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A STOCHASTIC THEORY FOR SELF-OTHER EXPECTATIONS

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A Stochastic Theory for Self-Other Expectations[1]

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I. Problem and Interaction Conditions

We shall propose a set of propositions describing the relation between specific and defined self-other expectations and the resolution of disagreements in a two man group.

We take as given a two man group with members A and B. Each member of the group holds a performance expectation for himself and his partner with respect to a specific task. Each expectation admits of only one of two values -- high [+] or low [-]. Thus a completely specified expectation structure might be one in which A holds high expectations as to what he can do -- anticipates that his performance on a given task will be of a high or superior quality, and holds low expectations as to what B can do -- anticipates that his performance on the given task will be of a low or inferior quality. At the same time B holds low expectations as to what he can do and high expectations as to what A can do.

A variety of concrete interpretations for the values [+] and [-] assigned to performance expectations are at this point admissible. Thus in the example given, A might be an individual who with respect to a specific task regards himself as the one who is likely to come up with the "best ideas," or highly creative solutions and his partner as the one

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1 The work reported in this paper was financed by a grant from the National Science Foundation. This represents the first report from a project concerned with (a) formulating a dynamic model for self-other expectation processes in the small-groups setting and (b) developing a standardized experimental situation for testing and elaborating such a model.

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who is likely to come up with "poor" or "inferior ideas, while B's expectations for himself and A are in agreement with those that A holds for A and B.

Given that each performance expectation has two referents -- self and other, and that each can take on only one of the two values -- high [+] and low [-], we distinguish the following four basic types:

Type I. The case in which the individual has high expectations for self and low expectations for his partner [+-];

Type II. The case in which the individual has low expectations for self and high expectations for partner [-+];

Type III. The case in which the individual has high expectations for both self and his partner [++]; and

Type IV. The case in which the individual holds low expectations as to what he and his partner can do with respect to a specific task [--].

We now

We now consider the situation where the members of a two man group are addressed to the solution of a typical evaluation problem used in small groups research, for example, a human relations problem. At the outset of their activities, each holds one of the four expectation patterns listed above and it is relevant to their task. That is, these expectations as to what each and his partner can do are precisely applicable to the problem at hand. For our purposes their task consists of a series of discrete decision stages. At each stage each subject has to decide between two alternatives -- different opinions, suggestions, or predictions concerning the problem. Each subject independently makes a preliminary decision from among the alternatives given to him. Information about preliminary decisions is then exchanged (via the experimenter), and subjects are then permitted to make their final decisions. Information about final decisions is not exchanged. The group's solution of the task consists of the final decisions the subjects have made over the entire process. The subjects are motivated to make the "best possible group solution," and are informed that the group's solution, which is the sum of their separate final decisions, can and is to be evaluated as to its quality. Finally at each decision stage, the members of the group are led to believe (via feigned information from the experimenter) that their partner's preliminary choice differs from their own. Consequently at each stage in the process, each subject is required to resolve a disagreement between himself and his partner.

We can now state more explicitly our problem: given a two man group with an initially defined expectation structure, and a decision making process such as characterized above, what predictions can we make concerning the relation between initial expectation structure and the pattern of disagreement resolutions?

We shall tackle the problem of describing the relations between specific self-other expectation patterns, which an individual holds, and the final resolutions he makes by considering what is involved in each of our four basic cases.

Before turning to this problem, it is important to summarize the characteristics of the task conditions given here: (1) The subjects are confronted with a problem which they are asked to solve as a two man team. (2) The "team's" solution consists of the total set of final decisions each man has made over the decision-making process. Each man's final decision at each stage constitutes one unit in this total set. (3) The subjects believe that the team's solution, i.e., the total set of final decisions, can and is to be evaluated as to its quality -- for example, in terms of the total number of "correct" or "good" final decisions made by both individuals over the decision making process.

(4) The subjects are motivated to achieve the best possible team solution -- for example, the highest total number of correct or good unit decisions. (5) There are no objective standards in terms of which a subject can actually evaluate, in a clear-cut and unambiguous manner, each decision. (6) There is the presence of disagreements between the members of the team at each stage in the process, and the necessity for resolving disagreements at each stage. (7) There is no feedback to the subjects as to how, in fact, disagreements have been resolved at each stage. The theory to be presented here is applicable to an "interaction" process for which these conditions hold.

II. Constructing a Set of Assumptions

Having set the conditions of this problem solving task, what predictions can we make concerning the relations of specific self-other expectation patterns and the final resolutions subjects will make?

