CALCULATION OF STEADY-STATE EVAPORATION FOR AN ARBITRARY MATRIC POTENTIAL AT GROUND SURFACE

A Thesis

by

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ABSTRACT

In the past few decades, many researchers have studied evaporation from soil columns in the presence of a water table. Several water retention functions have been developed to describe the water flow behavior in the field environment. While most studies involving these functions focus on analysis of water flow and solute transport in variably saturated porous media, there is a limited amount of research to estimate the evaporation rate at bare ground surface for an arbitrary matric potential head. In previous studies, Jury and Horton proposed a method of calculating the potential evaporation rate above a water table on the basis of the Haverkamp unsaturated hydraulic conductivity equation and an assumption that the potential evaporation rate is much less than the saturated hydraulic conductivity of the soil. In this thesis, I developed a new method to estimate the evaporation rate for an arbitrary matric potential head at bare soil surface. I also presented two programs to calculate evaporation rates for a wide range of depths and the fitting parameters of the Haverkamp equation. The results show that the evaporation rates calculated by this thesis fit well with the experimental data and can reproduce the result of potential evaporation rate calculated from previous equation under the special condition of an infinite matric potential head at bare soil surface.

DEDICATION

To My Family, Who Raised Me Up

ACKNOWLEDGEMENTS

I would like to thank my committee chair, Dr. Zhan for his invaluable guidance, patience and support throughout the course of this research. This research would not have been possible without his thoughts support. Although having a hard enough time, he still would spend time discussing and revising my thesis with me. Also, he usually taught me the way of doing research, combining with his own experiences.

I would also like to thank my committee members, Dr. Miller and Dr. Sparks for serving on my committee and for their vision and thoughts. Both of them gave me valuable advice on my thesis. Dr. Miller carefully corrected my thesis and gave me very important advice about realistic evaporation, evapotranspiration and pan evaporation rate. She helped me remedy a serious defect in my thesis. Dr. Sparks also carefully corrected the analytical process in my thesis and helped me optimize my study.

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1. INTRODUCTION

1.1 Background and motivation

Understanding the water loss from soil by evaporation and plant by transpiration due to the upward water flow from a water table is an important topics in many disciplines such as soil science, hydrology, and plant physiology. The evaporation at soil surface is usually an important component of the water balance. Direct measurement of soil evaporation is difficult and the most commonly used method involves a weighing lysimeter. Although water evaporation in actual field setting is a highly complex process, a nearly steady upward flow from a water table to a bare soil surface may be established if the daily evaporative demand is reasonably uniform for a long period of time (Jury and Horton, 2004). While the soil moisture and matric potential head at the ground surface depends on atmospheric conditions, the actual flux through the soil surface should be limited by the ability of the porous medium to transmit water from the unsaturated zone.

The unsaturated hydraulic conductivity is a nonlinear function of water content or matric potential head. There are several water retention functions developed to describe the unsaturated hydraulic conductivity. Jury and Horton (2004) proposed a method of calculating the potential evaporation rate above a groundwater table on the basis of the Haverkamp unsaturated hydraulic conductivity equation (Haverkamp et al., 1977) and an assumption that the potential evaporation rate is much less than the saturated hydraulic conductivity. In this thesis, I will develop a solution to calculate the general evaporation rate for arbitrary matric potential head by the Haverkamp model. I will also check if the assumption that the potential evaporation rate is much less than the saturated hydraulic conductivity is valid or not in actual field conditions. This research will provide a better and more accurate analytical method across calculating evaporation rate for a broad range field setting without involving possibly unrealistic *a priori* assumptions.

1.2 Review of previous work

Several researchers developed some steady-state water flow solutions through a soil profile from a water table to bare ground surface (e.g., Gardner, 1958; Warrick, 1988; Salvucci, 1993; Levine and Salvucci, 1999; Rose et al., 2005; Gowing et al., 2006). In many study of the steady-state water flow across the vadose zone during the past decades, Gardner's unsaturated hydraulic conductivity model (Gardner, 1958) has been used widely. From a soil column above a water table experimental, Gardner (1958) developed two unsaturated hydraulic conductivity models to show a relationship between soil hydraulic properties and depth to the water table. In this approach, the steady-state upward water flow across the soil profile follows the Buckingham–Darcy flux law (Burdine, 1953) as

$$z = -\int \frac{dh}{1+q/K(h)}, \tag{1}$$

where the z axis is positive upward with $z = 0$ at the water table, h is the matric potential head, $K(h)$ is the unsaturated hydraulic conductivity, and q is the upward water flux, which is usually equal to the value of evaporation rate at bare ground surface under steady-state flow condition. Eq. (1) was used to develop some analytical solutions for different unsaturated hydraulic conductivity models. Gardner (1958) developed two unsaturated hydraulic conductivity model $K(h) = K_s \exp(\alpha h)$, where K_s is the saturated hydraulic conductivity and α is a fitting parameter related to the pore size of the soil; and an algebraic unsaturated hydraulic conductivity model as the form $K(h)= a(h^N + b)^{-1}$, where a, b, and N are empirical factors related to soil texture (Warrick and Or, 2007). He used these relations to obtain analytical solutions of Eq. (1), with an empirical number of the fitting parameters b set equal to zero, and N set equal 1, 3/2, 2, 3, and 4. Warrick (1988) generalized an additional solution to estimate the upward flow for a range of N from 1.5 to 5. Analytical solutions of Eq. (1) show that the evaporation rate often depends on the depth to the water table, the matric potential at the soil surface, and the hydraulic conductivity of soil. Gardner (1958) revealed that when the matric potential head at the surface was changed to infinite, the evaporative flux can approach a maximum rate that is a function of the saturated hydraulic conductivity, the fitting parameters N, a, b in Eq.(1) and the depth to the water table.

The potential evaporation is usually estimated by means of meteorological data observed under non-potential conditions (Brutsaert, 2005). Because the air can interact with the subsurface soil, this is not the same rate as that which would be calculated or observed, if the soil water content of the ground surface was very high. Therefore, Brutsaert (2005) called this the “apparent” potential evaporation to reflect the fact that potential evaporation estimated on the basis of measurements carried out under non-potential conditions. Apparent potential evaporation can be estimated by means of an evaporation pan (Brutsaert, 2005). In general, the potential evaporation rate should be greater than apparent potential evaporation rate.

National Oceanic and Atmospheric Administration (NOAA) provided a compilation of monthly, seasonal, and annual averages of estimated pan evaporation based on observations from Class A pans and on meteorological measurements by the National Weather Service (NWS) and cooperating agencies (Farnsworth and Thomopson, 1983). The data set used for the evaporation atlas included, at most, 15 years of data record from 1956-1970.

The maximum pan evaporation rate in 50 states in USA is 22.76 inches/month in July in Amboy, California (34°32′, 115°42′). The minimum pan evaporation rate in 50 states is 0.55 inches/month in January in Sly Park, California (38°43′, 120°34′). The pan evaporation rate is ranging from 0.045 cm/day to 1.865 cm/day (Farnsworth and Thomopson, 1983). Usually, the highest evaporation rates in one year are observed in June and July, and the lowest evaporation rates in one year are observed in October to January.

Matric potential is defined as the difference in energy per unit volume or weight between standard water and soil water due to capillarity and adsorption. Gardner (1937) developed a filter-paper technique method to measure the soil water release characteristic. Smith (1991) improved the filter-paper technique method to measure soil matric potential in Australia. Depending on this technique method, the matric potential can be measured as a range from -1 kPa to -100 Mpa (Fawcett and Collis-George, 1967).

When the matrix potential approaches 0 kPa, the saturation of soil is approaching 1. In the case of a low-intensity precipitation, there is usually no apparent runoff on the surface, the matrix potential is generally in the range of -10 to -30 kPa. At a potential of -

33 kPa for sand, the soil moisture is viewed as the optimal condition for plant growth. At a potential of -1500 kPa, the soil is regarded as slightly dry, and the soil water is almost held by solid particles. When the matric potential is -100 MPa, it means that the soil is very dry. At a potential of -1000 MPa, the state of soil is oven dry (Wikipedia, web site: http://en.wikipedia.org/wiki/Water_potential). For the convenience of unit change, one may aware that a soil matric potential of -1 MPa is equivalent of a matric potential head of -10000 cm.

1.3 Problem statement

In their textbook on soil physics, Jury and Horton (2004) presented a one-dimensional model to describe water flow from a shallow water table upward to an evaporating surface. The simplest system to be considered is portrayed schematically in Figure 1. A water table is located at a distance L from the ground surface which is defined as z=0. The positive z axis is upward, thus the water table is at z=-L. The soil is homogeneous and flow in the soil is vertical.

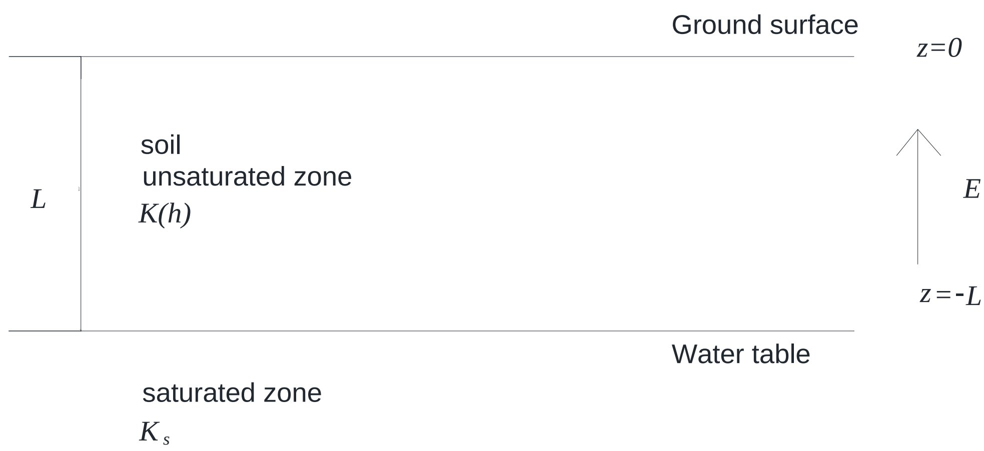

Figure 1: A schematic diagram of evaporation from a shallow water table.

