_{3c}p_{3i}(p_2D\epsilon_3)_j\epsilon_{2ab}I_5
\end{aligned}
$$

$$+2p_{2c}p_{3b}p_{3i}(p_2\epsilon_3)_a\epsilon_{2jk}I_9 - 2p_{2c}p_{2i}p_{3a}(p_2\epsilon_3)_b\epsilon_{2jk}I_9$$

$$+4p_{2c}p_{2i}p_{3a}(p_1N\epsilon_3)_b\epsilon_{2jk}I_{10} + 2p_{2b}p_{3c}p_{3i}(p_2D\epsilon_3)_a\epsilon_{2jk}I_5$$

$$-2p_{2c}p_{2i}p_{3a}(p_2D\epsilon_3)_b\epsilon_{2jk}I_5 - 2p_{2c}p_{2i}p_{3a}(p_3D\epsilon_3)_b\epsilon_{2jk}I_4'$$

$$-\left(p_2p_3\right)p_{2c}p_{2i}\epsilon_{2jk}\epsilon_{3ab}I_9 + 2\left(p_1Np_3\right)p_{2c}p_{2i}\epsilon_{2jk}\epsilon_{3ab}I_{10}$$

$$-\left(p_2Dp_3\right)p_{2c}p_{2i}\epsilon_{2jk}\epsilon_{3ab}I_5 + \left(p_2p_3\right)p_{2c}p_{3i}\epsilon_{2jk}\epsilon_{3ab}I_9$$

$$-\left(p_2Dp_3\right)p_{2c}p_{3i}\epsilon_{2jk}\epsilon_{3ab}I_5 - \left(p_2p_3\right)p_{2j}p_{3a}\epsilon_{2ki}\epsilon_{3bc}I_9$$

$$-\left(p_2Dp_3\right)p_{2j}p_{3a}\epsilon_{2ki}\epsilon_{3bc}I_5 + \left(p_2p_3\right)p_{3a}p_{3j}\epsilon_{2ki}\epsilon_{3bc}I_9$$

$$+2\left(p_1Np_2\right)p_{3a}p_{3j}\epsilon_{2ki}\epsilon_{3bc}I_{10} - \left(p_2Dp_3\right)p_{3a}p_{3j}\epsilon_{2ki}\epsilon_{3bc}I_5) + (2\leftrightarrow 3)\Big), \qquad (2.44)$$

$$\mathcal{A}^{(4)}_{C^{(p+1)}BB} = \frac{4i^{p(p+1)}\sqrt{2}}{(p-3)!}C^{ijkl}{}_{b_1\dots b_{p-3}}\varepsilon^{abcdb_1\dots b_{p-3}}p_{2d}p_{2j}p_{3a}p_{3i}\epsilon_{2kl}\epsilon_{3bc}I_{10} + (2\leftrightarrow 3). \qquad (2.45)$$

$$\mathcal{A}_{C^{(p+1)}hh} = \mathcal{A}^{(0)}_{C^{(p+1)}hh} + \mathcal{A}^{(1)}_{C^{(p+1)}hh} + \mathcal{A}^{(2)}_{C^{(p+1)}hh} + \mathcal{A}^{(3)}_{C^{(p+1)}hh} + \mathcal{A}^{(4)}_{C^{(p+1)}hh}. \qquad (2.46)$$

$$\mathcal{A}^{(0)}_{C^{(p+1)}hh} = \frac{2i^{p(p+1)}\sqrt{2}}{(p+1)!}C_{b_1\dots b_{p+1}}\varepsilon^{b_1\dots b_{p+1}}\left(-2\left(p_1N\epsilon_2p_3\right)\left(p_1N\epsilon_3p_2\right)I_2\right.$$

$$+2\left(p_1N\epsilon_3p_2\right)\left(p_2D\epsilon_2p_3\right)I_{20} + 2\left(p_1N\epsilon_2p_3\right)\left(p_2D\epsilon_3p_2\right)I_{14}$$

$$-2\left(p_1N\epsilon_2Dp_2\right)\left(p_2\epsilon_3p_2\right)I_{20} - 8\left(p_1N\epsilon_2Dp_2\right)\left(p_1N\epsilon_3p_2\right)I_6$$

$$-4\left(p_1N\epsilon_2Dp_2\right)\left(p_2\epsilon_3Dp_3\right)I_{18} - 2\left(p_1N\epsilon_2Dp_3\right)\left(p_2\epsilon_3p_2\right)I_{14}$$

$$+8\left(p_1N\epsilon_2Dp_3\right)\left(p_1N\epsilon_3p_2\right)I_8 - 2\left(p_1N\epsilon_2Dp_3\right)\left(p_2D\epsilon_3p_2\right)I_{15}$$

$$-2\left(p_1N\epsilon_2Dp_3\right)\left(p_2\epsilon_3Dp_3\right)I_{22} + 2\left(p_1N\epsilon_2Np_1\right)\left(p_2\epsilon_3p_2\right)I_2$$

$$-8\left(p_1N\epsilon_2Np_1\right)\left(p_1N\epsilon_3p_2\right)I_9 + 4\left(p_1N\epsilon_2Np_1\right)\left(p_2\epsilon_3Dp_3\right)I_6'$$

$$-4\left(p_1N\epsilon_2\epsilon_3p_2\right)I_{23} + 2\left(p_1N\epsilon_2\epsilon_3p_2\right)\left(p_2p_3\right)I_{16}$$

$$-8\left(p_1N\epsilon_2Dp_2\right)\left(p_1N\epsilon_3Dp_2\right)I_7 + 2\left(p_1N\epsilon_2Dp_3\right)\left(p_1N\epsilon_3Dp_2\right)I_1$$

$$-8\left(p_1N\epsilon_2Np_1\right)\left(p_1N\epsilon_3Dp_2\right)I_5 + 4\left(p_1N\epsilon_3Dp_3\right)\left(p_3D\epsilon_2p_3\right)I_{22}$$

$$+4\,(p_1N\epsilon_2Dp_2)\,(p_1N\epsilon_3Dp_3)\,I_3-4\,(p_1N\epsilon_2Np_1)\,(p_1N\epsilon_3Dp_3)\,I'_4$$

$$+4\,(p_1N\epsilon_3Np_1)\,(p_3D\epsilon_2p_3)\,I'_{11}+4\,(p_1N\epsilon_2Np_1)\,(p_1N\epsilon_3Np_1)\,I_{10}$$

$$-2\,(p_1N\epsilon_3Dp_2)\,(p_2D\epsilon_2Dp_3)\,I_{21}-4\,(p_1N\epsilon_3Dp_3)\,(p_2D\epsilon_2Dp_3)\,I_{19}$$

$$-4\,(p_1N\epsilon_3Np_1)\,(p_2D\epsilon_2Dp_3)\,I_7-2\,(p_1Np_3)\,(p_2D\epsilon_2\epsilon_3p_2)\,I_{20}$$

$$+2\,(p_1N\epsilon_2p_3)\,(p_2D\epsilon_3Dp_2)\,I_{15}-2\,(p_1N\epsilon_2Dp_2)\,(p_2D\epsilon_3Dp_2)\,I_{21}$$

$$+2\,(p_1N\epsilon_2Np_1)\,(p_2D\epsilon_3Dp_2)\,I_1+2\,(p_1N\epsilon_2p_3)\,(p_2D\epsilon_3Dp_3)\,I_{22}$$

$$+2\,(p_1Np_3)\,(p_2D\epsilon_3\epsilon_2p_3)\,I_{14}-2\,(p_1Np_2)\,(p_2\epsilon_3D\epsilon_2p_3)\,I_{14}$$

$$+2\,(p_1N\epsilon_2D\epsilon_3p_2)\,(p_2p_3)\,I_{14}-2\,(p_1N\epsilon_2\epsilon_3Dp_2)\,(p_2Dp_3)\,I_{15}$$

$$+2\,(p_1N\epsilon_2\epsilon_3Dp_3)\,(p_2p_3)\,I'_{20}-2\,(p_1N\epsilon_2\epsilon_3Dp_3)\,(p_2Dp_3)\,I_{22}$$

$$-2\,(p_1N\epsilon_2\epsilon_3Np_1)\,(p_2p_3)\,I_2+8\,(p_1N\epsilon_2\epsilon_3Np_1)\,(p_2Dp_3)\,I_0$$

$$-4\,(p_1N\epsilon_3D\epsilon_2p_3)\,(p_1Np_2)\,I'_{11}+4\,(p_1N\epsilon_3\epsilon_2Dp_3)\,(p_1Np_2)\,I'_{11}$$

$$-2\,(p_1Np_2)\,(p_2D\epsilon_3D\epsilon_2p_3)\,I_{15}+2\,(p_1Np_2)\,(p_2D\epsilon_3\epsilon_2Dp_3)\,I_{15}$$

$$+2\,(p_1Np_2)\,(p_3D\epsilon_2\epsilon_3Dp_3)\,I_{22}-2\,(p_1Np_2)\,(p_3D\epsilon_3D\epsilon_2p_3)\,I_{22}$$

$$+4\,(p_1N\epsilon_2D\epsilon_3Dp_2)\,I_{24}-2\,(p_1N\epsilon_2D\epsilon_3Dp_2)\,(p_2Dp_3)\,I_{17}$$

$$+2\,(p_1N\epsilon_2D\epsilon_3Dp_3)\,(p_2p_3)\,I_{22}-2\,(p_1N\epsilon_2D\epsilon_3Dp_3)\,(p_2Dp_3)\,I'_{21}$$

$$-8\,(p_1N\epsilon_2D\epsilon_3Np_1)\,(p_2p_3)\,I_0+2\,(p_1N\epsilon_2D\epsilon_3Np_1)\,(p_2Dp_3)\,I_1$$

$$+2\,(p_1Np_3)\,(p_2D\epsilon_2D\epsilon_3Dp_2)\,I_{21}+\mathrm{tr}(D\epsilon_2)\,(p_1Np_2)\,(p_2\epsilon_3p_2)\,I_{20}$$

$$+\mathrm{tr}(D\epsilon_2)\,(p_1Np_3)\,(p_2\epsilon_3p_2)\,I_{20}-2\,\mathrm{tr}(D\epsilon_2)\,(p_1N\epsilon_3p_2)\,(p_2p_3)\,I_{20}$$

$$+4\,\mathrm{tr}(D\epsilon_2)\,(p_1N\epsilon_3p_2)\,(p_1Np_2)\,I_6+2\,\mathrm{tr}(D\epsilon_2)\,(p_1Np_2)\,(p_2\epsilon_3Dp_3)\,I_{18}$$

$$+4\,\mathrm{tr}(D\epsilon_2)\,(p_1N\epsilon_3Dp_2)\,(p_1Np_2)\,I_7+2\,\mathrm{tr}(D\epsilon_2)\,(p_1N\epsilon_3Dp_2)\,(p_2Dp_3)\,I_{21}$$

$$-2\,\mathrm{tr}(D\epsilon_2)\,(p_1N\epsilon_3Dp_3)\,(p_2p_3)\,I_{18}-4\,\mathrm{tr}(D\epsilon_2)\,(p_1N\epsilon_3Dp_3)\,(p_1Np_2)\,I_3$$

$$+2\,\mathrm{tr}(D\epsilon_2)\,(p_1N\epsilon_3Dp_3)\,(p_2Dp_3)\,I_{19}-2\,\mathrm{tr}(D\epsilon_2)\,(p_1N\epsilon_3Np_1)\,(p_2p_3)\,I_6$$

$$+2\,\mathrm{tr}(D\epsilon_2)\,(p_1N\epsilon_3Np_1)\,(p_1Np_2)\,I_4$$

$$
\begin{aligned}
&+2\,\mathrm{tr}(D\epsilon_2)\,(p_1N\epsilon_3Np_1)\,(p_2Dp_3)\,I_7+\mathrm{tr}(D\epsilon_2)\,(p_1Np_2)\,(p_2D\epsilon_3Dp_2)\,I_{21}\\
&-\mathrm{tr}(D\epsilon_2)\,(p_1Np_3)\,(p_2D\epsilon_3Dp_2)\,I_{21}+2\,\mathrm{tr}(D\epsilon_2)\,(p_1Np_2)\,(p_2D\epsilon_3Dp_3)\,I_{19}\\
&-2\,\mathrm{tr}(D\epsilon_3)\,(p_1Np_3)\,(p_3D\epsilon_2p_3)\,I_{22}-\mathrm{tr}(D\epsilon_2)\,\mathrm{tr}(D\epsilon_3)\,(p_1Np_2)\,(p_2p_3)\,I_{18}\\
&+\mathrm{tr}(D\epsilon_2)\,\mathrm{tr}(D\epsilon_3)\,(p_1Np_2)\,(p_1Np_3)\,I_3-\mathrm{tr}(D\epsilon_2)\,\mathrm{tr}(D\epsilon_3)\,(p_1Np_2)\,(p_2Dp_3)\,I_{19}\\
&+2\,\mathrm{tr}(\epsilon_2\epsilon_3)\,(p_1Np_2)\,I_{23}-\mathrm{tr}(\epsilon_2\epsilon_3)\,(p_1Np_2)\,(p_2p_3)\,I_{16}\\
&+\mathrm{tr}(\epsilon_2\epsilon_3)\,(p_1Np_2)\,(p_1Np_3)\,I_2-\mathrm{tr}(\epsilon_2\epsilon_3)\,(p_1Np_2)\,(p_2Dp_3)\,I_{14}\\
&-2\,\mathrm{tr}(D\epsilon_2D\epsilon_3)\,(p_1Np_2)\,I_{24}+\mathrm{tr}(D\epsilon_2D\epsilon_3)\,(p_1Np_2)\,(p_2p_3)\,I_{15}\\
&-\mathrm{tr}(D\epsilon_2D\epsilon_3)\,(p_1Np_2)\,(p_1Np_3)\,I_1+\mathrm{tr}(D\epsilon_2D\epsilon_3)\,(p_1Np_2)\,(p_2Dp_3)\,I_{17})\\
&+(2\leftrightarrow 3),
\end{aligned}
\tag{2.47}
$$

$$
\begin{aligned}
\mathcal{A}^{(1)}_{C^{(p+1)}hh}=&\frac{2i^{p(p+1)}\sqrt{2}}{p!}C^{i}{}_{b_1\ldots b_p}\varepsilon^{ab_1\ldots b_p}\,(-2\,(p_2\epsilon_3D\epsilon_2p_3)\,p_{2a}p_{2i}I_{14}\\
&+2\,(p_2\epsilon_3\epsilon_2Dp_3)\,p_{2a}p_{2i}I_{14}-2\,(p_3D\epsilon_3\epsilon_2p_3)\,p_{2a}p_{2i}I'_{20}\\
&-4\,(p_1N\epsilon_3D\epsilon_2p_3)\,p_{2a}p_{2i}I'_{11}+4\,(p_1N\epsilon_3\epsilon_2Dp_3)\,p_{2a}p_{2i}I'_{11}\\
&-2\,(p_2D\epsilon_3D\epsilon_2p_3)\,p_{2a}p_{2i}I_{15}+2\,(p_2D\epsilon_3\epsilon_2Dp_3)\,p_{2a}p_{2i}I_{15}\\
&+2\,(p_3D\epsilon_2\epsilon_3Dp_3)\,p_{2a}p_{2i}I_{22}-2\,(p_3D\epsilon_3D\epsilon_2p_3)\,p_{2a}p_{2i}I_{22}\\
&+2\,(p_3D\epsilon_2D\epsilon_3Dp_3)\,p_{2a}p_{2i}I'_{21}+\mathrm{tr}(D\epsilon_2)\,(p_2\epsilon_3p_2)\,p_{2a}p_{2i}I_{20}\\
&+4\,\mathrm{tr}(D\epsilon_2)\,(p_1N\epsilon_3p_2)\,p_{2a}p_{2i}I_6+2\,\mathrm{tr}(D\epsilon_2)\,(p_2\epsilon_3Dp_3)\,p_{2a}p_{2i}I_{18}\\
&+4\,\mathrm{tr}(D\epsilon_2)\,(p_1N\epsilon_3Dp_2)\,p_{2a}p_{2i}I_7-4\,\mathrm{tr}(D\epsilon_2)\,(p_1N\epsilon_3Dp_3)\,p_{2a}p_{2i}I_3\\
&+2\,\mathrm{tr}(D\epsilon_2)\,(p_1N\epsilon_3Np_1)\,p_{2a}p_{2i}I_4+\mathrm{tr}(D\epsilon_2)\,(p_2D\epsilon_3Dp_2)\,p_{2a}p_{2i}I_{21}\\
&+2\,\mathrm{tr}(D\epsilon_2)\,(p_2D\epsilon_3Dp_3)\,p_{2a}p_{2i}I_{19}+\mathrm{tr}(D\epsilon_3)\,(p_3\epsilon_2p_3)\,p_{2a}p_{2i}I'_{20}\\
&-\mathrm{tr}(D\epsilon_3)\,(p_3D\epsilon_2Dp_3)\,p_{2a}p_{2i}I'_{21}-\mathrm{tr}(D\epsilon_2)\,\mathrm{tr}(D\epsilon_3)\,(p_2p_3)\,p_{2a}p_{2i}I_{18}\\
&+2\,\mathrm{tr}(D\epsilon_2)\,\mathrm{tr}(D\epsilon_3)\,(p_1Np_3)\,p_{2a}p_{2i}I_3
\end{aligned}
$$

$$- \operatorname{tr}(D\epsilon_2) \operatorname{tr}(D\epsilon_3) \, (p_2 D p_3) \, p_{2a} p_{2i} I_{19} + 2 \operatorname{tr}(\epsilon_2\epsilon_3) p_{2a} p_{2i} I_{23}$$

$$- \operatorname{tr}(\epsilon_2\epsilon_3) \, (p_2 p_3) \, p_{2a} p_{2i} I_{16} + 2 \operatorname{tr}(\epsilon_2\epsilon_3) \, (p_1 N p_3) \, p_{2a} p_{2i} I_2$$

$$- \operatorname{tr}(\epsilon_2\epsilon_3) \, (p_2 D p_3) \, p_{2a} p_{2i} I_{14} - 2 \operatorname{tr}(D\epsilon_2 D\epsilon_3) p_{2a} p_{2i} I_{24}$$

$$+ \operatorname{tr}(D\epsilon_2 D\epsilon_3) \, (p_2 p_3) \, p_{2a} p_{2i} I_{15} - 2 \operatorname{tr}(D\epsilon_2 D\epsilon_3) \, (p_1 N p_3) \, p_{2a} p_{2i} I_1$$

$$+ \operatorname{tr}(D\epsilon_2 D\epsilon_3) \, (p_2 D p_3) \, p_{2a} p_{2i} I_{17} - 2 \, (p_1 N \epsilon_2 \epsilon_3 p_2) \, p_{2i} p_{3a} I_2$$

$$+2 \, (p_1 N \epsilon_3 \epsilon_2 p_3) \, p_{2i} p_{3a} I_2 - 2 \, (p_2 \epsilon_3 D \epsilon_2 p_3) \, p_{2i} p_{3a} I_{14}$$

$$+2 \, (p_2 \epsilon_3 \epsilon_2 D p_3) \, p_{2i} p_{3a} I_{14} - 2 \, (p_3 D \epsilon_3 \epsilon_2 p_3) \, p_{2i} p_{3a} I'_{20}$$

$$-4 \, (p_1 N \epsilon_2 \epsilon_3 D p_3) \, p_{2i} p_{3a} I'_6 + 4 \, (p_1 N \epsilon_2 \epsilon_3 N p_1) \, p_{2i} p_{3a} I_9$$

$$-2 \, (p_1 N \epsilon_3 D \epsilon_2 p_3) \, p_{2i} p_{3a} I'_{11} + 2 \, (p_1 N \epsilon_3 \epsilon_2 D p_3) \, p_{2i} p_{3a} I'_{11}$$

$$-2 \, (p_2 D \epsilon_3 D \epsilon_2 p_3) \, p_{2i} p_{3a} I_{15} + 2 \, (p_2 D \epsilon_3 \epsilon_2 D p_3) \, p_{2i} p_{3a} I_{15}$$

$$+2 \, (p_3 D \epsilon_2 \epsilon_3 D p_3) \, p_{2i} p_{3a} I_{22} - 2 \, (p_3 D \epsilon_3 D \epsilon_2 p_3) \, p_{2i} p_{3a} I_{22}$$

$$-2 \, (p_1 N \epsilon_2 D \epsilon_3 D p_2) \, p_{2i} p_{3a} I_1 + 4 \, (p_1 N \epsilon_2 D \epsilon_3 D p_3) \, p_{2i} p_{3a} I'_7$$

$$+4 \, (p_1 N \epsilon_2 D \epsilon_3 N p_1) \, p_{2i} p_{3a} I_5 - 2 \, (p_1 N \epsilon_3 D \epsilon_2 D p_3) \, p_{2i} p_{3a} I_1$$

$$+2 \, (p_3 D \epsilon_2 D \epsilon_3 D p_3) \, p_{2i} p_{3a} I'_{21} + \operatorname{tr}(D\epsilon_2) \, (p_2 \epsilon_3 p_2) \, p_{2i} p_{3a} I_{20}$$

$$+2 \operatorname{tr}(D\epsilon_2) \, (p_1 N \epsilon_3 p_2) \, p_{2i} p_{3a} I_6 + 2 \operatorname{tr}(D\epsilon_2) \, (p_2 \epsilon_3 D p_3) \, p_{2i} p_{3a} I_{18}$$

$$+2 \operatorname{tr}(D\epsilon_2) \, (p_1 N \epsilon_3 D p_2) \, p_{2i} p_{3a} I_7 + \operatorname{tr}(D\epsilon_2) \, (p_2 D \epsilon_3 D p_2) \, p_{2i} p_{3a} I_{21}$$

$$+2 \operatorname{tr}(D\epsilon_2) \, (p_2 D \epsilon_3 D p_3) \, p_{2i} p_{3a} I_{19} + \operatorname{tr}(D\epsilon_3) \, (p_3 \epsilon_2 p_3) \, p_{2i} p_{3a} I'_{20}$$

$$+2 \operatorname{tr}(D\epsilon_3) \, (p_1 N \epsilon_2 p_3) \, p_{2i} p_{3a} I'_6 - 2 \operatorname{tr}(D\epsilon_3) \, (p_1 N \epsilon_2 D p_3) \, p_{2i} p_{3a} I'_7$$

$$- \operatorname{tr}(D\epsilon_3) \, (p_3 D \epsilon_2 D p_3) \, p_{2i} p_{3a} I'_{21} - \operatorname{tr}(D\epsilon_2) \operatorname{tr}(D\epsilon_3) \, (p_2 p_3) \, p_{2i} p_{3a} I_{18}$$

$$- \operatorname{tr}(D\epsilon_2) \operatorname{tr}(D\epsilon_3) \, (p_2 D p_3) \, p_{2i} p_{3a} I_{19} + 2tr \, (\epsilon_2 \epsilon_3) \, p_{2i} p_{3a} I_{23}$$

$$- \operatorname{tr}(\epsilon_2\epsilon_3) \, (p_2 p_3) \, p_{2i} p_{3a} I_{16} - \operatorname{tr}(\epsilon_2\epsilon_3) \, (p_2 D p_3) \, p_{2i} p_{3a} I_{14}$$

$$-2 \operatorname{tr}(D\epsilon_2 D\epsilon_3) p_{2i} p_{3a} I_{24} + \operatorname{tr}(D\epsilon_2 D\epsilon_3) \, (p_2 p_3) \, p_{2i} p_{3a} I_{15}$$

$$+\,\mathrm{tr}(D\epsilon_2 D\epsilon_3)\,(p_2 D p_3)\,p_{2i}p_{3a}I_{17} - 2\,(p_1 N\epsilon_3 D\epsilon_2 p_3)\,p_{2a}p_{3i}I'_{11}$$

$$-2\,(p_1 N\epsilon_3\epsilon_2 D p_3)\,p_{2a}p_{3i}I'_{11} - 2\,\mathrm{tr}(D\epsilon_3)\,(p_3 D\epsilon_2 p_3)\,p_{2a}p_{3i}I_{22}$$

$$-2\,\mathrm{tr}(D\epsilon_3)\,(p_3 D\epsilon_2 p_3)\,p_{3a}p_{3i}I_{22} + 2\,(p_1 N\epsilon_2 p_3)\,p_{2i}(p_2\epsilon_3)_a I_2$$

$$+2\,\mathrm{tr}(D\epsilon_2)\,(p_1 N p_3)\,p_{2i}(p_2\epsilon_3)_a I_6 + 2\,(p_1 N\epsilon_2 p_3)\,p_{3i}(p_2\epsilon_3)_a I_2$$

$$+4\,(p_1 N\epsilon_2 D p_2)\,p_{3i}(p_2\epsilon_3)_a I_6 + 2\,(p_1 N\epsilon_2 D p_3)\,p_{3i}(p_2\epsilon_3)_a I'_{11}$$

$$+4\,(p_1 N\epsilon_2 N p_1)\,p_{3i}(p_2\epsilon_3)_a I_9 - 2\,\mathrm{tr}(D\epsilon_2)\,(p_1 N p_2)\,p_{3i}(p_2\epsilon_3)_a I_6$$

$$-2\,(p_1 N\epsilon_2 p_3)\,p_{2a}(p_2\epsilon_3)_i I_2 + 2\,(p_2 D\epsilon_2 p_3)\,p_{2a}(p_2\epsilon_3)_i I_{20}$$

$$+2\,(p_3 D\epsilon_2 p_3)\,p_{2a}(p_2\epsilon_3)_i I_{14} - 2\,(p_3 D\epsilon_2 D p_3)\,p_{2a}(p_2\epsilon_3)_i I_{15}$$

$$-2\,\mathrm{tr}(D\epsilon_2)\,(p_2 p_3)\,p_{2a}(p_2\epsilon_3)_i I_{20} - 2\,\mathrm{tr}(D\epsilon_2)\,(p_1 N p_3)\,p_{2a}(p_2\epsilon_3)_i I_6$$

$$-2\,(p_1 N\epsilon_2 p_3)\,p_{3a}(p_2\epsilon_3)_i I_2 + 2\,(p_2 D\epsilon_2 p_3)\,p_{3a}(p_2\epsilon_3)_i I_{20}$$

$$+2\,(p_3 D\epsilon_2 p_3)\,p_{3a}(p_2\epsilon_3)_i I_{14} - 4\,(p_1 N\epsilon_2 D p_2)\,p_{3a}(p_2\epsilon_3)_i I_6$$

$$-4\,(p_1 N\epsilon_2 N p_1)\,p_{3a}(p_2\epsilon_3)_i I_9 - 2\,(p_3 D\epsilon_2 D p_3)\,p_{3a}(p_2\epsilon_3)_i I_{15}$$

$$-2\,\mathrm{tr}(D\epsilon_2)\,(p_2 p_3)\,p_{3a}(p_2\epsilon_3)_i I_{20} + 2\,\mathrm{tr}(D\epsilon_2)\,(p_1 N p_2)\,p_{3a}(p_2\epsilon_3)_i I_6$$

$$+2\,(p_1 N\epsilon_3 D p_2)\,p_{3i}(p_3\epsilon_2)_a I'_{11} - 2\,(p_1 N p_2)\,(p_2\epsilon_3)_i(p_3\epsilon_2)_a I_2$$

$$-2\,(p_1 N p_3)\,(p_2\epsilon_3)_i(p_3\epsilon_2)_a I_2 + 2\,(p_1 N\epsilon_3 D p_2)\,p_{2a}(p_3\epsilon_2)_i I'_{13}$$

$$+2\,(p_2 D\epsilon_3 D p_3)\,p_{2a}(p_3\epsilon_2)_i I_{22} + 2\,(p_1 N\epsilon_3 D p_2)\,p_{3a}(p_3\epsilon_2)_i I'_{11}$$

$$+2\,(p_2 D\epsilon_3 D p_3)\,p_{3a}(p_3\epsilon_2)_i I_{22} - 2\,(p_1 N p_2)\,(p_2 p_3)\,(\epsilon_2\epsilon_3)_{ai} I_2$$

$$+2\,(p_1 N p_3)\,(p_2 p_3)\,(\epsilon_2\epsilon_3)_{ai} I_2 + 4\,(p_1 N p_2)\,(p_1 N p_3)\,(\epsilon_2\epsilon_3)_{ai} I_9$$

$$+2\,(p_1 N p_3)\,(p_2 D p_3)\,(\epsilon_2\epsilon_3)_{ai} I'_{11} - 2\,(p_1 N p_3)\,(p_2 D p_3)\,(\epsilon_2\epsilon_3)_{ia} I'_{11}$$

$$-2\,(p_2\epsilon_3 p_2)\,p_{2i}(p_1 N\epsilon_2)_a I_2 + 8\,(p_1 N\epsilon_3 p_2)\,p_{2i}(p_1 N\epsilon_2)_a I_9$$

$$-4\,(p_2\epsilon_3 D p_3)\,p_{2i}(p_1 N\epsilon_2)_a I'_6 + 8\,(p_1 N\epsilon_3 D p_2)\,p_{2i}(p_1 N\epsilon_2)_a I_5$$

$$+4\,(p_1 N\epsilon_3 D p_3)\,p_{2i}(p_1 N\epsilon_2)_a I'_4 - 8\,(p_1 N\epsilon_3 N p_1)\,p_{2i}(p_1 N\epsilon_2)_a I_{10}$$

$$-2\,(p_2 D\epsilon_3 D p_2)\,p_{2_i}(p_1 N\epsilon_2)_a I_1 + 4\,(p_2 D\epsilon_3 D p_3)\,p_{2_i}(p_1 N\epsilon_2)_a I_7'$$

$$+2\,\mathrm{tr}(D\epsilon_3)\,(p_2 p_3)\,p_{2_i}(p_1 N\epsilon_2)_a I_6' - 2\,\mathrm{tr}(D\epsilon_3)\,(p_1 N p_3)\,p_{2_i}(p_1 N\epsilon_2)_a I_4'$$

$$-2\,\mathrm{tr}(D\epsilon_3)\,(p_2 D p_3)\,p_{2_i}(p_1 N\epsilon_2)_a I_7' - 2\,(p_2\epsilon_3 p_2)\,p_{3_i}(p_1 N\epsilon_2)_a I_2$$

$$+4\,(p_1 N\epsilon_3 p_2)\,p_{3_i}(p_1 N\epsilon_2)_a I_9 - 4\,(p_1 N\epsilon_3 D p_2)\,p_{3_i}(p_1 N\epsilon_2)_a I_5$$

$$+2\,(p_2 D\epsilon_3 D p_2)\,p_{3_i}(p_1 N\epsilon_2)_a I_1 - 2\,\mathrm{tr}(D\epsilon_3)\,(p_2 p_3)\,p_{3_i}(p_1 N\epsilon_2)_a I_6'$$

$$-2\,\mathrm{tr}(D\epsilon_3)\,(p_2 D p_3)\,p_{3_i}(p_1 N\epsilon_2)_a I_7' + 4\,(p_2 p_3)\,(p_1 N\epsilon_2)_a(p_2\epsilon_3)_i I_2$$

$$-4\,(p_1 N p_3)\,(p_1 N\epsilon_2)_a(p_2\epsilon_3)_i I_9 + 2\,(p_2\epsilon_3 p_2)\,p_{2_a}(p_1 N\epsilon_2)_i I_2$$

$$-8\,(p_1 N\epsilon_3 p_2)\,p_{2_a}(p_1 N\epsilon_2)_i I_9 + 4\,(p_2\epsilon_3 D p_3)\,p_{2_a}(p_1 N\epsilon_2)_i I_6'$$

$$-8\,(p_1 N\epsilon_3 D p_2)\,p_{2_a}(p_1 N\epsilon_2)_i I_5 - 4\,(p_1 N\epsilon_3 D p_3)\,p_{2_a}(p_1 N\epsilon_2)_i I_4'$$

$$+8\,(p_1 N\epsilon_3 N p_1)\,p_{2_a}(p_1 N\epsilon_2)_i I_{10} + 2\,(p_2 D\epsilon_3 D p_2)\,p_{2_a}(p_1 N\epsilon_2)_i I_1$$

$$-4\,(p_2 D\epsilon_3 D p_3)\,p_{2_a}(p_1 N\epsilon_2)_i I_7' - 2\,\mathrm{tr}(D\epsilon_3)\,(p_2 p_3)\,p_{2_a}(p_1 N\epsilon_2)_i I_6'$$

$$+2\,\mathrm{tr}(D\epsilon_3)\,(p_1 N p_3)\,p_{2_a}(p_1 N\epsilon_2)_i I_4' + 2\,\mathrm{tr}(D\epsilon_3)\,(p_2 D p_3)\,p_{2_a}(p_1 N\epsilon_2)_i I_7'$$

$$+2\,(p_2\epsilon_3 p_2)\,p_{3_a}(p_1 N\epsilon_2)_i I_2 - 4\,(p_1 N\epsilon_3 p_2)\,p_{3_a}(p_1 N\epsilon_2)_i I_9$$

$$+4\,(p_2\epsilon_3 D p_3)\,p_{3_a}(p_1 N\epsilon_2)_i I_6' - 4\,(p_1 N\epsilon_3 D p_2)\,p_{3_a}(p_1 N\epsilon_2)_i I_5$$

$$+2\,(p_2 D\epsilon_3 D p_2)\,p_{3_a}(p_1 N\epsilon_2)_i I_1 - 4\,(p_2 D\epsilon_3 D p_3)\,p_{3_a}(p_1 N\epsilon_2)_i I_7'$$

$$-2\,\mathrm{tr}(D\epsilon_3)\,(p_2 p_3)\,p_{3_a}(p_1 N\epsilon_2)_i I_6' + 2\,\mathrm{tr}(D\epsilon_3)\,(p_2 D p_3)\,p_{3_a}(p_1 N\epsilon_2)_i I_7'$$

$$-4\,(p_1 N p_3)\,(p_1 N\epsilon_2)_i(p_2\epsilon_3)_a I_9 - 4\,(p_3 D\epsilon_2 p_3)\,p_{3_i}(p_1 N\epsilon_3)_a I_{11}'$$

$$+4\,(p_2 p_3)\,(p_1 N\epsilon_2)_i(p_1 N\epsilon_3)_a I_9 + 4\,(p_2 D p_3)\,(p_1 N\epsilon_2)_i(p_1 N\epsilon_3)_a I_5$$

$$+4\,(p_3 D\epsilon_2 p_3)\,p_{2_a}(p_1 N\epsilon_3)_i I_{11}' + 4\,(p_3 D\epsilon_2 p_3)\,p_{3_a}(p_1 N\epsilon_3)_i I_{11}'$$

$$-2\,(p_2\epsilon_3 p_2)\,p_{2_a}(p_2 D\epsilon_2)_i I_{20} - 8\,(p_1 N\epsilon_3 p_2)\,p_{2_a}(p_2 D\epsilon_2)_i I_6$$

$$-4\,(p_2\epsilon_3 D p_3)\,p_{2_a}(p_2 D\epsilon_2)_i I_{18} - 8\,(p_1 N\epsilon_3 D p_2)\,p_{2_a}(p_2 D\epsilon_2)_i I_7$$

$$+8\,(p_1 N\epsilon_3 D p_3)\,p_{2_a}(p_2 D\epsilon_2)_i I_3 - 4\,(p_1 N\epsilon_3 N p_1)\,p_{2_a}(p_2 D\epsilon_2)_i I_4$$

$$-2\,(p_2D\epsilon_3Dp_2)\,p_{2a}(p_2D\epsilon_2)_iI_{21}-4\,(p_2D\epsilon_3Dp_3)\,p_{2a}(p_2D\epsilon_2)_iI_{19}$$

$$+2\,\mathrm{tr}(D\epsilon_3)\,(p_2p_3)\,p_{2a}(p_2D\epsilon_2)_iI_{18}-4\,\mathrm{tr}(D\epsilon_3)\,(p_1Np_3)\,p_{2a}(p_2D\epsilon_2)_iI_3$$

$$+2\,\mathrm{tr}(D\epsilon_3)\,(p_2Dp_3)\,p_{2a}(p_2D\epsilon_2)_iI_{19}-2\,(p_2\epsilon_3p_2)\,p_{3a}(p_2D\epsilon_2)_iI_{20}$$

$$-4\,(p_1N\epsilon_3p_2)\,p_{3a}(p_2D\epsilon_2)_iI_6-4\,(p_2\epsilon_3Dp_3)\,p_{3a}(p_2D\epsilon_2)_iI_{18}$$

$$-4\,(p_1N\epsilon_3Dp_2)\,p_{3a}(p_2D\epsilon_2)_iI_7-2\,(p_2D\epsilon_3Dp_2)\,p_{3a}(p_2D\epsilon_2)_iI_{21}$$

$$-4\,(p_2D\epsilon_3Dp_3)\,p_{3a}(p_2D\epsilon_2)_iI_{19}+2\,\mathrm{tr}(D\epsilon_3)\,(p_2p_3)\,p_{3a}(p_2D\epsilon_2)_iI_{18}$$

$$+2\,\mathrm{tr}(D\epsilon_3)\,(p_2Dp_3)\,p_{3a}(p_2D\epsilon_2)_iI_{19}-4\,(p_1Np_3)\,(p_2D\epsilon_2)_i(p_2\epsilon_3)_aI_6$$

$$+4\,(p_2p_3)\,(p_1N\epsilon_3)_a(p_2D\epsilon_2)_iI_6+4\,(p_2Dp_3)\,(p_1N\epsilon_3)_a(p_2D\epsilon_2)_iI_7$$

$$+2\,(p_1N\epsilon_2Dp_3)\,p_{2i}(p_2D\epsilon_3)_aI_1+2tr\,(D\epsilon_2)\,(p_1Np_3)\,p_{2i}(p_2D\epsilon_3)_aI_7$$

$$-2\,(p_1N\epsilon_2p_3)\,p_{3i}(p_2D\epsilon_3)_aI'_{11}+4\,(p_1N\epsilon_2Dp_2)\,p_{3i}(p_2D\epsilon_3)_aI_7$$

$$-2\,(p_1N\epsilon_2Dp_3)\,p_{3i}(p_2D\epsilon_3)_aI_1+4\,(p_1N\epsilon_2Np_1)\,p_{3i}(p_2D\epsilon_3)_aI_5$$

$$-2\,\mathrm{tr}(D\epsilon_2)\,(p_1Np_2)\,p_{3i}(p_2D\epsilon_3)_aI_7+2\,(p_1Np_3)\,(p_2D\epsilon_3)_a(p_3\epsilon_2)_iI'_{11}$$

$$-4\,(p_1Np_3)\,(p_1N\epsilon_2)_i(p_2D\epsilon_3)_aI_5-4\,(p_1Np_3)\,(p_2D\epsilon_2)_i(p_2D\epsilon_3)_aI_7$$

$$-2\,(p_3\epsilon_2p_3)\,p_{2a}(p_2D\epsilon_3)_iI_{14}+2\,(p_3D\epsilon_2p_3)\,p_{2a}(p_2D\epsilon_3)_iI_{15}$$

$$+2\,(p_1N\epsilon_2Dp_3)\,p_{2a}(p_2D\epsilon_3)_iI_1-2\,(p_2D\epsilon_2Dp_3)\,p_{2a}(p_2D\epsilon_3)_iI_{21}$$

$$+2\,\mathrm{tr}(D\epsilon_2)\,(p_1Np_3)\,p_{2a}(p_2D\epsilon_3)_iI_7+2\,\mathrm{tr}(D\epsilon_2)\,(p_2Dp_3)\,p_{2a}(p_2D\epsilon_3)_iI_{21}$$

$$-2\,(p_3\epsilon_2p_3)\,p_{3a}(p_2D\epsilon_3)_iI_{14}+2\,(p_3D\epsilon_2p_3)\,p_{3a}(p_2D\epsilon_3)_iI_{15}$$

$$-4\,(p_1N\epsilon_2Dp_2)\,p_{3a}(p_2D\epsilon_3)_iI_7+2\,(p_1N\epsilon_2Dp_3)\,p_{3a}(p_2D\epsilon_3)_iI_1$$

$$-4\,(p_1N\epsilon_2Np_1)\,p_{3a}(p_2D\epsilon_3)_iI_5-2\,(p_2D\epsilon_2Dp_3)\,p_{3a}(p_2D\epsilon_3)_iI_{21}$$

$$+2\,\mathrm{tr}(D\epsilon_2)\,(p_1Np_2)\,p_{3a}(p_2D\epsilon_3)_iI_7+2\,\mathrm{tr}(D\epsilon_2)\,(p_2Dp_3)\,p_{3a}(p_2D\epsilon_3)_iI_{21}$$

$$-2\,(p_1Np_3)\,(p_2D\epsilon_3)_i(p_3\epsilon_2)_aI'_{11}+4\,(p_1Np_3)\,(p_1N\epsilon_2)_a(p_2D\epsilon_3)_iI_5$$

$$-4\,(p_2Dp_3)\,(p_1N\epsilon_2)_a(p_2D\epsilon_3)_iI_1-2\,(p_1Np_3)\,p_{2i}(p_2\epsilon_3\epsilon_2)_aI_2$$

$$+2\,(p_1Np_2)\,p_{3i}(p_2\epsilon_3\epsilon_2)_aI_2-4p_{2a}(p_2\epsilon_3\epsilon_2)_iI_{23}+2\,(p_2p_3)\,p_{2a}(p_2\epsilon_3\epsilon_2)_iI_{16}$$

$$-2\,(p_1Np_3)\,p_{2a}(p_2\epsilon_3\epsilon_2)_iI_2-4p_{3a}(p_2\epsilon_3\epsilon_2)_iI_{23}+2\,(p_2p_3)\,p_{3a}(p_2\epsilon_3\epsilon_2)_iI_{16}$$

$$+2\,(p_1Np_2)\,p_{3a}(p_2\epsilon_3\epsilon_2)_iI_2+2\,(p_1N\epsilon_3p_2)\,p_{3i}(p_3D\epsilon_2)_aI'_{11}$$

$$-2\,(p_1Np_3)\,(p_2\epsilon_3)_i(p_3D\epsilon_2)_aI'_{11}+2\,(p_1Np_2)\,(p_2D\epsilon_3)_i(p_3D\epsilon_2)_aI_1$$

$$-2\,(p_1Np_3)\,(p_2D\epsilon_3)_i(p_3D\epsilon_2)_aI_1-2\,(p_1N\epsilon_3p_2)\,p_{2a}(p_3D\epsilon_2)_iI'_{13}$$

$$-2\,(p_2\epsilon_3Dp_3)\,p_{2a}(p_3D\epsilon_2)_iI_{22}-2\,(p_1N\epsilon_3p_2)\,p_{3a}(p_3D\epsilon_2)_iI'_{11}$$