Given the type of task the subject faces, where he must accept the initial ideas of one person and reject those of the second, we assume that he will come to attach differential valuations to these ideas. Forced to make this choice between his own act and the act of the other, he will come to "perceive" one suggestion as "good," "correct" and the other as "bad," "incorrect," or one proposal as representing a better decision than the second.[1] Once the subject has arrived at this differential evaluation of the two acts at a given step in the process, he will then choose between these acts on the basis of the relative valuation he has attached to them, i.e., the act which he comes to value more highly he will accept as the basis of his final decision while rejecting the less

[1] The implication here is that the need to resolve disagreements, of the type considered, is one of the principal conditions which leads to the differential evaluation of performances by an individual.

highly valued act. Now given the absence of "objective standards" in this task situation, we assume that the standard which the subject uses, to attach differential valuations to acts at a particular stage, is given by the expectations he holds for self and other at that stage. Consequently, the valuations which an individual attaches to the act of self and other will tend to agree with the values he assigns to his expectations for self and other. For example, if an individual holds high expectations for self and low expectations for other, at a particular stage in the decision making process, we would in general expect him to attach a relatively high valuation to the act of self and a low valuation to the act of other at that stage. Having attached these valuations to the two acts, the subject would then resolve the disagreement at this point by accepting the act of self and rejecting the act of other. Further, we assume that if such a resolution sequence occurs, it will serve to maintain the expectations which the individual holds. That is to say, if the subject's evaluation of acts at a given stage coincides with his evaluation of expectations at that stage, the pattern of expectations that he holds will continue, unchanged, to the next stage.[1]

We shall formulate in a more rigorous manner the assumptions we have been developing here. The first of our assumptions applies to each of the four types of expectation patterns we consider.

Assumption 1. The subject will accept that act on the nth decision stage to which he attaches a high valuation, and reject that act to which he attaches a low valuation on this stage.

1. It is clear that the argument employed here, i.e., that if evaluations of unit acts coincide with the expectation structure the individual holds for self and other the expectation structure will be maintained, is in accord with F. Heider's basic propositions concerning the maintenance of balanced interpersonal states. In Heiderian terms the subject, in this situation, holds a general positive evaluation of self and a negative evaluation of other. The subject associates himself with an act (selects a choice) which he positively evaluates and finds his partner associating himself with an act which the subject negatively evaluates. In terms of the theory of structural balance, this situation represents a balanced state and will tend to be maintained, see (3; 4, espec. Chap. 7).

The term act is interpreted to refer to "idea," "proposal," suggestion" which is made by the subject and his compatriot with respect to a decision unit in the decision making process. Assumption 1 simply make explicit the assertion that the individual will accept, as his final decision, that act to which he attributes the highest valuation at a given stage.

A. The Type I Expectation Pattern. Our next two assumptions are developed for the Type I expectation case. This is the situation in which the individual holds high expectations for self and low for his partner [+-]. The task situation with which the individual is confronted requires that he resolve a disagreement between himself and his partner at each stage. This we have assumed will lead him to make a differential evaluation between his own idea or proposal and that of his partner, and his expectation pattern will provide the basis for this evaluation. In the Type I case the subject's expectation structure does provide him with a basis for making differential evaluations as between his own act and the act of his partner. Since he holds high expectations for self and low for his partner, we in general expect him to attach a high valuation to his own act and a low valuation to that of his partner. We let ϕ represent the probability that this subject attaches a high valuation to his own act (while attaching a low valuation to that of his partner) on any stage in the decision making process on which he holds a Type I pattern. Then $1-\phi$ represents the probability that this subject will attach a high valuation to the idea of his partner (while attaching a low valuation to his own idea) at that stage. These ideas are embodied in our next assumption.

Assumption 2. If a subject holds high expectations for self and low for other [+-] at decision stage n, he will attach on this stage a high valuation to his own act with probability ϕ or a high valuation to the act of his partner with probability $1-\phi$.

We have already reasoned that if the subject's valuation of his own and his partner's acts, at any stage, coincides with the valuation he attaches to his expectations for self and other, his expectation structure will be maintained into the subsequent stage. In the Type I case we expect that the value of $\emptyset$ will be greater than the value of $1-\emptyset$. Thus in general we would expect that in this situation the resolution sequence which takes place is one which will lead to a maintenance of the individual's expectation structure. However, the possibility does exist that a subject with a Type I pattern does on a given stage attach a high valuation to the act of his partner (while attaching a low valuation to his own act). If this resolution sequence occurs, we reason that the subject with a Type I expectation structure is more likely to "entertain" the idea that his partner <u>may be as good as he is</u> than to entertain the conception that he <u>is as bad as his partner is</u> on this type of task. Thus as a consequence of this resolution sequence, we assume that the possibility exists that the expectation structure for this subject will change from Type I to Type III -- that is to one in which he holds high expectations for self and other. We let r represent the probability that the subject with a Type I pattern on the nth stage who makes a decision in favor of his partner comes to hold a Type III pattern on the n+1 stage. Then 1-r represents the probability that this subject's pattern remains the same on the next stage. These ideas are embodied in the following assumption:

Assumption 3: If a subject who holds high expectations for self and low for other [+-], attaches a high valuation to his own act on the nth decision stage, his expectations remain unchanged at the n + 1 stage. If this subject attaches a high valuation to the act of his partner, his expectations change to that of high for self and other [++] with probability r or remain unchanged with probability 1-r, in moving to the n + 1 stage.

