Evaluations and the Formation and Maintenance of Performance Expectations*

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I. The Problem

Bales and his associates (Bales et al., 1951; Bales, 1953; Bales and Slater, 1955; Heinicke and Bales, 1953) have shown that task performing groups whose members are the same in age, sex, race, education and occupation (that is, are initially status equals) tend to develop a stable power and prestige order. This power and prestige order is reflected in the inequalities which develop in activity initiated and received and in ratings members make of who had the best ideas, who guided the group discussion, and who demonstrated leadership. Others (Harvey, 1953; Sherif et al, 1955; Whyte, 1943) have found that an already established power and prestige order will determine the evaluations of performances, anticipations for future performances, and influence exercised.

Berger and Conner (1966) argue that such findings can be explained by assuming that the members of these groups come to develop, through time, stable conceptions of the performance capacities of each other. These conceptions, or performance expectations, are beliefs about the relative task abilities of individuals that the members of these groups come to hold. Typically these expectations are differentiated; that is, they represent conceptions of inequalities in the task abilities of group members. If differentiated, these performance expectations legitimate and determine inequalities in opportunities to perform, in performance rates, in evaluations of members' contributions, and in the relative influence of different members on the decisions of the group. Further, they argue that these inequalities in behavior, which are determined by performance expectations, operate to maintain these expectations. Thus, once established, the power and prestige order of such groups tends to be stable.[1]

But the assumption of a structure of performance expectations, to account for the known features of the power and prestige order of these groups, itself gives rise to a basic question: how are such performance expectations formed? What are the processes by which differentiated performance conceptions emerge in groups whose members are initially status equals, and how do these conceptions become stable? This is the problem to which this paper is addressed. More specifically, we shall be concerned with isolating and conceptualizing _one_ of the processes which we believe operates in the formation and maintenance of performance expectations. This is a process in which performance expectations are conceived of as emerging from, and being maintained by, the evaluations of performances individuals make in task-oriented situations. We shall refer to this as the _evaluation-expectation_ process.

In the next section we shall present a set of assumptions to describe the operation of the evaluation-expectation process as it occurs within a specific set of interaction conditions. Following this, a mathematical model for our theory is developed. This model enables us to describe the features of the evaluation-expectation process in a highly precise manner. In the remainder of this paper, an experimental test of our theory and model is described, and the status of our formulation is evaluated in the light of our findings from this investigation.

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[1]For an application and extension of this argument to the case of task-oriented groups whose members are initially differentiated in terms of socially valued status characteristics (sex, race, occupation, etc.) see Berger et al. (1966a).

II. Theory

A. Scope Conditions.--The theory to be presented is seen to apply to small task-focussed groups, whose members are initially undifferentiated and who are collectively oriented to solving some problem. Implicit in this type of characterization of groups is the idea of a social situation in which there obtains a particular set of initial status and task conditions. Our first task is to specify these conditions.

We imagine a group containing two or more actors, p, o_1, o_2 . . ., o_n. However, we view the group from the point of view of one actor, say p. Strictly speaking the other actors, o_1, o_2 . . ., o_n, are objects of orientation to p. For purposes of developing and experimentally testing our theory we shall confine our attention to a group with three persons, say p, o and q.

We assume p, o and q are engaged in the solution of some task, T, which for simplicity we view as having only two outcomes--"success" or "failure." T may be almost any kind of activity, but for the theory to apply it must involve a series of contributions or problem-solving attempts by one or more of the actors. Moreover, the members of the group are committed to the successful completion of the task, and it is both legitimate and crucial for them to take each other's behavior into account in order to achieve this goal. In this sense the group is "task-focussed," and its members are "collectively oriented" in solving their problem.

One way in which we may think about performance expectations is in terms of the idea of task ability. If a person were believed to have a great deal of task ability, then he would be expected to perform well, and vice versa. So we require as a condition for our situation that there be some ability or skill associated with successfully completing T. We will speak of a specific performance characteristic, C, which has two states, high and low. Ordinarily a person who possessed the high state of C would be good at the task while a person with the low state would be poor at the task. For example, if the task were to decide jointly a series of moves in a chess game, then C would be chess playing ability, and, as a theoretical simplification, we would think of there being only good players (those with the high state of C) and poor players (those with the low state of C).

Since we are concerned about the formation of beliefs about task ability we must insure that p, o and q initially have no such beliefs. Hence we require that two additional things be true of the actors and their situation. First, they must initially have no direct knowledge of their abilities--that is, no direct knowledge of the states of C they or the other members of the group possess. Second, they must not differ on other characteristics which have status value for them (e.g., occupation, age, race and sex) from which they could infer task ability. In this sense the members of the group are "initially undifferentiated" and are presumed to be status equals.[2]

B. Assumptions.--The process with which we are concerned is one in which expectations are formed and maintained as a consequence of their relations to the evaluation of performances. We imagine that in an open interaction situation as p and the others concern themselves with their task, they are continually providing each other with chances to perform and are continually making contributions directed at successfully completing the

[2] For a discussion of the nature of status characteristics and their effect upon performance expectations see Berger et al. (1966b).

task. As this takes place, p and the others are also engaged in evaluating each other's problem-solving attempts. On the bases of these evaluations they are communicating positive and negative reactions, and accepting and rejecting specific contributions. During the early phases of this process certain crucial events are seen to take place in a random manner--particularly the way in which chances to perform are distributed to group members and whether or not these chances are utilized to make problem-solving contributions. However, as the process unfolds, the evaluations (and differential evaluations) of performances become significant, and under certain circumstances p will generalize from these evaluations to the assignment of states of C to himself and others. Such an assignment represents the formation of performance-expectations. Should such an assignment occur, we assume that it will markedly affect p's future behavior. Specifically, it will affect the likelihood that he will give specific others chances to perform, that he will positively or negatively evaluate their contributions, and that he will or will not be influenced by them. Further, we believe that these behaviors of p, because of the way in which they are dependent upon his assignment of states of C, will in general operate to maintain his assignment of states of C. Thus, under the assumption that the task conditions are unchanged, we reason that, once formed, the performance expectations of p will be maintained.

In order to isolate the evaluation-expectation process from other processes which may affect the formation of expectations, we shall concern ourselves with a situation in which certain events of the open interaction situation are controlled. The assertions which follow allow us to describe the formation and maintenance of performance expectations in a situation where actors are repeatedly evaluating each other's performances. At the same time, other behaviors such as the giving or denying of performance opportunities or the differential utilization of such opportunities--behaviors which might affect the formation of expectations--have been controlled.

The situation has the following structure. Imagine that p, o and q are given a series of task problems and that they are to select the correct answer to each problem from the two alternative answers which are presented. Suppose further that their selection of an answer has several stages. First, each person makes a preliminary selection or initial choice between the alternatives. Next, after all have made their initial choice, each finds out what the others have selected. Last, each makes a private final selection. So for each task problem each person makes an initial choice of an answer, receives information about the initial choices of the others, and makes a private final choice of an answer.

From the standpoint of any one of our actors in this situation, say p, he is required to make an initial choice and communicate it to o and q. This is theoretically equivalent to his having been given a performance opportunity which he cannot decline. His initial choice is his performance output. No actor can receive more opportunities than another actor, and all must be responsible for the same number of performance outputs. Thus, inequalities in performance expectations cannot be inferred from inequalities in opportunities to perform or inequalities in performance rates.

If p happens to disagree with o and q about the correct answer (i.e., p initially selects a different alternative from o and q), then he must decide who is right. That is, he must decide whether to positively evaluate his own performance and negatively evaluate o's and q's performances, or vice versa. If he does the latter, that is, changes his mind, then he has been influenced by o and q; if he retains his original evaluations of the answers, he hasn't been influenced. We will assume that p disagrees with o and q on the answer to each task problem so that he must on repeated occasions decide either that "I'm right and they're wrong" or "They're right and I'm wrong." This can easily be arranged by appropriate experimental manipulation.

Let us look more carefully at each of the stages of p's selecting an answer to a task problem.[3] He must first select which of two possible answers (call them A and B) is the correct one. If he thinks A is correct, then we assume that he will actually choose A--that in this situation the alternative he selects and communicates to others will directly reflect his evaluation of that alternative. To put the principle more generally, any time p evaluates the alternative answers differentially, he will, if required to make a selection, choose that alternative he positively evaluates.

> Assumption 1: At any stage of the process: if p positively evaluates one alternative and negatively evaluates the second, then p will select the first and reject the second.

Once p has made an initial choice, he finds out that o and q have both chosen a different answer. We assume that p will suppose that o and q have both acted in accord with Assumption 1--that their behavior is neither random nor capricious but, in fact, reflects their evaluations.

> Assumption 2: At any stage of the process: p associates a disagreement between himself and the others on choice of alternative answers with different evaluations of alternatives by himself and the others.

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[3] We assume, for reasons that will be clear later in the discussion, that it makes no difference for the purpose of formulating our assumptions which particular task problem in the process is being considered.