$$-2\,(p_2\epsilon_3Dp_3)\,p_{3a}(p_3D\epsilon_2)_iI_{22}-2\,(p_1Np_3)\,(p_2\epsilon_3)_a(p_3D\epsilon_2)_iI'_{11}$$

$$+4\,(p_3D\epsilon_2p_3)\,p_{2a}(p_3D\epsilon_3)_iI_{22}+4\,(p_3D\epsilon_2p_3)\,p_{3a}(p_3D\epsilon_3)_iI_{22}$$

$$+2\,(p_1Np_3)\,(p_2p_3)\,(\epsilon_2D\epsilon_3)_{ai}I'_{11}-4\,(p_1Np_2)\,(p_1Np_3)\,(\epsilon_2D\epsilon_3)_{ai}I_5$$

$$+2\,(p_1Np_2)\,(p_2Dp_3)\,(\epsilon_2D\epsilon_3)_{ai}I_1+2\,(p_1Np_3)\,(p_2Dp_3)\,(\epsilon_2D\epsilon_3)_{ai}I_1$$

$$+2\,(p_1Np_3)\,(p_2p_3)\,(\epsilon_2D\epsilon_3)_{ia}I'_{11}-2\,(p_2p_3)\,p_{2i}(p_1N\epsilon_2\epsilon_3)_aI_2$$

$$+4\,(p_1Np_3)\,p_{2i}(p_1N\epsilon_2\epsilon_3)_aI_9+2\,(p_2p_3)\,p_{3i}(p_1N\epsilon_2\epsilon_3)_aI_2$$

$$+2\,(p_2Dp_3)\,p_{3i}(p_1N\epsilon_2\epsilon_3)_aI'_{11}-2\,(p_2p_3)\,p_{2a}(p_1N\epsilon_2\epsilon_3)_iI_2$$

$$+4\,(p_1Np_3)\,p_{2a}(p_1N\epsilon_2\epsilon_3)_iI_9-2\,(p_2p_3)\,p_{3a}(p_1N\epsilon_2\epsilon_3)_iI_2$$

$$-2\,(p_2Dp_3)\,p_{3i}(p_1N\epsilon_3\epsilon_2)_aI'_{11}-2\,(p_2Dp_3)\,p_{2a}(p_1N\epsilon_3\epsilon_2)_iI'_{12}$$

$$-2\,(p_2Dp_3)\,p_{3a}(p_1N\epsilon_3\epsilon_2)_iI'_{11}+2\,(p_2p_3)\,p_{2a}(p_2D\epsilon_2\epsilon_3)_iI_{20}$$

$$+4\,(p_1Np_3)\,p_{2a}(p_2D\epsilon_2\epsilon_3)_iI_6+2\,(p_2p_3)\,p_{3a}(p_2D\epsilon_2\epsilon_3)_iI_{20}$$

$$-2\,(p_2Dp_3)\,p_{2a}(p_2D\epsilon_3\epsilon_2)_iI_{15}-2\,(p_2Dp_3)\,p_{3a}(p_2D\epsilon_3\epsilon_2)_iI_{15}$$

$$+2\,(p_2p_3)\,p_{2a}(p_2\epsilon_3D\epsilon_2)_iI_{14}+2\,(p_2p_3)\,p_{3a}(p_2\epsilon_3D\epsilon_2)_iI_{14}$$

$$+2\,(p_1Np_3)\,p_{2i}(p_3D\epsilon_2\epsilon_3)_aI'_{11}-2\,(p_1Np_2)\,p_{3i}(p_3D\epsilon_2\epsilon_3)_aI'_{11}$$

$$+2\,(p_1Np_3)\,p_{2a}(p_3D\epsilon_2\epsilon_3)_iI'_{11}+2\,(p_1Np_2)\,p_{3a}(p_3D\epsilon_2\epsilon_3)_iI'_{11}$$

$$-2\,(p_2Dp_3)\,p_{2a}(p_3D\epsilon_3\epsilon_2)_iI_{22}-2\,(p_2Dp_3)\,p_{3a}(p_3D\epsilon_3\epsilon_2)_iI_{22}$$

$$-2\,(p_1Np_3)\,p_{2i}(p_3\epsilon_2D\epsilon_3)_aI'_{11}+2\,(p_1Np_2)\,p_{3i}(p_3\epsilon_2D\epsilon_3)_aI'_{11}$$

$$+2\,(p_1Np_3)\,p_{2a}(p_3\epsilon_2D\epsilon_3)_iI'_{11}-2\,(p_1Np_2)\,p_{3a}(p_3\epsilon_2D\epsilon_3)_iI'_{11}$$

$$
\begin{aligned}
&+4\,(p_1Np_3)\,p_{2i}(p_1N\epsilon_2D\epsilon_3)_aI_5-2\,(p_2Dp_3)\,p_{2i}(p_1N\epsilon_2D\epsilon_3)_aI_1\\
&-2\,(p_2p_3)\,p_{3i}(p_1N\epsilon_2D\epsilon_3)_aI'_{11}-2\,(p_2Dp_3)\,p_{3i}(p_1N\epsilon_2D\epsilon_3)_aI_1\\
&-4\,(p_1Np_3)\,p_{2a}(p_1N\epsilon_2D\epsilon_3)_iI_5+2\,(p_2Dp_3)\,p_{2a}(p_1N\epsilon_2D\epsilon_3)_iI_1\\
&+2\,(p_2Dp_3)\,p_{3a}(p_1N\epsilon_2D\epsilon_3)_iI_1-2\,(p_2p_3)\,p_{3i}(p_1N\epsilon_3D\epsilon_2)_aI'_{11}\\
&+2\,(p_2p_3)\,p_{2a}(p_1N\epsilon_3D\epsilon_2)_iI'_{12}+2\,(p_2p_3)\,p_{3a}(p_1N\epsilon_3D\epsilon_2)_iI'_{11}\\
&-4\,(p_1Np_3)\,p_{2a}(p_2D\epsilon_2D\epsilon_3)_iI_7-2\,(p_2Dp_3)\,p_{2a}(p_2D\epsilon_2D\epsilon_3)_iI_{21}\\
&-2\,(p_2Dp_3)\,p_{3a}(p_2D\epsilon_2D\epsilon_3)_iI_{21}+2\,(p_1Np_3)\,p_{2i}(p_2D\epsilon_3D\epsilon_2)_aI_1\\
&-2\,(p_1Np_2)\,p_{3i}(p_2D\epsilon_3D\epsilon_2)_aI_1+4p_{2a}(p_2D\epsilon_3D\epsilon_2)_iI_{24}\\
&+2\,(p_1Np_3)\,p_{2a}(p_2D\epsilon_3D\epsilon_2)_iI_1-2\,(p_2Dp_3)\,p_{2a}(p_2D\epsilon_3D\epsilon_2)_iI_{17}\\
&+4p_{3a}(p_2D\epsilon_3D\epsilon_2)_iI_{24}+2\,(p_1Np_2)\,p_{3a}(p_2D\epsilon_3D\epsilon_2)_iI_1\\
&-2\,(p_2Dp_3)\,p_{3a}(p_2D\epsilon_3D\epsilon_2)_iI_{17}+2\,(p_2p_3)\,p_{2a}(p_3D\epsilon_3D\epsilon_2)_iI_{22}\\
&+2\,(p_2p_3)\,p_{3a}(p_3D\epsilon_3D\epsilon_2)_iI_{22}+2\,(p_1Np_2)\,(p_2\epsilon_3p_2)\,\epsilon_{2ai}I_2\\
&+2\,(p_1Np_3)\,(p_2\epsilon_3p_2)\,\epsilon_{2ai}I_2-4\,(p_1N\epsilon_3p_2)\,(p_2p_3)\,\epsilon_{2ai}I_2\\
&-8\,(p_1N\epsilon_3p_2)\,(p_1Np_2)\,\epsilon_{2ai}I_9+4\,(p_1Np_2)\,(p_2\epsilon_3Dp_3)\,\epsilon_{2ai}I'_6\\
&-8\,(p_1N\epsilon_3Dp_2)\,(p_1Np_2)\,\epsilon_{2ai}I_5+4\,(p_1N\epsilon_3Dp_2)\,(p_2Dp_3)\,\epsilon_{2ai}I_1\\
&-4\,(p_1N\epsilon_3Dp_3)\,(p_2p_3)\,\epsilon_{2ai}I'_6-4\,(p_1N\epsilon_3Dp_3)\,(p_1Np_2)\,\epsilon_{2ai}I'_4\\
&-4\,(p_1N\epsilon_3Dp_3)\,(p_2Dp_3)\,\epsilon_{2ai}I'_7+4\,(p_1N\epsilon_3Np_1)\,(p_2p_3)\,\epsilon_{2ai}I_9\\
&+8\,(p_1N\epsilon_3Np_1)\,(p_1Np_2)\,\epsilon_{2ai}I_{10}-4\,(p_1N\epsilon_3Np_1)\,(p_2Dp_3)\,\epsilon_{2ai}I_5\\
&+2\,(p_1Np_2)\,(p_2D\epsilon_3Dp_2)\,\epsilon_{2ai}I_1-2\,(p_1Np_3)\,(p_2D\epsilon_3Dp_2)\,\epsilon_{2ai}I_1\\
&-4\,(p_1Np_2)\,(p_2D\epsilon_3Dp_3)\,\epsilon_{2ai}I'_7-2\,\mathrm{tr}(D\epsilon_3)\,(p_1Np_2)\,(p_2p_3)\,\epsilon_{2ai}I'_6\\
&+2\,\mathrm{tr}(D\epsilon_3)\,(p_1Np_3)\,(p_2p_3)\,\epsilon_{2ai}I'_6+2\,\mathrm{tr}(D\epsilon_3)\,(p_1Np_2)\,(p_1Np_3)\,\epsilon_{2ai}I'_4\\
&+2\,\mathrm{tr}(D\epsilon_3)\,(p_1Np_2)\,(p_2Dp_3)\,\epsilon_{2ai}I'_7+2\,\mathrm{tr}(D\epsilon_3)\,(p_1Np_3)\,(p_2Dp_3)\,\epsilon_{2ai}I'_7\\
&+4\,(p_1Np_3)\,(p_3D\epsilon_2p_3)\,\epsilon_{3ai}I'_{11})+(2\leftrightarrow 3), \qquad (2.48)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}^{(2)}_{C^{(p+1)}hh} =& \frac{2i^{p(p+1)}\sqrt{2}}{(p-1)!} C^{ij}{}_{b_1\ldots b_{p-1}} \varepsilon^{abb_1\ldots b_{p-1}} \big(\operatorname{tr}(D\epsilon_2) \operatorname{tr}(D\epsilon_3) p_{2b} p_{2j} p_{3a} p_{3i} I_3 \\
& + \operatorname{tr}(\epsilon_2\epsilon_3) p_{2b} p_{2j} p_{3a} p_{3i} I_2 - \operatorname{tr}(D\epsilon_2 D\epsilon_3) p_{2b} p_{2j} p_{3a} p_{3i} I_1 \\
& -2 \operatorname{tr}(D\epsilon_2) p_{2b} p_{2j} p_{3i} (p_2\epsilon_3)_a I_6 - 2 \operatorname{tr}(D\epsilon_2) p_{2j} p_{3b} p_{3i} (p_2\epsilon_3)_a I_6 \\
& +2 \operatorname{tr}(D\epsilon_2) p_{2b} p_{2j} p_{3a} (p_2\epsilon_3)_i I_6 + 2 \operatorname{tr}(D\epsilon_2) p_{2b} p_{3a} p_{3j} (p_2\epsilon_3)_i I_6 \\
& -2 p_{2b} p_{2j} (p_2\epsilon_3)_i (p_3\epsilon_2)_a I_2 - 2 p_{2j} p_{3b} (p_2\epsilon_3)_i (p_3\epsilon_2)_a I_2 \\
& -2 p_{2b} p_{3j} (p_2\epsilon_3)_i (p_3\epsilon_2)_a I_2 - 2 p_{3b} p_{3j} (p_2\epsilon_3)_i (p_3\epsilon_2)_a I_2 \\
& -4 \left(p_1 N p_3\right) p_{2b} p_{2i} (\epsilon_2\epsilon_3)_{aj} I_9 - 2 \left(p_2 p_3\right) p_{2b} p_{3i} (\epsilon_2\epsilon_3)_{aj} I_2 \\
& -2 \left(p_2 D p_3\right) p_{2b} p_{3i} (\epsilon_2\epsilon_3)_{aj} I'_{11} + 2 \left(p_2 p_3\right) p_{2j} p_{3a} (\epsilon_2\epsilon_3)_{bi} I_2 \\
& +2 \left(p_2 p_3\right) p_{3a} p_{3i} (\epsilon_2\epsilon_3)_{bj} I_2 + 4 \left(p_1 N p_2\right) p_{3a} p_{3i} (\epsilon_2\epsilon_3)_{bj} I_9 \\
& +2 \left(p_2 D p_3\right) p_{3a} p_{3i} (\epsilon_2\epsilon_3)_{bj} I'_{11} - 4 \left(p_2 D p_3\right) p_{2b} p_{3a} (\epsilon_2\epsilon_3)_{ij} I_0 \\
& +2 \left(p_2 D p_3\right) p_{2b} p_{3i} (\epsilon_2\epsilon_3)_{ja} I'_{11} - 2 \left(p_2 D p_3\right) p_{3a} p_{3i} (\epsilon_2\epsilon_3)_{jb} I'_{11} \\
& -4 p_{2i} p_{3j} (p_1 N\epsilon_2)_a (p_2\epsilon_3)_b I_9 + 4 p_{3b} p_{3i} (p_1 N\epsilon_2)_a (p_2\epsilon_3)_j I_9 \\
& -2 \operatorname{tr}(D\epsilon_3) p_{2j} p_{3a} p_{3i} (p_1 N\epsilon_2)_b I'_4 + 4 p_{2j} p_{3a} (p_1 N\epsilon_2)_b (p_2\epsilon_3)_i I_9 \\
& -2 \operatorname{tr}(D\epsilon_3) p_{2b} p_{3a} p_{3j} (p_1 N\epsilon_2)_i I'_4 - 4 p_{2b} p_{3j} (p_1 N\epsilon_2)_i (p_2\epsilon_3)_a I_9 \\
& -4 p_{3b} p_{3j} (p_1 N\epsilon_2)_i (p_2\epsilon_3)_a I_9 + 4 p_{2b} p_{3a} (p_1 N\epsilon_2)_i (p_2\epsilon_3)_j I_9 \\
& +4 p_{2j} p_{3i} (p_1 N\epsilon_2)_b (p_1 N\epsilon_3)_a I_{10} + 8 p_{2b} p_{3j} (p_1 N\epsilon_2)_i (p_1 N\epsilon_3)_a I_{10} \\
& +4 p_{2b} p_{3a} (p_1 N\epsilon_2)_j (p_1 N\epsilon_3)_i I_{10} + 4 \operatorname{tr}(D\epsilon_3) p_{2b} p_{3a} p_{3j} (p_2 D\epsilon_2)_i I_3 \\
& -4 p_{2b} p_{3j} (p_2 D\epsilon_2)_i (p_2\epsilon_3)_a I_6 - 4 p_{3b} p_{3j} (p_2 D\epsilon_2)_i (p_2\epsilon_3)_a I_6 \\
& +4 p_{2b} p_{3a} (p_2 D\epsilon_2)_i (p_2\epsilon_3)_j I_6 - 4 p_{2b} p_{3j} (p_1 N\epsilon_3)_a (p_2 D\epsilon_2)_i I_4 \\
& +4 p_{2b} p_{3a} (p_1 N\epsilon_3)_j (p_2 D\epsilon_2)_i I_4 - 2 \operatorname{tr}(D\epsilon_2) p_{2b} p_{2j} p_{3i} (p_2 D\epsilon_3)_a I_7 \\
& -2 \operatorname{tr}(D\epsilon_2) p_{2j} p_{3b} p_{3i} (p_2 D\epsilon_3)_a I_7 - 2 p_{2b} p_{3i} (p_2 D\epsilon_3)_a (p_3\epsilon_2)_j I'_{11} \\
& -2 p_{3b} p_{3i} (p_2 D\epsilon_3)_a (p_3\epsilon_2)_j I'_{11} - 4 p_{2j} p_{3i} (p_1 N\epsilon_2)_b (p_2 D\epsilon_3)_a I_5
\end{aligned}
$$

$$-4p_{2b}p_{3j}(p_1N\epsilon_2)_i(p_2D\epsilon_3)_aI_5+4p_{3b}p_{3i}(p_1N\epsilon_2)_j(p_2D\epsilon_3)_aI_5$$

$$-4p_{2b}p_{3j}(p_2D\epsilon_2)_i(p_2D\epsilon_3)_aI_7-4p_{3b}p_{3j}(p_2D\epsilon_2)_i(p_2D\epsilon_3)_aI_7$$

$$+2\,\mathrm{tr}(D\epsilon_2)p_{2b}p_{2j}p_{3a}(p_2D\epsilon_3)_iI_7+4p_{2j}p_{3a}(p_1N\epsilon_2)_b(p_2D\epsilon_3)_iI_5$$

$$-4p_{2b}p_{3a}(p_1N\epsilon_2)_j(p_2D\epsilon_3)_iI_5-4p_{2b}p_{3a}(p_2D\epsilon_2)_j(p_2D\epsilon_3)_iI_7$$

$$+2\,\mathrm{tr}(D\epsilon_2)p_{2b}p_{3a}p_{3i}(p_2D\epsilon_3)_jI_7+2p_{2b}p_{3i}(p_2D\epsilon_3)_j(p_3\epsilon_2)_aI'_{11}$$

$$+2p_{3b}p_{3i}(p_2D\epsilon_3)_j(p_3\epsilon_2)_aI'_{11}+4p_{2b}p_{3a}(p_2D\epsilon_3)_j(p_3\epsilon_2)_iI_8$$

$$-4p_{3b}p_{3i}(p_1N\epsilon_2)_a(p_2D\epsilon_3)_jI_5+2p_{2b}p_{2j}p_{3i}(p_2\epsilon_3\epsilon_2)_aI_2+2p_{2j}p_{3b}p_{3i}(p_2\epsilon_3\epsilon_2)_aI_2$$

$$-2p_{2b}p_{2i}p_{3a}(p_2\epsilon_3\epsilon_2)_jI_2-2p_{2b}p_{3a}p_{3i}(p_2\epsilon_3\epsilon_2)_jI_2+2p_{2b}p_{3i}(p_2\epsilon_3)_j(p_3D\epsilon_2)_aI'_{11}$$

$$+2p_{3b}p_{3i}(p_2\epsilon_3)_j(p_3D\epsilon_2)_aI'_{11}+2p_{2b}p_{2j}(p_2D\epsilon_3)_i(p_3D\epsilon_2)_aI_1$$

$$+2p_{2j}p_{3b}(p_2D\epsilon_3)_i(p_3D\epsilon_2)_aI_1+2p_{2b}p_{3i}(p_2D\epsilon_3)_j(p_3D\epsilon_2)_aI_1$$

$$+2p_{3b}p_{3i}(p_2D\epsilon_3)_j(p_3D\epsilon_2)_aI_1-2p_{2b}p_{3j}(p_2\epsilon_3)_a(p_3D\epsilon_2)_iI'_{11}$$

$$-2p_{3b}p_{3j}(p_2\epsilon_3)_a(p_3D\epsilon_2)_iI'_{11}+4p_{2b}p_{3a}(p_2D\epsilon_2)_j(p_3D\epsilon_3)_iI_3$$

$$+4\,(p_1Np_3)\,p_{2b}p_{2i}(\epsilon_2D\epsilon_3)_{aj}I_5-2\,(p_2Dp_3)\,p_{2b}p_{2i}(\epsilon_2D\epsilon_3)_{aj}I_1$$

$$-2\,(p_2p_3)\,p_{2b}p_{3i}(\epsilon_2D\epsilon_3)_{aj}I'_{11}-2\,(p_2Dp_3)\,p_{2b}p_{3i}(\epsilon_2D\epsilon_3)_{aj}I_1$$

$$-2\,(p_2Dp_3)\,p_{2j}p_{3a}(\epsilon_2D\epsilon_3)_{bi}I_1+2\,(p_2p_3)\,p_{3a}p_{3i}(\epsilon_2D\epsilon_3)_{bj}I'_{11}$$

$$-4\,(p_1Np_2)\,p_{3a}p_{3i}(\epsilon_2D\epsilon_3)_{bj}I_5+2\,(p_2Dp_3)\,p_{3a}p_{3i}(\epsilon_2D\epsilon_3)_{bj}I_1$$

$$-2\,(p_2p_3)\,p_{2b}p_{3i}(\epsilon_2D\epsilon_3)_{ja}I'_{11}+2\,(p_2p_3)\,p_{3a}p_{3i}(\epsilon_2D\epsilon_3)_{jb}I'_{11}$$

$$-4\,(p_2p_3)\,p_{2b}p_{3a}(\epsilon_2D\epsilon_3)_{ji}I_0+4p_{2j}p_{3a}p_{3i}(p_1N\epsilon_2\epsilon_3)_bI_9$$

$$+4p_{2b}p_{3a}p_{3i}(p_1N\epsilon_2\epsilon_3)_jI_9+4p_{2b}p_{3a}p_{3i}(p_2D\epsilon_2\epsilon_3)_jI_6-2p_{2b}p_{2j}p_{3i}(p_3D\epsilon_2\epsilon_3)_aI'_{11}$$

$$-2p_{2j}p_{3b}p_{3i}(p_3D\epsilon_2\epsilon_3)_aI'_{11}-2p_{2b}p_{2i}p_{3a}(p_3D\epsilon_2\epsilon_3)_jI'_{11}+2p_{2b}p_{3a}p_{3i}(p_3D\epsilon_2\epsilon_3)_jI'_{11}$$

$$+2p_{2b}p_{2j}p_{3i}(p_3\epsilon_2D\epsilon_3)_aI'_{11}+2p_{2j}p_{3b}p_{3i}(p_3\epsilon_2D\epsilon_3)_aI'_{11}+2p_{2b}p_{2i}p_{3a}(p_3\epsilon_2D\epsilon_3)_jI'_{11}$$

$$+2p_{2b}p_{3a}p_{3i}(p_3\epsilon_2D\epsilon_3)_jI'_{11}+4p_{2j}p_{3a}p_{3i}(p_1N\epsilon_2D\epsilon_3)_bI_5$$

$$+2\,(p_2p_3)\,p_{2b}p_{2i}(\epsilon_2\epsilon_3)_{aj}I_2$$

$$-4p_{2b}p_{3a}p_{3i}(p_1N\epsilon_2D\epsilon_3)_jI_5-4p_{2b}p_{3a}p_{3i}(p_2D\epsilon_2D\epsilon_3)_jI_7$$

$$-2p_{2b}p_{2j}p_{3i}(p_2D\epsilon_3D\epsilon_2)_aI_1-2p_{2j}p_{3b}p_{3i}(p_2D\epsilon_3D\epsilon_2)_aI_1$$

$$-2p_{2b}p_{2i}p_{3a}(p_2D\epsilon_3D\epsilon_2)_jI_1+2p_{2b}p_{3a}p_{3i}(p_2D\epsilon_3D\epsilon_2)_jI_1$$

$$+4\,(p_1Np_2)\,p_{3j}(p_2\epsilon_3)_b\epsilon_{2ai}I_9-4\,(p_2p_3)\,p_{2b}(p_2\epsilon_3)_j\epsilon_{2ai}I_2$$

$$+4\,(p_1Np_3)\,p_{2b}(p_2\epsilon_3)_j\epsilon_{2ai}I_9-4\,(p_2p_3)\,p_{3b}(p_2\epsilon_3)_j\epsilon_{2ai}I_2$$

$$-4\,(p_1Np_2)\,p_{3b}(p_2\epsilon_3)_j\epsilon_{2ai}I_9-4\,(p_2p_3)\,p_{3j}(p_1N\epsilon_3)_b\epsilon_{2ai}I_9$$

$$-8\,(p_1Np_2)\,p_{3j}(p_1N\epsilon_3)_b\epsilon_{2ai}I_{10}+4\,(p_2Dp_3)\,p_{3j}(p_1N\epsilon_3)_b\epsilon_{2ai}I_5$$

$$+4\,(p_2p_3)\,p_{2b}(p_1N\epsilon_3)_j\epsilon_{2ai}I_9-4\,(p_2Dp_3)\,p_{2b}(p_1N\epsilon_3)_j\epsilon_{2ai}I_5$$

$$+4\,(p_1Np_2)\,p_{3j}(p_2D\epsilon_3)_b\epsilon_{2ai}I_5-4\,(p_2p_3)\,p_{2b}(p_3D\epsilon_3)_j\epsilon_{2ai}I'_6$$

$$-4\,(p_2Dp_3)\,p_{2b}(p_3D\epsilon_3)_j\epsilon_{2ai}I'_7-2\,(p_2\epsilon_3p_2)\,p_{2b}p_{2i}\epsilon_{2aj}I_2$$

$$+8\,(p_1N\epsilon_3p_2)\,p_{2b}p_{2i}\epsilon_{2aj}I_9-4\,(p_2\epsilon_3Dp_3)\,p_{2b}p_{2i}\epsilon_{2aj}I'_6$$

$$+8\,(p_1N\epsilon_3Dp_2)\,p_{2b}p_{2i}\epsilon_{2aj}I_5+4\,(p_1N\epsilon_3Dp_3)\,p_{2b}p_{2i}\epsilon_{2aj}I'_4$$

$$-8\,(p_1N\epsilon_3Np_1)\,p_{2b}p_{2i}\epsilon_{2aj}I_{10}-2\,(p_2D\epsilon_3Dp_2)\,p_{2b}p_{2i}\epsilon_{2aj}I_1$$

$$+4\,(p_2D\epsilon_3Dp_3)\,p_{2b}p_{2i}\epsilon_{2aj}I'_7+2\,\mathrm{tr}(D\epsilon_3)\,(p_2p_3)\,p_{2b}p_{2i}\epsilon_{2aj}I'_6$$

$$-2\,\mathrm{tr}(D\epsilon_3)\,(p_1Np_3)\,p_{2b}p_{2i}\epsilon_{2aj}I'_4-2\,\mathrm{tr}(D\epsilon_3)\,(p_2Dp_3)\,p_{2b}p_{2i}\epsilon_{2aj}I'_7$$

$$-2\,(p_2\epsilon_3p_2)\,p_{2i}p_{3b}\epsilon_{2aj}I_2+4\,(p_1N\epsilon_3p_2)\,p_{2i}p_{3b}\epsilon_{2aj}I_9-4\,(p_2\epsilon_3Dp_3)\,p_{2i}p_{3b}\epsilon_{2aj}I'_6$$

$$+4\,(p_1N\epsilon_3Dp_2)\,p_{2i}p_{3b}\epsilon_{2aj}I_5-2\,(p_2D\epsilon_3Dp_2)\,p_{2i}p_{3b}\epsilon_{2aj}I_1$$

$$+4\,(p_2D\epsilon_3Dp_3)\,p_{2i}p_{3b}\epsilon_{2aj}I'_7+2\,\mathrm{tr}(D\epsilon_3)\,(p_2p_3)\,p_{2i}p_{3b}\epsilon_{2aj}I'_6$$

$$-2\,\mathrm{tr}(D\epsilon_3)\,(p_2Dp_3)\,p_{2i}p_{3b}\epsilon_{2aj}I'_7-2\,(p_2\epsilon_3p_2)\,p_{2b}p_{3i}\epsilon_{2aj}I_2$$

$$+4\,(p_1N\epsilon_3p_2)\,p_{2b}p_{3i}\epsilon_{2aj}I_9-4\,(p_1N\epsilon_3Dp_2)\,p_{2b}p_{3i}\epsilon_{2aj}I_5$$

$$+2\,(p_2D\epsilon_3Dp_2)\,p_{2b}p_{3i}\epsilon_{2aj}I_1-2\,\mathrm{tr}(D\epsilon_3)\,(p_2p_3)\,p_{2b}p_{3i}\epsilon_{2aj}I'_6$$

$$-2\,\mathrm{tr}(D\epsilon_3)\,(p_2Dp_3)\,p_{2b}p_{3i}\epsilon_{2aj}I'_7-2\,(p_2\epsilon_3p_2)\,p_{3b}p_{3i}\epsilon_{2aj}I_2$$

$$+2\,(p_2D\epsilon_3Dp_2)\,p_{3b}p_{3i}\epsilon_{2aj}I_1+4\,(p_1Np_3)\,p_{2i}(p_2\epsilon_3)_b\epsilon_{2aj}I_9$$

$$-4\,(p_2p_3)\,p_{2i}(p_1N\epsilon_3)_b\epsilon_{2aj}I_9-4\,(p_2Dp_3)\,p_{2i}(p_1N\epsilon_3)_b\epsilon_{2aj}I_5$$

$$+4\,(p_1Np_3)\,p_{2i}(p_2D\epsilon_3)_b\epsilon_{2aj}I_5+4\,(p_1Np_3)\,p_{2b}(p_2D\epsilon_3)_i\epsilon_{2aj}I_5$$

$$-4\,(p_2Dp_3)\,p_{2b}(p_2D\epsilon_3)_i\epsilon_{2aj}I_1+4\,(p_1Np_2)\,p_{3b}(p_2D\epsilon_3)_i\epsilon_{2aj}I_5$$

$$-4\,(p_2Dp_3)\,p_{3b}(p_2D\epsilon_3)_i\epsilon_{2aj}I_1+2\,\mathrm{tr}(D\epsilon_3)\,(p_2p_3)\,p_{3a}p_{3i}\epsilon_{2bj}I_6'$$

$$+2\,\mathrm{tr}(D\epsilon_3)\,(p_1Np_2)\,p_{3a}p_{3i}\epsilon_{2bj}I_4'+2\,\mathrm{tr}(D\epsilon_3)\,(p_2Dp_3)\,p_{3a}p_{3i}\epsilon_{2bj}I_7'$$

$$+4\,(p_2p_3)\,p_{3a}(p_1N\epsilon_3)_i\epsilon_{2bj}I_9+8\,(p_1Np_2)\,p_{3a}(p_1N\epsilon_3)_i\epsilon_{2bj}I_{10}$$

$$-4\,(p_2Dp_3)\,p_{3a}(p_1N\epsilon_3)_i\epsilon_{2bj}I_5-4\,(p_2p_3)\,p_{3a}(p_3D\epsilon_3)_i\epsilon_{2bj}I_6'$$

$$-4\,(p_1Np_2)\,p_{3a}(p_3D\epsilon_3)_i\epsilon_{2bj}I_4'-4\,(p_2Dp_3)\,p_{3a}(p_3D\epsilon_3)_i\epsilon_{2bj}I_7'$$

$$-4\,(p_3D\epsilon_2p_3)\,p_{2b}p_{3i}\epsilon_{3aj}I_{11}'-4\,(p_3D\epsilon_2p_3)\,p_{3b}p_{3i}\epsilon_{3aj}I_{11}'$$

$$+4\,(p_1Np_2)\,(p_2p_3)\,\epsilon_{2aj}\epsilon_{3bi}I_9-4\,(p_1Np_2)\,(p_1Np_3)\,\epsilon_{2aj}\epsilon_{3bi}I_{10}$$

$$+4\,(p_1Np_2)\,(p_2Dp_3)\,\epsilon_{2aj}\epsilon_{3bi}I_5\Big)+(2\leftrightarrow 3), \tag{2.49}$$

$$\mathcal{A}^{(3)}_{C^{(p+1)}hh}=\frac{4i^{p(p+1)}\sqrt{2}}{(p-2)!}C^{ijk}{}_{b_1\ldots b_{p-2}}\varepsilon^{abcb_1\ldots b_{p-2}}\Big(-2p_{2c}p_{2j}p_{3a}p_{3i}(\epsilon_2\epsilon_3)_{bk}I_9$$

$$+2p_{2c}p_{2j}p_{3a}p_{3i}(\epsilon_2D\epsilon_3)_{bk}I_5-2p_{2c}p_{2i}p_{3j}(p_2\epsilon_3)_b\epsilon_{2ak}I_9$$

$$-2p_{2i}p_{3c}p_{3j}(p_2\epsilon_3)_b\epsilon_{2ak}I_9+4p_{2c}p_{2i}p_{3j}(p_1N\epsilon_3)_b\epsilon_{2ak}I_{10}$$

$$-2p_{2c}p_{2i}p_{3j}(p_2D\epsilon_3)_b\epsilon_{2ak}I_5-2p_{2i}p_{3c}p_{3j}(p_2D\epsilon_3)_b\epsilon_{2ak}I_5$$

$$+2p_{2c}p_{2i}p_{3a}(p_2\epsilon_3)_k\epsilon_{2bj}I_9-2p_{2a}p_{3c}p_{3i}(p_2\epsilon_3)_k\epsilon_{2bj}I_9$$

$$-4p_{2c}p_{2i}p_{3a}(p_1N\epsilon_3)_k\epsilon_{2bj}I_{10}+2p_{2c}p_{2i}p_{3a}(p_2D\epsilon_3)_k\epsilon_{2bj}I_5$$

$$-2p_{2c}p_{3a}p_{3i}(p_2D\epsilon_3)_k\epsilon_{2bj}I_5+2p_{2c}p_{2i}p_{3a}(p_3D\epsilon_3)_k\epsilon_{2bj}I_4'$$

$$-\mathrm{tr}(D\epsilon_3)p_{2c}p_{2j}p_{3a}p_{3i}\epsilon_{2bk}I_4'-2\,(p_2p_3)\,p_{2c}p_{2i}\epsilon_{2aj}\epsilon_{3bk}I_9$$

$$+4\,(p_1 N p_3)\,p_{2c}p_{2i}\epsilon_{2aj}\epsilon_{3bk}I_{10} - 2\,(p_2 D p_3)\,p_{2c}p_{2i}\epsilon_{2aj}\epsilon_{3bk}I_5$$

$$+2\,(p_2 p_3)\,p_{2c}p_{3i}\epsilon_{2aj}\epsilon_{3bk}I_9 - 2\,(p_2 D p_3)\,p_{2c}p_{3i}\epsilon_{2aj}\epsilon_{3bk}I_5\Big) + (2\leftrightarrow 3), \qquad (2.50)$$

$$\mathcal{A}^{(4)}_{C^{(p+1)}hh} = \frac{8i^{p(p+1)}\sqrt{2}}{(p-3)!}C^{ijkl}{}_{b_1\ldots b_{p-3}}\varepsilon^{abcdb_1\ldots b_{p-3}}p_{2d}p_{2j}p_{3a}p_{3i}\epsilon_{2bk}\epsilon_{3cl}I_{10} + (2\leftrightarrow 3). \qquad (2.51)$$

2.2.4 $C^{(p-1)}$ amplitudes

$$\mathcal{A}_{C^{(p-1)}Bh} = \mathcal{A}^{(0)}_{C^{(p-1)}Bh} + \mathcal{A}^{(1)}_{C^{(p-1)}Bh} + \mathcal{A}^{(2)}_{C^{(p-1)}Bh} + \mathcal{A}^{(3)}_{C^{(p-1)}Bh}. \qquad (2.52)$$

$$\begin{aligned}
\mathcal{A}^{(0)}_{C^{(p-1)}Bh} =& \frac{2i^{p(p+1)}\sqrt{2}}{(p-1)!}C_{b_1\ldots b_{p-1}}\varepsilon^{abb_1\ldots b_{p-1}}\,\big(2\,(p_1 N\epsilon_2\epsilon_3 p_2)\,p_{2b}p_{3a}I_2 \\
&+2\,(p_1 N\epsilon_3\epsilon_2 p_3)\,p_{2b}p_{3a}I_2 - 2\,(p_2 D\epsilon_2\epsilon_3 p_2)\,p_{2b}p_{3a}I_{20} \\
&-2\,(p_2 D\epsilon_3\epsilon_2 p_3)\,p_{2b}p_{3a}I_{14} + 2\,(p_2\epsilon_3\epsilon_2 D p_3)\,p_{2b}p_{3a}I_{14} \\
&+2\,(p_1 N\epsilon_2 D\epsilon_3 p_2)\,p_{2b}p_{3a}I_{11} + 2\,(p_1 N\epsilon_2\epsilon_3 D p_2)\,p_{2b}p_{3a}I_{11} \\
&+4\,(p_1 N\epsilon_2\epsilon_3 D p_3)\,p_{2b}p_{3a}I'_6 - 4\,(p_1 N\epsilon_2\epsilon_3 N p_1)\,p_{2b}p_{3a}I_9 \\
&-2\,(p_1 N\epsilon_3 D\epsilon_2 p_3)\,p_{2b}p_{3a}I'_{11} + 4\,(p_1 N\epsilon_3\epsilon_2 D p_2)\,p_{2b}p_{3a}I_6 \\
&+2\,(p_1 N\epsilon_3\epsilon_2 D p_3)\,p_{2b}p_{3a}I'_{11} + 2\,(p_2 D\epsilon_2 D\epsilon_3 p_2)\,p_{2b}p_{3a}I'_{22} \\
&+2\,(p_2 D\epsilon_2\epsilon_3 D p_2)\,p_{2b}p_{3a}I'_{22} - 4\,(p_2 D\epsilon_2\epsilon_3 D p_3)\,p_{2b}p_{3a}I_{18} \\
&-2\,(p_2 D\epsilon_3 D\epsilon_2 p_3)\,p_{2b}p_{3a}I_{15} + 2\,(p_2\epsilon_3 D\epsilon_2 D p_3)\,p_{2b}p_{3a}I_{15} \\
&-2\,(p_3 D\epsilon_2\epsilon_3 D p_3)\,p_{2b}p_{3a}I_{22} + 2\,(p_1 N\epsilon_2 D\epsilon_3 D p_2)\,p_{2b}p_{3a}I_1 \\
&-4\,(p_1 N\epsilon_2 D\epsilon_3 D p_3)\,p_{2b}p_{3a}I'_7 - 4\,(p_1 N\epsilon_2 D\epsilon_3 N p_1)\,p_{2b}p_{3a}I_5 \\
&+4\,(p_1 N\epsilon_3 D\epsilon_2 D p_2)\,p_{2b}p_{3a}I_7 - 2\,(p_1 N\epsilon_3 D\epsilon_2 D p_3)\,p_{2b}p_{3a}I_1 \\
&-2\,(p_2 D\epsilon_2 D\epsilon_3 D p_2)\,p_{2b}p_{3a}I_{21} - 4\,(p_2 D\epsilon_2 D\epsilon_3 D p_3)\,p_{2b}p_{3a}I_{19}
\end{aligned}$$

$$-2\,\mathrm{tr}(D\epsilon_3)\,(p_1N\epsilon_2p_3)\,p_{2b}p_{3a}I'_6+2\,\mathrm{tr}(D\epsilon_3)\,(p_2D\epsilon_2p_3)\,p_{2b}p_{3a}I_{18}$$

$$+2\,\mathrm{tr}(D\epsilon_3)\,(p_3D\epsilon_2p_3)\,p_{2b}p_{3a}I_{22}+2\,\mathrm{tr}(D\epsilon_3)\,(p_1N\epsilon_2Dp_3)\,p_{2b}p_{3a}I'_7$$

$$+2\,\mathrm{tr}(D\epsilon_3)\,(p_2D\epsilon_2Dp_3)\,p_{2b}p_{3a}I_{19}+2\,(p_3D\epsilon_3\epsilon_2p_3)\,p_{2a}p_{3b}I'_{20}$$

$$+2\,(p_3D\epsilon_3D\epsilon_2p_3)\,p_{2a}p_{3b}I_{22}+2\,(p_3D\epsilon_2D\epsilon_3Dp_3)\,p_{2a}p_{3b}I'_{21}$$

$$-2\,(p_1N\epsilon_2p_3)\,p_{2b}(p_2\epsilon_3)_aI_2+2\,(p_2D\epsilon_2p_3)\,p_{2b}(p_2\epsilon_3)_aI_{20}$$

$$+2\,(p_3D\epsilon_2p_3)\,p_{2b}(p_2\epsilon_3)_aI_{14}-2\,(p_1N\epsilon_2Dp_3)\,p_{2b}(p_2\epsilon_3)_aI_{11}$$

$$-2\,(p_2D\epsilon_2Dp_3)\,p_{2b}(p_2\epsilon_3)_aI'_{22}-2\,(p_1N\epsilon_2p_3)\,p_{3b}(p_2\epsilon_3)_aI_2$$

$$+2\,(p_2D\epsilon_2p_3)\,p_{3b}(p_2\epsilon_3)_aI_{20}+2\,(p_3D\epsilon_2p_3)\,p_{3b}(p_2\epsilon_3)_aI_{14}$$

$$-2\,(p_2D\epsilon_2Dp_3)\,p_{3b}(p_2\epsilon_3)_aI'_{22}+2\,(p_1N\epsilon_2Dp_3)\,p_{3a}(p_2\epsilon_3)_bI_{12}$$

$$-2\,(p_1N\epsilon_3p_2)\,p_{2b}(p_3\epsilon_2)_aI_2+2\,(p_2D\epsilon_3p_2)\,p_{2b}(p_3\epsilon_2)_aI_{14}$$

$$+2\,(p_2\epsilon_3Dp_3)\,p_{2b}(p_3\epsilon_2)_aI'_{20}+2\,(p_1N\epsilon_3Dp_2)\,p_{2b}(p_3\epsilon_2)_aI'_{13}$$

$$-4\,(p_1N\epsilon_3Dp_3)\,p_{2b}(p_3\epsilon_2)_aI'_6+4\,(p_1N\epsilon_3Np_1)\,p_{2b}(p_3\epsilon_2)_aI_9$$

$$+2\,(p_2D\epsilon_3Dp_2)\,p_{2b}(p_3\epsilon_2)_aI_{15}+2\,(p_2D\epsilon_3Dp_3)\,p_{2b}(p_3\epsilon_2)_aI_{22}$$

$$-2\,\mathrm{tr}(D\epsilon_3)\,(p_2p_3)\,p_{2b}(p_3\epsilon_2)_aI'_{20}+2\,\mathrm{tr}(D\epsilon_3)\,(p_1Np_3)\,p_{2b}(p_3\epsilon_2)_aI'_6$$

$$-2\,(p_1N\epsilon_3p_2)\,p_{3b}(p_3\epsilon_2)_aI_2+2\,(p_2D\epsilon_3p_2)\,p_{3b}(p_3\epsilon_2)_aI_{14}$$