B. <u>The Type II Expectation Pattern</u>. We next consider the case of the subject with a Type II expectation structure [-+] who is confronted with this decision making task. Reasoning in a manner similar to that in the Type I case, we argue that because the subject in this situation is required, at each step in the task, to resolve a disagreement which entails accepting the act of one (self or his partner) and rejecting the act of other, he will assign differing values to the two acts. The basis for the assignment of these values is given by the expectation structure he holds on the given decision stage. As in the previous case, the fact that the subject with a Type II pattern holds *differential* expectations for self as compared with other provides him with a basis for attaching differential values to the acts of self and other at a given stage; and that the differential values he attaches to these acts will, in general, tend to coincide with the differential expectations he holds. We let ψ represent the probability that this subject attaches a high valuation to his own act (with a low valuation being attached to the act of his partner) on any specific decision making stage on which he holds a Type II pattern. Then $1-\psi$ represents the probability that he will attach a high valuation to the idea of his partner (with a low valuation being attached to his own idea) on that stage. These ideas are stated in the following assumption:

Assumption 4. If a subject holds low expectations for self and high for other [-+] at decision stage n, he will attach, on this stage, a high valuation to his own act with probability ψ, or a high valuation to the act of his partner with probability $1-\psi$.

In general we expect that the value of $1-\psi$ will be greater than the value of ψ; and that if the subject's evaluation of acts on a particular stage *does* coincide with his expectations for self and other at that stage, then his expectation structure continues unchanged into the subsequent stage. But again the possibility exists, although it may be a relatively small one, that a subject with a Type II pattern [-+] on a given stage, comes to attach a high valuation to his own act and a low valuation to the act of his partner at that stage. We reason that if this resolution sequence should occur, the subject holding low expectations for self and high for other is likely to "entertain" the conception that his partner is <u>no better than he is</u> on this type of task.[1] Thus we assume that given this type of resolution sequence the possibility exists that the expectation structure for this individual will change from Type II to Type IV -- to one in which the individual holds low expectations for self and other. We let d represent the probability that a subject who holds a Type II pattern and attaches a high valuation to his own act on a given stage comes to hold a Type IV pattern on the subsequent stage. Then 1-d represents the probability that this subject's expectations do not change in moving to the next stage of the process. These assertions are stated in Assumption 5.

Assumption 5. If a subject holding low expectations for self and high for other [-+], attaches a high valuation to the act of his partner on the nth decision stage, his expectations remain unchanged at the n + 1 stage. If this subject attaches a high valuation to his own act, his expectations change to that of low for self and other [--] with probability d or remain unchanged with probability 1-d, in moving to the n + 1 stage.

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1. It is of interest to compare this situation with one in which a subject with a Type II pattern [-+] finds himself in <u>agreement</u> with other. If the subject is in agreement with other, we would assume that he may conceive of himself as being "as good as his partner" on the problem, and therefore be capable of moving to a [++] pattern. In this case, the valuation of the subject's act is <u>validated</u> by its agreement with an individual for whom he holds high expectations. In the situation we are considering above, the subject <u>rejects</u> the act of the individual for whom he holds high expectations. Thus, we argue, this subject is likely to entertain the idea that his partner is "no better than himself" and is capable of moving to a [--] pattern.

C. <u>The Type III Expectation Pattern</u>. This is the case in which an individual holds high expectations for self and other [++]. Unlike the individual with a Type I or Type II structure, in this situation the subject does not have a <u>differentiated</u> expectation structure which can provide a basis for assigning different values to the acts of self and other. In spite of this fact, his task is such that he must accept and reject acts at each stage and thus, we reason, he is led to attach differential values to these acts at each stage. We assume that where a subject does not have a differentiated expectation pattern, which provides the basis for a <u>more or less likely</u> assignment of values to acts, he will attach a high valuation to one act (while attaching a low valuation to the second) in a random manner. This idea is presented in Assumption 6, which is applicable to the case in which the subject holds either a Type IV pattern [--] or a Type III pattern [++].

Assumption 6. If a subject holds high expectations for self and other [++] or low expectations for self and other [--] at the nth decision stage, he will randomly attach either a high valuation to his own act or a high valuation to his partner's act on the nth stage.

Since the expectation pattern of a subject holding a Type III structure is not differentiated, his differential evaluations of acts <u>will not coincide</u> with the expectations he holds for self and other. Consequently, this subject is assumed to be in an unstable situation.[1]

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1. In terms of Heider's theory the situation we are describing here represents an imbalanced state. The subject holds a general positive evaluation of self and other; associates himself with an act for which he has a positive evaluation and finds his partner selecting a choice which the subject negatively evaluates. According to Heider, if an imbalanced state exists , then forces will arise to change this state, "either through action..(on the part of the subject)..or through cognitive reorganization." If change is not possible, "the state of imbalance will produce tension" (3). A similar conception has been developed by Festinger as part of his theory of cognitive dissonance (2).

Since he is forced to assign different values to acts at each stage, it is assumed that these valuations will "impinge" on his undifferentiated structure, and that a tendency will exist, at each stage, to change the subject's undifferentiated structure to a differentiated one. In this situation, where the subject holds high expectations for self and other and he is forced at each stage to assign high or low values to the acts of self in comparison to the acts of other, we assume that he is more likely to change his high expectations for other to low expectations for other than he is to change his high self expectations to low self expectations. In particular, we expect this change to be likely to occur at each stage in which a subject actually does assign a high value to his own act and a low value to the act of other, otherwise his expectation structure will remain unchanged. These considerations are given in Assumption 7. In this assumption we let p represent the probability that the subject holding high expectations for self and other [++] on the nth stage, comes to hold high expectations for self and low for other [+-] on the n+1 stage, and we let 1-p represent the probability that this subject's expectation pattern is unchanged in moving to the n + 1 stage.