P is now forced to make a final decision. We believe that there are two activities going on simultaneously, or possibly alternating with each other at this stage. P probably is trying to decide which alternative is right, A or B; and also who is right, himself or the others. He might first decide who is correct and then what is correct, or he might first decide what is correct and then who is correct.

If p initially is unable to decide who is right, he is still required to choose between alternatives. Either he continues to view his preliminary choice as correct, or he changes his evaluations of the alternatives and makes a selection that accords with the preliminary decision of others. These responses provide an observable indication of whether p has been influenced or not on a given step of the process, and we define them accordingly:

> Definition 1: P makes an s-response at any stage of the process if his final selection of an alternative is the same as his preliminary selection. P makes an o-response at any stage of the process if his final selection of an alternative is the same as the preliminary selection of the others.

We now assume that as a consequence of making a final decision , p will also assign unit evaluations to persons that are consistent with the final evaluations of alternatives that he has made. Unit evaluations of persons are positive or negative evaluations of himself or others that are relevant to a given step of the process. Thus, for example, if he makes an s-response, he will come to believe, "This time I was right and they were wrong;" or if he makes an o-response, "This is one they got, and I missed."

If p initially is able to decide who is right, then we claim he will evaluate the alternatives A or B in accord with these unit evaluations of persons, and by Assumption 1 his final decision is determined. Thus, on any step of the process, if p evaluates and chooses among alternatives, he will then evaluate persons. If he evaluates persons, he will then evaluate and choose among alternatives; and such evaluations of persons and alternatives will be consistent. These ideas are embodied in Assumption 3.

> Assumption 3: At any stage of the process: if p assigns unit evaluations to alternatives then he will assign unit evaluations to persons, or if he assigns unit evaluations to persons then he will assign unit evaluations to alternatives; and such evaluations of persons and alternatives will be consistent.

Now consider the impact of p's having made unit evaluations of persons at a particular stage. On the basis of such unit evaluations, p may come to believe that he and the others differ with respect to the ability required for the task, and, more important, that they differ in a particular manner. Thus, for example, from "I was right and they were wrong in this case" p may be led to believe "I am better at this than they are," which is equivalent to assigning the positively evaluated state of C to himself and the negatively evaluated state of C to the others. However, we do not believe that p's having evaluated persons assures that he will, on any given stage of the process, actually assign states of C. The consequence of p's unit evaluations of persons, on any stage, is that the possibility then exists which did not exist previously for him to assign states of C. Moreover, if he does assign states of C, the positively evaluated state of C will always be assigned to the person who was given the positive unit evaluation, and the negatively evaluated state of C will be assigned to the negatively evaluated person.

> Assumption 4: At any stage of the process: given p has not a assigned states of C, if p assigns unit evaluations to persons then the possibility exists that p will also assign states of C to self and others and his assignment of states will be consistent with his unit evaluations.

Once p has assigned states of C to self and others, we believe that this assignment will be stable. The stability of the assignment of expectation states is not problematical if p believes himself to be more competent than the others and makes an s-response, or if he believes himself to be less competent and makes an o-response, because his unit evaluations of persons in these cases are consistent with his assignment of states of C. If, however, p believes himself more competent and makes an o-response, or believes himself less competent and makes an s-response--possibilities which we do not exclude in our formulation--his unit evaluations of persons are inconsistent with his assignment of states of C. Thus it is not self-evident that p's assignment of states will remain unchanged. We argue that this stability in the assignment of expectation states is a function of at least two important features of the evaluation-expectation process: (1) the way in which the assignment of states of C affects the <u>subsequent</u> assignment of unit evaluations of persons, and (2) a change in significance of unit evaluations of persons given an assignment of expectation states.

Given that p has already assigned states of C to self and others, he now has a basis other than the properties of the task for assigning unit evaluations to persons. In fact, we assume that the assignment of states of C, once it has occurred as a <u>consequence</u> of unit evaluations, will <u>in turn</u> affect the way in which p subsequently assigns unit evaluations to persons.

> <u>Assumption 5</u>: At any stage of the process: if p has assigned positively and negatively evaluated states of C to self and others, then he will tend to assign positive and negative unit evaluations to self and others consistent with his assignment of states of C.

Assumption 5 has several important implications which are relevant to the issue of the stability of expectation states. First, once p has assigned states of C to self and others, his process of making final decisions is <u>more likely</u> to be structured in the order <u>who</u> is right <u>and then</u> <u>what</u> is right than was true before he assigned states of C. Second, taken with earlier assumptions, 5 implies that if p believes he possesses the high state of C and others the low, he is more likely to make s-responses than a p who has not assigned states of C. Similarly, if p believes he possesses the low state of C and others the high, he is more likely to make o-responses than a p who has not assigned states of C. <u>Thus, once p has assigned states of C, he is expected to be more frequently making those very responses which are consistent with these assigned states</u>.

Aside from the process described by Assumption 5, stability of assignment is also seen to be related to a change in the significance of unit-evaluations of persons. Given an assignment of C, inconsistent unit evaluations are more likely to be subject by p to special interpretations. Thus, for example, if p believes himself more competent than others and makes an o-response, this behavior is more likely to be dismissed, minimized, or rationalized as a "special event" than the case where a p has not yet assigned states of C. As a consequence, after assignment of states of C, inconsistent evaluations tend to become irrelevant to changes in p's beliefs about task ability. Therefore, we argue,

> <u>Assumption 6</u>: At any stage of the process: if p has assigned positively and negatively evaluated states of C, this assignment is maintained.

C. <u>The Mathematical Model</u>.--It is now possible on the basis of our assumptions and arguments to begin the construction of a mathematical model for more precisely describing the formation of p's expectations and the resultant changes in whether he is influenced by the others. Let us begin by labelling the three possibilities for p's assignment of states of C. First, p may believe he possesses the high state of C and o and q the low state. We will designate this by the symbol [+-] where the first entry in the bracket denotes p's expectations for himself and the second entry his expectations for the others. If p believes he possesses the negatively evaluated state of C and the others the positively evaluated state then we will designate that by [-+]. Finally, p may not have assigned states of C and we will designate that by [0 0].

In line with Definition 1, we shall continue to employ a short designation for whether p is influenced or not. If p is not influenced, we say that he made an s-response; if he is influenced, that he made an o-response.

Our sbustantive formulation says that if p begins the series of task problems in [0 0], he may at some time change to [+-] or [-+] as a result of his decisions about who is right. If he changes from [0 0] to [+-] he will make more s-responses, and if he changes to [-+] he will make more o-responses. The model we have formulated asserts in addition that in order to understand p's decision behavior for any particular task problem we need know only what his expectations were before he began to work on that problem (and not what his previous decisions about who is right were) and how his decision about who is right on the current problem will change his expectations before the next problem is presented.

Suppose that before he begins to solve a particular task problem p is in [0 0]. We know that when faced with making his final choice p will sometimes make an s-response and at other times make an o-response. We assert that there is a specifiable and stable probability (call it α_1) that he will make an s-response on that problem which does not depend on his past behavior but only on the fact that he is in [0 0]. Thus, no matter which problem p is attempting to solve and no matter how p has solved previous problems, the model asserts that he will make an s-response with a fixed probability if he is in state [0 0]. Similarly, we assert that if p is in [+-] he will make an s-response with probability α_2 and if he is in [-+] with probability α_3. As a first approximation we assume that α's do not differ across individuals.

If p is in [0 0] and makes an s-response, Assumption 4 in conjunction with Assumption 1 tells us that the possibility exists for him to assign states of C which are consistent with his decision. That is, the possibility exists for him to infer from his feeling that "This time, I was right and they were wrong," that he is better than the others. Hence, there is a possibility of his moving to the [+-] state following his s-response. We assert that there will be a specifiable and stable probability (call it r) that p will move to [+-] and that the value of the probability will not depend on his prior responses. By a similar line of reasoning we assert that if p is in [0 0] and makes an o-response, then he will move to [-+] with probability d.

Once p has moved to either [+-] or [-+] Assumption 6 tells us that he will remain in that state for the remainder of the series of problems. Moreover, Assumption 5 taken in conjunction with earlier assumptions implies that the probability that p will make an s-response once he is in a new state will be different than previously. In particular, α_2 will be greater than α_1, and α_3 will be less than α_1.

The possibilities and their probabilities of occurrence for a particular task problem can be represented by a set of three tree diagrams, one for each kind of assignment of the states of C that are possible for p before he begins to solve a problem. Figure 1 shows the diagrams for this process.

Figure 1 about here

This formulation of the process means that we have a Markov Chain with the expectations that p holds for himself and the others as the states of the chain. The one step transition matrix for the chain can be easily computed from Figure 1 and is given in Figure 2.

Figure 2 about here

The initial vector of the chain is also easily obtained. We assume that everyone begins the process in [0 0]. Thus the initial distribution vector contains a 1 for that state and zeros for the other states.