The unsaturated hydraulic conductivity of the soil is a function of the matric potential head h [L] (negative) (Haverkamp et al., 1977):

$$K(h) = \frac{K_s}{1 + (h/a)^N}, \tag{2}$$

where a is a characteristic length [L] (negative) and N is a positive constant. Eq. (2) is called the Haverkamp model hereinafter. When steady-state flow is of concern, applying the Buckingham-Darcy law to vertical flow, one has:

$$J_w = -K(h)\left(\frac{dh}{dz} + 1\right), \tag{3}$$

where J_w is the vertical specific discharge. h is only a function of z under steady-state flow condition, resulting in a constant J_w. Reorganizing Eq. (3) into an integral, one has:

$$\int_{h_1}^{h_2} \frac{dh}{1 + J_w / K(h)} = -\int_{z_1}^{z_2} dz = z_1 - z_2, \tag{4}$$

where $h_1=h(z_1)$ and $h_2=h(z_2)$ are two matric potential heads at two different elevations z_1 and z_2, respectively. In the problem studied below, we set $z_1=-L$ (water table) and $h_1(-L)=0$; $z_2=0$ (ground surface) and $h_2(0)=h_0$, which is the constant matric potential head at ground surface. Therefore,

$$\int_{0}^{h_0} \frac{dh}{1 + (E/K_s)\left[1 + (h/a)^N\right]} = -L, \tag{5}$$

where $E=J_w$ is the evaporation rate, which must be greater than zero. Defining a new parameter $y=h/a$ and $y_0=h_0/a$ which are positive, and substituting them into Eq. (5), one has:

$$\int_0^{y_0} \frac{dy}{1+(E/K_s)+(E/K_s)y^N} = -L/a \, . \tag{6}$$

Be aware that $-L/a$ is positive because a is a negative constant. Defining the following new parameters: $\varepsilon = \left(\frac{E/K_s}{1+E/K_s}\right)^{1/N}$, $x=\varepsilon y$ and $x_0 = \varepsilon y_0$, one transforms Eq. (6) into:

$$\int_0^{x_0} \frac{dx}{1+x^N} = \left(-L/a\right)\varepsilon(1+E/K_s) \, . \tag{7}$$

For the special case of calculating the potential evaporation rate, one may apply the negative infinite matric potential head at ground surface or a positive infinite x_0 in Eq. (7). Under this condition, one can employ the following identity (Abramowitz and Stegun, 1970):

$$\int_0^{\infty} \frac{dx}{1+x^N} = \frac{\pi}{N\sin(\pi/N)} \, . \tag{8}$$

Substituting Eq. (8) into Eq. (7) will lead to the following equation:

$$\left(\frac{E_p}{K_s}\right)^{\frac{1}{N}}\left(1+\frac{E_p}{K_s}\right)^{1-\frac{1}{N}} = \frac{-a\pi}{NL\sin(\pi/N)} , \tag{9}$$

where E_p in Eq. (9) represents the potential evaporation rate hereinafter. Eq. (9) can be used to calculate the potential evaporation rate. The form of Eq. (9) does not permit a direct analytical estimation of E_p for a general soil type, and one has to seek help from a numerical root-searching method. Under the special condition that E_p/K_s is much less than 1, one can obtain a closed-form solution for E_p:

$$E_p = K_s \left(\frac{-a\pi}{NL \sin(\pi / N)} \right)^N . \tag{10}$$

The purpose of Eq. (10) is to simplify the process of the calculation of Eq. (9) so that one can obtain a closed-form analytical solution. However, there is no discussion for the special condition of relative evaporation rate in the book of soil physics (Jury and Horton, 2004), it is necessary to check the pre-assumption of Eq. (10) (E_p/K_s is much less than 1) before its use.

1.4 Research objectives

In this thesis, I plan to conduct a combined analytical and numerical investigation to achieve the following objectives:

i. I want to check if the assumption that E/K_s is much less than 1 is valid or not when the water table is relatively shallow (usually less than 1 m for clay). I will also check the serviceable water table range for different types of soils that we can use Eq. (10) to calculate the potential evaporation rate in steady-state flux problems.

ii. I will develop a new solution of the steady-state evaporation rate with an arbitrary matric potential head at the bare ground surface.

iii. I will apply the widely used Brooks and Corey (1964) retention equation instead of the Haverkamp equation to calculate the steady-state evaporation rate. The numerical simulation will be carried out using the HYDRUS-1D, which is a suite of Windows-based modeling software that can be used for analysis of water flow, heat and solute transport in variably saturated porous media (Simunek et al., 2005).

2. THE STEADY-STATE EVAPORATION RATE AT BARE SURFACE CALCULATION

2.1 Determination of the range of relative evaporation rate (E/K_s)

In the past decades, several researchers (e.g., Warrick, 1988; Salvucci, 1993; Rose, 2005) have studied the effect of water table depth on evaporation from ground water. Salvucci (1993) discussed that when the fitting parameter N (see Eq. (2)) increases, the magnitude of relative evaporation rate should decrease.

The solution by Jury and Horton (2004) is shown in Eq. (10). It presents the potential evaporation rate E_p as a function of the distance L between the soil surface and the water table. To check the assumption of "E_p/K_s is much less than 1" used by Jury and Horton (2004), I developed a numerical program (Appendix A) to find the value of E_p/K_s in Eq. (9) by using the fitting parameters collected from previous studies (see Table 1).

The results in Table 2 give the calculated relative evaporation rate (E/K_s) values for water table depths ranging from 10 cm to 1000 cm. This Table shows a few important findings. First, for a very shallow water table such as 10 cm, all the cases violate the assumption of a low ratio of E_p/K_s. For instance, the Pachappa soil has the greatest E_p/K_s value of 7.07 which is much greater than 1. Even the lowest E_p/K_s value for Yolo Light Clay (2.38) is considerably greater than 1. This clearly shows that Eq. (10) cannot be used for the four example soil types when the water table depth is as shallow as 10 cm. Second, when the water table depth changes to 50 cm, one finds that although the values of E_p/K_s for all four cases are less than 1, they still may not satisfy "the much less than 1 assumption" needed for Eq. (10). For instance, the lowest E_p/K_s value found for the Buckeye or Yolo Light Clay is 0.289. The highest E_p/K_s value is actually quite close to 1 at 0.958. When the water table is 100 cm, the assumption of a much less E_p/K_s value may be acceptable for the cases of Buckeye sand, but it may not be acceptable for the cases of Chino Clay, Pachappa sandy loam, and Yolo Light Clay. When the water table depth is greater than 300 cm, one may safely use the assumption of a much less E_p/K_s value and Eq. (10) to perform the calculation of evaporation rate.

To estimate the discrepancy of values produced by Eqs. (9) and (10), one may use the following formula: $\varepsilon = |E_9 - E_{10}| / E_9$, where E_9 and E_{10} represent the potential evaporate rates calculated from Eq. (9) and Eq. (10), respectively. The results of discrepancy for five different soil types are listed in Table 3. Previous experimental data suggested the *N* values to be 2, 3, 4,4, 5, and the *a* values to be -20.8 cm, -86.7 cm, -17 cm, -10.9 cm and -44.7 cm, respectively for clay loam, silty loam, sandy loam, coarse sand and fine sand in Table 3. The hydraulic properties of soils were measured by Ashraf (1997, 2000), Rijtema (1969) and van Hylckama (1966).

Table 1: Haverkamp Modeled Soil Parameters used in this study.

Soil site/type	Parameter value	Reference
Chino Clay	N=2, a=-23.8 cm	Gardner and Fireman (1958)
Pachappa (fine sandy loam)	N=3, a=-63.83cm	Gardner and Fireman (1958)
Buckeye (fine sand)	N=5, a=-44.7cm	van Hylckama (1966)
Yolo Light Clay	N=1.77, a=-15.3cm	Haverkamp (1977)

Table 2: The E_p/Ks values from the different water table depth and soil types.

L(cm)	10	50	100	300	500	1000
Chino Clay, E_p/K_s	3.27	0.399	0.124	0.015	0.0056	<0.0001
Pachappa (fine sandy loam), E_p/K_s	7.07	0.958	0.280	0.016	0.004	0.00045
Buckeye (fine sand), E_p/K_s	4.00	0.289	0.023	0.0001	<0.0001	<0.0001
Yolo Light Clay E_p/K_s	2.38	0.289	0.096	0.014	0.006	0.002

Table 3: The discrepancy ratio ($\varepsilon = |E_9 - E_{10}| / E_9$) of results calculated from Eqs. (9) and (10) , the fitting parameters of soils were shown in the text.

E/K_s / Soil	0.05	0.01	0.005	0.001	0.0001	0.00001
Buckeye (fine sand)	17.763%	3.939%	1.961%	0.398%	0.040%	0.004%
clay loam	4.762%	0.990%	0.498%	0.100%	0.010%	0.001%
silty loam	9.297%	1.970%	0.990%	0.200%	0.020%	0.002%
sandy loam	13.644%	2.941%	1.488%	0.299%	0.030%	0.003%
coarse sand	13.644%	2.941%	1.488%	0.299%	0.030%	0.003%

Figure 2 shows the values of E_p/K_s for a range of both N and $-a/L$ which calculated by Eq. (9). In Figure 2, six different contours of E_p/K_s ranging from 0.05 to 0.00001 are plotted. This figure may be used to quickly estimate the range of evaporation rate based on the soil type parameters a and N for a given water table L. By knowing the range of E_p/K_s, one can subsequently estimate the discrepancy range of the results obtained from Eqs. (9) and (10) (see Table 3). Such a discrepancy range will allow us to decide if Eq. (10) or Eq. (9) should be used. In this study, I will choose 5% discrepancy as the threshold, meaning that if the discrepancy is greater than 5%, Eq. (10) is not recommended to use and one has to use Eq. (9); if the discrepancy is less than 5%, one can use Eq. (10) as a good approximation of Eq. (9). For instance, when E_p/K_s are 0.05 and 0.01, the discrepancy ratios between Eqs. (9) and (10) for Buckeye soil (fine sand) are 17.76% and 3.94%, respectively. Then one can conclude that Eq. (10) may be applicable when E_p/K_s is 0.01, but not applicable when E_p/K_s is 0.05. However, for clay loam soil, when E_p/K_s

are 0.005 and 0.01, the discrepancy ratios between Eqs. (9) and (10) are 4.76% and 0.99% , respectively. Therefore, Eq. (10) may be applicable for both E_p/K_s of 0.005 and 0.01.

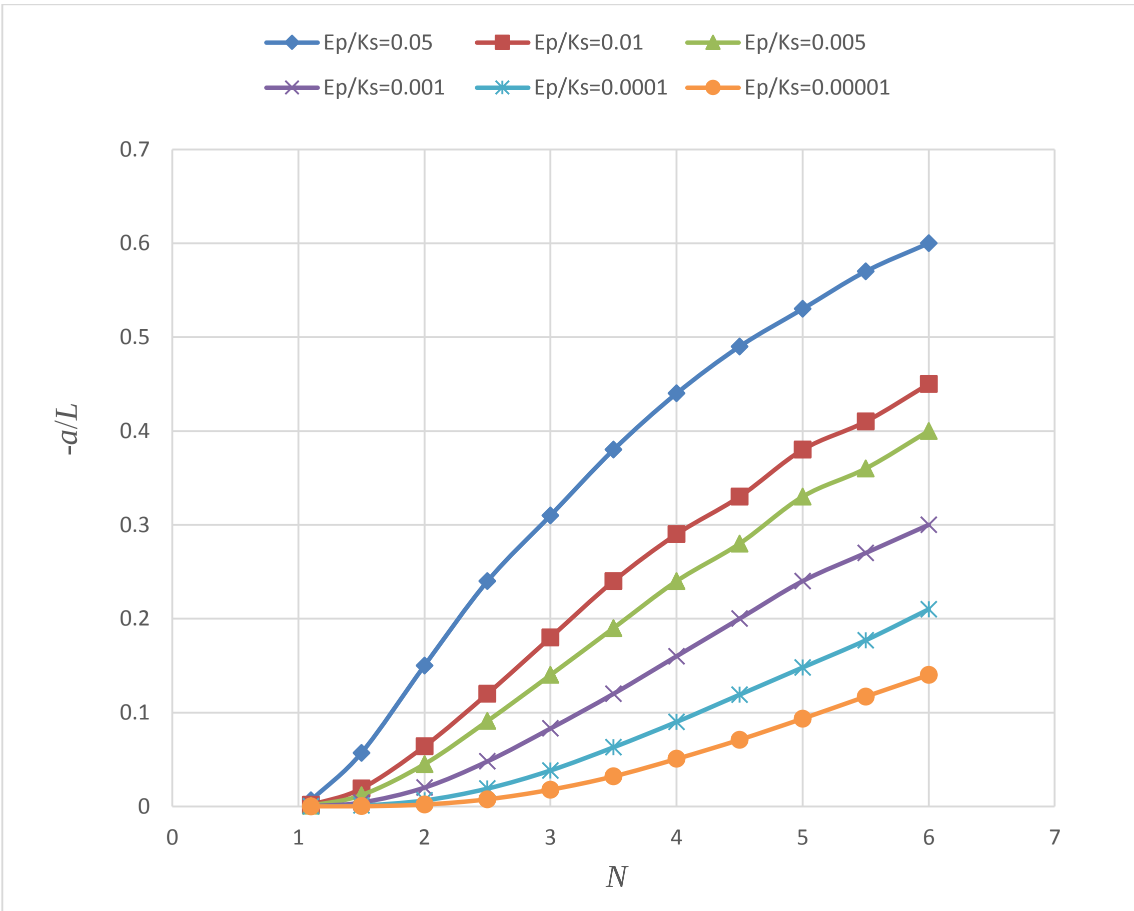

Figure 2: The different contours of E_p/K_s value for N and $-a/L$ in Eq. (9).

