APPLICATION OF HISTORY MATCHING QUALITY INDEX WITH MOVING

LINEAR REGRESSION ANALYSIS

A Thesis

by

HYE YOUNG JUNG

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Chair of Committee, Co-chair of Committee, Michael J. King Committee Member, Head of Department,

Akhil Datta-Gupta Yalchin Efendiev A. Daniel Hill

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ABSTRACT

History Matching is the process of calibrating uncertain parameters of a reservoir model in order to reach the best plausible match with the observed data. By integrating the dynamic data of a reservoir, the reservoir properties can be estimated so that it is a key step in developing reservoir performance, which is normally time consuming and computationally infeasible.

With the rise of global energy demand, the reliance on enhanced oil recovery (EOR) has increased and the model calibration for chemical flooding also becomes significant. However, due to large amounts of uncertain parameters and complicated relationships among them, it is hard to apply a traditional manual history matching with a single deterministic model to chemical flooding. Instead, a stochastic method using the genetic algorithm (GA) can be efficient in that it can consider several parameters simultaneously. However, this probabilistic-assisted history matching generates several updated models, all of which have a potential to be good matches. Therefore, there is a need to evaluate history matching results consistently without any subjectivity.

In addition, the assessment of results from model calibration is also difficult when it comes to large field cases, which are involved with a number of wells and different types of objectives. Since each well and objective presents contrasting results, a comprehensive decision making for selecting a better history matching model is necessarily complicated. However, current approaches mostly rely on reviewers' experience, which is too subjective or uses a misfit function without any consideration for the data. We first introduce a History Match Quality Index (HMQI) in assessing the quality of history matching and ranking among those results. This method assigns index a value of either 0 or 1 based on the quality of the match. Moreover, combining the HMQI with a Moving Linear Regression Analysis (MLRA) provides the more robust assessment by removing outliers which come from a variety of sources of errors.

Secondly, we apply the HMQI to the synthetic case of alkaline-surfactant-polymer (ASP) flooding as well as that of polymer flooding. Moreover, we compare the results with other method for evaluating the quality of history match to prove the feasibility of our approach. Lastly, field-scale simulations are conducted to demonstrate the reliability and robustness of our methodologies.

The HMQI with the MLRA has proven its ability to identify outliers of data using the case study from synthetic to field. In addition, in comparison with the misfit calculation, it has been shown to eliminate subjectivity, using normalized values without the bias toward outliers.

DEDICATION

To my beloved family and friends.

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NOMENCLATURE

BHP	Bottomhole Pressure	
HMQI	History Matching Quality Index	
MLRA	Moving Linear Regression Analysis	
GA	Genetic Algorithm	
WCT	Water Cut	
d_i^{obs}	Observed Values at Time Step i	
$d_i^{\ calc.}$	Simulated Values at Time Step i	
Wi	Weighting Factor	

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CHAPTER I

INTRODUCTION AND STUDY OBJECTIVE

Enhanced oil recovery (EOR), the last stage of an oil and gas field, plays an increasingly pivotal role in crude oil development. Especially since the infrastructure for chemical flooding does not require additional facilities after waterflooding, chemical EOR is more beneficial, compared to other thermal or gas EORs, in that most oil fields have been already under waterflooding. Consequently, it becomes more significant to reproduce past historical performance in chemical flooding to characterize the reservoir. This is where assisted history matching using the genetic algorithm comes into play. Due to large amounts of parameter affecting each other, the deterministic history matching method, which depends strongly on the initial model is not appropriate for chemical flooding. In contrast, genetic algorithms have been known to be highly effective global search techniques and give us multiple solutions.

However, one of the challenging aspects of history matching using the stochastic method is to evaluate how well the calibrated model matches with the measured data among several simulation runs. Therefore, the main focus of this research is to assess the quality of history matching results with a new approach and its application. Traditionally, evaluation has been conducted by visually or comparing sum of square of error (SSE). Since the former usually relies on the experience of experts, which is too subjective and the latter sometimes cannot account for the general trend of the observed dynamic data, more systematic methods should be considered. Except for the standard way to plot the observed data values versus time, along with corresponding simulation results, some statistical methods have been proposed for history matching evaluation (Uldrich et al. 2002). All of the methods anticipate computing a set of deviation values, each of which is defined to be a calibrated model minus the corresponding production history. Although those methods provide an indicator of confidence whether the reservoir description in the simulator properly represents the actual reservoir or not, it gives only a relative comparison of the degree of match between two or more simulation runs.

1.1 Overview of History Matching

In order to develop reliable reservoir models, the successful integration of dynamic data is a critical step. Since reservoir models are affected by a large number of subsurface uncertainties, only considering the static data cannot help to reproduce past historical performance. Therefore, adjustments to the model should be made until simulated results and the past observation are well-matched.

Conventional manual history matching usually uses a single deterministic model to reconcile the model with observed data, which is time consuming and subjective with a trial-and-error approach (Williams et al. 1998; Williams et al. 2004). Due to impracticability of this manual process, an automatic history matching, which is the process of calibrating parameters of a reservoir model by way of an automated algorithm, has been heavily investigated (Landa et al. 2003). In assisted history matching (AHM), the production history is compared to the simulation results by a misfit function, trying to minimize the misfit so that the best reservoir model can be achieved. Over the last years, several approaches to the minimization process has been widely demonstrated in the literature: gradient-based, sensitivity-based and derivative-free methods. Compared to gradient-based methods, sensitivity-based methods converge rapidly (Bissell et al. 1992). Although the derivative-free methods are easy to implement, they are limited to only a small number of parameters due to computational issue. Lastly, gradient-based methods have the slower rate of convergence than sensitivity-based methods (Gill et al. 1981; McCormick 1972). Since local search techniques have the problem of convergence to local optimum nearest to the initial point, global search algorithms, such as genetic algorithm (Holland 1992) and the simulated annealing (Ouenes et al. 1994; Kirkpatrick et al. 1983; Galassi et al. 2009), have been known to be highly powerful for history-matching process. In addition, these stochastic search methods do not require a smooth response space or complicated differential equations. The limitation of those methods is, however, that it is computationally expensive when having a large number of parameters.

1.2 Overview of Statistical Methods for Evaluation of a History Match

Typically, comparing the quality of history match for different simulation runs has been conducted by the expert opinion or using weighted root mean square error. Even if the experience of experts provides sufficient assessment based on visual inspection by graph, the different knowledge of reviewers can influence their evaluation, which is too subjective. In order to avoid that subjectiveness, it is better to have as many experts as possible with enough experience. Weighted root mean square error, frequently regarded as the sum of square of errors (SSE), represents the data misfit by calculating the cumulative square of error between observed data and simulation results.

$$SSE = \sum_{i=1}^{n} w_i (d_i^{\ obs} - d_i^{\ calc})^2$$
(1.1)

Although this method is quantitative, compared to the expert opinion, it can be easily affected by the data outliers, which might lead to an erroneous conclusion.

Uldrich (2002) proposed methods using statistics for an evaluation of history on the basis of deviation values. Deviation values can be defined from the observed and predicted value, and then plotted in two ways: the deviation distribution plot and the deviation band plot. The deviation distribution plot quickly gives us whether one case is better matched than the other. Finding positive or negative bias in the match is vital when comparing history match results in order to decide what changes to make next for the history match. The deviation band plot is also a convenient way to demonstrate the degree of match by evaluating the total amount of deviation, regardless of whether the simulator is over or under-predicting the production history. However, the above approaches enable only a relative comparison among several simulation runs. They do not suggest any minimum criteria to be satisfied for declaring a model sufficiently matched.

1.3 Research Objectives and Thesis Outline

The objective of this research is to present the History Matching Quality Index (HMQI) with the Moving Linear Regression Analysis (MLRA), and an emphasis is placed on the application to chemical flooding and large field cases with implementation of the

proposed method to evaluate the results of history matching. Now, we will outline the specific procedure of this thesis in Chapters II-V.

This research consists of three main parts. First, in Chapter II, we present a background of history matching with the Genetic Algorithm and give a summary of History Matching Quality Index to evaluate the history matching results.