$$+2\,(p_2\epsilon_3Dp_3)\,p_{3b}(p_3\epsilon_2)_aI'_{20}+2\,(p_1N\epsilon_3Dp_2)\,p_{3b}(p_3\epsilon_2)_aI'_{11}$$

$$+2\,(p_2D\epsilon_3Dp_2)\,p_{3b}(p_3\epsilon_2)_aI_{15}+2\,(p_2D\epsilon_3Dp_3)\,p_{3b}(p_3\epsilon_2)_aI_{22}$$

$$-2\,\mathrm{tr}(D\epsilon_3)\,(p_2p_3)\,p_{3b}(p_3\epsilon_2)_aI'_{20}-2\,\mathrm{tr}(D\epsilon_3)\,(p_1Np_2)\,p_{3b}(p_3\epsilon_2)_aI'_6$$

$$-2\,(p_1Np_2)\,(p_2\epsilon_3)_b(p_3\epsilon_2)_aI_2-2\,(p_1Np_3)\,(p_2\epsilon_3)_b(p_3\epsilon_2)_aI_2$$

$$-2\,(p_1Np_2)\,(p_2p_3)\,(\epsilon_2\epsilon_3)_{ab}I_2+2\,(p_1Np_3)\,(p_2p_3)\,(\epsilon_2\epsilon_3)_{ab}I_2$$

$$+4\,(p_1Np_2)\,(p_1Np_3)\,(\epsilon_2\epsilon_3)_{ab}I_9-2\,(p_1Np_2)\,(p_2Dp_3)\,(\epsilon_2\epsilon_3)_{ab}I_{11}$$

$$+2\,(p_1Np_3)\,(p_2Dp_3)\,(\epsilon_2\epsilon_3)_{ab}I'_{11}+2\,(p_2\epsilon_3p_2)\,p_{2b}(p_1N\epsilon_2)_aI_2$$

$$-8\,(p_1N\epsilon_3p_2)\,p_{2b}(p_1N\epsilon_2)_aI_9+4\,(p_2D\epsilon_3p_2)\,p_{2b}(p_1N\epsilon_2)_aI_{11}$$

$$+4\,(p_2\epsilon_3Dp_3)\,p_{2b}(p_1N\epsilon_2)_aI'_6-8\,(p_1N\epsilon_3Dp_2)\,p_{2b}(p_1N\epsilon_2)_aI_5$$

$$-4\,(p_1N\epsilon_3Dp_3)\,p_{2b}(p_1N\epsilon_2)_aI'_4+8\,(p_1N\epsilon_3Np_1)\,p_{2b}(p_1N\epsilon_2)_aI_{10}$$

$$+2\,(p_2D\epsilon_3Dp_2)\,p_{2b}(p_1N\epsilon_2)_aI_1-4\,(p_2D\epsilon_3Dp_3)\,p_{2b}(p_1N\epsilon_2)_aI'_7$$

$$-2\,\mathrm{tr}(D\epsilon_3)\,(p_2p_3)\,p_{2b}(p_1N\epsilon_2)_aI'_6+2\,\mathrm{tr}(D\epsilon_3)\,(p_1Np_3)\,p_{2b}(p_1N\epsilon_2)_aI'_4$$

$$+2\,\mathrm{tr}(D\epsilon_3)\,(p_2Dp_3)\,p_{2b}(p_1N\epsilon_2)_aI'_7-4\,(p_1N\epsilon_3p_2)\,p_{3b}(p_1N\epsilon_2)_aI_9$$

$$+4\,(p_2D\epsilon_3p_2)\,p_{3b}(p_1N\epsilon_2)_aI_{11}+4\,(p_2\epsilon_3Dp_3)\,p_{3b}(p_1N\epsilon_2)_aI'_6$$

$$-4\,(p_1N\epsilon_3Dp_2)\,p_{3b}(p_1N\epsilon_2)_aI_5-4\,(p_2D\epsilon_3Dp_3)\,p_{3b}(p_1N\epsilon_2)_aI'_7$$

$$-2\,\mathrm{tr}(D\epsilon_3)\,(p_2p_3)\,p_{3b}(p_1N\epsilon_2)_aI'_6+2\,\mathrm{tr}(D\epsilon_3)\,(p_2Dp_3)\,p_{3b}(p_1N\epsilon_2)_aI'_7$$

$$-4\,(p_1Np_3)\,(p_1N\epsilon_2)_a(p_2\epsilon_3)_bI_9-2\,(p_2\epsilon_3p_2)\,p_{3a}(p_1N\epsilon_2)_bI_2$$

$$-2\,(p_2D\epsilon_3Dp_2)\,p_{3a}(p_1N\epsilon_2)_bI_1+4\,(p_1N\epsilon_2p_3)\,p_{2b}(p_1N\epsilon_3)_aI_9$$

$$+4\,(p_2D\epsilon_2p_3)\,p_{2b}(p_1N\epsilon_3)_aI_6+4\,(p_3D\epsilon_2p_3)\,p_{2b}(p_1N\epsilon_3)_aI'_{11}$$

$$+4\,(p_1N\epsilon_2Dp_3)\,p_{2b}(p_1N\epsilon_3)_aI_5+4\,(p_2D\epsilon_2Dp_3)\,p_{2b}(p_1N\epsilon_3)_aI_7$$

$$-4\,(p_2p_3)\,(p_1N\epsilon_3)_a(p_3\epsilon_2)_bI_2-4\,(p_1Np_2)\,(p_1N\epsilon_3)_a(p_3\epsilon_2)_bI_9$$

$$+4\,(p_2p_3)\,(p_1N\epsilon_2)_a(p_1N\epsilon_3)_bI_9+4\,(p_2Dp_3)\,(p_1N\epsilon_2)_a(p_1N\epsilon_3)_bI_5$$

$$-2\,(p_2\epsilon_3p_2)\,p_{2b}(p_2D\epsilon_2)_aI_{20}-8\,(p_1N\epsilon_3p_2)\,p_{2b}(p_2D\epsilon_2)_aI_6$$

$$+4\,(p_2D\epsilon_3p_2)\,p_{2b}(p_2D\epsilon_2)_aI'_{22}-4\,(p_2\epsilon_3Dp_3)\,p_{2b}(p_2D\epsilon_2)_aI_{18}$$

$$-8\,(p_1N\epsilon_3Dp_2)\,p_{2b}(p_2D\epsilon_2)_aI_7+8\,(p_1N\epsilon_3Dp_3)\,p_{2b}(p_2D\epsilon_2)_aI_3$$

$$-4\,(p_1N\epsilon_3Np_1)\,p_{2b}(p_2D\epsilon_2)_aI_4-2\,(p_2D\epsilon_3Dp_2)\,p_{2b}(p_2D\epsilon_2)_aI_{21}$$

$$-4\,(p_2D\epsilon_3Dp_3)\,p_{2b}(p_2D\epsilon_2)_aI_{19}+2\,\mathrm{tr}(D\epsilon_3)\,(p_2p_3)\,p_{2b}(p_2D\epsilon_2)_aI_{18}$$

$$-4\,\mathrm{tr}(D\epsilon_3)\,(p_1Np_3)\,p_{2b}(p_2D\epsilon_2)_aI_3+2\,\mathrm{tr}(D\epsilon_3)\,(p_2Dp_3)\,p_{2b}(p_2D\epsilon_2)_aI_{19}$$

$$-2\,(p_2\epsilon_3p_2)\,p_{3b}(p_2D\epsilon_2)_aI_{20}-4\,(p_1N\epsilon_3p_2)\,p_{3b}(p_2D\epsilon_2)_aI_6$$

$$+4\,(p_2D\epsilon_3p_2)\,p_{3b}(p_2D\epsilon_2)_aI'_{22} - 4\,(p_2\epsilon_3Dp_3)\,p_{3b}(p_2D\epsilon_2)_aI_{18}$$

$$-4\,(p_1N\epsilon_3Dp_2)\,p_{3b}(p_2D\epsilon_2)_aI_7 - 2\,(p_2D\epsilon_3Dp_2)\,p_{3b}(p_2D\epsilon_2)_aI_{21}$$

$$-4\,(p_2D\epsilon_3Dp_3)\,p_{3b}(p_2D\epsilon_2)_aI_{19} + 2\,\mathrm{tr}(D\epsilon_3)\,(p_2p_3)\,p_{3b}(p_2D\epsilon_2)_aI_{18}$$

$$+2\,\mathrm{tr}(D\epsilon_3)\,(p_2Dp_3)\,p_{3b}(p_2D\epsilon_2)_aI_{19} - 4\,(p_1Np_3)\,(p_2D\epsilon_2)_a(p_2\epsilon_3)_bI_6$$

$$+4\,(p_2p_3)\,(p_1N\epsilon_3)_b(p_2D\epsilon_2)_aI_6 + 4\,(p_2Dp_3)\,(p_1N\epsilon_3)_b(p_2D\epsilon_2)_aI_7$$

$$-2\,(p_1N\epsilon_2p_3)\,p_{2b}(p_2D\epsilon_3)_aI_{11} - 2\,(p_2D\epsilon_2p_3)\,p_{2b}(p_2D\epsilon_3)_aI'_{22}$$

$$+2\,(p_3D\epsilon_2p_3)\,p_{2b}(p_2D\epsilon_3)_aI_{15} - 2\,(p_1N\epsilon_2Dp_3)\,p_{2b}(p_2D\epsilon_3)_aI_1$$

$$+2\,(p_2D\epsilon_2Dp_3)\,p_{2b}(p_2D\epsilon_3)_aI_{21} - 2\,(p_2D\epsilon_2p_3)\,p_{3b}(p_2D\epsilon_3)_aI'_{22}$$

$$+2\,(p_3D\epsilon_2p_3)\,p_{3b}(p_2D\epsilon_3)_aI_{15} - 2\,(p_1N\epsilon_2Dp_3)\,p_{3b}(p_2D\epsilon_3)_aI_1$$

$$+2\,(p_2D\epsilon_2Dp_3)\,p_{3b}(p_2D\epsilon_3)_aI_{21} + 4\,(p_1Np_3)\,(p_2D\epsilon_2)_b(p_2D\epsilon_3)_aI_7$$

$$+2\,(p_1N\epsilon_2p_3)\,p_{3a}(p_2D\epsilon_3)_bI_{12} - 2\,(p_1Np_2)\,(p_2D\epsilon_3)_b(p_3\epsilon_2)_aI_{11}$$

$$+2\,(p_1Np_3)\,(p_2D\epsilon_3)_b(p_3\epsilon_2)_aI'_{11} - 4\,(p_1Np_3)\,(p_1N\epsilon_2)_a(p_2D\epsilon_3)_bI_5$$

$$-4p_{2b}(p_2\epsilon_3\epsilon_2)_aI_{23} + 2\,(p_2p_3)\,p_{2b}(p_2\epsilon_3\epsilon_2)_aI_{16} - 2\,(p_1Np_3)\,p_{2b}(p_2\epsilon_3\epsilon_2)_aI_2$$

$$+4p_{3a}(p_2\epsilon_3\epsilon_2)_bI_{23} - 2\,(p_2p_3)\,p_{3a}(p_2\epsilon_3\epsilon_2)_bI_{16} - 2\,(p_1Np_2)\,p_{3a}(p_2\epsilon_3\epsilon_2)_bI_2$$

$$-2\,(p_2\epsilon_3p_2)\,p_{2b}(p_3D\epsilon_2)_aI_{14} - 2\,(p_1N\epsilon_3p_2)\,p_{2b}(p_3D\epsilon_2)_aI'_{13}$$

$$-2\,(p_2D\epsilon_3p_2)\,p_{2b}(p_3D\epsilon_2)_aI_{15} - 2\,(p_2\epsilon_3Dp_3)\,p_{2b}(p_3D\epsilon_2)_aI_{22}$$

$$+2\,(p_1N\epsilon_3Dp_2)\,p_{2b}(p_3D\epsilon_2)_aI_1 - 4\,(p_1N\epsilon_3Dp_3)\,p_{2b}(p_3D\epsilon_2)_aI'_7$$

$$-4\,(p_1N\epsilon_3Np_1)\,p_{2b}(p_3D\epsilon_2)_aI_5 - 2\,(p_2D\epsilon_3Dp_3)\,p_{2b}(p_3D\epsilon_2)_aI'_{21}$$

$$+2\,\mathrm{tr}(D\epsilon_3)\,(p_1Np_3)\,p_{2b}(p_3D\epsilon_2)_aI'_7+2\,\mathrm{tr}(D\epsilon_3)\,(p_2Dp_3)\,p_{2b}(p_3D\epsilon_2)_aI'_{21}$$

$$-2\,(p_2\epsilon_3p_2)\,p_{3b}(p_3D\epsilon_2)_aI_{14}-2\,(p_1N\epsilon_3p_2)\,p_{3b}(p_3D\epsilon_2)_aI'_{11}$$

$$-2\,(p_2D\epsilon_3p_2)\,p_{3b}(p_3D\epsilon_2)_aI_{15}-2\,(p_2\epsilon_3Dp_3)\,p_{3b}(p_3D\epsilon_2)_aI_{22}$$

$$+2\,(p_1N\epsilon_3Dp_2)\,p_{3b}(p_3D\epsilon_2)_aI_1-2\,(p_2D\epsilon_3Dp_3)\,p_{3b}(p_3D\epsilon_2)_aI'_{21}$$

$$+2\,\mathrm{tr}(D\epsilon_3)\,(p_1Np_2)\,p_{3b}(p_3D\epsilon_2)_aI'_7+2\,\mathrm{tr}(D\epsilon_3)\,(p_2Dp_3)\,p_{3b}(p_3D\epsilon_2)_aI'_{21}$$

$$-2\,(p_1Np_2)\,(p_2\epsilon_3)_b(p_3D\epsilon_2)_aI_{11}-2\,(p_1Np_3)\,(p_2\epsilon_3)_b(p_3D\epsilon_2)_aI'_{11}$$

$$-2\,(p_1Np_2)\,(p_2D\epsilon_3)_b(p_3D\epsilon_2)_aI_1+2\,(p_1Np_3)\,(p_2D\epsilon_3)_b(p_3D\epsilon_2)_aI_1$$

$$-4\,(p_1Np_2)\,(p_1N\epsilon_3)_a(p_3D\epsilon_2)_bI_5+4\,(p_2Dp_3)\,(p_1N\epsilon_3)_a(p_3D\epsilon_2)_bI_1$$

$$-4p_{2b}(p_3\epsilon_2\epsilon_3)_aI_{23}+2\,(p_2p_3)\,p_{2b}(p_3\epsilon_2\epsilon_3)_aI_{16}-2\,(p_1Np_3)\,p_{2b}(p_3\epsilon_2\epsilon_3)_aI_2$$

$$+4p_{3a}(p_3\epsilon_2\epsilon_3)_bI_{23}-2\,(p_2p_3)\,p_{3a}(p_3\epsilon_2\epsilon_3)_bI_{16}-2\,(p_1Np_2)\,p_{3a}(p_3\epsilon_2\epsilon_3)_bI_2$$

$$+2\,(p_1Np_2)\,(p_2p_3)\,(\epsilon_2D\epsilon_3)_{ba}I_{11}+2\,(p_1Np_3)\,(p_2p_3)\,(\epsilon_2D\epsilon_3)_{ba}I'_{11}$$

$$-4\,(p_1Np_2)\,(p_1Np_3)\,(\epsilon_2D\epsilon_3)_{ba}I_5+2\,(p_1Np_2)\,(p_2Dp_3)\,(\epsilon_2D\epsilon_3)_{ba}I_1$$

$$+2\,(p_1Np_3)\,(p_2Dp_3)\,(\epsilon_2D\epsilon_3)_{ba}I_1+2\,(p_2p_3)\,p_{2b}(p_1N\epsilon_2\epsilon_3)_aI_2$$

$$-4\,(p_1Np_3)\,p_{2b}(p_1N\epsilon_2\epsilon_3)_aI_9+2\,(p_2Dp_3)\,p_{2b}(p_1N\epsilon_2\epsilon_3)_aI_{11}$$

$$-2\,(p_2p_3)\,p_{3a}(p_1N\epsilon_2\epsilon_3)_bI_2-2\,(p_2Dp_3)\,p_{3a}(p_1N\epsilon_2\epsilon_3)_bI_{12}$$

$$-2\,(p_2p_3)\,p_{2b}(p_1N\epsilon_3\epsilon_2)_aI_2-2\,(p_2Dp_3)\,p_{2b}(p_1N\epsilon_3\epsilon_2)_aI'_{12}$$

$$+2\,(p_2p_3)\,p_{3a}(p_1N\epsilon_3\epsilon_2)_bI_2+4\,(p_1Np_2)\,p_{3a}(p_1N\epsilon_3\epsilon_2)_bI_9$$

$$+2\,(p_2Dp_3)\,p_{3a}(p_1N\epsilon_3\epsilon_2)_bI'_{11}-2\,(p_2p_3)\,p_{2b}(p_2D\epsilon_2\epsilon_3)_aI_{20}$$

$$-4\,(p_1Np_3)\,p_{2b}(p_2D\epsilon_2\epsilon_3)_aI_6+2\,(p_2Dp_3)\,p_{2b}(p_2D\epsilon_2\epsilon_3)_aI'_{22}$$

$$+2\,(p_2p_3)\,p_{3a}(p_2D\epsilon_2\epsilon_3)_bI_{20}-2\,(p_2Dp_3)\,p_{3a}(p_2D\epsilon_2\epsilon_3)_bI'_{22}$$

$$+2\,(p_1Np_3)\,p_{2b}(p_2D\epsilon_3\epsilon_2)_aI_{11}-2\,(p_2Dp_3)\,p_{2b}(p_2D\epsilon_3\epsilon_2)_aI_{15}$$

$$+2\,(p_1Np_2)\,p_{3b}(p_2D\epsilon_3\epsilon_2)_aI_{11}-2\,(p_2Dp_3)\,p_{3b}(p_2D\epsilon_3\epsilon_2)_aI_{15}$$

$$+2\,(p_2p_3)\,p_{2b}(p_2\epsilon_3D\epsilon_2)_aI_{14}-2\,(p_1Np_3)\,p_{2b}(p_2\epsilon_3D\epsilon_2)_aI_{11}$$

$$+2\,(p_2p_3)\,p_{3b}(p_2\epsilon_3D\epsilon_2)_aI_{14}+2\,(p_1Np_2)\,p_{3b}(p_2\epsilon_3D\epsilon_2)_aI_{11}$$

$$-2\,(p_1Np_3)\,p_{2b}(p_3D\epsilon_2\epsilon_3)_aI'_{11}-2\,(p_2Dp_3)\,p_{2b}(p_3D\epsilon_2\epsilon_3)_aI_{15}$$

$$-2\,(p_1Np_2)\,p_{3b}(p_3D\epsilon_2\epsilon_3)_aI'_{11}-2\,(p_2Dp_3)\,p_{3b}(p_3D\epsilon_2\epsilon_3)_aI_{15}$$

$$+2\,(p_2p_3)\,p_{2b}(p_3D\epsilon_3\epsilon_2)_aI'_{20}-2\,(p_2Dp_3)\,p_{2b}(p_3D\epsilon_3\epsilon_2)_aI_{22}$$

$$+2\,(p_2p_3)\,p_{3b}(p_3D\epsilon_3\epsilon_2)_aI'_{20}+4\,(p_1Np_2)\,p_{3b}(p_3D\epsilon_3\epsilon_2)_aI'_{6}$$

$$-2\,(p_2Dp_3)\,p_{3b}(p_3D\epsilon_3\epsilon_2)_aI_{22}+2\,(p_2p_3)\,p_{2b}(p_3\epsilon_2D\epsilon_3)_aI_{14}$$

$$+2\,(p_1Np_3)\,p_{2b}(p_3\epsilon_2D\epsilon_3)_aI'_{11}+2\,(p_2p_3)\,p_{3b}(p_3\epsilon_2D\epsilon_3)_aI_{14}$$

$$-2\,(p_1Np_2)\,p_{3b}(p_3\epsilon_2D\epsilon_3)_aI'_{11}+2\,(p_2p_3)\,p_{2b}(p_1N\epsilon_2D\epsilon_3)_aI_{11}$$

$$-4\,(p_1Np_3)\,p_{2b}(p_1N\epsilon_2D\epsilon_3)_aI_5+2\,(p_2Dp_3)\,p_{2b}(p_1N\epsilon_2D\epsilon_3)_aI_1$$

$$-2\,(p_2p_3)\,p_{3a}(p_1N\epsilon_2D\epsilon_3)_bI_{12}-2\,(p_2Dp_3)\,p_{3a}(p_1N\epsilon_2D\epsilon_3)_bI_1$$

$$+2\,(p_2p_3)\,p_{2b}(p_1N\epsilon_3D\epsilon_2)_aI'_{12}+2\,(p_2Dp_3)\,p_{2b}(p_1N\epsilon_3D\epsilon_2)_aI_1$$

$$-2\,(p_2p_3)\,p_{3a}(p_1N\epsilon_3D\epsilon_2)_bI'_{11}+4\,(p_1Np_2)\,p_{3a}(p_1N\epsilon_3D\epsilon_2)_bI_5$$

$$-2\,(p_2Dp_3)\,p_{3a}(p_1N\epsilon_3D\epsilon_2)_bI_1+2\,(p_2p_3)\,p_{2b}(p_2D\epsilon_2D\epsilon_3)_aI'_{22}$$

$$-4\,(p_1Np_3)\,p_{2b}(p_2D\epsilon_2D\epsilon_3)_aI_7-2\,(p_2Dp_3)\,p_{2b}(p_2D\epsilon_2D\epsilon_3)_aI_{21}$$

$$-2\,(p_2p_3)\,p_{3a}(p_2D\epsilon_2D\epsilon_3)_bI'_{22}+2\,(p_2Dp_3)\,p_{3a}(p_2D\epsilon_2D\epsilon_3)_bI_{21}$$

$$+4p_{2b}(p_2D\epsilon_3D\epsilon_2)_aI_{24}+2\,(p_1Np_3)\,p_{2b}(p_2D\epsilon_3D\epsilon_2)_aI_1$$

$$-2\,(p_2Dp_3)\,p_{2b}(p_2D\epsilon_3D\epsilon_2)_aI_{17}-4p_{3a}(p_2D\epsilon_3D\epsilon_2)_bI_{24}$$

$$-2\,(p_1Np_2)\,p_{3a}(p_2D\epsilon_3D\epsilon_2)_bI_1+2\,(p_2Dp_3)\,p_{3a}(p_2D\epsilon_3D\epsilon_2)_bI_{17}$$

$$+4p_{2b}(p_3D\epsilon_2D\epsilon_3)_aI_{24}+2\,(p_1Np_3)\,p_{2b}(p_3D\epsilon_2D\epsilon_3)_aI_1$$

$$-2\,(p_2Dp_3)\,p_{2b}(p_3D\epsilon_2D\epsilon_3)_aI_{17}-4p_{3a}(p_3D\epsilon_2D\epsilon_3)_bI_{24}$$

$$-2\,(p_1Np_2)\,p_{3a}(p_3D\epsilon_2D\epsilon_3)_bI_1+2\,(p_2Dp_3)\,p_{3a}(p_3D\epsilon_2D\epsilon_3)_bI_{17}$$

$$+2\,(p_2p_3)\,p_{2b}(p_3D\epsilon_3D\epsilon_2)_aI_{22} - 2\,(p_2Dp_3)\,p_{2b}(p_3D\epsilon_3D\epsilon_2)_aI'_{21}$$
$$+2\,(p_2p_3)\,p_{3b}(p_3D\epsilon_3D\epsilon_2)_aI_{22} - 4\,(p_1Np_2)\,p_{3b}(p_3D\epsilon_3D\epsilon_2)_aI'_7$$
$$-2\,(p_2Dp_3)\,p_{3b}(p_3D\epsilon_3D\epsilon_2)_aI'_{21} - (p_1Np_2)\,(p_2\epsilon_3p_2)\,\epsilon_{2ab}I_2$$
$$-\,(p_1Np_3)\,(p_2\epsilon_3p_2)\,\epsilon_{2ab}I_2 + 2\,(p_1N\epsilon_3p_2)\,(p_2p_3)\,\epsilon_{2ab}I_2$$
$$+4\,(p_1N\epsilon_3p_2)\,(p_1Np_2)\,\epsilon_{2ab}I_9 - 2\,(p_1Np_2)\,(p_2D\epsilon_3p_2)\,\epsilon_{2ab}I_{11}$$
$$-2\,(p_1Np_2)\,(p_2\epsilon_3Dp_3)\,\epsilon_{2ab}I'_6 + 4\,(p_1N\epsilon_3Dp_2)\,(p_1Np_2)\,\epsilon_{2ab}I_5$$
$$-2\,(p_1N\epsilon_3Dp_2)\,(p_2Dp_3)\,\epsilon_{2ab}I_1 + 2\,(p_1N\epsilon_3Dp_3)\,(p_2p_3)\,\epsilon_{2ab}I'_6$$
$$+2\,(p_1N\epsilon_3Dp_3)\,(p_1Np_2)\,\epsilon_{2ab}I'_4 + 2\,(p_1N\epsilon_3Dp_3)\,(p_2Dp_3)\,\epsilon_{2ab}I'_7$$
$$-2\,(p_1N\epsilon_3Np_1)\,(p_2p_3)\,\epsilon_{2ab}I_9 - 4\,(p_1N\epsilon_3Np_1)\,(p_1Np_2)\,\epsilon_{2ab}I_{10}$$
$$+2\,(p_1N\epsilon_3Np_1)\,(p_2Dp_3)\,\epsilon_{2ab}I_5 - (p_1Np_2)\,(p_2D\epsilon_3Dp_2)\,\epsilon_{2ab}I_1$$
$$+\,(p_1Np_3)\,(p_2D\epsilon_3Dp_2)\,\epsilon_{2ab}I_1 + 2\,(p_1Np_2)\,(p_2D\epsilon_3Dp_3)\,\epsilon_{2ab}I'_7$$
$$+\,\mathrm{tr}(D\epsilon_3)\,(p_1Np_2)\,(p_2p_3)\,\epsilon_{2ab}I'_6 - \mathrm{tr}(D\epsilon_3)\,(p_1Np_3)\,(p_2p_3)\,\epsilon_{2ab}I'_6$$
$$-\,\mathrm{tr}(D\epsilon_3)\,(p_1Np_2)\,(p_1Np_3)\,\epsilon_{2ab}I'_4 - \mathrm{tr}(D\epsilon_3)\,(p_1Np_2)\,(p_2Dp_3)\,\epsilon_{2ab}I'_7$$
$$-\,\mathrm{tr}(D\epsilon_3)\,(p_1Np_3)\,(p_2Dp_3)\,\epsilon_{2ab}I'_7)\,, \tag{2.53}$$

$$\mathcal{A}^{(1)}_{C^{(p-1)}Bh} = \frac{2i^{p(p+1)}\sqrt{2}}{(p-2)!}C^i{}_{b_1\ldots b_{p-2}}\varepsilon^{abcb_1\ldots b_{p-2}}\left(-2\,\mathrm{tr}(D\epsilon_3)p_{2c}p_{3b}p_{3i}(p_3\epsilon_2)_aI'_6\right.$$
$$-2p_{2c}p_{2i}(p_2\epsilon_3)_b(p_3\epsilon_2)_aI_2 - 2p_{2i}p_{3c}(p_2\epsilon_3)_b(p_3\epsilon_2)_aI_2$$
$$-2p_{2c}p_{3i}(p_2\epsilon_3)_b(p_3\epsilon_2)_aI_2 - 2p_{3c}p_{3i}(p_2\epsilon_3)_b(p_3\epsilon_2)_aI_2$$
$$+2\,\mathrm{tr}(D\epsilon_3)p_{2c}p_{2i}p_{3a}(p_3\epsilon_2)_bI'_6 - 2\,(p_2p_3)\,p_{2c}p_{2i}(\epsilon_2\epsilon_3)_{ab}I_2$$
$$+4\,(p_1Np_3)\,p_{2c}p_{2i}(\epsilon_2\epsilon_3)_{ab}I_9 - 2\,(p_2Dp_3)\,p_{2c}p_{2i}(\epsilon_2\epsilon_3)_{ab}I_{11}$$
$$+2\,(p_2p_3)\,p_{2c}p_{3i}(\epsilon_2\epsilon_3)_{ab}I_2 + 2\,(p_2Dp_3)\,p_{2c}p_{3i}(\epsilon_2\epsilon_3)_{ab}I'_{11}$$

$$+2\,(p_2p_3)\,p_{3_c}p_{3_i}(\epsilon_2\epsilon_3)_{ab}I_2+4\,(p_1Np_2)\,p_{3_c}p_{3_i}(\epsilon_2\epsilon_3)_{ab}I_9$$

$$+2\,(p_2Dp_3)\,p_{3_c}p_{3_i}(\epsilon_2\epsilon_3)_{ab}I'_{11}-2\,(p_2p_3)\,p_{2_i}p_{3_a}(\epsilon_2\epsilon_3)_{bc}I_2$$

$$-2\,(p_2Dp_3)\,p_{2_i}p_{3_a}(\epsilon_2\epsilon_3)_{bc}I_{11}-8\,(p_2Dp_3)\,p_{2_c}p_{3_a}(\epsilon_2\epsilon_3)_{bi}I_0$$

$$+8\,(p_2Dp_3)\,p_{2_c}p_{3_a}(\epsilon_2\epsilon_3)_{ib}I_0-2\,\mathrm{tr}(D\epsilon_3)p_{2_c}p_{3_b}p_{3_i}(p_1N\epsilon_2)_aI'_4$$

$$-4p_{2_c}p_{3_i}(p_1N\epsilon_2)_a(p_2\epsilon_3)_bI_9-4p_{3_c}p_{3_i}(p_1N\epsilon_2)_a(p_2\epsilon_3)_bI_9$$

$$+4p_{2_c}p_{3_b}(p_1N\epsilon_2)_a(p_2\epsilon_3)_iI_9-4p_{2_c}p_{2_i}(p_1N\epsilon_3)_a(p_3\epsilon_2)_bI_9$$

$$+8p_{2_b}p_{3_i}(p_1N\epsilon_2)_c(p_1N\epsilon_3)_aI_{10}+4p_{2_c}p_{3_i}(p_1N\epsilon_3)_b(p_3\epsilon_2)_aI_9$$

$$+4p_{2_c}p_{3_a}(p_1N\epsilon_3)_i(p_3\epsilon_2)_bI_9-8p_{2_c}p_{3_b}(p_1N\epsilon_2)_a(p_1N\epsilon_3)_iI_{10}$$

$$+4\,\mathrm{tr}(D\epsilon_3)p_{2_c}p_{3_b}p_{3_i}(p_2D\epsilon_2)_aI_3-4p_{2_c}p_{3_i}(p_2D\epsilon_2)_a(p_2\epsilon_3)_bI_6$$

$$-4p_{3_c}p_{3_i}(p_2D\epsilon_2)_a(p_2\epsilon_3)_bI_6+4p_{2_c}p_{3_b}(p_2D\epsilon_2)_a(p_2\epsilon_3)_iI_6$$

$$-4p_{2_c}p_{3_i}(p_1N\epsilon_3)_b(p_2D\epsilon_2)_aI_4+4p_{2_c}p_{3_b}(p_1N\epsilon_3)_i(p_2D\epsilon_2)_aI_4$$

$$+2p_{2_c}p_{2_i}(p_2D\epsilon_3)_a(p_3\epsilon_2)_bI_{11}+2p_{2_i}p_{3_c}(p_2D\epsilon_3)_a(p_3\epsilon_2)_bI_{11}$$

$$-4p_{2_b}p_{3_i}(p_1N\epsilon_2)_c(p_2D\epsilon_3)_aI_5+4p_{2_c}p_{3_i}(p_2D\epsilon_2)_b(p_2D\epsilon_3)_aI_7$$

$$+4p_{3_c}p_{3_i}(p_2D\epsilon_2)_b(p_2D\epsilon_3)_aI_7+2p_{2_c}p_{3_i}(p_2D\epsilon_3)_b(p_3\epsilon_2)_aI'_{11}$$

$$+2p_{3_c}p_{3_i}(p_2D\epsilon_3)_b(p_3\epsilon_2)_aI'_{11}+8p_{2_c}p_{3_a}(p_2D\epsilon_3)_b(p_3\epsilon_2)_iI_0$$

$$-4p_{3_c}p_{3_i}(p_1N\epsilon_2)_a(p_2D\epsilon_3)_bI_5-4p_{2_c}p_{3_a}(p_2D\epsilon_3)_i(p_3\epsilon_2)_bI_8$$

$$+4p_{2_c}p_{3_b}(p_1N\epsilon_2)_a(p_2D\epsilon_3)_iI_5+4p_{2_c}p_{3_b}(p_2D\epsilon_2)_a(p_2D\epsilon_3)_iI_7$$

$$-2p_{2_c}p_{2_i}p_{3_a}(p_2\epsilon_3\epsilon_2)_bI_2-2p_{2_c}p_{3_a}p_{3_i}(p_2\epsilon_3\epsilon_2)_bI_2$$

$$-2\,\mathrm{tr}(D\epsilon_3)p_{2_c}p_{3_b}p_{3_i}(p_3D\epsilon_2)_aI'_7-2p_{2_c}p_{3_i}(p_2\epsilon_3)_b(p_3D\epsilon_2)_aI'_{11}$$

$$-2p_{3_c}p_{3_i}(p_2\epsilon_3)_b(p_3D\epsilon_2)_aI'_{11}-4p_{2_c}p_{3_b}(p_2\epsilon_3)_i(p_3D\epsilon_2)_aI_8$$

$$-4p_{2_c}p_{3_i}(p_1N\epsilon_3)_b(p_3D\epsilon_2)_aI_5+4p_{2_c}p_{3_b}(p_1N\epsilon_3)_i(p_3D\epsilon_2)_aI_5$$

$$+2p_{2_c}p_{3_i}(p_2D\epsilon_3)_b(p_3D\epsilon_2)_aI_1+2p_{3_c}p_{3_i}(p_2D\epsilon_3)_b(p_3D\epsilon_2)_aI_1$$

$$-2\,\mathrm{tr}(D\epsilon_3)p_{2c}p_{2i}p_{3a}(p_3D\epsilon_2)_bI'_7+2p_{2c}p_{2i}(p_2\epsilon_3)_a(p_3D\epsilon_2)_bI_{11}$$

$$+2p_{2i}p_{3c}(p_2\epsilon_3)_a(p_3D\epsilon_2)_bI_{11}-4p_{2c}p_{2i}(p_1N\epsilon_3)_a(p_3D\epsilon_2)_bI_5$$

$$+2p_{2c}p_{2i}(p_2D\epsilon_3)_a(p_3D\epsilon_2)_bI_1+2p_{2i}p_{3c}(p_2D\epsilon_3)_a(p_3D\epsilon_2)_bI_1$$

$$+8p_{2c}p_{3a}(p_2\epsilon_3)_b(p_3D\epsilon_2)_iI_0+4p_{2c}p_{3b}(p_3D\epsilon_3)_i(p_3\epsilon_2)_aI'_6$$

$$+4p_{2c}p_{3b}(p_1N\epsilon_2)_a(p_3D\epsilon_3)_iI'_4-8p_{2c}p_{3b}(p_2D\epsilon_2)_a(p_3D\epsilon_3)_iI_3$$

$$+4p_{2c}p_{3b}(p_3D\epsilon_2)_a(p_3D\epsilon_3)_iI'_7-2p_{2c}p_{2i}p_{3a}(p_3\epsilon_2\epsilon_3)_bI_2$$

$$-2p_{2c}p_{3a}p_{3i}(p_3\epsilon_2\epsilon_3)_bI_2-2\,(p_2p_3)\,p_{2c}p_{2i}(\epsilon_2D\epsilon_3)_{ab}I_{11}$$

$$+4\,(p_1Np_3)\,p_{2c}p_{2i}(\epsilon_2D\epsilon_3)_{ab}I_5-2\,(p_2Dp_3)\,p_{2c}p_{2i}(\epsilon_2D\epsilon_3)_{ab}I_1$$

$$+2\,(p_2p_3)\,p_{2c}p_{3i}(\epsilon_2D\epsilon_3)_{ba}I'_{11}+2\,(p_2Dp_3)\,p_{2c}p_{3i}(\epsilon_2D\epsilon_3)_{ba}I_1$$

$$+2\,(p_2p_3)\,p_{3c}p_{3i}(\epsilon_2D\epsilon_3)_{ba}I'_{11}-4\,(p_1Np_2)\,p_{3c}p_{3i}(\epsilon_2D\epsilon_3)_{ba}I_5$$

$$+2\,(p_2Dp_3)\,p_{3c}p_{3i}(\epsilon_2D\epsilon_3)_{ba}I_1-2\,(p_2p_3)\,p_{2i}p_{3a}(\epsilon_2D\epsilon_3)_{bc}I_{11}$$

$$-2\,(p_2Dp_3)\,p_{2i}p_{3a}(\epsilon_2D\epsilon_3)_{bc}I_1+8\,(p_2p_3)\,p_{2c}p_{3a}(\epsilon_2D\epsilon_3)_{bi}I_0$$

$$+8\,(p_2p_3)\,p_{2c}p_{3a}(\epsilon_2D\epsilon_3)_{ib}I_0-4p_{2c}p_{3a}p_{3i}(p_1N\epsilon_2\epsilon_3)_bI_9$$

$$+4p_{2c}p_{2i}p_{3a}(p_1N\epsilon_3\epsilon_2)_bI_9-4p_{2c}p_{3a}p_{3i}(p_2D\epsilon_2\epsilon_3)_bI_6-2p_{2c}p_{2i}p_{3a}(p_2D\epsilon_3\epsilon_2)_bI_{11}$$

$$+2p_{2c}p_{3a}p_{3i}(p_2D\epsilon_3\epsilon_2)_bI_{11}-2p_{2c}p_{2i}p_{3a}(p_2\epsilon_3D\epsilon_2)_bI_{11}-2p_{2c}p_{3a}p_{3i}(p_2\epsilon_3D\epsilon_2)_bI_{11}$$

$$+2p_{2c}p_{2i}p_{3a}(p_3D\epsilon_2\epsilon_3)_bI'_{11}-2p_{2c}p_{3a}p_{3i}(p_3D\epsilon_2\epsilon_3)_bI'_{11}-4p_{2c}p_{2i}p_{3a}(p_3D\epsilon_3\epsilon_2)_bI'_6$$

$$+2p_{2c}p_{2i}p_{3a}(p_3\epsilon_2D\epsilon_3)_bI'_{11}+2p_{2c}p_{3a}p_{3i}(p_3\epsilon_2D\epsilon_3)_bI'_{11}$$

$$-4p_{2c}p_{3a}p_{3i}(p_1N\epsilon_2D\epsilon_3)_bI_5+4p_{2c}p_{2i}p_{3a}(p_1N\epsilon_3D\epsilon_2)_bI_5$$

$$-4p_{2c}p_{3a}p_{3i}(p_2D\epsilon_2D\epsilon_3)_bI_7-2p_{2c}p_{2i}p_{3a}(p_2D\epsilon_3D\epsilon_2)_bI_1$$

$$+2p_{2c}p_{3a}p_{3i}(p_2D\epsilon_3D\epsilon_2)_bI_1-2p_{2c}p_{2i}p_{3a}(p_3D\epsilon_2D\epsilon_3)_bI_1$$

$$+2p_{2c}p_{3a}p_{3i}(p_3D\epsilon_2D\epsilon_3)_bI_1+4p_{2c}p_{2i}p_{3a}(p_3D\epsilon_3D\epsilon_2)_bI'_7-(p_2\epsilon_3p_2)\,p_{2c}p_{2i}\epsilon_{2ab}I_2$$

$$+4\,(p_1N\epsilon_3p_2)\,p_{2c}p_{2i}\epsilon_{2ab}I_9-2\,(p_2D\epsilon_3p_2)\,p_{2c}p_{2i}\epsilon_{2ab}I_{11}$$

$$-2\,(p_2\epsilon_3 Dp_3)\,p_{2c}p_{2i}\epsilon_{2ab}I'_6+4\,(p_1N\epsilon_3Dp_2)\,p_{2c}p_{2i}\epsilon_{2ab}I_5$$

$$+2\,(p_1N\epsilon_3Dp_3)\,p_{2c}p_{2i}\epsilon_{2ab}I'_4-4\,(p_1N\epsilon_3Np_1)\,p_{2c}p_{2i}\epsilon_{2ab}I_{10}$$

$$-\,(p_2D\epsilon_3Dp_2)\,p_{2c}p_{2i}\epsilon_{2ab}I_1+2\,(p_2D\epsilon_3Dp_3)\,p_{2c}p_{2i}\epsilon_{2ab}I'_7$$

$$+\,\mathrm{tr}(D\epsilon_3)\,(p_2p_3)\,p_{2c}p_{2i}\epsilon_{2ab}I'_6-\mathrm{tr}(D\epsilon_3)\,(p_1Np_3)\,p_{2c}p_{2i}\epsilon_{2ab}I'_4$$

$$-\,\mathrm{tr}(D\epsilon_3)\,(p_2Dp_3)\,p_{2c}p_{2i}\epsilon_{2ab}I'_7-(p_2\epsilon_3p_2)\,p_{2i}p_{3c}\epsilon_{2ab}I_2$$

$$+2\,(p_1N\epsilon_3p_2)\,p_{2i}p_{3c}\epsilon_{2ab}I_9-2\,(p_2D\epsilon_3p_2)\,p_{2i}p_{3c}\epsilon_{2ab}I_{11}$$

$$-2\,(p_2\epsilon_3Dp_3)\,p_{2i}p_{3c}\epsilon_{2ab}I'_6+2\,(p_1N\epsilon_3Dp_2)\,p_{2i}p_{3c}\epsilon_{2ab}I_5$$

$$-\,(p_2D\epsilon_3Dp_2)\,p_{2i}p_{3c}\epsilon_{2ab}I_1+2\,(p_2D\epsilon_3Dp_3)\,p_{2i}p_{3c}\epsilon_{2ab}I'_7$$

$$+\,\mathrm{tr}(D\epsilon_3)\,(p_2p_3)\,p_{2i}p_{3c}\epsilon_{2ab}I'_6-\mathrm{tr}(D\epsilon_3)\,(p_2Dp_3)\,p_{2i}p_{3c}\epsilon_{2ab}I'_7$$

$$-\,(p_2\epsilon_3p_2)\,p_{2c}p_{3i}\epsilon_{2ab}I_2+2\,(p_1N\epsilon_3p_2)\,p_{2c}p_{3i}\epsilon_{2ab}I_9$$