2.2 The Haverkamp modeling approach and solution

The E_p/K_s discussed in Figure 2 shows that Eq. (10) is not consistently valid when the water table depth L is not deep enough. To estimate the general evaporation rate, the suction/matric potential head at bare soil surface should be determined by the water content of soil. The development of new methodologies and tools that enable the determination of the water content in soil is of great importance for agronomic knowledge and scientific research (Beraldo et al., 2012). By knowing the water moisture profile above the water table to the ground surface, one can employ a certain type of equation such as the Haverkamp model or the Brooks-Corey model to calculate the matric potential head, which enable the calculation of the evaporation rate at ground surface.

In the following, we will explain the procedures for this endeavor, starting from Eq. (7). For an arbitrary matric potential head h, $x_0 = \left(\frac{E/K_s}{1+E/K_s}\right)^{1/N} \frac{h}{a}$, and x_0 is a finite positive number. For the special case of an infinite matric potential head, x_0 goes to positive infinity, which was assumed in tuhe development of Eqs. (9) and (10).

If x_0>1, Eq. (7) can be written as:

$$\int_0^{x_0} \frac{dx}{1+x^N} = \int_1^{x_0} x^{-N} \frac{dx}{1+x^{-N}} + \int_0^1 \frac{dx}{1+x^N} dx$$

$$= \int_1^{x_0} x^{-N} \sum_{n=0}^{\infty} (-x^{-N})^n \, dx + \int_0^1 \sum_{n=0}^{\infty} (-x^{-N})^n dx \; , \qquad (11)$$

$$\int_0^{x_0} \frac{dx}{1+x^N} = \sum_{n=0}^{\infty} \frac{(-1)^n {x_0}^{-Nn-N+1}}{-Nn-N+1} + \sum_{n=0}^{\infty} \frac{(-1)^n(-2Nn-N)}{(Nn+1)(-Nn-N+1)} = (-L/h_0)x_0(1+\frac{E}{K_s}) \ . \quad (12)$$

If x_0<1, one can similarly obtain:

$$\int_0^{x_0} \frac{dx}{1+x^N} = \sum_{n=0}^{\infty} (-1)^n \frac{x_0^{Nn+1}}{Nn+1} = (-L/h_0)x_0(1+\frac{E}{K_s}) \ . \quad (13)$$

To find the solution in Eq. (12) and Eq. (13), I developed a numerical root-searching program (Appendix B). On the basis of Eqs. (12) and (13), we calculated the evaporation rate for three types of soils under different surface suction values.

In this steady-state water flow problem, although the water table depths are different types of soils are the same, the soil moisture content at ground surface is not the same.

In a steady-state water flow condition and the shallowest water table case as L is a range of 0 cm to 20 cm, the water table is fairly close to the ground surface and the soil moisture content at the ground surface should be very high. The absolute value of matric potential head h at the bare ground surface in this case should be very small. For the shallow water case of vadose zone which is filled by sandy soil, owing to the hydraulic conductivity of sandy soil is usually great, the soil moisture content at ground surface is also very high. The calculation of evaporation rate for this case must be carefully with the absolute value of matric potential head.

In general, the evaporation rate at bare ground surface should be not great than evapotranspiration rate or pan evaporation rate. In view that our solution of evaporation is constant 24 hours a day and the observed value of pan evaporation rate concludes the low level of evaporation in night. The potential evaporation rate could not exceed three times of the maximum pan evaporation value, which is 1.865 cm/day.

The results of the first soil type (Chino Clay) are presented in Figure 3 and Table 4. For this case, the saturated hydraulic conductivity is 1.95 cm/day, the fitting parameters a is -23.8 cm and N is 2. The results of the second soil type (Pachappa fine sandy loam) are presented in Figure 4 and Table 5. For this case, the saturated hydraulic conductivity is 12.31 cm/day, the fitting parameters a is -63.83 cm and N is 3. The results for the third soil type (Buckeye fine sand) are presented in Figure 5 and Table 6. The saturated hydraulic conductivity is 417 cm/day, the fitting parameters a is -44.7 cm and N is 5.

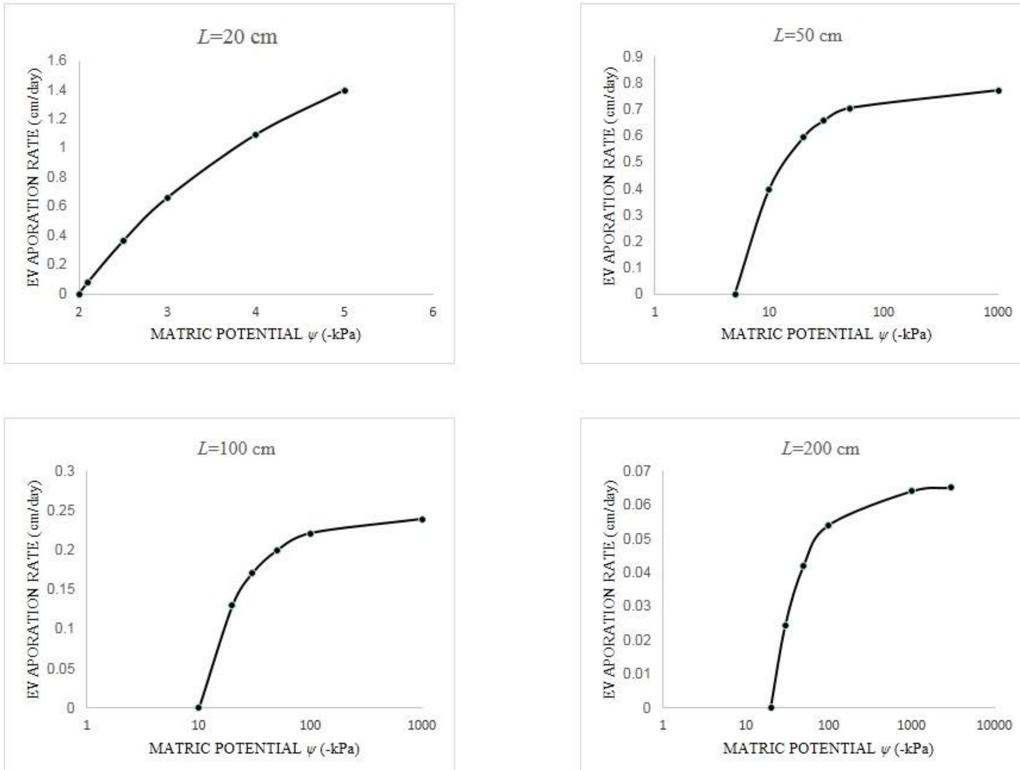

Figure 3: Influence of water table depth and matric potential on estimated evaporation rate for the Chino Clay**.**

Table 4: the steady-state evaporation rate (cm/d) on different water table depth (L) and matric potential head (h) / matric potential (ψ) for Chino Clay in vadose zone.

Ψ(kPa)	L(cm) / h(cm)	20	50	100	200
-2	-20	0	-	-	-
-2.1	-21	0.078	-	-	-
-2.5	-25	0.361	-	-	-
-3	-30	0.658	-	-	-
-4	-40	1.09	-	-	-
-5	-50	1.394	0	-	-
-10	-100	-	0.398	0	0.13
-20	-200	-	0.594	0.13	0
-30	-300	-	0.656	0.17	0.0244
-50	-500	-	0.704	0.199	0.0421
-100	-1000	-	-	0.221	0.054
-1000	-10000	-	0.772	0.239	0.064
-3000	-30000	-	-	-	0.065

The results in Table 4 give the calculated evaporation rate values under different matric potential head values at bare ground surface for four different water table depths ranging from 20 cm to 200 cm. For the shallow water table as the water depth L equals 20 cm, the saturation of soil at bare ground surface is close to 1, and the soil moisture content is very high. The absolute value of matric potential head at bare ground surface in this case should be small. According to the limit of evaporation rate (usually E is around 3 cm/day at ground surface), the matric potential heads and the evaporation rates are show in Table 4.

When the water table depths are 50 cm, 100 cm and 200 cm, and the absolute values of matric potential head increase to a large level, we can find the evaporation rates should be converged. It appears that the evaporation rate E=0.77 cm/day for h=-10000 cm and L=50 cm in Table 4 agrees reasonably well with the solution E_p=0.79 cm/day obtained from Eq. (9), suggesting that h=-10000 cm can be regarded as close to the maximum matric potential head since the actual evaporation rate is very similar to the potential evaporation rate for this case.

The results in Tables 5 and 6 also give the calculated evaporation rate values under different matric potential head values at bare ground surface for four different water table depths ranging from 20 cm to 200 cm for Pachappa soil and Buckeye soil. Differently, when the water table depths equal 20 cm, 50 cm and 100 cm, the potential evaporation rates are limited by the high soil moisture content. The absolute values of matric potential head at bare ground surface for these cases would not very large. For the case of water table depth 200 cm, the potential evaporation rate is equal 0.644 cm/day, when the soil of ground surface is very dry.

Figures 3 to 5 depict the changes of evaporation rate accord to a range of matric potential for four different water table depths for each of the three soils types mentioned above (Chino Clay, Pachappa soil, and Buckeye soil). In most cases of shallow water table, the absolute value of matric potential would not exceed the range of the curve. For the cases that the evaporation rates have been converged, Eqs. (12) and (13) can be unlimited used to calculate the evaporation rate.

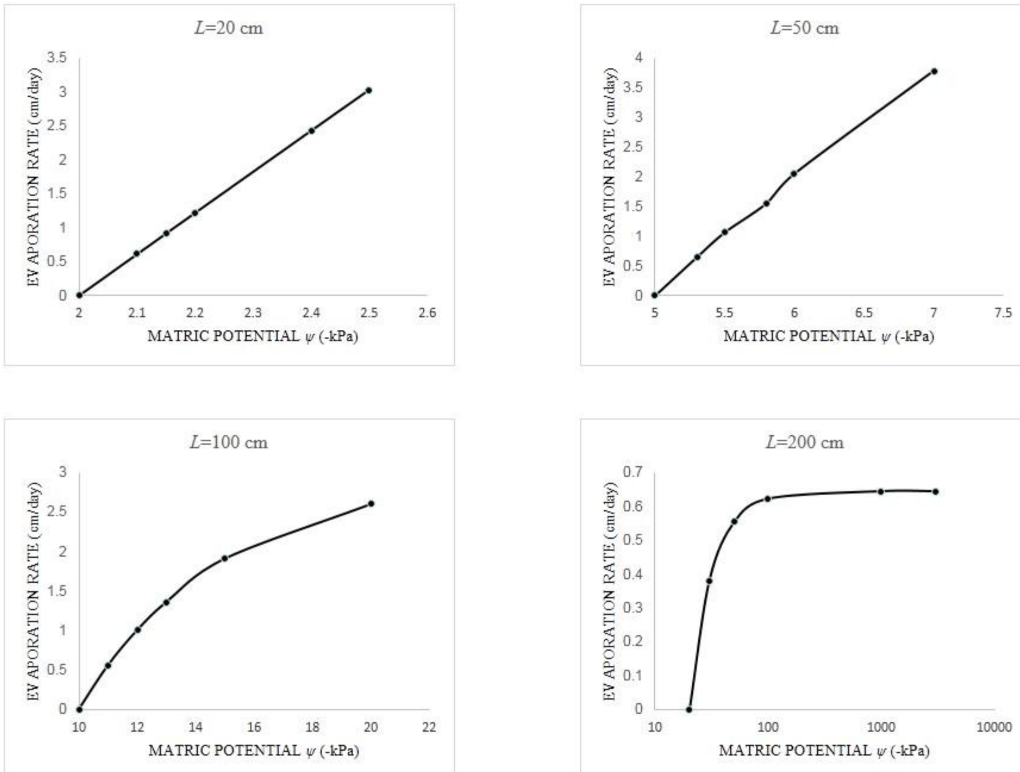

Figure 4: Influence of water table depth and matric potential on estimated evaporation rate for the Pachappa soil (fine sandy loam).