Secondly, in Chapter III, we implement this algorithm and demonstrate the capability of the HMQI with the synthetic case of polymer flooding and ASP flooding. We will run the polymer flooding with different degrees of noise, and compare the results with the sum of square error to show whether the HMQI method accounts for eliminating outliers efficiently or not. In addition, we apply this new ranking method to an Alkaline-Surfactant-Polymer flooding to do assessment on the quality of history matching results. We analyze the results with the HMQI and other methodsz in order to demonstrate the ability of this approach to consider different types of objective functions.

In Chapter IV, two field cases will be presented to test applicability of the HMQI with MLRA. This chapter includes details of model description, history matching results and evaluation of history matching results on the field case study.

In Chapter V, the research is concluded with a summary of the key findings. Recommendations and proposals for further research are also presented.

CHAPTER II

HISTORY MATCHING USING GENETIC ALGORITHM AND HISTORY MATCHING QUALITY INDEX

In recent years, understanding the uncertainty of subsurface, and calibrating static and dynamic parameters, have been done with a probabilistic method. This stochastic approach has a variety of advantages over the deterministic one, which uses a single initial geological model. The stochastic method has been known to be effective since global search algorithms do not require complicated differential equations, and can avoid the problem of convergence to local optimum nearest to the initial starting point (Cheng et al. 2008). Although they have been extensively applied to many history-matching problems because of their powerful ability to deal with several sets of parameters, stochastic search algorithms give us multiple solutions that are need to be assessed in terms of which history matching results could be the best match (Bittencourt and Horne 1997; Floris et al. 2001; Romero and Carter 2001; Schulze-Riegert et al. 2002; Williams et al. 2004).

Even if there has been a few ways to investigate history matching results, it is still challenging for all reservoir engineers to evaluate whether the results are good enough to be matched with the observed data. Lack of systematic and well-organized methods to evaluate on the history matching results gives rise to the demand for History Match Quality Index (HMQI).

In this chapter, we have a discussion about a stochastic method using the Genetic Algorithm (GA) for model calibration. The Genetic Algorithm, one of the derivative-free

global techniques, shows its global search nature and the advantage of calibrating diverse types of parameters simultaneously. After the assisted history matching process using the Genetic Algorithm, we introduce the concept of the History Match Quality Index with the Moving Linear Regression Analysis for data and error analysis. The coupling of HMQI and MLRA together during an evaluation of history matching results leads to an increase on the reliability of analyzing history matching results.

2.1 History Matching Using Proxy-Assisted Genetic Algorithm

We have mainly adopted the Genetic Algorithm (GA) to conduct model calibration in order to adjust parameters. The GA is favorable to take into account a variety of parameters at the same time, and hence it provides more flexibility when it comes to the choice of parameters (Xie et al. 2014). In addition, the GA is applicable to carry out the analysis of uncertainty, as well as the history matching, since it works with the population of the parameters. As one of the evolutionary algorithms, the GA follows the survival of the fittest describing the mechanism of natural selection. It applies the biological principle of evolution to the selection process during the history matching. The flowchart in Figure 2.1 below shows the outline of model calibration with the Genetic Algorithm, which is followed by the explanation of several essential concepts for the GA.



Figure 2.1 Flowchart of GLOBAL.

• Key parameter selection by sensitivity analysis

The purpose of history matching is to match the dynamic data considering only key parameters, which have great impact on the objective function. Hence, sensitivity analysis should be performed to distinguish influential parameters to be calibrated. This parameter screening step needs several simulation runs with the high and low values of each parameter. To begin with, only one parameter is changed into either low or high value from the base model. Afterwards, the effects of each parameter on predefined objective functions will be ranked to identify sensitive parameters and remove insensitive parameters for the following history matching process. Parameters that has a large range of objective function with upper and lower boundaries of value are significant to be kept as main parameters.

Construction of objective function

One of the ways to evaluate the quality of history matching is to define objective function. During performing inverse problem, the simulation case, which has smaller value of objective function will have higher opportunity to be selected from the genetic algorithm. It quantifies the quality of history matching results from the weighted sum of squares difference between historical dynamic data and calculated production response from the simulation. The weighting factors (w_i) are incorporated since they take account into the importance of the observed values. x represents uncertain parameters.

$$f = \sum_{i=1}^{n_{obs}} w_i (f_i^{obs}(x) - f_i^{calc}(x))^2; \sum_{i=1}^{n_{obs}} w_i = 1$$
(2.1)

However, multi-objective problems usually have difficulty in determining the weighting factors since they are not readily available. Instead, objective function is defined by using sum of logarithm of the absolute misfits rather than weighting.

$$f(m_i) = f(m_1, m_2, m_3 \cdots, m_N) = \ln|\Delta p| + \ln|\Delta Q| +$$
(2.2)

where m is the list of global variables. By using the sum of logarithm residual, not only overall objective misfits are reduced continuously, but also conflicting objectives will be reconciled automatically.

Initial proxy construction by experimental design

Although GA has been expected to outperform other deterministic methods, a number of simulations required, especially for field cases with large sets of parameters, are challenging when it comes to computational costs. A proxy model allows the response surface to be used to filter potential cases, an objective function of which is larger than a predefined threshold. This procedure prevents redundant forward simulation, which enhances the feasibility of stochastic algorithms. In this research, Latin Hypercube design, one of the methods of experimental design, contributes to establishing the initial proxy as it gives the methods to cover a full range of each variable.

Response surface construction through Kriging

A response surface, the proxy model, is used to represent a true model or its simulation. It is indispensable when evaluating a random sample directly is computationally unreasonable. A response surface becomes more desirable to some degree as a new simulation takes place. However, as more and more experiments are conducted, the further improvement of a proxy model will be restricted if we use all of the current samples. Therefore, only a small number of experiments should be considered for Kriging in order to construct an accurate response surface.

• Operators to reproduce the generation

Evolutionary algorithms are determined by an iterative sequence of variation and selection operations (Schulze-Riegert et al. 2002). First, the selection process is performed by the combination of an objective function and the fitness of genomes that consist of binary strings of 0's or 1's (GA uses the binary strings to represent the solutions). It is encouraged to maximize the fitness while minimizing the misfit function between the observed and calculated data, which is equivalent to maximizing a fitness function $g(m_i)$. The probability of model m_i is given by:

$$P(m_i) = \frac{\exp[-\frac{f(m_i)}{T_n}]}{\sum_i \exp[-\frac{f(m_i)}{T_n}]}; \ T_n = \lambda^n T_0; \ 0 < \lambda < 1$$
(2.3)

If the models are fitter, they are much more likely to be chosen to reproduce the next generation. Secondly, crossover and mutation operators have been used for the diversity of population. By recombining the samples of the previous generation, crossover plays a pivotal role in producing various models. There are several algorithms to conduct the crossover based on different ways to choose the position where the binary bits can be switched. As the process of selection and crossover has been repeated, however, the variability of a polulation will be reduced inevitably. Mutation leads to increasing the diversity of a generation by giving a genome the opportunity to flip the bits with the very low probability.

• Cluster analysis

After the simulation has been finished, the stochastic method provides a reduced range of each parameter with history matching results. A number of updated models can be subdivided by a similarity among parameters. From the subset of calibrated cases, representative models will be selected for further comparison between the history matching results and the reference model.

The genetic algorithm starts with a first generation of a population, which consists of randomly generated individuals. The fitness of individuals in the population is evaluated to select possible individuals for a new population of a next generation. In general, the process of calibration will be completed if either it reaches the maximum number of generation or a certain fitness level has been achieved.