Assumption 7. If a subject holds high expectations for self and other [++] at the nth decision stage, and attaches a high valuation to his own act on this stage, his expectations change to high for self and low for other [+-] with probability p, or remain unchanged with probability 1-p in moving to the n + 1 stage. If this subject attahces a high valuation to the act of his partner on this stage, his expectations remain unchanged in moving to the n + 1 stage.

D. The Type IV Expectation Pattern. The final case we consider is one in which the subject holds low expectations for self and for other [--]. As in the case of the subject who holds a Type III pattern, the expectation structure of this subject is <u>undifferentiated</u> and we assume, assplying Assumption 6, that as long as this subject holds to this structure he will randomly assign high (and concomitantly low) values to the acts of self and other. As in the previous case, since the subject's differential valuations of acts at each stage in the process cannot coincide with the values he assigns to self and other expectations, we assume that the subject is in an unstable situation. Once again we reason that the differential valuations of acts, which this subject is forced to make at each stage, in order to resolve disagreements, impinges upon his expectation structure. Consequently, there is a tendency at each stage to change his undifferentiated expectation structure to a differentiated one. What kind of change is to be expected in this situation? We reason as follows: if this subject were to move in the direction of changing his self-expectations from low to high (without changing those for other) he would be moving into a position from which he would have to assume a major share of personal responsibility for the quality of the team's solution. Movement in this direction can be expected to induce anxiety in this subject, over and above that which it is reasonable to believe he already has by virtue of his initially low self-expectations. We assume that in this situation the subject will move in a direction which will <u>decrease</u> his initial anxiety rather than <u>increase</u> it, and that he will act in an "ego defensive" manner. Consequently, we expect this subject to be likely to change his low expectations for other to high expectations rather than to change his own self-expectations. Therefore we assert that, if on a given decision stage this subject actually does attach a high value to the idea of other (while attaching a low value to his own idea), the possibility exists that this subject's pattern will change from [--] to [-+] in moving to the next stage; otherwise this subject's pattern will remain unchanged. The reasoning developed here is embodied in Assumption 8, in which q represents the probability that a subject with a [--] pattern on the nth decision stage comes to hold a [-+] pattern on the next stage, and 1-q represents the probability that this subject's pattern does not change.

Assumption 8. If a subject holds low expectations for self and other [--] at the nth decision stage, and attaches a high valuation to the act of his partner at this stage, his expectations change to low for self and high for other [-+] with probability q, or remain unchanged, with probability 1-q, in moving to the n + 1 stage. If this subject attaches a high valuation to his own act on this stage, his expectations remain unchanged in moving to the n + 1 stage.

Our task is now completed. Assumptions 1 to 8 as a set, constitute a simple theory for self-other expectation patterns in the two man group. In particular, they describe the effect of the expectation pattern, which an individual holds, on the manner in which he will resolve a disagreement between self and other, and <u>in turn</u> the effect of these resolutions upon his expectation pattern. At this stage, the theory is formulated to apply to the particular type of intereaction situation we have described, and no greater generality with respect to task situation and interaction process is assumed.

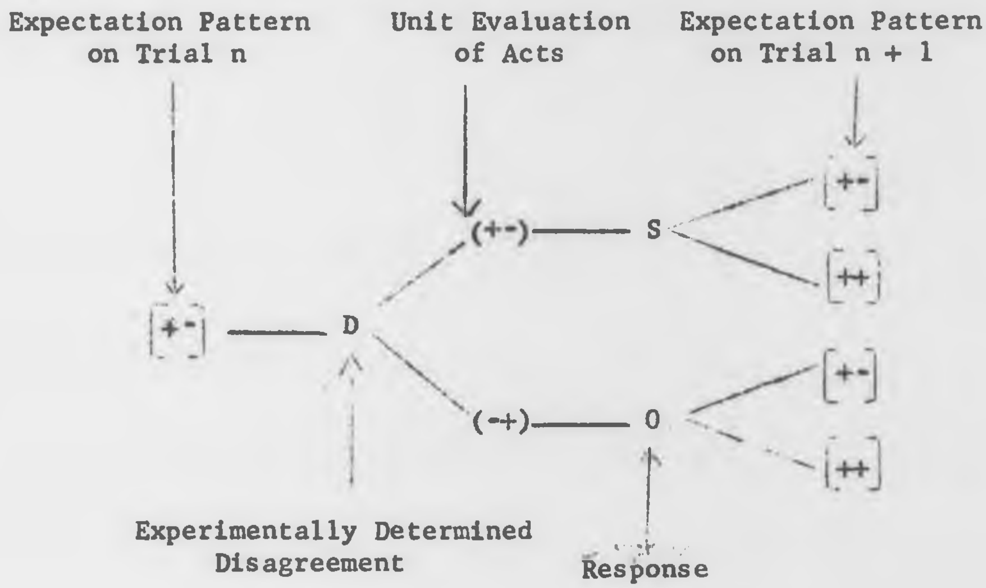
III. Some Mathematical Properties of the Theory

In this section we shall formulate this theory so as to be able to use it in a precise manner to predict relations between the expectations which an individual holds and the resolutions he will make. To do this we shall first examine the structure of the action process we have constructed in developing this theory. To begin with, a subject on any trial n is seen to hold one of the basic expectation patterns [++] [+-] [-+] [--]. On each trial he is confronted with a disagreement (D) which, given the assumed task conditions, leads him to assign either a high value to his own unit act and a low value to the unit act of his partner (+-) or the reverse (-+). On the basis of this differential evaluation of unit acts he resolves the initial disagreement either in favor of self (S) or other (O), and in turn moves to one of the basic expectation patterns on the n + 1 step. The structure of the action process we have formulated is represented diagramtically in Figure 1 for the case in which a subject holds a [+-] pattern on trial n.

