

A GAME THEORY BASED MODEL OF HUMAN DRIVING WITH APPLICATION  
TO AUTONOMOUS AND MIXED DRIVING

A Dissertation

by

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## ABSTRACT

In this work, I consider the development of a driver model to better understand human drivers' various behaviors in the upcoming mixed situation of human drivers and autonomous vehicles. For this, my current effort focuses on modeling the driver's decisions and corresponding driving behaviors.

First, I study an individual driver's reasoning process through game theoretic investigation. The driver decision model is modeled as the Stackelberg game, which is based on the backward information propagation. In the driver decision model, I focus on the drivers' insensible desires and corresponding unwanted traffic situations. With the comparison of the model and the field data, it is shown that the model reproduces the relationship between the driver's inattentiveness and collisions in the real world.

Next, the driving behavior control is presented. I propose a human-like predictive perception model of potential collision with an adjacent vehicle. The model is based on hybrid systematic approach. In turn, with the predictive perceptions, a driving safety controller is designed based on model predictive control. The model shows adequate predictive responses against the other vehicles with respect to the driver's rationality.

In sum, I present a driver model that corresponds to and predicts traffic situations according to a human driver's irrationality factor. This model shows a meaningful similarity to the real-world crashes and predictive behaviors according to the driver's irrationality.

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## 1. INTRODUCTION

Understanding human driving behaviors is one of the traffic topics that have been studied by researchers [1-9]. This is because human behaviors play a critical role in traffic safety and efficiency, in conjunction with environmental factors. Many researchers have developed a variety of driving models that realistically respond to the given traffic conditions and determine the actions of the vehicles [10-14]. There are many decisions to be made by a driver at every instant on the highway; for example, which lane to go, when to change a lane, or whether to accelerate. The various factors of the road, the general traffic, or the driver's intentions and dispositions can exert strong influences on such driver's responsive decisions. A driver determines how he manipulates the steering wheel and the accelerator so as to drive the vehicle to the driver's desired motion. In turn, the desired motion of the vehicle such as lanes, speed, or etc. is also grounded in the driver's goals and other traffic environments. Thus, microscopic traffic models have been improved to better understand or reproduce a driver's responsive decisions and corresponding reactions.

At the same time, the attempts to employ autonomous vehicles on a road also have been made. The California Partners for Advanced Transportation Technology (PATH) has provided an integrated methodology for an autonomous highway driving [15-19]. Defense Advanced Research Projects Agency (DARPA) Urban Challenge has also shown the possibility of autonomous driving along with challenges and potential improvements [20-26]. Currently, Google has been showing and testing its autonomous

car in real driving situations [27-30]. Corresponding regulations have been enacted in California and Florida to make the driving of autonomous vehicles legal. With these considerations in mind, there is no doubt of the fact that someday soon we will see a combination of autonomous vehicles and manually operated vehicles on the road, where it will be necessary for an autonomous vehicle to predict the actions of the other cars as human drivers do. This is one of the biggest challenges that the autonomous vehicles are encountering in pursuance of proliferation, as Campbell's report [21] informed. Thus, modeling a various driver's behaviors is essential to understanding other drivers' styles as well as to providing an autonomous vehicle with human-like prediction ability.

However, what we need to notice in the development of a driver model and an autonomous vehicle is that drivers are not perfect. That is to say, human factors have a significant influence on driving. Driving is a kind of social interaction among drivers. Yet the drivers may be inexperienced, inattentive, or even aggressive. As a result, 85 percent of the accidents in major cities are caused by the failure to take note of the driving world due to so called road rages [31-33]. Also, driver distraction is regarded as an another major factor in errors that can lead to a crash [34]. National Highway Traffic Safety Administration (NHTSA) estimated that in nearly 80 percent of vehicle crashes, the vehicle driver had looked away from the roadway just prior to the crash [3]. Accordingly, it is also necessary to encompass a driver's unreasonable or aggressive behaviors, as well as a normally reasonable/efficient driving in a driver model. The study of irrational behavior has been emphasized in the areas of economics and psychology [35-37]. It is believed that a mechanism that gives rational behaviors in normal

conditions can bring about irrational behaviors in abnormal conditions. Since behaviors are affected by how people perceive the world, irrational behaviors can come from illogical perceptions and belief systems [38]. In this spirit, human irrational driving behaviors can be modeled through a logical decision method that is made, based on unreasonable perceptions or desires.

With these in mind, I develop a highway driver model that serves to develop better formal understanding of human drivers, which is essential in determining how driving behavior leads to anomalous situations such as accidents. The model will highly rely on game theory and a hybrid dynamic system: game theory is utilized as a logical decision method that can result in rational and irrational outcomes with regard to unreasonable perceptions in a multi-agent situation. Hybrid systematic approach is used to take account of the discrete decisions and continuous vehicle dynamics at the same time. Understanding human driver rational and irrational behaviors together can help develop a more effective approach for implementing autonomous vehicles that need to coexist and interact with human drivers.

### **1.1. Literature Review**

As briefly stated earlier, there have been previous studies about the driver model, traffic simulation, and vehicle autonomy, as well as safety designs. These will be reviewed below to delineate the grounds and the contributions of this work.

### 1.1.1. Microscopic Traffic Model

An accurate driving/driver model is increasingly interesting to transportation researchers [39]. Several models have been developed to characterize human driving behaviors. Two main aspects describe the human driving in longitudinal and lateral directions: a car-following model in a single lane and an extension to a multi-lane model with lane-change and merging operations.

#### 1.1.1.1 Car-Following Models

First, the driver's longitudinal behavior has been developed to express the driver's car following characteristics. For this, researchers have provided a wide range of continuous time car following models [40-44]. The goal of these models is to produce an adequate acceleration of the vehicle, which can result in an efficient and realistic traffic simulation, based on essential physical attributes, such as relative position, relative velocity, the velocity of the subject vehicle, time delay to perceive the situation. The well-known intelligent driver model (IDM) [14] is shown below as an example. IDM produces an acceleration as a continuous function of the velocity  $v_\alpha$ , the velocity difference  $\Delta v$ , the desired gap  $s^*$  and the gap  $s$ . Maximum acceleration  $a$  and acceleration exponent  $\delta$  are model parameters.

$$\dot{v}_\alpha = a \left[ 1 - \left( \frac{v_\alpha}{v_0} \right)^\delta - \left( \frac{s^*(v_\alpha, \Delta v_\alpha)}{s_\alpha} \right)^2 \right] \quad (1.1)$$

With the use of IDM, Treiber et al. showed that congested traffic propagates upstream and the oscillation of the traffic flow grows [44].

### 1.1.1.2 Lane Change Models

The driver's lateral behavior has also been studied in some works [10-12], in terms of lane-change or merge operations. These works have focused on revealing and arranging heuristics that induce a driver's lane-change or merge desire, as the basis to create effective traffic flow modeling tools. Gipps, for instance, has addressed a set of factors that cause a driver to change lanes [11]. Since a lane-change depends on multiple objectives, such as higher speed, obstacles, turning, the model should take into account a number of decisions and their corresponding outcomes. Ahmed [10] has presented a gap acceptance model to assess whether an adjacent gap is acceptable. In the model, drivers have minimum acceptable gaps between the subject vehicle and leading/following vehicles. Likewise, Hidas [12] has proposed a notion of *driver courtesy*<sup>1</sup>, a concept of cooperation among drivers, in modeling lane-change and merge operations. This is aimed to improve the weakness of the previous lane-change models, which is that drivers change lanes only when they have physically safe gaps. Figure 1.1 and 1.2 show an individual driver's heuristic for lane changing and merging in his work. However, these methods that are reviewed above depend on several rules and corresponding flowcharts as shown in Figure 1.2 to subsume the whole possible cases. Thus, it is

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<sup>1</sup> If a gap in the target lane is not enough while changing lanes, the model perform a forced lane-change that requests the subsequent vehicles to slow down.

necessary to introduce a framework to synthetically deal with the numerous desires that are used in decision making.

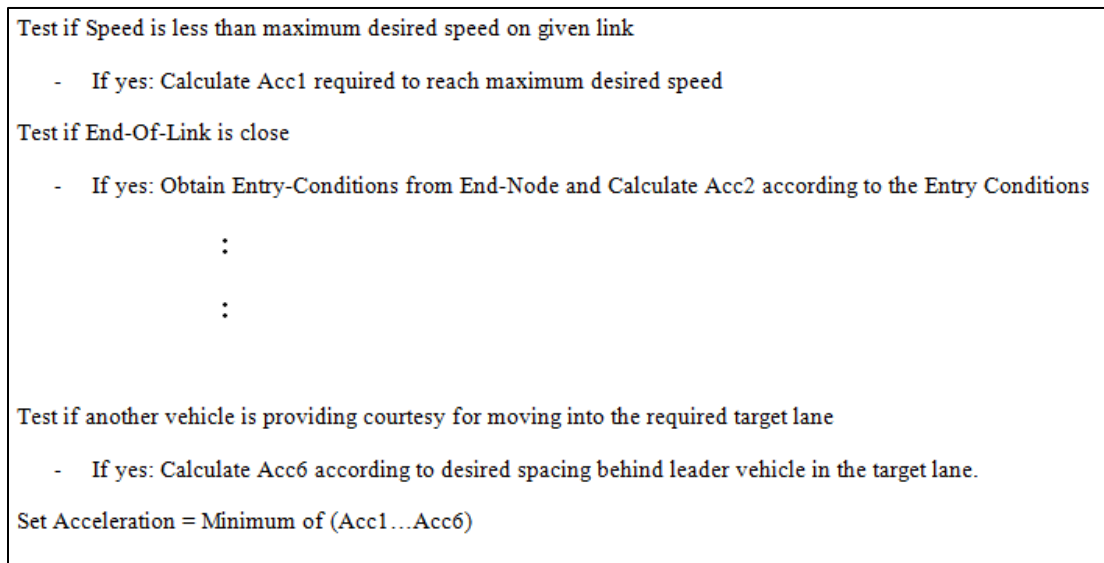


Figure 1.1. Vehicle acceleration heuristic by Hidas [12]

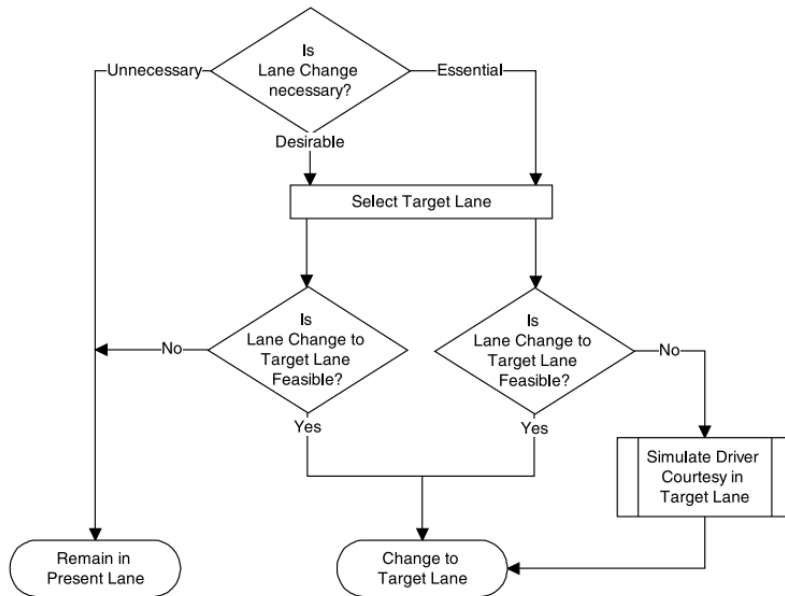


Figure 1.2. Flowchart of the lane-change excerpted from [12]

### 1.1.1.3 Integrated Models and Human Factors in the Models

In addition to works with regard to the longitudinal and lateral driving behaviors, researchers have improved the models from the perspective of a good approximation of the driver. Kesting et al. [39] proposed an enhanced IDM that eliminates unrealistic behaviors of IDM in cut-in situations. Kim and Langari [45] also proposed a modified IDM to deal with the conflicts among a group of vehicles when there is not a sufficient gap to merge in. The target that the subject vehicle tracks is newly arranged to cut in, which is a forced merging model in the sense that the vehicle in the mainline needs to yield right of way to the merging vehicle. Whereas, some researchers paid attention to



the model adaptation to reflect human attributes. MacAdam [13] has studied human attributes from the perspective of degradation factors of the driver, physical limitations such as human time delay, visual characteristics, as well as physical attributes such as preview and adaptiveness. Recent works [46, 47] to reflect neuromuscular properties can be understood in the context of the model adaptation. However, as noted in [13], human factors, like driver distraction are regarded as different areas although it is expected to cause directional instabilities. In this sense, Salvucci [9] developed a computational model of driver behaviors in a cognitive architecture, which intends to provide a framework that incorporates psychological/cognitive architecture. The model is based on Adaptive Control of Thought-Rational (ACT-R). The ACT-R [48] is a framework to understand how people organize knowledge and produce intelligent behavior, based on *declarative knowledge* (facts or information) and *procedural knowledge* (condition-action rules). The ACT-R integrated driver model showed a procedure of information and decisions in *Monitor*, *Control*, and *Decide of Drive* in the cognitive model. For example, Figure 1.3 shows the information and decision structure during lane-changes.