2.2 History Match Quality Index with Moving Linear Regression Analysis

2.2.1 Introduction

Evaluating how well the results of a simulation match observed field behavior is often controversial when it comes to history matching process. There has been qualitative methodologies, such as the subject matter expert, or quantitative methodologies, such as the sum of square of error, used for assessing the quality of history match. Each method has been performed quite successfully but cannot satisfy in all aspects. For a qualitative approach conducted by sophisticated engineers, reviewers do careful examination on the data visually and classify the results. This evaluation may vary depending on individuals` experience and their subjectivity, which leads to different judgements on every evaluation. On the other hand, more consistent assessments are conducted by calculating the misfit between historical dynamic data and calculated results. Although this approach allows us to obtain the degree of mismatch quantitatively, it can be readily influenced by outliers of the data, which originate in diverse sources of error. The following proposed method, the History Matching Quality Index (HMQI), offers a much more robust and reliable process to decide the quality of history matching results. The HMQI functions with the Moving Linear Regression Analysis (MLRA) in order to manage data outliers. In Figure 2.2, the outline for the combination of the MLRA and HMQI is summarized.



Figure 2.2 Workflow of MLRA and HMQI.

2.2.2 Moving Linear Regression Analysis

In general, random noises in the data make interpretation of history matching results less precise. Ideally, we would like to remove outliers while keeping the trend of original observed data before evaluating the history matching results. By removing possible outliers, the assessment of history matching results could be improved. Even if some data noises, which should be removed are discernible and straightforward to detect, many others may not. In addition, there has not been efficient way to examine possible outliers automatically and thus mostly one should have studied history data to screen out those outliers case-by-case. In this section, we introduce the systematic method to exclude plausible data outliers while preserving the initial tendency of measured data.

• Linear Regression (Jensen et al. 1997)

Regression is the methodology to identify and estimate the relationships between a dependent variable and independent variable. Linear Regression, one of widely used type of regression approaches, is that dependent variable can be represent as a linear combination of predictors like following equation (2.4).

$$Y = \beta_0 + \beta_1 X + \varepsilon \tag{2.4}$$

In constructed regression model above, the X is the predictor; Y is the response; ε is a random error; and β_0 and β_1 are the parameters of the regression model. Since the variation of X will have an impact on the result of changing Y, the Y is called a dependent variable. Consequently, the measurement of X and its accuracy are closely related to the errors. Sometimes, we have to use simplistic regression

models because it is difficult to measure the predictor correctly so that mostly prediction error should be considered. However, regression can still be the reliable approach for establishing a prediction model. Since the Y is not independent of X, we use conditional operator to obtain the mean value and variance of Y. Accordingly, we take the conditional expectation of Eq. (2.4) to give

$$E(Y|X = X_0) = \beta_0 + \beta_1 X_0 \tag{2.5}$$

We achieve Eq. (2.5) by assuming that $E(\varepsilon) = 0$. Similarly, the variance can be

$$Var(Y|X = X_0) = Var(\beta_0 + \beta_1 X + \varepsilon | X = X_0) = \sigma_{\varepsilon}^2$$
(2.6)

The equation (2.6) yields the variance of errors because the term, $\beta_0 + \beta_1 X$, is constant at $X = X_0$ and only the errors are not affected by variable X. Therefore, only the error term is left after taking variance.

• t distribution (Devore et al. 1999)

When we analyze the population, a sample from the population can be used to characterize the mean and standard deviation of the data. If we assume that the population has normal distribution, the sampling distribution is also normal for any size of sample. Accordingly, standardized variable $z, z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$, follows a standard normal distribution, which can be represented by z distribution curve. For large number of sample n, σ can be replaced by sample variance (s) since this substitution does not cause a lot of variability, which indicates that $\frac{\bar{x}-\mu}{s/\sqrt{n}}$ has generally normal distribution. On the other hand, it cannot be applied for small number of sample n since the value of sample variance may differ in every case of

sampling. Therefore, a new type of probability distribution has been needed, which is suitable for a small number of sample.

If $x_1, x_2, x_3 \cdots, x_n$ is a set of random sample from the population that has a normal distribution, the standardized variable t can be represented as follows.

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}} \tag{2.7}$$

Equation (2.7) has a type of probability distribution called t-distribution with n-1 degrees of freedom. The spread of t curves is affected by the number of degree of freedom. As the degree of freedom increases, t curves are approaching z curve (Figure 2.3). t critical values are required to take certain either central or cumulative areas under t curves and they are used to calculate t confidence interval. A t value is specified with certain value of confidence level, corresponding to the particular t curve area and degree of freedom.



Figure 2.3 Comparison of t curves to z curve

• Tolerance (Jensen et al. 1997)

The confidence interval for predetermined confidence level and certain degree of freedom could be used as the tolerance for excluding data outliers during the process of MLRA. The following shows that how the tolerance is obtained from the procedure of regression.

If we estimate $E(Y|X = X_0)$, an unbiased point estimator is

$$\widehat{Y}_0 = \widehat{\beta_0} + \widehat{\beta_1} X_0 \tag{2.8}$$

 \widehat{Y}_0 has variance

(The

$$Var(\widehat{Y}_0) = Var(\widehat{\beta}_0 + \widehat{\beta}_1 X_0) = Var(\overline{Y} + \widehat{\beta}_1 (X_0 - \overline{X}))$$

(Since $\hat{\beta}_0 = \overline{Y} - \hat{\beta}_1 \overline{X}$ by linear relationship)

$$= \sigma^2_{\varepsilon} \left(\frac{1}{l} + \frac{(X_0 - \bar{X})^2}{\sum (X_i - \bar{X})^2} \right)$$
(2.9)

(The variance of $\hat{\beta}_1$ is given by $\operatorname{Var}(\beta_1) = \frac{\sigma_{\varepsilon}^2}{\sum (x_i - \bar{x})^2}$)

When we replace σ_{ε}^2 by its estimate, this provides the following sample variance:

$$s_{\overline{Y_0}}^2 = MS_{\varepsilon}(\frac{1}{I} + \frac{(X_0 - \overline{X})^2}{\sum(X_i - \overline{X})^2})$$
(2.10)
estimate of σ_{ε}^2 is given by $MS_{\varepsilon} = \frac{\sum(y_i - \hat{y}_i)^2}{I - 2}$

The sample variance $(s_{\widehat{Y_0}}^2)$, the estimate for $Var(\widehat{Y_0})$, is then used to determine the $(1 - \alpha)$ confidence intervals for the estimate of $E(Y|X = X_0)$:

$$\widehat{Y}_{0} \pm t(\frac{\alpha}{2}, I-2)s_{\widehat{Y}_{0}}$$
 (2.11)

To take a confidence interval into account regarding a new response (X_0, Y^*) , we incorporate the variability of error $(Var(\varepsilon))$.

$$Var(Y^*) = Var(\widehat{Y}_0) + Var(\varepsilon) = \sigma^2_{\varepsilon} \left(1 + \frac{1}{I} + \frac{(X_0 - \overline{X})^2}{\sum (X_i - \overline{X})^2} \right)$$
(2.12)

Since the error (ϵ) is independent, the variance of the error can be included. Using the sample values, this results in the confidence interval:

$$\widehat{Y}_{0} \pm t(\frac{\alpha}{2}, I-2) \sqrt{MS_{\varepsilon}(1+\frac{1}{I}+\frac{(X_{0}-\bar{X})^{2}}{\sum(X_{i}-\bar{X})^{2}})}$$
(2.13)

- Steps for Moving Linear Regression Analysis (Darman et al. 2010)
 - (1) Determine the size of window on which to perform linear regression analysis. The size of the span is estimated by checking the measured data. Within that interval, estimated value behaves linearly assuming that a linear relationship can adequately represent the production history.
 - (2) Repeating until all the data points have been considered:
 - Conduct a linear regression analysis for the points preceding and following the evaluation point to calculate the estimated value and confidence interval.
 - b. Compute the difference between the observed data and the estimated value.
 - c. Compare the difference with the confidence radius of the estimated value to determine if the point is outlier which will be excluded.
 - (3) Keep the confidence interval to define a tolerance level for subsequent procedure to assign the History Match Quality Index

2.2.3 History Match Quality Index

One of the conventional methods of assessing the matching quality is the sum of square of error (SSE):

$$SSE = \sum_{i=1}^{n} w_i (d_i^{obs} - d_i^{calc.})^2$$
(2.14)

where d_i^{obs} and $d_i^{calc.}$ are the observed data and the calculated response at time step i; w_i is the weighting factor. This cumulative square of error has been widely used as an objective function to be minimized in the history matching process. Despite the fact that this standard approach allows an evaluation of history matching results to be judged from quantitative values, it cannot serve as a direct indication of match quality. Since it does not represent normalized values but relative values, we have to compare between more than two cases to determine if the simulation is well matched or not. The alternative way to do evaluation of match quality more conveniently uses the History Matching Quality Index (HMQI), which gives not only quantitative indication but also normalized values.

$$HMQI = \frac{1}{n} \sum_{i=1}^{n} w_i$$
 (2.15)

Where
$$w_i = \begin{cases} 1 & if \ abs(d_i^{\ obs} - d_i^{\ calc.}) < tol \\ 0 & if \ abs(d_i^{\ obs} - d_i^{\ calc.}) > tol \end{cases}$$

tol is the predefined tolerance level obtained from the procedure of MLRA.