Table 5: the steady-state evaporation rate (cm/d) on different water table depth (L) and matric potential head (h) / matric potential (ψ) for Pachappa soil in vadose zone.

Ψ(kPa)	L(cm) \ h(cm)	20	50	100	200
-2	-20	0	-	-	-
-2.1	-21	0.61	-	-	-
-2.15	-21.5	0.915	-	-	-
-2.2	-22	1.219	-	-	-
-2.4	-24	2.43	-	-	-
-2.5	-25	3.032	-	-	-
-5	-50	-	0	-	-
-5.3	-53	-	0.647	-	-
-5.5	-55	-	1.063	-	-
-5.8	-58	-	1.554	-	-
-6	-60	-	2.051	-	-
-7	-70	-	3.782	-	-
-10	-100	-	-	0	-
-11	-110	-	-	0.56	-
-12	-120	-	-	1.004	-
-13	-130	-	-	1.36	-
-15	-150	-	-	1.91	-
-20	-200	-	-	2.602	0
-30	-300	-	-	-	0.381
-50	-500	-	-	-	0.554
-100	-1000	-	-	-	0.622
-1000	-10000	-	-	-	0.644
-3000	-30000	-	-	-	0.644

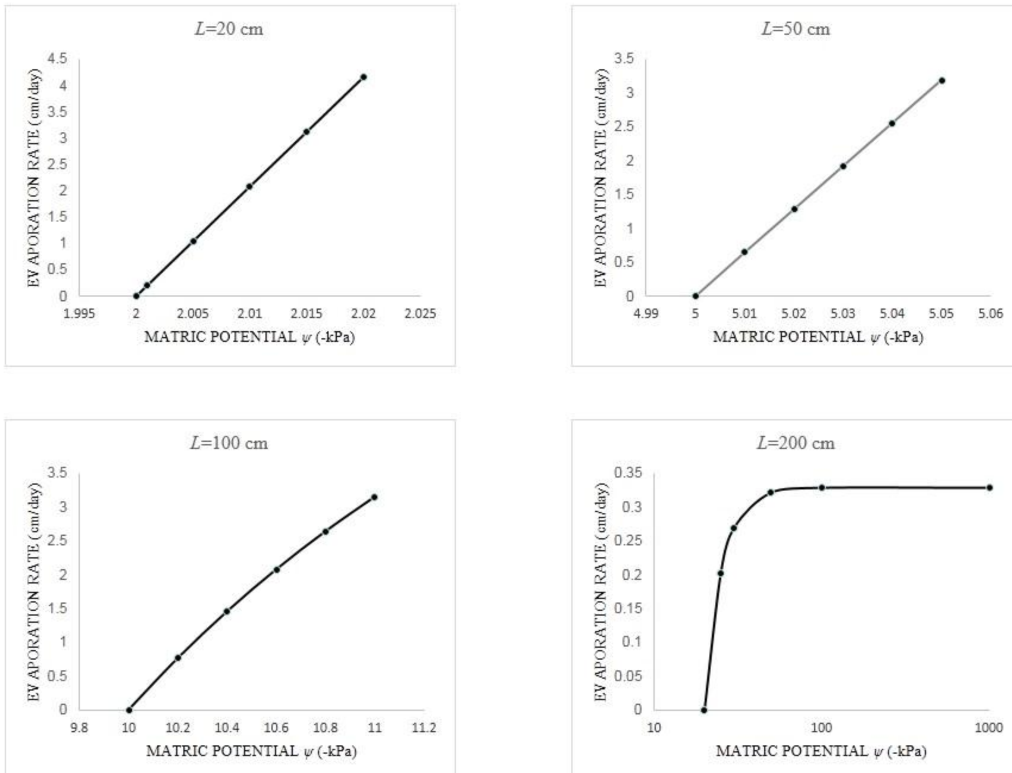

Figure 5: Influence of water table depth and matric potential on estimated evaporation rate for the Buckeye soil (fine sand).

Table 6: the steady-state evaporation rate (cm/d) on different water table depth (L) and matric potential head (h) / matric potential (ψ) for Buckeye soil in vadose zone.

Ψ(kPa)	L(cm) / h(cm)	20	50	100	200
-2	-20	0	-	-	-
-2.001	-20.01	0.207	-	-	-
-2.005	-20.05	1.039	-	-	-
-2.01	-20.1	2.078	-	-	-
-2.015	-20.15	3.114	-	-	-
-2.02	-20.2	4.153	-	-	-
-5	-50	-	0	-	-
-5.01	-50.1	-	0.647	-	-
-5.02	-50.2	-	1.283	-	-
-5.03	-50.3	-	1.922	-	-
-5.04	-50.4	-	2.558	-	-
-5.05	-50.5	-	3.192	-	-
-10	-100	-	-	0	-
-10.2	-102	-	-	0.765	-
-10.4	-104	-	-	1.455	-
-10.6	-106	-	-	2.077	-
-10.8	-108	-	-	2.64	-
-11	-110	-	-	3.15	-
-20	-200	-	-	-	0
-25	-250	-	-	-	0.202
-30	-300	-	-	-	0.269
-50	-500	-	-	-	0.321
-100	-1000	-	-	-	0.328
-1000	-10000	-	-	-	0.328

2.3 The Brooks-Corey modeling approach and solution

The Brooks-Corey function is also widely used for water flow in unsaturated zone. It is commonly associated with Burdine's pore-size distribution model (Burdine, 1953), leading to the hydraulic conductivity function:

$$K(S) = K_s S^{p+2+2/\lambda}, \tag{14}$$

$$S = (\frac{h_v}{h})^{\lambda}, h_v > h, \tag{15}$$

$$\mathrm{S} = 1, h_v \leq h. \tag{16}$$

where *p* (positive) is a soil specific parameter which accounts for the tortuosity of the flow [dimensionless], λ (positive) is the pore size distribution index [dimensionless], *S* is the degree of saturation [dimensionless] and h_v (negative) [L] is the air-entry value of *h* (negative). The *p* value assumed to be 1.0 in the original study of Brooks and Corey (1964).

Instead of using the Haverkamp equation (Haverkamp et al., 1977), one can use the Brooks and Corey (1964) function in Eq. (4) to get：

$$\int_0^{h_0} \frac{dh}{1+E/(K_s(S)^{p+2+2/\lambda})} = -L, \tag{17}$$

where $E=J_w>0$ is the evaporation rate, and $h_v<h$. Therefore:

$$\int_0^{h_0} \frac{dh}{1+E/\left(K_s\left(\frac{h_v}{h}\right)^{p+2+2/\lambda}\right)} = -L, \tag{18}$$

where $h_0 < h_v$ at $z < -(h_0 - h_v) L / h_0$ represents a region above the water table which has the largest saturation value "1" in Eq. (16) . Defining the following new parameters: $w=p\lambda+2\lambda+2$, the integral in Eq. (18) can be transformed to:

$$\int_{hv}^{h_0} \frac{dh}{1+\frac{Eh^w}{K_s {h_v}^w}} + \int_0^{h_v} \frac{dh}{1+\frac{E}{K_s}} = -L. \tag{19}$$

The second part on the left side of Eq. (19) is easy to calculate as follows:

$$\int_{hv}^{h_0} \frac{dh}{1+\frac{Eh^w}{K_s {h_v}^w}} = -L - \frac{h_v K_s}{E+K_s}. \tag{20}$$

Defining the following new parameters: $\varepsilon = \left(\frac{E}{K_s}\right)^{1/w}$, $x = \frac{h}{h_v}\left(\frac{E}{K_s}\right)^{1/w}$ and $x_0 = \frac{h_0}{h_v}\left(\frac{E}{K_s}\right)^{1/w}$, one transforms Eq. (20) into:

$$\int_{\varepsilon}^{x_0} \frac{dx}{1+x^w} = \left(-\frac{L}{h_0}\right) x_0 - \frac{x_0 h_v}{h_0 + h_0 K_s \left(\frac{x_0 h_v}{h_0}\right)^w}. \tag{21}$$

Using similar method of dealing with Eq. (7) for the left side of Eq. (21), when $x_0>1$, one has:

$$\int_{\varepsilon}^{x_0} \frac{dx}{1+x^w} = \sum_{n=0}^{\infty} \frac{(-1)^n {x_0}^{-wn-w+1}}{-wn-w+1} + \sum_{n=0}^{\infty} \frac{(-1)^n(-2wn-w)}{(wn+1)(-wn-w+1)} - \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{x_0 h_v}{h_0}\right)^{wn+1}}{wn+1} =$$

$$(-L/h_0)x_0 - \frac{x_0 h_v}{h_0 + h_0 \left(\frac{x_0 h_v}{h_0}\right)^w}. \tag{22a}$$

When $x_0<1$, one has:

$$\int_{\varepsilon}^{x_0} \frac{dx}{1+x^w} = \sum_0^{\infty}(-1)^n \frac{x_0^{wn+1}}{wn+1} - \sum_{n=0}^{\infty} \frac{(-1)^n\left(\frac{x_0 h_v}{h_0}\right)^{wn+1}}{wn+1} = \left(-\frac{L}{h_0}\right) x_0 - \frac{x_0 h_v}{h_0 + h_0\left(\frac{x_0 h_v}{h_0}\right)^w} . \quad (22b)$$

To find the solution in Eqs. (22a) and (22b) for x_0, we developed a numerical program (Appendix C).

In a realistic problem, when we want to estimate an evaporation rate of bare ground surface for a known water table depth and soil type of vadose zone, we can measured the soil water content θ and then calculate the matric potential head h from $S = \frac{\theta - \theta_r}{\theta_s - \theta_r}$ and Eq. (15). The evaporation rate can be calculated by Eqs. (23a) and (23b).

Sadeghi et al. (2012) suggested that the Brooks-Corey soil parameters h_v equaled the Haverkamp fitting parameters a, and $p\lambda+2\lambda+2$ equaled the Haverkamp fitting parameter N for Chino Clay. The results of evaporation rate calculated by the Haverkamp model and the Brooks-Corey model are shown in Figure 6 and Table 7. One can see that the evaporation rate-matric potential head relationship can be nicely fitted with logarithm function $y = b\ln(x) - c$, where x and y represent the ($-h$) and E, respectively, and b and c are two fitting constants. It is interesting to see that the E versus ($-h$) relationship shown in Figure 6 (a logarithmic function) is quite different from the E-L relationship (a power-law function) shown in Figure 3 for Chino Clay. One thing to note is that the value of evaporation rate in Figure 3 would be close to converged for the matric potential head h=-500 cm. If you want to estimate the evaporation rate quickly by Figure 3, the logarithm function can be used only for matric potential head ranging -500 cm to 0 cm. The logarithm function cannot be used, when the matric potential head is smaller than -500 cm.