A similar representation can be given for subjects holding each of the other patterns on trial n.

Figure 1

Structure of Action Process
for Subject Holding [+-] Pattern on Trial n



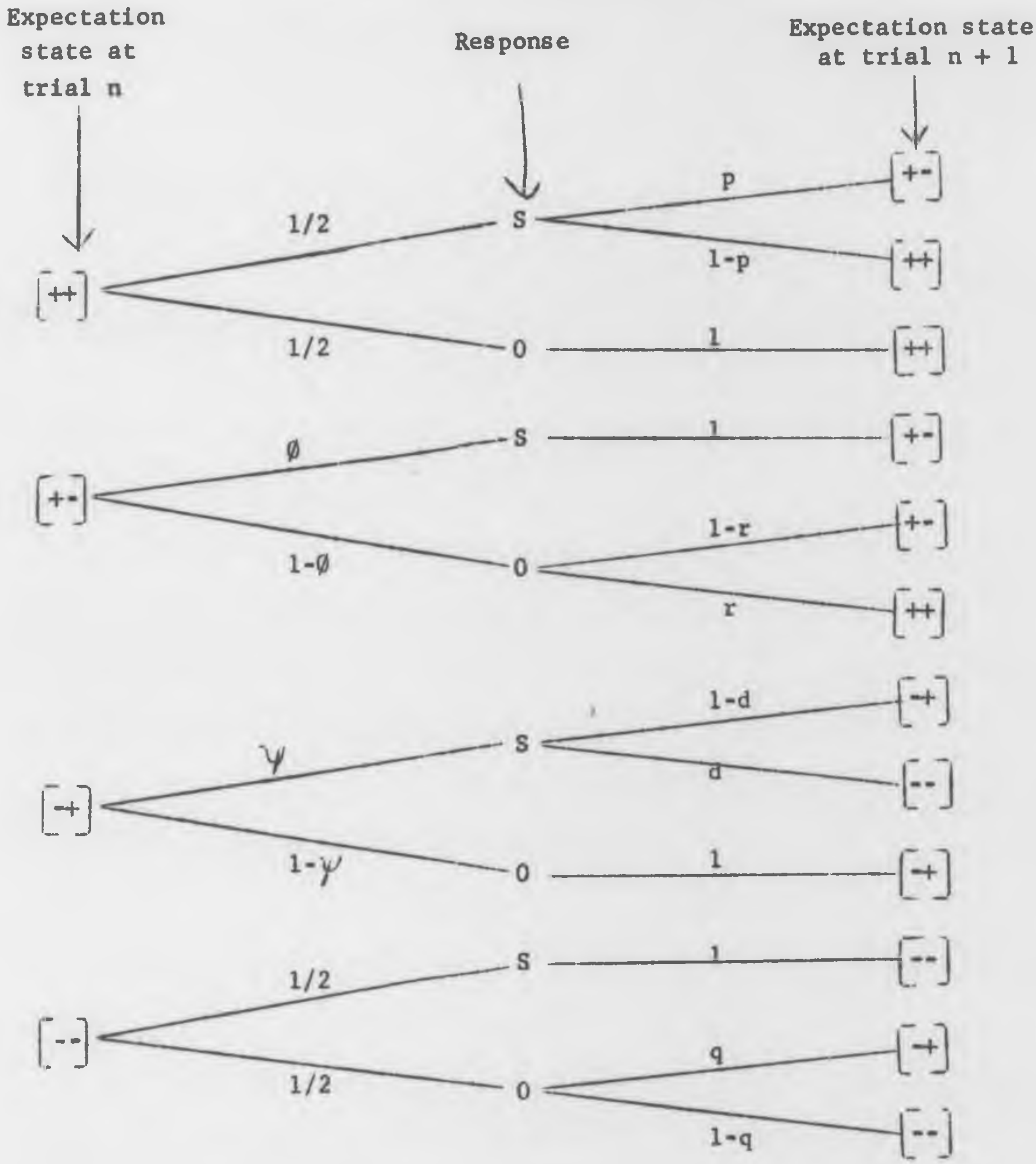
In formalizing our theory we take as our basic unit of analysis the four types of expectation patterns an individual can hold, and the two types of resolution behavior in which he can engage, i.e., accepting the act of self, S (while rejecting that of other) or accepting the act of other, 0 (while rejecting that of self).

As the experiment proceeds the subject makes a sequence of responses S or 0 and moves through the basic expectation patterns, [++], [+-], [-+], [--]. We shall assume that movement through the expectation patterns can be described by means of a Markov chain whose states are the four expectation patterns. To specify this chain we must specify the transition probabilities p_{ij}, that is, the probability that the subject moves to state j on the next step when he is in state i. We compute these probabilities by making use of our assumptions A1-A8. For each expectation state the possibilities and probabilities for his next response and expectation state are given by the tree diagram in Figure 2.