III. The Experiment

Recall from our previous discussion that the theory applies to a situation in which three individuals, p and two others, are engaged in the solution of a task, T, which has two outcomes--success and failure. An ability is associated with the solution of T, and initially p and the others have no opinion of each other's ability and are not aware of any external status differences between themselves. Through time, as p and the others attempt to complete T, they evaluate the contributions which each is making toward the solution of T, and on the basis of those evaluations accept and reject these contributions. The theory asserted that each person would

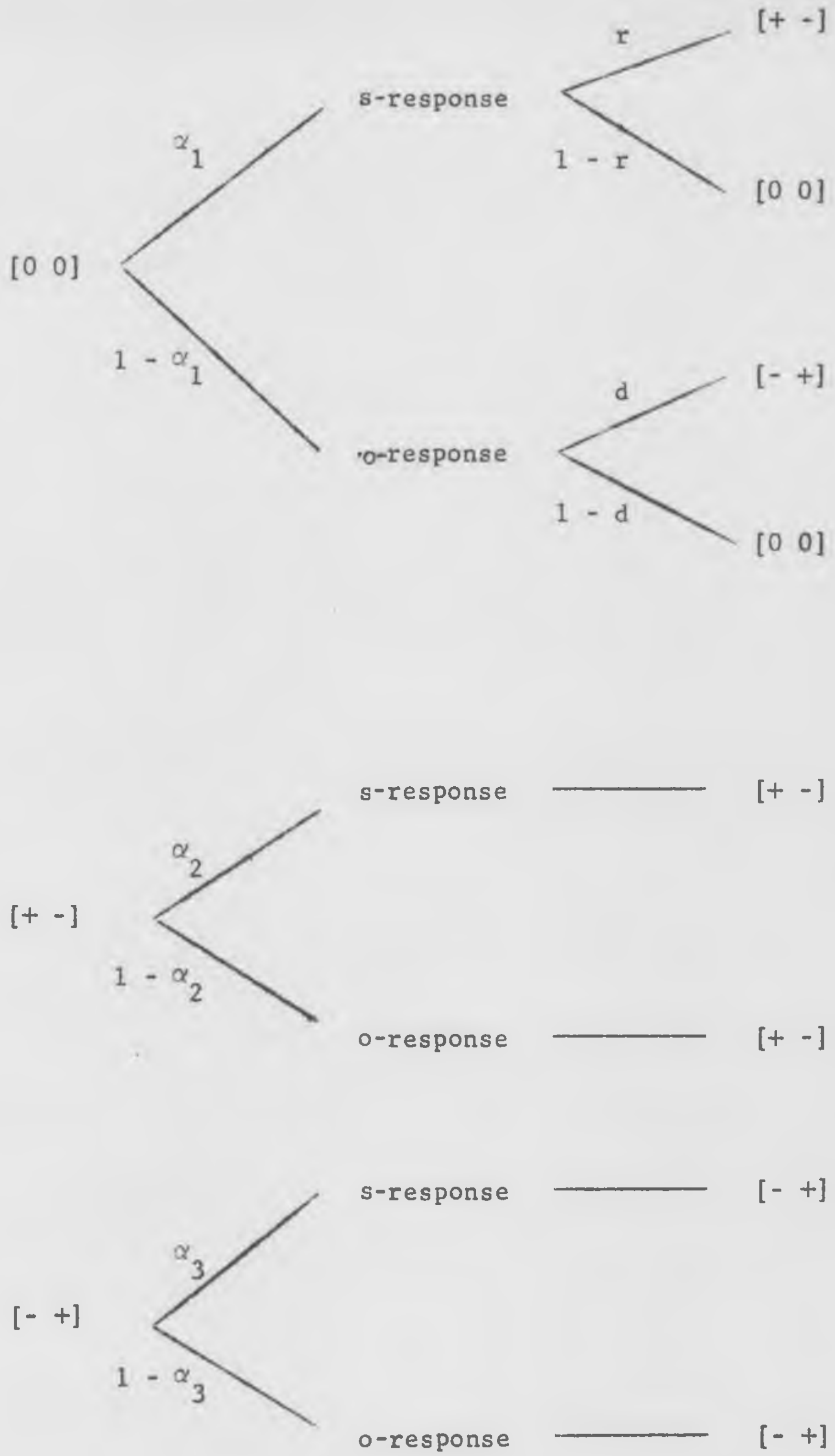

Figure 1. Tree diagrams showing probabilities of each kind of response given the expectation state and probabilities of state changes following responses.

$$
\begin{array}{c} \\ [0\ 0] \\ [+\ -] \\ [-\ +] \end{array}
\begin{array}{c} \begin{array}{ccc} [0\ 0] & [+\ -] & [-\ +] \end{array} \\
\begin{bmatrix} 1 - \alpha_1 r - (1 - \alpha_1)d & \alpha_1 r & (1 - \alpha_1)d \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}
$$

Figure 2. One step transition matrix for change of expectation states.

generalize from performance evaluations to beliefs about relative ability and that their beliefs about ability would then come to govern their evaluations of performances and the acceptance of influence.

The experiment which was carried out focussed upon the evaluations of contributions and the acceptance of influence. Three subjects were confronted with an n-step decision process in which each made an initial choice between binary alternatives, received information about the other persons' initial choices, and, on the basis of evaluations of those choices, made a private final choice between the alternatives. Communicated initial choices were taken to be equivalent to performance outputs where the person was always given an opportunity to perform and always accepted that opportunity. Thus differences in opportunities to perform and in rates of performances, which might occur in the open interaction situation, were controlled. The communication of initial choices was further controlled by the experimenter so that each person would continually disagree with the others on initial choices, and, hence, would always have to differentially evaluate them. The private final choices of each person were taken to be equivalent to acceptance and rejection of influence from the others.

There were other restrictions placed upon the experimental situation. Each binary choice was required to be "nearly veridical," meaning that in each case there would exist a perceivable "correct" choice and sufficient ambiguity about the choice to create uncertainty. Each subject was to be task-focussed; that is, motivated to make the correct final choice.

There were 42 trials in the experiment. For each trial the subjects were seated in booths with separate panels of lights and buttons so that none could see the movements of the others. To make his initial choice a subject pushed one of two labelled buttons. After having pressed his button, one of a set of two lights came on informing him which button the other subjects had pushed. To make his final choice, a subject pushed one of another set of two buttons. The buttons on each panel were connected to a master control panel so that choices could be monitored and recorded and so that which one of the pair of information lights came on could be controlled. As indicated above, it was arranged that on all trials, with the exception of two preliminary trials, a subject was led to believe that the other subjects' preliminary choices differed from his own.

Subjects were instructed to make what they felt to be the correct preliminary choice, and after having taken the information from the other subjects into consideration, to make what they felt was the correct final choice. To operationalize collective-orientation it was repeatedly emphasized to the subjects that it should be of no importance whether their initial and final choices coincided, that the utilization of advice and information from others was both legitimate and crucial, and that it was primarily important that they make a correct final choice.

To operationally define "success" and "failure" at the decision-making task, a set of "standards" with respect to number of correct final choices was presented to each set of subjects. A score of 31-40 correct final decisions was defined as "good" and a score of from 0-30 correct final decisions was defined as ranging from "poor" to only "fair."

The actual task used was a variant of a previously developed visual perception task (see Moore, 1965; and Conner, 1966). Subjects viewed a series of rectangles which were divided, checkerboard fashion, into smaller, equal sized rectangles, either black or white in color. Each larger rectangle was projected from a 35mm slide to a screen, and subjects were asked to choose whether there were more white or more black smaller rectangles within the larger one. As already indicated, the decision with respect to any particular slide was a near veridical decision in the sense that a high proportion of the time it is expected that a subject would choose the correct alternative (i.e., the color which did in fact cover more of the area) although there is enough ambiguity about the decision that influence is possible.

To operationalize the idea of a performance characteristic or ability which is instrumental to the successful completion of the task, subjects were told that the ability to choose the correct answers to the slides was a newly discovered ability called "spatial judgment" ability, and that the ability was unrelated to other skills that they might already possess. The latter instruction was given to insure that a subject would not use knowledge of special skills he already had to infer his spatial judgment ability.

In order to control for any lack of homogeneity between task slides, and hence to control for possible spurious effects due to task properties, the order of presentation of the slides was specified by a two-stage randomization. First, the 40 slides were randomly assigned a number from 1 to 40 and ordered according to those numbers. The resulting <u>relative</u> order was <u>fixed</u> for all experiments. Second, for each experiment the slide which was presented first was randomly selected. Thus, if the initial slide was selected to be 23, the actual order of presentation in the experiment would be 23, 24, 25, . . ., 38, 39, 40, 1, 2, 3, . . ., 20, 21, 22.

The order of events as they occurred in the experiment began with the reading by the experimenter of the instructions for the experiment. The instructions explained the routine mechanics of the experiment, the nature of the task and of the decisions, and other special requirements or features such as scoring standards, emphasis on the final choice, etc.