The advantage of normalization is also suitable for analyzing the history matching results between different objective functions, such as BHP and water cut, which have

different magnitude of scales. We can incorporate the score of each objective function into one composite value for a particular simulation run, not considering the weighting factors. The following figure (Figure 2.4) shows the outline of how to obtain a single composite value for the simulation case.



Figure 2.4 Outline of obtaining the hierarchical HMQI.

CHAPTER III

APPLICATION TO CHEMICAL FLOODING

As reserves have been depleted, an efficient way to enhance oil recovery becomes significant. About 60% of oil is still remaining after the water flooding; chemical EOR has moved from a potential technique in producing remaining oil to one of the major interests regarding enhanced oil recovery. Therefore, reservoir management, such as model calibration, should be considered for chemical flooding at the same time. However, due to large amount of parameters related to complex mechanisms during the chemical flooding process, it is crucial to efficiently constrain subsurface uncertainties. In order to take several parameters into account together, we use the Genetic Algorithm to calibrate a model, which generates diverse plausible updated models. Accordingly, history matching should be followed by the evaluation of a number of results to determine which simulation case is closer to the true model.

This chapter is organized in mainly two parts. In subchapter 3.1, we implement the combination of the MLRA and the HMQI on the polymer flooding synthetic case after the history matching has been done. The results of HMQI are compared with the sum of square of error to discuss how adequately outliers are removed, which contributes to accurate history matching assessments. In subchapter 3.2, the ASP flooding synthetic case is used to exhibit the effectiveness of the HMQI in terms of different scales of objectives.

3.1 Application to Polymer Flooding

3.1.1 Introduction to Polymer Flooding

For hydrocarbon reservoirs in which the mobility of the displacing fluids is higher than the one of the displaced fluids, injecting water only into the reservoir cannot achieve a sufficient recovery factor. Polymer flooding has been commonly applied to chemical enhanced oil recovery techniques with a higher success rate. Shao et al. (2008) demonstrates that the incremental oil recovery is 15.3% and the oil production increased by polymer mass per ton is 75.14 at the area of northwest. In addition to the practicability of polymer flooding, favorable oil and polymer prices allow polymer flooding to be performed in many hydrocarbon reservoirs. The main objective of polymer flooding is to increase sweep efficiency by improving the mobility ratio of oil compared to water. Injected polymer helps water to have more viscosity, which will result in better oil displacement efficiency. The significant mechanism in polymer flooding is reducing viscous fingering; improving the water-injection profile (Sorbie 1991); reducing the relative permeability of water flow more than the permeability of oil flow through disproportionate permeability reduction (Sheng 2015). Furthermore, waterflooding becomes more practical with less water injected and produced since permeability has been remained reduced after polymer flooding (Sheng 2011). On top of that, polymer contributes to the synergy with other components when it is added into the combination process.

3.1.2 Model Description

This synthetic case shows a three-dimensional two-phase reservoir consisting of $15 \times 15 \times 3$ grid blocks (1640.5 ft × 1640.5 ft × 10.8 ft), with one injector and four producers perforated throughout the whole vertical layers. The area is heterogeneous permeability within each layer (Figure 3.1). Five-spot pattern is applied to flood this area with polymer for 762 days and total simulation time is 1540 days. UTCHEM, developed by the University of Texas at Austin, is used to run the simulations as the chemical flood simulator. Since polymer flooding typically does not change displacement efficiency whereas it contributes to improving sweep efficiency, the initial oil saturation before polymer flooding should be higher than the residual oil saturation at the final time steps (Figure 3.2). Description of major parameters for this synthetic case is given below (Table 3.1)



Figure 3.1 Permeability distribution in x direction.



Figure 3.2 (a) Initial oil saturation (b) Final oil saturation.

Parameter		Value
Well data	4 Producer	Pressure constrained at 100 psi
	1 Injector	Rate constrained at 1412.59 ft^3 /day
Components		Water, Oil, Surfactant, Polymer, Chloride, Calcium, Alcohol
Reservoir porosity		Avg. = 30%
Reservoir permeability		Avg. $k_h = 1552.5$ md, Avg. $k_v = 1548.5$ md
Water viscosity		0.73 cp
Oil viscosity		40 cp
Figure 3.3 and Figure 3.4 shows the measured data from the producer in the true model. Figure 3.3 represents cumulative injected polymer and cumulative produced polymer. One of the reasons for big drop from injected to produced polymer is polymer adsorption, which results from the fact that the porous medium adsorbs the polymer molecules. This adsorption can cause not only decreasing the mobility of aqueous phases but also delaying the propagation of the polymer front. In Figure 3.4, you can see that oil cut increases with regard to polymer injection and after polymer flooding oil cut gradually decreases.



Figure 3.3 Cumulative produced and injected polymer.



Figure 3.4 Oil cut and injected polymer.

3.1.3 Procedure

One of advantages of the HMQI with the MLRA over the SSE method is that during the process of MLRA, outliers that possibly come from errors can be screened out efficiently and automatically. That would lead to a more precise evaluation of history matching results, still preserving the tendency of observed data. On the other hand, the SSE method calculates misfit on every data point without reviewing historical dynamic data, which has an opportunity to result in inaccurate conclusion due to influence of outliers. In this subchapter, we demonstrate how the noisy observed data can impact on the assessment of history matching results. First, we add the some degree of noise to our synthetic polymer-flooding case in order to show whether the MLRA can conveniently exclude outliers. The case of having some suspicious points in history is able to cause unreliable evaluation on history matching results, compared to the original polymerflooding case. Therefore, by comparing the results of the SSE and the HMQI, we can validate how easily the HMQI can prevent biased history matching evaluation from the reference models with outliers. A whole workflow for a model calibration and an interpretation of history matching is shown below in Figure 3.5. A specific description for each step follows after the flowchart.



Figure 3.5 Flowchart of model calibration and history matching evaluation.

Step 1: Add noise to observe data

The main purpose of adding unnecessary data into the observed response is to demonstrate the impact of noises on history matching and a subsequent evaluation of the quality of history matching results. By introducing certain degree of aberrant data points randomly into a reference model, we can estimate not only the significance of removing outliers, but also the effectiveness of our HMQI method with the MLRA. Hence, a noisy polymer-flooding case (Case 2) has been used for the purpose of comparison with the original model (Case 1). After modifying the initial observed data, the Case 1 and Case 2 are shown in Figure 3.6.

Case 1	Case 2
Original model	Add noise at every 3 point

 Table 3.2 Description of Case 1 and Case 2.



Figure 3.6 Observed data after adding noise (every 3 points).

Step 2: Identify parameters to be matched

Before carrying out the history matching, key variables to be matched for the GA should be specified through the literature review of polymer flooding. In this polymer

flooding case, possible variables, such as relative permeability, permeability reduction due to polymer and effective porosity for polymer, has been considered over an objective function of oil cut. The endpoint (kr1low and kr2low) and the exponent (e1low and e2low) for water and oil phase at low trapping number are selected regarding relative permeability. Also, the effect of permeability reduction (crk) has been chosen since it affects reducing the mobility of the polymer rich phase. Lastly, effective porosity for polymer (ephi4) is used to account for inaccessible pore volume in the case of polymer. Table 3.3 describes the final list of parameters, including the value of low and high boundaries as well as that of initial and true model.