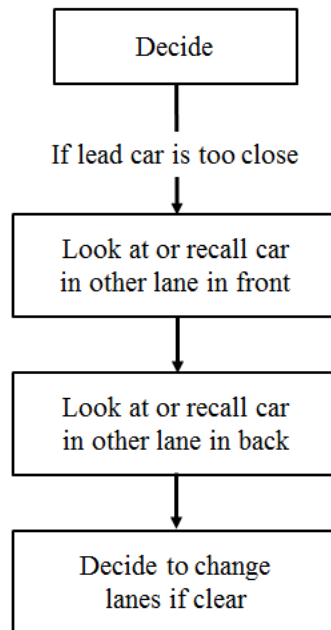


Figure 1.3. Information and decision structure during lane-changes excerpted from [9]

### *1.1.2. Intelligent Transportation Systems (ITS) and Autonomous Driving*

ITS incorporates many-sided complex components, which include advanced transportation management systems (ATMS), advanced traveler information systems (ATIS), advanced vehicle control and safety systems (AVCSS), advanced public transportation systems (APTS) [49-51]. The elementary ITS components, such as electronic toll system, navigation system, and advanced cruise control (ACC), are becoming common and widely equipped in the ground transportation system.

Specifically, ATMS, ATIS and AVCSS have been investigated to enhance mobility and safety, together with increasing communication technologies and Driver Assistant

Systems (DAS) [52, 53]. One can see a rapid development of DAS such as Intelligent/Adaptive Cruise Control (I/ACC), Forward Collision Warning (FCW) and Lane Departure Warning (LDW)/ Lane Keeping Systems (LKS). Decreasing costs of electronic devices and the conversely increasing computing power and sensor technologies favor the adoption of DAS in an increasingly broad range of vehicles [52]. Also, emerging concepts such as vehicle to vehicle (V2V) and vehicle to infrastructure (V2I) communication, with the advent of dedicated short range communication (DSRC) in North America and Japan as well as Europe [54], are expected to broaden the scope of ITS. In this context, a number of works have been tried to address and implement a certain level of driving autonomy.

The first well-known example of these trials is the California PATH program [17, 19]. They developed and limitedly demonstrated Automated Highway System (AHS) in the 1990s. They developed a big picture for AHS with the concept of platooning; a group of vehicles acting as an agent. Through a multi-layer architecture, they formed and controlled the platoons and the corresponding individual vehicles. Increased capacity and safety were demonstrated by controlling vehicle platoons [55-57]. In addition, Bose and Ioannou performed an analysis to examine the transient behavior of traffic flow with mixed manual and semi-automated vehicles [58]. Next, another effort at autonomous driving was made in the 2007 DARPA Urban Challenge. Many teams participated and competed with their autonomous vehicles in urban driving situations where human drivers coexist, which has presented the remaining challenges in autonomous driving beyond their current achievements [21]. How the autonomous

vehicle predicts the behaviors of human drivers and interacts with them, when autonomous vehicle and human drivers coexist, is one of the challenges that are listed in [21], which must be investigated. Dresner et al.'s work [59] is one of the investigations that heuristically derives a policy for such a situation where autonomous cars encounter human drivers, especially at intersections. From more practical perspectives, human drivers, although supported by DAS, still remain involved in each driving as the final arbiters that can override the controls of DAS. Thus, understanding and corresponding prediction of the human driver behaviors is essential regardless of the level of automation.

### *1.1.3. Human Factors in Traffic Research*

There is no doubting the fact that human factors have a significant influence on driving safety. To substantiate human factors that affect the transport system, meaningful research has been studied [3, 33, 60]. Shinar and Compton analyzed the relationship between observable factors and aggressive driving, such as gender, age, etc [60]. They concluded that aggressive driving is correlated with both situational influences and individual human aspects. A research performed by NHTSA [3] also observed that drivers' attentiveness has an explicit relation with driving safety. These effects of individual characteristics to driving safety can be interpreted in connection with Fuller's definition [61] on the risk of accidents. It is defined that there are three basic terms in the risk of accidents: objective risk, subjective risk estimate, and the feeling of risk. Objective risk represents the objective probability that a person is involved in an

accident. Subjective risk estimate is a driver's awareness and judgment for the objective collision probability in the cognitive level. Feeling of risk refers to a driver's emotional status. Thus, it can be easily recognized that the subjective risk estimate suggests again the proposition that human characteristics have momentous correlation with the driving safety and the observations that are collected and analyzed by the transportation researchers.

#### *1.1.4. Game Theory*

Since its inception by Borel in the 1930s [62] and subsequent works by von Neumann and Nash [63-65], game theory has been used as a reasonable model of decision-making in many areas of social science as well as in engineering because it presents optimal solution(s) during the multiple players' interactions [66-70]. In particular, note that game theory can be used to model human reasoning that makes a logical selection among numerous alternatives in decision processes [66, 67].

##### **1.1.4.1 Noncooperative Game Theory**

To enhance understanding on game theory, the basic concepts of game theory are reviewed in this section, with using a simple example that has a strategic form<sup>2</sup>. The game depicted in Table 1.1 has two players ( $A, B$ ), and these players have choices of strategies ( $a, b$ ), and ( $c, d$ ), respectively. Each player has different payoffs according to

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<sup>2</sup> A strategic form of a game in game theory denotes a game that is typically represented by a matrix. The matrix shows the players and their strategies as well as corresponding payoffs.

his/her own strategy and the other player's choice(s). For instance, choice  $a$  by player  $A$  and choice  $c$  by player  $B$  lead to equal payoffs of 4 units for the two players, whereas, choices  $a$  and  $d$  by the respective players produce 7 units of payoff for player  $A$  and 2 units for player  $B$ . Next, we consider how game theory offers a solution to this game. We start by a discussion of alternate equilibrium conditions commonly used in game theory where cooperation is not considered.

Table 1.1. 2×2 matrix game payoffs table

A's payoff, B's payoff		Player B	
		$c$	$d$
Player A	$a$	4,4	7,2
	$b$	1,1	5,3

#### 1.1.4.1.1. Nash equilibrium

The concept of Nash equilibrium refers to a strategy pair such that no player can obtain a better payoff by individually deviating from the given pair. In the game given in Table 1.1, strategy pair  $(a,c)$  is the Nash equilibrium because both players  $A$  and  $B$  have higher payoffs compared to strategy pairs  $(b,c)$  and  $(a,d)$  respectively, assuming the core

assumption underlying the existence of a Nash equilibrium in a noncooperative game. As stated earlier, an equilibrium condition cannot be improved upon by a given player by individually deviating from the strategy choices that resulted in it.

#### *1.1.4.1.2. Stackelberg equilibrium*

Since the Stackelberg game is grounded on the presence of a hierarchical decision order among players, let's assume that player  $A$  is a leader who can commit to his/her strategy before the follower (player  $B$ ). In the Stackelberg game, the follower maximizes his/her payoff based on the strategy that the leader chooses. If the leader chooses  $(a)$ , the follower will choose  $(c)$ . Otherwise, the follower will choose  $(d)$ . Thus, it is evident that as a rational player,  $A$ , the leader, will choose  $b$  to get the pair  $(b,d)$  between strategy pairs  $(a,c)$  and  $(b,d)$ . The pair  $(b,d)$  is the Stackelberg equilibrium in this game.

#### **1.1.4.2 Game Theory in Traffic Research**

Fisk [71] has pointed out that two behavioral models from game theory can be used in some transportation modeling, such as intercity passenger travel and signal optimization problem. The two equilibrium concepts, Nash and Stackelberg respectively, can be used to discover which action set or strategy is optimal for every participant in the game. Optimality in this context is evaluated on the basis of payoffs resulting from the decisions by (and interaction among) the participants [72]. The Nash game delivers the optimal solution in non-cooperative games such as the well-known prisoner's

dilemma [73]. On the other hand, the Stackelberg game guarantees the best payoff for every player when there exists a hierarchical structure among players, namely when players are divided into a leader, who has the power to choose his/her strategy first, and a follower, who should choose his/her action after the leader's decision has been made [72].

Specifically, Hollander [74] classified transportation research works that utilize game theory as a main problem-solving method into 4 groups: Games against a demon, Games between travelers, Games between authorities, and Games between travelers and authority, which are illustrated by a driver's travel cost minimizing problem [75], traffic network problem [76], and conflicts between flow control and capacity control [77], respectively. Although game theory is applied in various ways to study the effects of policy, decisions, and/or the actions of individual agents in studies regarding transportation system as shown above, these studies can be broadly understood as two approaches: infrastructural regulation studies (traffic control problem) and agent-oriented studies (vehicle placement or route decisions).

In the first category, researchers have employed game theory on dynamic traffic control or assignment problems. For instance, Chen and Ben-Akiva [78] adopted a non-cooperative game model to study the interaction between a traffic regulation system and traffic flows to optimally assign traffic flow on a highway or an intersection while Li and Chen [69] addressed the ramp-metering problem via Stackelberg game theory. Su et al. [79] used game theory to simulate the evolution of the traffic network. The point in the



first category is that regulations and traffic flow are considered individual players and the games are used to derive an optimal traffic policy.

Second, game theory has also been utilized as a controller that plans optimal placement or routes among multiple unmanned vehicles [68, 70, 80]. Only vehicles are regarded as the game participants. Kita [81] has adopted a game theoretic analysis to consider a merging-giveway interaction between a through car and a merging car, which is modeled as a two-person non-zero sum non-cooperative game. Kita's approach can be regarded as a game theoretic interpretation of Hidas' *driver courtesy* [12] from the viewpoint that the vehicles share the payoffs or heuristics on the lane changes, which is a reasonable traffic model but fails to assign uncertainties resulting from the action of the other human drivers. Moreover, one cannot guarantee that the counterpart would act as determined in the game since the counterpart may be able to consider other factors that the subject driver cannot take into account. Accordingly, it is necessary to design an individual driver model that does not share their payoffs in the decision making processes to reflect such an uncertainty. This approach, as we shall see later, facilitates a more realistic model of driver behavior in traffic situations.

In addition, most game theoretic works in traffic research deal with collective drivers to apply a mixed strategy game where the solution is the form of probability distribution. However, the probabilistic solution of the mixed strategy game is not applicable for the deterministic model.

### **1.1.4.3. Game Theoretic Approach to the Relationship between Traffic Safety and Human Disposition**

Among many game theoretic approaches to the transportation system, there is an important work that tried to clarify the relationship between driver's aggressiveness and traffic safety. Pederson [82] set games between *Dove* (timid driver) and *Hawk* (aggressive driver) to derive a psychological proposition. Note that Pederson assumed that decision priority among game participants (drivers) is determined by the participants' aggressiveness. For instance, it is assumed that an aggressive driver has precedence in decision making, compared with timid driver. Therefore, three different games are defined with respect to the combination of drivers: the Nash game between *Dove* and *Dove*, the Stackelberg game between *Dove* and *Hawk*, and the Nash game between *Hawk* and *Hawk*. It is observed that an aggressive driver get better payoffs throughout the games, which leads to the secondary conclusion. The secondary conclusion is that aggressive driving is advantageous, which leads to the moral hazard effect (increase the number of aggressive drivers) for every participant and, in turn, increases *traffic unsafety*. It is a remarkable research result that can show the negative effects of aggressive drivers to traffic safety. However, as noted in [74], competition among drivers who have different aggressiveness, presented in Pedersen's game, can be evolved to a quantitative game if distribution of aggressiveness across drivers is specified. Moreover, considering that an aggressive driver cannot always have priority in time domain against a timid driver, it is necessary to find an objective priority among

drivers and apply it to analyze the effects of the competition among drivers who have different aggressiveness.