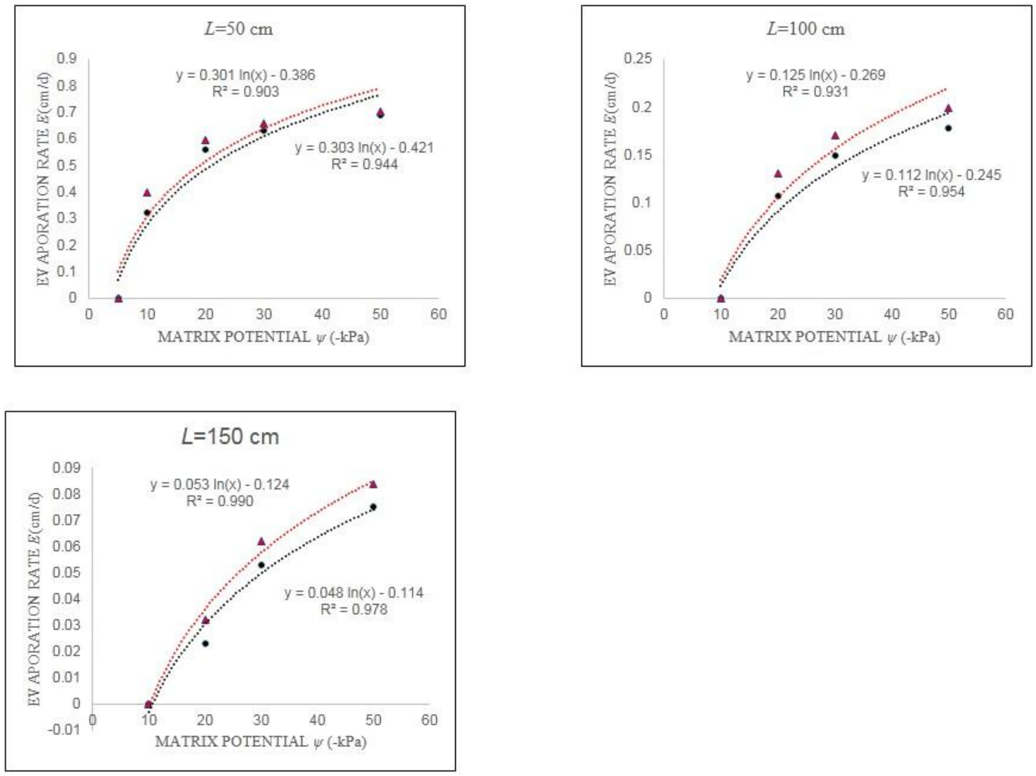

Figure 6: The evaporation rate (cm/d) calculated by Haverkamp model (red triangle) and Brooks-Corey model (black dot) versus matric potential (-kPa) for the Chino Clay**.**

Table 7: the steady-state evaporation rate (cm/d) calculated by Haverkamp model and Brooks-Corey model on different water table depth (*L*) and matric potential head (*h*) for Chino Clay vadose zone.

h(cm) / *L*(cm)	-50	-100	-200	-300	-500	-10000
B-C(L=50)	0	0.323	0.560	0.630	0.690	0.775
Haverkamp(L=50)	0	0.398	0.594	0.656	0.704	0.770
B-C(L=100)	0	0	0.107	0.149	0.178	0.218
Haverkamp(L=100)	0	0	0.130	0.170	0.199	0.239
B-C(L=150)	0	0	0.023	0.053	0.075	0.103
Haverkamp(L=150)	0	0	0.032	0.062	0.084	0.112

The ratios of the *E* values calculated by the Brooks-Corey and the Haverkamp models for a water table depth of 50 cm and the matric potential heads of -100 cm, -200 cm, -300 cm, -500 cm and -10000 cm are 81%, 94%, 96%, 98% and 100%, respectively. The reason to include -10000 cm of matric potential head is to simulate the potential evaporation rate (E_p). If changing the water table depth to 100 cm, the ratios of the *E* values calculated by the Brooks-Corey and the Haverkamp models are 82%, 88%, 89% and 91% for the matric potential heads of -200 cm, -300 cm, -500 cm and -10000 cm. If further changing the water table depth to 150 cm, the ratios of the *E* values calculated by the Brooks-Corey versus the Haverkamp models are 72%, 85%, 89%, 92% for the matric potential heads of -200 cm, -300 cm, -500 cm, -10000 cm, respectively.

A few observations can made for the comparison of the Brooks-Corey versus the Haverkamp models. First, the calculated *E* values from both models are not too far apart, even for the relatively small matric potential head at the surface. The smallest ratio of the E values between the Brooks-Corey model and the Haverkamp model is 72% for the water table depth of 150 cm and a matric potential head of – 200 cm. Second, such a ratio increases with the magnitude of the matric potential head for a given water table depth. Third, the E_p values (corresponding to the -10000 cm matric potential head) calculated from these two models are very close to each other. For instance, for the shallower water table depth of 50 cm, the E_p values calculated from both models are essentially the same, resulting in a 100% ratio. The greatest discrepancy of the E_p ratio for the water table depth of 150 cm is only 8%.

The Brooks-Corey parameters used above are closely related to the Haverkamp parameters. However, this is not always applicable for some soil types. For example, Rose et al. (2005) summarized the Brooks-Corey fitting parameters for four types of soils with the details listed Table 8. Substituting the Brooks-Corey parameters of Table 8 into Eqs. (22a) and (22b), we calculated the evaporation rate for four types of soils under different matric potential heads at ground surface, and the results are shown in Tables 9 to 13 and Figures 7 to 10. For shallow water table condition, sometimes owing to great hydraulic conductivity, the soil at the bare ground surface is very moist, and the absolute value of matric potential head is low. The potential evaporation rate is also lower than pan evaporation rate. In this thesis, the evaporation rate calculated by Equations can regarded an instant rate for the soil moisture condition. The maximum value of evaporation rates can be considered as maximum evaporation in one day. That may be not greater than three times of the observed value of pan evaporation rate. Because in night and early morning, the pan evaporation rate should decrease to a low level, the pan evaporation value is a mean value of one day.

Table 8: The parameters for soil properties (Brooks and Corey, 1964), soil used in the program (Rose et al., 2005).

Soil Type	K_s(cm/d)	h_v(cm)	λ	θ_r(m^3m^{-3})	θ_s(m^3m^{-3})
Clay loam	0.976	-25.9	0.194	0.000	0.45
Silty loam	65.3	-20.7	0.211	0.001	0.52
Sandy loam	1253	-8.69	0.474	0.001	0.49
Coarse sand	11232	-4.92	0.592	0.001	0.41

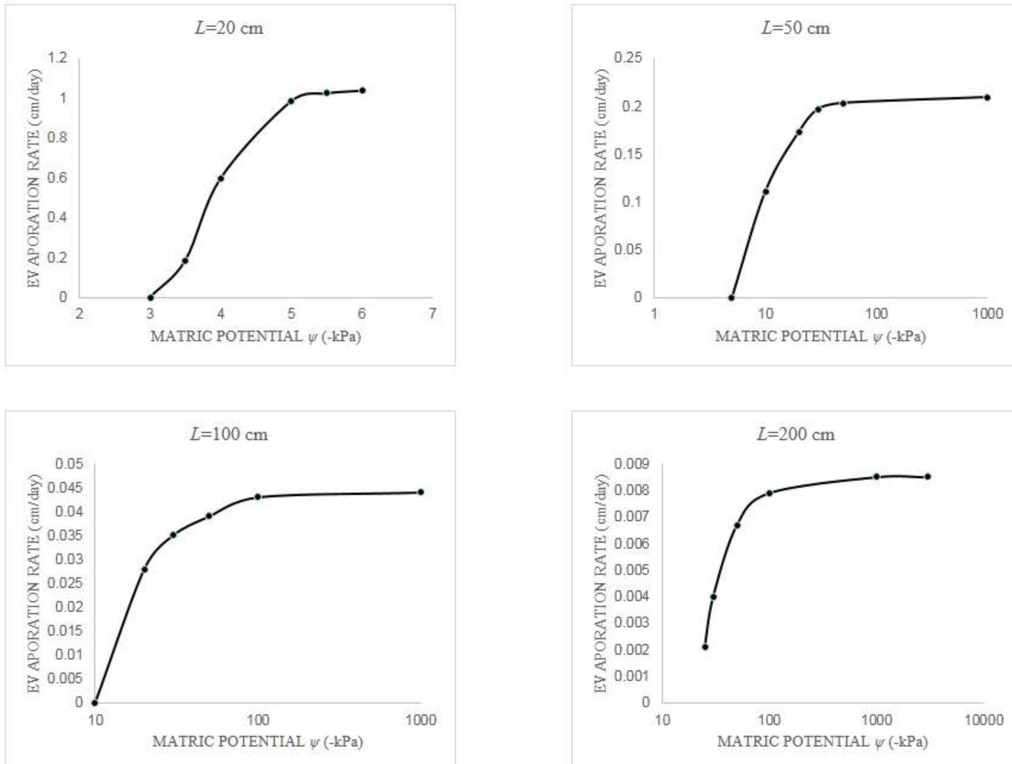

Figure 7: Influence of water table depth and matric potential on estimated evaporation rate for the clay loam.

Table 9: The steady-state evaporation rate (cm/d) on different water table depth (L) and matric potential head (h) / matric potential (ψ) for clay loam in vadose zone.

θ(m^3m^{-3})	ψ(kPa)	L(cm) \ h(cm)	20	50	100	200
0.437	-3	-30	0	-	-	-
0.424	-3.5	-35	0.186	-	-	-
0.414	-4	-40	0.599	-	-	-
0.396	-5	-50	0.986	0	-	-
0.389	-5.5	-55	1.025	-	-	-
0.382	-6	-60	1.038	-	-	-
0.346	-10	-100	-	0.111	0	-
0.303	-20	-200	-	0.173	0.028	-
0.290	-25	-250	-	-	-	0.0021
0.280	-30	-300	-	0.197	0.035	0.0040
0.253	-50	-500	-	0.203	0.039	0.0067
0.222	-100	-1000	-	-	0.043	0.0079
0.142	-1000	-10000	-	0.209	0.044	0.0085
0.115	-3000	-30000	-	-	-	0.0085

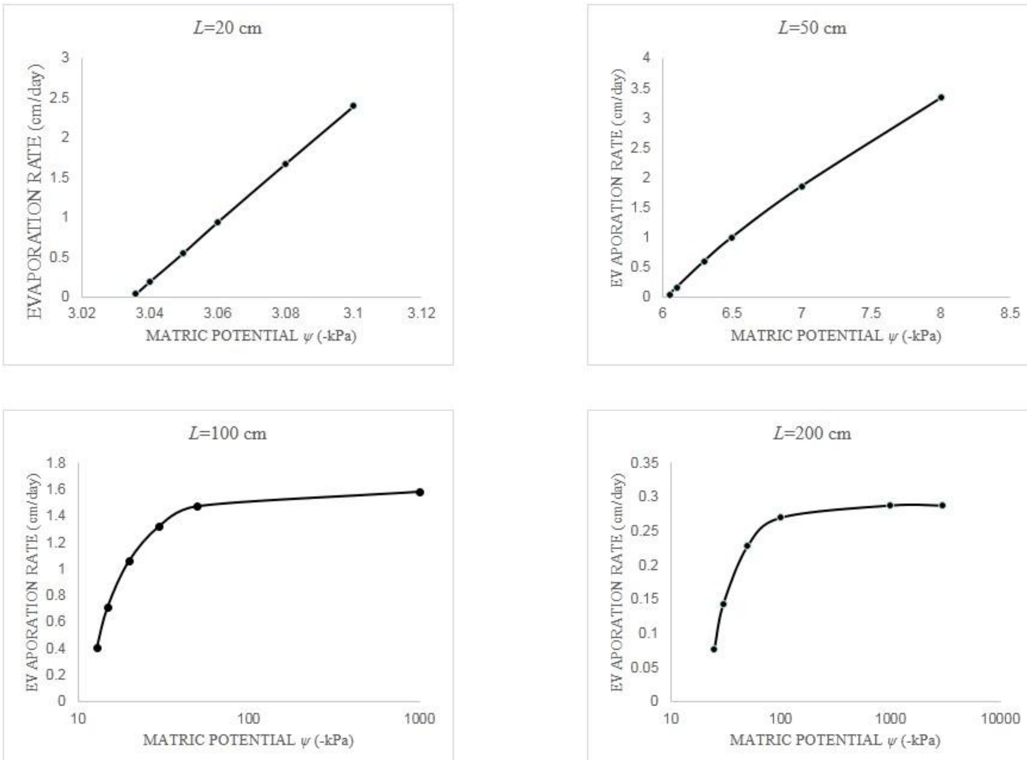

Figure 8: Influence of water table depth and matric potential on estimated evaporation rate from the silty loam**.**

Table 10: The steady-state evaporation rate (cm/d) on different water table depth (L) and matric potential head (h) / matric potential (ψ) for silty loam in vadose zone.