	Uncertainty	Reference	Base	Low	High
Relative Permeability	kr1low	0.2	0.8	0.1	1.0
	kr2low	1.0	0.2	0.1	1.0
	e1low	1.5	2.0	1.0	7.0
	e2low	2.0	3.5	1.0	7.0
Permeability Reduction	crk	0.2	0.35	0.0	0.6
Effective Porosity for Polymer	ephi4	0.85	0.95	0.5	1.2

Table 3.3 List of key parameters and their ranges (Polymer flooding).

Step 3: History matching results

Based on the selected parameters in the previous step, we have performed history matching process using the GLOBAL (software by MCERI group), which takes the Genetic Algorithm as a global search algorithm. Once the convergence criterion is met, updated models are selected as history matching results. Among updated simulation models, we have chosen five cases that have different values of uncertain parameters in order to rank the results. As shown in Figure 3.7 and Figure 3.8, although both cases have initial huge discrepancies in oil cut between the base and true model, eventually the updated models agree well with the observed response. Compared to the Case 1, selected simulation models for the Case 2 have larger difference at the early time because of included outliers. In addition, the overall history matching results of Case 2 slightly vary with those of Case 1 due to the effect of aberrant data points.



Figure 3.7 History matching results (Case 1).



Figure 3.8 History matching results (Case 2).

Step 4: Results in MLRA

Before evaluating history matching results, a thorough inspection on production history is required. Identifying possible outliers from the history data enables more improved assessment of updated models. While the MLRA is performed, every data point is able to calculate the confidence interval of the estimated value with certain size of span (n) and t-value (Table 3.4). The size of window (n) for linear regression can be chosen by examining the observed data. For this case, t-value has been selected with the degree of freedom (n=10) and 90% confidence interval.

Case 1			
Window (n)	8		
t-value (90% confidence interval)	1.860		
Case 2			
Window (n)	10		
t-value (90% confidence interval)	1.812		

Table 3.4 Key input parameters for the MLRA (Polymer flooding).

That confidence interval serves not only as the tolerance level for screening out outliers but also as the criteria when assigning the quality index at the next step. The observed data, which is located above upper bound (red dash line) or below lower bound (blue dash line) from the estimated value, has an opportunity of being errors. Accordingly, they need to be marked, which are no longer considered for the following steps. As we can expect, the Case1 does not have any suspicious data (Figure 3.9) whereas the Case 2 has some abnormal data that are successfully excluded during the process of the MLRA (Figure 3.10)



Figure 3.9 Result from MLRA (Case 1).



Figure 3.10 Result from MLRA (Case 2).

Step 5: Results in HMQI

Once the several well-matched history matching are selected, the quality index is assigned on each data point, depending on the tolerance level that is predefined at the previous step. If the difference between the measured and calculated data is within the tolerance level, that data point obtains the index of 1. Otherwise, the index of 0 is assigned, which implies that simulated response is located in unacceptable range. After assigning the index on every point, overall scores of wells can be achieved by averaging. These scores can be used as a criteria to decide the quality of history matching well by well. Furthermore, if we average indexes from all wells to obtain one single composite value for the simulation run, it becomes proper standards with respect to the comparison of history matching results. Figure 3.11 represents the averaged HMQI from all wells, indicating the quality of history matching for oil cut. Here, since we only use one objective, this HMQI is identical to the indication of history matching quality for each simulation run. Although Case 2 has a lot of noisy data, the result of HMQI of the Case 2 shows higher value than that of Case 1. Because the value of the HMQI is affected by t value, which is determined by confidence level and the degree of freedom, the value of HMQI means that how many points are located within the predefined tolerance. Due to a large number of outliers in observed data, the size of span for Case 2 is bigger than for Case 1, which leads to wider confidence interval. It allows the Case 2 to have a bit more higher value of HMQI.



Figure 3.11 Results from HMQI (Oil Cut): Case 1 (left) and Case 2 (right).

3.1.4 Results and Discussion

Figure 3.12 and Figure 3.13 show the comparison of the HMQI to the SSE method and the L1-norm for the Case 1 and Case 2 respectively, which have different degrees of noises in their observed data. The effect of data outliers can be noticed from the distinct results of the Case 1 and Case 2. In the Case 1, the result of the HMQI is consistent with that of the SSE. In contrast, the HMQI indicates a different simulation case as a good history matching result for the Case 2, which is different from the SSE method. Since the Case 2 has more data outliers than the Case 1, the calculation of the data misfit has been significantly affected by the outliers. In Figure 3.14, it is noticed that the considerable portion of data misfit derives from the doubtful data in Case 2 when calculating the SSE. This perceptible amount of misfit from outliers implies that those possible noisy data can substantially influence evaluation of history matching results, based on the data misfit calculation. In addition, it is possible that worse history matching results that are close to noise points can result in smaller data misfit, which causes an erroneous conclusion as a better history matching model. However, the HMQI with the MLRA ignores aberrant data before the evaluation of history matching so that the quality of history matching results can be determined only by the meaningful observed data. Table 3.5 shows the list of global variables and comparison of two realizations (model 143 and 144) with the reference model for Case 2. Updated parameters of model 143, which has been ranked as the best case according to the HMQI, are closer to those of true model than model 144. This indicates that the HMQI are more effective to determine the best-matched history matching result than the calculation of misfit, including the ability to screen-out the suspicious data automatically.



Figure 3.12 Comparison of the HMQI to the SSE (Case 1).



Figure 3.13 Comparison of the HMQI to the SSE (Case 2).





	Uncertainty	Reference	143	144
	kr1low	0.2	0.34	0.34
Relative Permeability	kr2low	1.0	0.94	0.82
	e1low	1.5	2.6	2.6
	e2low	2.0	1.0	1.0
Permeability Reduction	crk	0.2	0.28	0.32
Effective Porosity for Polymer	ephi4	0.85	0.87	0.87

Table 3.5 Comparison of key global parameters.

3.2 Application to ASP Flooding

3.2.1 Introduction to ASP Flooding

Despite the considerable costs of chemicals, ASP (Alkaline-Surfactant-Polymer) flooding has been proposed as one of the promising enhanced oil recovery techniques. Typically, chemical processes have been failed to overcome inherent disadvantages, such as significant amounts of surfactant loss due to adsorption. The ASP flooding is carried out by injecting the combination of alkali, surfactant and polymer together to solve those problems and also obtains the synergy of those components. The primary objective of ASP flooding is to mobilize the remaining oil mainly by reducing the oil-water interfacial tension (IFT). First of all, alkali reacts with the acid component in a crude oil, generating the surfactant in situ. This in-situ surfactant, petroleum soap, is different from an injected, artificial surfactant. In addition, alkali prevents the retention of surfactant on the rock

surface, which allows small amounts of surfactant to be injected during the ASP process. Secondly, surfactant, including the one generated from the alkali, plays a significant role in lowering the interfacial tension, which enhances the microscopic sweep efficiency (Nelson et al., 1984). As surfactant is added to the system, the hydrophilic part of the surfactant is directed into aqueous phase whereas hydrophobic part toward hydrocarbon phases. Decrease in IFT can be achieved by the increase of the solubility between water and oil resulting from above mechanism. At last, the addition of polymer supports the progress of the sweep efficiency and makes emulsions, which are created by soap and surfactant, stable because of its high viscosity to delay coalescence (Sheng 2013). When it comes to the adsorption, alkaline and polymer injections with surfactant help to reduce the surfactant adsorption significantly, thereby decreasing the required amount of chemicals to be used. As an additional advantage, the mixture of surfactant with the soap generated in situ allows a wider range of salinity in which the interfacial tension is low since the petroleum soap and synthetic surfactant have different optimum salinity. The synergy of ASP flooding has been demonstrated by comparing with other chemical flooding methods showing that the oil recovery of the ASP flooding is even much higher than the sum of other methods (Olsen et al. 1990). The recovery factors from ASP, Alkaline and Polymer flooding are 45.3%, 10% and 11.6% respectively.