From the tree diagrams in Figure 2 we can easily find the desired transition matrix, P, for the expectation process.

Figure 2

Tree Diagrams for
Expectation Transitions[1]



1. For purposes of simplifying these diagrams certain features of the action process have been omitted: (a) the representation of a disagreement (D) which by stipulated task condition is taken to occur on each trial, (b) the representation of a differential unit evaluation of acts (+-) and (-+) which by Assumption 1 (Page 5) are assumed to stand respectively in a one to one correspondence with self and other responses, see Figure 1.

Matrix of One-Step Transition
Probabilities for the Expectation Process

$P =$

n \ n+1	[++]	[+-]	[-+]	[--]
[++]	$1-(1/2)p$	$(1/2)p$	0	0
[+-]	$r(1-\phi)$	$(1-r) + r\phi$	0	0
[-+]	0	0	$1-d\psi$	$d\psi$
[--]	0	0	$(1/2)q$	$1-(1/2)q$

We shall refer to this Markov chain as the expectation process. Of course we do not observe the expectation patterns during the course of the decision making process but rather the responses. Hence we are also interested in a process which describes these responses. We obtain such a process in the following manner. We first form a Markov chain in which the states are [++S], [++0], [+-S], [+-0], [-+S], [-+0], [--S], [--0]. These represent the expectation pattern and the responses made by the subject on a trial. For example, he is in state [++S] when he is in the expectation state [++] and attaches a high evaluation to his own act. We shall call this process the total process. The transition matrix can again be computed from the tree diagram of figure 2. This transition matrix, P, for the total process is given below. Note that we are using the assumption that the response of a subject depends only on the expectation pattern that he holds and not on his previous response.

Matrix[1] of One-Step Transition Probabilities
for Total Process

n+1

	(1) [++S]	(2) [++0]	(3) [+-S]	(4) [+-0]	(5) [-+S]	(6) [-+0]	(7) [--S]	(8) [--0]
(1) [++S]	$\frac{1}{2}(1-p)$	$\frac{1}{2}(1-p)$	$p\phi$	$p(1-\phi)$	0	0	0	0
(2) [++0]	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0	0	0
(3) [+-S]	0	0	ϕ	$1-\phi$	0	0	0	0
(4) [+-0]	$\frac{1}{2}r$	$\frac{1}{2}r$	$(1-r)\phi$	$(1-r)(1-\phi)$	0	0	0	0
(5) [-+S]	0	0	0	0	$(1-d)\psi$	$(1-d)(1-\psi)$	$\frac{1}{2}d$	$\frac{1}{2}d$
(6) [-+0]	0	0	0	0	ψ	$1-\psi$	0	0
(7) [--S]	0	0	0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$
(8) [--0]	0	0	0	0	$q\psi$	$q(1-\psi)$	$\frac{1}{2}(1-q)$	$\frac{1}{2}(1-q)$

From the total process we obtain the response process by combining certain states into a single state. That is, for states [++S], [+-S], [-+S], [--S], we record a single state S. Similarly, for the states [++0], [+-0], [-+0], [--0], we record a single state 0.

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1. Matrix P in fact can be partitioned into four sub-matrices: a matrix Q whose entries represent the one-step probabilities of moving from and to states (1) through (4); a matrix R whose entries represent the one-step probabilities of moving from and to states (5) through (8); and two matrices D and Z whose entries, respectively, represent the one-step probabilities of moving from states (1) through (4) to states (5) through (8), and (5) through (8) to (1) through (4). In the present theoretical formulation of the total process, matrices D and Z are 0 matrices. Consequently, matrices Q and R can be used in lieu of P in describing properties of the total process. This same partitioning occurs in the expectation transition matrix.

The response process is not in general a Markov chain. That is, it is not true that: the probability of a subject making an S or 0 response on the n + 1 trial depends only on the response he made on the nth trial and does not depend on his previous sequence of responses.[1] This may be seen, for example, by computing the probabilities of a subject making a particular response given different previous response histories.

To illustrate this, let us consider a subject who starts out initially in a $[++]$ expectation state. Let R_n represent the response he makes on the nth decision unit. R_n is equal to either S or 0. What is the probability, for example, that his response on the third state is S, given that he made an S response both on the second and on the first decision unit? We can compute this probability from our tree diagrams properly extended to describe the first three decision stages.

$$\Pr[R_3 = S \mid R_2 = S \wedge R_1 = S] = \frac{\Pr[R_3 = S \wedge R_2 = S \wedge R_1 = S]}{\Pr[R_2 = S \wedge R_1 = S]}$$

$$= \frac{4p\emptyset^2 + 2p\emptyset(1-p) + (1-p)^2}{4p\emptyset + 2(1-p)}$$

We can now compare this with the probability that this subject will make an S response on the third stage given that he made an S response on the second and an 0 response on the initial stage.

$$\Pr[R_3 = S \mid R_2 = S \wedge R_1 = 0] = \frac{\Pr[R_3 = S \wedge R_2 = S \wedge R_1 = 0]}{\Pr[R_2 = S \wedge R_1 = 0]}$$

$$= p\emptyset + \tfrac{1}{2}(1-p)$$

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1. For a discussion of the assumptions of a Markov chains, see Kemeny and Snell (5).