#### 1.1.5. Hybrid System

Hybrid dynamical system can be defined as a system such that the state of the system can move along both discrete transitions and continuous flows. That is to say, it can be understood as a combination of discrete and continuous dynamics [83]. Thus, in order to express all the characteristics, the hybrid system is typically defined by a hybrid automaton  $\mathbf{H}(Q, X, Init, F, E, G, R)$  with  $Q$ : a set of discrete states  $q$ ,  $X$ : a set of continuous states  $x$ ,  $Init$ : initial state,  $F$ : a set of vector fields  $f_q$  along which the continuous state flows,  $E$ : a set of discrete transitions (also referred to jump),  $G$ : a set of guards (the condition in which the jump occurs), and  $R$ : a set of reset relations. Figure 1.4 shows an example of a hybrid model with a single discrete state. We can see that state  $x$  starts from  $x_0$ . Then the state flows along the continuous dynamics  $f$  until it meets the guard condition  $G$ , which makes the state jumps to another state (represented by reset relation  $R$ ). In this example, the state is reset to another state in the original discrete state space when it jumped. When the system has multiple discrete states, the state of the system can jump to the state that belongs to another discrete state space as in Figure 1.5. Since hybrid system typically has multiple discrete states, the solution of hybrid system is basically defined as a tuple of time, discrete state, and continuous state. The solution flows along continuous dynamics within corresponding discrete state space and jumps to another state with the reset relations.

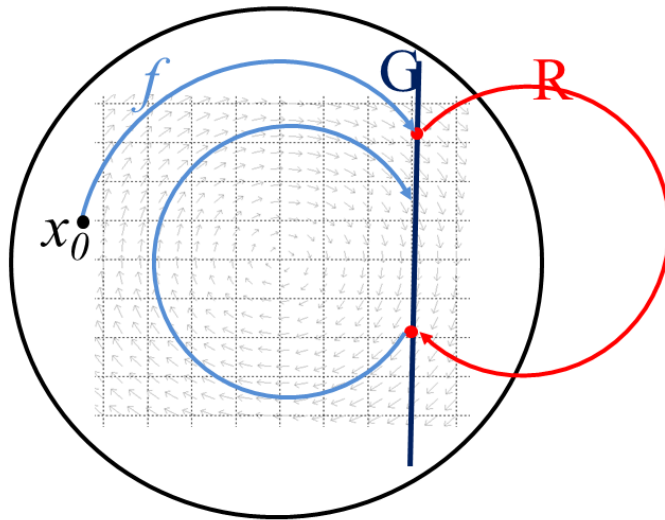


Figure 1.4. Hybrid system with a single discrete state

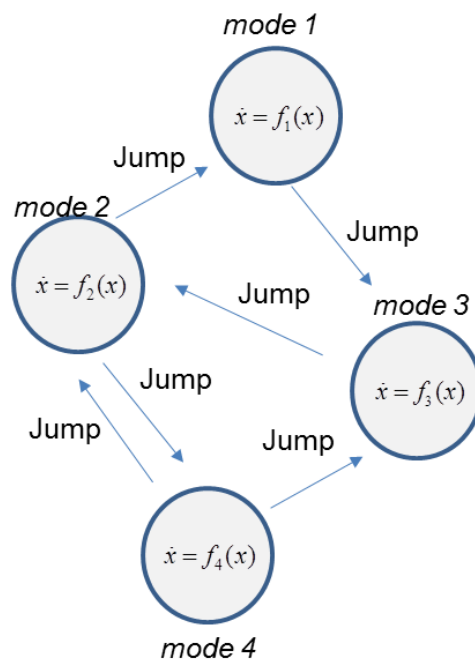


Figure 1.5. Hybrid system with multiple discrete state

Such discrete event systems where each discrete state has its own continuous dynamics are found in several fields of engineering. The following can be modelled via hybrid systems: collisions and transmission gear shifts in mechanical engineering, switches in electrical engineering, and valves and pumps in chemical engineering as well as state machines in computer science.

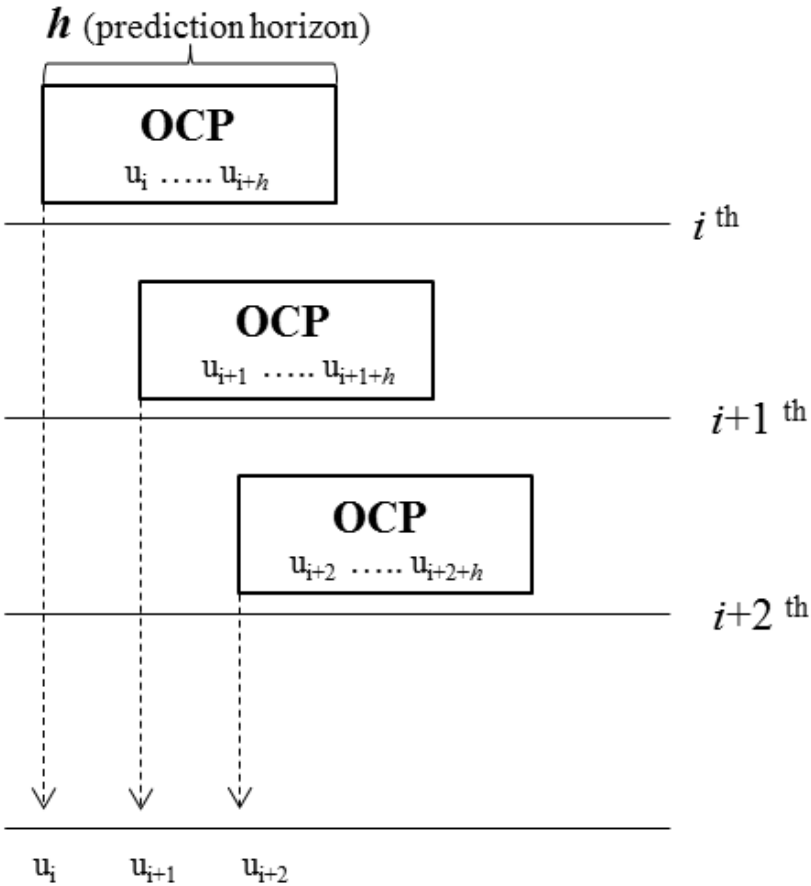


Figure 1.6. MPC scheme

### 1.1.6. Model Predictive Control

Model Predictive Control (MPC) is also referred to Receding Horizon Control (RHC) [84] because MPC solves an Optimal Control Problem (OCP) in receding predictive horizon. That is to say, the MPC solves OCP to minimize a specific cost function on the finite time horizon  $[t, t+h]$ , where  $h$  is the predictive horizon. Thus, the MPC keeps calculating the optimal solution at every time step predefined. As shown in Figure 1.6, one uses respective first control inputs from the finite OCPs in receding predictive horizon. For instance, a MPC controller can be expressed as

$$\begin{aligned} & \min_{u(\cdot)} \int_t^{t+h} \text{Cost Function } (J) \, d\tau \\ & \text{subject to system dynamics, } \dot{x} = f(x, u), \\ & \quad \text{Initial conditions, Trajectory constraints,} \\ & \quad \text{and/or Terminal constraints} \end{aligned} \tag{1.2}$$

where  $x$  and  $u$  are is state and control input vectors. Although many benefits of MPC, such as stability, adaptation, and tracking performance, are listed in [84], I give notice to “*constraint handling capability*” of MPC in this work. MPC allows us to consider several constraints, such as initial condition, trajectory constraints, and/or terminal constraints, in the process of design. We can imagine a mobile robot that navigates while avoiding obstacles and directing to the goal.

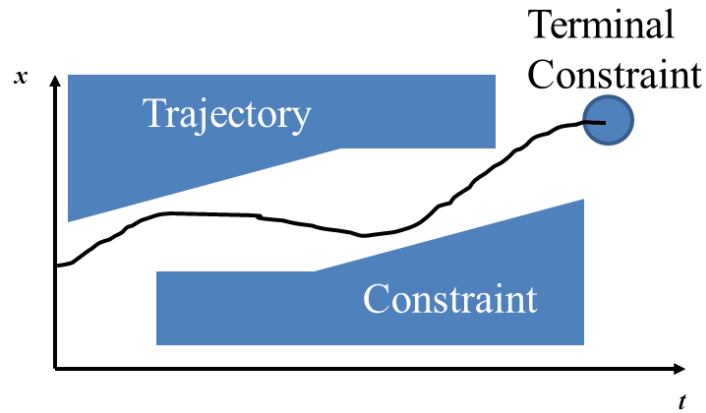


Figure 1.7. Trajectory constraints and terminal constraints in time domain

### 1.1.7. Driving Safety

There have been multiple approaches to obtain collision free trajectory and control as described below. First, the car following models listed above basically considered the longitudinal safety by controlling the gap between the subject vehicle and the predecessor. In addition, in the lateral direction, the safety design has been investigated for lane-change maneuvers. Swaroop and Yoon have developed an emergency lane change maneuver in response to the presence of obstacles under the concept of platooning [85]. Jula et al. have performed an analysis of the kinematics of lane-change maneuvers and presented minimum longitudinal spacing for no crash [86]. Kanaris and Ioannou proposed Minimum Safety Spacing for Lane Changing (MSSLC) for lane-changing and merging in automated highway systems [87]. Some researchers have provided collision-free paths, designed with elastic band theory [88, 89].

Meanwhile, a variety of probabilistic/deterministic collision detections have been proposed to compute the future collision possibility or avoid the collision. Broadhurst et al. developed Monte Carlo path planning to generate a probability distribution for the future motions and assess their danger [90]. Reachability set computation was applied to an unmanned aircraft so as to attain the obstacle avoidance. Althoff et al. proposed an approach in collision detection, the analysis of stochastic reachable sets [91]. They evaluated the crash probability of planned trajectory of autonomous cars by calculating probabilistic forward reachable sets of the subject vehicle and the other participant along the planned trajectory. Safety assurance design or control has also been studied with backward reachable set computation based on differential games; e.g. pursuit-evasion games. This approach has been intensely implemented by some researchers, related to hybrid system that consists of both continuous and discrete dynamics [92-96]. One can obtain absolute (regardless of the counterpart) safety against the counterpart through the backward evolution of a final unsafe state, with the help of the game theoretic techniques [97, 98] that exclude all possibilities that lead to an unsafe state. Lygeros et al. built a hybrid model and a safety controller for the AHS in the context of platooning [93]. Other researchers, Verma and Vecchio, have focused on designing a safety controller to avoid a collision at an intersection against a human driven vehicle with no knowledge of the human driven vehicle's mode (acceleration and deceleration) [96]. They computed a collision-free speed in the intersection of two circulating single lanes. The backward reachable set computation has also been used in an air traffic management problem [95] and a safe maneuver design of an autonomous quadrotor [99]. Although the backward



reachable set computation provides guaranteed safety property, it cannot be solved for numerous possible final states. If we are not able to confirm the unsafe final set, this may lead to a situation in which all surfaces are covered by the reachable sets.

## **1.2. Contributions of the Work**

### *1.2.1. Stackelberg Game based Decision Model of an Individual Driver*

First, the model serves to develop better formal understanding of human drivers. Human driver's behavior is manifested by the driver's intention and manipulation of his/her respective vehicle. The human intention is a decision-making process dependent on the driver's inherent reasoning and the nature of the surroundings. In this work, I discuss the application of game theory to individual driver's reasoning. Most traffic research handled collective drivers with a few games or regarded a whole transportation system as a game. Specifically, I develop an individual driver decision model<sup>3</sup> based on the Stackelberg game theory because the theory is pertinent to the sequential structure of information on a highway<sup>4</sup>. On a road, drivers basically respond to what is right behind them seen through rear-view or side-view mirrors, as well as what is ahead. However, there is no doubt of the fact that the traffic information that may affect driving such as speed limits, free headways, traffic accidents, congestion, etc. is propagated backwards.