$$-2\,(p_1N\epsilon_3Dp_2)\,p_{2c}p_{3i}\epsilon_{2ab}I_5+(p_2D\epsilon_3Dp_2)\,p_{2c}p_{3i}\epsilon_{2ab}I_1$$

$$-\,\mathrm{tr}(D\epsilon_3)\,(p_2p_3)\,p_{2c}p_{3i}\epsilon_{2ab}I'_6-\mathrm{tr}(D\epsilon_3)\,(p_2Dp_3)\,p_{2c}p_{3i}\epsilon_{2ab}I'_7$$

$$-\,(p_2\epsilon_3p_2)\,p_{3c}p_{3i}\epsilon_{2ab}I_2+(p_2D\epsilon_3Dp_2)\,p_{3c}p_{3i}\epsilon_{2ab}I_1$$

$$+2\,(p_1Np_3)\,p_{2i}(p_2\epsilon_3)_c\epsilon_{2ab}I_9-2\,(p_1Np_2)\,p_{3i}(p_2\epsilon_3)_c\epsilon_{2ab}I_9$$

$$+2\,(p_2p_3)\,p_{2c}(p_2\epsilon_3)_i\epsilon_{2ab}I_2-2\,(p_1Np_3)\,p_{2c}(p_2\epsilon_3)_i\epsilon_{2ab}I_9$$

$$+2\,(p_2p_3)\,p_{3c}(p_2\epsilon_3)_i\epsilon_{2ab}I_2+2\,(p_1Np_2)\,p_{3c}(p_2\epsilon_3)_i\epsilon_{2ab}I_9$$

$$-2\,(p_2p_3)\,p_{2i}(p_1N\epsilon_3)_c\epsilon_{2ab}I_9-2\,(p_2Dp_3)\,p_{2i}(p_1N\epsilon_3)_c\epsilon_{2ab}I_5$$

$$+2\,(p_2p_3)\,p_{3i}(p_1N\epsilon_3)_c\epsilon_{2ab}I_9+4\,(p_1Np_2)\,p_{3i}(p_1N\epsilon_3)_c\epsilon_{2ab}I_{10}$$

$$-2\,(p_2Dp_3)\,p_{3i}(p_1N\epsilon_3)_c\epsilon_{2ab}I_5-2\,(p_2p_3)\,p_{2c}(p_1N\epsilon_3)_i\epsilon_{2ab}I_9$$

$$+2\,(p_2Dp_3)\,p_{2c}(p_1N\epsilon_3)_i\epsilon_{2ab}I_5+2\,(p_1Np_3)\,p_{2i}(p_2D\epsilon_3)_c\epsilon_{2ab}I_5$$

$$-2\,(p_1Np_2)\,p_{3i}(p_2D\epsilon_3)_c\epsilon_{2ab}I_5+2\,(p_1Np_3)\,p_{2c}(p_2D\epsilon_3)_i\epsilon_{2ab}I_5$$

$$-2\,(p_2Dp_3)\,p_{2c}(p_2D\epsilon_3)_i\epsilon_{2ab}I_1+2\,(p_1Np_2)\,p_{3c}(p_2D\epsilon_3)_i\epsilon_{2ab}I_5$$

$$
\begin{aligned}
&-2\,(p_2Dp_3)\,p_{3c}(p_2D\epsilon_3)_i\epsilon_{2ab}I_1+2\,(p_2p_3)\,p_{2c}(p_3D\epsilon_3)_i\epsilon_{2ab}I_6'\\
&+2\,(p_2Dp_3)\,p_{2c}(p_3D\epsilon_3)_i\epsilon_{2ab}I_7'-\mathrm{tr}(D\epsilon_3)\,(p_2p_3)\,p_{3a}p_{3i}\epsilon_{2bc}I_6'\\
&-\mathrm{tr}(D\epsilon_3)\,(p_1Np_2)\,p_{3a}p_{3i}\epsilon_{2bc}I_4'-\mathrm{tr}(D\epsilon_3)\,(p_2Dp_3)\,p_{3a}p_{3i}\epsilon_{2bc}I_7'\\
&-2\,(p_2p_3)\,p_{3a}(p_1N\epsilon_3)_i\epsilon_{2bc}I_9-4\,(p_1Np_2)\,p_{3a}(p_1N\epsilon_3)_i\epsilon_{2bc}I_{10}\\
&+2\,(p_2Dp_3)\,p_{3a}(p_1N\epsilon_3)_i\epsilon_{2bc}I_5+2\,(p_2p_3)\,p_{3a}(p_3D\epsilon_3)_i\epsilon_{2bc}I_6'\\
&+2\,(p_1Np_2)\,p_{3a}(p_3D\epsilon_3)_i\epsilon_{2bc}I_4'+2\,(p_2Dp_3)\,p_{3a}(p_3D\epsilon_3)_i\epsilon_{2bc}I_7'\\
&-4\,(p_2p_3)\,p_{2c}(p_3\epsilon_2)_b\epsilon_{3ai}I_2+4\,(p_1Np_3)\,p_{2c}(p_3\epsilon_2)_b\epsilon_{3ai}I_9\\
&-4\,(p_2p_3)\,p_{3c}(p_3\epsilon_2)_b\epsilon_{3ai}I_2-4\,(p_1Np_2)\,p_{3c}(p_3\epsilon_2)_b\epsilon_{3ai}I_9\\
&-4\,(p_2p_3)\,p_{2c}(p_2D\epsilon_2)_b\epsilon_{3ai}I_6-4\,(p_1Np_3)\,p_{2c}(p_2D\epsilon_2)_b\epsilon_{3ai}I_4\\
&-4\,(p_2Dp_3)\,p_{2c}(p_2D\epsilon_2)_b\epsilon_{3ai}I_7-4\,(p_1Np_3)\,p_{2c}(p_3D\epsilon_2)_b\epsilon_{3ai}I_5\\
&+4\,(p_2Dp_3)\,p_{2c}(p_3D\epsilon_2)_b\epsilon_{3ai}I_1-4\,(p_1Np_2)\,p_{3c}(p_3D\epsilon_2)_b\epsilon_{3ai}I_5\\
&+4\,(p_2Dp_3)\,p_{3c}(p_3D\epsilon_2)_b\epsilon_{3ai}I_1+4\,(p_1N\epsilon_2p_3)\,p_{2c}p_{3a}\epsilon_{3bi}I_9\\
&+4\,(p_2D\epsilon_2p_3)\,p_{2c}p_{3a}\epsilon_{3bi}I_6+4\,(p_3D\epsilon_2p_3)\,p_{2c}p_{3a}\epsilon_{3bi}I_{11}'\\
&+4\,(p_1N\epsilon_2Dp_3)\,p_{2c}p_{3a}\epsilon_{3bi}I_5+4\,(p_2D\epsilon_2Dp_3)\,p_{2c}p_{3a}\epsilon_{3bi}I_7\\
&+4\,(p_2p_3)\,p_{2c}(p_1N\epsilon_2)_a\epsilon_{3bi}I_9-8\,(p_1Np_3)\,p_{2c}(p_1N\epsilon_2)_a\epsilon_{3bi}I_{10}\\
&+4\,(p_2Dp_3)\,p_{2c}(p_1N\epsilon_2)_a\epsilon_{3bi}I_5-4\,(p_2p_3)\,p_{3a}(p_2D\epsilon_2)_c\epsilon_{3bi}I_6\\
&-4\,(p_2Dp_3)\,p_{3a}(p_2D\epsilon_2)_c\epsilon_{3bi}I_7-4\,(p_2p_3)\,p_{3b}(p_1N\epsilon_2)_a\epsilon_{3ci}I_9\\
&-4\,(p_2Dp_3)\,p_{3b}(p_1N\epsilon_2)_a\epsilon_{3ci}I_5+2\,(p_1Np_2)\,(p_2p_3)\,\epsilon_{2ab}\epsilon_{3ci}I_9\\
&-2\,(p_1Np_3)\,(p_2p_3)\,\epsilon_{2ab}\epsilon_{3ci}I_9-4\,(p_1Np_2)\,(p_1Np_3)\,\epsilon_{2ab}\epsilon_{3ci}I_{10}\\
&+2\,(p_1Np_2)\,(p_2Dp_3)\,\epsilon_{2ab}\epsilon_{3ci}I_5+2\,(p_1Np_3)\,(p_2Dp_3)\,\epsilon_{2ab}\epsilon_{3ci}I_5\Big)\,,
\end{aligned}
\tag{2.54}
$$

$$
\mathcal{A}^{(2)}_{C^{(p-1)}Bh}=\frac{4i^{p(p+1)}\sqrt{2}}{(p-3)!}C^{ij}{}_{b_1\ldots b_{p-3}}\varepsilon^{abcdb_1\ldots b_{p-3}}\Big(2p_{2d}p_{2j}p_{3a}p_{3i}(\epsilon_2\epsilon_3)_{bc}I_9
$$

$$+2p_{2d}p_{2j}p_{3a}p_{3i}(\epsilon_2 D\epsilon_3)_{bc}I_5 + p_{2d}p_{2i}p_{3j}(p_2\epsilon_3)_c\epsilon_{2ab}I_9$$

$$+p_{2i}p_{3d}p_{3j}(p_2\epsilon_3)_c\epsilon_{2ab}I_9 - 2p_{2d}p_{2i}p_{3j}(p_1N\epsilon_3)_c\epsilon_{2ab}I_{10}$$

$$+p_{2d}p_{2i}p_{3j}(p_2D\epsilon_3)_c\epsilon_{2ab}I_5 + p_{2i}p_{3d}p_{3j}(p_2D\epsilon_3)_c\epsilon_{2ab}I_5$$

$$-\frac{1}{2}\,\mathrm{tr}(D\epsilon_3)p_{2d}p_{2j}p_{3a}p_{3i}\epsilon_{2bc}I'_4 + p_{2d}p_{2j}p_{3a}(p_2\epsilon_3)_i\epsilon_{2bc}I_9 + p_{2d}p_{3a}p_{3j}(p_2\epsilon_3)_i\epsilon_{2bc}I_9$$

$$-2p_{2d}p_{2j}p_{3a}(p_1N\epsilon_3)_i\epsilon_{2bc}I_{10} + p_{2d}p_{2j}p_{3a}(p_2D\epsilon_3)_i\epsilon_{2bc}I_5$$

$$+p_{2d}p_{3a}p_{3i}(p_2D\epsilon_3)_j\epsilon_{2bc}I_5 + p_{2d}p_{2j}p_{3a}(p_3D\epsilon_3)_i\epsilon_{2bc}I'_4 + 2p_{2d}p_{2i}p_{3a}(p_3\epsilon_2)_c\epsilon_{3bj}I_9$$

$$-2p_{2a}p_{3d}p_{3i}(p_3\epsilon_2)_c\epsilon_{3bj}I_9 - 2p_{2d}p_{3a}p_{3i}(p_2D\epsilon_2)_c\epsilon_{3bj}I_4$$

$$+2p_{2d}p_{2i}p_{3a}(p_3D\epsilon_2)_c\epsilon_{3bj}I_5 + 2p_{2a}p_{3d}p_{3i}(p_3D\epsilon_2)_c\epsilon_{3bj}I_5$$

$$+4p_{2d}p_{3b}p_{3i}(p_1N\epsilon_2)_a\epsilon_{3cj}I_{10} - (p_2p_3)\,p_{2d}p_{2i}\epsilon_{2ab}\epsilon_{3cj}I_9$$

$$+2\,(p_1Np_3)\,p_{2d}p_{2i}\epsilon_{2ab}\epsilon_{3cj}I_{10} - (p_2Dp_3)\,p_{2d}p_{2i}\epsilon_{2ab}\epsilon_{3cj}I_5$$

$$+\,(p_2p_3)\,p_{2d}p_{3i}\epsilon_{2ab}\epsilon_{3cj}I_9 - (p_2Dp_3)\,p_{2d}p_{3i}\epsilon_{2ab}\epsilon_{3cj}I_5 - (p_2p_3)\,p_{2j}p_{3a}\epsilon_{2bc}\epsilon_{3di}I_9$$

$$-\,(p_2Dp_3)\,p_{2j}p_{3a}\epsilon_{2bc}\epsilon_{3di}I_5 + (p_2p_3)\,p_{3a}p_{3j}\epsilon_{2bc}\epsilon_{3di}I_9$$

$$+2\,(p_1Np_2)\,p_{3a}p_{3j}\epsilon_{2bc}\epsilon_{3di}I_{10} - (p_2Dp_3)\,p_{3a}p_{3j}\epsilon_{2bc}\epsilon_{3di}I_5\big)\,, \tag{2.55}$$

$$\mathcal{A}^{(3)}_{C^{(p-1)}Bh} = \frac{8i^{p(p+1)}\sqrt{2}}{(p-4)!}C^{ijk}{}_{b_1\ldots b_{p-4}}\varepsilon^{abcdeb_1\ldots b_{p-4}}p_{2e}p_{2j}p_{3a}p_{3i}\epsilon_{2bc}\epsilon_{3dk}I_{10}. \tag{2.56}$$

2.2.5 $C^{(p-3)}$ amplitudes

$$\mathcal{A}_{C^{(p-3)}BB} = \mathcal{A}^{(0)}_{C^{(p-3)}BB} + \mathcal{A}^{(1)}_{C^{(p-3)}BB} + \mathcal{A}^{(2)}_{C^{(p-3)}BB}. \tag{2.57}$$

$$\mathcal{A}^{(0)}_{C^{(p-3)}BB} = \frac{2i^{p(p+1)}\sqrt{2}}{(p-3)!}C_{b_1\ldots b_{p-3}}\varepsilon^{abcdb_1\ldots b_{p-3}}\big(-4\,(p_2Dp_3)\,p_{2d}p_{3a}(\epsilon_2\epsilon_3)_{bc}I_0$$

$$+4p_{2d}p_{3b}(p_1N\epsilon_2)_a(p_2\epsilon_3)_cI_9 + 4p_{2c}p_{3a}(p_1N\epsilon_2)_d(p_1N\epsilon_3)_bI_{10}$$

$$+4p_{2d}p_{3b}(p_2D\epsilon_2)_a(p_2\epsilon_3)_cI_6 + 4p_{2d}p_{3b}(p_1N\epsilon_3)_c(p_2D\epsilon_2)_aI_4$$

$$
\begin{aligned}
&-4p_{2c}p_{3a}(p_1N\epsilon_2)_d(p_2D\epsilon_3)_bI_5+4p_{2d}p_{3a}(p_2D\epsilon_2)_c(p_2D\epsilon_3)_bI_7\\
&-4p_{2d}p_{3a}(p_2D\epsilon_3)_c(p_3\epsilon_2)_bI_8-4p_{2d}p_{3a}(p_2D\epsilon_2)_c(p_3D\epsilon_3)_bI_3\\
&-4\left(p_2p_3\right)p_{2d}p_{3a}(\epsilon_2D\epsilon_3)_{cb}I_0+2\left(p_2p_3\right)p_{2d}(p_2\epsilon_3)_c\epsilon_{2ab}I_2\\
&-2\left(p_1Np_3\right)p_{2d}(p_2\epsilon_3)_c\epsilon_{2ab}I_9+2\left(p_2p_3\right)p_{3d}(p_2\epsilon_3)_c\epsilon_{2ab}I_2\\
&+2\left(p_1Np_2\right)p_{3d}(p_2\epsilon_3)_c\epsilon_{2ab}I_9-2\left(p_2p_3\right)p_{2d}(p_1N\epsilon_3)_c\epsilon_{2ab}I_9\\
&+2\left(p_2Dp_3\right)p_{2d}(p_1N\epsilon_3)_c\epsilon_{2ab}I_5+2\left(p_2p_3\right)p_{2d}(p_3D\epsilon_3)_c\epsilon_{2ab}I_6'\\
&+2\left(p_2Dp_3\right)p_{2d}(p_3D\epsilon_3)_c\epsilon_{2ab}I_7'-2\left(p_1N\epsilon_3p_2\right)p_{2a}p_{3d}\epsilon_{2bc}I_9\\
&-2\left(p_2\epsilon_3Dp_3\right)p_{2a}p_{3d}\epsilon_{2bc}I_6'+2\left(p_1N\epsilon_3Dp_2\right)p_{2a}p_{3d}\epsilon_{2bc}I_5\\
&-2\left(p_2D\epsilon_3Dp_3\right)p_{2a}p_{3d}\epsilon_{2bc}I_7'+2\left(p_2p_3\right)p_{3a}(p_1N\epsilon_3)_d\epsilon_{2bc}I_9\\
&+4\left(p_1Np_2\right)p_{3a}(p_1N\epsilon_3)_d\epsilon_{2bc}I_{10}-2\left(p_2Dp_3\right)p_{3a}(p_1N\epsilon_3)_d\epsilon_{2bc}I_5\\
&+2\left(p_1Np_3\right)p_{2d}(p_2D\epsilon_3)_a\epsilon_{2bc}I_5-2\left(p_2Dp_3\right)p_{2d}(p_2D\epsilon_3)_a\epsilon_{2bc}I_1\\
&+2\left(p_1Np_2\right)p_{3d}(p_2D\epsilon_3)_a\epsilon_{2bc}I_5-2\left(p_2Dp_3\right)p_{3d}(p_2D\epsilon_3)_a\epsilon_{2bc}I_1\\
&-2\left(p_2p_3\right)p_{3a}(p_3D\epsilon_3)_d\epsilon_{2bc}I_6'-2\left(p_1Np_2\right)p_{3a}(p_3D\epsilon_3)_d\epsilon_{2bc}I_4'\\
&-2\left(p_2Dp_3\right)p_{3a}(p_3D\epsilon_3)_d\epsilon_{2bc}I_7'+2\left(p_3D\epsilon_2p_3\right)p_{2d}p_{3a}\epsilon_{3bc}I_{11}'\\
&-\left(p_1Np_2\right)\left(p_2p_3\right)\epsilon_{2ab}\epsilon_{3cd}I_9+\left(p_1Np_2\right)\left(p_1Np_3\right)\epsilon_{2ab}\epsilon_{3cd}I_{10}\\
&-\left(p_1Np_2\right)\left(p_2Dp_3\right)\epsilon_{2ab}\epsilon_{3cd}I_5)+(2\leftrightarrow 3),
\end{aligned}
\tag{2.58}
$$

$$
\begin{aligned}
\mathcal{A}^{(1)}_{C^{(p-3)}BB}=&\frac{2i^{p(p+1)}\sqrt{2}}{(p-4)!}C^i{}_{b_1\ldots b_{p-4}}\varepsilon^{abcdeb_1\ldots b_{p-4}}\left(-2p_{2e}p_{2i}p_{3a}(p_2\epsilon_3)_d\epsilon_{2bc}I_9\right.\\
&+2p_{2a}p_{3e}p_{3i}(p_2\epsilon_3)_d\epsilon_{2bc}I_9+4p_{2e}p_{2i}p_{3a}(p_1N\epsilon_3)_d\epsilon_{2bc}I_{10}\\
&-2p_{2e}p_{2i}p_{3a}(p_2D\epsilon_3)_d\epsilon_{2bc}I_5-2p_{2a}p_{3e}p_{3i}(p_2D\epsilon_3)_d\epsilon_{2bc}I_5\\
&-2p_{2e}p_{2i}p_{3a}(p_3D\epsilon_3)_d\epsilon_{2bc}I_4'-\left(p_2p_3\right)p_{2e}p_{2i}\epsilon_{2ab}\epsilon_{3cd}I_9
\end{aligned}
$$

$$+2\left(p_1Np_3\right)p_{2e}p_{2i}\epsilon_{2ab}\epsilon_{3cd}I_{10}-\left(p_2Dp_3\right)p_{2e}p_{2i}\epsilon_{2ab}\epsilon_{3cd}I_5$$

$$+\left(p_2p_3\right)p_{2e}p_{3i}\epsilon_{2ab}\epsilon_{3cd}I_9-\left(p_2Dp_3\right)p_{2e}p_{3i}\epsilon_{2ab}\epsilon_{3cd}I_5)+(2\leftrightarrow 3), \tag{2.59}$$

$$\mathcal{A}^{(2)}_{C^{(p-3)}BB}=\frac{2\sqrt{2}i^{p(p+1)}}{(p-5)!}C^{ij}{}_{b_1\ldots b_{p-5}}\varepsilon^{abcdefb_1\ldots b_{p-5}}\epsilon_{2bc}\epsilon_{3de}p_{2f}p_{2j}p_{3a}p_{3i}I_{10}+(2\leftrightarrow 3), \tag{2.60}$$

$$\mathcal{A}_{C^{(p-3)}hh}=\mathcal{A}^{(0)}_{C^{(p-3)}hh}. \tag{2.61}$$

$$\mathcal{A}^{(0)}_{C^{(p-3)}hh}=\frac{8\sqrt{2}i^{p(p+1)}}{(p-3)!}C_{b_1\ldots b_{p-3}}p_{2d}p_{3a}\varepsilon^{abcdb_1\ldots b_{p-3}}\left(\left(p_2Dp_3\right)\left(\epsilon_2\epsilon_3\right)_{bc}I_0\right.$$

$$\left.-2(p_2D\epsilon_3)_c(p_3\epsilon_2)_bI_0-\left(p_2p_3\right)\left(\epsilon_2D\epsilon_3\right)_{cb}I_0\right)+(2\leftrightarrow 3). \tag{2.62}$$

2.3 Algorithm to Covariantize the Amplitudes

The amplitudes obtained in the previous section are expressed as an S-matrix of the elementary fields C, B, h. This form can be useful in some cases. However for the purpose of construction of the effective couplings (or performing T-duality), it would be better to rewrite the amplitudes such that (1) the Ramond field is expressed in terms of the gauge field strength $F = dC$; (2) the two NSNS fields are symmetric under exchange. This is closer to the feature of the off-shell effective action.

As dicussed in [15], no matter how we distribute the picture charge, at the formalism level, we either have explicit gauge invariance or explicit exchange invariance, but cannot have both at same time. With our choice of the picture charge in the previous section, the Ramond field is automatically gauge invariant and expressed in terms of the T symbol. We would like to have an algorithm to make the exchange invariance explicit. To do this, we need the Bianchi identity:

$$0 = \epsilon^{a_1 \ldots a_k b_1 \ldots b_{p+1-k}} p_{[a_k} F_{b_1 \ldots b_{p+1-k} i_1 \ldots i_l]}. \tag{2.63}$$

In terms of the T symbol, it becomes

$$p_{a_k} T^{a_1 \ldots a_k}{}_{i_1 \ldots i_l} + l p_{[i_1} T^{a_1 \ldots a_{k-1}}{}_{i_2 \ldots i_l]} = 0. \tag{2.64}$$

If there are no normal indices in T, we simply have

$$p_{a_k} T^{a_1 \ldots a_k} = 0. \tag{2.65}$$

The algorithm is the following:

1. We start with the amplitudes that have the largest number of normal indices,

for example the $T^{abcijkl}$ term in all $C^{(p+1)}BB$ amplitudes. We decompose it into the $2 \leftrightarrow 3$ exchange symmetric part and antisymmetric part.

2. The antisymmetric part is also in the form of $p_{a_k} T^{a_1 \ldots a_k}{}_{i_1 \ldots i_l}$. We then apply the Bianchi identity to reduce the number of normal indices by 1. For example, the $T^{abcijkl}$ terms become T^{abijk} terms.

3. We combine the terms from (2) with similar terms from Wick contraction, then perform step (1) and step (2) again. Again this procedure reduces the number of normal indices by 1.

4. Repeating the above steps, we will eventually reach the amplitude that has the least number of normal indices, for example the T^i part in all $C^{(p+1)}BB$ amplitudes. However we observe that the antisymmetric part of those terms is always zero.

5. We obtain the final result by collecting the symmetric part in each step.

Note that in all of the above steps, we will need to impose on-shell conditions and the minimal basis condtion.

In the following sections, we list all the covariantized amplitudes. The amplitudes are expressed in terms of the T symbol, and classified by the number of indices in the T tensor. For example, $\mathcal{A}^{\left(T^{(7)}\right)}_{\left(C^{(p+5)}BB\right)}$ denotes the terms that contain $T^{aijklmn}$. Our results are consistent with the partial terms calculated in the existing literature [21] [22].

$$\mathcal{A}_{C^{(p+5)}BB} = \mathcal{A}^{T^{(7)}}_{C^{(p+5)}BB} + \mathcal{A}^{T^{(5)}}_{C^{(p+5)}BB}. \tag{2.66}$$

$$\mathcal{A}^{T^{(7)}}_{C^{(p+5)}BB} = -\frac{i}{8\sqrt{2}} T^{aijklmn} p_{2_a} p_{2_j} p_{3_i} \epsilon_{2mn} \epsilon_{3kl} I_{10} + (2 \leftrightarrow 3), \tag{2.67}$$

$$\begin{aligned}\mathcal{A}^{T^{(5)}}_{C^{(p+5)}BB} =& \frac{i}{8\sqrt{2}} T^{ijklm} (2p_{2_m} p_{3_i} (p_1 N \epsilon_3)_l \epsilon_{2jk} I_{10} - 2p_{2_i} p_{3_j} (p_2 \epsilon_3)_m \epsilon_{2kl} I_9 \\ &+ 2p_{2_j} p_{3_i} (p_2 D \epsilon_3)_m \epsilon_{2kl} I_5 + (p_2 p_3)\, p_{2_i} \epsilon_{2lm} \epsilon_{3jk} I_9 \\ &- (p_1 N p_3)\, p_{2_i} \epsilon_{2lm} \epsilon_{3jk} I_{10} + (p_2 D p_3)\, p_{2_i} \epsilon_{2lm} \epsilon_{3jk} I_5) + (2 \leftrightarrow 3).\end{aligned} \tag{2.68}$$

$$\mathcal{A}_{C^{(p+3)}Bh} = \mathcal{A}^{T^{(7)}}_{C^{(p+3)}Bh} + \mathcal{A}^{T^{(5)}}_{C^{(p+3)}Bh} + \mathcal{A}^{T^{(3)}}_{C^{(p+3)}Bh}. \tag{2.69}$$

$$\mathcal{A}^{T^{(7)}}_{C^{(p+3)}Bh} = \frac{i}{4\sqrt{2}} T^{abijklm} (p_{2_a} p_{2_i} p_{3_j} \epsilon_{2lm} \epsilon_{3bk} I_{10} + p_{2_j} p_{3_a} p_{3_i} \epsilon_{2kl} \epsilon_{3bm} I_{10}) + (2 \leftrightarrow 3), \tag{2.70}$$

$$\begin{aligned}\mathcal{A}^{T^{(5)}}_{C^{(p+3)}Bh} =& \frac{i}{16\sqrt{2}} T^{aijkl} (-4p_{2_j} p_{3_a} p_{3_i} (\epsilon_2 \epsilon_3)_{kl} I_9 + 4p_{2_a} p_{2_i} p_{3_j} (\epsilon_2 \epsilon_3)_{lk} I_9 \\ &+ 4p_{2_j} p_{3_a} p_{3_i} (\epsilon_2 D \epsilon_3)_{kl} I_5 - 4p_{2_a} p_{2_i} p_{3_j} (\epsilon_2 D \epsilon_3)_{lk} I_5 \\ &- 2p_{2_i} p_{3_a} (p_2 \epsilon_3)_l \epsilon_{2jk} I_9 + 2p_{2_a} p_{3_i} (p_2 \epsilon_3)_l \epsilon_{2jk} I_9 \\ &- 4p_{2_i} p_{3_l} (p_1 N \epsilon_3)_a \epsilon_{2jk} I_{10} + 4p_{2_i} p_{3_a} (p_1 N \epsilon_3)_l \epsilon_{2jk} I_{10}\end{aligned}$$

$$
\begin{aligned}
&- 2p_{2i}p_{3a}(p_2D\epsilon_3)_l\epsilon_{2jk}I_5 - 2p_{2a}p_{3i}(p_2D\epsilon_3)_l\epsilon_{2jk}I_5 \\
&+ 2p_{3a}p_{3i}(p_2D\epsilon_3)_l\epsilon_{2jk}I_5 + 2p_{2a}p_{2i}(p_3D\epsilon_3)_l\epsilon_{2jk}I_4' \\
&- 2p_{2i}p_{3a}(p_3D\epsilon_3)_l\epsilon_{2jk}I_4' - \mathrm{tr}(D\epsilon_3)p_{2j}p_{3a}p_{3i}\epsilon_{2kl}I_4' \\
&- \mathrm{tr}(D\epsilon_3)p_{2a}p_{2i}p_{3j}\epsilon_{2kl}I_4' - 4p_{2j}p_{3i}(p_2\epsilon_3)_a\epsilon_{2kl}I_9 \\
&- 2p_{2a}p_{2j}(p_2\epsilon_3)_i\epsilon_{2kl}I_9 + 2p_{3a}p_{3j}(p_2\epsilon_3)_i\epsilon_{2kl}I_9 \\
&+ 4p_{2a}p_{2j}(p_1N\epsilon_3)_i\epsilon_{2kl}I_{10} - 4p_{2j}p_{3i}(p_2D\epsilon_3)_a\epsilon_{2kl}I_5 \\
&- 2p_{2a}p_{2j}(p_2D\epsilon_3)_i\epsilon_{2kl}I_5 + 8p_{2l}p_{3i}(p_1N\epsilon_2)_k\epsilon_{3aj}I_{10} \\
&- 4\left(p_2p_3\right)p_{2i}\epsilon_{2kl}\epsilon_{3aj}I_9 + 4\left(p_1Np_3\right)p_{2i}\epsilon_{2kl}\epsilon_{3aj}I_{10} \\
&- 4\left(p_2Dp_3\right)p_{2i}\epsilon_{2kl}\epsilon_{3aj}I_5 + 4\left(p_2p_3\right)p_{3i}\epsilon_{2kl}\epsilon_{3aj}I_9 \\
&+ 4\left(p_1Np_2\right)p_{3i}\epsilon_{2kl}\epsilon_{3aj}I_{10} - 4\left(p_2Dp_3\right)p_{3i}\epsilon_{2kl}\epsilon_{3aj}I_5 \\
&- 8p_{2i}p_{3j}(p_3\epsilon_2)_l\epsilon_{3ak}I_9 - 8p_{2j}p_{3i}(p_3D\epsilon_2)_l\epsilon_{3ak}I_5),
\end{aligned}
\tag{2.71}
$$

$$
\begin{aligned}
\mathcal{A}^{T^{(3)}}_{C^{(p+3)}Bh} =& \frac{i}{16\sqrt{2}}T^{ijk}(-4p_{3j}(p_2\epsilon_3)_k(p_3\epsilon_2)_iI_2 + 4p_{2i}(p_2\epsilon_3)_k(p_3\epsilon_2)_jI_2 \\
&- 4\,\mathrm{tr}(D\epsilon_3)p_{2j}p_{3i}(p_3\epsilon_2)_kI_6' + 4\left(p_2p_3\right)p_{2i}(\epsilon_2\epsilon_3)_{kj}I_2 \\
&- 4\left(p_1Np_3\right)p_{2i}(\epsilon_2\epsilon_3)_{kj}I_9 + 4\left(p_2Dp_3\right)p_{2i}(\epsilon_2\epsilon_3)_{kj}I_{11} \\
&- 4\left(p_2p_3\right)p_{3i}(\epsilon_2\epsilon_3)_{kj}I_2 - 4\left(p_1Np_2\right)p_{3i}(\epsilon_2\epsilon_3)_{kj}I_9 \\
&- 4\left(p_2Dp_3\right)p_{3i}(\epsilon_2\epsilon_3)_{kj}I_{11}' - 4p_{3j}(p_1N\epsilon_2)_i(p_2\epsilon_3)_kI_9 \\
&+ 2\,\mathrm{tr}(D\epsilon_3)p_{2k}p_{3i}(p_1N\epsilon_2)_jI_4' + 4p_{2i}(p_1N\epsilon_2)_j(p_2\epsilon_3)_kI_9 \\
&- 8p_{2j}(p_1N\epsilon_3)_i(p_3\epsilon_2)_kI_9 + 8p_{2i}(p_1N\epsilon_2)_k(p_1N\epsilon_3)_jI_{10} \\
&+ 4p_{2j}(p_2D\epsilon_3)_i(p_3\epsilon_2)_kI_{11} + 4p_{2j}(p_1N\epsilon_2)_k(p_2D\epsilon_3)_iI_5 \\
&+ 4p_{3i}(p_2D\epsilon_3)_k(p_3\epsilon_2)_jI_{11}' - 4p_{3i}(p_1N\epsilon_2)_j(p_2D\epsilon_3)_kI_5 \\
&- 4p_{2j}p_{3i}(p_2\epsilon_3\epsilon_2)_kI_2 - 4p_{3j}(p_2\epsilon_3)_k(p_3D\epsilon_2)_iI_{11}'
\end{aligned}
$$

$$- 4p_{3j}(p_2D\epsilon_3)_k(p_3D\epsilon_2)_iI_1 - 4\,\mathrm{tr}(D\epsilon_3)p_{2i}p_{3k}(p_3D\epsilon_2)_jI'_7$$

$$- 4p_{2j}(p_2\epsilon_3)_i(p_3D\epsilon_2)_kI_{11} + 8p_{2j}(p_1N\epsilon_3)_i(p_3D\epsilon_2)_kI_5$$

$$- 4p_{2j}(p_2D\epsilon_3)_i(p_3D\epsilon_2)_kI_1 + 8p_{2j}(p_3D\epsilon_3)_i(p_3\epsilon_2)_kI'_6$$

$$+ 4p_{2j}(p_1N\epsilon_2)_k(p_3D\epsilon_3)_iI'_4 + 8p_{2j}(p_3D\epsilon_2)_k(p_3D\epsilon_3)_iI'_7$$

$$- 4p_{2j}p_{3i}(p_3\epsilon_2\epsilon_3)_kI_2 + 4\,(p_2p_3)\,p_{2i}(\epsilon_2D\epsilon_3)_{jk}I_{11}$$

$$- 4\,(p_1Np_3)\,p_{2i}(\epsilon_2D\epsilon_3)_{jk}I_5 + 4\,(p_2Dp_3)\,p_{2i}(\epsilon_2D\epsilon_3)_{jk}I_1$$

$$- 4\,(p_2p_3)\,p_{3i}(\epsilon_2D\epsilon_3)_{kj}I'_{11} + 4\,(p_1Np_2)\,p_{3i}(\epsilon_2D\epsilon_3)_{kj}I_5$$

$$- 4\,(p_2Dp_3)\,p_{3i}(\epsilon_2D\epsilon_3)_{kj}I_1 + 4p_{2k}p_{3i}(p_1N\epsilon_2\epsilon_3)_jI_9$$

$$- 4p_{2i}p_{3j}(p_1N\epsilon_3\epsilon_2)_kI_9 - 4p_{2j}p_{3i}(p_2D\epsilon_3\epsilon_2)_kI_{11}$$

$$+ 4p_{2j}p_{3i}(p_2\epsilon_3D\epsilon_2)_kI_{11} - 4p_{2j}p_{3i}(p_3D\epsilon_2\epsilon_3)_kI'_{11}$$

$$- 4p_{2j}p_{3i}(p_3\epsilon_2D\epsilon_3)_kI'_{11} - 4p_{2k}p_{3i}(p_1N\epsilon_2D\epsilon_3)_jI_5$$

$$- 4p_{2i}p_{3k}(p_1N\epsilon_3D\epsilon_2)_jI_5 + 4p_{2j}p_{3i}(p_2D\epsilon_3D\epsilon_2)_kI_1$$

$$- 4p_{2j}p_{3i}(p_3D\epsilon_2D\epsilon_3)_kI_1 - 6\,(p_1N\epsilon_3Dp_2)\,p_{2k}\epsilon_{2ij}I_5$$

$$- 2\,(p_1N\epsilon_3Dp_3)\,p_{2k}\epsilon_{2ij}I'_4 + 4\,(p_1N\epsilon_3Np_1)\,p_{2k}\epsilon_{2ij}I_{10}$$

$$- 2\,(p_1N\epsilon_3p_2)\,p_{3k}\epsilon_{2ij}I_9 + 2\,(p_1N\epsilon_3Dp_2)\,p_{3k}\epsilon_{2ij}I_5$$

$$- 4\,(p_2p_3)\,(p_2\epsilon_3)_k\epsilon_{2ij}I_2 - 2\,(p_1Np_2)\,(p_2\epsilon_3)_k\epsilon_{2ij}I_9$$

$$+ 2\,(p_1Np_3)\,(p_2\epsilon_3)_k\epsilon_{2ij}I_9 - 2\,(p_1Np_2)\,(p_2D\epsilon_3)_k\epsilon_{2ij}I_5$$

$$- 2\,(p_1Np_3)\,(p_2D\epsilon_3)_k\epsilon_{2ij}I_5 + 4\,(p_2Dp_3)\,(p_2D\epsilon_3)_k\epsilon_{2ij}I_1$$

$$- 4\,(p_2p_3)\,(p_3D\epsilon_3)_k\epsilon_{2ij}I'_6 - 2\,(p_1Np_2)\,(p_3D\epsilon_3)_k\epsilon_{2ij}I'_4$$

$$- 4\,(p_2Dp_3)\,(p_3D\epsilon_3)_k\epsilon_{2ij}I'_7 + 2\,(p_2\epsilon_3p_2)\,p_{2i}\epsilon_{2jk}I_2$$

$$- 6\,(p_1N\epsilon_3p_2)\,p_{2i}\epsilon_{2jk}I_9 + 4\,(p_2D\epsilon_3p_2)\,p_{2i}\epsilon_{2jk}I_{11}$$

$$+ 4\,(p_2\epsilon_3Dp_3)\,p_{2i}\epsilon_{2jk}I'_6 + 2\,(p_2D\epsilon_3Dp_2)\,p_{2i}\epsilon_{2jk}I_1$$

$$- 4\,(p_2D\epsilon_3Dp_3)\,p_{2i}\epsilon_{2jk}I'_7 - 2\,\mathrm{tr}(D\epsilon_3)\,(p_2p_3)\,p_{2i}\epsilon_{2jk}I'_6$$

$$
\begin{aligned}
&+ \operatorname{tr}(D\epsilon_3)\,(p_1Np_3)\,p_{2i}\epsilon_{2jk}I_4' + 2\operatorname{tr}(D\epsilon_3)\,(p_2Dp_3)\,p_{2i}\epsilon_{2jk}I_7' \\
&+ 2\,(p_2\epsilon_3p_2)\,p_{3i}\epsilon_{2jk}I_2 - 2\,(p_2D\epsilon_3Dp_2)\,p_{3i}\epsilon_{2jk}I_1 \\
&+ 2\operatorname{tr}(D\epsilon_3)\,(p_2p_3)\,p_{3i}\epsilon_{2jk}I_6' + \operatorname{tr}(D\epsilon_3)\,(p_1Np_2)\,p_{3i}\epsilon_{2jk}I_4' \\
&+ 2\operatorname{tr}(D\epsilon_3)\,(p_2Dp_3)\,p_{3i}\epsilon_{2jk}I_7' + 4\,(p_2p_3)\,(p_1N\epsilon_3)_i\epsilon_{2jk}I_9 \\
&+ 4\,(p_1Np_2)\,(p_1N\epsilon_3)_i\epsilon_{2jk}I_{10} - 4\,(p_2Dp_3)\,(p_1N\epsilon_3)_i\epsilon_{2jk}I_5).
\end{aligned} \tag{2.72}
$$

$$
\mathcal{A}_{C^{(p+1)}BB} = \mathcal{A}^{T^{(7)}}_{C^{(p+1)}BB} + \mathcal{A}^{T^{(5)}}_{C^{(p+1)}BB} + \mathcal{A}^{T^{(3)}}_{C^{(p+1)}BB} + \mathcal{A}^{T^{(1)}}_{C^{(p+1)}BB}. \tag{2.73}
$$

$$
\mathcal{A}^{T^{(7)}}_{C^{(p+1)}BB} = \frac{i}{8\sqrt{2}}T^{abcijkl}(p_{2a}p_{2j}p_{3i}\epsilon_{2kl}\epsilon_{3bc}I_{10} - p_{2j}p_{3a}p_{3i}\epsilon_{2kl}\epsilon_{3bc}I_{10}) + (2 \leftrightarrow 3), \tag{2.74}
$$

$$
\begin{aligned}
\mathcal{A}^{T^{(5)}}_{C^{(p+1)}BB} =& \frac{i}{8\sqrt{2}} T^{abijk} \big(2p_{2j}p_{3a}p_{3i}(\epsilon_2\epsilon_3)_{bk}I_9 + 2p_{2a}p_{2i}p_{3j}(\epsilon_2\epsilon_3)_{bk}I_9 \\
&+ 2p_{2j}p_{3a}p_{3i}(\epsilon_2 D\epsilon_3)_{bk}I_5 + 2p_{2a}p_{2i}p_{3j}(\epsilon_2 D\epsilon_3)_{bk}I_5 \\
&+ 2p_{2k}p_{3i}(p_2\epsilon_3)_j\epsilon_{2ab}I_9 - 2p_{2k}p_{3i}(p_1 N\epsilon_3)_j\epsilon_{2ab}I_{10} \\
&+ 2p_{2k}p_{3i}(p_2 D\epsilon_3)_j\epsilon_{2ab}I_5 - p_{2b}p_{2i}(p_2\epsilon_3)_a\epsilon_{2jk}I_9 \\
&- p_{2b}p_{3i}(p_2\epsilon_3)_a\epsilon_{2jk}I_9 + p_{3b}p_{3i}(p_2\epsilon_3)_a\epsilon_{2jk}I_9 \\
&- p_{2i}p_{3a}(p_2\epsilon_3)_b\epsilon_{2jk}I_9 + 2p_{2b}p_{2i}(p_1 N\epsilon_3)_a\epsilon_{2jk}I_{10} \\
&+ 2p_{2i}p_{3a}(p_1 N\epsilon_3)_b\epsilon_{2jk}I_{10} - p_{2b}p_{2i}(p_2 D\epsilon_3)_a\epsilon_{2jk}I_5 \\
&+ p_{2b}p_{3i}(p_2 D\epsilon_3)_a\epsilon_{2jk}I_5 - p_{2i}p_{3a}(p_2 D\epsilon_3)_b\epsilon_{2jk}I_5 \\
&+ p_{3a}p_{3i}(p_2 D\epsilon_3)_b\epsilon_{2jk}I_5 - p_{2b}p_{2i}(p_3 D\epsilon_3)_a\epsilon_{2jk}I'_4 \\
&- p_{2i}p_{3a}(p_3 D\epsilon_3)_b\epsilon_{2jk}I'_4 - (p_2p_3)\, p_{2i}\epsilon_{2jk}\epsilon_{3ab}I_9 \\
&+ (p_1 N p_3)\, p_{2i}\epsilon_{2jk}\epsilon_{3ab}I_{10} - (p_2 D p_3)\, p_{2i}\epsilon_{2jk}\epsilon_{3ab}I_5
\end{aligned}
$$