Following the reading of the instructions the experimenter presented 42 slides, although the subjects were told that there were only 40. The extra 2 slides were included in order to be able to arrange agreement trials at the beginning to allay suspicion of the manipulation of the information exchange. On the remaining 40 trials the subjects continually disagreed on preliminary choices. At the end of the slide series, a short questionnaire was administered which asked each subject to rate his and his partners' performances on the test and to predict future performance for himself and his partners on a similar test in which each worked separately. A post session interview was then conducted in which the attempt was made to ascertain if experimental manipulations were successful, if the subject became suspicious of the manipulated disagreements, and if the subject's perception of and behavior in the situation coincided with the interpretation which the experimenter was making of it. The interviewer also fully explained to each subject the purpose of the study and made him aware of the aspects of the experiment which involved deception. Each subject was asked to not discuss the experiment with his friends.

IV. Analysis of Results

A. Subject Population.--Ninety-five subjects, each a male undergraduate from a local junior college, participated in the experiment. We eliminated 32 of these from the analysis because they became suspicious of one or more of the deceptions. We decided a subject had become suspicious if:

1. He volunteered the information in the post session interview that he thought the exchange of information was "rigged."

2. He had read previously about deception experiments (such as the Asch conformity experiments) and thought the present experiment was similar.
3. He had heard from others that there was deception in the present experiment.
4. He had participated previously in a deception experiment and thought the present experiment was similar.

Three subjects were eliminated from the analysis when post session interview revealed that they had become confused by the experimental procedure and did not understand what kind of information was being furnished to them about the choice behavior of the other members of their groups. Another two subjects were eliminated from the analysis because they represented a violation of the initial condition of the theory requiring that each subject begin the process equal in status to the other subject. Data from any subject who had an obvious physical characteristic which could be interpreted as a status characteristic (such as being a Negro) or who participated with someone else who had such a characteristic was not included. This left 58 subjects whose response data could be examined to test the theory.

B. Models to be Examined.--The most general form of our model is the 5-parameter version, which allows movement to either of the two differentiated expectation states. This is the form of the model we are most interested in and the one we believe will describe the process for this particular situation.

However, we will investigate two other versions of the model--a 1-parameter Bernoulli process model, and a 3-parameter model. The 1-parameter model is obtained when both r and d are zero. This model assumes the occurrence of a process in which there is no change of behavior as a function of the evaluational activities of the members of the group. Thus, although technically a special case of our general model, the substantive claims of the 1-parameter model are different from those developed in the assumptions of our evaluation-expectation theory. Therefore, we examine it as a baseline model from which to compare the predictions of our other models. Two different forms of 3-parameter models can be obtained from our general model, one by setting d=0 and restricting movement only to the [+-] state, and the second by setting r=0 thereby restricting movement only to the [-+] state. Both of these forms do assume the occurrence of a process in which expectations emerge as a function of behavior and changes in behavior occur as a function of the formation of expectations. However, they differ in their characterization of the <u>particular form</u> of the evaluation-expectation process. In an experiment reported by Conner (1966) which was identical to the present one except that each subject was confronted with a single other subject, it was found that change from the undefined expectation state was only to the [+-] state. It is possible for that to have happened in the present experiment even though each subject was confronted by two other subjects. But it is also possible for the majority effect to have become of overriding importance restricting movement to only the [-+] state. Since at this stage we do not know the specific form of the evaluation-expectation process for the <u>particular case</u> involved in our experiment, we shall also consider these two 3-parameter versions of our general model in examining the results of this experiment.

C. <u>Parameter Estimation</u>.--Our analysis of the response data will consist of a comparison of the empirically obtained values of a list of quantities with the values of those same quantities obtained from computer simulations of the process. The simulation values represent approximations of the values that would have been arrived at from analytic expressions for the quantities. As with many models like ours, however, these analytic expressions are difficult, and in some cases impossible, to obtain.

Before simulations could be conducted, estimates for each of the parameters of the model had to be obtained. The procedure we used is described in the appendix of this paper and is discussed in detail in Berger et al. (1968). For our present purposes it is sufficient to state that in each case a series of expressions involving the parameters was obtained for the expected frequencies of certain response events. Because the expressions are complicated, values for the parameters could not be obtained by setting the expressions equal to empirical quantities and analytically solving the resulting system of equations. Rather, approximate values were arrived at by numerically solving the system of equations with the help of a computer. For the 5-parameter model it was found that $\alpha_1 = .690$ (α_1 is the probability of an s-response in the [0 0] state), $\alpha_2 = .846$ (α_2 is the probability of an s-response in the [+-] state), $\alpha_3 = .320$ (α_3 is the probability of an s-response in the [-+] state), r = .030 (r is the probability of moving on one trial from the [0 0] to the [+-] state), and d = .025 (d is the probability of moving on one trial from the [0 0] to the [-+] state). Since the values obtained for r and d were so similar we decided to work with a simpler model that assumes r = d. We obtained new estimates for this 4-parameter model and they are given below:

$$\alpha_1 = .698$$

$$\alpha_2 = .846$$

$$\alpha_3 = .332$$

$$r = d = .028$$

Notice that the estimates are in accord with our theoretical expectations that α_2 would be greater than α_1 which would in turn be greater than α_3.

For the simpler 3-parameter model we were also able to obtain estimates. The estimates for the three parameters are given below:

$$\alpha_1 = .462$$

$$\alpha_2 = .782$$

$$r = .186$$

Because the estimation procedure is independent of the content of the one state to which movement is allowed, and since $\alpha_2 > \alpha_1$, the only 3-parameter model that is possible is one which restricts movement to only the [+-] state. Therefore, we can already conclude that a model which allows movement to only the [-+] state cannot describe the observed process.

In estimating the parameter in the Bernoulli model it was not necessary to use the complicated procedure above. Rather the simple maximum likelihood estimate based on the proportion of s-responses per trial for all trials and subjects was computed. It was found that:

$$\alpha_1 = .696$$

D. Principal Results.--Since there are no absolute rules for deciding on an adequate list of model testing quantities, there is a certain degree of arbitrariness in selecting empirical features of the data to examine. We did attempt to select (1) quantities that would characterize what we believe are substantively significant features of the data as well as (2) quantities which would allow us to discriminate between the three models. The second of these criteria is of special significance since the evaluation of any specific model is based in part on how adequate it is in comparison to some theoretically relevant second model.

Since changes in the rate of acceptance of influence are of particular substantive importance, the first quantity we will examine is the proportion of s-responses on successive trials. We will examine both the observed curve and simulation curves based on each of the models. The Simulations were standard Monte-Carlo simulations in which a computer generated pseudo-random numbers whose values determined responses and state changes for a fixed number of "subjects." We generated 40 different sets of data, each set based on 58 subjects and 40 trials. From each set we calculated the value of the quantities being examined and then calculated the average of those values over the 40 sets of data.

Let us first consider the predictions that each of these models make for the curve of s-responses. The predictions of the 1-parameter model are straightforward. Since the process postulated here is one in which no change of behavior is assumed to have occurred, the curve for the mean proportion of s-responses should be <u>constant</u> through time. The predictions of the 3-parameter model for this quantity are also straightforward but markedly different. In the process postulated by this model, our subjects are initially in an undifferentiated state in which they are making s-responses at a rate of 46%. As the process unfolds, these subjects move into high-low states in which they are now making s-responses at a rate of 78%. Further, since the estimated change of state parameter is relatively large ($r = .186$) given the number of trials involved, almost all subjects will have moved to the high-low state by the end of the experiment. Therefore the 3-parameter model predicts a sharply <u>increasing</u> curve of s-responses. The situation for the 4-parameter model is considerably more subtle. In the process assumed to have occurred under this model, all subjects start out in an undifferentiated state where the rate of s-responses is near 70%. As the experiment continues, some move to [+-] where the s-response rate is 85% and some to the [-+] where the rate is only 33%. Thus the decrease in the rate of s-responses for those moving into the low-high state is approximately 2.4 times the size of the increase in these responses for those moving to the high-low state. However, the relative likelihood of moving into one type of differentiated state as compared to a second is a function of the rate at which responses, consistent with these expectation states, are occurring in the undifferentiated state. The estimates for this model tell us that while the subject is in the undifferentiated state, s-responses, which are consistent with a high-low state, are occurring at approximately 2.3 times the rate of o-responses (70% vs. 30%). Therefore, we should expect to find roughly the same difference in the numbers who have moved into high-low states as compared to low-high states. Thus the effect of differences in the change of response rate is compensated for by the effect of differences in the change of state frequencies. As a consequence, while postulating the occurrence of a considerably more complex process than the 1-parameter model, the 4-parameter model makes essentially the same prediction for this quantity; namely, that the s-response curve will be <u>constant</u>.

Figure 3 shows the average proportion of s-responses for successive blocks of eight trials for both observed and simulated responses. The observed curve is based on the responses of 58 subjects, and each simulated curve is based on the average of 40 sets of the responses of 58 subjects--in effect 2,320 subjects.