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<sup>3</sup> Note that Stackelberg game based driver model is not time continuous traffic model but the driver decision model.

<sup>4</sup> This is supported by the backward propagation of the information and a driver's recognition order (from front to back) that are reviewed earlier.

To make the discussion clearer, the drivers behind may make their decisions before the drivers ahead in this framework. However, their decisions are based on the information propagated from ahead for the most part. Note that the hierarchical decision making process does not refer to the time sequence of the decisions but the propagation of the information. This backward propagation of information supports the hierarchical decision making process, which is best represented by the Stackelberg game theory.

### *1.2.2. Study on the Relationship between Driver's Dispositions and Traffic Unsafty*

Moreover, this model pays attention to a traffic uncertainty (unsafe outcomes) that can be caused by driver's insensible decisions. It specifically focuses on determining how driving behavior leads to anomalous situations such as accidents. To this end, I consider the effects of human factors as a form of insensible payoffs. The payoffs are designed to consider the driver's irrationality, such as road-rage. The driver's irrationality can be expressed by a deterioration of human functioning according to driver dispositions (e.g. aggressiveness or loss of attentiveness), which is not shared by the other drivers. Thus, an irrational driver is designed to make a decision through a rational reasoning based on irrational payoffs. In this paper, drivers are assumed to be selfish without cooperation as the first step to create an irrational driver model under an uncertainty that vehicles do not share their payoffs. Cooperation is indirectly considered according to the driver's disposition, such as having a larger headway or following a regulated speed. Subsequently, meaningful factors, such as the possibility of accidents, are assessed according to the driver disposition and traffic situation. The model is

validated via Monte Carlo simulations, which correlate the possibility of collision with level of aggressiveness of the drivers and inter-vehicular distances, which agrees with qualitative observations done by other researchers. Moreover, risk of aggressive drivers that is simulated by the proposed model is validated by comparing the result of traffic simulation and the relation between driver's inattentiveness and *traffic unsafety* that is observed in the real world. These validation steps are the precursor to the future use of this model in assessing the implementation of naturalistic driving models in which autonomous vehicles and human drivers can run together. Its other usages may also include driver education campaigns and transportation policy analysis.

### *1.2.3. From Psychological Collision Risks To Collision Prediction Using Hybrid System and Game Theory*

Starting from the psychological argument that an interpretation of the collision risk should be identified differently between objective prospect and subjective recognition, I present a collision risk estimation model for individual drivers who have different safety criteria. I propose a hybrid model for lane-changes that expresses the trajectory of the vehicle during lane-changes, with respect to the multiple phases of the process. Moreover, the hybrid model for a lane-change is designed to produce various driving trajectories according to the driver's aggressiveness. Based on the hybrid model for a lane-change, I develop an objective prediction of the collision in a framework of the multi-agent hybrid system, which is based on the forward reachable set of the collision area that is established in a relative coordinate. The objective prediction of the

collision suppose a situation with no guarantee of other drivers' rationality. The probabilistic information that the objective collision prediction must possess is realized using mixed strategy Nash game. The proposing driver model adopts two different game concepts: the Stackelberg game against players who the subject can assert his/her priority and the Nash game when the subject needs to recognize the traffic situation conservatively against the preceding players. Finally, I design a subjective collision perception that can mirror the effect of a driver's aggressiveness to the collision risk recognition, which is intended to consider illogical desires that may result in an irrational driving behavior in spite of the reasonable causality.

#### *1.2.4. Driver Driving Control Using MPC*

Based on the subjective perception of the collision that differs among drivers, I design a controller that drives the vehicle to stay outside the subjectively anticipated collision. The controller is designed with the consideration of a rational (optimal) method along with illogical perceptions. For the local optimality of the pursuit of the safety within the driver's prediction horizon, I design an MPC on the receding horizon, with the use of trajectory parameterization and collocation. The developed controller based on the risk estimation was combined with the previous driver decision model based on the Stackelberg game and showed additional responses to the threats from the adjacent front vehicles. Also, the different responses to the adjacent front vehicles made different interactions of the vehicles. These interactions can be explained from the viewpoint of driver's prediction ability. Driver's behaviors and corresponding traffic

interactions can vary if their perceptions are irrational, even if the drivers handle the vehicle optimally. Also, the upcoming mixed situation of autonomous vehicles and human drivers is of considerable significance. In this regard, the model I propose can be applied widely, from an aggressive human driver model that has a certain level of uncertainty to an autonomous vehicle that pursues maximum driving safety without guarantee of other human driven cars' rational responses.

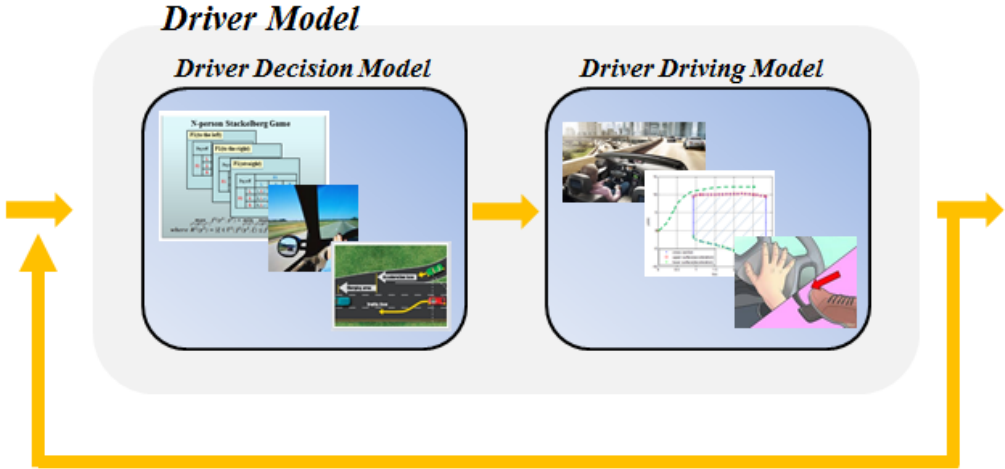


Figure 1.8. Schematic diagram of the proposing driver model

**1.3. Outline of the Dissertation**

The driver model is developed with two steps as shown in Figure 1.8; a driver decision model and a driver driving model. The driver decision model formulates a

driver's intention on lane-changes or merges with the consideration of drivers' irrationality. Section 2 describes the development of the driver decision model for highway driving and merging. The driver decision model is validated by comparing the model and the real world data. Next, Section 3 gives a formulation of driver's collision estimation procedure and corresponding driving control. This section materializes the psychological argument of the collision risk and corresponding evasive driving. It will enable the driving control to be used in from a driver model to an autonomous vehicle. Lastly, a summary of this work will be viewed in Section 4.

## 2. DRIVER DECISION MODEL<sup>\*</sup>

Game theory has been widely used to represent a reasonable model of decision making since its inception [62-65]. In particular, Stackelberg game theory is pertinent to derive decisions among multiple players when the game has a sequential structure of decisions among players as noted previously [67, 72]. We reviewed works that show the fact that the information of highway is propagated backwards for the most part<sup>5</sup> and drivers' cognitive procedures also move from front to back [9]. This supports the hierarchical structure in the drivers' decision making processes in highway. For example, when there are 4 vehicles (Vehicle 1, Vehicle 2, Vehicle 3, and Vehicle 4) as shown in Figure 2.1, Vehicle 1 has an information priority to Vehicle 2, so does Vehicle 2 to Vehicle 3, and so on. Therefore, in Section 2, I have considered the application of Stackelberg game theory to individual driver's decision modeling in highway settings. The game theoretic decision is made by every driver, respectively, in consideration of the follower's responsive actions. The followers' actions are assumed to base on the information from the vehicles that have higher priorities than itself, where the follower means the vehicle that has a lower priority. Thus, we have the same number of games as the number of drivers. For instance, we have 4 different games for 4 players as in the Figure 2.2.

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<sup>\*</sup> Reprinted with permission from "Stackelberg Game Based Model of Highway Driving," by Je Hong Yoo; Reza Langari, 2012. *DSCC*, Vol. 1, pp. 499-508, Copyright 2012 by ASME and "A Stackelberg Game Theoretic Driver Model for Merging," by Je Hong Yoo; Reza Langari, 2013. *DSCC*, Vol. 2, Paper number DSCC2013-3882, Copyright 2013 by ASME.

<sup>5</sup> Note that this does not mean that a front vehicle does not use the information of rear vehicles.

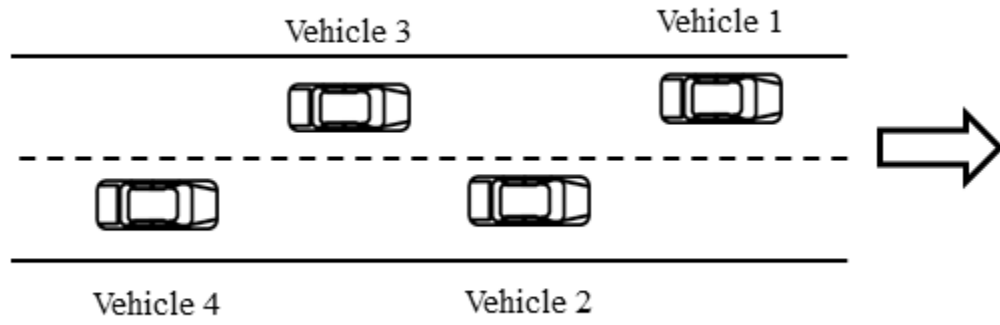


Figure 2.1. Scheme for ordered vehicles

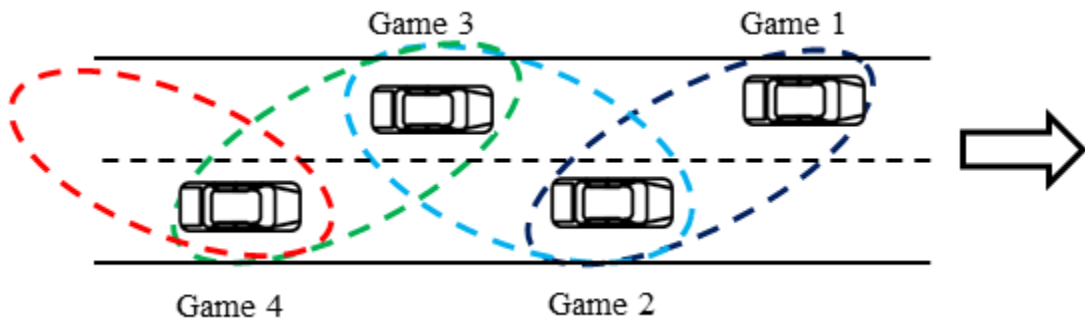


Figure 2.2. Multiple games on a two-lane road



Lastly, this model is intended to make a decision with regard to the selection of lanes; in detail, whether to change lanes or merge, when to change lanes or merge, which lane to choose and whether to accelerate or decelerate before merging.

## **2.1. System Configuration**

For a vehicle, the traffic situation mainly consists of three components: the environment, the traffic regulation system, and the vehicle itself that includes a driver, as shown in Figure 2.3. This paper mainly deals with two components, the environment and the vehicle, in order to focus on the vehicles' interactions. The traffic regulations are considered to be constant. Normal traffic regulations such as speed limit are assumed to exist although these are not subject to game theory analysis. The driver's decision, where to go or which vehicle to follow, is made based on the vicinity recognition. The given vehicle's actions following the driver's decisions are caused by the manipulation controller and, in turn, embodied by the vehicle dynamics in conjunction with the nature of the surrounding environment.

We propose a driver decision model, which does not generate continuous time traffic properties, such as speed, lane, or longitudinal distance of the vehicle. Thus, it is necessary to produce such traffic properties by adopting adequate substitutes for driver's manipulation and the vehicle. To this end, two Proportional Derivative (PD) controllers calculates human driver's inputs such as manipulation of a steering wheel and an accelerator and the vehicle dynamic model translates the human driver's inputs to physical vehicle data like velocity, position, and the yaw of the vehicle.