The fact that these probabilities differ means that if one were to consider the subject's responses alone, we would, in general, expect to find that they do depend on his history of past responses.[1] That, by virtue of our assumptions, this is expected to be the case, appears to be an intuitively desirable property of this formulation.

This result (i.e., that the observable response process is not expected to be a Markov chain) points to the fact that expectation patterns and the behavior of these patterns, as described in our assumptions, operate on the level of an inferential process in this theory.

Finally, it is to be observed that if we combine states in the total process by combining states with the same expectation pattern, we obtain the expectation process which is a Markov chain.

IV. Uses and Development of the Formal Model

By virtue of the fact that the expectation process and the total process are Markov chains, we can use the results of this stochastic model to derive substantively meaningful consequences from our assumptions.[2] We shall illustrate this by considering one of the quantities, that can be formally derived, which is of particular importance in testing and further developing this theory. This quantity is the predicted sequence of observable responses.

Let P be the transition matrix for the expectation process. Then the entries of the matrix P raised to the nth power, P^n, have a simple

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1. This does not overlook the fact that for special cases, i.e., particular values of our parameters, the subject's responses may be independent of his history of responses.

2. For a detailed discussion of the types of quantities which can be obtained from this type of stochastic model, see (5).

probabilistic interpretation. The entry $p_{ij}^{(n)}$ is the probability that the chain which started in state i will be in state j at time n. Let us assume that we start in state $\boxed{++}$ or $\boxed{+-}$ so that we can restrict out attention to the transition matrix

$$P = \begin{array}{c} \\ \boxed{++} \\ \boxed{+-} \end{array} \begin{array}{c} \begin{array}{cc} \boxed{++} & \boxed{+-} \end{array} \\ \begin{pmatrix} 1-(1/2)p & (1/2)p \\ r(1-\emptyset) & (1-r)+r\emptyset \end{pmatrix} \end{array}$$

The nth power of this matrix is

$$P^n = \begin{pmatrix} \frac{r(1-\emptyset)}{r(1-\emptyset)+\frac{1}{2}p} & \frac{(\frac{1}{2})p}{r(1-\emptyset)+\frac{1}{2}p} \\ \frac{r(1-\emptyset)}{r(1-\emptyset)+\frac{1}{2}p} & \frac{(\frac{1}{2})p}{r(1-\emptyset)+\frac{1}{2}p} \end{pmatrix} + \left(1 - r(1-\emptyset) - (\tfrac{1}{2})p\right)^n \begin{pmatrix} \frac{(\frac{1}{2})p}{r(1-\emptyset)+\frac{1}{2}p} & \frac{-\frac{1}{2}p}{r(1-\emptyset)+\frac{1}{2}p} \\ \frac{-r(1-\emptyset)}{r(1-\emptyset)+\frac{1}{2}p} & \frac{r(1-\emptyset)}{r(1-\emptyset)+\frac{1}{2}p} \end{pmatrix}$$

The factor $\left(1 - r(1-\theta) - \frac{1}{2}p\right)^n$ which multiplies the second matrix is less than one in absolute value and hence as n increases this matrix tends to the 0 matrix. This means that the probability of being in each of the expectation states approach a limiting value given by the first matrix. The fact that the rows of this matrix are the same means that this limiting probability of being in each of the states does not depend on the starting state. These results are typical for this type of Markov chain.

We can also obtain the probabilities for an S response on each trial. When the expectation process is in state $\boxed{++}$ the probability of an S response is 1/2. When it is in state $\boxed{+-}$ it is $\emptyset$. Let $p_{++,s}^{(n)}$ be the probability of an S response after the nth state of the expectation process.[1] Then

$$(1) \quad p^{(n)}_{++,s} = (1/2)p^{(n)}_{++,++} + \emptyset\, p^{(n)}_{++,+-}$$

$$= \frac{1/2\left(r(1-\emptyset) + p\emptyset\right)}{r(1-\emptyset) + \frac{1}{2}p} + \left(1 - r(1-\emptyset) - \tfrac{1}{2}p\right)^n \left(\frac{\frac{1}{2}p(\frac{1}{2}-\emptyset)}{r(1-\emptyset)+\frac{1}{2}p}\right)$$

$$(2) \quad p^{(n)}_{+-,s} = (1/2)p^{(n)}_{+-,++} + \emptyset\, p^{(n)}_{+-,+-}$$

$$= \frac{1/2\left(r(1-\emptyset) + \frac{1}{2}p\right)}{r(1-\emptyset) + \frac{1}{2}p} + \left(1 - r(1-\emptyset) - \tfrac{1}{2}p\right)^n \left(\frac{(1-\emptyset)(\emptyset r - \frac{1}{2}r)}{r(1-\emptyset) + \frac{1}{2}p}\right)$$

Again we see that the second terms tend toward 0 (zero) and we have a limiting probability for an S response which is independent of the starting expectation pattern.

To illustrate these response probabilities let us take $\emptyset = .9$, $p = .4$, and $r = .5$.[2] Then from (1) and (2) we obtain theoretically predicted curves for the observable responses for the cases in which the subject starts in expectation pattern [++] and pattern [+-]. These curves are shown in figure 3. Note that after about 15 trials the probabilities for responses remain constant. We say that the process has reached <u>equilibrium</u> at this time.