3.2.2 Model Description

The ASP flooding case, the pilot application, is history matched through the calibration of reservoir properties, especially with regard to permeability and adsorption.

The three-dimensional grid consists of $15 \times 15 \times 36$ cells with the pilot area of 492 ft × 492 ft × 157.5 ft. The initial oil saturation in the reservoir model with the pattern of 5 wells—4 injectors and 1 producers— is shown in Figure 3.15. Four peripheral injectors flood this area, starting from water injection for the first 122 days through ASP flooding for 150 days, and lastly polymer has been injected for 300 days with the total simulation time of 578 days. In order to capture the essential chemical and flow properties of an ASP flood, a comprehensive reservoir-simulation model is built by the chemical compositional simulator UTCHEM, developed by the University of Texas at Austin. It has been known as the most advanced chemical flood simulator, which is used to run simulation for multiphase, multicomponent and three-dimensional in the displacement processes at both laboratory and field cases. The permeability and porosity distribution are shown in Figure 3.16. Other key parameters as an input for the simulation are summarized in Table 3.6.



Figure 3.15 Initial oil saturation.



Figure 3.16 (a) Permeability distribution (b) Porosity distribution.

Par	Parameter Value	
Well data	1 Producer	Pressure constrained at 1300 psi
	4 Injectors	Rate constrained at 2105.5 ft^3/day
Component	с с	Water, Oil, Surfactant, Polymer, Anion, Calcium,
Component	5	Sodium, Hydrogen, Alkali
Reservoir p	ermeability	Avg. = 3511 md, $\frac{k_v}{k_h} = 0.3$
Initial reser	voir pressure	1436 psia
Depth		2632 ft
Water visco	osity	0.48 cp
Oil viscosit	У	17 cp

Table 3.6 Input parameters for the simulation model.

The production of oil obviously increase between 0.19 PV and 0.43 PV, which is during the ASP flooding and about 54% of the initial amount of oil has been recovered (Figure 3.17). It implies that after significant drop in terms of oil cut with the waterflooding, the ASP flooding definitely helps to improve the oil recovery. At the beginning stage of polymer flooding, oil cut goes to zero because of a large amount of microeumulsion. One of the main issues in chemical flooding is adsorption of surfactant and polymer, which are shown in Figure 3.18 below. The difference between cumulative surfactant injected and produced, the same as for the polymer, represents the amount of adsorption.



Figure 3.17 Oil cut, oil saturation and oil recovery.



Figure 3.18 Adsorption of surfactant (above) and polymer (below) in reference model.

3.2.3 Procedure

The history matching process for chemical flooding, which is involved with plenty of uncertain parameters, often needs the investigation of objective functions on the basis of the production history. After determining which objectives are more global and dominant among several types of objectives, we can identify corresponding parameters to a certain objective. Eventually, by classifying objective functions, we do history matching in hierarchical manner in order to achieve more efficient way of model calibration. However, while evaluating history matching results in terms of several objective functions, the scale of each objective is not negligible. The SSE of all objective functions is affected by mainly the one having large scale, not considering the influence of every objective equivalently. In order to reduce the impact of different magnitude of scale, an objective function should take a proper weight factor that is normally not available or objectives have to be normalized before calculating the misfit. Therefore, in this subchapter, we make a comparison between the SSE method and the HMQI to show how effectively the HMQI approach manages objective functions, which has a various magnitude of scale. All the procedure regarding evaluation of history matching results is shown in Figure 3.19.



Figure 3.19 Flowchart of model calibration and history matching evaluation.

Step 1: Identify parameters to be matched

In order to evaluate the influence of possible parameters on production history, a set of variables, regarding relative permeability and adsorption in ASP flooding, has been selected. Sensitivity analysis has been already performed over the objective functions, such as oil cut, chloride concentration history, surfactant concentration and polymer concentration (Zhang 2014). The outcome of sensitivity analysis is displayed in Figure 3.20 with the range of each parameter used for the sensitivity studies in Table 3.7. According to the results of sensitivity analysis, the model calibration has been divided in a hierarchical manner. Since the uncertain parameters of relative permeability like the endpoint and exponent for water, oil and microemulsion phase at low trapping number (kr1low, kr2low, kr3low, e1low and e3low) are dominant for all the objective functions, those parameters has been matched at the first stage of history matching over the objective functions of oil cut and chloride concentration. On top of that, due to increasing impact of parameters, associated with adsorption (ad31, b3, ad41 and b4) on history data of surfactant and polymer concentration, the second stage of history matching has been conducted on those uncertain parameters although they still are not influential, compared to relative permeability.

	Uncertainty	Reference	Base	Low	High
	kr1low	0.6	0.3	0.1	0.7
	kr2low	0.93	0.5	0.1	1.0
Relative Permeability	kr3low	0.6	0.3	0.1	1.0
	e1low	2.5	5	2	6
	e3low	2.5	5	1	7
	ad31	1.7	1.4	1.2	2.0
Surfactant Adsorption	ad32	0.05	0.08	0.01	0.09
	b3	1000	700	500	1500
	ad41	2.0	2.3	1.5	2.5
Polymer Adsorption	ad42	0.1	0.06	0.05	0.15
	b4	100	70	50	150

Table 3.7 List of key parameters and their ranges (ASP flooding).



Figure 3.20 Tornado charts for sensitivity analysis: oil cut, surfactant concentration, chloride concentration and polymer concentration.

Step 2: Results from first stage of history matching

Based on the selected parameters in the previous step, we have performed history matching using the GLOBAL (software by MCERI group), which takes the GA as a global search algorithm. Once the convergence criterion is met, updated models are selected as a history matching results. As shown in Figure 3.21, although there are initially huge discrepancies in oil cut and chloride concentration between the base and true model, most of updated simulation cases show significant improvement.



Figure 3.21 History matching results (a) Oil cut and (b) Chloride concentration.

Step 3: Results from second stage of history matching

After the first stage of history matching, local and less dominating parameters regarding surfactant and polymer adsorption have been matched while honoring the results of dominating parameters in the previous history matching with the calibrated range. In addition, the surfactant and polymer concentration has been included as the new objective functions for the second stage. Figure 3.22 shows history matching results for all the objective functions in the second stage of history matching. Oil cut and chloride concentration are already well-matched with the initial model due to the first stage of calibration. In contrast, it has been noticed that the updated models for surfactant and polymer concentration has been close to the production history, compared to the base model, which indicates a good agreement with the observed data.



Figure 3.22 History matching results (a) Oil cut, (b) Chloride conc., (c) Surfactant conc. and (d) Polymer conc.

Step 4: Results in MLRA

The results from the process of MLRA can be seen in Figure 3.23. Each graph shows the result of each objective function. Since this ASP flooding case has only one

production well, we do not show the result of the well particularly. Reference model does not have any data errors so that all the observed data are placed within the range of the confidence interval. Red and blue dashed line represent upper-bound and lower-bound of confidence interval respectively. Although this step does not contribute to the examination of probable data outliers, it provides the tolerance that is needed for the subsequent step. The size of window (n) for linear regression can be chosen by examining the observed data. For this case, t-value has been selected with the degree of freedom (n=10) and 90% confidence interval for both 1st stage and 2nd stage of history matching evaluation (Table 3.8).

Table 3.8 Key input parameters for the MLRA (ASP flooding).

Window (n)	10
t-value (90% confidence interval)	1.812



Figure 3.23 Results from MLRA.

Step 5: Results in HMQI

Similar to the polymer-flooding case, we evaluate the history matching results by comparing the results of a reference and simulation models. After assigning the index on every data point, the single number of quality index of each simulation run is shown in Figure 3.24 by averaging for all the objectives. As a result of HMQI, the simulation cases, #158 and #20, have been selected as the best-matched history matching result respectively for each stage. Furthermore, since this ASP flooding case uses four objective functions that have different scales for model calibration, overall HMQI of one simulation run can be indicative of the history matching quality, integrating results of all the objectives

readily with the normalized value. Regardless of the scale of each objective, which ranges from the order of 10^{-4} to 10^{-1} , the total HMQI is within the range of 0 to 1.