θ(m^{3}m^{-3})	ψ(cm)	L(cm) / h(cm)	20	50	100	200
0.479	-3.036	-30.36	0.038	-	-	-
0.479	-3.04	-30.4	0.190	-	-	-
0.478	-3.05	-30.5	0.556	-	-	-
0.478	-3.06	-30.6	0.939	-	-	-
0.477	-3.08	-30.8	1.674	-	-	-
0.477	-3.1	-31	2.396	-	-	-
0.414	-6.05	-60.5	-	0.036	-	-
0.413	-6.1	-61	-	0.153	-	-
0.410	-6.3	-63	-	0.596	-	-
0.408	-6.5	-65	-	0.998	-	-
0.401	-7	-70	-	1.857	-	-
0.390	-8	-80	-	3.343	-	-
0.352	-13	-130	-	-	0.404	-
0.342	-15	-150	-	-	0.708	-
0.322	-20	-200	-	-	1.056	-
0.307	-25	-250	-	-	1.318	0.077
0.295	-30	-300	-	-	1.318	0.142
0.265	-50	-500	-	-	1.470	0.228
0.229	-100	-1000	-	-	-	0.270
0.141	-1000	-10000	-	-	1.580	0.288
0.112	-3000	-30000	-	-	-	0.288

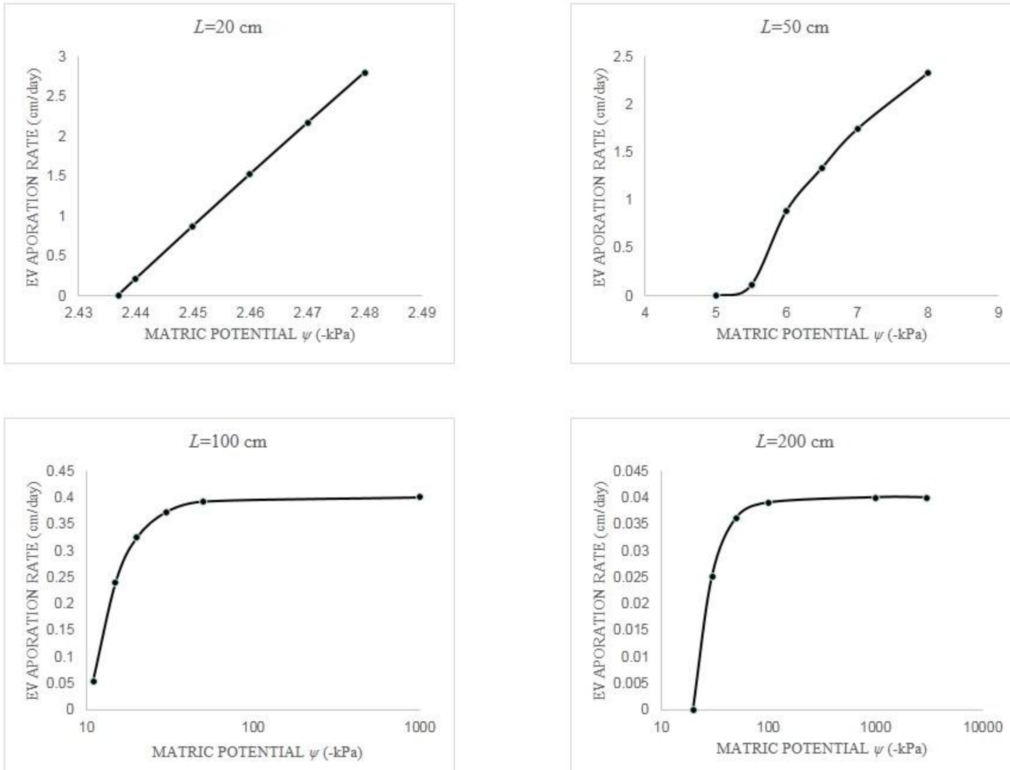

Figure 9: Influence of water table depth and matric potential on estimated evaporation rate for sandy loam.

Table 11: The steady-state evaporation rate (cm/d) on different water table depth (L) and matric potential head (h) / matric potential (ψ) for sandy loam in vadose zone.

θ(m^{3}m^{-3})	ψ(cm)	L(cm) / h(cm)	20	50	100	200
0.300	-2.437	-24.37	0.004	-	-	-
0.300	-2.44	-24.4	0.206	-	-	-
0.299	-2.45	-24.5	0.871	-	-	-
0.299	-2.46	-24.6	1.526	-	-	-
0.298	-2.47	-24.7	2.170	-	-	-
0.297	-2.48	-24.8	2.805	-	-	-
0.213	-5	-50	-	0	-	-
0.204	-5.5	-55	-	0.112	-	-
0.196	-6	-60	-	0.888	-	-
0.188	-6.5	-65	-	1.335	-	-
0.182	-7	-70	-	1.742	-	-
0.171	-8	-80	-	2.328	-	-
0.147	-11	-110	-	-	0.054	-
0.127	-15	-150	-	-	0.240	-
0.111	-20	-200	-	-	0.324	0
0.091	-30	-300	-	-	0.373	0.025
0.072	-50	-500	-	-	0.393	0.036
0.052	-100	-1000	-	-	-	0.039
0.017	-1000	-10000	-	-	0.401	0.040
0.010	-3000	-30000	-	-	-	0.040

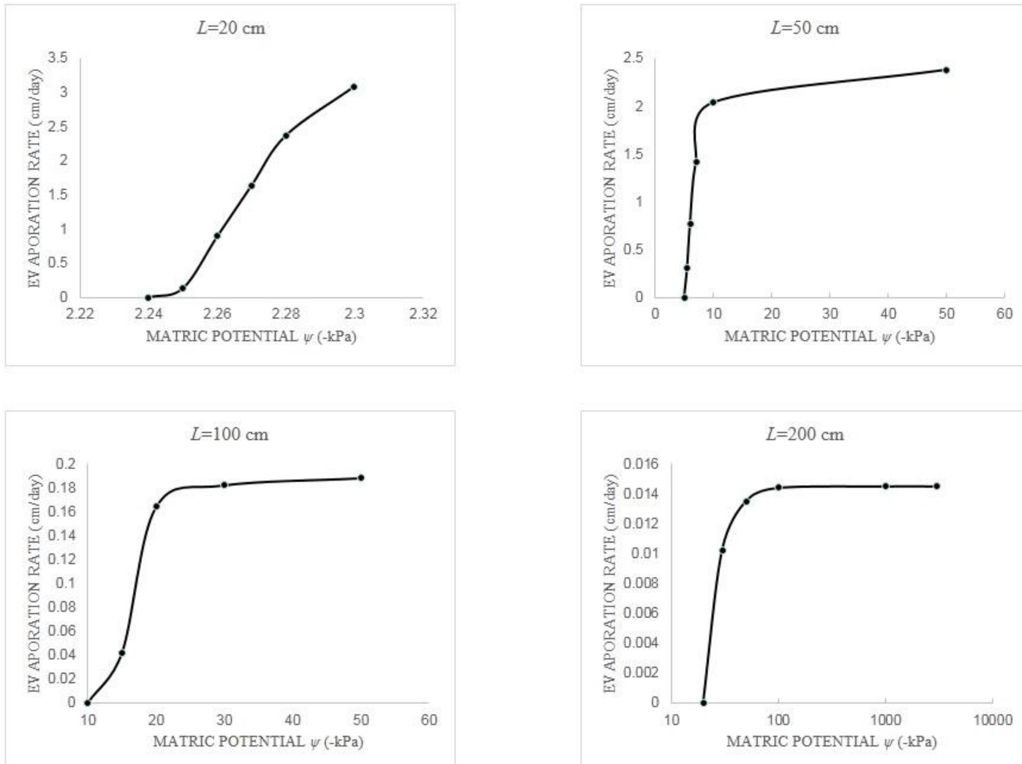

Figure 10: Influence of water table depth and matric potential on estimated evaporation rate for coarse loam.

Table 12: The steady state evaporation rate (cm/d) on different water table depth (L) and matric potential head (h) / matric potential (ψ) for coarse sand in vadose zone.

θ(m^3m^{-3})	ψ(cm)	L(cm) \ h(cm)	20	50	100	200
0.167	-2.24	-22.4	0	-	-	-
0.166	-2.25	-22.5	0.134	-	-	-
0.166	-2.26	-22.6	0.894	-	-	-
0.165	-2.27	-22.7	1.639	-	-	-
0.165	-2.28	-22.8	2.370	-	-	-
0.164	-2.3	-23	3.088	-	-	-
0.104	-5	-50	-	0	-	-
0.098	-5.5	-55	-	0.302	-	-
0.093	-6	-60	-	0.763	-	-
0.085	-7	-70	-	1.413	-	-
0.069	-10	-100	-	2.036	-	-
0.065	-11	-110	-	-	0.035	-
0.054	-15	-150	-	-	0.042	-
0.046	-20	-200	-	-	0.164	0
0.036	-30	-300	-	-	0.182	0.0102
0.027	-50	-500	-	2.378	0.188	0.0135
0.018	-100	-1000	-	-	-	0.0144
0.005	-1000	-10000	-	-	0.190	0.0145
0.002	-3000	-30000	-	-	-	0.0145

In realistic study of evaporation, the evaporation rate should be limited by high water moisture content at ground surface for shallow water table cases. In clay loam problem, when the water table depth equals 20 cm, the evaporation rate will vary by a ranging matric potential from -6 kPa to -30 kPa at bare ground surface. The soil water content at ground surface may range from 0.382 to 0.437. And for the other three types of soils, the evaporation rate should be also limited by a range of matric potential. The water content at ground surface for silty loam is ranging from 0.477 to 0.479, and the matric potentials are ranging from -3.036 kPa to -3.1 kPa. The water content at ground surface for sandy loam is ranging from 0.297 to 0.300, and the matric potentials are ranging from -2.437 kPa to -2.48 kPa. The water content at ground surface for silty loam is ranging from 0.164 to 0.167, and the matric potentials are ranging from -2.24 kPa to -2.30 kPa.

For instance, when the water table depth equals 20 cm and the soil water content θ at ground surface is 0.424 m^3m^{-3} for clay loam vadose zone. We can calculate the matric potential head h=-35 cm from Eq. (15). And then the evaporation rate can be calculated as 0.186 cm/day by Eq. (23b).

When the water table depth is 50 cm, the soils of silty loam, sandy loam and coarse sand should not be very dry. The water content of silty loam at ground surface could range from 0.390 to 0.414. The water content of sandy loam at ground surface could range from 0.171 to 0.213. The water content of coarse sand at ground surface could range from 0.027 to 0.104. The soil of clay loam could be very dry and very wet. Because the hydraulic conductivity of clay loam is very small and the water of surface soil could be recharged quickly.