Figure 3 about here

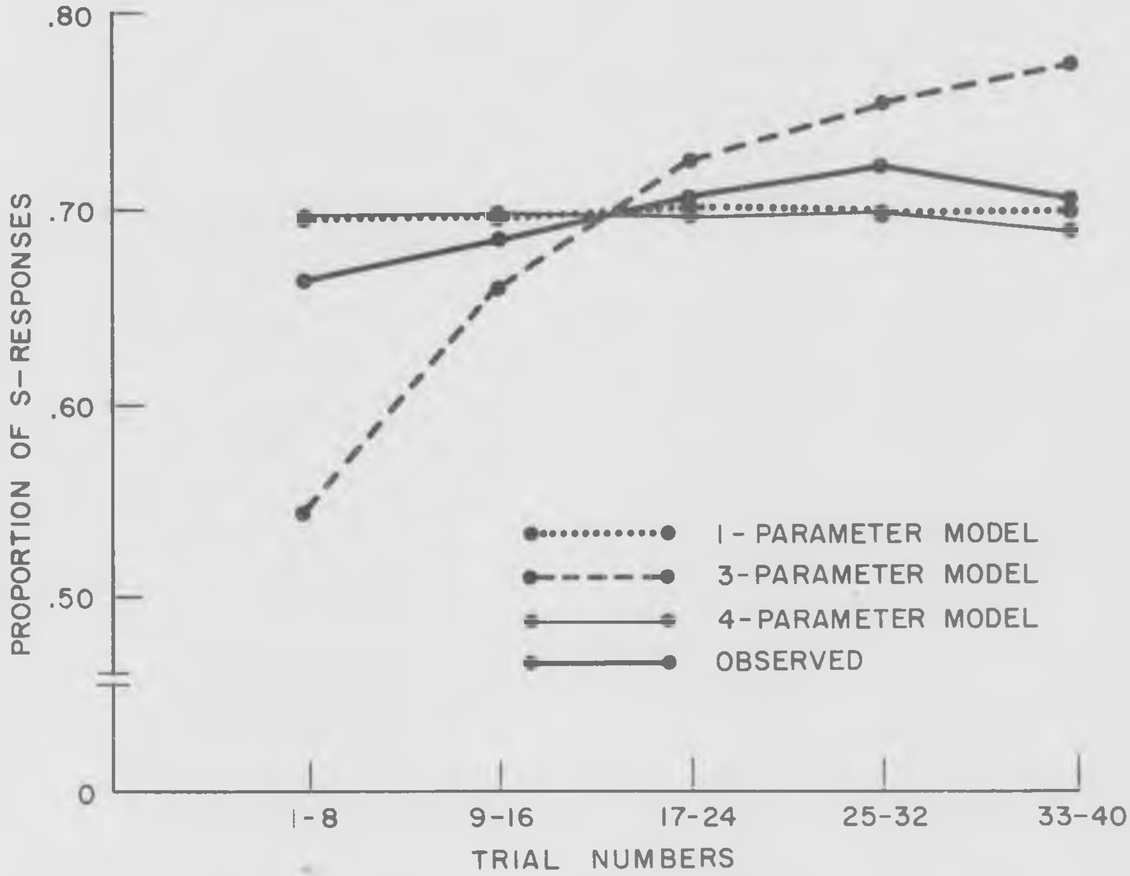

Figure 3. Proportion of s-responses per trial, computed for blocks of eight trials.

The observed curve is clearly more consistent with the curves of the 1- and 4-parameter models and is within the limits of variation of those curves. The 3-parameter curve is definitely not an accurate description of the observed curve.

The second quantity we will examine is alternations. An alternation is a pair of adjacent responses where one is an s-response and the other an o-response. In particular we are interested in changes in alternating behavior through time. Such a quantity provides us with information on changes in the relative stability and instability of the subject's response behavior as the process evolves. The predictions from the 1-parameter model are again straightforward. It predicts no change in the degree of consistency in s-responses; therefore, the curve for the mean proportion alternations through time should be <u>constant</u>. In the case of the 3-parameter model, however, a change is predicted. This follows only in part from the fact that this model assumes a process in which a change of state has occurred. What is relevant here is that in moving into a high-low state the subject is now making a particular type of response, for example s-responses, at a rate closer to 100% than was the case while he was in the undifferentiated state (78% vs. 48%). As a consequence, this model predicts a general <u>decrease</u> through time in the mean proportion of alternations. The predictions from the 4-parameter model are similar to those from the 3-parameter one, although the argument is slightly more complicated. For the subjects who have moved into the low-high state there should be no change in their rate of alternating behavior. The rate at which these subjects are making their most frequent responses in the low-high state (o-responses, 67% of the time) is not significantly closer to (or farther from) the 100% level than the rate of their most frequent response while in the undifferentiated state (s-responses, 70% of the time). Therefore, for these subjects the model claims that there was no increase in the degree of consistency in their behavior. On the other hand, for those subjects who have moved from the undifferentiated state into the high-low state, the rate of their most frequent response, s-responses, has shifted significantly close to the 100% level (70% vs. 85%). For these subjects change of state also involves increase in the consistency of their behavior. Consequently, the overall prediction of this model is that the mean proportion of alternations decreases through time.

Figure 4 shows the average proportion of alternations for successive blocks of transitions for both observed and simulated data.

Figure 4 about here

The curves predicted by the 3-parameter and the 4-parameter model show the expected decrease of alternations through time and are in good agreement with the observed curve. The 1-parameter curve, as expected, is flat and is clearly not an adequate representation of the observed curve. So although the 1-parameter model could predict the s-response curve it does not predict the alternations curve, and while the 3-parameter model failed to predict the s-response curve it does predict the alternations curve. The 4-parameter model is consistent with both curves.

The third quantity we will examine is the variance among subjects at different times in the process in their likelihood of making s-responses. We examined blocks of eight trials and computed the variance of the number of s-responses per subject for each block. The predictions of the three models

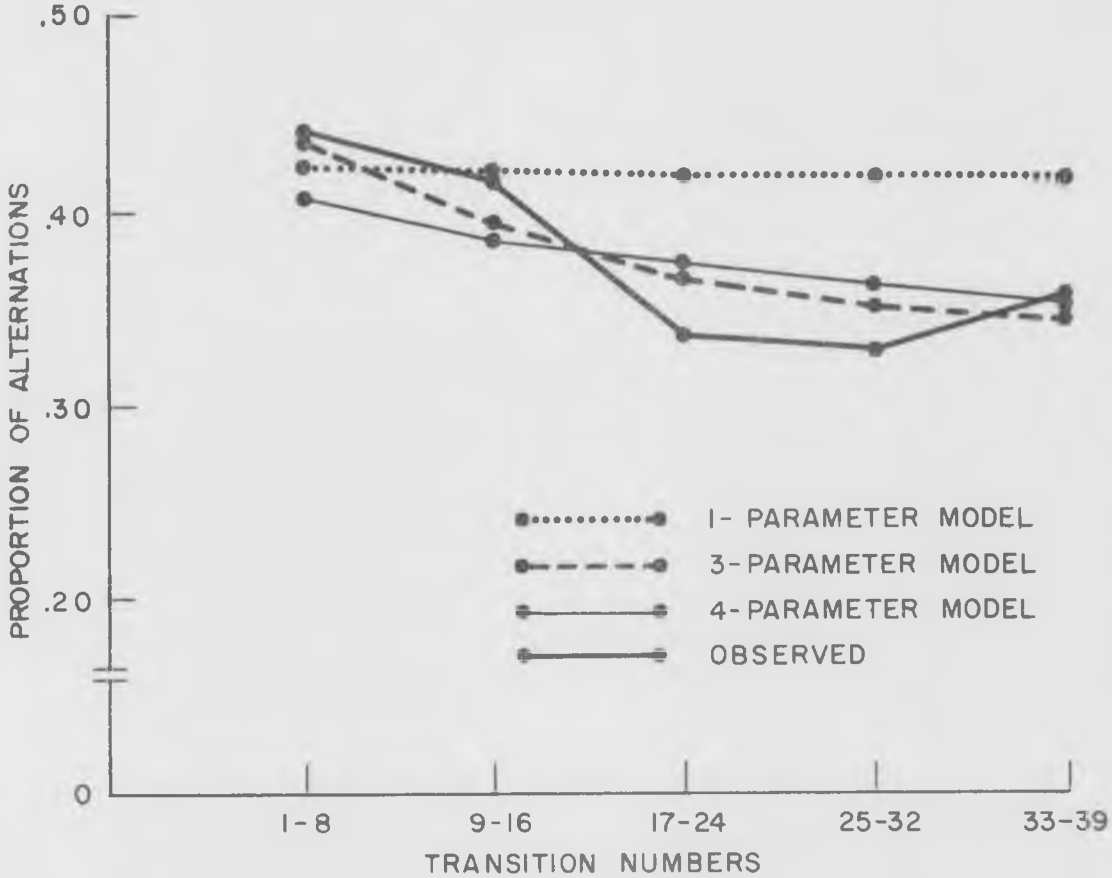

Figure 4. Proportion of alternations per transition, computed for blocks of 8, 8, 8, 8, and 7 transitions.

for this quantity are quite different. The 3-parameter model claims that during the earlier phases of the process there are subjects in the undifferentiated state, and because the value of r is so large, there are some subjects who have already moved into a high-low state. Thus, during these phases s-responses are being generated by two populations, one at a rate near 46% and the other at a rate near 78%. However, the large value of r also means that by the end of the process almost all subjects will have moved to the high-low state and thus will be making s-responses at the same general rate. Therefore, this model predicts that overall the variance among subjects should decrease through time. The 4-parameter model claims that during the earlier phases of the process most subjects are still in the undifferentiated state making s-responses at a rate near 70%. Further, since the values of the change of state parameters in this case are small, at the end of the process we should find three groups of subjects: those who have moved to high-low, those who have moved to low-high, and those still in the undifferentiated state. Subjects in each of these states will be making s-responses at different rates (70%, 85%, and 33%). Hence this model predicts that overall the variance among subjects should increase with time. For the 1-parameter model, since no change of state is assumed to occur, no change in the number of subpopulations producing s-responses at different rates is predicted. As a consequence, the variance of s-responses among subjects should be constant through time.