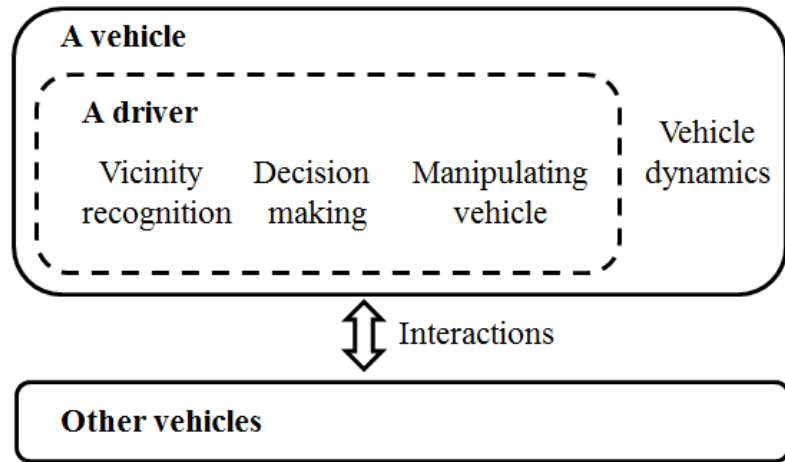


Figure 2.3. System configurations

### 2.1.1. Vicinity Recognition

This function classifies the given vehicle's surrounding into vehicles ahead, the *leading* vehicles, and vehicles behind, the *following* vehicles, according to their longitudinal positions in each lane. Then the nearest vehicles in each lane are chosen as the vicinity vehicles. In reality, the vicinity recognition degenerates due to the internal and/or external conditions. In this paper, we focus on the degradation of the information of the vicinity vehicles due to the internal factor, rather than the external conditions such as bad weather. We add artificial errors to approximate the human uncertainty in recognizing their surroundings according to the driver's disposition.

### *2.1.2. Driver's Manipulation*

In reality, the driver controls his/her vehicle by steering and accelerating it to go where the driver wishes it to go. The goal of this function is to precisely execute these driver's actions determined by the driver's intention, namely the decision maker. Two PD controllers in both the longitudinal and lateral directions are used to reproduce the driver's low-level controls of headway or the speed as well as the steering angle in consideration of the lane to go. The longitudinal controller is designed to produce adequate acceleration or deceleration to follow the target. A PD controller is used to control the longitudinal motion of the vehicle, such as time-headway between the front vehicle and itself, or the speed of the vehicle. We apply two tracking controllers (the velocity tracking controller and the combined tracking controller) according to the road density assumptions. In the case of combined tracking controller, we assume that an aggressive driver tends to follow the front vehicle with a certain time-headway and a timid driver tends to maintain a certain speed (within the highway speed limit). Thus, the resultant acceleration is determined by the weighted mean of two accelerations according to the driver's disposition that shows how aggressive the driver is. Likewise, in order to track its lane or lane-change path, the lateral controller is designed to produce a proper steering angle by using the estimated lateral error as input. For this purpose, another PD controller is used in changing the steering wheel angle, which leads to the change of the rotational velocity of the vehicle and consequently the lateral position of the vehicle. Both controllers are limited by physical factors and additional parameters that express the driver's characteristics. That is to say, an aggressive driver will have

more drastic longitudinal and lateral acceleration than a normal or timid driver. Thus, the vehicle acceleration  $a$  is given by

$$a = \min(K_{pg} \cdot e_{v,d} + K_{dg} \cdot \dot{e}_{v,d}, g_l, g_{pl}) \quad (2.1)$$

where  $e_{v,d}$  is the error between the reference velocity and the velocity of the vehicle or the error between the reference relative distance and the relative distance between the given vehicle and the vehicle immediately ahead:  $K_{pg}$  is the proportional gain of the longitudinal controller,  $K_{dg}$  is the derivative gain of the longitudinal controller,  $g_l$  is the acceleration limit reflecting the driver's disposition, and  $g_{pl}$  is the physical limitation of the vehicle during acceleration or deceleration. Likewise, the steering angle is defined by

$$\delta = \min(K_{pl} \cdot e_{lat} + K_{dl} \cdot \dot{e}_{lat}, \delta_{lat}, \delta_{pl}) \quad (2.2)$$

where  $e_{lat}$  is the error between the reference lateral position and the lateral position of the vehicle,  $K_{pl}$  is the proportional gain of the lateral controller,  $K_{dl}$  is the derivative gain of the longitudinal controller,  $\delta_{lat}$  is the steering angle limit that can be altered by the driver's disposition, and  $\delta_{pl}$  is the physical limitation of the steering angle. Here,  $\delta_{lat}$  can be obtained by using the following ratio which is known as the lateral acceleration gain [100]:

$$\frac{a_{yl}}{\delta_{lat}} = \frac{v^2}{57.3Lg + K_{us}v^2} \quad (2.3)$$

where  $a_{yl}$  denotes the lateral acceleration limit that can be changed by the driver's disposition,  $L$  is the wheelbase,  $K_{us}$  is the understeer gradient of the vehicle, and  $g$  is the gravitational acceleration. Additionally, both the longitudinal and lateral controllers are limited by additional parameters that express the driver's dispositions. That is to say, an aggressive driver will have more drastic longitudinal and lateral acceleration than a normal or timid driver. Both the controllers are tuned to be slightly overdamped to avoid oscillation at the execution level. In addition, the physical limits for both controllers prevent oscillation of the vehicles' relative positions. If the decision is accomplished quickly beyond the physical limitations of the vehicle<sup>6</sup>, the continuous switching may occur in the lateral or longitudinal direction. For example, two vehicles may keep on changing their lanes instantly while overtaking each other. This can be prevented in the proposed model since it would imply drivers behaving erratically beyond the physical limitations.

### 2.1.3. Vehicle Dynamic Model

In order to simulate the motion of the vehicles involved in this study, a two-wheel vehicle model (1) [101] in Figure 2.4 is used to generate the yaw velocity  $r$  and

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<sup>6</sup> The velocity, acceleration, steering angle, steering velocity of the vehicle are limited within reasonable thresholds. Thus, a certain action cannot be finished in a comparatively small time whereas the decision can be instantly decided, which in turn prevents continuous decision switching.

lateral velocity  $v_{lat}$  of the vehicle according to its velocity,  $v$ , and steering angle,  $\delta$ , which leads to the vehicle pose (2);  $(x,y)$  in Cartesian coordinate system as well as the heading angle,  $\theta$ .

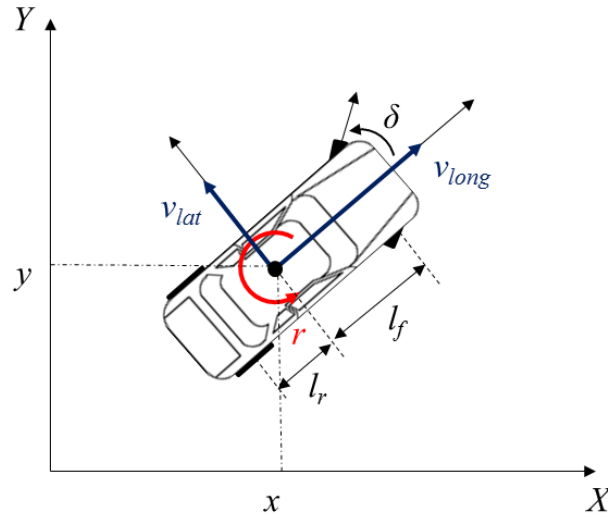


Figure 2.4. Planar view of vehicle in motion

$$\begin{bmatrix} \dot{v}_{lat} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} -\frac{C_{af} + C_{ar}}{mv_{long}} & \frac{-l_f C_{af} + l_r C_{ar}}{mv_{long}} - v_{long} \\ \frac{l_f C_{af} - l_r C_{ar}}{I_z v_{long}} & \frac{l_f^2 C_{af} + l_r^2 C_{ar}}{I_z v_{long}} \end{bmatrix} \begin{bmatrix} v_{lat} \\ r \end{bmatrix} + \begin{bmatrix} \frac{C_{af}}{m} \\ \frac{l_f C_{af}}{I_z} \end{bmatrix} \delta \quad (2.4)$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} v \cdot \cos \theta \\ v \cdot \sin \theta \\ r \end{bmatrix} \quad (2.5)$$

where  $m$  denotes the mass of the vehicle,  $I_z$  the moment of inertia,  $v_{long}$  the longitudinal velocity,  $l_f/l_r$  the distance between the front/rear wheel and its center of mass, and  $C_{af}/C_{ar}$  the front/rear cornering stiffness.

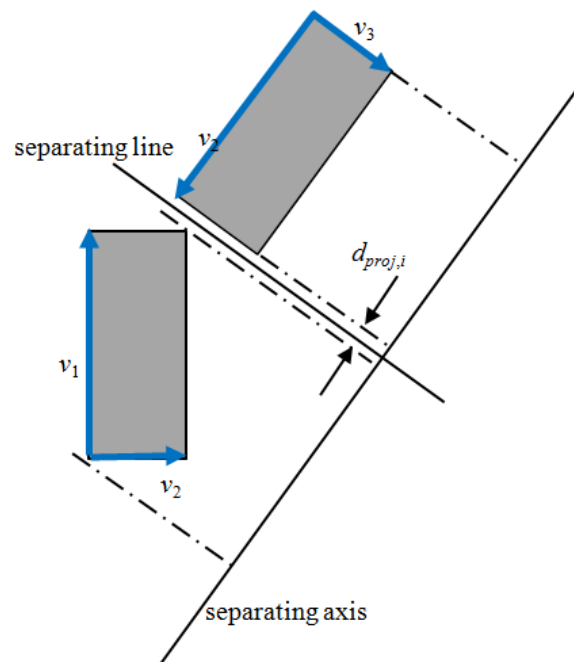


Figure 2.5. Separating axis theorem

### 2.1.4. Collision Detection

The vehicles are assumed to be rectangles that have certain widths and lengths. Thus, the collision detection between two vehicles can be replaced by the detection of the overlapping area between two rectangles. Since the rectangles are 2-dimensional convex shapes, the calculation of the overlap is easily done by using the Separating Axis Theorem shown in Figure 2.5 [102]. We define the collision possibility index (3) with use of projection to the axes.

$$I_{col} = e^{-\|\Delta d_{proj}\|_2} \quad (2.6)$$

where  $I_{col}$  denotes the collision possibility index and the gap of the two rectangles along the separating axis,  $\Delta d_{proj}$  is defined by

$$\Delta d_{proj(v,i)} = \begin{cases} \min\left(\left|\frac{v_i \cdot v}{v_i}\right|\right) & \text{if } \forall \frac{v_i \cdot v}{|v_i|} < 0 \\ \min\left(\left|\frac{v_i \cdot v}{v_i}\right| - |v_i|\right) & \text{if } \forall \frac{v_i \cdot v}{|v_i|} > |v_i| \\ 0 & \text{otherwise} \end{cases} \quad (2.7)$$

The collision possibility index is 1 when two rectangles are overlapped and 0 when the gap between them approaches infinity.



## 2.2. Highway Driving

First, the investigation starts by developing a 3-person Stackelberg game that simulates a typical driver's reasoning in view of his/her disposition so that the driver's reaction in response to roadway traffic is appropriately considered. This is intended to simulate realistic characteristics of a variety of drivers with different sensible levels or rationalities, i.e. aggressiveness. The aggressiveness is an important factor of drivers to understand the drivers' interaction and traffic safety [33, 60]. Thus, the desire and reluctance to choose lanes is limited or altered by the driver's aggressiveness. The interaction among multiple vehicles, especially among drivers that have different aggressiveness is studied. The test case of the least number of vehicles is initially studied to find the basic correlations properties. Monte Carlo simulation follows in order to determine the general effects of the interaction, which is utilized to assess the highest possibility of collision between the given vehicles as a function of their relative distances and the disposition of the drivers in terms of their levels of aggressiveness through fuzzy nonlinear classification method; Adaptive Neuro-Fuzzy Inference System (ANFIS). The results indicate a direct link between aggressiveness and possibility of collision, which is further enhanced when the reduction in the inter-vehicle distance. Finally, we conduct a series of traffic simulations with different vehicle configurations, based on the proposed Stackelberg game based driver decision model. It is shown that first, the 3-person game works as a driver decision model in the three-lane situation, second, the crash rate according to aggressiveness is evaluated by comparing the results of the model with the field data, and, finally, the aforementioned trend between the vehicles in the two-vehicle

driving scenario remains valid regardless of the number of vehicles. This validates that the proposed driver decision model can be effectively used in creating the *traffic unsafety* and may indeed be extended to assess the impact of naturalistic driving models in future autonomous vehicles in mixed traffic.