The other cases of the water table depth 100 cm and 200 cm, the soil moisture content at ground surface can change from lowest to highest. The evaporation rate can be calculated unlimited by Eqs. (23a) and (23b).

3 A COMPARISON OF THE HAVERKAMP MODEL, THE BROOKS-COREY MODEL AND HYDRUS-1D

3.1 Introduction

HYDRUS-1D is a software for simulating water, heat and solute movement in one-dimensional variably saturated media (Simunek et al., 2005). The software consists of the HYDRUS program, and the HYDRUS-1D interactive graphics-based user interface (Simunek et al., 2005). The HYDRUS-1D code numerically solves the Richards equation for variably saturated water flow (Jury and Horton, 2004):

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left(K(\theta)\frac{\partial h}{\partial z} + K(\theta)\right), \tag{23}$$

where z (cm) is the vertical coordinate positive upward and t (day) is time. In this study, the HYDRUS-1D created 100 sections to solve the differential equation by 101 nodes for the distance between ground surface and water table.

3.2 Numerical examples

In the study of steady-state water flow between the water table and bare ground surface, we developed two methods based on the Haverkamp model and the Brooks-Corey model to calculate the E value for an arbitrary matric potential head at ground surface. In the HYDRUS-1D program, we can select the Brooks-Corey module to simulate the water flow, and compare the simulated E values with those calculated by Eqs. (22a) and (22b). The types of soils selected for illustration are clay loam, silty loam and sandy loam and coarse sand, and the fitting parameters are shown in Table 8.

The results for the water table depths of 50 cm and 100 cm were shown in Figures 11 and 12. The results in Figures 11 and 12 give the calculated and simulated E values under different matric potential head values at bare ground surface. The E values calculated by Eqs. (22a) and (22b) are shown in Tables 9 to 12 for different matric potential heads. For the matric potential -10 kPa and soil water content 0.346 at the bare ground surface, the evaporation rate of clay loam vadose zone is 0.111 cm/day by calculated and 0.118 cm/day by simulated. For the matric potential -15 kPa and soil water content 0.342 at bare ground surface, the evaporation rate of silty loam is 0.708 cm/day by calculated and 0.703 cm/day by simulated. Such calculated E values are very close to their simulated counterparts by HYDRUS-1D for the cases of clay loam and silty loam.

For the cases of sandy loam and coarse sand, the calculated E values are smaller than their simulated counterparts by HYDRUS-1D. The discrepancy ratio of evaporation rate by calculated and simulated for the water table depth 50 cm and sandy loam is ranging from 5.9% to 12.0%. The discrepancy ratio of evaporation rate by calculated and simulated for the water table depth 50 cm and coarse sand is ranging from 388% to 596%.

Figures 11 and 12 indicate that the evaporation rates calculated fit very well with the results of HYDRUS-1D for clay loam and silty loam, but very bad for sandy loam and coarse sand. From these two figures, we can find that when the hydraulic conductivity is greater and the effective porosity of the soil is larger, the results by calculated will fit worse with those by simulated.

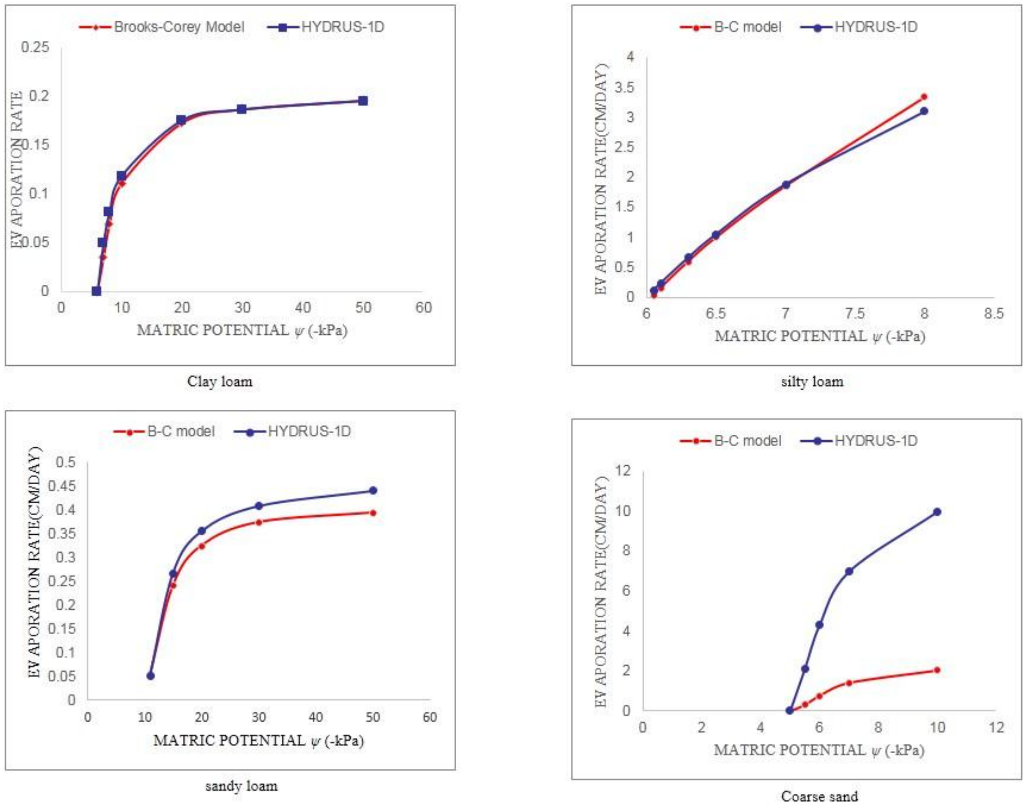

Figure 11: A comparison of Brooks-Corey solution and HYDRUS-1D for the water table depth equal 50 cm

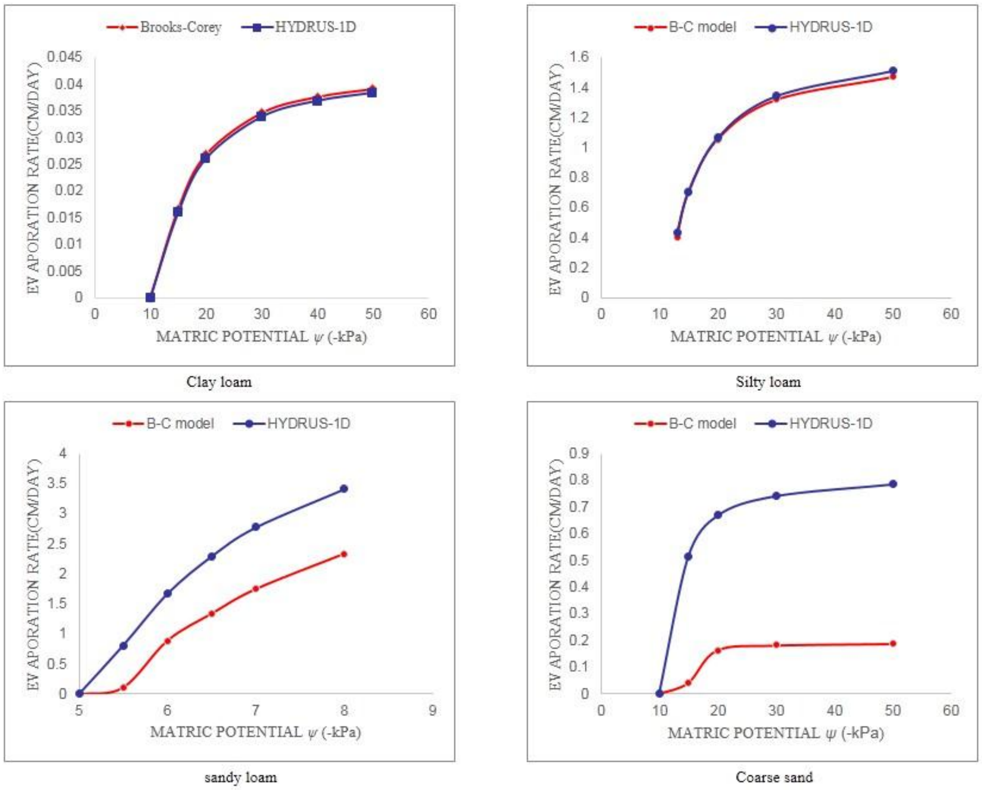

Figure 12: A comparison of Brooks-Corey solution and HYDRUS-1D for the water able depth equal 100cm.

4 CONCLUSIONS AND FUTURE STUDY

4.1 Conclusions

In this thesis, the Haverkamp model, the Brooks-Corey model and HYDRUS-1D have been used to analytically and semi-analytically calculate the steady-state evaporation rate for an arbitrary matric potential head at ground surface with the presence of a water table, as the Haverkamp model and Brooks-Corey model are widely used in vadose zone study. This is different from most previous analytical and semi-analytical studies which usually focused on estimating the potential evaporation rate at ground surface (with infinitely large matric suction at ground surface). In actual field conditions, the surface suction may be affected by the humid climate, invalidating the infinity matric suction assumption, or the actual evaporation rate is much less than the potential evaporation rate. This study fills a gap for providing an analytical and semi-analytical method to calculate the evaporation rate under an arbitrary surface suction. The new model established here may also be used to estimate the difference of the potential and actual evaporation rates for a variety of conditions.

This study selects four types of soils to demonstrate the application of the proposed method for the Haverkamp model and the Brooks-Corey model. For some soils where the fitting parameters of a and N in the Haverkamp model are directly related to the fitting parameters of p, λ, h_v of the Brooks-Corey model in a fashion of $N=p\lambda+2\lambda+2$ and $h_v=a$ (Sadeghi et al., 2012), the E values obtained from these two models can fit very well with each other (Figure 6). The compassion between the Brooks-Corey model and the

HYDRUS-1D simulation is very good for the cases of clay loam and silty loam (see Figures 11 and 12). The results show the method I developed in this thesis is useful for general evaporation rate estimation for the soil which concludes a large percentage of clay.

4.2 Future study

I have developed two methods to calculate the steady-state evaporation rate at bare ground surface above a homogeneous vadose zone. In future study, we can calculate the evaporation rate for more kinds of soils and more water table depths. We can also employ two or three layers model, which is probably more realistic, for unsaturated zone to calculate the upward flux. Because the soil which is below 5 cm of the ground surface usually is looser than the deeper soil, and the clay, silt and sand percentages of soil is not consistently in the unsaturated zone. The analytical solutions of two or three layers will be more difficult. The van Genuchten model for unsaturated hydraulic conductivity is another widely used model in soil physics, and we can try to develop a new solution based on the van Genuchten model (1980) and to compare the result with the experimental data and the van Genuchten module of HYDRUS-1D.