$$
\begin{aligned}
&+ (p_2p_3)\, p_{3i}\epsilon_{2jk}\epsilon_{3ab}I_9 + (p_1 N p_2)\, p_{3i}\epsilon_{2jk}\epsilon_{3ab}I_{10} \\
&- (p_2 D p_3)\, p_{3i}\epsilon_{2jk}\epsilon_{3ab}I_5 \big) + (2 \leftrightarrow 3), \qquad (2.75)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}^{T^{(3)}}_{C^{(p+1)}BB} =& \frac{i}{8\sqrt{2}} T^{aij} (\mathrm{tr}(-\epsilon_2\epsilon_3) p_{2j} p_{3a} p_{3i} I_2 - \mathrm{tr}(D\epsilon_2 D\epsilon_3) p_{2j} p_{3a} p_{3i} I_1 \\
& + 2p_{2_j} (p_2\epsilon_3)_i (p_3\epsilon_2)_a I_2 + 2p_{3_j} (p_2\epsilon_3)_i (p_3\epsilon_2)_a I_2 \\
& - 2\,(p_2 p_3)\, p_{2i} (\epsilon_2\epsilon_3)_{aj} I_2 + 2\,(p_1 N p_3)\, p_{2i} (\epsilon_2\epsilon_3)_{aj} I_9 \\
& + 2\,(p_2 p_3)\, p_{3i} (\epsilon_2\epsilon_3)_{aj} I_2 + 2\,(p_1 N p_2)\, p_{3i} (\epsilon_2\epsilon_3)_{aj} I_9 \\
& + 2\,(p_2 D p_3)\, p_{3i} (\epsilon_2\epsilon_3)_{aj} I'_{11} + 4\,(p_2 D p_3)\, p_{3a} (\epsilon_2\epsilon_3)_{ij} I_0 \\
& - 2\,(p_2 D p_3)\, p_{3i} (\epsilon_2\epsilon_3)_{ja} I'_{11} - 4p_{3_i} (p_1 N\epsilon_2)_a (p_2\epsilon_3)_j I_9 \\
& + 2p_{2_j} (p_1 N\epsilon_2)_i (p_2\epsilon_3)_a I_9 + 2p_{3_j} (p_1 N\epsilon_2)_i (p_2\epsilon_3)_a I_9 \\
& + 4p_{2_i} (p_1 N\epsilon_2)_j (p_1 N\epsilon_3)_a I_{10} - 4p_{3_i} (p_2 D\epsilon_2)_a (p_2\epsilon_3)_j I_6 \\
& - 2p_{3_i} (p_1 N\epsilon_3)_j (p_2 D\epsilon_2)_a I_4 - 2p_{3_i} (p_2 D\epsilon_3)_a (p_3\epsilon_2)_j I'_{11} \\
& - 2p_{2_i} (p_1 N\epsilon_2)_j (p_2 D\epsilon_3)_a I_5 + 2p_{3_i} (p_1 N\epsilon_2)_j (p_2 D\epsilon_3)_a I_5 \\
& + 4p_{2_a} (p_2 D\epsilon_3)_i (p_3\epsilon_2)_j I_0 + 2p_{3_i} (p_2 D\epsilon_3)_j (p_3\epsilon_2)_a I'_{11} \\
& + 4p_{3_a} (p_2 D\epsilon_3)_j (p_3\epsilon_2)_i I_0 - 4p_{3_i} (p_1 N\epsilon_2)_a (p_2 D\epsilon_3)_j I_5 \\
& - 4p_{3_i} (p_2 D\epsilon_2)_a (p_2 D\epsilon_3)_j I_7 - 2p_{2_j} p_{3_i} (p_2\epsilon_3\epsilon_2)_a I_2 \\
& - p_{2_a} p_{2_i} (p_2\epsilon_3\epsilon_2)_j I_2 + p_{2_i} p_{3_a} (p_2\epsilon_3\epsilon_2)_j I_2 \\
& - p_{2_a} p_{3_i} (p_2\epsilon_3\epsilon_2)_j I_2 + p_{3_a} p_{3_i} (p_2\epsilon_3\epsilon_2)_j I_2 \\
& - 2p_{3_i} (p_2\epsilon_3)_j (p_3 D\epsilon_2)_a I'_{11} + 2p_{2_j} (p_2 D\epsilon_3)_i (p_3 D\epsilon_2)_a I_1 \\
& + 2p_{3_i} (p_2 D\epsilon_3)_j (p_3 D\epsilon_2)_a I_1 + 2p_{3_j} (p_2\epsilon_3)_a (p_3 D\epsilon_2)_i I'_{11} \\
& + 2\,(p_1 N p_3)\, p_{2i} (\epsilon_2 D\epsilon_3)_{aj} I_5 - 2\,(p_2 D p_3)\, p_{2i} (\epsilon_2 D\epsilon_3)_{aj} I_1 \\
& - 2\,(p_2 p_3)\, p_{3i} (\epsilon_2 D\epsilon_3)_{aj} I'_{11} + 2\,(p_1 N p_2)\, p_{3i} (\epsilon_2 D\epsilon_3)_{aj} I_5
\end{aligned}
$$

$$
\begin{aligned}
&- 2\,(p_2 D p_3)\, p_{3_i} (\epsilon_2 D \epsilon_3)_{aj} I_1 - 4\,(p_2 p_3)\, p_{2_a} (\epsilon_2 D \epsilon_3)_{ij} I_0 \\
&- 2\,(p_2 p_3)\, p_{3_i} (\epsilon_2 D \epsilon_3)_{ja} I'_{11} + 2 p_{2_j} p_{3_i} (p_1 N \epsilon_2 \epsilon_3)_a I_9 \\
&+ 2 p_{2_a} p_{3_i} (p_1 N \epsilon_2 \epsilon_3)_j I_9 - 2 p_{3_a} p_{3_i} (p_1 N \epsilon_2 \epsilon_3)_j I_9 \\
&+ 2 p_{2_a} p_{3_i} (p_2 D \epsilon_2 \epsilon_3)_j I_6 - 2 p_{3_a} p_{3_i} (p_2 D \epsilon_2 \epsilon_3)_j I_6 \\
&+ 2 p_{2_j} p_{3_i} (p_3 D \epsilon_2 \epsilon_3)_a I'_{11} - p_{2_a} p_{2_i} (p_3 D \epsilon_2 \epsilon_3)_j I'_{11} \\
&+ p_{2_i} p_{3_a} (p_3 D \epsilon_2 \epsilon_3)_j I'_{11} + p_{2_a} p_{3_i} (p_3 D \epsilon_2 \epsilon_3)_j I'_{11} \\
&- p_{3_a} p_{3_i} (p_3 D \epsilon_2 \epsilon_3)_j I'_{11} + 2 p_{2_j} p_{3_i} (p_3 \epsilon_2 D \epsilon_3)_a I'_{11} \\
&- p_{2_a} p_{2_i} (p_3 \epsilon_2 D \epsilon_3)_j I'_{11} + p_{2_i} p_{3_a} (p_3 \epsilon_2 D \epsilon_3)_j I'_{11} \\
&- p_{2_a} p_{3_i} (p_3 \epsilon_2 D \epsilon_3)_j I'_{11} + p_{3_a} p_{3_i} (p_3 \epsilon_2 D \epsilon_3)_j I'_{11} \\
&- 2 p_{2_j} p_{3_i} (p_1 N \epsilon_2 D \epsilon_3)_a I_5 + 2 p_{2_a} p_{3_i} (p_1 N \epsilon_2 D \epsilon_3)_j I_5 \\
&- 2 p_{3_a} p_{3_i} (p_1 N \epsilon_2 D \epsilon_3)_j I_5 + 2 p_{2_a} p_{3_i} (p_2 D \epsilon_2 D \epsilon_3)_j I_7 \\
&- 2 p_{3_a} p_{3_i} (p_2 D \epsilon_2 D \epsilon_3)_j I_7 - 2 p_{2_j} p_{3_i} (p_2 D \epsilon_3 D \epsilon_2)_a I_1 \\
&+ p_{2_a} p_{2_i} (p_2 D \epsilon_3 D \epsilon_2)_j I_1 - p_{2_i} p_{3_a} (p_2 D \epsilon_3 D \epsilon_2)_j I_1 \\
&- p_{2_a} p_{3_i} (p_2 D \epsilon_3 D \epsilon_2)_j I_1 + p_{3_a} p_{3_i} (p_2 D \epsilon_3 D \epsilon_2)_j I_1 \\
&- (p_1 N \epsilon_3 p_2)\, p_{2_a} \epsilon_{2ij} I_9 - (p_2 \epsilon_3 D p_3)\, p_{2_a} \epsilon_{2ij} I'_6 \\
&+ (p_1 N \epsilon_3 D p_2)\, p_{2_a} \epsilon_{2ij} I_5 - (p_2 D \epsilon_3 D p_3)\, p_{2_a} \epsilon_{2ij} I'_7 \\
&+ (p_1 N \epsilon_3 p_2)\, p_{3_a} \epsilon_{2ij} I_9 + (p_2 \epsilon_3 D p_3)\, p_{3_a} \epsilon_{2ij} I'_6 \\
&- (p_1 N \epsilon_3 D p_2)\, p_{3_a} \epsilon_{2ij} I_5 + (p_2 D \epsilon_3 D p_3)\, p_{3_a} \epsilon_{2ij} I'_7 \\
&+ 2\,(p_2 p_3)\,(p_2 \epsilon_3)_a \epsilon_{2ij} I_2 + (p_1 N p_2)\,(p_2 \epsilon_3)_a \epsilon_{2ij} I_9 \\
&- (p_1 N p_3)\,(p_2 \epsilon_3)_a \epsilon_{2ij} I_9 - 2\,(p_2 p_3)\,(p_1 N \epsilon_3)_a \epsilon_{2ij} I_9 \\
&- 2\,(p_1 N p_2)\,(p_1 N \epsilon_3)_a \epsilon_{2ij} I_{10} + 2\,(p_2 D p_3)\,(p_1 N \epsilon_3)_a \epsilon_{2ij} I_5 \\
&+ (p_1 N p_2)\,(p_2 D \epsilon_3)_a \epsilon_{2ij} I_5 + (p_1 N p_3)\,(p_2 D \epsilon_3)_a \epsilon_{2ij} I_5 \\
&- 2\,(p_2 D p_3)\,(p_2 D \epsilon_3)_a \epsilon_{2ij} I_1 + 2\,(p_2 p_3)\,(p_3 D \epsilon_3)_a \epsilon_{2ij} I'_6
\end{aligned}
$$

$$
\begin{aligned}
&+ (p_1Np_2)\,(p_3D\epsilon_3)_a\epsilon_{2ij}I'_4 + 2\,(p_2Dp_3)\,(p_3D\epsilon_3)_a\epsilon_{2ij}I'_7\\
&- (p_3D\epsilon_2p_3)\,p_{2_a}\epsilon_{3ij}I'_{11} + (p_3D\epsilon_2p_3)\,p_{3_a}\epsilon_{3ij}I'_{11}) + (2\leftrightarrow 3),
\end{aligned}
\tag{2.76}
$$

$$
\begin{aligned}
\mathcal{A}^{T^{(1)}}_{C^{(p+1)}BB} =& \frac{i}{8\sqrt{2}}T^i(-(p_1N\epsilon_2\epsilon_3p_2)\,p_{2_i}I_2 + (p_1N\epsilon_3\epsilon_2p_3)\,p_{2_i}I_2\\
&+ 2\,(p_2\epsilon_3D\epsilon_2p_3)\,p_{2_i}I_{14} + 2\,(p_2\epsilon_3\epsilon_2Dp_3)\,p_{2_i}I_{14}\\
&- 2\,(p_3D\epsilon_3\epsilon_2p_3)\,p_{2_i}I'_{20} - 2\,(p_1N\epsilon_2\epsilon_3Dp_3)\,p_{2_i}I'_6\\
&+ 2\,(p_1N\epsilon_2\epsilon_3Np_1)\,p_{2_i}I_9 + 3\,(p_1N\epsilon_3D\epsilon_2p_3)\,p_{2_i}I'_{11}\\
&+ 3\,(p_1N\epsilon_3\epsilon_2Dp_3)\,p_{2_i}I'_{11} + 2\,(p_2D\epsilon_3D\epsilon_2p_3)\,p_{2_i}I_{15}\\
&+ 2\,(p_2D\epsilon_3\epsilon_2Dp_3)\,p_{2_i}I_{15} + 2\,(p_3D\epsilon_2\epsilon_3Dp_3)\,p_{2_i}I_{22}\\
&+ 2\,(p_3D\epsilon_3D\epsilon_2p_3)\,p_{2_i}I_{22} + (p_1N\epsilon_2D\epsilon_3Dp_2)\,p_{2_i}I_1\\
&- 2\,(p_1N\epsilon_2D\epsilon_3Dp_3)\,p_{2_i}I'_7 - 2\,(p_1N\epsilon_2D\epsilon_3Np_1)\,p_{2_i}I_5\\
&+ (p_1N\epsilon_3D\epsilon_2Dp_3)\,p_{2_i}I_1 - 2\,(p_3D\epsilon_2D\epsilon_3Dp_3)\,p_{2_i}I'_{21}\\
&+ 2\,\mathrm{tr}(\epsilon_2\epsilon_3)p_{2_i}I_{23} - \mathrm{tr}(\epsilon_2\epsilon_3)\,(p_2p_3)\,p_{2_i}I_{16}\\
&+ \mathrm{tr}(\epsilon_2\epsilon_3)\,(p_1Np_3)\,p_{2_i}I_2 - \mathrm{tr}(\epsilon_2\epsilon_3)\,(p_2Dp_3)\,p_{2_i}I_{14}\\
&+ 2\,\mathrm{tr}(D\epsilon_2D\epsilon_3)p_{2_i}I_{24} - \mathrm{tr}(D\epsilon_2D\epsilon_3)\,(p_2p_3)\,p_{2_i}I_{15}\\
&+ \mathrm{tr}(D\epsilon_2D\epsilon_3)\,(p_1Np_3)\,p_{2_i}I_1 - \mathrm{tr}(D\epsilon_2D\epsilon_3)\,(p_2Dp_3)\,p_{2_i}I_{17}\\
&+ (p_1N\epsilon_3D\epsilon_2p_3)\,p_{3_i}I'_{11} - (p_1N\epsilon_3\epsilon_2Dp_3)\,p_{3_i}I'_{11}\\
&+ 2\,(p_1N\epsilon_2p_3)\,(p_2\epsilon_3)_iI_2 - 2\,(p_2D\epsilon_2p_3)\,(p_2\epsilon_3)_iI_{20}\\
&- 2\,(p_3D\epsilon_2p_3)\,(p_2\epsilon_3)_iI_{14} + 2\,(p_1N\epsilon_3Dp_2)\,(p_3\epsilon_2)_iI'_{25}\\
&- 2\,(p_2D\epsilon_3Dp_3)\,(p_3\epsilon_2)_iI_{22} + 2\,(p_1N\epsilon_3p_2)\,(p_1N\epsilon_2)_iI_9\\
&+ 2\,(p_2\epsilon_3Dp_3)\,(p_1N\epsilon_2)_iI'_6 - 2\,(p_1N\epsilon_3Dp_2)\,(p_1N\epsilon_2)_iI_5\\
&+ 2\,(p_2D\epsilon_3Dp_3)\,(p_1N\epsilon_2)_iI'_7 - 2\,(p_3D\epsilon_2p_3)\,(p_1N\epsilon_3)_iI'_{11}
\end{aligned}
$$

$$
\begin{aligned}
&- 2\left(p_3 D\epsilon_2 p_3\right)\left(p_2 D\epsilon_3\right)_i I_{15} + 2\left(p_1 N\epsilon_2 D p_3\right)\left(p_2 D\epsilon_3\right)_i I_1 \\
&- 2\left(p_2 D\epsilon_2 D p_3\right)\left(p_2 D\epsilon_3\right)_i I_{21} - 4(p_2\epsilon_3\epsilon_2)_i I_{23} \\
&+ 2\left(p_2 p_3\right)\left(p_2\epsilon_3\epsilon_2\right)_i I_{16} + \left(p_1 N p_2\right)\left(p_2\epsilon_3\epsilon_2\right)_i I_2 \\
&- \left(p_1 N p_3\right)\left(p_2\epsilon_3\epsilon_2\right)_i I_2 + 2\left(p_1 N\epsilon_3 p_2\right)\left(p_3 D\epsilon_2\right)_i I'_{25} \\
&- 2\left(p_2\epsilon_3 D p_3\right)\left(p_3 D\epsilon_2\right)_i I_{22} - 2\left(p_2 p_3\right)\left(p_1 N\epsilon_2\epsilon_3\right)_i I_2 \\
&+ 2\left(p_1 N p_3\right)\left(p_1 N\epsilon_2\epsilon_3\right)_i I_9 - 2\left(p_2 D p_3\right)\left(p_1 N\epsilon_3\epsilon_2\right)_i I'_{25} \\
&+ 2\left(p_2 p_3\right)\left(p_2 D\epsilon_2\epsilon_3\right)_i I_{20} + 2\left(p_1 N p_3\right)\left(p_2 D\epsilon_2\epsilon_3\right)_i I_6 \\
&- 2\left(p_2 D p_3\right)\left(p_2 D\epsilon_3\epsilon_2\right)_i I_{15} - 2\left(p_2 p_3\right)\left(p_2\epsilon_3 D\epsilon_2\right)_i I_{14} \\
&+ \left(p_1 N p_2\right)\left(p_3 D\epsilon_2\epsilon_3\right)_i I'_{11} + \left(p_1 N p_3\right)\left(p_3 D\epsilon_2\epsilon_3\right)_i I'_{11} \\
&- 2\left(p_2 D p_3\right)\left(p_3 D\epsilon_3\epsilon_2\right)_i I_{22} + \left(p_1 N p_2\right)\left(p_3\epsilon_2 D\epsilon_3\right)_i I'_{11} \\
&- \left(p_1 N p_3\right)\left(p_3\epsilon_2 D\epsilon_3\right)_i I'_{11} + 2\left(p_1 N p_3\right)\left(p_1 N\epsilon_2 D\epsilon_3\right)_i I_5 \\
&- 2\left(p_2 D p_3\right)\left(p_1 N\epsilon_2 D\epsilon_3\right)_i I_1 - 2\left(p_2 p_3\right)\left(p_1 N\epsilon_3 D\epsilon_2\right)_i I'_{25} \\
&+ 2\left(p_1 N p_3\right)\left(p_2 D\epsilon_2 D\epsilon_3\right)_i I_7 + 2\left(p_2 D p_3\right)\left(p_2 D\epsilon_2 D\epsilon_3\right)_i I_{21} \\
&- 4(p_2 D\epsilon_3 D\epsilon_2)_i I_{24} - \left(p_1 N p_2\right)\left(p_2 D\epsilon_3 D\epsilon_2\right)_i I_1 \\
&- \left(p_1 N p_3\right)\left(p_2 D\epsilon_3 D\epsilon_2\right)_i I_1 + 2\left(p_2 D p_3\right)\left(p_2 D\epsilon_3 D\epsilon_2\right)_i I_{17} \\
&- 2\left(p_2 p_3\right)\left(p_3 D\epsilon_3 D\epsilon_2\right)_i I_{22}\big) + (2 \leftrightarrow 3).
\end{aligned} \tag{2.77}
$$

$$
\mathcal{A}_{C^{(p+1)}hh} = \mathcal{A}^{T^{(7)}}_{C^{(p+1)}hh} + \mathcal{A}^{T^{(5)}}_{C^{(p+1)}hh} + \mathcal{A}^{T^{(3)}}_{C^{(p+1)}hh} + \mathcal{A}^{T^{1}}_{C^{(p+1)}hh}. \tag{2.78}
$$

$$
\mathcal{A}^{T^{(7)}}_{C^{(p+1)}hh} = \frac{i}{2\sqrt{2}} T^{abcijkl} p_{2a} p_{2j} p_{3i} \epsilon_{2cl} \epsilon_{3bk} I_{10} + (2 \leftrightarrow 3), \tag{2.79}
$$

$$
\begin{aligned}
\mathcal{A}^{T^{(5)}}_{C^{(p+1)}hh} =& \frac{i}{8\sqrt{2}} T^{abijk} (-2p_{2j}p_{3a}p_{3i}(\epsilon_2\epsilon_3)_{bk} I_9 - 2p_{2a}p_{2i}p_{3j}(\epsilon_2\epsilon_3)_{bk} I_9 \\
& + 2p_{2j}p_{3a}p_{3i}(\epsilon_2 D\epsilon_3)_{bk} I_5 + 2p_{2a}p_{2i}p_{3j}(\epsilon_2 D\epsilon_3)_{bk} I_5 \\
& + 4p_{2k}p_{3i}(p_1 N\epsilon_3)_b \epsilon_{2aj} I_{10} - 4p_{2i}p_{3j}(p_2\epsilon_3)_b \epsilon_{2ak} I_9 \\
& + 4p_{2j}p_{3i}(p_2 D\epsilon_3)_b \epsilon_{2ak} I_5 + 2p_{2i}p_{3a}(p_2\epsilon_3)_k \epsilon_{2bj} I_9 \\
& - 2p_{2a}p_{3i}(p_2\epsilon_3)_k \epsilon_{2bj} I_9 - 4p_{2i}p_{3a}(p_1 N\epsilon_3)_k \epsilon_{2bj} I_{10} \\
& + 2p_{2i}p_{3a}(p_2 D\epsilon_3)_k \epsilon_{2bj} I_5 + 2p_{2a}p_{3i}(p_2 D\epsilon_3)_k \epsilon_{2bj} I_5 \\
& - 2p_{3a}p_{3i}(p_2 D\epsilon_3)_k \epsilon_{2bj} I_5 - 2p_{2a}p_{2i}(p_3 D\epsilon_3)_k \epsilon_{2bj} I'_4 \\
& + 2p_{2i}p_{3a}(p_3 D\epsilon_3)_k \epsilon_{2bj} I'_4 - \mathrm{tr}(D\epsilon_3) p_{2j}p_{3a}p_{3i}\epsilon_{2bk} I'_4 \\
& - \mathrm{tr}(D\epsilon_3) p_{2a}p_{2i}p_{3j}\epsilon_{2bk} I'_4 - 2p_{2a}p_{2j}(p_2\epsilon_3)_i \epsilon_{2bk} I_9 \\
& + 2p_{3a}p_{3j}(p_2\epsilon_3)_i \epsilon_{2bk} I_9 + 4p_{2a}p_{2j}(p_1 N\epsilon_3)_i \epsilon_{2bk} I_{10} \\
& - 2p_{2a}p_{2j}(p_2 D\epsilon_3)_i \epsilon_{2bk} I_5 - 4\,(p_2p_3)\, p_{2i}\epsilon_{2bk}\epsilon_{3aj} I_9 \\
& + 4\,(p_1 N p_3)\, p_{2i}\epsilon_{2bk}\epsilon_{3aj} I_{10} - 4\,(p_2 D p_3)\, p_{2i}\epsilon_{2bk}\epsilon_{3aj} I_5) + (2 \leftrightarrow 3), \quad (2.80)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}^{T^{(3)}}_{C^{(p+1)}hh} =& \frac{i}{8\sqrt{2}} T^{aij} (\mathrm{tr}(D\epsilon_2)\,\mathrm{tr}(D\epsilon_3) p_{2j}p_{3a}p_{3i} I_3 + \mathrm{tr}(\epsilon_2\epsilon_3) p_{2j}p_{3a}p_{3i} I_2 \\
& - \mathrm{tr}(D\epsilon_2 D\epsilon_3) p_{2j}p_{3a}p_{3i} I_1 - 2\,\mathrm{tr}(D\epsilon_2) p_{2j}p_{3i}(p_2\epsilon_3)_a I_6 \\
& - \mathrm{tr}(D\epsilon_2) p_{2a}p_{2j}(p_2\epsilon_3)_i I_6 + \mathrm{tr}(D\epsilon_2) p_{2j}p_{3a}(p_2\epsilon_3)_i I_6 \\
& + \mathrm{tr}(D\epsilon_2) p_{3a}p_{3j}(p_2\epsilon_3)_i I_6 + \mathrm{tr}(D\epsilon_2) p_{2a}p_{3i}(p_2\epsilon_3)_j I_6 \\
& - 2p_{2j}(p_2\epsilon_3)_i (p_3\epsilon_2)_a I_2 - 2p_{3j}(p_2\epsilon_3)_i (p_3\epsilon_2)_a I_2 \\
& + 2\,(p_2p_3)\, p_{2i}(\epsilon_2\epsilon_3)_{aj} I_2 - 2\,(p_1 N p_3)\, p_{2i}(\epsilon_2\epsilon_3)_{aj} I_9 \\
& - 2\,(p_2p_3)\, p_{3i}(\epsilon_2\epsilon_3)_{aj} I_2 - 2\,(p_1 N p_2)\, p_{3i}(\epsilon_2\epsilon_3)_{aj} I_9 \\
& - 2\,(p_2 D p_3)\, p_{3i}(\epsilon_2\epsilon_3)_{aj} I'_{11} - 4\,(p_2 D p_3)\, p_{3a}(\epsilon_2\epsilon_3)_{ij} I_0 \\
& + 2\,(p_2 D p_3)\, p_{3i}(\epsilon_2\epsilon_3)_{ja} I'_{11} + \mathrm{tr}(D\epsilon_3) p_{2j}p_{3i}(p_1 N\epsilon_2)_a I'_4
\end{aligned}
$$

$$+ 2p_{2i}(p_1N\epsilon_2)_a(p_2\epsilon_3)_j I_9 + 2p_{3i}(p_1N\epsilon_2)_a(p_2\epsilon_3)_j I_9$$

$$+ \operatorname{tr}(D\epsilon_3)p_{2a}p_{3j}(p_1N\epsilon_2)_i I_4' - \operatorname{tr}(D\epsilon_3)p_{3a}p_{3j}(p_1N\epsilon_2)_i I_4'$$

$$- 4p_{3j}(p_1N\epsilon_2)_i(p_2\epsilon_3)_a I_9 + 2p_{3a}(p_1N\epsilon_2)_i(p_2\epsilon_3)_j I_9$$

$$+ 2p_{2a}(p_1N\epsilon_2)_j(p_2\epsilon_3)_i I_9 + 4p_{3j}(p_1N\epsilon_2)_i(p_1N\epsilon_3)_a I_{10}$$

$$- 4p_{2a}(p_1N\epsilon_2)_j(p_1N\epsilon_3)_i I_{10} - 2\operatorname{tr}(D\epsilon_3)p_{2a}p_{3j}(p_2D\epsilon_2)_i I_3$$

$$+ 2\operatorname{tr}(D\epsilon_3)p_{3a}p_{3j}(p_2D\epsilon_2)_i I_3 - 4p_{3j}(p_2D\epsilon_2)_i(p_2\epsilon_3)_a I_6$$

$$+ 2p_{3a}(p_2D\epsilon_2)_i(p_2\epsilon_3)_j I_6 - 2p_{3j}(p_1N\epsilon_3)_a(p_2D\epsilon_2)_i I_4$$

$$+ 2p_{3a}(p_1N\epsilon_3)_j(p_2D\epsilon_2)_i I_4 + 2p_{2a}(p_2D\epsilon_2)_j(p_2\epsilon_3)_i I_6$$

$$+ 2p_{2a}(p_1N\epsilon_3)_i(p_2D\epsilon_2)_j I_4 - 2\operatorname{tr}(D\epsilon_2)p_{2j}p_{3i}(p_2D\epsilon_3)_a I_7$$

$$- 2p_{3i}(p_2D\epsilon_3)_a(p_3\epsilon_2)_j I_{11}' - 4p_{3j}(p_1N\epsilon_2)_i(p_2D\epsilon_3)_a I_5$$

$$- 4p_{3j}(p_2D\epsilon_2)_i(p_2D\epsilon_3)_a I_7 - \operatorname{tr}(D\epsilon_2)p_{2a}p_{2j}(p_2D\epsilon_3)_i I_7$$

$$+ \operatorname{tr}(D\epsilon_2)p_{2j}p_{3a}(p_2D\epsilon_3)_i I_7 + 2p_{2a}(p_2D\epsilon_3)_i(p_3\epsilon_2)_j I_8$$

$$- 2p_{2j}(p_1N\epsilon_2)_a(p_2D\epsilon_3)_i I_5 + 2p_{2a}(p_1N\epsilon_2)_j(p_2D\epsilon_3)_i I_5$$

$$+ 2p_{2a}(p_2D\epsilon_2)_j(p_2D\epsilon_3)_i I_7 - 2p_{3a}(p_2D\epsilon_2)_j(p_2D\epsilon_3)_i I_7$$

$$- \operatorname{tr}(D\epsilon_2)p_{2a}p_{3i}(p_2D\epsilon_3)_j I_7 + \operatorname{tr}(D\epsilon_2)p_{3a}p_{3i}(p_2D\epsilon_3)_j I_7$$

$$+ 2p_{3i}(p_2D\epsilon_3)_j(p_3\epsilon_2)_a I_{11}' + 2p_{3a}(p_2D\epsilon_3)_j(p_3\epsilon_2)_i I_8$$

$$- 2p_{3i}(p_1N\epsilon_2)_a(p_2D\epsilon_3)_j I_5 + 2p_{3a}(p_1N\epsilon_2)_i(p_2D\epsilon_3)_j I_5$$

$$+ 2p_{2j}p_{3i}(p_2\epsilon_3\epsilon_2)_a I_2 + p_{2a}p_{2i}(p_2\epsilon_3\epsilon_2)_j I_2$$

$$- p_{2i}p_{3a}(p_2\epsilon_3\epsilon_2)_j I_2 + p_{2a}p_{3i}(p_2\epsilon_3\epsilon_2)_j I_2$$

$$- p_{3a}p_{3i}(p_2\epsilon_3\epsilon_2)_j I_2 + 2p_{3i}(p_2\epsilon_3)_j(p_3D\epsilon_2)_a I_{11}'$$

$$+ 2p_{2j}(p_2D\epsilon_3)_i(p_3D\epsilon_2)_a I_1 + 2p_{3i}(p_2D\epsilon_3)_j(p_3D\epsilon_2)_a I_1$$

$$- 2p_{3j}(p_2\epsilon_3)_a(p_3D\epsilon_2)_i I_{11}' - 4p_{2a}(p_2D\epsilon_2)_j(p_3D\epsilon_3)_i I_3$$

$$+ 2\,(p_1Np_3)\,p_{2i}(\epsilon_2D\epsilon_3)_{aj} I_5 - 2\,(p_2Dp_3)\,p_{2i}(\epsilon_2D\epsilon_3)_{aj} I_1$$

$$
\begin{aligned}
&-2\,(p_2p_3)\,p_{3i}(\epsilon_2D\epsilon_3)_{aj}I'_{11}+2\,(p_1Np_2)\,p_{3i}(\epsilon_2D\epsilon_3)_{aj}I_5\\
&-2\,(p_2Dp_3)\,p_{3i}(\epsilon_2D\epsilon_3)_{aj}I_1-4\,(p_2p_3)\,p_{2a}(\epsilon_2D\epsilon_3)_{ij}I_0\\
&-2\,(p_2p_3)\,p_{3i}(\epsilon_2D\epsilon_3)_{ja}I'_{11}-2p_{2j}p_{3i}(p_1N\epsilon_2\epsilon_3)_aI_9\\
&-2p_{2a}p_{3i}(p_1N\epsilon_2\epsilon_3)_jI_9+2p_{3a}p_{3i}(p_1N\epsilon_2\epsilon_3)_jI_9\\
&-2p_{2a}p_{3i}(p_2D\epsilon_2\epsilon_3)_jI_6+2p_{3a}p_{3i}(p_2D\epsilon_2\epsilon_3)_jI_6\\
&-2p_{2j}p_{3i}(p_3D\epsilon_2\epsilon_3)_aI'_{11}+p_{2a}p_{2i}(p_3D\epsilon_2\epsilon_3)_jI'_{11}\\
&-p_{2i}p_{3a}(p_3D\epsilon_2\epsilon_3)_jI'_{11}-p_{2a}p_{3i}(p_3D\epsilon_2\epsilon_3)_jI'_{11}\\
&+p_{3a}p_{3i}(p_3D\epsilon_2\epsilon_3)_jI'_{11}+2p_{2j}p_{3i}(p_3\epsilon_2D\epsilon_3)_aI'_{11}\\
&-p_{2a}p_{2i}(p_3\epsilon_2D\epsilon_3)_jI'_{11}+p_{2i}p_{3a}(p_3\epsilon_2D\epsilon_3)_jI'_{11}\\
&-p_{2a}p_{3i}(p_3\epsilon_2D\epsilon_3)_jI'_{11}+p_{3a}p_{3i}(p_3\epsilon_2D\epsilon_3)_jI'_{11}\\
&-2p_{2j}p_{3i}(p_1N\epsilon_2D\epsilon_3)_aI_5+2p_{2a}p_{3i}(p_1N\epsilon_2D\epsilon_3)_jI_5\\
&-2p_{3a}p_{3i}(p_1N\epsilon_2D\epsilon_3)_jI_5+2p_{2a}p_{3i}(p_2D\epsilon_2D\epsilon_3)_jI_7\\
&-2p_{3a}p_{3i}(p_2D\epsilon_2D\epsilon_3)_jI_7-2p_{2j}p_{3i}(p_2D\epsilon_3D\epsilon_2)_aI_1\\
&+p_{2a}p_{2i}(p_2D\epsilon_3D\epsilon_2)_jI_1-p_{2i}p_{3a}(p_2D\epsilon_3D\epsilon_2)_jI_1\\
&-p_{2a}p_{3i}(p_2D\epsilon_3D\epsilon_2)_jI_1+p_{3a}p_{3i}(p_2D\epsilon_3D\epsilon_2)_jI_1\\
&-6\,(p_1N\epsilon_3Dp_2)\,p_{2j}\epsilon_{2ai}I_5+4\,(p_1N\epsilon_3Np_1)\,p_{2j}\epsilon_{2ai}I_{10}\\
&-2\,(p_1N\epsilon_3p_2)\,p_{3j}\epsilon_{2ai}I_9-4\,(p_2p_3)\,(p_2\epsilon_3)_j\epsilon_{2ai}I_2\\
&-2\,(p_1Np_2)\,(p_2\epsilon_3)_j\epsilon_{2ai}I_9+2\,(p_1Np_3)\,(p_2\epsilon_3)_j\epsilon_{2ai}I_9\\
&-2\,(p_1Np_2)\,(p_2D\epsilon_3)_j\epsilon_{2ai}I_5-2\,(p_1Np_3)\,(p_2D\epsilon_3)_j\epsilon_{2ai}I_5\\
&+4\,(p_2Dp_3)\,(p_2D\epsilon_3)_j\epsilon_{2ai}I_1-4\,(p_2p_3)\,(p_3D\epsilon_3)_j\epsilon_{2ai}I'_6\\
&-2\,(p_1Np_2)\,(p_3D\epsilon_3)_j\epsilon_{2ai}I'_4-4\,(p_2Dp_3)\,(p_3D\epsilon_3)_j\epsilon_{2ai}I'_7\\
&-2\,(p_2\epsilon_3p_2)\,p_{2i}\epsilon_{2aj}I_2+6\,(p_1N\epsilon_3p_2)\,p_{2i}\epsilon_{2aj}I_9\\
&-4\,(p_2\epsilon_3Dp_3)\,p_{2i}\epsilon_{2aj}I'_6+2\,(p_1N\epsilon_3Dp_3)\,p_{2i}\epsilon_{2aj}I'_4
\end{aligned}
$$

$$
\begin{aligned}
&- 2\,(p_2 D\epsilon_3 D p_2)\, p_{2i}\epsilon_{2aj} I_1 + 4\,(p_2 D\epsilon_3 D p_3)\, p_{2i}\epsilon_{2aj} I'_7 \\
&+ 2\,\mathrm{tr}(D\epsilon_3)\,(p_2 p_3)\, p_{2i}\epsilon_{2aj} I'_6 - \mathrm{tr}(D\epsilon_3)\,(p_1 N p_3)\, p_{2i}\epsilon_{2aj} I'_4 \\
&- 2\,\mathrm{tr}(D\epsilon_3)\,(p_2 D p_3)\, p_{2i}\epsilon_{2aj} I'_7 - 2\,(p_2\epsilon_3 p_2)\, p_{3i}\epsilon_{2aj} I_2 \\
&- 2\,(p_1 N\epsilon_3 D p_2)\, p_{3i}\epsilon_{2aj} I_5 + 2\,(p_2 D\epsilon_3 D p_2)\, p_{3i}\epsilon_{2aj} I_1 \\
&- 2\,\mathrm{tr}(D\epsilon_3)\,(p_2 p_3)\, p_{3i}\epsilon_{2aj} I'_6 - \mathrm{tr}(D\epsilon_3)\,(p_1 N p_2)\, p_{3i}\epsilon_{2aj} I'_4 \\
&- 2\,\mathrm{tr}(D\epsilon_3)\,(p_2 D p_3)\, p_{3i}\epsilon_{2aj} I'_7 - 4\,(p_2 p_3)\,(p_1 N\epsilon_3)_i \epsilon_{2aj} I_9 \\
&- 4\,(p_1 N p_2)\,(p_1 N\epsilon_3)_i \epsilon_{2aj} I_{10} + 4\,(p_2 D p_3)\,(p_1 N\epsilon_3)_i \epsilon_{2aj} I_5 \\
&- 4\,(p_3 D\epsilon_2 p_3)\, p_{3i}\epsilon_{3aj} I'_{11}) + (2 \leftrightarrow 3),
\end{aligned}
\tag{2.81}
$$

$$
\begin{aligned}
\mathcal{A}^{T^{(1)}}_{C^{(p+1)}hh} =& \frac{i}{8\sqrt{2}} T^i ((p_1 N\epsilon_2\epsilon_3 p_2)\, p_{2i} I_2 - (p_1 N\epsilon_3\epsilon_2 p_3)\, p_{2i} I_2 \\
&+ 2\,(p_2\epsilon_3 D\epsilon_2 p_3)\, p_{2i} I_{14} - 2\,(p_2\epsilon_3\epsilon_2 D p_3)\, p_{2i} I_{14} \\
&+ 2\,(p_3 D\epsilon_3\epsilon_2 p_3)\, p_{2i} I'_{20} + 2\,(p_1 N\epsilon_2\epsilon_3 D p_3)\, p_{2i} I'_6 \\
&- 2\,(p_1 N\epsilon_2\epsilon_3 N p_1)\, p_{2i} I_9 + 3\,(p_1 N\epsilon_3 D\epsilon_2 p_3)\, p_{2i} I'_{11} \\
&- 3\,(p_1 N\epsilon_3\epsilon_2 D p_3)\, p_{2i} I'_{11} + 2\,(p_2 D\epsilon_3 D\epsilon_2 p_3)\, p_{2i} I_{15} \\
&- 2\,(p_2 D\epsilon_3\epsilon_2 D p_3)\, p_{2i} I_{15} - 2\,(p_3 D\epsilon_2\epsilon_3 D p_3)\, p_{2i} I_{22} \\
&+ 2\,(p_3 D\epsilon_3 D\epsilon_2 p_3)\, p_{2i} I_{22} + (p_1 N\epsilon_2 D\epsilon_3 D p_2)\, p_{2i} I_1 \\
&- 2\,(p_1 N\epsilon_2 D\epsilon_3 D p_3)\, p_{2i} I'_7 - 2\,(p_1 N\epsilon_2 D\epsilon_3 N p_1)\, p_{2i} I_5 \\
&+ (p_1 N\epsilon_3 D\epsilon_2 D p_3)\, p_{2i} I_1 - 2\,(p_3 D\epsilon_2 D\epsilon_3 D p_3)\, p_{2i} I'_{21} \\
&- \mathrm{tr}(D\epsilon_2)\,(p_2\epsilon_3 p_2)\, p_{2i} I_{20} - 3\,\mathrm{tr}(D\epsilon_2)\,(p_1 N\epsilon_3 p_2)\, p_{2i} I_6 \\
&- 2\,\mathrm{tr}(D\epsilon_2)\,(p_2\epsilon_3 D p_3)\, p_{2i} I_{18} - 3\,\mathrm{tr}(D\epsilon_2)\,(p_1 N\epsilon_3 D p_2)\, p_{2i} I_7 \\
&+ 2\,\mathrm{tr}(D\epsilon_2)\,(p_1 N\epsilon_3 D p_3)\, p_{2i} I_3 - \mathrm{tr}(D\epsilon_2)\,(p_1 N\epsilon_3 N p_1)\, p_{2i} I_4 \\
&- \mathrm{tr}(D\epsilon_2)\,(p_2 D\epsilon_3 D p_2)\, p_{2i} I_{21} - 2\,\mathrm{tr}(D\epsilon_2)\,(p_2 D\epsilon_3 D p_3)\, p_{2i} I_{19}
\end{aligned}
$$

$$- \operatorname{tr}(D\epsilon_3)\,(p_3\epsilon_2 p_3)\,p_{2i} I'_{20} - \operatorname{tr}(D\epsilon_3)\,(p_1 N\epsilon_2 p_3)\,p_{2i} I'_6$$

$$+ \operatorname{tr}(D\epsilon_3)\,(p_1 N\epsilon_2 D p_3)\,p_{2i} I'_7 + \operatorname{tr}(D\epsilon_3)\,(p_3 D\epsilon_2 D p_3)\,p_{2i} I'_{21}$$

$$+ \operatorname{tr}(D\epsilon_2)\operatorname{tr}(D\epsilon_3)\,(p_2 p_3)\,p_{2i} I_{18} - \operatorname{tr}(D\epsilon_2)\operatorname{tr}(D\epsilon_3)\,(p_1 N p_3)\,p_{2i} I_3$$

$$+ \operatorname{tr}(D\epsilon_2)\operatorname{tr}(D\epsilon_3)\,(p_2 D p_3)\,p_{2i} I_{19} - 2\operatorname{tr}(\epsilon_2\epsilon_3) p_{2i} I_{23}$$

$$+ \operatorname{tr}(\epsilon_2\epsilon_3)\,(p_2 p_3)\,p_{2i} I_{16} - \operatorname{tr}(\epsilon_2\epsilon_3)\,(p_1 N p_3)\,p_{2i} I_2$$