### 2.2.1. Design of a Game for Highway Driving

#### 2.2.1.1. Game definition

We establish a game theoretic model to simulate driver behavior in the traffic situation. Thus, we configure a straight road with three lanes as the smallest meaningful traffic setting for the purpose at hand, namely driver behavior during lane changes. This setting offers three basic choices: changing lane to left, going straight, and changing lane to the right. We assume the road to be occupied by two kinds of vehicles: vehicles that incorporate decision makers or have intentions and vehicles that follow given set paths. The vehicles following given paths act as props and construct the boundary of the simulation.

In the present study, we formulate a vehicle to execute a game that has three players as shown in Figure 2.6: the vehicle itself<sup>7</sup>, serving as the *lead* vehicle and the two *follower* vehicles in the two adjacent lanes with our assumption of backward propagation of information on the highway, which is nothing but an extension to three

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<sup>7</sup> This and subsequent simplifications are meant to bracket the problem to a manageable form. Future work will remove some of these assumptions.

lanes of the game previously shown in the Figure 2.6. This implies that the subject vehicle does not try to control the vehicles ahead but utilize the information from ahead.

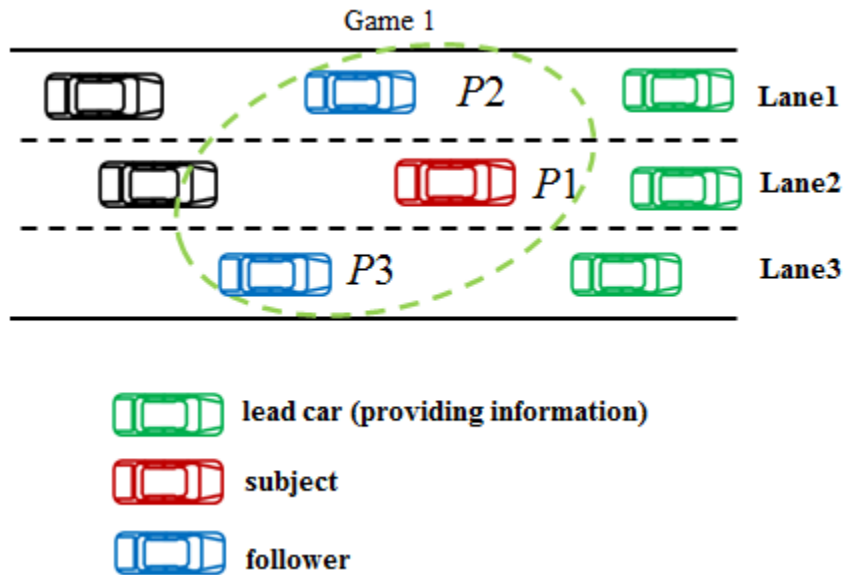


Figure 2.6. Formulation of 3-person Stackelberg game

However, notice that it does not mean that the vehicles ahead do not respond to the subject vehicle. The Stackelberg game is therefore defined as the three-person finite game with three levels of hierarchy:

*Players* :  $P1, P2, P3$

*Strategy space* :  $I_1 \times \Gamma_1 \times \Gamma_2 \times \Gamma_3$  (2.8)

$I_1 = \{1, 2, 3\}, \Gamma_{1,2,3} = \{l, s, r\}$

where  $P1$  designates the first leader,  $P2$  the second leader,  $P3$  the follower,  $I_1$  the  $P1$ 's lane number among lanes 1,2, and 3, and  $\Gamma_{1,2,3}$  the actions of  $P1, P2,$  and  $P3$ : going left,  $l$ , going straight,  $s$ , and going right,  $r$ . Moreover, the game is considered to be dynamic, in the sense that the payoffs corresponding to the strategies continuously change with the driving situation, as we shall see in a later section. Finally we should point out that the players do not share their payoff matrices. This is an important fact in that it implies that each player may have a different perspective on the game that s/he is involved, and each player may perceive its optimal strategy in a way that may or may not be consistent with the way in which other players view theirs. This also means that a vehicle ahead may also consider the subject vehicle as a game player in a different game and respond to the subject vehicle's actions. We believe this adds realism to the individual driver modeling and eliminates the doubt of why the drivers ahead do not consider the subject vehicle.

#### **2.2.1.2. Utility design**

Drivers' lane selection procedures are observed by the following two steps [10, 11, 103]: Do I need to change lanes? Is the gap large enough for me to change lanes successfully? In order to reproduce what drivers will mainly consider when they drive, we define two utility functions: basic *positive* utility and basic *negative* utility. The two

utilities are related to the two factors among multiple factors that Gipps [11] considered; *speed advantage* and *unacceptable collision risk*. Next, those utility functions are designed to be adjusted by the drivers' dispositions. In this paper, we use a disposition index that represents how aggressive a given driver is since the driver's aggressiveness is an important factor that affects the drivers' interactions and corresponding traffic safety [60]. For instance, if the driver is completely aggressive, the index is 100%. If the driver is normally attentive, the index is 50%, and if the driver is completely timid, the index is 0%. The use of the aggressiveness is also intended to reflect the human driver's recklessness or distraction, as well as a willfully aggressive driving, which may be measured by means of the studies estimating driver's sleepiness, cognitive mental load, or emotional state [104-106].

### **2.2.1.3. Basic positive utility**

We assume that every driver wants to have more headway in front<sup>8</sup>. Larger distances between the given vehicle and the foregoing vehicle in an adjacent lane will motivate the vehicle to change its lane; otherwise it will maintain its lane. Thus, the basic *positive* utility or motivation for a given vehicle is defined by the free distance in front of the vehicle. However, the simplification associated with this assumption cannot always be true. Thus, the assumption is weakened according to the driver's disposition in later section and moreover, the longitudinal acceleration is determined to track desired

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<sup>8</sup> We realize the simplification associated with this assumption and will address it in a future extension to this study.

speed as well as headway<sup>9</sup>. Since the game has the strategy space that is difficult to visualize, it is necessary to construct a method to check the entire strategy space without any loss of meaning. We design a cell formulation that is composed of the lanes and the players; i.e. the vehicles. In Figure 2.7, the rows designate the hierarchy among the players and the columns show the probable lane selections. Note that being located in the same row or column does not signify that those vehicles have the same longitudinal or lateral positions. Every strategy combination is marked on the cells by laterally changing the players' lanes. Since we defined a game that has three highway lanes, the left area,  $C_L$ , of the leftmost lane and the right area,  $C_R$ , of the rightmost lane are assumed to be areas where changing lanes is impossible, such as the centerline and the edgeline. Therefore, the vehicles have the least payoffs for the strategies that make them enter these areas. Also, since the human driver's visibility is bounded by physical limitation, the vehicles beyond the visibility distance are not considered.

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<sup>9</sup> When the desire to have larger headway is weakened, the driver tends to follow his/her desired speed.

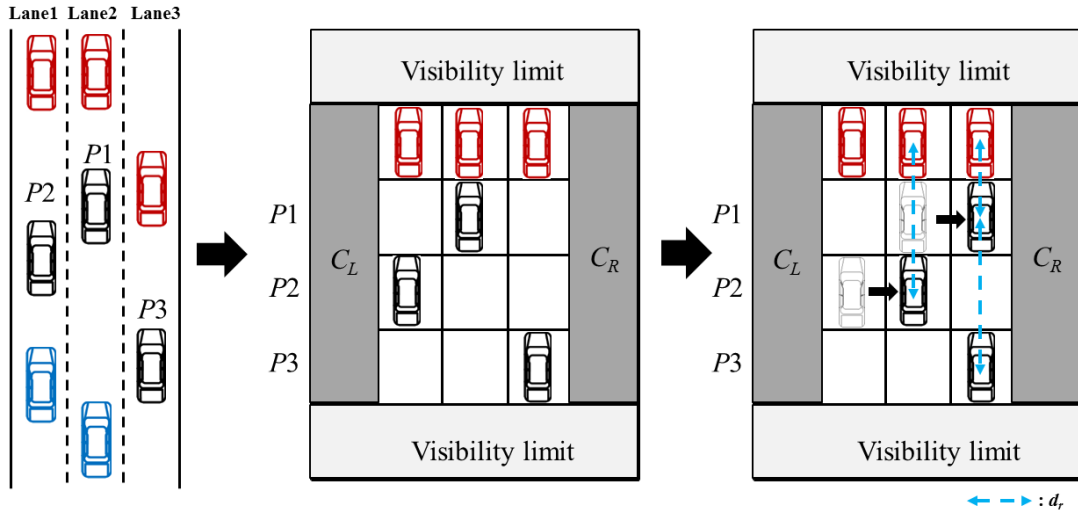


Figure 2.7. Cell formulation

Based on each combination of the players' strategies, the basic *positive* utility is calculated as the free headway. For instance, Figure 2.7 depicts the case when  $P1$ ,  $P2$  and  $P3$  choose  $R$ ,  $R$ , and  $S$  respectively as their strategies and the utilities are determined by their physical longitudinal positions.

$$U_{pos} = \begin{cases} \min(d_r, d_v), & \text{if there exists a vehicle ahead} \\ d_v & , \text{ otherwise} \end{cases} \quad (2.9)$$

where  $d_v$  denotes the visibility distance,  $d_r$  the relative distance between the vehicle ahead and the subject vehicle. Note that Figure 2.7 represents one possible strategy pair

in the strategy space. As we can see in the left figure, the strategy pair is not likely to lead to better payoffs for  $P1$ , rather than the payoffs of other strategy pairs.

#### 2.2.1.4. Basic negative utility

Once the driver has determined the lane to which it wants to move, another important consideration before actually changing lanes is to look to the side to see if there is a vehicle approaching in the adjacent lane, and, if so, how probable it is to collide with the approaching vehicle. If the vehicle in the adjacent lane is fast enough to overtake the given vehicle, the driver will not change lanes even though the lane provides the driver with the larger free space in front. To factor in this reluctance to change lanes, we consider the following threat evaluation from the approaching vehicle in the adjacent lane.

$$U_{neg} = d_r - v_r \cdot T - D_{suf} \quad (2.10)$$

where  $d_r$  denotes the relative distance between the player and the vehicle behind,  $v_r$  the relative velocity,  $T$  the prediction time, and  $D_{suf}$  the distance essential to change lanes: i.e. the diagonal length of the vehicle. The *negative* utility is considered in case the rear vehicles stay within the rearward visibility distance. The prediction time varies according to the driver's aggressiveness, which will be described in a later section.



### 2.2.1.5. Drivers' uncertainty

One may suspect that we cannot simulate drivers' behaviors with only the above two factors because, in reality, human drivers have numerous motivations and distractions. Thus, to make the drivers' utility functions more realistic, the following modifications to the basic utilities are considered.

#### 2.2.1.5.1. Recognition distance

Human drivers have limited visibility. Young drivers have about 70m to 200m for various traffic signs, and elderly people have comparatively smaller visibility distances [107]. And many studies have shown that driving errors are related to human factors, such as vigilance, age, fatigue, etc. [108]. From the results of previous studies, we settle on a normal visibility distance for a normal driver and scale this distance according to the driver's disposition. This means that unaccustomed or timid drivers will have restrictions in discerning objects and thus, they will have a smaller recognition distance than the normal visibility distance. The restricted recognition distance is used to limit the drivers' desire to select lanes:

$$d_{vr} = \alpha_{visibility}(q_a) \cdot d_v \quad (2.11)$$

$$U'_{pos} = \min(U_{pos}, d_{vr}) \quad (2.12)$$

where  $d_{vr}$  denotes the restricted visibility distance (e.g. 150m),  $\alpha_{visibility}$  the degree of restriction with a range of [0.33, 1],  $q_a$  the index of the driver's aggressiveness with a range of [0, 1] or [0, 100]%, and  $U'_{pos}$  the modified *positive* utility.

#### 2.2.1.5.2. Neglectful side-viewing

The gap between a given vehicle and its immediately following vehicle in adjacent lane that makes people feel safe to change lanes is different for every driver. However, in general, aggressive drivers tend to change lanes even if the space is comparatively tight and, conversely, unaccustomed or timid drivers prefer a larger gap as the prerequisite to change lanes. We thus have

$$U'_{neg} = d_r - v_r \cdot T(q_a) - D_{suf} \quad (2.13)$$

where  $U'_{neg}$  denotes the modified *negative* utility and  $T$  the prediction time (e.g. 3 sec is set for a normal driver and linearly change with respect to the driver's aggressiveness) with a range of [1, 5] sec, and  $D_{suf}$  the distance essential to change lanes: e.g. the diagonal length of the vehicle.