Figure 5 shows the observed and simulated curves.

Figure 5 about here

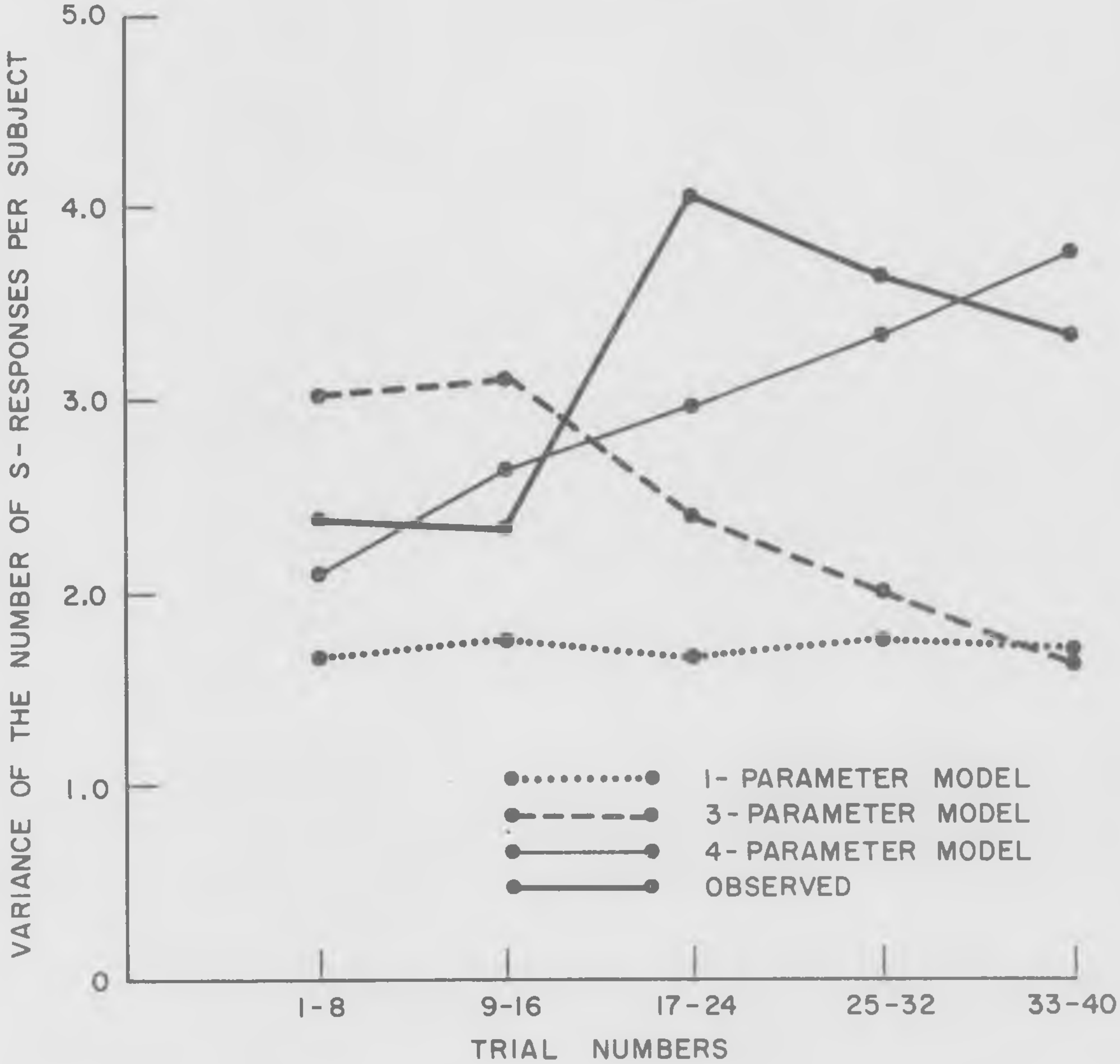

Figure 5. Variance of the number of s-responses per subject, computed for blocks of eight trials.

It is clear that neither the 3-parameter nor the 1-parameter curve is consistent with the observed curve. The 4-parameter curve is not an exact match to the observed curve but is certainly the most nearly consistent curve. It is not known whether the departures in this latter case are attributable to sampling variation or not.

E. Overall Assessment of Fit.--The 4-parameter model seems to have provided a much better account of the three observed quantities we examined than did the 3-parameter model or the 1-parameter model. Table 1 presents in summary form the particular results of our analysis.

Table 1 about here

The only model which predicted all quantities was the 4-parameter model, although it was not as successful as we would like on the blocked variance curve. The 1-parameter model failed in predicting the alternations curve and the blocked variance curve. The 3-parameter model also failed in predicting two quantities--the s-response curve and the blocked variance curve.[4]

The significance of these findings merits some further discussion. The finding that the 4-parameter model is in general more adequate than the 3-parameter model in accounting for our results is, in the first instance, of *factual* significance. It tells us that for this particular case of p interacting with two others--as contrasted with the cases, for example, in which he is interacting with one other or three others--the assumption of a process in which two differentiated states emerge in differing proportions is more tenable than that of a process in which only one differentiated state (either [-+] or [+-]) is formed. In what ways the evaluation-expectation process is

[4]For the results of a more extensive analysis and comparison of the relative fits of the 3- and 4-parameter models, see Berger, et al., 1968.

Table 1

Summary of whether each model was or was not successful in predicting the process trends of the three quantities which were examined.

	Models		
Quantities	1-parameter	3-parameter	4-parameter
Proportion of s-responses per trial	yes	no	Yes
Proportion of alternations per transition	no	yes	yes
Variance of number of s-responses per subject	no	no	yes

affected by increases in the size of the group (as one possible variation on this experiment) can only be answered by further experimentation.

On the other hand, the superiority of the 4-parameter model to the 1-parameter model provides us with information of a different nature. It tells us that the assumption of a process in which there is no change of behavior as a consequence of the evaluational activities of the members of the group is inadequate (in a comparative sense) in accounting for these experimental results. Since changes of state and behavior are basic features of our evaluation-expectation theory, the inadequacy of the no change model is a result of general <u>theoretical</u> significance.

V. Summary

We began our investigation with the general argument that the known features of power and prestige orders which emerge in task performing groups can be accounted for by assuming that the members of these groups come to hold stable and typically differentiated conceptions of the performance capacities of each other. This argument, in turn, poses the problem of: how do these stable differentiated conceptions, performance expectations, develop in task groups? We believe that <u>one</u> of the ways in which performance expectations are formed involves the generalization of evaluations made by group members of each other's problem-solving attempts. In order to isolate and investigate this process we have constructed a theory which describes its operation in a situation where actors are continually evaluating and accepting or rejecting each other's performances but where other behaviors, which might affect the formation of expectations, have been controlled. On the basis of the assertions of our theory, we reason that the occurrence of differential performance evaluations will lead the actor to form differentiated performance expectations. Once these expectations are formed, the actor's evaluations of subsequent performances will tend to be consistent with these expectations, and the rate at which he is influenced by others will be accordingly changed. Finally, since his behavior will tend to be in accord with his expectations the process becomes self-maintaining. Thus, under the given task conditions, his expectations, once formed, will remain unchanged.

An experiment was conducted in order to investigate the process described by our theory. The experiment consisted of a series of forty trials on each of which subjects made an initial choice between two alternative answers to a task problem presented, exchanged information with the other two subjects about initial choices, and made a private final choice. The exchange of information was controlled by the experimenter so that on each trial each subject believed that he had selected a different alternative from the one the other subjects selected. Each subject's initial choice was his performance output for that trial, his evaluation (or reevaluation) of the choice alternatives after exchanging information was his unit performance evaluation, and his final choice indicated his acceptance or rejection of influence on that trial. It was predicted that the evaluations each subject made of his own and the others' performances on each trial would lead him to form performance expectations for himself and for the others. Since he was always in disagreement with the others, we assumed that either he would come to believe himself better at the task than the others or worse. In the former case his rate of acceptance of influence would drop while in the latter case his rate would rise.