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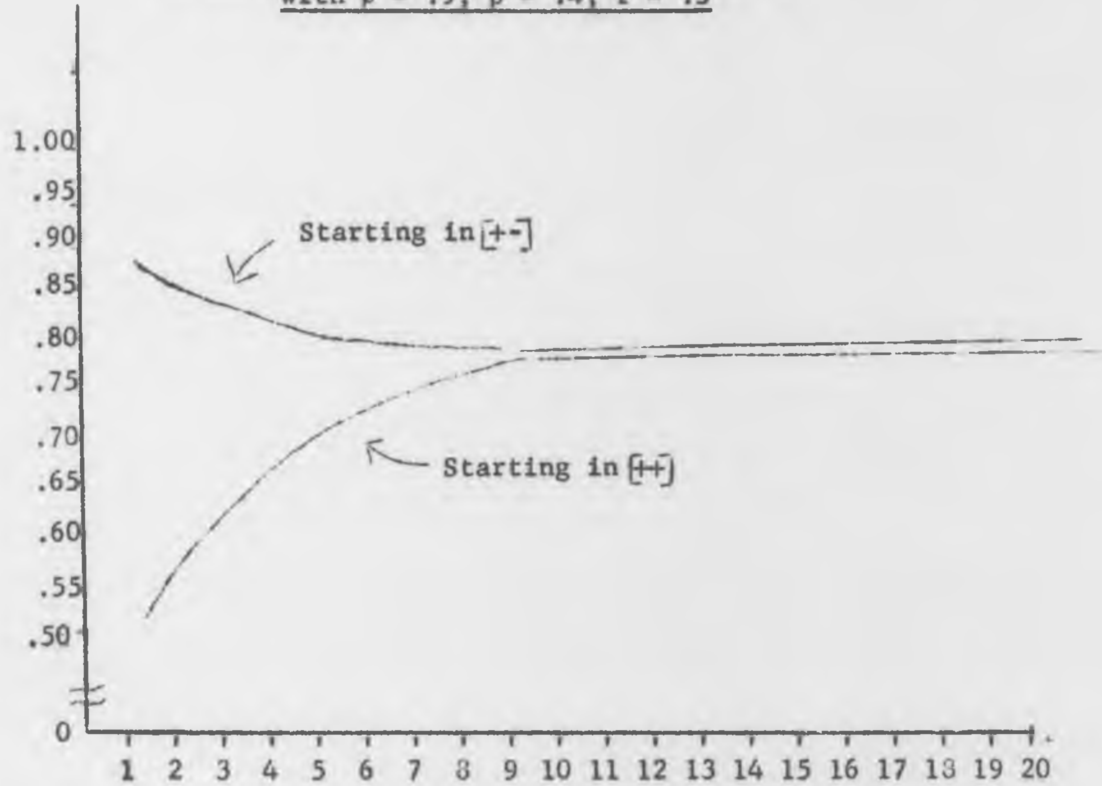
1. By experimental manipulation the investigator knows which expectation state the subject is initially in: [++], [+-], [-+], or [--].

2. It is to be observed that if r = 0, the expectation process reduces to an absorbing chain for subjects starting with either a [++] or [+-] state. Similarly, if d = 0, this process reduces itself to an absorbing chain for subjects starting from a [--] or [-+] state. See <u>Matrix of One Step Transformation Probabilities for the Expectation Process</u>, p. 17.

Figure 4

Predicted Response Process
Given Different Initial Starting States,
with $\emptyset = .9$, $p = .4$, $r = .5$



In general, these theoretically predicted response curves will depend upon the subject's initial state and the values of the parameters for the specific experimental situation.

More detailed information concerning the response process is needed for estimation and testing. This information is most easily obtained from the total process using the fact that the response process may be obtained from this process by combining states. Quantities which may be computed using standard Markov chain methods include (a) the probability of obtaining a specific response sequence, (b) the mean and variance for length of runs of a particular type of response, (c) the mean and variance for the number of times the subject makes a particular type of response given that he started off in a specified expectation state, and

(d) the mean and variance of the number of times the subject moved from one response to a second. Such derived quantities enable us to describe, in a highly precise manner, the various aspects and features of the response predicted from this theory. For a particular experimental situation the relevant parameters of the model ($\emptyset$, p, and r, if the subject is initially in one of the expectation states [++], [+-], and ψ, q, and d, if he is initially in one of the state [-+], [--]) can be estimated from a subset of these derived quantities (for example, using maximum likelihood methods we can estimate them from (a) alone). The model itself can then be tested against the remaining quantities which can be formally derived for this process described by our assumption.

It is clear that in its present form the theory, which we have presented, includes a number of simplifying assumptions, and that any decision as to the utility of this theory, as a basis for further elaboration, must await the results of extensive experimental study.[1] The development of a rigorous theory to describe the effects of expectations on the interaction process of individuals has long been a problem of interest in our field. It is to be hoped that this theory and model may stimulate a new set of theoretical and experimental approaches to this problem.

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1. Research is presently under way to develop an experimental situation which meets the conditions we have stipulated for this action process and which, it is anticipated, will provide the type of test data required by our conceptualization.

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