#### 2.2.1.5.3. Prediction and response

Time-headway, unless too small, is not the sole factor for collisions. The driver's reaction times as well as poor prediction of other vehicles' actions are also crucial factors that lead to accidents [109-112]. To consider these issues, we apply another

degradation factor that can show delayed recognition due to ill prediction and late response. In the recognition step, a decision maker identifies the lane of every vehicle in its vicinity. However, when a vehicle cuts in from an adjacent lane, the amount of time it takes the driver to recognize the vehicle varies according to the driver's disposition. Time to Line Crossing (TLC) is proposed in [113]. TLC is defined as the time period before a driver cross the line.

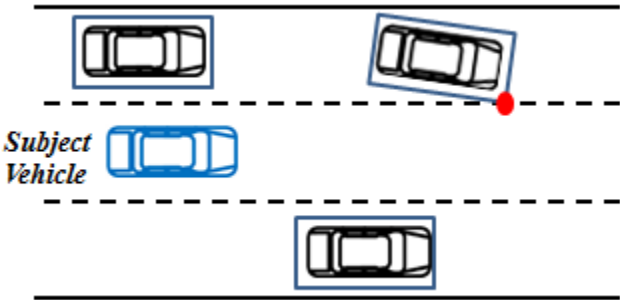


Figure 2.8. Response delay

In this paper, we propose a consideration on the response delay, which is similar to TLC. This is implemented by changing the decision maker's recognition point that is used to number the lanes occupied by the vehicles. The reference on which the subject driver recognizes the lane number of the other vehicle changes according to the driver's

attentiveness. When the closest front corner of the rectangle around the other vehicle meets the criterion line as shown in Figure 2.8, the subject driver recognizes the line-crossing of the other vehicle. If the driver's response is comparatively slow, it is assumed that the magnification ratio of the rectangle around the vehicle would decrease, which linearly moves from 1 to 1.3 with respect to aggressiveness.

### 2.2.1.6. The Solution of Stackelberg Game

Each player's total utility is determined by summing the modified *positive* and *negative* utilities:

$$U = U'_{pos} + U'_{neg} \quad (2.14)$$

Next, appropriate action should be chosen among the final utility pairs in order to simulate the driver's behavior. Since every vehicle follows the Stackelberg game, the solution  $(\gamma^{1*}, \gamma^{2*}, \gamma^{3*})$  of the game is obtained by the following 3-person Stackelberg equilibrium equations with the designed utility values,

$$\min_{\gamma^2 \in S^2(\gamma^{1*})} \min_{\gamma^3 \in S^3(\gamma^{1*}; \gamma^2)} U^1(\gamma^{1*}, \gamma^2, \gamma^3) = \max_{\gamma^1 \in \Gamma^1} \min_{\gamma^2 \in S^2(\gamma^{1*})} \min_{\gamma^3 \in S^3(\gamma^{1*}; \gamma^2)} U^1(\gamma^1, \gamma^2) \stackrel{def}{=} U^{1*} \quad (2.15)$$

$$S^2(\gamma^1) \stackrel{def}{=} \{\xi \in \Gamma^2 : \min_{\gamma^3 \in S^3(\gamma^1; \xi)} U^2(\gamma^1, \xi, \gamma^3) \geq \min_{\gamma^3 \in S^3(\gamma^1; \gamma^2)} U^2(\gamma^1, \gamma^2, \gamma^3), \forall \gamma^2 \in \Gamma^2\} \quad (2.16)$$

$$S^3(\gamma^1; \gamma^2) \stackrel{def}{=} \{\xi \in \Gamma^3 : U^3(\gamma^1, \gamma^2, \xi) \geq U^3(\gamma^1, \gamma^2, \gamma^3), \forall \gamma^3 \in \Gamma^3\} \quad (2.17)$$

where  $U^1$ ,  $U^2$ , and  $U^3$  denote the respective utilities of the first leader, the second leader, and the follower;  $\gamma^1$  is the possible action of the first leader.  $\gamma^2$  and  $\gamma^3$  are respective reactions of the second leader and the follower. Every two-person finite Stackelberg game admits a strategy for the leader [72], which can be extended to the three-person finite Stackelberg game with three levels of hierarchy because it can be understood as two successive two-person games. The set of  $\gamma^3$ ,  $S^3$ , is first obtained as the argument of the maximum payoff for the follower with respect to  $\gamma^1$  and  $\gamma^2$ . In turn,  $\gamma^2$  is obtained in the same manner for a given  $\gamma^1$  with the consideration of the follower's rational reaction. Since the follower's (the second player against the first leader and the follower against the first and second leaders) payoff is unique but the strategy is not necessarily unique [72], we establish an order among the strategies that have the same payoffs, such that the driver chooses "going straight" if the payoffs of "going straight" and "changing lanes" are the same, and the driver in the second lane chooses the first lane when the first and third lanes offer the driver the same payoffs. Thus, every possible  $\gamma^1$  has the accompanying unique reactions of the second leader and the follower. Accordingly,  $\gamma^{2*} \in S^2(\gamma^{1*})$  and  $\gamma^{3*} \in S^3(\gamma^{1*}; \gamma^{2*})$  are determined as the optimal strategies of the second leader and the follower corresponding to  $\gamma^{1*}$  and the pair  $(\gamma^{1*}, \gamma^{2*})$ , respectively. In simple terms, the driver in our proposed model predicts the two followers' responses and chooses the best strategy based on that prediction. Note that since every driver has his/her own Stackelberg game-based decision model and does not share the utilities, the best responses for the followers are not guaranteed.

### 2.2.2. Simulations

Two evaluation scenarios are tested. First, unit tests with a few vehicles are run to check if the model works properly and derive some meaningful factors to analyze the traffic situation. To be exact, 2 vehicles are employed in the unit test, which is sufficient to identify the factors affect the *traffic unsafety* since more than 85% of all road accidents involve only two vehicles [114]. Next, traffic simulation with a number of vehicles will be run to check if the model can reflect real world data effectively, and if so, to check if the model reproduces the observational results on the traffic situation in relation to aggressiveness.

#### 2.2.2.1 Unit test scenarios

We test a specific scenario consisting of two vehicles, in addition to the three front dummy vehicles that form a boundary of the simulation area. The purpose of the scenario is to focus on the interaction between two vehicles when they use the Stackelberg game theoretic decision model. Also, note that we focus on two vehicles to assess the collision between vehicles [114], although the model considers 3 players in normal traffic situation<sup>10</sup>. Vehicles are assumed to travel in longitudinal direction, Y-axis, and the lanes are set in lateral direction, X-axis of the Cartesian coordinate; Lane 1 (0 m), Lane 2 (3.3m), and Lane 3 (6.6m). There are two initial settings for the vehicles. We need to set the initial positions, velocities, and the drivers' dispositions for every vehicle to determine the impact of the driver's disposition in conjunction with the design

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<sup>10</sup> Usage of 3-person game will be validated in the section of traffic simulation for a number of vehicles.

of the aforementioned utility functions. One vehicle in the third lane, among the front prop vehicles, is located slightly ahead in order to bring about a lane change for the subject vehicles. Vehicle 1 and 2 are initially in the second and third lanes, respectively. In order to test the lane change situation when there is a fast approaching vehicle, Vehicle 2 is located 50 *m* behind Vehicle 1, but with a higher velocity. There are four aggressiveness combinations to be tested for Vehicle 1 and 2. The test cases 1 to 4 represent the interactions of two normal drivers, an aggressive driver and a timid driver, two aggressive drivers, and two timid drivers in that order.

Table 2.1. The initial positions of the vehicles in the two vehicles test case

	Initial conditions		
	$x_0 (m)$	$y_0 (m)$	$v_0 (km/h)$
Vehicle 1	3.3	0	100
Vehicle 2	6.6	-50	130

### 2.2.2.2 Unit test results

The unit test results with a specific setting are shown in Figure 2.9a and Figure 2.9b. Panels (a), (b), (c), and (d) depict the results corresponding to the test cases

mentioned before. The two normal driver test case (a) and the two aggressive driver test case (c) show that the driver of Vehicle 1 in the second lane changes its lane to the third lane, which initially has a larger space in front, and then Vehicle 2 changes its lane to the second lane because its free space is now restricted by Vehicle 1. Compared with Case (a), Case (c) shows that the aggressive drivers' lane changes happens sooner than the normal drivers'. In Case (b), the timid driver of Vehicle 2 does not try to overtake the aggressive driver's and maintains a safer relative distance. In Case (d), the combination of the two timid drivers, no drivers changes their lane. The most dangerous instant comes from Case (c). The reason for this result is that the more aggressive the drivers are, the smaller the headway they set. Additionally, in Case (c), the solution of "changing lanes" is derived without much consideration of the approaching vehicles. Thus, although both Vehicle 1 and Vehicle 2 have smaller headway in front, their lane changes occur, which results in higher collision possibility (3). Conversely, if one driver is timid, even though the other is aggressive like Case (b), they hardly have a high collision possibility.



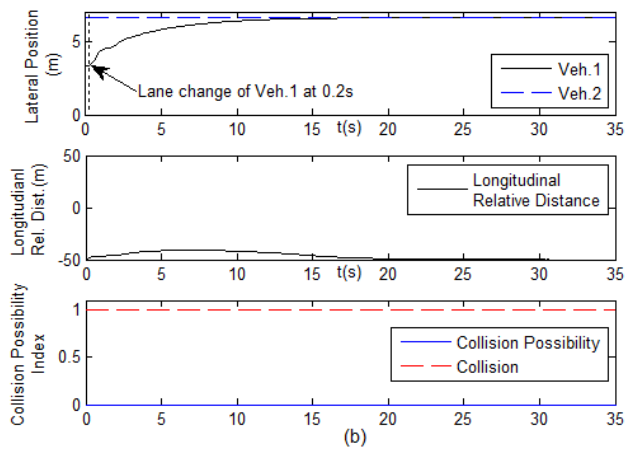
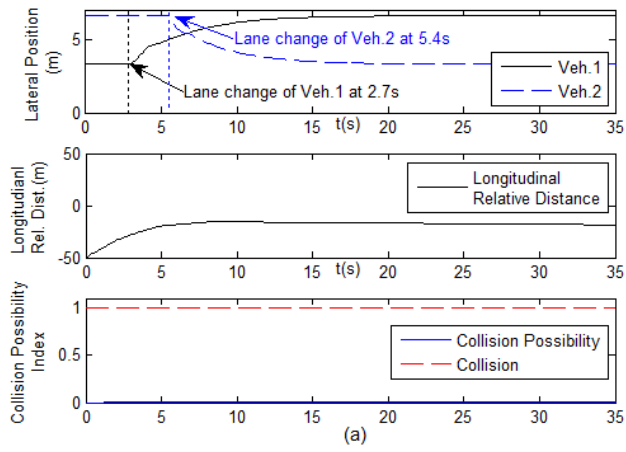


Figure 2.9a. Unit test results (a) and (b)

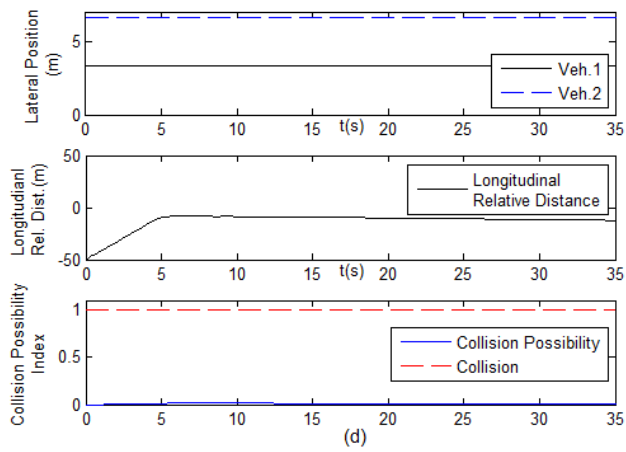
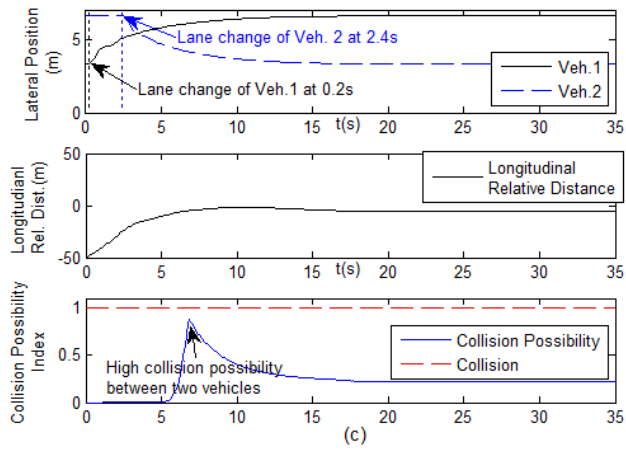