We constructed a Markov chain model which formalizes the process described by our theory. The states of the chain were the expectations p could hold for himself and the others. Either his expectations would be unformed, state [0 0]; or he would hold high expectations for himself and low for the others, state [+-]; or he would hold low expectations for himself and high for the others, state [-+]. For each state we assigned a probability of not accepting influence (s-response) on any trial for any person in that state. Movement between states was hypothesized to be restricted to either moving with a fixed probability from [0 0] to [+-] on any one trial after making an s-response, or moving with a fixed probability from [0 0] to [-+] on any one trial after making an o-response (accepting influence). This results in a model with five parameters which are restated below:

P (s-response in [0 0]) = α_1

P (s-response in [+-]) = α_2

P (s-response in [-+]) = α_3

P (moving from [0 0] to [+-] after an s-response) = r

P (moving from [0 0] to [-+] after an o-response) = d

We also considered two other models--a 3-parameter model in which movement either to [+-] is not allowed (r = 0) or in which movement to [-+] is not allowed (d = 0), and a 1-parameter model which did not allow any change of state.

Estimates of the parameter values for each of the models was carried out and we were able to immediately reject the 3-parameter model with r = 0 because it was found that the probability of an s-response increases if movement is restricted to only one state. For the remaining version of the 3-parameter model, the 5-parameter model, and the 1-parameter model, reasonable estimates for all parameters were obtained. Additionally, it was found that for the 5-parameter model, $r \cong d$. Consequently we simplified it to a 4-parameter model with the same parameter governing movement to either [+-] or [-+].

The evaluation of the fit of the three models was carried out by comparing the observed values of three empirical quantities with the values of those same quantities obtained by computer simulation of the response process specified by each model. We found that the predictions of the 4-parameter model were clearly in greater accord with the observed data than those of either the 3-parameter model or the 1-parameter model.

These findings enable us to conclude (1) that the assumption of a process in which there are no changes in behavior will not adequately describe the observed process; and (2) the assumption of an evaluation-expectation process in which <u>two</u> differentiated states are seen to emerge, in differing proportions, does provide a generally adequate basis for characterizing the observed behavior in this particular case.

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APPENDIX

PARAMETER ESTIMATION

Introduction

In this appendix we describe the parameter estimation technique for the 3-, 4-, and 5-parameter models considered in Section IV. The experiment on which the estimation of parameters is based, it will be recalled, consists of observing s subjects for t trials with $s = 58$ and $t = 40$ for the experiment considered in this report. Each subject gives a sequence of s-responses and o-responses; the observation for a subject i is the vector $(x_{i1}, x_{i2}, \ldots x_{it})$ where

$$x_{ij} = \begin{cases} 1 & \text{if the } i^{th} \text{ subject's response on trial } j \text{ is an s-response,} \\ 0 & \text{if the } i^{th} \text{ subject's response on trial } j \text{ is an o-response.} \end{cases}$$

Given s observations of this type, the problem is to estimate the parameters of the 3-, 4-, and 5-parameter models.

Section 1. An Estimator Based on the Expectation of Tuples

A strategy to specify an appropriate estimation technique is relatively simple: choose those parameter values as estimates which give the "best" fit for the observed values of some statistics computed from the observed data to the statistics' theoretical expressions evaluated using the chosen parameter values. This strategy requires us to specify which quantities are to be employed along with some criterion specifying what constitutes "best" fit.

The expected number of 3 tuples of a given type in a response sequence was chosen as an appropriate quantity. Since the response sequence is binary there are 8 different types of 3 tuples, that is, 000, 001, . . . , 111. Let us denote these tuples in the order given as $T_1, T_2, \ldots, T_8$. By expected number of 3 tuples in a response sequence we refer to $E(T_i)$ which gives the expected number of tuples of the type T_i in a hypothetical response sequence. The theoretical expressions for $E(T_i)$ will be derived in Section 2. The observed values of these quantities may be obtained by tabulating instances of each event T_i across trials by the usual "overlapping" tabulating technique and across subjects. Let the observed frequency of these events be given by $N(T_i)$. The observed mean number of tuples, T_i, in a response sequence is then given by

$$\frac{N(T_i)}{s} ,$$

where s is the number of subjects.

The criterion for the "best" fit is a least-square measure of discrepancy between observed and predicted of the form,

$$\left(E(T_i) - \frac{N(T_i)}{s} \right)^2 .$$

Finding the "best" fit involves finding that set of parameter values which minimizes the sum of the discrepancies over all permissible values of i. Thus, we want to minimize the function,

$$f(\alpha_1, \alpha_2, \alpha_3, r, d) = \sum_{i=1}^{8} \left(E(T_i) - \frac{N(T_i)}{s} \right)^2 . \tag{1}$$

To minimize this function we need to take the partial derivative with respect to each of the parameters, set the partials equal to 0, and solve the resulting set of equations. For the 5-parameter model we would have 5 equations in

5 unknowns, the unknowns being the parameters α_1, α_2, α_3, r, d. Again the modifications necessary to handle the 3- and 4-parameter models are straightforward and will not be considered.

Due to the complexity of the partial derivatives, however, in practice the minimization is accomplished through the use of a numerical routine implemented in a computer program. Since most well established computer languages now have library programs which will minimize a function of several variables, the technical details of this minimization are omitted. The set of parameters which are found to give the function minimum are taken to be the parameter estimates. The parameter estimates given in Section IVC for the 3-, 4-, and 5-parameter models were obtained by this method. In the following section the estimator obtained by the strategy described in this section will be referred to as the tuple estimator.

Section 2. Derivation of $E(T_i)$

The theoretical expressions derived here are for the 5-parameter model. Theoretical expressions for the 3 and 4 parameter models are obtained by taking the expressions for the 5-parameter model and setting d = 0 or r = 0 for a particular 3-parameter model and setting r = d for the 4-parameter model. To derive expressions for $E(T_i)$, note that this expression may be written in terms of the underlying states of the expectation process as

$$E(T_i) = \sum_{k=1}^{3} Pr(T_i/\text{state } k)E(\text{state } k) , \qquad (2)$$

where E(state k) refers to the expected number of steps in state k. Equation 2 states that the expected number of tuples of a particular type is the probability of the tuple given the state times the expected number of steps in the given state summed over all states.

First, let us derive the expected number of steps in each of the states. The transition matrix for the 5-parameter model is

$$P = \begin{array}{c} [+-] \\ [0\ 0] \\ [-+] \end{array} \overset{\begin{array}{ccc} [+-] & [0\ 0] & [-+] \end{array}}{\begin{bmatrix} 1 & 0 & 0 \\ \alpha_1 r & \theta & \bar{\alpha}_1 d \\ 0 & 0 & 1 \end{bmatrix}},$$

Where $\bar{\alpha}_1 = 1 - \alpha_1$ and θ is defined as $1 - \alpha_1 r - \bar{\alpha}_1 d = \alpha_1 \bar{r} + \bar{\alpha}_1 \bar{d}$. The initial distribution vector is assumed to be given by

$$\vec{p}^{\,o} = (0,1,0)\ .$$

It is well known that $\vec{p}^{\,j}$, the vector giving the probability that the process will be in each of the states after j steps, is given by

$$\vec{p}^{\,j} = \vec{p}^{\,o}\, P^j\ .$$

Thus, to obtain $\vec{p}^{\,j}$ we must find the expression for P^j. It can easily be shown that raising P to the j^{th} power gives

$$P^j = \begin{array}{c} [+-] \\ [0\ 0] \\ [-+] \end{array} \overset{\begin{array}{ccc} [+-] & [0\ 0] & [-+] \end{array}}{\begin{bmatrix} 1 & 0 & 0 \\ \frac{\alpha_1 r(1-\theta^j)}{1-\theta} & \theta^j & \frac{\bar{\alpha}_1 d(1-\theta^j)}{1-\theta} \\ 0 & 0 & 1 \end{bmatrix}}.$$

To obtain $\vec{p}^{\,j}$ we take the product $\vec{p}^{\,o} P^j$ which gives

$$\vec{p}^{\,j} = \left(\frac{\alpha_1 r(1-\theta^j)}{1-\theta}\ ,\ \theta^j\ ,\ \frac{\bar{\alpha}_1 d(1-\theta^j)}{1-\theta} \right).$$

This expression gives the probability vector containing the probabilities that the process will be in state k after j steps (k = 2, 1, and 3, respectively). However, the state transitions occur after the subject makes an s-response or an o-response. That is, we assume that the completion of the subject's response is prior to the state transitions; thus, the first state transition occurs after the subject's first s-response or o-response, the second state transition occurs after the subject's second s-response or o-response, and so forth. In terms of the trial number where $\vec{p}^{\,j}$ is redefined as giving the probability vector for trial j we have

$$\vec{p}^{\,j} = \left(\frac{\alpha_1 r(1-\theta^{j-1})}{1-\theta}, \theta^{j-1}, \frac{\bar{\alpha}_1 d(1-\theta^{j-1})}{1-\theta} \right).$$

The expected number of steps in state k may now be obtained by summing the probabilities of being in state k over trials to t-2, where, assuming the first 3 tuple is considered to begin on trial 1, the last 3 tuple begins on trial t-2. Performing this operation for each of the states we obtain

$$\text{(3)} \qquad E(\text{state } 1) = \frac{1-\theta^{t-2}}{1-\theta},$$

$$\text{(4)} \qquad E(\text{state } 2) = (t-2)\left(\frac{\alpha_1 r}{1-\theta}\right) - \frac{\alpha_1 r(1-\theta^{t-2})}{(1-\theta)^2}, \qquad \text{and}$$

$$\text{(5)} \qquad E(\text{state } 3) = (t-2)\left(\frac{\bar{\alpha}_1 d}{1-\theta}\right) - \frac{\bar{\alpha}_1 d(1-\theta^{t-2})}{(1-\theta)^2}.$$

Thus we have derived the expected number of times the process will be in each state for a sequence t-2 trials long.