Figure 3.24 Results from HMQI (1st stage).





3.2.4 Results and Discussion

Obtaining not only the overall index of each objective, but also overall one simulation model can be achieved by averaging. The averaging of HMQI among different objectives is not difficult because HMQI provides already normalized value, while additional normalizing processes should be required for the calculation of misfit. This advantage also allows absolute evaluation on history matching results as well as relative comparison among several simulation cases. In other words, a score of 0.9 implies that 90% of data points are located within the predefined interval, which is tolerance. Therefore, the HMQI approach successfully functions as the indicator of history matching quality for the evaluation of one simulation case. On the other hand, although we normalize all objectives in order to reduce the effect of different scale of objectives, the total value of SSE is meaningless when considering only one simulation case. Another way to rank the results of history matching, Sum of Square of Error (SSE), is shown in Figure 3.26. Due to the large difference of the scale of objectives, especially between oil cut and surfactant concentration, normalization has been performed before calculating the misfit. However, the overall SSE is still affected by certain objective. In other words, the ranking of the simulation models primarily reflects oil cut and surfactant concentration, not considering every objectives identically since the sum of the square of errors has different degree of value (Figure 3.28). It could result in an inaccurate conclusion during the evaluation of history matching quality. On the other hand, the advantage of HMQI, which already has the normalized value between 0 and 1, provides more consistent and robust way to rank the simulation models since the magnitude of unit of objectives does

not impact on the evaluation. In Figure 3.26 and Figure 3.27, the smallest SSE is not the case that has the highest value of HMQI. The SSE method may fail to reach the right conclusion because its result does not take all the objectives into account evenly. For most of the parameters, the highest ranking Case 158 from the HMQI shows better improvement than Case 61 from the SSE at the 1st stage (Table 3.9). Similarly, best-matched Case 20 from the HMQI at the 2nd stage has closer parameters to the reference model (Table 3.10).



Figure 3.26 Comparison of the results between HMQI (left) and SSE (right) (1st stage).



Figure 3.27 Comparison of the results between HMQI (left) and SSE (right) (2nd stage).





	Uncertainty	Reference	Case 158	Case 61
Relative Permeability	kr1low 0.6		0.46	0.42
	kr2low 0.93 0.94		0.94	0.82
	kr3low	0.6	0.34	0.52
	e1low	2.5	2.27	2.27
	e3low	2.5	2.2	2.2

Table 3.9 Comparison of key global parameters (1st stage).

Table 3.10 List of key parameters and their ranges (2nd stage)

	Uncertainty	Reference	Case 20	Case 94
Surfactant Adsorption	ad31	1.7	1.71	1.68
	b3	1000	746	733
Polymer Adsorption	ad41	2.0	2.16	2.4
	b4	100	95	123

CHAPTER IV

FIELD APPLICATION

This chapter gives two field applications of the History Matching Quality Index with the Moving Linear Regression Analysis. Using those methods, we show the capacity of this approach when it comes to readily evaluating history matching results. We implement the HMQI in field cases that were calibrated by a structured history matching. Each example performs the global calibration, followed by the local calibration, such as water cut matching. Similar to the synthetic case, we apply the MLRA in order to exclude possible data outliers. Then, we also use the HMQI to quantify the degree of history matching results and thus compare several simulation models to determine the best matched case efficiently. These two cases validate the convenience and simplicity of this approach and show the feasibility and practicability by applying the method to the real field cases successfully.

This chapter also includes the usability of this new evaluating method with regard to displaying the improvement of history matching results quantitatively. In contrast to the Sum of Square of Error, the HMQI represents the quality of a simulated model as a normalized number, allowing us to observe the progress of the history matched model from the initial model.

4.1 Application to Channelized Reservoir

The data regarding this field has been provided by an oil and gas company for the purpose of research and education. Due to confidentiality, the references of information related to this field, such as the location and production history, are not given. This reservoir has 7 producers and 2 water injectors, which is characterized as sand-field channels. The model of the reservoir contains 50 by 30 by 20 cells and the geologic model consists of 7 geological regions that are divided by seismic amplitude difference. The structure and well location of the reservoir are given in Figure 4.1.



Figure 4.1 Structure and well location of the reservoir (permeability distribution).

4.1.1 Procedure

The history matching workflow for this channelized reservoir case is divided into two stages: global calibration and local calibration. In the global calibration, we use the Genetic Algorithm to match the global parameters that are related to reservoir energy, such as pore volume multipliers, transmissibility multipliers and aquifer strength. In order to calibrate the model at a global level, field total fluid productions, bottomhole pressure (BHP) and modular dynamic tester (MDT) are selected as objectives. The Following stages for local calibration use local parameters to match well by well water-cut response. Globally updated models are calibrated again using a streamline-based history matching software in MCERI (Destiny). Evaluation has been conducted on each stage to determine the overall ranking of the history matching results. Figure 4.2 illustrates a complete flowchart of the evaluation of history matching results, as well as the model calibration process, followed by detailed explanation and its result of each step.



Figure 4.2 Flowchart of model calibration and history matching evaluation.

Global calibration

Step 1: Define uncertain parameters and their range for GLOBAL

Table 4.1 shows the selected list of global parameters and their ranges for global model calibration. Multipliers for horizontal permeability and pore volume are used instead of real values and also aquifer strength is included.

Uncertainty (Multipliers)	Base	Low	High
PERMX3	1	0.1	1
PERMX5	1	0.1	2
PERMX6	1	0.1	1
K_V/K_H	1	0.05	1
MULTPV5	1	0.1	1.0
MULTPV6	1	0.1	1.5
AQUIFER	300	100	500

Table 4.1 List of key global parameters and their ranges.

Step 2: History matching results

After global model calibration, bottomhole pressure shows improvement from the initial response. Figure 4.3 represents some of the enhanced well responses, which is closer to the history data than initial response. Originally, BHP of the base model is higher than the actual data. However, the overall BHP has been adjusted to the production history after we update global key parameters.


Figure 4.3 History matching results after global calibration (BHP).

Step 3: Results in MLRA (BHP)

For the purpose of obtaining the tolerance and excluding suspicious data, the MLRA has been performed. The major input parameters to implement the MLRA are shown in Table 4.2. Appropriate size of the span (n) should be selected after examining observed data. With the size of the window (n) and the certain level of confidence interval, the t-value is determined for the regression. Figure 4.4 shows that the outcome after conducting the MLRA. The history data of BHP is relatively smooth and does not have many outliers, which means most of the observed data are located within the confidence

interval on the basis of estimated values (between upper-bound and lower-bound, which are a red dashed line and a blue dashed line respectively).

Window (n)	8
t-value (90% confidence interval)	1.860









Step 4: Results in HMQI (BHP)

Based on the tolerance from the previous step, each data point of history matching results has been evaluated whether the difference between the actual and updated response is smaller than the tolerance. Consequently, one simulation run has a single composite HMQI value by averaging indexes of every point. We use this HMQI value to assess the history matching quality. Not only does HMQI represent the relative comparison between several simulation models, but also it indicates the absolute evaluation. For the objective of bottomhole pressure, the simulation of #610 has been chosen as a well matched model. It will be considered for overall objectives together after the local calibration.



Figure 4.5 Results from HMQI (BHP).

Local calibration

Step 5: Perform Destiny

Further history matching procedures have been conducted in order to match the well-level parameters. Compared to the initial response, updated models show a reasonable match although well A9 has not been improved sufficiently.



Figure 4.6 History matching results after local calibration (WWCT).

Step 6: Results in MLRA (WCT)

The process of MLRA is the same as the application in the BHP. Table 4.3 shows the list of the size of the window and corresponding t-value with the certain confidence interval. Contrary to bottomhole pressure, water cut has some doubtful data that has been excluded by the MLRA for accurate history matching evaluation (Figure.4.7). Black unfilled dots indicate possible data outliers and black filled dots are preserved data.