$$+ \operatorname{tr}(\epsilon_2\epsilon_3)\,(p_2 D p_3)\,p_{2i} I_{14} + 2\operatorname{tr}(D\epsilon_2 D\epsilon_3) p_{2i} I_{24}$$

$$- \operatorname{tr}(D\epsilon_2 D\epsilon_3)\,(p_2 p_3)\,p_{2i} I_{15} + \operatorname{tr}(D\epsilon_2 D\epsilon_3)\,(p_1 N p_3)\,p_{2i} I_1$$

$$- \operatorname{tr}(D\epsilon_2 D\epsilon_3)\,(p_2 D p_3)\,p_{2i} I_{17} + (p_1 N\epsilon_3 D\epsilon_2 p_3)\,p_{3i} I'_{11}$$

$$+ (p_1 N\epsilon_3\epsilon_2 D p_3)\,p_{3i} I'_{11} + 2\operatorname{tr}(D\epsilon_3)\,(p_3 D\epsilon_2 p_3)\,p_{3i} I_{22}$$

$$+ 2\,(p_1 N\epsilon_2 p_3)\,(p_2\epsilon_3)_i I_2 - 2\,(p_2 D\epsilon_2 p_3)\,(p_2\epsilon_3)_i I_{20}$$

$$- 2\,(p_3 D\epsilon_2 p_3)\,(p_2\epsilon_3)_i I_{14} + 2\,(p_1 N\epsilon_2 D p_2)\,(p_2\epsilon_3)_i I_6$$

$$+ 2\,(p_1 N\epsilon_2 N p_1)\,(p_2\epsilon_3)_i I_9 + 2\,(p_3 D\epsilon_2 D p_3)\,(p_2\epsilon_3)_i I_{15}$$

$$+ 2\operatorname{tr}(D\epsilon_2)\,(p_2 p_3)\,(p_2\epsilon_3)_i I_{20} - \operatorname{tr}(D\epsilon_2)\,(p_1 N p_2)\,(p_2\epsilon_3)_i I_6$$

$$+ \operatorname{tr}(D\epsilon_2)\,(p_1 N p_3)\,(p_2\epsilon_3)_i I_6 - 4\,(p_1 N\epsilon_3 D p_2)\,(p_3\epsilon_2)_i I'_{26}$$

$$- 2\,(p_2 D\epsilon_3 D p_3)\,(p_3\epsilon_2)_i I_{22} - 2\,(p_2\epsilon_3 p_2)\,(p_1 N\epsilon_2)_i I_2$$

$$+ 6\,(p_1 N\epsilon_3 p_2)\,(p_1 N\epsilon_2)_i I_9 - 4\,(p_2\epsilon_3 D p_3)\,(p_1 N\epsilon_2)_i I'_6$$

$$+ 6\,(p_1 N\epsilon_3 D p_2)\,(p_1 N\epsilon_2)_i I_5 + 2\,(p_1 N\epsilon_3 D p_3)\,(p_1 N\epsilon_2)_i I'_4$$

$$- 4\,(p_1 N\epsilon_3 N p_1)\,(p_1 N\epsilon_2)_i I_{10} - 2\,(p_2 D\epsilon_3 D p_2)\,(p_1 N\epsilon_2)_i I_1$$

$$+ 4\,(p_2 D\epsilon_3 D p_3)\,(p_1 N\epsilon_2)_i I'_7 + 2\operatorname{tr}(D\epsilon_3)\,(p_2 p_3)\,(p_1 N\epsilon_2)_i I'_6$$

$$- \operatorname{tr}(D\epsilon_3)\,(p_1 N p_3)\,(p_1 N\epsilon_2)_i I'_4 - 2\operatorname{tr}(D\epsilon_3)\,(p_2 D p_3)\,(p_1 N\epsilon_2)_i I'_7$$

$$- 4\,(p_3 D\epsilon_2 p_3)\,(p_1 N\epsilon_3)_i I'_{11} + 2\,(p_2\epsilon_3 p_2)\,(p_2 D\epsilon_2)_i I_{20}$$

$$+ 6\,(p_1 N\epsilon_3 p_2)\,(p_2 D\epsilon_2)_i I_6 + 4\,(p_2\epsilon_3 D p_3)\,(p_2 D\epsilon_2)_i I_{18}$$

$$+ 6\,(p_1 N\epsilon_3 D p_2)\,(p_2 D\epsilon_2)_i I_7 - 4\,(p_1 N\epsilon_3 D p_3)\,(p_2 D\epsilon_2)_i I_3$$

$$
\begin{aligned}
&+ 2\,(p_1 N \epsilon_3 N p_1)\,(p_2 D \epsilon_2)_i I_4 + 2\,(p_2 D \epsilon_3 D p_2)\,(p_2 D \epsilon_2)_i I_{21} \\
&+ 4\,(p_2 D \epsilon_3 D p_3)\,(p_2 D \epsilon_2)_i I_{19} - 2\,\mathrm{tr}(D\epsilon_3)\,(p_2 p_3)\,(p_2 D \epsilon_2)_i I_{18} \\
&+ 2\,\mathrm{tr}(D\epsilon_3)\,(p_1 N p_3)\,(p_2 D \epsilon_2)_i I_3 - 2\,\mathrm{tr}(D\epsilon_3)\,(p_2 D p_3)\,(p_2 D \epsilon_2)_i I_{19} \\
&+ 2\,(p_3 \epsilon_2 p_3)\,(p_2 D \epsilon_3)_i I_{14} - 2\,(p_3 D \epsilon_2 p_3)\,(p_2 D \epsilon_3)_i I_{15} \\
&+ 2\,(p_1 N \epsilon_2 D p_2)\,(p_2 D \epsilon_3)_i I_7 - 2\,(p_1 N \epsilon_2 D p_3)\,(p_2 D \epsilon_3)_i I_1 \\
&+ 2\,(p_1 N \epsilon_2 N p_1)\,(p_2 D \epsilon_3)_i I_5 + 2\,(p_2 D \epsilon_2 D p_3)\,(p_2 D \epsilon_3)_i I_{21} \\
&- \mathrm{tr}(D\epsilon_2)\,(p_1 N p_2)\,(p_2 D \epsilon_3)_i I_7 - \mathrm{tr}(D\epsilon_2)\,(p_1 N p_3)\,(p_2 D \epsilon_3)_i I_7 \\
&- 2\,\mathrm{tr}(D\epsilon_2)\,(p_2 D p_3)\,(p_2 D \epsilon_3)_i I_{21} + 4(p_2 \epsilon_3 \epsilon_2)_i I_{23} \\
&- 2\,(p_2 p_3)\,(p_2 \epsilon_3 \epsilon_2)_i I_{16} - (p_1 N p_2)\,(p_2 \epsilon_3 \epsilon_2)_i I_2 \\
&+ (p_1 N p_3)\,(p_2 \epsilon_3 \epsilon_2)_i I_2 + 4\,(p_1 N \epsilon_3 p_2)\,(p_3 D \epsilon_2)_i I'_{26} \\
&+ 2\,(p_2 \epsilon_3 D p_3)\,(p_3 D \epsilon_2)_i I_{22} - 4\,(p_3 D \epsilon_2 p_3)\,(p_3 D \epsilon_3)_i I_{22} \\
&+ 2\,(p_2 p_3)\,(p_1 N \epsilon_2 \epsilon_3)_i I_2 - 2\,(p_1 N p_3)\,(p_1 N \epsilon_2 \epsilon_3)_i I_9 \\
&+ 2\,(p_2 D p_3)\,(p_1 N \epsilon_3 \epsilon_2)_i I'_{25} - 2\,(p_2 p_3)\,(p_2 D \epsilon_2 \epsilon_3)_i I_{20} \\
&- 2\,(p_1 N p_3)\,(p_2 D \epsilon_2 \epsilon_3)_i I_6 + 2\,(p_2 D p_3)\,(p_2 D \epsilon_3 \epsilon_2)_i I_{15} \\
&- 2\,(p_2 p_3)\,(p_2 \epsilon_3 D \epsilon_2)_i I_{14} - (p_1 N p_2)\,(p_3 D \epsilon_2 \epsilon_3)_i I'_{11} \\
&- (p_1 N p_3)\,(p_3 D \epsilon_2 \epsilon_3)_i I'_{11} + 2\,(p_2 D p_3)\,(p_3 D \epsilon_3 \epsilon_2)_i I_{22} \\
&+ (p_1 N p_2)\,(p_3 \epsilon_2 D \epsilon_3)_i I'_{11} - (p_1 N p_3)\,(p_3 \epsilon_2 D \epsilon_3)_i I'_{11} \\
&+ 2\,(p_1 N p_3)\,(p_1 N \epsilon_2 D \epsilon_3)_i I_5 - 2\,(p_2 D p_3)\,(p_1 N \epsilon_2 D \epsilon_3)_i I_1 \\
&- 2\,(p_2 p_3)\,(p_1 N \epsilon_3 D \epsilon_2)_i I'_{25} + 2\,(p_1 N p_3)\,(p_2 D \epsilon_2 D \epsilon_3)_i I_7 \\
&+ 2\,(p_2 D p_3)\,(p_2 D \epsilon_2 D \epsilon_3)_i I_{21} - 4(p_2 D \epsilon_3 D \epsilon_2)_i I_{24} \\
&- (p_1 N p_2)\,(p_2 D \epsilon_3 D \epsilon_2)_i I_1 - (p_1 N p_3)\,(p_2 D \epsilon_3 D \epsilon_2)_i I_1 \\
&+ 2\,(p_2 D p_3)\,(p_2 D \epsilon_3 D \epsilon_2)_i I_{17} - 2\,(p_2 p_3)\,(p_3 D \epsilon_3 D \epsilon_2)_i I_{22}) + (2 \leftrightarrow 3)
\end{aligned}
\tag{2.82}
$$

$$\mathcal{A}_{C^{(p-1)}Bh} = \mathcal{A}^{T^{(7)}}_{C^{(p-1)}Bh} + \mathcal{A}^{T^{(5)}}_{C^{(p-1)}Bh} + \mathcal{A}^{T^{(3)}}_{C^{(p-1)}Bh} + \mathcal{A}^{T^{(1)}}_{C^{(p-1)}Bh} \tag{2.83}$$

$$\mathcal{A}^{T^{(7)}}_{C^{(p-1)}Bh} = \frac{i}{4\sqrt{2}} T^{abcdijk}(-p_{2_a}p_{2_i}p_{3_j}\epsilon_{2cd}\epsilon_{3bk}I_{10} - p_{2_j}p_{3_a}p_{3_i}\epsilon_{2bc}\epsilon_{3dk}I_{10}) \tag{2.84}$$

$$\begin{aligned}
\mathcal{A}^{T^{(5)}}_{C^{(p-1)}Bh} =& \frac{i}{16\sqrt{2}} T^{abcij}(-4p_{2_j}p_{3_a}p_{3_i}(\epsilon_2\epsilon_3)_{bc}I_9 + 4p_{2_a}p_{2_i}p_{3_j}(\epsilon_2\epsilon_3)_{cb}I_9 \\
&- 4p_{2_j}p_{3_a}p_{3_i}(\epsilon_2 D\epsilon_3)_{bc}I_5 + 4p_{2_a}p_{2_i}p_{3_j}(\epsilon_2 D\epsilon_3)_{cb}I_5 \\
&- 4p_{2_i}p_{3_j}(p_2\epsilon_3)_c\epsilon_{2ab}I_9 + 4p_{2_i}p_{3_j}(p_1 N\epsilon_3)_c\epsilon_{2ab}I_{10} \\
&- 4p_{2_i}p_{3_j}(p_2 D\epsilon_3)_c\epsilon_{2ab}I_5 + \mathrm{tr}(D\epsilon_3)p_{2_j}p_{3_a}p_{3_i}\epsilon_{2bc}I'_4 \\
&+ \mathrm{tr}(D\epsilon_3)p_{2_a}p_{2_i}p_{3_j}\epsilon_{2bc}I'_4 + 2p_{2_a}p_{2_j}(p_2\epsilon_3)_i\epsilon_{2bc}I_9 \\
&- 2p_{2_j}p_{3_a}(p_2\epsilon_3)_i\epsilon_{2bc}I_9 - 2p_{3_a}p_{3_j}(p_2\epsilon_3)_i\epsilon_{2bc}I_9 \\
&- 2p_{2_a}p_{3_i}(p_2\epsilon_3)_j\epsilon_{2bc}I_9 - 4p_{2_a}p_{2_j}(p_1 N\epsilon_3)_i\epsilon_{2bc}I_{10} \\
&+ 4p_{2_j}p_{3_a}(p_1 N\epsilon_3)_i\epsilon_{2bc}I_{10} + 2p_{2_a}p_{2_j}(p_2 D\epsilon_3)_i\epsilon_{2bc}I_5 \\
&- 2p_{2_j}p_{3_a}(p_2 D\epsilon_3)_i\epsilon_{2bc}I_5 + 2p_{2_a}p_{3_i}(p_2 D\epsilon_3)_j\epsilon_{2bc}I_5 \\
&- 2p_{3_a}p_{3_i}(p_2 D\epsilon_3)_j\epsilon_{2bc}I_5 + 2p_{2_a}p_{2_j}(p_3 D\epsilon_3)_i\epsilon_{2bc}I'_4 \\
&- 2p_{2_j}p_{3_a}(p_3 D\epsilon_3)_i\epsilon_{2bc}I'_4 + 4\,(p_2p_3)\,p_{2_i}\epsilon_{2bc}\epsilon_{3aj}I_9 \\
&- 4\,(p_1Np_3)\,p_{2_i}\epsilon_{2bc}\epsilon_{3aj}I_{10} + 4\,(p_2Dp_3)\,p_{2_i}\epsilon_{2bc}\epsilon_{3aj}I_5 \\
&- 4\,(p_2p_3)\,p_{3_i}\epsilon_{2bc}\epsilon_{3aj}I_9 - 4\,(p_1Np_2)\,p_{3_i}\epsilon_{2bc}\epsilon_{3aj}I_{10} \\
&+ 4\,(p_2Dp_3)\,p_{3_i}\epsilon_{2bc}\epsilon_{3aj}I_5 - 4p_{2_i}p_{3_a}(p_3\epsilon_2)_c\epsilon_{3bj}I_9 \\
&+ 4p_{2_a}p_{3_i}(p_3\epsilon_2)_c\epsilon_{3bj}I_9 + 8p_{2_a}p_{3_i}(p_1 N\epsilon_2)_c\epsilon_{3bj}I_{10} \\
&- 4p_{2_a}p_{3_i}(p_2 D\epsilon_2)_c\epsilon_{3bj}I_4 + 4p_{3_a}p_{3_i}(p_2 D\epsilon_2)_c\epsilon_{3bj}I_4 \\
&+ 4p_{2_a}p_{2_i}(p_3 D\epsilon_2)_c\epsilon_{3bj}I_5 - 4p_{2_i}p_{3_a}(p_3 D\epsilon_2)_c\epsilon_{3bj}I_5
\end{aligned}$$

$$
\begin{aligned}
&- 4p_{2a}p_{3i}(p_3D\epsilon_2)_c\epsilon_{3bj}I_5 + 4p_{2b}p_{2i}(p_3\epsilon_2)_a\epsilon_{3cj}I_9 \\
&- 4p_{3b}p_{3i}(p_3\epsilon_2)_a\epsilon_{3cj}I_9 - 8p_{3b}p_{3i}(p_1N\epsilon_2)_a\epsilon_{3cj}I_{10} \\
&+ 4p_{3b}p_{3i}(p_3D\epsilon_2)_a\epsilon_{3cj}I_5)
\end{aligned} \tag{2.85}
$$

$$
\begin{aligned}
\mathcal{A}^{T^{(3)}}_{C^{(p-1)}Bh} =& \frac{i}{8\sqrt{2}}T^{abi}\left(\mathrm{tr}(D\epsilon_3)p_{2b}p_{2i}(p_3\epsilon_2)_aI'_6 - \mathrm{tr}(D\epsilon_3)p_{3b}p_{3i}(p_3\epsilon_2)_aI'_6\right. \\
&-2p_{2i}(p_2\epsilon_3)_b(p_3\epsilon_2)_aI_2 - 2p_{3i}(p_2\epsilon_3)_b(p_3\epsilon_2)_aI_2 \\
&+\mathrm{tr}(D\epsilon_3)p_{2i}p_{3a}(p_3\epsilon_2)_bI'_6 - \mathrm{tr}(D\epsilon_3)p_{2a}p_{3i}(p_3\epsilon_2)_bI'_6 \\
&+2\,(p_2p_3)\,p_{2i}(\epsilon_2\epsilon_3)_{ba}I_2 - 2\,(p_1Np_3)\,p_{2i}(\epsilon_2\epsilon_3)_{ba}I_9 \\
&+2\,(p_2Dp_3)\,p_{2i}(\epsilon_2\epsilon_3)_{ba}I_{11} - 2\,(p_2p_3)\,p_{3i}(\epsilon_2\epsilon_3)_{ba}I_2 \\
&-2\,(p_1Np_2)\,p_{3i}(\epsilon_2\epsilon_3)_{ba}I_9 - 2\,(p_2Dp_3)\,p_{3i}(\epsilon_2\epsilon_3)_{ba}I'_{11} \\
&+4\,(p_2Dp_3)\,p_{2a}(\epsilon_2\epsilon_3)_{bi}I_0 - 4\,(p_2Dp_3)\,p_{3a}(\epsilon_2\epsilon_3)_{bi}I_0 \\
&-4\,(p_2Dp_3)\,p_{2a}(\epsilon_2\epsilon_3)_{ib}I_0 + 4\,(p_2Dp_3)\,p_{3a}(\epsilon_2\epsilon_3)_{ib}I_0 \\
&-\mathrm{tr}(D\epsilon_3)p_{3b}p_{3i}(p_1N\epsilon_2)_aI'_4 - 4p_{3i}(p_1N\epsilon_2)_a(p_2\epsilon_3)_bI_9 \\
&+2p_{3b}(p_1N\epsilon_2)_a(p_2\epsilon_3)_iI_9 - \mathrm{tr}(D\epsilon_3)p_{2a}p_{3i}(p_1N\epsilon_2)_bI'_4 \\
&+2p_{2a}(p_1N\epsilon_2)_b(p_2\epsilon_3)_iI_9 - 2p_{2i}(p_1N\epsilon_3)_a(p_3\epsilon_2)_bI_9 \\
&-2p_{3i}(p_1N\epsilon_3)_a(p_3\epsilon_2)_bI_9 - 4p_{3i}(p_1N\epsilon_2)_b(p_1N\epsilon_3)_aI_{10} \\
&-2p_{3b}(p_1N\epsilon_3)_i(p_3\epsilon_2)_aI_9 - 2p_{2a}(p_1N\epsilon_3)_i(p_3\epsilon_2)_bI_9 \\
&-4p_{3b}(p_1N\epsilon_2)_a(p_1N\epsilon_3)_iI_{10} - 4p_{2a}(p_1N\epsilon_2)_b(p_1N\epsilon_3)_iI_{10} \\
&+2\,\mathrm{tr}(D\epsilon_3)p_{3b}p_{3i}(p_2D\epsilon_2)_aI_3 - 4p_{3i}(p_2D\epsilon_2)_a(p_2\epsilon_3)_bI_6 \\
&+2p_{3b}(p_2D\epsilon_2)_a(p_2\epsilon_3)_iI_6 - 2p_{3i}(p_1N\epsilon_3)_b(p_2D\epsilon_2)_aI_4 \\
&+2p_{3b}(p_1N\epsilon_3)_i(p_2D\epsilon_2)_aI_4 + 2\,\mathrm{tr}(D\epsilon_3)p_{2a}p_{3i}(p_2D\epsilon_2)_bI_3 \\
&+2p_{2a}(p_2D\epsilon_2)_b(p_2\epsilon_3)_iI_6 + 2p_{2a}(p_1N\epsilon_3)_i(p_2D\epsilon_2)_bI_4
\end{aligned}
$$

$$+2p_{2i}(p_2D\epsilon_3)_a(p_3\epsilon_2)_bI_{11}+4p_{2b}(p_2D\epsilon_3)_a(p_3\epsilon_2)_iI_0$$

$$+4p_{3i}(p_2D\epsilon_2)_b(p_2D\epsilon_3)_aI_7+2p_{3i}(p_2D\epsilon_3)_b(p_3\epsilon_2)_aI'_{11}$$

$$+4p_{3a}(p_2D\epsilon_3)_b(p_3\epsilon_2)_iI_0-4p_{3i}(p_1N\epsilon_2)_a(p_2D\epsilon_3)_bI_5$$

$$+2p_{2a}(p_2D\epsilon_3)_i(p_3\epsilon_2)_bI_8-2p_{3a}(p_2D\epsilon_3)_i(p_3\epsilon_2)_bI_8$$

$$+2p_{3b}(p_1N\epsilon_2)_a(p_2D\epsilon_3)_iI_5+2p_{2a}(p_1N\epsilon_2)_b(p_2D\epsilon_3)_iI_5$$

$$+2p_{3b}(p_2D\epsilon_2)_a(p_2D\epsilon_3)_iI_7+2p_{2a}(p_2D\epsilon_2)_b(p_2D\epsilon_3)_iI_7$$

$$+p_{2a}p_{2i}(p_2\epsilon_3\epsilon_2)_bI_2-p_{2i}p_{3a}(p_2\epsilon_3\epsilon_2)_bI_2$$

$$+p_{2a}p_{3i}(p_2\epsilon_3\epsilon_2)_bI_2-p_{3a}p_{3i}(p_2\epsilon_3\epsilon_2)_bI_2$$

$$-\operatorname{tr}(D\epsilon_3)p_{3b}p_{3i}(p_3D\epsilon_2)_aI'_7-2p_{3i}(p_2\epsilon_3)_b(p_3D\epsilon_2)_aI'_{11}$$

$$-2p_{3b}(p_2\epsilon_3)_i(p_3D\epsilon_2)_aI_8-2p_{3i}(p_1N\epsilon_3)_b(p_3D\epsilon_2)_aI_5$$

$$+2p_{3b}(p_1N\epsilon_3)_i(p_3D\epsilon_2)_aI_5+2p_{3i}(p_2D\epsilon_3)_b(p_3D\epsilon_2)_aI_1$$

$$+\operatorname{tr}(D\epsilon_3)p_{2a}p_{2i}(p_3D\epsilon_2)_bI'_7-\operatorname{tr}(D\epsilon_3)p_{2i}p_{3a}(p_3D\epsilon_2)_bI'_7$$

$$-\operatorname{tr}(D\epsilon_3)p_{2a}p_{3i}(p_3D\epsilon_2)_bI'_7+2p_{2i}(p_2\epsilon_3)_a(p_3D\epsilon_2)_bI_{11}$$

$$-2p_{2a}(p_2\epsilon_3)_i(p_3D\epsilon_2)_bI_8-2p_{2i}(p_1N\epsilon_3)_a(p_3D\epsilon_2)_bI_5$$

$$+2p_{2a}(p_1N\epsilon_3)_i(p_3D\epsilon_2)_bI_5+2p_{2i}(p_2D\epsilon_3)_a(p_3D\epsilon_2)_bI_1$$

$$-4p_{2a}(p_2\epsilon_3)_b(p_3D\epsilon_2)_iI_0+4p_{3a}(p_2\epsilon_3)_b(p_3D\epsilon_2)_iI_0$$

$$+2p_{3b}(p_3D\epsilon_3)_i(p_3\epsilon_2)_aI'_6+2p_{2a}(p_3D\epsilon_3)_i(p_3\epsilon_2)_bI'_6$$

$$+2p_{3b}(p_1N\epsilon_2)_a(p_3D\epsilon_3)_iI'_4+2p_{2a}(p_1N\epsilon_2)_b(p_3D\epsilon_3)_iI'_4$$

$$-4p_{3b}(p_2D\epsilon_2)_a(p_3D\epsilon_3)_iI_3-4p_{2a}(p_2D\epsilon_2)_b(p_3D\epsilon_3)_iI_3$$

$$+2p_{3b}(p_3D\epsilon_2)_a(p_3D\epsilon_3)_iI'_7+2p_{2a}(p_3D\epsilon_2)_b(p_3D\epsilon_3)_iI'_7$$

$$+p_{2a}p_{2i}(p_3\epsilon_2\epsilon_3)_bI_2-p_{2i}p_{3a}(p_3\epsilon_2\epsilon_3)_bI_2$$

$$+p_{2a}p_{3i}(p_3\epsilon_2\epsilon_3)_bI_2-p_{3a}p_{3i}(p_3\epsilon_2\epsilon_3)_bI_2$$

$$-2\,(p_2p_3)\,p_{2i}(\epsilon_2D\epsilon_3)_{ab}I_{11}+2\,(p_1Np_3)\,p_{2i}(\epsilon_2D\epsilon_3)_{ab}I_5$$

$$-2\,(p_2Dp_3)\,p_{2i}(\epsilon_2D\epsilon_3)_{ab}I_1+2\,(p_2p_3)\,p_{3i}(\epsilon_2D\epsilon_3)_{ba}I'_{11}$$

$$-2\,(p_1Np_2)\,p_{3i}(\epsilon_2D\epsilon_3)_{ba}I_5+2\,(p_2Dp_3)\,p_{3i}(\epsilon_2D\epsilon_3)_{ba}I_1$$

$$-4\,(p_2p_3)\,p_{2a}(\epsilon_2D\epsilon_3)_{bi}I_0+4\,(p_2p_3)\,p_{3a}(\epsilon_2D\epsilon_3)_{bi}I_0$$

$$-4\,(p_2p_3)\,p_{2a}(\epsilon_2D\epsilon_3)_{ib}I_0+4\,(p_2p_3)\,p_{3a}(\epsilon_2D\epsilon_3)_{ib}I_0$$

$$+2p_{2a}p_{3i}(p_1N\epsilon_2\epsilon_3)_bI_9-2p_{3a}p_{3i}(p_1N\epsilon_2\epsilon_3)_bI_9$$

$$-2p_{2a}p_{2i}(p_1N\epsilon_3\epsilon_2)_bI_9+2p_{2i}p_{3a}(p_1N\epsilon_3\epsilon_2)_bI_9$$

$$+2p_{2a}p_{3i}(p_2D\epsilon_2\epsilon_3)_bI_6-2p_{3a}p_{3i}(p_2D\epsilon_2\epsilon_3)_bI_6$$

$$+p_{2a}p_{2i}(p_2D\epsilon_3\epsilon_2)_bI_{11}-p_{2i}p_{3a}(p_2D\epsilon_3\epsilon_2)_bI_{11}$$

$$-p_{2a}p_{3i}(p_2D\epsilon_3\epsilon_2)_bI_{11}+p_{3a}p_{3i}(p_2D\epsilon_3\epsilon_2)_bI_{11}$$

$$+p_{2a}p_{2i}(p_2\epsilon_3D\epsilon_2)_bI_{11}-p_{2i}p_{3a}(p_2\epsilon_3D\epsilon_2)_bI_{11}$$

$$+p_{2a}p_{3i}(p_2\epsilon_3D\epsilon_2)_bI_{11}-p_{3a}p_{3i}(p_2\epsilon_3D\epsilon_2)_bI_{11}$$

$$-p_{2a}p_{2i}(p_3D\epsilon_2\epsilon_3)_bI'_{11}+p_{2i}p_{3a}(p_3D\epsilon_2\epsilon_3)_bI'_{11}$$

$$+p_{2a}p_{3i}(p_3D\epsilon_2\epsilon_3)_bI'_{11}-p_{3a}p_{3i}(p_3D\epsilon_2\epsilon_3)_bI'_{11}$$

$$+2p_{2a}p_{2i}(p_3D\epsilon_3\epsilon_2)_bI'_6-2p_{2i}p_{3a}(p_3D\epsilon_3\epsilon_2)_bI'_6$$

$$-p_{2a}p_{2i}(p_3\epsilon_2D\epsilon_3)_bI'_{11}+p_{2i}p_{3a}(p_3\epsilon_2D\epsilon_3)_bI'_{11}$$

$$-p_{2a}p_{3i}(p_3\epsilon_2D\epsilon_3)_bI'_{11}+p_{3a}p_{3i}(p_3\epsilon_2D\epsilon_3)_bI'_{11}$$

$$+2p_{2a}p_{3i}(p_1N\epsilon_2D\epsilon_3)_bI_5-2p_{3a}p_{3i}(p_1N\epsilon_2D\epsilon_3)_bI_5$$

$$-2p_{2a}p_{2i}(p_1N\epsilon_3D\epsilon_2)_bI_5+2p_{2i}p_{3a}(p_1N\epsilon_3D\epsilon_2)_bI_5$$

$$+2p_{2a}p_{3i}(p_2D\epsilon_2D\epsilon_3)_bI_7-2p_{3a}p_{3i}(p_2D\epsilon_2D\epsilon_3)_bI_7$$

$$+p_{2a}p_{2i}(p_2D\epsilon_3D\epsilon_2)_bI_1-p_{2i}p_{3a}(p_2D\epsilon_3D\epsilon_2)_bI_1$$

$$-p_{2a}p_{3i}(p_2D\epsilon_3D\epsilon_2)_bI_1+p_{3a}p_{3i}(p_2D\epsilon_3D\epsilon_2)_bI_1$$

$$
\begin{aligned}
&+p_{2a}p_{2i}(p_3D\epsilon_2D\epsilon_3)_bI_1 - p_{2i}p_{3a}(p_3D\epsilon_2D\epsilon_3)_bI_1\\
&-p_{2a}p_{3i}(p_3D\epsilon_2D\epsilon_3)_bI_1 + p_{3a}p_{3i}(p_3D\epsilon_2D\epsilon_3)_bI_1\\
&-2p_{2a}p_{2i}(p_3D\epsilon_3D\epsilon_2)_bI_7' + 2p_{2i}p_{3a}(p_3D\epsilon_3D\epsilon_2)_bI_7'\\
&-(p_2\epsilon_3p_2)\,p_{2i}\epsilon_{2ab}I_2 + 3\,(p_1N\epsilon_3p_2)\,p_{2i}\epsilon_{2ab}I_9\\
&-2\,(p_2D\epsilon_3p_2)\,p_{2i}\epsilon_{2ab}I_{11} - 2\,(p_2\epsilon_3Dp_3)\,p_{2i}\epsilon_{2ab}I_6'\\
&+3\,(p_1N\epsilon_3Dp_2)\,p_{2i}\epsilon_{2ab}I_5 + (p_1N\epsilon_3Dp_3)\,p_{2i}\epsilon_{2ab}I_4'\\
&-2\,(p_1N\epsilon_3Np_1)\,p_{2i}\epsilon_{2ab}I_{10} - (p_2D\epsilon_3Dp_2)\,p_{2i}\epsilon_{2ab}I_1\\
&+2\,(p_2D\epsilon_3Dp_3)\,p_{2i}\epsilon_{2ab}I_7' + \mathrm{tr}(D\epsilon_3)\,(p_2p_3)\,p_{2i}\epsilon_{2ab}I_6'\\
&-\frac{1}{2}\,\mathrm{tr}(D\epsilon_3)\,(p_1Np_3)\,p_{2i}\epsilon_{2ab}I_4' - \mathrm{tr}(D\epsilon_3)\,(p_2Dp_3)\,p_{2i}\epsilon_{2ab}I_7'\\
&-(p_2\epsilon_3p_2)\,p_{3i}\epsilon_{2ab}I_2 + (p_1N\epsilon_3p_2)\,p_{3i}\epsilon_{2ab}I_9\\
&-(p_1N\epsilon_3Dp_2)\,p_{3i}\epsilon_{2ab}I_5 + (p_2D\epsilon_3Dp_2)\,p_{3i}\epsilon_{2ab}I_1\\
&-\mathrm{tr}(D\epsilon_3)\,(p_2p_3)\,p_{3i}\epsilon_{2ab}I_6' - \frac{1}{2}\,\mathrm{tr}(D\epsilon_3)\,(p_1Np_2)\,p_{3i}\epsilon_{2ab}I_4'\\
&-\mathrm{tr}(D\epsilon_3)\,(p_2Dp_3)\,p_{3i}\epsilon_{2ab}I_7' + 2\,(p_2p_3)\,(p_2\epsilon_3)_i\epsilon_{2ab}I_2\\
&+(p_1Np_2)\,(p_2\epsilon_3)_i\epsilon_{2ab}I_9 - (p_1Np_3)\,(p_2\epsilon_3)_i\epsilon_{2ab}I_9\\
&-2\,(p_2p_3)\,(p_1N\epsilon_3)_i\epsilon_{2ab}I_9 - 2\,(p_1Np_2)\,(p_1N\epsilon_3)_i\epsilon_{2ab}I_{10}\\
&+2\,(p_2Dp_3)\,(p_1N\epsilon_3)_i\epsilon_{2ab}I_5 + (p_1Np_2)\,(p_2D\epsilon_3)_i\epsilon_{2ab}I_5\\
&+(p_1Np_3)\,(p_2D\epsilon_3)_i\epsilon_{2ab}I_5 - 2\,(p_2Dp_3)\,(p_2D\epsilon_3)_i\epsilon_{2ab}I_1\\
&+2\,(p_2p_3)\,(p_3D\epsilon_3)_i\epsilon_{2ab}I_6' + (p_1Np_2)\,(p_3D\epsilon_3)_i\epsilon_{2ab}I_4'\\
&+2\,(p_2Dp_3)\,(p_3D\epsilon_3)_i\epsilon_{2ab}I_7' - 4\,(p_2p_3)\,(p_3\epsilon_2)_b\epsilon_{3ai}I_2\\
&-2\,(p_1Np_2)\,(p_3\epsilon_2)_b\epsilon_{3ai}I_9 + 2\,(p_1Np_3)\,(p_3\epsilon_2)_b\epsilon_{3ai}I_9\\
&-4\,(p_2p_3)\,(p_1N\epsilon_2)_b\epsilon_{3ai}I_9 + 4\,(p_1Np_3)\,(p_1N\epsilon_2)_b\epsilon_{3ai}I_{10}\\
&-4\,(p_2Dp_3)\,(p_1N\epsilon_2)_b\epsilon_{3ai}I_5 - 4\,(p_2p_3)\,(p_2D\epsilon_2)_b\epsilon_{3ai}I_6
\end{aligned}
$$

$$
\begin{aligned}
&-2\left(p_1Np_3\right)\left(p_2D\epsilon_2\right)_b\epsilon_{3ai}I_4-4\left(p_2Dp_3\right)\left(p_2D\epsilon_2\right)_b\epsilon_{3ai}I_7\\
&-2\left(p_1Np_2\right)\left(p_3D\epsilon_2\right)_b\epsilon_{3ai}I_5-2\left(p_1Np_3\right)\left(p_3D\epsilon_2\right)_b\epsilon_{3ai}I_5\\
&+4\left(p_2Dp_3\right)\left(p_3D\epsilon_2\right)_b\epsilon_{3ai}I_1-2\left(p_1N\epsilon_2p_3\right)p_{2_a}\epsilon_{3bi}I_9\\
&-2\left(p_2D\epsilon_2p_3\right)p_{2_a}\epsilon_{3bi}I_6-2\left(p_3D\epsilon_2p_3\right)p_{2_a}\epsilon_{3bi}I'_{11}\\
&-2\left(p_1N\epsilon_2Dp_3\right)p_{2_a}\epsilon_{3bi}I_5-2\left(p_2D\epsilon_2Dp_3\right)p_{2_a}\epsilon_{3bi}I_7\\
&+2\left(p_1N\epsilon_2p_3\right)p_{3_a}\epsilon_{3bi}I_9+2\left(p_2D\epsilon_2p_3\right)p_{3_a}\epsilon_{3bi}I_6\\
&+2\left(p_3D\epsilon_2p_3\right)p_{3_a}\epsilon_{3bi}I'_{11}+2\left(p_1N\epsilon_2Dp_3\right)p_{3_a}\epsilon_{3bi}I_5\\
&+2\left(p_2D\epsilon_2Dp_3\right)p_{3_a}\epsilon_{3bi}I_7) \qquad (2.86)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}^{T^{(1)}}_{C^{(p-1)}Bh}=&\frac{i}{8\sqrt{2}}T^a(-\left(p_1N\epsilon_2\epsilon_3p_2\right)p_{2_a}I_2-\left(p_1N\epsilon_3\epsilon_2p_3\right)p_{2_a}I_2\\
&+\left(p_2D\epsilon_2\epsilon_3p_2\right)p_{2_a}I_{20}+\left(p_2D\epsilon_3\epsilon_2p_3\right)p_{2_a}I_{14}\\
&-\left(p_2\epsilon_3\epsilon_2Dp_3\right)p_{2_a}I_{14}+\left(p_3D\epsilon_3\epsilon_2p_3\right)p_{2_a}I'_{20}\\
&-\left(p_1N\epsilon_2D\epsilon_3p_2\right)p_{2_a}I_{11}-\left(p_1N\epsilon_2\epsilon_3Dp_2\right)p_{2_a}I_{11}\\
&-2\left(p_1N\epsilon_2\epsilon_3Dp_3\right)p_{2_a}I'_6+2\left(p_1N\epsilon_2\epsilon_3Np_1\right)p_{2_a}I_9\\
&+\left(p_1N\epsilon_3D\epsilon_2p_3\right)p_{2_a}I'_{11}-2\left(p_1N\epsilon_3\epsilon_2Dp_2\right)p_{2_a}I_6\\
&-\left(p_1N\epsilon_3\epsilon_2Dp_3\right)p_{2_a}I'_{11}-\left(p_2D\epsilon_2D\epsilon_3p_2\right)p_{2_a}I'_{22}\\
&-\left(p_2D\epsilon_2\epsilon_3Dp_2\right)p_{2_a}I'_{22}+2\left(p_2D\epsilon_2\epsilon_3Dp_3\right)p_{2_a}I_{18}\\
&+\left(p_2D\epsilon_3D\epsilon_2p_3\right)p_{2_a}I_{15}-\left(p_2\epsilon_3D\epsilon_2Dp_3\right)p_{2_a}I_{15}\\
&+\left(p_3D\epsilon_2\epsilon_3Dp_3\right)p_{2_a}I_{22}+\left(p_3D\epsilon_3D\epsilon_2p_3\right)p_{2_a}I_{22}\\
&-\left(p_1N\epsilon_2D\epsilon_3Dp_2\right)p_{2_a}I_1+2\left(p_1N\epsilon_2D\epsilon_3Dp_3\right)p_{2_a}I'_7\\
&+2\left(p_1N\epsilon_2D\epsilon_3Np_1\right)p_{2_a}I_5-2\left(p_1N\epsilon_3D\epsilon_2Dp_2\right)p_{2_a}I_7\\
&+\left(p_1N\epsilon_3D\epsilon_2Dp_3\right)p_{2_a}I_1+\left(p_2D\epsilon_2D\epsilon_3Dp_2\right)p_{2_a}I_{21}
\end{aligned}
$$

$$+\,2\,(p_2 D\epsilon_2 D\epsilon_3 D p_3)\,p_{2_a} I_{19} + (p_3 D\epsilon_2 D\epsilon_3 D p_3)\,p_{2_a} I'_{21}$$

$$+\,\mathrm{tr}(D\epsilon_3)\,(p_1 N\epsilon_2 p_3)\,p_{2_a} I'_6 - \mathrm{tr}(D\epsilon_3)\,(p_2 D\epsilon_2 p_3)\,p_{2_a} I_{18}$$

$$-\,\mathrm{tr}(D\epsilon_3)\,(p_3 D\epsilon_2 p_3)\,p_{2_a} I_{22} - \mathrm{tr}(D\epsilon_3)\,(p_1 N\epsilon_2 D p_3)\,p_{2_a} I'_7$$

$$-\,\mathrm{tr}(D\epsilon_3)\,(p_2 D\epsilon_2 D p_3)\,p_{2_a} I_{19} + (p_1 N\epsilon_2\epsilon_3 p_2)\,p_{3_a} I_2$$

$$+\,(p_1 N\epsilon_3\epsilon_2 p_3)\,p_{3_a} I_2 - (p_2 D\epsilon_2\epsilon_3 p_2)\,p_{3_a} I_{20}$$

$$-\,(p_2 D\epsilon_3\epsilon_2 p_3)\,p_{3_a} I_{14} + (p_2\epsilon_3\epsilon_2 D p_3)\,p_{3_a} I_{14}$$

$$-\,(p_3 D\epsilon_3\epsilon_2 p_3)\,p_{3_a} I'_{20} + (p_1 N\epsilon_2 D\epsilon_3 p_2)\,p_{3_a} I_{11}$$

$$+\,(p_1 N\epsilon_2\epsilon_3 D p_2)\,p_{3_a} I_{11} + 2\,(p_1 N\epsilon_2\epsilon_3 D p_3)\,p_{3_a} I'_6$$

$$-\,2\,(p_1 N\epsilon_2\epsilon_3 N p_1)\,p_{3_a} I_9 - (p_1 N\epsilon_3 D\epsilon_2 p_3)\,p_{3_a} I'_{11}$$

$$+\,2\,(p_1 N\epsilon_3\epsilon_2 D p_2)\,p_{3_a} I_6 + (p_1 N\epsilon_3\epsilon_2 D p_3)\,p_{3_a} I'_{11}$$

$$+\,(p_2 D\epsilon_2 D\epsilon_3 p_2)\,p_{3_a} I'_{22} + (p_2 D\epsilon_2\epsilon_3 D p_2)\,p_{3_a} I'_{22}$$

$$-\,2\,(p_2 D\epsilon_2\epsilon_3 D p_3)\,p_{3_a} I_{18} - (p_2 D\epsilon_3 D\epsilon_2 p_3)\,p_{3_a} I_{15}$$

$$+\,(p_2\epsilon_3 D\epsilon_2 D p_3)\,p_{3_a} I_{15} - (p_3 D\epsilon_2\epsilon_3 D p_3)\,p_{3_a} I_{22}$$

$$-\,(p_3 D\epsilon_3 D\epsilon_2 p_3)\,p_{3_a} I_{22} + (p_1 N\epsilon_2 D\epsilon_3 D p_2)\,p_{3_a} I_1$$

$$-\,2\,(p_1 N\epsilon_2 D\epsilon_3 D p_3)\,p_{3_a} I'_7 - 2\,(p_1 N\epsilon_2 D\epsilon_3 N p_1)\,p_{3_a} I_5$$

$$+\,2\,(p_1 N\epsilon_3 D\epsilon_2 D p_2)\,p_{3_a} I_7 - (p_1 N\epsilon_3 D\epsilon_2 D p_3)\,p_{3_a} I_1$$