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APPENDIX A

The code to solve Eq. (9):

```
import java.lang.Math;
public class Equation1 extends Equation {
	public Equation1() {
		super();
	}
	public Equation1(double rootAccuracy, double N, double L, double a,
			double increment, double leftBoundary, double rightBoundary) {
		super(rootAccuracy);
		this.N = N;
		this.L = L;
		this.a = a;
		this.increment = increment;
		this.leftBoundary = leftBoundary;
		this.rightBoundary = rightBoundary;
	}
	public double findRoot() {
		double root = leftBoundary;
		double result = 0;
		for(int i=0;i<(rightBoundary - leftBoundary)/increment; i++){

			if (Math.abs(getLeftResult(root) - getRightResult()) <
rootAccuracy) {
				this.flag = "Result Found";
				result = root;
				break;
			}
			root += increment;
		}
		return result;
	}
      public double getLeftResult(double root) {
		double result = root * Math.PI / (N * L * Math.sin(Math.PI / N));
		return result;
	}
	  public double getRightResult() {
		  double result = Math.pow(a, 1 / N) * Math.pow(1 + a, 1 - 1 / N);
		  return result;
	}
	public static void main(String[] args) {
```

```
		double rootAccuracy = 0.0000001;
		double N = 1.1;
		double L = 1;
		double a = 0.0001;
		double increment = 0.00000001;
		double left = 0;
		double right = 1;
		Equation1 test = new Equation1(rootAccuracy, N, L, a, increment, left,
right);
		System.out.println(test.findRoot());
		System.out.println(test.flag);
	}
}
```

APPENDIX B

The code (JAVA) to solve Eq. (13):

```
public class Equation2 extends Equation {

	public Equation2() {
		super();
	}

	public Equation2(double rootAccuracy, double N, double L, double h,
			double increment, double leftBoundary, double rightBoundary,
			double convergencyAccuracy) {

		super(rootAccuracy);
		this.N = N;
		this.L = L;
		this.h = h;
		this.increment = increment;
		this.leftBoundary = leftBoundary;
		this.rightBoundary = rightBoundary;
		this.convergencyAccuracy = convergencyAccuracy;
	}

	public double findRoot() {
		double root = leftBoundary;
		double result = 0;
		for (int i = 0; i < (rightBoundary - leftBoundary) / increment; i++) {

			if (Math.abs(getLeftResult(root) - getRightResult(root)) <
rootAccuracy) {
				this.flag = "Result Found";
				result = root;
				break;
			}
			root += increment;
		}
```

```
		calculateE(result);

		return result;
	}

	public double getLeftResult(double root) {
		double result = sum(root);
		return result;
	}

	public double getRightResult(double root) {
		double result = (-L / h) * root * Math.pow(h, N) / (Math.pow(h, N) -
Math.pow(a * root, N));
		return result;
	}

	public double sum(double root) {

		double result = 0;
		double previous = 0;
		double previous2 = 0;
		int n = 0;
		for (n = 0; n < 10000; n++) {
			double r1 = Math.pow(-1, n) * Math.pow(root, -N * n - N + 1)
					/ (-N * n - N + 1);
			double r2 = Math.pow(-1, n) * (-2 * N * n - N)
					/ ((N * n + 1) * (-N * n - N + 1));

			result = result + r1 + r2;

			if (Math.abs(r1 + previous) < convergencyAccuracy &&
Math.abs(r2 + previous2) < convergencyAccuracy && n >= 1) {
				break;
			} else {
				previous = r1;
				previous2 = r2;
			}
```

```
        }
        this.convergency = n;

        return result;
    }

    public void calculateE(double root) {
        setE(ks / (Math.pow(a*root/h, -N) - 1));
    }

    public static void main(String[] args) {

        double rootAccuracy = 0.001;
        double N = 4.74;
        double L = 50;
        double h = -5000;
        double increment = 0.001;
        double left = 1;
        double right = 1000;
        double convergencyAccuracy = 0.0001;
        double ks = 816;
        double a = -19.1;

        Equation2 test = new Equation2(rootAccuracy, N, L, h, increment, left,
                    right, convergencyAccuracy);
        test.setKs(ks);
        test.setA(a);

        System.out.println("The root is " + test.findRoot());
        System.out.println("The series is converged at " + test.convergency);
        System.out.println("Status: " + test.flag);
        System.out.println(test.E);
    }


}
```

The code to solve Eq. (14):

```
public class Equation3 extends Equation {

	public Equation3() {
		super();
	}

	public Equation3(double rootAccuracy, double N, double L, double h,
			double increment, double leftBoundary, double rightBoundary,
			double convergencyAccuracy) {

		super(rootAccuracy);
		this.N = N;
		this.L = L;
		this.h = h;
		this.increment = increment;
		this.leftBoundary = leftBoundary;
		this.rightBoundary = rightBoundary;
		this.convergencyAccuracy = convergencyAccuracy;
	}

	public double findRoot() {
		double root = leftBoundary + 0.00000000000000000001;
		double result = 0;
		for (int i = 0; i < (rightBoundary - leftBoundary) / increment; i++) {

			if (Math.abs(getLeftResult(root) - getRightResult(root)) <
rootAccuracy) {
				this.flag = "Result Found";
				result = root;
				break;
			}
			root += increment;
		}

		calculateE(result);
		return result;
	}
```

```
	public double getLeftResult(double root) {
		double result = sum(root);
		return result;
	}

	public double getRightResult(double root) {
		double result = (-L / h) * Math.pow(h, N) / (Math.pow(h, N) -
Math.pow(a * root, N));
		return result;
	}

	public double sum(double root) {
		double result = 0;
		for (int n = 0; n < 10000; n++) {
			double r1 = Math.pow(-1, n) * Math.pow(root, N * n)
					/ (N * n + 1);
			double r2 = Math.pow(-1, n + 1) * Math.pow(root, N * (n + 1))
					/ (N * (n + 1) + 1);

			result = result + r1;



			if (Math.abs(r1 + r2) < convergencyAccuracy

			) {
				this.convergency = n+1;
				result += r2;
				break;
			}
		}
		return result;
	}

	public void calculateE(double root) {
		setE(ks / (Math.pow(a*root/h, -N) - 1));
	}
```

```
	public static void main(String[] args) {

		double rootAccuracy = 0.0001;
		double N = 5;
		double L = 300;
		double h = -300;
		double increment = 0.0001;
		double left = 0;
		double right = 1;
		double convergencyAccuracy = 0.0001;
		double ks = 417;
		double a = -44.7;

		Equation3 test = new Equation3(rootAccuracy, N, L, h, increment, left,
				right, convergencyAccuracy);
		test.setKs(ks);
		test.setA(a);
		System.out.println("The root is " + test.findRoot());
		System.out.println("The series is converged at " + test.convergency);
		System.out.println("Status: " + test.flag);
		System.out.println(test.E);

	}


}
```

APPENDIX C

The code to solve Eq. (22a):

```
public class Equation4 extends Equation {

	public Equation4() {
		super();
	}

	public Equation4(double rootAccuracy, double P, double L, double h,
				double ks, double hv, double lambda, double increment,
				double leftBoundary, double rightBoundary,
				double convergencyAccuracy) {

		super(rootAccuracy);
		this.P = P;
		this.L = L;
		this.h = h;
		this.ks = ks;
		this.hv = hv;
		this.lambda = lambda;
		this.increment = increment;
		this.leftBoundary = leftBoundary;
		this.rightBoundary = rightBoundary;
		this.convergencyAccuracy = convergencyAccuracy;
	}

	public double findRoot() {
		double root = leftBoundary;
		double result = 0;
		for (int i = 0; i < (rightBoundary - leftBoundary) / increment; i++) {

			if (Math.abs(getLeftResult(root) - getRightResult(root)) <
rootAccuracy) {
				this.flag = "Result Found";
				result = root;
				break;
			}
			root += increment;

		}
```

```
		calculateE(result);
		return result;
	}

	public double getLeftResult(double root) {
		double result = sum(root);
		return result;
	}

	public double getRightResult(double root) {
		double result = ((h -  0 * hv) / h) * (-L / h) * root - root * hv / (h * (1 +
Math.pow(root * hv / h , N)));
		return result;
	}
	public double sum(double root) {

		double N = P * lambda + 2 * lambda + 2;
		double result = 0;
		double previous = 0;
		double previous2 = 0;
		int n = 0;
		for (n = 0; n < 10000; n++) {
			double r1 = Math.pow(-1, n) * Math.pow(root, -N * n - N + 1)
					/ (-N * n - N + 1) - Math.pow(-1, n) *
Math.pow((root * hv) / h, N * n + 1) / (N * n + 1);
			double r2 = Math.pow(-1, n) * (-2 * N * n - N)
					/ ((N * n + 1) * (-N * n - N + 1));

			result = result + r1 + r2;

			if (Math.abs(r1 + previous) < convergencyAccuracy &&
Math.abs(r2 + previous2) < convergencyAccuracy && n >= 1) {
				break;
			} else {
				previous = r1;
				previous2 = r2;
			}

		}
		this.convergency = n;

		return result;
	}
```

```
	public void calculateE(double root) {
		double a = P * lambda + 2 * lambda + 2;
		setE(Math.pow(root * hv / h, a) * ks);
	}

	public static void main(String[] args) {

		double rootAccuracy = 0.0001;
		double P = 1;
		double L = 50;
		double h = -100;
		double ks = 0.976;
		double hv = -25.9;
		double lambda = 0.194;
		double increment = 0.0001;
		double left = 1;
		double right = 1000;
		double convergencyAccuracy = 0.0001;

		Equation4 test = new Equation4(rootAccuracy, P, L, h, ks, hv, lambda,
				increment, left, right, convergencyAccuracy);
		System.out.println("The root is " + test.findRoot());
		System.out.println("The series is converged at " + test.convergency);
		System.out.println("Status: " + test.flag);
		System.out.println(test.E);

	}
```

The code to solve Eq. (22b):

```
public class Equation5 extends Equation {

	public Equation5() {
		super();
	}

	public Equation5(double rootAccuracy, double P, double L, double h,
			double ks, double hv, double lambda, double increment,1.339
			double leftBoundary, double rightBoundary,
			double convergencyAccuracy) {

		super(rootAccuracy);
		this.P = P;
		this.L = L;
		this.h = h;
```

```
		this.ks = ks;
		this.hv = hv;
		this.lambda = lambda;
		this.increment = increment;
		this.leftBoundary = leftBoundary;
		this.rightBoundary = rightBoundary;
		this.convergencyAccuracy = convergencyAccuracy;
	}

	public double findRoot() {
		double root = leftBoundary;
		double result = 0;
		for (int i = 0; i < (rightBoundary - leftBoundary) / increment; i++) {

			if (Math.abs(getLeftResult(root) - getRightResult(root)) <
rootAccuracy) {
				this.flag = "Result Found";
				result = root;
				break;
			}
			root += increment;
		}

		calculateE(result);
		return result;
	}

	public double getLeftResult(double root) {
		double result = sum(root);
		return result;
	}

	public double getRightResult(double root) {
		double result = -L / h - hv / (h * (1 + ks * Math.pow(root * hv / h , N)));
		return result;
	}

	public double sum(double root) {


		double N = P * lambda + 2 * lambda + 2;
		double result = 0;
		double previous = 0;
		for (int n = 0; n < 10000; n++) {
```

```
                    double r1 = Math.pow(-1, n) * Math.pow(root, N * n)
                                / (N * n + 1) - Math.pow(-1, n) * Math.pow( hv / h,
N * n + 1)
                                / (N * n + 1) *  Math.pow(root, N * n)
                                / (N * n + 1);

                    result = result + r1;

                    if (Math.abs(r1 + previous) < convergencyAccuracy && n>=1) {
                          this.convergency = n;
                          break;
                    }
                    else{
                          previous = r1;
                    }
              }
              return result;
        }

        public void calculateE(double root) {
              double a = P * lambda + 2 * lambda + 2;
              setE(Math.pow(root * hv / h, a) * ks);
        }

        public static void main(String[] args) {

              double rootAccuracy = 0.0001;
              double P = 1;
              double L = 50;
              double h = -60;
              double ks = 0.976;
              double hv = -25.9;
              double lambda = 0.194;
              double increment = 0.0001;
              double left = 0;
              double right = 1;
              double convergencyAccuracy = 0.0001;

              Equation5 test = new Equation5(rootAccuracy, P, L, h, ks, hv, lambda,
                    increment, left, right, convergencyAccuracy);
              System.out.println("Start calculating...");
              System.out.println("The root is " + test.findRoot());
              System.out.println("The series is converged at " + test.convergency);
              System.out.println("Status: " + test.flag);
```

```
        System.out.println(test.E);
    }
}
```