Window (n)	10
t-value (90% confidence interval)	1.812

Table 4.3 Key input parameters for the MLRA (WWCT).



A5

A9



A8

Figure 4.7 Result from MLRA (WWCT).

Step 7: Results in HMQI (WCT)

Based on the tolerance given by the process of MLRA, Index has been assigned on each data point by comparing the difference between simulated and observed data. Similar to bottomhole pressure, each simulation model has one HMQI value, which can be used as a criteria of ranking the history matching results. The simulation of #610 has been selected as the best matched model.



Figure 4.8 Results from HMQI (WWCT).

4.1.2 Results and Discussion

In this subchapter, the HMQI approach is successfully applied to a field case. First, we eliminate suspicious data outliers from the observed data by performing the Moving Linear Regression. Secondly, our proposed approach provides a consistent way to make a judgement on the history matching results for a large number of wells. Although experts can visually assess the history matching results well by well on the basis of their experience, they have no clear criteria to integrate the assessment for overall wells. Thirdly, water-cut matching and bottomhole pressure matching, which have a distinguishable difference in their scale, have been evaluated equally. In Figure 4.9, the ranking of history matching results from the HMQI does not agree with the SSE method. This phenomenon might occur because the SSE method does not consider bottomhole pressure and water cut identical when calculating misfit. Figure 4.10 shows that the amount of misfit for water-cut matching is much larger than that of misfit for bottomhole pressure even if each objective is already normalized before calculating misfit. Accordingly, the total sum of square of error is significantly affected by water-cut misfit, resulting in biased evaluation for history matching results. At last, the overall index from the HMQI approach not only allows relative comparison among several simulation cases but also implies absolute quality of history matching. For example, the Case 610, relatively the best-matched model among updated models, has an index of 0.8473, which means that the model is within the predefined tolerance with approximately 85% possibility.



Figure 4.9 Results from HMQI (left) and SSE (right) for all objectives.



Figure 4.10 Results of overall SSE (top), BHP SSE (bottom left) and WWCT (bottom right).

4.2 Application to Norne Field

All grid information, associated with the properties and history of production data, were provided by the operator. The Norne field was discovered in 1991 and began producing in November 1997. The geologic model of the Norne field consists of five zones; the porosity ranges from 25 to 30%; the permeability has been within 20 to 2500mD (Steffensen and Karstade, 1995; Osdal et al. 2006). The reservoir model includes 113,334 cells (active cells: 44,927) and has 27 producers and 9 injectors (Figure 4.11) The history matching period was selected from 1997 to 2006 for the objective functions of water cut and bottomhole pressure.

Previously, Kam (2015) has already matched history data using joint inversion with a multiscale approach to capture from larger- to smaller-scale heterogeneity efficiently. He proposed a novel semi-analytic approach to compute the sensitivity of the bottomhole pressure data in order to solve the problem that calibration through a streamline-based method does not maintain pressure matches. In this section, we apply the HMQI to quantify the degree of history matching quality, compared to the initial model. By performing the HMQI to this field, which has a lot of producing wells and different scale of objective functions, we could assess simulated models quantitatively and conveniently.



Figure 4.11 Structure and well location of the Norne field.

4.2.1 Procedure



Figure 4.12 Flowchart of model calibration and history matching evaluation.

Step 1: History matching results

After model calibration, some of well responses for bottomhole pressure and water cut are shown in Figure 4.13. Compared to an initial model, the final updated model shows improvement or maintains the initial response if it was already close to the history data.



Figure 4.13 History matching results: bottomhole pressure (left column) and water cut (right column).

Step 2: Results in MLRA

For the purpose of obtaining the tolerance and excluding suspicious data, the MLRA has been performed. The major input parameters to implement the MLRA are shown in Table 4.4. Appropriate size of the span (n) should be selected after examining observed data. With the size of window (n) and certain level of confidence interval, t-value is determined for the regression. Figure 4.14 shows the outcome after conducting the MLRA. The history data has some outliers, which are located outside of the confidence interval on the basis of estimated values.

Bottomhole pressure	
Window (n)	8
t-value (90% confidence interval)	1.860
Water cut	
Window (n)	10
t-value (90% confidence interval)	1.812

Table 4.4 Key input parameters for the MLRA (Norne field).



Figure 4.14 Result from MLRA: bottomhole pressure (left column) and water cut (right column).

Step 3: Results in HMQI

Based on the tolerance from the previous step, each data point of a history matching result has been evaluated whether the difference between actual and updated response is smaller than the tolerance. Consequently, one simulation run has a single composite HMQI value by averaging indexes of every point. We use the value of HMQI to assess the history matching quality. In Figure 4.15, the comparison of the HMQI between an initial model and a final history matched model is given for each objective function.



Figure 4.15 Results from HMQI: BHP (left) and WWCT (right).

4.2.2 Results and Discussion

In this field application, we also show feasibility of the HMQI with the MLRA method. This field case is composed of a larger number of producing wells, which requires a more careful and consistent tool to evaluate each well response. The overall HMQI of both initial and history matched models are compared in Figure. 4.16. Since total HMQI

itself represents the quality of history matching results, further processing is not needed to quantify the improvement of an updated model. Therefore, it is noticed that the final history matched model shows about 18% progress, based on the HMQI. In Figure 4.17, the result of HMQI is then compared from the normalized data misfit. For the bottomhole pressure, the History Matching Quality Index shows less improvement than the normalized data misfit does. When the HMQI is assigned, relying on the predefined tolerance, all the history matching results that are not within the tolerance are regarded as the same. This approach does not have exhaustive tools to differentiate among data points, which are not within the tolerance. In particular, bottomhole pressure responses for both the initial and final models have some points that are not located within the tolerance even if the final model is relatively closer to the observed data than the initial one. As a result, the HMQI cannot take the difference between the base and history matched model into account, considering them identically as an index of 0.



Figure 4.16 HMQI for the initial model (left) and the final updated model (right).



Figure 4.17 HMQI (left) and normalized data misfit (right).

CHAPTER V

CONCLUSION AND RECOMMENDATIONS

In this work, we have presented and summarized the History Matching Quality Index (HMQI) for evaluation of history matching results, including identifying and preparing the observed data efficiently with the Moving Linear Regression Analysis (MLRA). The HMQI has proven its ability to cleanse data of outliers and to eliminate subjectivity with the case study from synthetic to field cases. The Sum of Square of Error (SSE) method has been selected for a comparison to validate the feasibility of the HMQI.

First, we demonstrated that the MLRA has efficiently excluded suspicious data points such as outliers. In order to prevent bias toward outliers in the evaluation of history matching results, careful analysis of the observed data and screening out outliers are required. With a certain degree of freedom and confidence level, the t-value is determined to perform the moving linear regression, which results in the confidence interval (tolerance). Depending on this confidence interval, the observed data that are not located within that interval was marked as the suspicious data outliers. Accordingly, marked points are not used to conduct the following steps when evaluating the history matching results. Furthermore, this process allows us to identify the trend of observed data simultaneously, not simply quantifying the degree of history matching quality.

Second, we considered objective functions that have a different magnitude of scale equally through the HMQI. We tested the HMQI approach with the ASP flooding case, which used four types of objective functions, it leads to easier ranking, compared to the SSE method. Since the value of HMQI is already normalized, it does not require additional processes when comparing the results among several simulation cases. Consequently, the HMQI enables comprehensive decisions to be made, including all objectives as well as wells with one single composite value for the simulation run.

Finally, the practicability of the HMQI with MLRA has been confirmed from field applications. For both channelized reservoir and Norne field applications, the HMQI showed advantages over the SSE method when it comes to excluding outliers, normalized values and a large number of wells. However, in the field case study that shows less improvement of history matching results, compared to the synthetic case, the different degrees of quality of history matching results are not well captured when they are outside of the predefined tolerance. Once the history matching results are not located within the tolerance, they are considered all the same way as an index of 0. The developed HMQI could benefit from the consideration of more detailed evaluation from this issue.

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