$$-\,(p_2 D\epsilon_2 D\epsilon_3 D p_2)\,p_{3_a} I_{21} - 2\,(p_2 D\epsilon_2 D\epsilon_3 D p_3)\,p_{3_a} I_{19}$$

$$-\,(p_3 D\epsilon_2 D\epsilon_3 D p_3)\,p_{3_a} I'_{21} - \mathrm{tr}(D\epsilon_3)\,(p_1 N\epsilon_2 p_3)\,p_{3_a} I'_6$$

$$+\,\mathrm{tr}(D\epsilon_3)\,(p_2 D\epsilon_2 p_3)\,p_{3_a} I_{18} + \mathrm{tr}(D\epsilon_3)\,(p_3 D\epsilon_2 p_3)\,p_{3_a} I_{22}$$

$$+\,\mathrm{tr}(D\epsilon_3)\,(p_1 N\epsilon_2 D p_3)\,p_{3_a} I'_7 + \mathrm{tr}(D\epsilon_3)\,(p_2 D\epsilon_2 D p_3)\,p_{3_a} I_{19}$$

$$-\,2\,(p_1 N\epsilon_2 p_3)\,(p_2\epsilon_3)_a I_2 + 2\,(p_2 D\epsilon_2 p_3)\,(p_2\epsilon_3)_a I_{20}$$

$$+\,2\,(p_3 D\epsilon_2 p_3)\,(p_2\epsilon_3)_a I_{14} - 2\,(p_1 N\epsilon_2 D p_3)\,(p_2\epsilon_3)_a I_{25}$$

$$- 2\,(p_2 D\epsilon_2 D p_3)\,(p_2\epsilon_3)_a I'_{22} - 2\,(p_1 N\epsilon_3 p_2)\,(p_3\epsilon_2)_a I_2$$

$$+ 2\,(p_2 D\epsilon_3 p_2)\,(p_3\epsilon_2)_a I_{14} + 2\,(p_2\epsilon_3 D p_3)\,(p_3\epsilon_2)_a I'_{20}$$

$$+ 4\,(p_1 N\epsilon_3 D p_2)\,(p_3\epsilon_2)_a I'_{26} - 2\,(p_1 N\epsilon_3 D p_3)\,(p_3\epsilon_2)_a I'_6$$

$$+ 2\,(p_1 N\epsilon_3 N p_1)\,(p_3\epsilon_2)_a I_9 + 2\,(p_2 D\epsilon_3 D p_2)\,(p_3\epsilon_2)_a I_{15}$$

$$+ 2\,(p_2 D\epsilon_3 D p_3)\,(p_3\epsilon_2)_a I_{22} - 2\,\mathrm{tr}(D\epsilon_3)\,(p_2 p_3)\,(p_3\epsilon_2)_a I'_{20}$$

$$- \mathrm{tr}(D\epsilon_3)\,(p_1 N p_2)\,(p_3\epsilon_2)_a I'_6 + \mathrm{tr}(D\epsilon_3)\,(p_1 N p_3)\,(p_3\epsilon_2)_a I'_6$$

$$+ 2\,(p_2\epsilon_3 p_2)\,(p_1 N\epsilon_2)_a I_2 - 6\,(p_1 N\epsilon_3 p_2)\,(p_1 N\epsilon_2)_a I_9$$

$$+ 4\,(p_2 D\epsilon_3 p_2)\,(p_1 N\epsilon_2)_a I_{11} + 4\,(p_2\epsilon_3 D p_3)\,(p_1 N\epsilon_2)_a I'_6$$

$$- 6\,(p_1 N\epsilon_3 D p_2)\,(p_1 N\epsilon_2)_a I_5 - 2\,(p_1 N\epsilon_3 D p_3)\,(p_1 N\epsilon_2)_a I'_4$$

$$+ 4\,(p_1 N\epsilon_3 N p_1)\,(p_1 N\epsilon_2)_a I_{10} + 2\,(p_2 D\epsilon_3 D p_2)\,(p_1 N\epsilon_2)_a I_1$$

$$- 4\,(p_2 D\epsilon_3 D p_3)\,(p_1 N\epsilon_2)_a I'_7 - 2\,\mathrm{tr}(D\epsilon_3)\,(p_2 p_3)\,(p_1 N\epsilon_2)_a I'_6$$

$$+ \mathrm{tr}(D\epsilon_3)\,(p_1 N p_3)\,(p_1 N\epsilon_2)_a I'_4 + 2\,\mathrm{tr}(D\epsilon_3)\,(p_2 D p_3)\,(p_1 N\epsilon_2)_a I'_7$$

$$+ 2\,(p_1 N\epsilon_2 p_3)\,(p_1 N\epsilon_3)_a I_9 + 2\,(p_2 D\epsilon_2 p_3)\,(p_1 N\epsilon_3)_a I_6$$

$$+ 2\,(p_3 D\epsilon_2 p_3)\,(p_1 N\epsilon_3)_a I'_{11} + 2\,(p_1 N\epsilon_2 D p_3)\,(p_1 N\epsilon_3)_a I_5$$

$$+ 2\,(p_2 D\epsilon_2 D p_3)\,(p_1 N\epsilon_3)_a I_7 - 2\,(p_2\epsilon_3 p_2)\,(p_2 D\epsilon_2)_a I_{20}$$

$$- 6\,(p_1 N\epsilon_3 p_2)\,(p_2 D\epsilon_2)_a I_6 + 4\,(p_2 D\epsilon_3 p_2)\,(p_2 D\epsilon_2)_a I'_{22}$$

$$- 4\,(p_2\epsilon_3 D p_3)\,(p_2 D\epsilon_2)_a I_{18} - 6\,(p_1 N\epsilon_3 D p_2)\,(p_2 D\epsilon_2)_a I_7$$

$$+ 4\,(p_1 N\epsilon_3 D p_3)\,(p_2 D\epsilon_2)_a I_3 - 2\,(p_1 N\epsilon_3 N p_1)\,(p_2 D\epsilon_2)_a I_4$$

$$- 2\,(p_2 D\epsilon_3 D p_2)\,(p_2 D\epsilon_2)_a I_{21} - 4\,(p_2 D\epsilon_3 D p_3)\,(p_2 D\epsilon_2)_a I_{19}$$

$$+ 2\,\mathrm{tr}(D\epsilon_3)\,(p_2 p_3)\,(p_2 D\epsilon_2)_a I_{18} - 2\,\mathrm{tr}(D\epsilon_3)\,(p_1 N p_3)\,(p_2 D\epsilon_2)_a I_3$$

$$+ 2\,\mathrm{tr}(D\epsilon_3)\,(p_2 D p_3)\,(p_2 D\epsilon_2)_a I_{19} - 2\,(p_1 N\epsilon_2 p_3)\,(p_2 D\epsilon_3)_a I_{25}$$

$$- 2\,(p_2 D\epsilon_2 p_3)\,(p_2 D\epsilon_3)_a I'_{22} + 2\,(p_3 D\epsilon_2 p_3)\,(p_2 D\epsilon_3)_a I_{15}$$

$$- 2\,(p_1N\epsilon_2Dp_3)\,(p_2D\epsilon_3)_a I_1 + 2\,(p_2D\epsilon_2Dp_3)\,(p_2D\epsilon_3)_a I_{21}$$

$$- 4(p_2\epsilon_3\epsilon_2)_a I_{23} + 2\,(p_2p_3)\,(p_2\epsilon_3\epsilon_2)_a I_{16}$$

$$+ (p_1Np_2)\,(p_2\epsilon_3\epsilon_2)_a I_2 - (p_1Np_3)\,(p_2\epsilon_3\epsilon_2)_a I_2$$

$$- 2\,(p_2\epsilon_3p_2)\,(p_3D\epsilon_2)_a I_{14} - 4\,(p_1N\epsilon_3p_2)\,(p_3D\epsilon_2)_a I'_{26}$$

$$- 2\,(p_2D\epsilon_3p_2)\,(p_3D\epsilon_2)_a I_{15} - 2\,(p_2\epsilon_3Dp_3)\,(p_3D\epsilon_2)_a I_{22}$$

$$+ 2\,(p_1N\epsilon_3Dp_2)\,(p_3D\epsilon_2)_a I_1 - 2\,(p_1N\epsilon_3Dp_3)\,(p_3D\epsilon_2)_a I'_7$$

$$- 2\,(p_1N\epsilon_3Np_1)\,(p_3D\epsilon_2)_a I_5 - 2\,(p_2D\epsilon_3Dp_3)\,(p_3D\epsilon_2)_a I'_{21}$$

$$+ \operatorname{tr}(D\epsilon_3)\,(p_1Np_2)\,(p_3D\epsilon_2)_a I'_7 + \operatorname{tr}(D\epsilon_3)\,(p_1Np_3)\,(p_3D\epsilon_2)_a I'_7$$

$$+ 2\operatorname{tr}(D\epsilon_3)\,(p_2Dp_3)\,(p_3D\epsilon_2)_a I'_{21} - 4(p_3\epsilon_2\epsilon_3)_a I_{23}$$

$$+ 2\,(p_2p_3)\,(p_3\epsilon_2\epsilon_3)_a I_{16} + (p_1Np_2)\,(p_3\epsilon_2\epsilon_3)_a I_2$$

$$- (p_1Np_3)\,(p_3\epsilon_2\epsilon_3)_a I_2 + 2\,(p_2p_3)\,(p_1N\epsilon_2\epsilon_3)_a I_2$$

$$- 2\,(p_1Np_3)\,(p_1N\epsilon_2\epsilon_3)_a I_9 + 2\,(p_2Dp_3)\,(p_1N\epsilon_2\epsilon_3)_a I_{25}$$

$$- 2\,(p_2p_3)\,(p_1N\epsilon_3\epsilon_2)_a I_2 - 2\,(p_1Np_2)\,(p_1N\epsilon_3\epsilon_2)_a I_9$$

$$- 2\,(p_2Dp_3)\,(p_1N\epsilon_3\epsilon_2)_a I'_{25} - 2\,(p_2p_3)\,(p_2D\epsilon_2\epsilon_3)_a I_{20}$$

$$- 2\,(p_1Np_3)\,(p_2D\epsilon_2\epsilon_3)_a I_6 + 2\,(p_2Dp_3)\,(p_2D\epsilon_2\epsilon_3)_a I'_{22}$$

$$+ (p_1Np_2)\,(p_2D\epsilon_3\epsilon_2)_a I_{11} + (p_1Np_3)\,(p_2D\epsilon_3\epsilon_2)_a I_{11}$$

$$- 2\,(p_2Dp_3)\,(p_2D\epsilon_3\epsilon_2)_a I_{15} + 2\,(p_2p_3)\,(p_2\epsilon_3D\epsilon_2)_a I_{14}$$

$$+ (p_1Np_2)\,(p_2\epsilon_3D\epsilon_2)_a I_{11} - (p_1Np_3)\,(p_2\epsilon_3D\epsilon_2)_a I_{11}$$

$$- (p_1Np_2)\,(p_3D\epsilon_2\epsilon_3)_a I'_{11} - (p_1Np_3)\,(p_3D\epsilon_2\epsilon_3)_a I'_{11}$$

$$- 2\,(p_2Dp_3)\,(p_3D\epsilon_2\epsilon_3)_a I_{15} + 2\,(p_2p_3)\,(p_3D\epsilon_3\epsilon_2)_a I'_{20}$$

$$+ 2\,(p_1Np_2)\,(p_3D\epsilon_3\epsilon_2)_a I'_6 - 2\,(p_2Dp_3)\,(p_3D\epsilon_3\epsilon_2)_a I_{22}$$

$$+ 2\,(p_2p_3)\,(p_3\epsilon_2D\epsilon_3)_a I_{14} - (p_1Np_2)\,(p_3\epsilon_2D\epsilon_3)_a I'_{11}$$

$$+ (p_1Np_3)\,(p_3\epsilon_2D\epsilon_3)_a I'_{11} + 2\,(p_2p_3)\,(p_1N\epsilon_2D\epsilon_3)_a I_{25}$$
$$- 2\,(p_1Np_3)\,(p_1N\epsilon_2D\epsilon_3)_a I_5 + 2\,(p_2Dp_3)\,(p_1N\epsilon_2D\epsilon_3)_a I_1$$
$$+ 2\,(p_2p_3)\,(p_1N\epsilon_3D\epsilon_2)_a I'_{25} - 2\,(p_1Np_2)\,(p_1N\epsilon_3D\epsilon_2)_a I_5$$
$$+ 2\,(p_2Dp_3)\,(p_1N\epsilon_3D\epsilon_2)_a I_1 + 2\,(p_2p_3)\,(p_2D\epsilon_2D\epsilon_3)_a I'_{22}$$
$$- 2\,(p_1Np_3)\,(p_2D\epsilon_2D\epsilon_3)_a I_7 - 2\,(p_2Dp_3)\,(p_2D\epsilon_2D\epsilon_3)_a I_{21}$$
$$+ 4(p_2D\epsilon_3D\epsilon_2)_a I_{24} + (p_1Np_2)\,(p_2D\epsilon_3D\epsilon_2)_a I_1$$
$$+ (p_1Np_3)\,(p_2D\epsilon_3D\epsilon_2)_a I_1 - 2\,(p_2Dp_3)\,(p_2D\epsilon_3D\epsilon_2)_a I_{17}$$
$$+ 4(p_3D\epsilon_2D\epsilon_3)_a I_{24} + (p_1Np_2)\,(p_3D\epsilon_2D\epsilon_3)_a I_1$$
$$+ (p_1Np_3)\,(p_3D\epsilon_2D\epsilon_3)_a I_1 - 2\,(p_2Dp_3)\,(p_3D\epsilon_2D\epsilon_3)_a I_{17}$$
$$+ 2\,(p_2p_3)\,(p_3D\epsilon_3D\epsilon_2)_a I_{22} - 2\,(p_1Np_2)\,(p_3D\epsilon_3D\epsilon_2)_a I'_7$$
$$- 2\,(p_2Dp_3)\,(p_3D\epsilon_3D\epsilon_2)_a I'_{21}). \tag{2.87}$$

$$\mathcal{A}_{C^{(p-3)}BB} = \mathcal{A}^{T^{(7)}}_{C^{(p-3)}BB} + \mathcal{A}^{T^{(5)}}_{C^{(p-3)}BB} + \mathcal{A}^{T^{(3)}}_{C^{(p-3)}BB}. \tag{2.88}$$

$$\mathcal{A}^{T^{(7)}}_{C^{(p-3)}BB} = \frac{i}{8\sqrt{2}} T^{abcdeij}(-p_{2a}p_{2j}p_{3i}\epsilon_{2de}\epsilon_{3bc})I_{10} + (2 \leftrightarrow 3), \tag{2.89}$$

$$\mathcal{A}^{T^{(5)}}_{C^{(p-3)}BB} = \frac{i}{8\sqrt{2}} T^{abcdi}(p_{2i}p_{3a}(p_2\epsilon_3)_d\epsilon_{2bc}I_9 - p_{2a}p_{3i}(p_2\epsilon_3)_d\epsilon_{2bc}I_9$$
$$- 2p_{2i}p_{3a}(p_1N\epsilon_3)_d\epsilon_{2bc}I_{10} + p_{2i}p_{3a}(p_2D\epsilon_3)_d\epsilon_{2bc}I_5$$
$$+ p_{2a}p_{3i}(p_2D\epsilon_3)_d\epsilon_{2bc}I_5 - p_{3a}p_{3i}(p_2D\epsilon_3)_d\epsilon_{2bc}I_5$$
$$+ p_{2i}p_{3a}(p_3D\epsilon_3)_d\epsilon_{2bc}I'_4 + p_{2b}p_{2i}(p_2\epsilon_3)_a\epsilon_{2cd}I_9$$
$$- p_{3b}p_{3i}(p_2\epsilon_3)_a\epsilon_{2cd}I_9 - 2p_{2b}p_{2i}(p_1N\epsilon_3)_a\epsilon_{2cd}I_{10}$$

$$
\begin{aligned}
&+ p_{2b}p_{2i}(p_2D\epsilon_3)_a\epsilon_{2cd}I_5 + p_{2b}p_{2i}(p_3D\epsilon_3)_a\epsilon_{2cd}I'_4 \\
&+ (p_2p_3)\, p_{2i}\epsilon_{2cd}\epsilon_{3ab}I_9 - (p_1Np_3)\, p_{2i}\epsilon_{2cd}\epsilon_{3ab}I_{10} \\
&+ (p_2Dp_3)\, p_{2i}\epsilon_{2cd}\epsilon_{3ab}I_5) + (2 \leftrightarrow 3),
\end{aligned} \tag{2.90}
$$

$$
\begin{aligned}
\mathcal{A}^{T^{(7)}}_{C^{(p-3)}BB} =& \frac{i}{8\sqrt{2}} T^{abc}(4\,(p_2Dp_3)\, p_{3a}(\epsilon_2\epsilon_3)_{bc}I_0 - 2p_{3b}(p_1N\epsilon_2)_a(p_2\epsilon_3)_cI_9 \\
&- 2p_{2b}(p_1N\epsilon_2)_c(p_2\epsilon_3)_aI_9 + 4p_{2b}(p_1N\epsilon_2)_c(p_1N\epsilon_3)_aI_{10} \\
&- 2p_{3b}(p_2D\epsilon_2)_a(p_2\epsilon_3)_cI_6 - 2p_{3b}(p_1N\epsilon_3)_c(p_2D\epsilon_2)_aI_4 \\
&- 2p_{2b}(p_2D\epsilon_2)_c(p_2\epsilon_3)_aI_6 - 2p_{2b}(p_1N\epsilon_3)_a(p_2D\epsilon_2)_cI_4 \\
&- 2p_{2b}(p_2D\epsilon_3)_a(p_3\epsilon_2)_cI_8 - 2p_{2b}(p_1N\epsilon_2)_c(p_2D\epsilon_3)_aI_5 \\
&- 2p_{2b}(p_2D\epsilon_2)_c(p_2D\epsilon_3)_aI_7 - 2p_{3b}(p_2D\epsilon_3)_c(p_3\epsilon_2)_aI_8 \\
&- 2p_{3b}(p_1N\epsilon_2)_a(p_2D\epsilon_3)_cI_5 - 2p_{3b}(p_2D\epsilon_2)_a(p_2D\epsilon_3)_cI_7 \\
&- p_{2a}p_{3b}(p_2\epsilon_3\epsilon_2)_cI_2 + 4p_{2b}(p_2D\epsilon_2)_c(p_3D\epsilon_3)_aI_3 \\
&+ 4\,(p_2p_3)\, p_{2a}(\epsilon_2D\epsilon_3)_{bc}I_0 + p_{2a}p_{3b}(p_1N\epsilon_2\epsilon_3)_cI_9 \\
&+ p_{2a}p_{3b}(p_3D\epsilon_2\epsilon_3)_cI'_{11} + p_{2a}p_{3b}(p_3\epsilon_2D\epsilon_3)_cI'_{11} \\
&- p_{2a}p_{3b}(p_1N\epsilon_2D\epsilon_3)_cI_5 - p_{2a}p_{3b}(p_2D\epsilon_3D\epsilon_2)_cI_1 \\
&- 2\,(p_2p_3)\,(p_2\epsilon_3)_c\epsilon_{2ab}I_2 - (p_1Np_2)\,(p_2\epsilon_3)_c\epsilon_{2ab}I_9 \\
&+ (p_1Np_3)\,(p_2\epsilon_3)_c\epsilon_{2ab}I_9 - (p_1Np_2)\,(p_2D\epsilon_3)_c\epsilon_{2ab}I_5 \\
&- (p_1Np_3)\,(p_2D\epsilon_3)_c\epsilon_{2ab}I_5 + 2\,(p_2Dp_3)\,(p_2D\epsilon_3)_c\epsilon_{2ab}I_1 \\
&+ (p_1N\epsilon_3p_2)\, p_{2a}\epsilon_{2bc}I_9 + (p_2\epsilon_3Dp_3)\, p_{2a}\epsilon_{2bc}I'_6 \\
&- (p_1N\epsilon_3Dp_2)\, p_{2a}\epsilon_{2bc}I_5 + (p_2D\epsilon_3Dp_3)\, p_{2a}\epsilon_{2bc}I'_7 \\
&- (p_1N\epsilon_3p_2)\, p_{3a}\epsilon_{2bc}I_9 - (p_2\epsilon_3Dp_3)\, p_{3a}\epsilon_{2bc}I'_6 \\
&+ (p_1N\epsilon_3Dp_2)\, p_{3a}\epsilon_{2bc}I_5 - (p_2D\epsilon_3Dp_3)\, p_{3a}\epsilon_{2bc}I'_7
\end{aligned}
$$

$$+\,2\,(p_2p_3)\,(p_1N\epsilon_3)_a\epsilon_{2bc}I_9+2\,(p_1Np_2)\,(p_1N\epsilon_3)_a\epsilon_{2bc}I_{10}$$
$$-\,2\,(p_2Dp_3)\,(p_1N\epsilon_3)_a\epsilon_{2bc}I_5-2\,(p_2p_3)\,(p_3D\epsilon_3)_a\epsilon_{2bc}I_6'$$
$$-\,(p_1Np_2)\,(p_3D\epsilon_3)_a\epsilon_{2bc}I_4'-2\,(p_2Dp_3)\,(p_3D\epsilon_3)_a\epsilon_{2bc}I_7'$$
$$+\,(p_3D\epsilon_2p_3)\,p_{2_a}\epsilon_{3bc}I_{11}'-(p_3D\epsilon_2p_3)\,p_{3_a}\epsilon_{3bc}I_{11}')+(2\leftrightarrow 3). \tag{2.91}$$

$$\begin{aligned}\mathcal{A}^{T^{(7)}}_{C^{(p-3)}hh}=&\frac{i}{16\sqrt{2}}T^{abc}(-2\,\mathrm{tr}(D\epsilon_2)p_{2c}p_{3a}(p_2\epsilon_3)_bI_6-8\,(p_2Dp_3)\,p_{3a}(\epsilon_2\epsilon_3)_{bc}I_0\\
&-\mathrm{tr}(D\epsilon_3)p_{2a}p_{3c}(p_1N\epsilon_2)_bI_4'-8p_{2b}(p_2D\epsilon_3)_a(p_3\epsilon_2)_cI_0\\
&-2\,\mathrm{tr}(D\epsilon_2)p_{2c}p_{3a}(p_2D\epsilon_3)_bI_7-8p_{3b}(p_2D\epsilon_3)_c(p_3\epsilon_2)_aI_0\\
&+2p_{2_a}p_{3_b}(p_2\epsilon_3\epsilon_2)_cI_2+8\,(p_2p_3)\,p_{2_a}(\epsilon_2D\epsilon_3)_{bc}I_0\\
&-2p_{2_a}p_{3_b}(p_1N\epsilon_2\epsilon_3)_cI_9-2p_{2_a}p_{3_b}(p_3D\epsilon_2\epsilon_3)_cI_{11}'\\
&+2p_{2_a}p_{3_b}(p_3\epsilon_2D\epsilon_3)_cI_{11}'-2p_{2_a}p_{3_b}(p_1N\epsilon_2D\epsilon_3)_cI_5\\
&-2p_{2_a}p_{3_b}(p_2D\epsilon_3D\epsilon_2)_cI_1)\end{aligned} \tag{2.92}$$

2.5 Some Integrals

The integrals appearing in the amplitudes section are defined as

$$I_n=\int_{|z_i|\leq 1}d^2z_2d^2z_3A_n\mathcal{K} \tag{2.93}$$

where A_n are listed below.

$$A_0=\frac{(-\bar{z}_3z_2+\bar{z}_2z_3)^{\,2}}{2\,|z_2|^{\,2}\,|1-\bar{z}_3z_2|^{\,2}\,|z_2-z_3|^{\,2}\,|z_3|^{\,2}} \tag{2.94}$$

$$A_1 = \frac{|1 + \bar{z}_3 z_2|^2}{|z_2|^2 |1 - \bar{z}_3 z_2|^2 |z_3|^2} \tag{2.95}$$

$$A_2 = \frac{|z_2 + z_3|^2}{|z_2|^2 |z_2 - z_3|^2 |z_3|^2} \tag{2.96}$$

$$A_3 = \frac{(1 + |z_2|^2)(1 + |z_3|^2)}{|z_2|^2 (1 - |z_2|^2) |z_3|^2 (1 - |z_3|^2)} \tag{2.97}$$

$$A_4 = \frac{2(1 + |z_2|^2)}{|z_2|^2 (1 - |z_2|^2) |z_3|^2} \tag{2.98}$$

$$A_5 = \frac{1 - |z_2|^2 |z_3|^2}{|z_2|^2 |1 - \bar{z}_3 z_2|^2 |z_3|^2} \tag{2.99}$$

$$A_6 = \frac{(-1 - |z_2|^2)(|z_2|^2 - |z_3|^2)}{|z_2|^2 (1 - |z_2|^2) |z_2 - z_3|^2 |z_3|^2} \tag{2.100}$$

$$A_7 = \frac{(-1 - |z_2|^2)(1 - |z_2|^2 |z_3|^2)}{|z_2|^2 (1 - |z_2|^2) |1 - \bar{z}_3 z_2|^2 |z_3|^2} \tag{2.101}$$

$$A_8 = \frac{(|z_2|^2 - |z_3|^2)(1 - |z_2|^2 |z_3|^2)}{|z_2|^2 |1 - \bar{z}_3 z_2|^2 |z_2 - z_3|^2 |z_3|^2} \tag{2.102}$$

$$A_9 = \frac{|z_2|^2 - |z_3|^2}{|z_2|^2 |z_2 - z_3|^2 |z_3|^2} \tag{2.103}$$

$$A_{10} = \frac{1}{|z_2|^2 |z_3|^2} \tag{2.104}$$

$$\begin{aligned} A_{11} &= \frac{(|z_2|^2 - |z_3|^2)(1 - |z_2|^2 |z_3|^2)}{|z_2|^2 |1 - \bar{z}_3 z_2|^2 |z_2 - z_3|^2 |z_3|^2} - \frac{(-\bar{z}_3 z_2 + \bar{z}_2 z_3)^2}{|z_2|^2 |1 - \bar{z}_3 z_2|^2 |z_2 - z_3|^2 |z_3|^2} \\ &= A_8 - 2A_0 \end{aligned} \tag{2.105}$$

$$\begin{aligned} A_{12} &= \frac{(|z_2|^2 - |z_3|^2)(1 - |z_2|^2 |z_3|^2)}{|z_2|^2 |1 - \bar{z}_3 z_2|^2 |z_2 - z_3|^2 |z_3|^2} - \frac{3(-\bar{z}_3 z_2 + \bar{z}_2 z_3)^2}{|z_2|^2 |1 - \bar{z}_3 z_2|^2 |z_2 - z_3|^2 |z_3|^2} \\ &= A_8 - 6A_0 \end{aligned} \tag{2.106}$$

$$\begin{aligned} A_{13} &= \frac{3(|z_2|^2 - |z_3|^2)(1 - |z_2|^2 |z_3|^2)}{|z_2|^2 |1 - \bar{z}_3 z_2|^2 |z_2 - z_3|^2 |z_3|^2} - \frac{(-\bar{z}_3 z_2 + \bar{z}_2 z_3)^2}{|z_2|^2 |1 - \bar{z}_3 z_2|^2 |z_2 - z_3|^2 |z_3|^2} \\ &= 3A_8 - 2A_0 \end{aligned} \tag{2.107}$$

$$A_{14} = \frac{(1 - |z_2|^2 |z_3|^2) |z_2 + z_3|^2}{|z_2|^2 |z_2 - z_3|^2 |z_3|^2 |-1 + \bar{z}_2 z_3|^2} \tag{2.108}$$

$$A_{15} = \frac{(|z_2|^2 - |z_3|^2) |1 + \bar{z}_2 z_3|^2}{|z_2|^2 |z_2 - z_3|^2 |z_3|^2 |-1 + \bar{z}_2 z_3|^2} \tag{2.109}$$

$$A_{16} = \frac{(|z_2|^2 - |z_3|^2) |z_2 + z_3|^2}{|z_2|^2 |z_2 - z_3|^4 |z_3|^2} \tag{2.110}$$

$$A_{17} = \frac{(1 - |z_2|^2 |z_3|^2) |1 + \bar{z}_2 z_3|^2}{|z_2|^2 |z_3|^2 |-1 + \bar{z}_2 z_3|^4} \tag{2.111}$$

$$A_{18} = \frac{(1+|z_2|^2)(|z_2|^2-|z_3|^2)(1+|z_3|^2)}{|z_2|^2(-1+|z_2|^2)|z_2-z_3|^2|z_3|^2(-1+|z_3|^2)} \tag{2.112}$$

$$A_{19} = \frac{(1+|z_2|^2)(1+|z_3|^2)(1-|z_2|^2|z_3|^2)}{|z_2|^2(-1+|z_2|^2)|z_3|^2(-1+|z_3|^2)|-1+\bar{z}_2 z_3|^2} \tag{2.113}$$

$$A_{20} = \frac{(1+|z_2|^2)|z_2+z_3|^2}{|z_2|^2(1-|z_2|^2)|z_2-z_3|^2|z_3|^2} \tag{2.114}$$

$$A_{21} = \frac{(1+|z_2|^2)|1+\bar{z}_2 z_3|^2}{|z_2|^2(1-|z_2|^2)|z_3|^2|-1+\bar{z}_2 z_3|^2} \tag{2.115}$$

$$A_{22} = \frac{(1+|z_3|^2)((|z_2|^2-|z_3|^2)(1-|z_2|^2|z_3|^2)+(\bar{z}_3 z_2-\bar{z}_2 z_3)^2)}{|z_2|^2|z_2-z_3|^2|z_3|^2(1-|z_3|^2)|-1+\bar{z}_2 z_3|^2} \tag{2.116}$$

$$A_{23} = \frac{(|z_2|^2-|z_3|^2)(\bar{z}_3 z_2+\bar{z}_2 z_3)}{|z_2|^2|z_2-z_3|^4|z_3|^2} \tag{2.117}$$

$$A_{24} = \frac{(1-|z_2|^2|z_3|^2)(\bar{z}_3 z_2+\bar{z}_2 z_3)}{|z_2|^2|z_3|^2|-1+\bar{z}_2 z_3|^4} \tag{2.118}$$

$$\begin{aligned} A_{25} &= \frac{(|z_2|^2-|z_3|^2)(1-|z_2|^2|z_3|^2)}{|z_2|^2|1-\bar{z}_3 z_2|^2|z_2-z_3|^2|z_3|^2} - \frac{2(-\bar{z}_3 z_2+\bar{z}_2 z_3)^2}{|z_2|^2|1-\bar{z}_3 z_2|^2|z_2-z_3|^2|z_3|^2} \\ &= A_8 - 4A_0 \end{aligned} \tag{2.119}$$

$$A_{26} = \frac{(|z_2|^2 - |z_3|^2)(1 - |z_2|^2 |z_3|^2)}{|z_2|^2 |1 - \bar{z}_3 z_2|^2 |z_2 - z_3|^2 |z_3|^2} - \frac{(-\bar{z}_3 z_2 + \bar{z}_2 z_3)^2}{2 |z_2|^2 |1 - \bar{z}_3 z_2|^2 |z_2 - z_3|^2 |z_3|^2}$$
$$= A_8 - A_0 \tag{2.120}$$

Each of these expressions could be expanded out in terms of the integral $I_{a,b,c,d,e,f}$ defined in section 2.1.2, but it is these combinations which appear naturally from the contractions.

Finally, when we put a prime on an integral, I'_n, we obtain it from I_n by exchanging z_2 with z_3 in A_n before performing the integration, so for instance

$$I'_4 = \int_{|z_i| \le 1} d^2 z_2 d^2 z_3 \frac{2(1 + |z_3|^2)}{|z_2|^2 (1 - |z_3|^2) |z_3|^2}. \tag{2.121}$$

3. HIGHER DERIVATIVE EFFECTIVE COUPLINGS

3.1 Effective Couplings from the String Scattering Amplitudes

There are different ways to derive supergravity effective couplings from string theory, such as by requiring the beta function of the worldsheet theory vanishes. However when the Ramond field is present, it is most convenient to derive the effective couplings by comparing the string scattering amplitudes with the field theory Feynman diagrams. Some computations on two point couplings can be find in the earlier literature [17, 18, 19, 20].

In this section, we construct the four-derivative $C^{(p+5)}BB$ effective coupling from the disc amplitude.

The basic principle is that the string scattering amplitudes contain all the on-shell field theory results. We need to evaluate all possible field theory Feynman diagrams from the known vertices, then isolate the new couplings appearing in the string amplitudes.

The detailed strategy is the following:

1. We start with the ten dimensional supergravity action and calculate all possible zero-order RR-NSNS Feynman diagrams using the D-brane vertices from the Wess-Zumino term.

2. We compute the RR-NSNS disc amplitudes and expand them up to four derivatives.

3. By comparing the results in step 1 and step 2, we can find the RR-NSNS (two points) four-derivative correction to the Wess-Zumino action.

4. Using the Wess-Zumino action plus the corrected two-point couplings from step 3, we calculate all possible RR-NSNS-NSNS Feynman diagrams up to four derivatives.

5. We calculate the RR-NSNS-NSNS string scattering amplitude, and expand it up to four derivatives.

6. By comparing the results in step 4 and step 5, we can isolate the RR-NSNS-NSNS (three points) four-derivative correction to the Wess-Zumino action.

In addition, we should keep in mind that the string amplitudes calculation is determined up to a overall constant. Thus we also need to fix this constant by comparing the zero momentum terms in field theory and string theory.

3.2 Review of Ten Dimensional Supergravity

We use same notations as in [15]. The ten dimensional supergravity action has the following form:

$$S_{10} = -\frac{1}{2}\int d^{10}x \sum_n \frac{1}{(n+1)!}\left((n+1)\partial_{[\mu_1} C^{(n)}_{\mu_2\cdots\mu_{n+1}]} + 2\kappa\frac{(n+1)!}{3!(n-2)!}H_{[\mu_1\mu_2\mu_3} C^{(n-2)}_{\mu_4\cdots\mu_{n+1}]}\right)^2 \tag{3.1}$$

where κ is gravitational constant and n is the rank of C form.

Isolating the $C^{(n)}C^{(n-2)}B$ coupling, we obtain

$$S_{10}^{\left(C^{(n)}C^{(n-2)}B\right)} = -\int d^{10}x\kappa\frac{(n+1)}{(n-2)!2!}\partial_{[\mu_1} C^{(n)}_{\mu_2\cdots\mu_{n+1}]}\left(\partial^{\mu_1}B^{\mu_2\mu_3}\right)C^{(n-2)\mu_4\cdots\mu_{n+1}}. \tag{3.2}$$

In momentum space, this becomes

$$S_{10}^{\left(C^{(n)}C^{(n-2)}B\right)} = -\kappa\frac{1}{(n-2)!3!}\int dp dp_1 dp_2 2\pi\delta\left(p+p_1+p_2\right)F_{\mu_1\cdots\mu_{n+1}}H^{\mu_1\cdots\mu_3}C^{(n-2)\mu_4\cdots\mu_{n+1}} \tag{3.3}$$

where $H = dB$, $F = dC$.

The part relevant for the propagator is

$$S_{10}^{\left(C^{(n)}C^{(n)}\right)} = \frac{1}{2}\int dp(2\pi)\frac{(n+1)^2}{(n+1)!}(p)_{[\mu_1}C^{(n)}_{\mu_2...\mu_{n+1}]}(p)^{[\mu_1}C^{(n)\,\mu_2...\mu_{n+1}]}. \tag{3.4}$$

The equation of motion (EOM) is[1]

$$\begin{aligned} 0 &= \frac{1}{n!}\frac{\delta}{\delta C^{(n)\boldsymbol{\nu}}}S_{10}^{\left(C^{(n)}C^{(n)}\right)} \\ &= (2\pi)\frac{(n+1)}{(n+1)!}\left((p)_{\nu_1}C^{(n)}_{\nu_2...\nu_{n+1}}(p)^{\nu_1} - n(p)_{\nu_2}C^{(n)}_{\nu_1...\nu_{n+1}}(p)^{\nu_1}\right) \\ &= (2\pi)\frac{(n+1)}{(n+1)!}p^2C^{(n)}_{\nu_2...\nu_{n+1}}, \end{aligned} \tag{3.5}$$

where $\boldsymbol{\nu} = (v_1...v_{n+1})$ and we made a gauge choice $p^\mu C_\mu... = 0$.

The two point function is defined as

$$\begin{aligned} \left\langle C^{(n)\boldsymbol{\mu}}(p_1)\,C^{(n)}{}_{\boldsymbol{\nu}}(p_2)\right\rangle &= i\frac{1}{n!}\frac{\delta}{\delta C^{(n)\boldsymbol{\nu}}}\frac{1}{n!}\frac{\delta}{\delta C^{(n)\boldsymbol{\mu}}}S_{10}^{\left(C^{(n)}C^{(n)}\right)} \\ &= i(2\pi)\delta\left(p_1+p_2\right)\frac{1}{n!}p^2\delta^{\boldsymbol{\mu}}_{\boldsymbol{\nu}}. \end{aligned} \tag{3.6}$$

The propagator $G_{\boldsymbol{\mu}}{}^{\boldsymbol{\nu}}$ is defined through

$$G_{\boldsymbol{\mu}}{}^{\boldsymbol{\nu}}\left\langle C^{(n)\boldsymbol{\mu}}(p_1)\,C^{(n)}{}_{\boldsymbol{\nu}}(p_2)\right\rangle = (2\pi)\delta\left(p_1+p_2\right), \tag{3.7}$$

which can be written in the simpler form

$$G_{\boldsymbol{\mu}}{}^{\boldsymbol{\nu}} = n!\frac{-i}{p^2}\delta^{\boldsymbol{\nu}}_{\boldsymbol{\mu}}. \tag{3.8}$$

[1]To make the expression simpler, we will use a bold letter to denote a sequence of indices in this work. The precise definition of the functional derivative using the bold indices is in the section 3.4.

At zero order, the fields on the brane are described by the Wess-Zumino action:

$$S_{p+1} = \sqrt{2}\mu_p\kappa \int_{W_{p+1}} Ce^{2\kappa B + \frac{1}{\sqrt{\mu_p}}F} \tag{3.9}$$

where $\int_{W_{p+1}}$ denotes the integration on the Dp-brane and F is the field strength for the $U(1)$ gauge field.

This action is understood in the following way:

1. The indices in C, B, F are space-time indices. We expand the $Ce^{\cdots}$ structure to get all differential forms that live in the bulk and could be rank higher than $p+1$.

2. We use the pullback map $X^\mu = X^\mu(x^a)$, where X^μ is the spacetime coordinate and x^a is the brane coordinate, to pull all the field to the brane, then perform the integration.

Note that any form with rank higher than $p+1$, will vanish on the brane after the pullback. Therefore when we expand the $Ce^{\cdots}$ structure, we should select forms of rank $p+1$ or lower.

We work out the two point coupling in the Wess-Zumino action. To simplify the calculation, we redefine the field's normalization

$$\begin{aligned}
\sqrt{2}\mu_p\kappa C &\to C, \\
2\kappa B &\to B, \\
\frac{1}{\sqrt{\mu_p}}F &\to F.
\end{aligned} \tag{3.10}$$

We will restore the dimensional coefficient in the final step.

In terms of the redefined field, the action is

$$
\begin{aligned}
S_{p+1} &= \int Ce^{B+F} \\
&= \int C^{(p+1)} + \int C^{(p-1)} \wedge B + \int C^{(p-1)} \wedge F + \frac{1}{2}\int C^{(p-3)} \wedge B \wedge B \\
&\quad + \int C^{(p-3)} \wedge B \wedge F + \frac{1}{2}\int C^{(p-3)} \wedge F \wedge F + ... \qquad (3.11)
\end{aligned}
$$

We denote the map $X^\mu(x) = \delta^\mu{}_a x^a + \delta^\mu{}_i X^i(x)$ as f.

Pulling back the C field to the brane, we see

$$
\begin{aligned}
C^{(p+1)} &= \frac{1}{(p+1)!} f_* C^{(p+1)}{}_{\mu_1...\mu_{p+1}}(X) dX^{\mu_1} \wedge ... \wedge dX^{\mu_{p+1}} \\
&= \frac{1}{(p+1)!} C^{(p+1)}{}_{\mu_1...\mu_{p+1}}(x) \frac{\partial X^{\mu_1}}{\partial x^{a_1}} ... \frac{\partial X^{\mu_{p+1}}}{\partial x^{a_{p+1}}} dx^{a_1} \wedge ... dx^{a_{p+1}}, \qquad (3.12)
\end{aligned}
$$

where f_* is the induced pullback map on the forms.