We must now obtain expressions for $Pr(T_i/\text{state } k)$ for each state k and each T_i. If we have state 2, [+-], or state 3, [-+], we have a Bernoulli process with parameter α_2 and α_3, respectively. Table A1 gives the appropriate expressions for $Pr(T_i/\text{state } k)$, k = 2 or 3.

Table A1 about here

For state 1, however, we must resort to a tree diagram to obtain expressions for each type of 3 tuple. Figure A1 gives the tree diagram needed to obtain $Pr(T_i/\text{state } 1)$.

Figure A1 about here

Table A2 gives the probabilities associated with each possible 3 tuple given state 1.

Table A2 about here

Substituting the results of Equations 3, 4, 5 and the results given in Tables A1 and A2 into Equation 2, we may calculate the expectation of any 3 tuple, $E(T_i)$. Given these theoretical expressions we are able to estimate parameters as described in Section 1.

Table A1

Expressions for Pr(T_i/state 2) and Pr(T_i/state 3) *

i	Pr(T_i/state 2)	Pr(T_i/state 3)
1	$\bar{\alpha}_2^3$	$\bar{\alpha}_3^3$
2	$\bar{\alpha}_2^2\ \alpha_2$	$\bar{\alpha}_3^2\ \alpha_3$
3	$\bar{\alpha}_2^2\ \alpha_2$	$\bar{\alpha}_3^2\ \alpha_3$
4	$\bar{\alpha}_2\ \alpha_2^2$	$\bar{\alpha}_3\ \alpha_3^2$
5	$\alpha_2\ \bar{\alpha}_2^2$	$\alpha_3\ \bar{\alpha}_3^2$
6	$\alpha_2^2\ \bar{\alpha}_2$	$\alpha_3^2\ \bar{\alpha}_3$
7	$\alpha_2^2\ \bar{\alpha}_2$	$\alpha_3^2\ \bar{\alpha}_3$
8	α_2^3	α_3^3

* In this table $\bar{\alpha}_2 = 1-\alpha_2$ and $\bar{\alpha}_3 = 1-\alpha_3$.

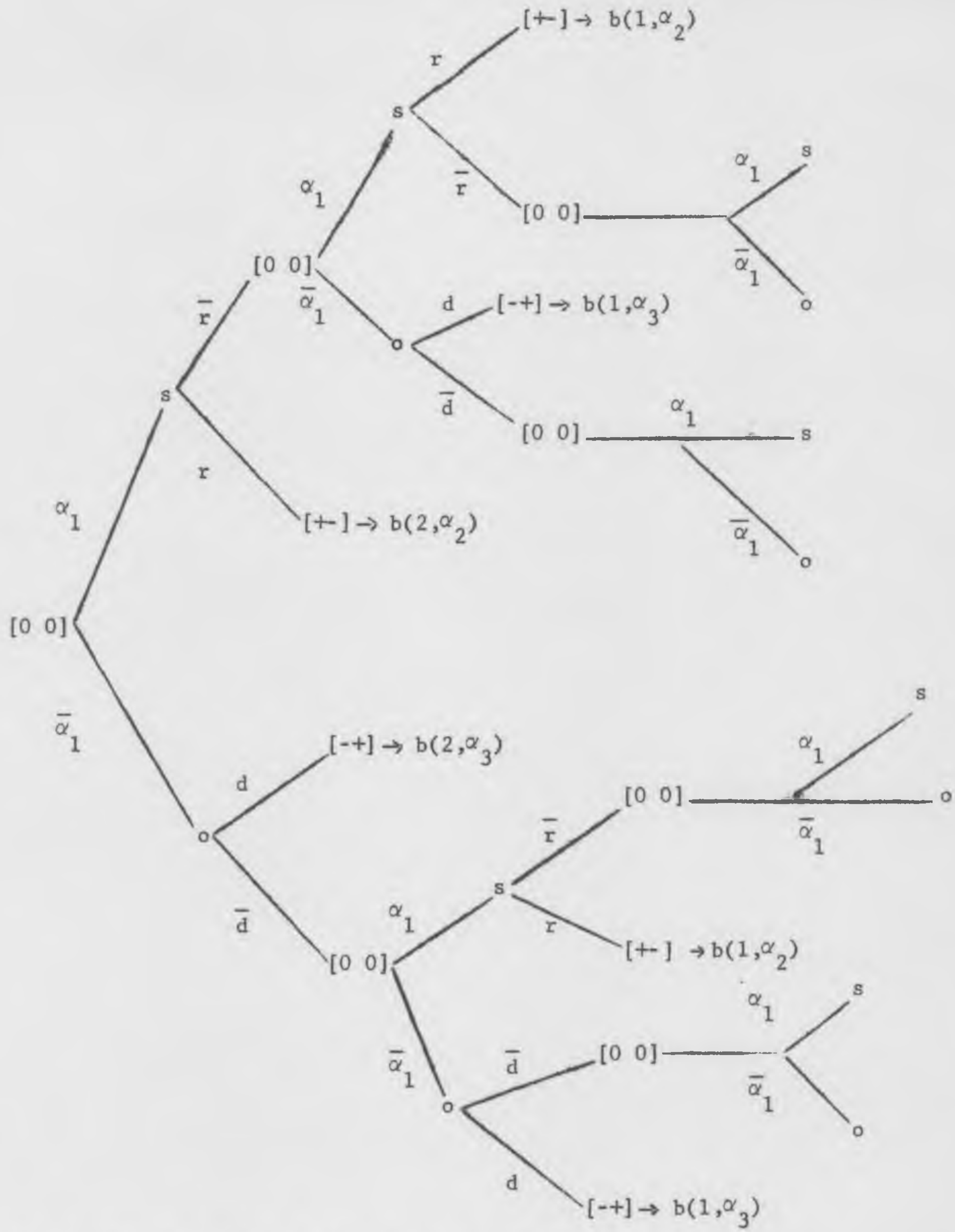

Figure A1. Tree diagram for obtaining $Pr(T_1/\text{state } 1)$.*

* In the tree diagram $b(n,\alpha_i)$ represents a Bernoulli process for n trials with parameter α_i.

Table A2

Expressions for $Pr(T_i/\text{state } 1)$*

i	$Pr(T_i/\text{state } 1)$
1	$\bar{\alpha}_1 d\bar{\alpha}_3^2 + \bar{\alpha}_1^2\bar{d}d\bar{\alpha}_3 + \bar{\alpha}_1^3\bar{d}^2$
2	$\bar{\alpha}_1 d\bar{\alpha}_3\alpha_3 + \bar{\alpha}_1^2\bar{d}d\alpha_3 + \bar{\alpha}_1^2\alpha_1\bar{d}^2$
3	$\bar{\alpha}_1 d\alpha_3\bar{\alpha}_3 + \bar{\alpha}_1\alpha_1\bar{d}r\bar{\alpha}_2 + \bar{\alpha}_1^2\alpha_1\overline{dr}$
4	$\bar{\alpha}_1 d\alpha_3^2 + \bar{\alpha}_1\alpha_1\bar{d}r\alpha_2 + \bar{\alpha}_1\alpha_1^2\overline{dr}$
5	$\alpha_1 r\bar{\alpha}_2^2 + \alpha_1\bar{\alpha}_1\bar{r}d\bar{\alpha}_3 + \alpha_1\bar{\alpha}_1^2\bar{r}\bar{d}$
6	$\alpha_1 r\bar{\alpha}_2\alpha_2 + \alpha_1\bar{\alpha}_1\bar{r}d\alpha_3 + \alpha_1^2\bar{\alpha}_1\bar{r}\bar{d}$
7	$\alpha_1 r\alpha_2\bar{\alpha}_2 + \alpha_1^2\bar{r}r\bar{\alpha}_2 + \alpha_1^2\bar{\alpha}_1\bar{r}^2$
8	$\alpha_1 r\alpha_2^2 + \alpha_1^2\bar{r}r\alpha_2 + \alpha_1^3\bar{r}^2$

* In this table $\bar{\alpha}_1 = 1-\alpha_1$, $\bar{\alpha}_2 = 1-\alpha_2$, $\bar{\alpha}_3 = 1-\alpha_3$, $\bar{r} = 1-r$, and $\bar{d} = 1-d$.