In first order, we expand partial derivatives as

$$
\frac{\partial X^\mu}{\partial x^a} = \delta^\mu{}_a + \delta^\mu{}_i \partial_a X^i(x) \qquad (3.13)
$$

and are left with the expression

$$
\begin{aligned}
(p+1)! C^{(p+1)}(X) =& C^{(p+1)}{}_{a_1...a_{p+1}}(X) dx^{a_1} \wedge ... dx^{a_{p+1}} \\
&+ (p+1) C^{(p+1)}{}_{i_1 a_2...a_{p+1}}(X) \partial_{a_1} X^i(x) dx^{a_1} \wedge ... dx^{a_{p+1}}. \qquad (3.14)
\end{aligned}
$$

Here $C(X)$ still contains fields at higher order. Up to two-field order, we have

$$
C^{(p+1)}{}_{a_1...a_{p+1}}(X) = C^{(p+1)}{}_{a_1...a_{p+1}}(x) + \partial_i C^{(p+1)}{}_{a_1...a_{p+1}}(x) X^i. \qquad (3.15)
$$

Therefore the first term in the amplitude (3.11) is

$$\int \left(\frac{1}{(p+1)!} C^{(p+1)}{}_{a_1 \ldots a_{p+1}} + \frac{1}{(p+1)!} \partial_i C^{(p+1)}{}_{a_1 \ldots a_{p+1}} X^i \right. \\ \left. + \frac{1}{p!} C^{(p+1)}{}_{i_1 a_2 \ldots a_{p+1}} \partial_{a_1} X^i \right) \epsilon^{a_1 \ldots, a_{p+1}} d^{p+1} x. \tag{3.16}$$

Up to second order in the fields, the 2nd and 3rd terms in (3.11) are

$$\int C^{(p-1)} \wedge B = \int \frac{1}{(p-1)!2!} C^{(p+1)}{}_{a_1 \ldots a_{p-1}} B_{a_p a_{p+1}} \epsilon^{a_1 \ldots, a_{p+1}} d^{p-1} x \\ \int C^{(p-1)} \wedge F = \int \frac{1}{(p-1)!} \epsilon^{a_1 \ldots, a_{p+1}} C^{(p-1)}{}_{a_1 \ldots a_{p-1}} \partial_{a_p} A_{a_{p+1}} d^{p+1} x. \tag{3.17}$$

The rest of the terms in (3.11) contain at least three fields and can be neglected. Putting all pieces together and restoring the dimensional coefficient; at two-field order the two-point couplings on the D-brane are

$$S = \int d^{p+1} x \epsilon^{a_1 \ldots, a_{p+1}} \left(\frac{\sqrt{2}\mu_p \kappa}{(p+1)!} C^{(p+1)}{}_{a_1 \ldots a_{p+1}} + \frac{\sqrt{2}\mu_p \kappa}{(p+1)!} \partial_i C^{(p+1)}{}_{a_1 \ldots a_{p+1}} X^i \right. \\ + \frac{\sqrt{2}\mu_p \kappa}{p!} C^{(p+1)}{}_{i_1 a_2 \ldots a_{p+1}} \partial_{a_1} X^i + \frac{2\sqrt{2}\mu_p \kappa^2}{(p-1)!2!} C^{(p-1)}{}_{a_1 \ldots a_{p-1}} B_{a_p a_{p+1}} d^{p+1} x \\ \left. + \frac{\sqrt{2}\sqrt{\mu_p}\kappa}{(p-1)!} C^{(p-1)}{}_{a_1 \ldots a_{p-1}} \partial_{a_p} A_{a_{p+1}} \right). \tag{3.18}$$

3.3 Construction of Four-Derivative Effective Couplings for $C^{p+5}BB$

We follow the strategy listed at the beginning of this section. The three point RR-NSNS-NSNS string amplitudes contain following the Feynman diagrams:

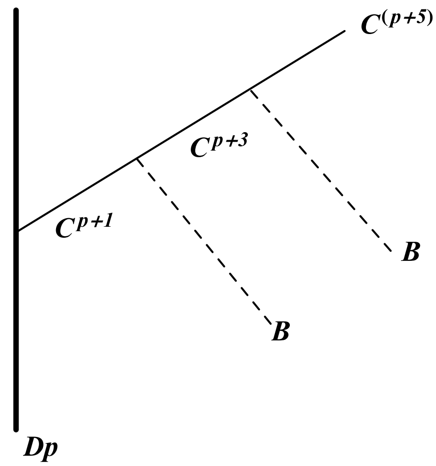

Figure 3.1: Zero-derivative diagram for $C^{(p+5)}BB$

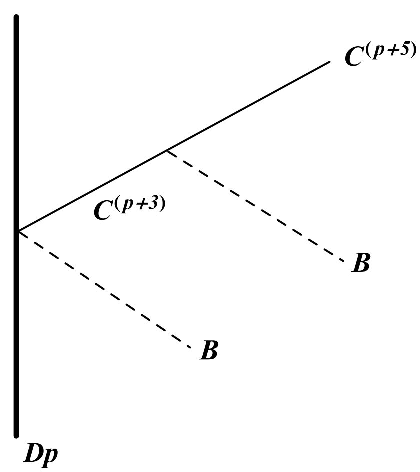

Figure 3.2: Four-derivative diagram for $C^{(p+5)}BB$, with four-derivative two point D-brane vertex

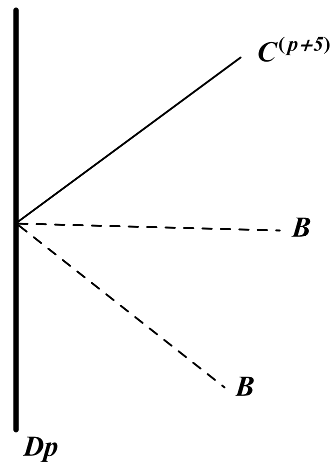

Figure 3.3: Four-derivative diagram for $C^{(p+5)}BB$ (the contact term)

In the above diagrams, the thick lines represent the D-brane; the dashed lines represent the B field; and solid lines represent the Ramond gauge field C. Figure 3.3 is the four-derivative correction to the Wess-Zumino term, which is what we want to calculate. In Figure 3.2, the D-brane vertex comes from the Wess-Zumino term, therefore this term is zero-order. In Figure 3.1, the D-brane vertex is unknown yet, because there is no $C^{(p+3)}B$ coupling at lowest order. However we can evaluate this vertex from the two point RR-NSNS string amplitude. It contains the following field theory diagrams:

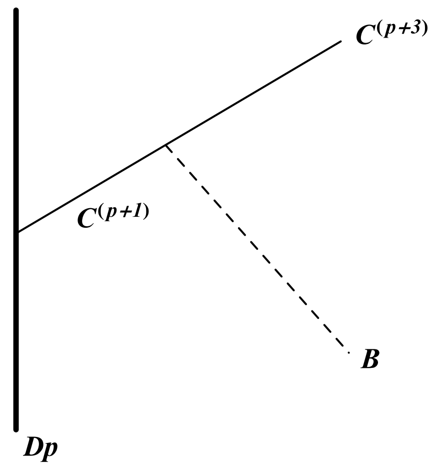

Figure 3.4: Zero-derivative order effective coupling for $C^{(p+3)}B$

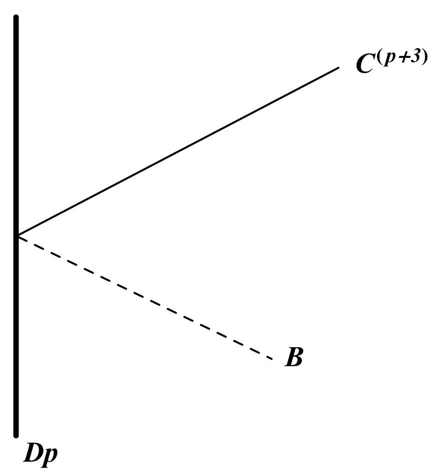

Figure 3.5: Four-derivative order effective coupling for $C^{(p+3)}B$

Figure 3.4 is the four-derivative correction to the two point coupling $C^{(p+3)}B$. This is the brane vertex in figure (3.2). In the following section, we will evaluate those diagrams.

3.3.1 $C^{p+3}B$ couplings in zero momentum

The amplitude corresponding to Feynman diagram (3.4) has the following form:

$$A^{(0)}_{C^{p+3}B} := C^{(p+3)}{}_{\boldsymbol{\mu}} \frac{1}{(p+3)!} \frac{\delta}{\delta C^{(p+3)}{}_{\boldsymbol{\mu}}} \frac{1}{(p+1)!} \frac{\delta}{\delta C^{(p+1)}{}_{\boldsymbol{b}}} B \frac{\delta}{\delta B} S_{10} G^{\left(C^{(p+1)}\right)}{}_{\boldsymbol{b},\boldsymbol{a}} \times \frac{1}{(p+1)!} \frac{\delta}{\delta C^{(p+1)}{}_{\boldsymbol{a}}} S_{p+1}, \tag{3.19}$$

where the bold indices are defined as

$$\begin{aligned} \boldsymbol{\mu} &= (\mu_1, ...\mu_{p+4}), \\ \boldsymbol{a} &= (a_1, ...a_{p+1}), \\ \boldsymbol{b} &= (b_1, ...b_{p+1}). \end{aligned} \tag{3.20}$$

Using $G^{\left(C^{(p+1)}\right)}{}_{\boldsymbol{a},\boldsymbol{b}} \epsilon^{\boldsymbol{b}} = \frac{-i}{p^2} \epsilon_{\boldsymbol{a}} (p+1)!$, one can check the brane action takes the form

$$G^{\left(C^{(p+1)}\right)}{}_{\boldsymbol{a},\boldsymbol{b}} \frac{1}{(p+1)!} \frac{\delta}{\delta C^{(p+1)}{}_{\boldsymbol{a}}} S_{p+1} = \sqrt{2} \mu_p \kappa \frac{-i}{p^2} \epsilon_{\boldsymbol{b}}, \tag{3.21}$$

while the bulk part is

$$\begin{aligned} & C^{(p+3)}{}_{\boldsymbol{\mu}} \frac{1}{(p+3)!} \frac{\delta}{\delta C^{(p+3)}{}_{\boldsymbol{\mu}}} \frac{1}{(p+1)!} \frac{\delta}{\delta C^{(p+1)}{}_{\boldsymbol{b}}} B \frac{\delta}{\delta B} S_{10} \\ &= \kappa \frac{(p+4)}{(p+1)!2!} (p_1)_{[i} C^{(p+3)}_{jk\boldsymbol{c}]} \left((p_2)^{i} B^{jk} \right) \eta^{\boldsymbol{c},\boldsymbol{b}}. \end{aligned} \tag{3.22}$$

Here p, p_1, p_2 are momenta for $C^{(p+3)}, C^{(p+1)}, B$ respectively and the delta function $\delta(p + p_1 + p_2)$ is implicit.

The brane is $p+1$ dimensional, so μ_1, μ_2, μ_3 have to be normal indices. Thus the amplitude takes the form

$$\begin{aligned} A^{(0)}_{C^{p+3}B} &= \kappa \frac{(p+4)}{(p+1)!2!} (p_1)_{[i} C^{(p+3)}_{jk\boldsymbol{c}]} \left((p_2)^{\,i} B^{jk} \right) \eta^{\boldsymbol{c},\boldsymbol{b}} \left(\sqrt{2}\mu_p \kappa \frac{-i}{p^2} \right) \epsilon_{\boldsymbol{b}} \\ &= \frac{\sqrt{2}\mu_p \kappa^2}{(p+1)!2!} \frac{-i}{p^2} \frac{-1}{3} (-1)^{(p+3)} F^{(p+4)ijk}_{\boldsymbol{c}} H_{ijk} \epsilon^{\boldsymbol{c}}. \end{aligned} \tag{3.23}$$

We define kinematic variables

$$\begin{aligned} s &= p_1.D.p_1, \\ t &= p_1.p_2, \\ r &= t + \frac{s}{2}, \end{aligned} \tag{3.24}$$

in terms of which the two-point function takes the form

$$\left\langle C^{(p+3)} B \right\rangle := i A^{(0)}_{C^{p+3}B} = \frac{\sqrt{2}\mu_p \kappa^2}{(p+1)!2!} \frac{1}{6} \frac{1}{t} (-1)^p F^{(p+4)ijk}_{\boldsymbol{c}} H_{ijk} \epsilon^{\boldsymbol{c}}. \tag{3.25}$$

3.3.2 The string disc amplitude for $C^{p+3}B$

At zero momentum order, the relevant RR-NSNS amplitude is [15]

$$\frac{i\pi}{2\sqrt{2}} \frac{\Gamma(t+1)\Gamma(s+1)}{\Gamma(t+s+1)} \left(\frac{-1}{t} \right) (\epsilon D)_{\mu\nu} \left(D p_2 \right)_\rho T^{\mu\nu\rho}. \tag{3.26}$$

Plugging in the defintion for D and T , we have

$$(\epsilon D)_{ij} \left(D p_2 \right)_k = -\epsilon_{ij} p_{2k} \tag{3.27}$$

$$T^{ijk} = -i^{p(p-1)} \frac{32}{(p+1)!} \epsilon_{b_1 \ldots b_{p+1}} F^{b_1 \ldots b_{p+1} ijk}, \tag{3.28}$$

where we take all the indices in T to be normal to get the gauge field strength for $C^{(p+3)}$.

The amplitude then becomes

$$-\frac{\pi}{6\sqrt{2}} i^{p(p-1)} \frac{32}{(p+1)!} \frac{\Gamma(t+1)\Gamma(s+1)}{\Gamma(t+s+1)} \frac{1}{t} \epsilon_{b_1 \dots b_{p+1}} F^{b_1 \dots b_{p+1} ijk} H_{ijk}. \tag{3.29}$$

We expand the Γ factor up to four momentum order:

$$\frac{\Gamma(t+1)\Gamma(s+1)}{\Gamma(t+s+1)} = 1 - \frac{\pi^2}{6} st + ..., \tag{3.30}$$

while at zero order in momentum we have

$$-\frac{\pi}{6\sqrt{2}} i^{p(p-1)} \frac{32}{(p+1)!} \frac{1}{t} \epsilon_{b_1 \dots b_{p+1}} F^{b_1 \dots b_{p+1} ijk} H_{ijk}. \tag{3.31}$$

We see[2] the amplitudes differ by an overall factor

$$\text{QFT} = \text{String} \times \left(\frac{(i\kappa)^2 \mu_p}{32\pi i^{(1+p)p}} \right). \tag{3.32}$$

At four momentum order, the amplitude takes the form

$$\frac{\sqrt{2}\mu_p \kappa^2}{(p+1)!2!} (-1)^p \frac{1}{6} \frac{1}{t} \left(-\frac{\pi^2}{6} st \right) F_{\boldsymbol{c}}^{(p+4)ijk} H_{ijk} \epsilon^{\boldsymbol{c}}. \tag{3.33}$$

More explicitly

$$\frac{1}{t}(st) F_{\boldsymbol{c}}^{(p+4)ijk} H_{ijk} = 2 F_{\boldsymbol{c}}^{(p+4)ijk} (p_1)_a (p_1)^a H_{ijk} = -2 F_{\boldsymbol{c}}^{(p+4)ijk} \nabla_a \nabla^a H_{ijk}. \tag{3.34}$$

[2]Each point will contribute a dimensional coefficient $(i\kappa)$. So in case of the three-point function, the factor would be $\frac{(i\kappa)^3 \mu_p}{32\pi i^{(1+p)p}}$. As we will see later, this is indeed the case.

where we used the on-shell condition

$$(p_1 + p_2)_a = 0, \tag{3.35}$$

$$p_1{}^{\mu} p_{1\mu} = p_2{}^{\mu} p_{2\mu} = 0. \tag{3.36}$$

Therefore the four-derivative couplings for $C^{(p+3)}B$ on the brane are

$$\begin{aligned} S^{(4)}_{\left(C^{(p+3)}B\right)} &= \frac{\sqrt{2}\mu_p \kappa^2}{(p+1)!3!}(-1)^p \left(\frac{\pi^2}{6}\right) F_{\boldsymbol{c}}^{(p+4)ijk} \nabla_a \nabla^a H_{ijk} \\ &= \frac{\sqrt{2}\mu_p \kappa^2}{(p+1)!2!}(-1)^p \left(\frac{\pi^2}{6}\right) (p+4)\, (p_1)_{[c_1}\, C_{c_2 \ldots c_{p+1} ijk]}\, (p_2)_a\, (p_2)^{\,a}\, (p_2)^{\,[i} B^{jk]} \epsilon^{\boldsymbol{c}}. \end{aligned} \tag{3.37}$$

3.3.3 $C^{(p+5)}BB$ couplings at zero momentum

The Feynman diagram is shown in Figure 3.1. The corresponding field theory amplitude is

$$\delta_{C^{(p+3)}{}_{\boldsymbol{\sigma}}} \delta_{C^{(p+3)\boldsymbol{\nu}}} B_2 \delta_{B_2} S_{10} G^{\left(C^{(p+1)}\right)^{\boldsymbol{\nu}}}{}_{\boldsymbol{\mu}} \delta_{C^{(p+3)}{}_{\boldsymbol{\mu}}} \delta_{C^{(p+1)\boldsymbol{b}}} B_3 \delta_{B_3} S_{10} G^{\left(C^{(p+1)}\right)^{\boldsymbol{b}}}{}_{\boldsymbol{a}} \delta_{C^{(p+1)}{}_{\boldsymbol{a}}} S_{p+1}. \tag{3.38}$$

where $\boldsymbol{\sigma}, \boldsymbol{\mu}, \boldsymbol{\nu}$ are rank $p+3$ indices (i.e. $\boldsymbol{\sigma} = (\sigma_1, ...\sigma_{p+3})$) and $\boldsymbol{a}, \boldsymbol{b}$ are rank $p+1$ indices.

Momentum convervation on the brane gives

$$\begin{aligned} p_4 &= -p_1 - p_2, \\ (p_4)^{\,a} &= (p_3)^{\,a}. \end{aligned} \tag{3.39}$$

Thus

$$\delta_{C^{(n)}\boldsymbol{\nu}}\delta_{C^{(n-2)\boldsymbol{\mu}}}B\delta_B S_{10} = \kappa\frac{(n+1)}{(n-2)!2!}\left(p_1\right)_{[\rho_1}\ \delta^{\nu_2\ldots\nu_{n+1}}_{\rho_2\ldots\rho_{n+1}]}\left(p_2\right)^{\rho_1}B^{\rho_2\rho_3}\delta^{\rho_4\ldots\rho_{n+1}}_{\boldsymbol{\mu}}. \tag{3.40}$$

The amplitude can be rewritten as

$$(-1)\left(\frac{1}{2!}\frac{1}{2!}\frac{\sqrt{2}\mu_p\kappa^3}{(p+1)!}(p+6)(p+4)\right)$$
$$\times\left(\left(p_1\right)_{[\sigma_1}\left(C^{(p+5)}\right)_{\sigma_2\sigma_3\boldsymbol{\mu}]}\left(p_2\right)^{\sigma_1}B_2{}^{\sigma_2\sigma_3}\frac{1}{\left(p_4\right)^2}\left(p_4\right)_{[\rho_1}\ \delta^{\boldsymbol{\mu}}_{\rho_2\rho_3\boldsymbol{c}]}\left(p_3\right)^{\rho_1}B_3{}^{\rho_2\rho_3}\frac{1}{p^2}\epsilon^{\boldsymbol{c}}\right). \tag{3.41}$$

We now have two cases in the above expression: (1) $\boldsymbol{\mu}$ contains $p+1$ tangential indices, and 2 normal indices, (2) $\boldsymbol{\mu}$ contains p tangential indices, and 3 normal indices.

Case 1:

The number of upper and lower normal indices in the delta function must match, otherwise the Kroneker delta vanishes. Therefore in this case all indices $\rho_1\rho_2\rho_3$ and $\sigma_1\sigma_2\sigma_3$ need to be normal indices.

$$\begin{aligned}(-1)\left(\frac{1}{2!}\frac{1}{2!}\frac{\sqrt{2}\mu_p\kappa^3}{(p+1)!}(p+6)(p+4)\right)&\left(\left(p_1\right)_{[\sigma_1}C^{(p+5)}{}_{\sigma_2\sigma_3\boldsymbol{\mu}]}\left(p_2\right)^{\sigma_1}B_2{}^{\sigma_2\sigma_3}\frac{1}{\left(p_4\right)^2}\right.\\ &\left.\left(p_4\right)_{[\rho_1}\ \delta^{\boldsymbol{\mu}}_{\rho_2\rho_3\boldsymbol{c}]}\left(p_3\right)^{\rho_1}B_3{}^{\rho_2\rho_3}\frac{1}{p^2}\epsilon^{\boldsymbol{c}}\right)\\ =\left(\frac{3}{(p+4)}\right)&\left(\left(p_1\right)_{[r}C^{(p+5)}{}_{stj k\boldsymbol{c}]}\left(p_2\right)^{[r}B_2{}^{st]}\frac{1}{\left(p_4\right)^2}\left(p_4\right)_i\left(p_3\right)^{[i}B_3{}^{jk]}\frac{1}{p^2}\epsilon^{\boldsymbol{c}}\right)\end{aligned} \tag{3.42}$$

where we used identities appearing in the appendix.

Rewriting in terms of the T symbol we have

$$\left(\frac{1}{i}\right)\left(\frac{1}{3!}\frac{1}{3!}\frac{\sqrt{2}\mu_p\kappa^3}{32i^{p(1+p)}}\right)(3)\left(T^{rstjk}\left(H_2\right)_{rst}\frac{1}{\left(p_4\right)^2}\left(p_4\right)^i\left(H_3\right)_{ijk}\frac{1}{p^2}\right). \tag{3.43}$$

Case 2:

There are $\binom{p+3}{3}$ choices for $\boldsymbol{\mu}$. But no matter which positions we choose, we always shift all the normal indices to the front and denote them as $ijk\boldsymbol{e}$, where $\boldsymbol{e}$ denotes p tangential indices.

$$\begin{aligned}
&(-1)\left(\frac{1}{2!}\frac{1}{2!}\frac{\sqrt{2}\mu_p\kappa^3}{(p+1)!}(p+6)(p+4)\right)\times\\
&\left(\left(p_1\right)_{[\sigma_1}C^{(p+5)}{}_{\sigma_2\sigma_3\boldsymbol{\mu}]}\left(p_2\right)^{\sigma_1}B_2{}^{\sigma_2\sigma_3}\frac{1}{\left(p_4\right)^2}\left(p_4\right)_{[\rho_1}\delta^{\boldsymbol{\mu}}{}_{\rho_2\rho_3\boldsymbol{c}]}\left(p_3\right)^{\rho_1}B_3{}^{\rho_2\rho_3}\frac{1}{p^2}\epsilon^{\boldsymbol{c}}\right)\\
&=\left((-1)\frac{(p+1)}{(p+4)}\right)\left(\left(p_1\right)_{[\sigma_1}C^{(p+5)}{}_{\sigma_2\sigma_3ijk\boldsymbol{e}]}\left(p_2\right)^{[\sigma_1}B_2{}^{\sigma_2\sigma_3]}\frac{1}{\left(p_4\right)^2}\left(p_4\right)_c\left(p_3\right)^{[i}B_3{}^{jk]}\frac{1}{p^2}\epsilon^{c\boldsymbol{e}}\right)\\
&=\left(\frac{1}{i}\right)\left(\frac{1}{3!}\frac{1}{3!}\frac{\sqrt{2}\mu_p\kappa^3}{p!}\right)(-1)\left(F^{(p+5)}{}_{\sigma_1\sigma_2\sigma_3ijk\boldsymbol{e}}\left(H_2\right)^{\sigma_1\sigma_2\sigma_3}\frac{1}{\left(p_4\right)^2}\left(p_4\right)_c\left(H_3\right)^{ijk}\frac{1}{p^2}\epsilon^{c\boldsymbol{e}}\right).
\end{aligned} \tag{3.44}$$

Now there are two cases, either $\sigma_1\sigma_2\sigma_3$ are all normal indices, or 2 normal indices and 1 tangential index.

In terms of the T symbol, the first case yields

$$\left(\frac{1}{i}\right)\left(\frac{1}{3!}\frac{1}{3!}\frac{\sqrt{2}\mu_p\kappa^3}{32i^{p(1+p)}}\right)(-1)\left(T^c{}_{lmnijk}H_2{}^{lmn}\frac{1}{\left(p_4\right)^2}\left(p_4\right)_c H_3{}^{ijk}\frac{1}{p^2}\right) \tag{3.45}$$

while the second case yields

$$\left(\frac{1}{i}\right)\left(\frac{1}{3!}\frac{1}{3!}\frac{\sqrt{2}\mu_p\kappa^3}{32i^{p(1+p)}}\right)(3)\left(T_{mnijk}H_2{}^{amn}\frac{1}{\left(p_4\right)^2}\left(p_4\right)_a H_3{}^{ijk}\frac{1}{p^2}\right). \tag{3.46}$$

Collecting all pieces, we have

$$\left(\frac{1}{i}\right)\left(\frac{1}{3!}\frac{1}{3!}\frac{\sqrt{2}\mu_p\kappa^3}{32i^{p(1+p)}}\right)\left(3T^{rstjk}\left(H_2\right)_{rst}\frac{1}{\left(p_4\right)^2}\left(p_4\right)^i\left(H_3\right)_{ijk}\frac{1}{p^2}\right.$$
$$\left.-T^c{}_{lmnijk}H_2{}^{lmn}\frac{1}{\left(p_4\right)^2}\left(p_4\right)_c H_3{}^{ijk}\frac{1}{p^2}+3T_{mnijk}H_2{}^{amn}\frac{1}{\left(p_4\right)^2}\left(p_4\right)_a H_3{}^{ijk}\frac{1}{p^2}\right)+(2\leftrightarrow 3). \tag{3.47}$$

We added a $(2 \leftrightarrow 3)$ at the end, because there will be a similar expression with 2 and 3 exchanged.

3.3.4 The string disc amplitude for $C^{(p+5)}BB$ at zero momentum

In this section, we evaluate the $C^{(p+5)}BB$ amplitudes up to four derivatives. We use the following expansion for the integral [16]

$$I_9=\frac{\pi^2}{p^2\left(p_1.p_3\right)}\left(1-\frac{\pi^2}{6}p^2\left(p_2.D.p_2\right)\right)-\frac{\pi^2}{p^2\left(p_1.p_2\right)}\left(1-\frac{\pi^2}{6}p^2\left(p_3.D.p_3\right)\right)+... \tag{3.48}$$

$$I_5=I_{10}=\frac{\pi^2}{p^2\left(p_1.p_3\right)}\left(1-\frac{\pi^2}{6}p^2\left(p_2.D.p_2\right)\right)+\frac{\pi^2}{p^2\left(p_1.p_2\right)}\left(1-\frac{\pi^2}{6}p^2\left(p_3.D.p_3\right)\right)+... \tag{3.49}$$

where $p^2 = p_1.p_2 + p_1.p_3 + p_2.p_3$.

At zero momentum order, the terms with six normal indices are

$$\mathcal{A}^{(6),0}=\frac{i\pi^2}{8\sqrt{2}}T^{aijklmn}\frac{1}{p^2}\left(-\frac{p_{2a}p_{2j}p_{3i}\epsilon_{2mn}\epsilon_{3kl}}{(p_1p_2)}-\frac{p_{2a}p_{2j}p_{3i}\epsilon_{2mn}\epsilon_{3kl}}{(p_1p_3)}\right)+(2\leftrightarrow 3). \tag{3.50}$$

The terms with five normal indices take the form

$$
\begin{aligned}
\mathcal{A}^{(5),0} =& \frac{i\pi^2}{8\sqrt{2}} T^{ijklm} \frac{1}{p^2} \left(\frac{2p_{2m}p_{3i}(p_1N\epsilon_3)_l\epsilon_{2jk}}{(p_1p_2)} + \frac{2p_{2m}p_{3i}(p_1N\epsilon_3)_l\epsilon_{2jk}}{(p_1p_3)} \right. \\
&+ \frac{2p_{2j}p_{3i}(p_2D\epsilon_3)_m\epsilon_{2kl}}{(p_1p_2)} + \frac{2p_{2j}p_{3i}(p_2D\epsilon_3)_m\epsilon_{2kl}}{(p_1p_3)} \\
&+ \frac{2p_{2i}p_{3j}(p_2\epsilon_3)_m\epsilon_{2kl}}{(p_1p_2)} - \frac{2p_{2i}p_{3j}(p_2\epsilon_3)_m\epsilon_{2kl}}{(p_1p_3)} \\
&- \frac{(p_1Np_3)\, p_{2i}\epsilon_{2lm}\epsilon_{3jk}}{(p_1p_2)} - \frac{(p_1Np_3)\, p_{2i}\epsilon_{2lm}\epsilon_{3jk}}{(p_1p_3)} \\
&+ \frac{(p_2Dp_3)\, p_{2i}\epsilon_{2lm}\epsilon_{3jk}}{(p_1p_2)} + \frac{(p_2Dp_3)\, p_{2i}\epsilon_{2lm}\epsilon_{3jk}}{(p_1p_3)} \\
&\left. - \frac{(p_2p_3)\, p_{2i}\epsilon_{2lm}\epsilon_{3jk}}{(p_1p_2)} + \frac{(p_2p_3)\, p_{2i}\epsilon_{2lm}\epsilon_{3jk}}{(p_1p_3)} \right) + (2 \leftrightarrow 3).
\end{aligned} \tag{3.51}
$$

Using the definition for the NSNS field strength $H_{\mu lm} = 3p_{[\mu}\epsilon_{lm]}$ and the Bianchi identity $0 = p_{[\mu}H_{lmn]}$, we can rewrite all of the above terms as

$$
\begin{aligned}
\mathcal{A}^0 =& \frac{i\pi^2 p_4{}^a H_{2aij}H_{3klm}T^{ijklm}}{6\sqrt{2}p^2\,(p_4)^{\,2}} + \frac{i\pi^2 p_4{}^n H_{2ijk}H_{3nlm}T^{ijklm}}{6\sqrt{2}p^2\,(p_4)^{\,2}} \\
&+ \frac{i\pi^2 p_{4a}H_{2jkl}H_{3imn}T^{aijklmn}}{18\sqrt{2}p^2\,(p_4)^{\,2}} + (2 \leftrightarrow 3),
\end{aligned} \tag{3.52}
$$

where $p_4 = p_1 + p_2$ and so that $(p_4)^{\,2} = 2p_1.p_2$, $(p_4)_c = -(p_3)_c$.

Comparing this with the field theory result, we see

$$
\text{QFT} = \text{String} \times \frac{(i\kappa)^3\mu_p}{32\pi^2 i^{p(1+p)}}. \tag{3.53}
$$

3.3.5 The field theory diagram for $C^{(p+5)}BB$ at four momentum

The relevant Feynman diagram is Figure 3.2. We should take the brane vertex to be the four-derivative correction from (3.37). The expression for this diagram is

$$C^{(p+5)}{}_{\boldsymbol{\sigma}}\delta_{C^{(p+5)}{}_{\boldsymbol{\sigma}}}\delta_{C^{(p+3)\boldsymbol{\nu}}}B_2\delta_{B_2}S_{10}G^{\left(C^{(p+3)}\right)^{\boldsymbol{\nu}}}{}_{\boldsymbol{\mu}}\delta_{C^{(p+3)}{}_{\boldsymbol{\mu}}}B_3\delta_{B_3}S^{(4)}_{p+1} \tag{3.54}$$

Let p_1, p_2, p_3, p_4 be momenta for $C^{(p+5)}, B_2, B_3, C^{(p+3)}$ respectively. The above expression can be written as

$$\begin{aligned}&(-1)\left(\frac{1}{2!}\frac{1}{2!}\frac{\sqrt{2}\mu_p\kappa^3}{(p+1)!}(p+6)(p+4)\right)\left((-1)\frac{\pi^2}{6}\right)\\&\left((p_1)_{[\sigma_1}C^{(p+5)}_{\sigma_2\sigma_3\boldsymbol{\mu}]}(p_2)^{\sigma_1}B_2{}^{\sigma_2\sigma_3}\right)\left((p_4)_{[i}\delta^{\boldsymbol{\mu}}_{jk\boldsymbol{c}]}(p_3)^{[i}B_3{}^{jk]}\epsilon^{\boldsymbol{c}}\right)\left((p_3)_a(p_3)^a\frac{1}{(p_4)^2}\right).\end{aligned} \tag{3.55}$$

If we compare this with the zero momentum expression, we see they are almost the same except for a overall factor $\left((-1)\frac{\pi^2}{6}\right)$ and $\frac{1}{p^2}$ being replaced by $(p_3)_a(p_3)^a$. Since all the Kronecker delta structure is exactly the same, the calculation is the same as for the zero momentum case. We therefore know the four momentum field theory result is

$$\begin{aligned}&\left(\frac{1}{i}\right)\left(\frac{1}{3!}\frac{1}{3!}\frac{\sqrt{2}\mu_p\kappa^3}{32i^{p(1+p)}}\right)\left(-\frac{\pi^2}{6}\right)\frac{1}{(p_4)^2}(p_3)_a(p_3)^a\\&\Big(3T^{rstjk}(H_2)_{rst}(p_4)^i(H_3)_{ijk}-T^c{}_{lmnijk}H_2{}^{lmn}(p_4)_cH_3{}^{ijk}\\&\quad+3T_{mnijk}H_2{}^{amn}(p_4)_aH_3{}^{ijk}\Big)+(2\leftrightarrow 3).\end{aligned} \tag{3.56}$$

3.3.6 The string disc amplitude for $C^{(p+5)}BB$ at four momentum

The four momentum terms are $\mathcal{A}^{(6),4} + \mathcal{A}^{(5),4}$, where

$$\mathcal{A}^{(6),4} = \frac{i\pi^4}{48\sqrt{2}} T^{aijklmn} \left(\frac{(p_2 D p_2)\, p_{2a} p_{2j} p_{3i} \epsilon_{2mn} \epsilon_{3kl}}{(p_1 p_3)} + \frac{(p_3 D p_3)\, p_{2a} p_{2j} p_{3i} \epsilon_{2mn} \epsilon_{3kl}}{(p_1 p_2)} \right), \tag{3.57}$$

$$\begin{aligned}
\mathcal{A}^{(5),4} = \frac{i\pi^4}{48\sqrt{2}} T^{ijklm} \Bigg(& -\frac{2\,(p_2 D p_2)\, p_{2m} p_{3i} (p_1 N \epsilon_3)_l \epsilon_{2jk}}{(p_1 p_3)} - \frac{2\,(p_3 D p_3)\, p_{2m} p_{3i} (p_1 N \epsilon_3)_l \epsilon_{2jk}}{(p_1 p_2)} \\
& - \frac{2\,(p_2 D p_2)\, p_{2j} p_{3i} (p_2 D \epsilon_3)_m \epsilon_{2kl}}{(p_1 p_3)} - \frac{2\,(p_3 D p_3)\, p_{2j} p_{3i} (p_2 D \epsilon_3)_m \epsilon_{2kl}}{(p_1 p_2)} \\
& + \frac{2\,(p_2 D p_2)\, p_{2i} p_{3j} (p_2 \epsilon_3)_m \epsilon_{2kl}}{(p_1 p_3)} - \frac{2\,(p_3 D p_3)\, p_{2i} p_{3j} (p_2 \epsilon_3)_m \epsilon_{2kl}}{(p_1 p_2)} \\
& + \frac{(p_1 N p_3)\,(p_2 D p_2)\, p_{2i} \epsilon_{2lm} \epsilon_{3jk}}{(p_1 p_3)} - \frac{(p_2 D p_2)\,(p_2 D p_3)\, p_{2i} \epsilon_{2lm} \epsilon_{3jk}}{(p_1 p_3)} \\
& - \frac{(p_2 D p_2)\,(p_2 p_3)\, p_{2i} \epsilon_{2lm} \epsilon_{3jk}}{(p_1 p_3)} + \frac{(p_1 N p_3)\,(p_3 D p_3)\, p_{2i} \epsilon_{2lm} \epsilon_{3jk}}{(p_1 p_2)} \\
& - \frac{(p_2 D p_3)\,(p_3 D p_3)\, p_{2i} \epsilon_{2lm} \epsilon_{3jk}}{(p_1 p_2)} + \frac{(p_2 p_3)\,(p_3 D p_3)\, p_{2i} \epsilon_{2lm} \epsilon_{3jk}}{(p_1 p_2)} \Bigg).
\end{aligned} \tag{3.58}$$

Note this is just like the zero momentum result except that the $\frac{1}{p_1 \cdot p_2}$ pole terms have an extra factor $\left(-\frac{\pi^2}{6}\right)(p_2.D.p_2)\, p^2$ and the $\frac{1}{p_1 \cdot p_3}$ pole terms have an extra factor $\left(-\frac{\pi^2}{6}\right)(p_3.D.p_3)\, p^2$. Therefore the final result is

$$\begin{aligned}
I_1 = & \left(\frac{1}{i}\right) \left(\frac{1}{3!} \frac{1}{3!} \frac{\sqrt{2}\mu_p \kappa^3}{32 i^{p(1+p)}} \right) \left(-\frac{\pi^2}{6} \right) \frac{1}{(p_4)^2} (p_3.D.p_3) \frac{1}{2} \\
& \left(3T^{rstjk} (H_2)_{rst} (p_4)^i (H_3)_{ijk} - T^c{}_{lmnijk} H_2{}^{lmn} (p_4)_c H_3{}^{ijk} + 3T_{mnijk} H_2{}^{amn} (p_4)_a H_3{}^{ijk} \right)
\end{aligned} \tag{3.59}$$

But this is just what we get from the field theory calculation! Therefore we conclude the four-derivative correction to $C^{(p+5)}BB$ vanishes and the amplitude obeys a non-renormalization theorem.

3.4 Some Identities for Antisymmetrization

In this appendix, we list some identities that we used in the Feynman diagram calculation in previous section.

Statement 1:

$$A_{[c_1...c_p}\,\epsilon_{a_1...a_{m-p}]b_1...b_p} = \frac{p!(m-p)!}{m!} A_{[b_1...b_p]}\epsilon_{a_1...a_{m-p}c_1...c_p}. \tag{3.60}$$

where ϵ is a antisymmetric rank p tensor and indices a_i, b_i take values in $1, ...p$.

Statement 2: Assume all indices take values in $1, ...m$, and A, T, ϵ are antisymmetric tensors, then

$$A_{c_1...c_p}T_{a_1...a_{m-p}b_1...b_p}\epsilon^{c_1...c_p a_1...a_{m-p}} = \frac{(m-p)!p!}{m!} A_{b_1...b_p}T_{a_1...a_{m-p}c_1...c_p}\epsilon^{c_1...c_p a_1...a_{m-p}}. \tag{3.61}$$

The case with $p = 1$ is

$$A_c T_{a_1...a_{m-1}b}\epsilon^{a_1...a_{m-1}c} = \frac{1}{m} A_b T_{a_1...a_{m-1}c}\epsilon^{a_1...a_{m-1}c}. \tag{3.62}$$

Let $A_{a_1...a_p}$ be the antisymmetric tensor field. Denote the indices as $\boldsymbol{a} = (a_1...a_p)$ Define $\delta_{\boldsymbol{a}}^{\boldsymbol{b}} = \delta_{a_1}^{[b_1}...\delta_{a_p}^{b_p]} = \delta_{[a_1}^{[b_1}...\delta_{a_p]}^{b_p]}$.

Statement 3:

$$\frac{\delta}{\delta A_{\boldsymbol{b}}(x)} A_{a_1...a_p}(y) = p!\delta_{\boldsymbol{a}}^{\boldsymbol{b}}\delta(x-y). \tag{3.63}$$

Statement 4:

$$\int dx A_{\boldsymbol{b}}(x) \frac{\delta}{\delta A_{\boldsymbol{b}}(x)} A_{a_1 \dots a_p}(y) = p! A_{a_1 \dots a_p}(y). \tag{3.64}$$

Statement 5:

$$\delta^{\boldsymbol{b}}_{\boldsymbol{a}} A^{\boldsymbol{a}} = A^{\boldsymbol{b}}. \tag{3.65}$$

Statement 6: Let $i_1 \dots$, $j_1 \dots$, ... be one type of index ("normal "), $a_1 \dots$, $b_1 \dots$ be another type ("tangential"). For indices of different type $\delta^i_a = 0$. Then

$$A_{[j_1 \dots j_p} \delta^{k_1 \dots k_m b_1 \dots b_n}_{i_1 \dots i_m a_1 \dots a_n]} = \frac{(p+m)!(m+n)!}{(p+n+m)!m!} A_{[j_1 \dots j_p} \delta^{k_1 \dots k_m b_1 \dots b_n}_{i_1 \dots i_m] a_1 \dots a_n}. \tag{3.66}$$

Statement 7: Let $i_1 \dots$, $j_1 \dots$, ... be one type of index ("normal"), $a_1 \dots$, $b_1 \dots$ be another type ("tangential"). For different type of index $\delta^i_a = 0$. Then

$$\epsilon^{a_1 \dots a_{n+l}} A_{[j_1 \dots j_p} \delta^{k_1 \dots k_m b_1 \dots b_n}_{i_1 \dots i_{m-l} a_1 \dots a_{n+l}]} = \frac{\binom{m+n}{n}\binom{p}{i}}{\binom{n+m+p}{n+i}} (-1)^{l(p+m-l)} \epsilon^{a_1 \dots a_{n+l}} \times$$
$$A_{a_1 \dots a_l \left[j_1 \dots j_{p-l} \right.} \delta^{k_1 \dots k_m b_1 \dots b_n}_{\left. j_{p-l+1} \dots j_p i_1 \dots i_{m-l} \right] a_{l+1} \dots a_{l+n}}, \tag{3.67}$$

where $\binom{p}{i}$ is the binomial coefficient.

Statement 8: Let $i_1 \dots$, $j_1 \dots$, ... be one type of indices ("normal"), $a_1 \dots$, $b_1 \dots$ be another type ("tangential").

Let A, B be the antisymmetric tensors. Then

$$A_{[a_1} B_{a_2 \dots a_m i_1 \dots i_n]} = \frac{m}{m+n} A_{[a_1} B_{a_2 \dots a_m] i_1 \dots i_n} + \frac{n}{m+n} (-1)^m A_{[i_1} B_{|a_1 \dots a_m| i_2 \dots i_n]}. \tag{3.68}$$

Statement 9: Let H_i be an antisymmetric tensor, then

$$p_{[i_1} V_{i_2 \ldots i_{3n}]} {H_1}^{i_1 i_2 i_3} \ldots {H_n}^{i_{3n-2} i_{3n-1} i_{3n}} =$$
$$\frac{1}{n} \sum_{m=1}^{n} (-1)^{3m-3} p_{i_{3m-2}} V_{i_1 \ldots i_{3m-3} i_{3m-1} i_{3m} i_{3m+1} \ldots i_{3n}} {H_1}^{i_1 i_2 i_3} \ldots {H_n}^{i_{3n-2} i_{3n-1} i_{3n}}. \tag{3.69}$$

4. CONCLUSION

This thesis describes calculations in perturbative string theory. In chapter 2, we evaluated the entire tree level RR-NSNS-NSNS disc amplitudes. We presented the amplitudes with explicit Ramond gauge invariance as well as exchange invariance among NSNS fields. There are still many interesting questions regarding these amplitudes. In particular, it would be nice to have B-field gauge invariance manifest. However, so far this can only be derived in the case of $C^{(p+5)}BB$, and it is not straightforward. We do not have a generic algorithm that works for all amplitudes. Another interesting question is that whether we could generalize the algorithm in section 2.3 to generic n-point function. If this is the case, we could formulate string theory in a way that is closer to the features of low energy effective theory.

In chapter 3, we evaluated the four-derivative effective couplings from the $C^{(p+5)}BB$ amplitude. We show this coupling vanishes and points to an interesting non-renormalization theorem. The surprising observation is that the heaviest calculation needed is actually not for the contact diagram that we are interested in. Rather, the four-derivative correction can be calculated easily by recycling the zero order result. In our calculation we observe that relative coefficients of four-derivative terms to zero-derivative terms are similar in both three-point and two-point disc amplitudes. It is this similarity that cancel out most terms and renders the effective coupling zero. If a lot of terms cancel each other, it often indicates that there might be a better method or formulation to start with. We suspect that there might be a recursive relation between higher point string amplitudes and lower point ones. A complete evaluation of the $C^{(p+3)}$, $C^{(p+1)}$, $C^{(p-1)}$ four-derivative couplings might give more insight into this question. We hope to return to these questions in the future.

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