Figure 2.9b. Unit test results (c) and (d)

### 2.2.2.3 Monte Carlo simulation

A Monte Carlo simulation is used to verify the propagation of uncertainties from inputs to outputs through a given deterministic model. A Monte Carlo simulation empirically executes this by using random samples [115]. The above results of unit tests indicate that the interaction among the vehicles depends on the level of aggressiveness of the drivers, as well as the initial conditions of the vehicles. In order to determine the general effects of the aggressiveness of the drivers, we perform a Monte Carlo simulation involving randomized longitudinal positions and construct a model that estimates the collision possibility given the longitudinal positions and aggressiveness combinations of the drivers. We test 100 cases with random longitudinal positions of the two vehicles. The longitudinal positions of Vehicle 1 and 2 are uniformly distributed in the range of 0 to 50 *m* and 0 to -50 *m*, respectively. Three aggressive combinations (Normal/Normal, Aggressive/Timid, and Aggressive/Aggressive) are demonstrated; the case that both drivers are timid has no meaningful results. Figure 2.10 shows the number of potential collisions at every second. Although the highest level of potential collision is different for every test case, the pattern of potential collisions with respect to time appears similar to the unit test results.

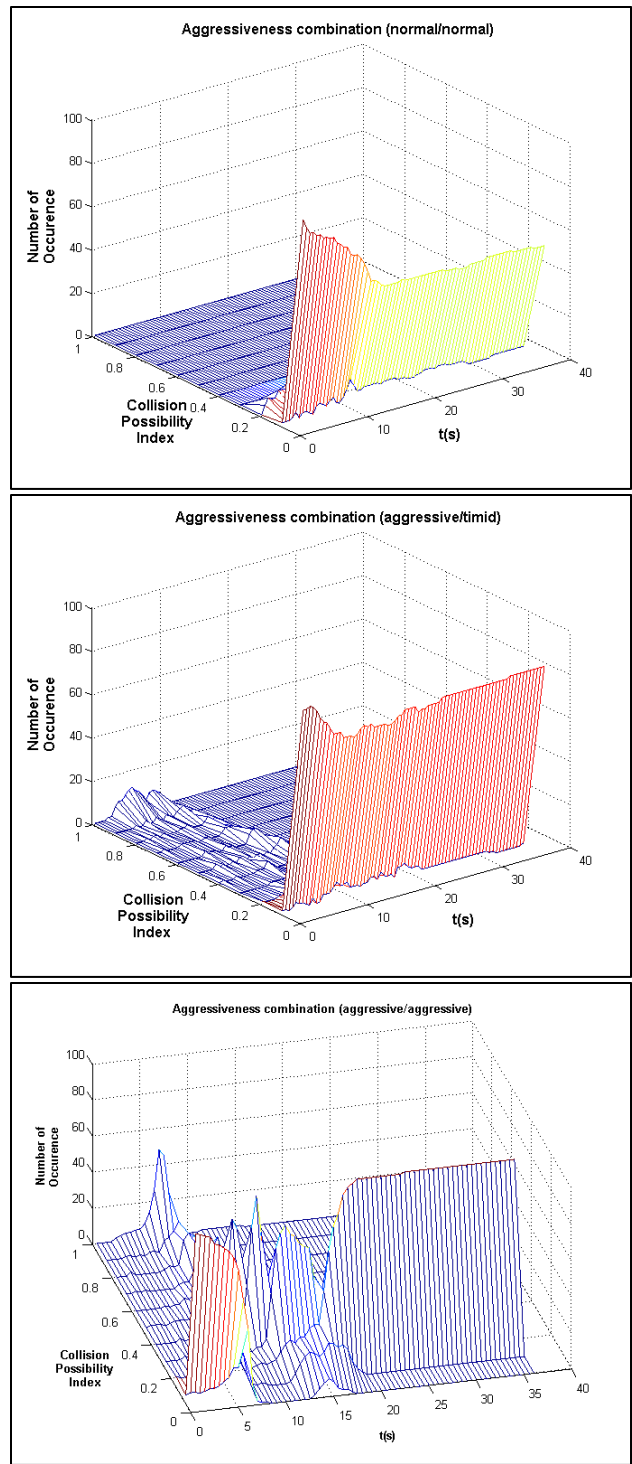


Figure 2.10. Monte Carlo simulation results for combined and velocity tracking controller

Table 2.2. ANFIS settings in MATLAB

Input MF (membership function) Type	trapezoidal
Input Number of MFs	3
Output MF Type	linear
Optimization Method	hybird
Epochs	3

#### 2.2.2.4. ANFIS modeling

ANFIS (Adaptive Neuro-Fuzzy Inference System) [116] is a nonlinear modeling method that employs two complementary techniques: neural networks and fuzzy logic. Neural networks provide adaptive learning that fuzzy logic can use for linguistic expression via if-then rules. Using ANFIS, we build a comprehensive model to represent the vehicles' potential collisions according to the vehicles' relative position and aggressiveness combinations. We use Monte Carlo simulation results to associate collision possibilities with given relative positions and aggressive combinations. The collision possibility model is learned from two inputs and one output. Two inputs are the initial longitudinal relative positions and aggressiveness combinations. We define aggressiveness combinations (Normal/Normal, Aggressive/Timid, and Aggressive/Aggressive) as Mode 1, 2, and 3, respectively. The output used is the

highest collision possibility value of every test case. The setting for the ANFIS is listed in Table 2.2.

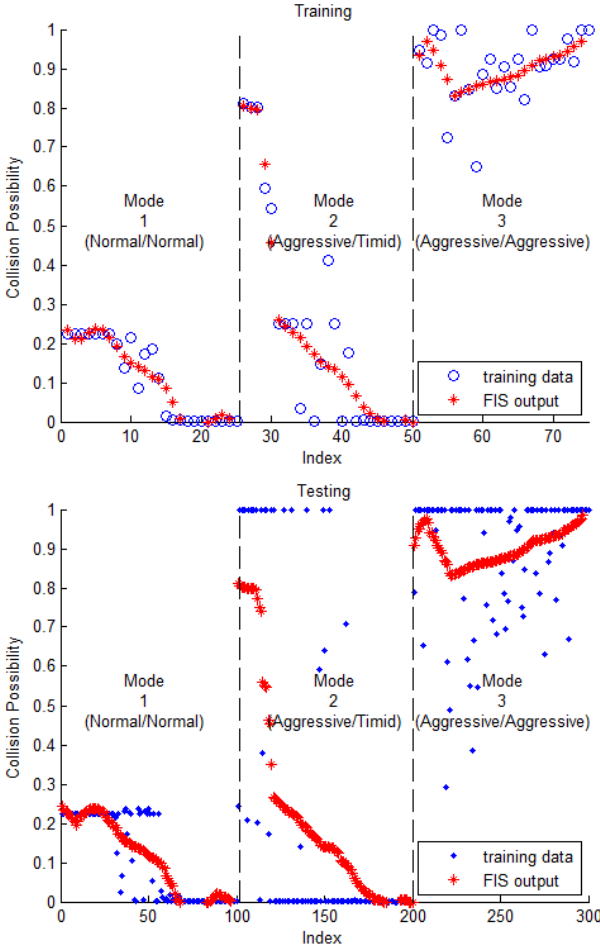


Figure 2.11. ANFIS training and testing results

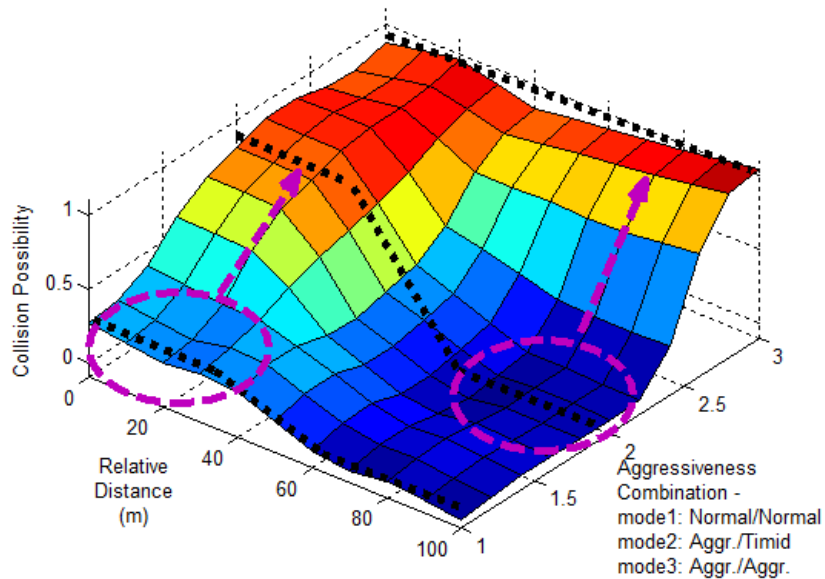


Figure 2.12. Rule surface of ANFIS collision possibility model

The ANFIS model is trained with one fourth of the data with a training error, 2%. When the model is tested with the total data, the modeling error is 6%. The error mainly occurs in Mode 2. However, the model is sufficient to show the tendency of the collision possibility as shown in Figure 2.11. Figure 2.12 shows the collision possibility model. At the combination of two normal drivers (Mode 1), the collision possibility remains low even though it increases to 0.25 when the relative distance is less than 50 m. However, if one driver becomes aggressive and another driver becomes timid, the collision possibility when the relative distance is less than 50 m increases to 0.8. Moreover, if

both drivers are aggressive, the collision possibility stays at the highest level, regardless of the relative distance.

#### 2.2.2.5. Traffic flow simulation

In this section, the Stackelberg game based highway driver decision model is used as the basis for a microscopic traffic simulation. To accomplish this, we conduct multiple vehicle simulations for the evaluation of the crash occurrence and the cumulative collision possibility according to the drivers' aggressiveness combinations. A 200 m section of a three-lane highway is simulated as shown in Figure 2.13. When one vehicle goes out of the section, another vehicle comes in the section. Thus, the density of the vehicle in the section is maintained, which can be translated to the flow rate of the vehicles. For example, if 10 groups of the vehicles pass through the section for 1 minute when the density of the vehicles is 10 veh/section, the flow rate is 100 veh/min.

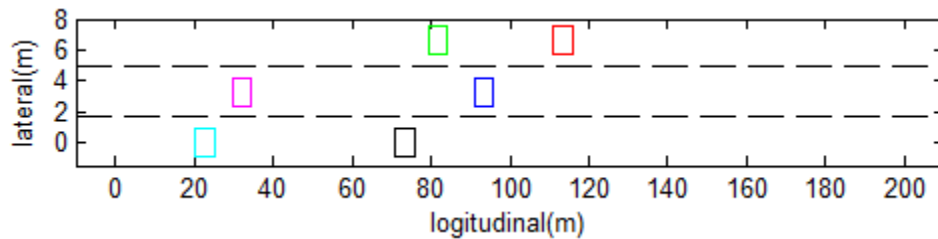


Figure 2.13. Configuration of the traffic flow simulation



To validate the *traffic unsafety* (i.e. crashes) that Stackelberg game based driver decision model leads to according to the driver's aggressiveness<sup>11</sup>, we compare the result of the traffic simulation results based on our driver decision model with the crash data from [3]. The occurrence of crash is counted in our result when the collision possibility index is 1. Also, when the collision possibility index exceeds 0.5, near crash is counted. The density of the traffic is 6 veh/section. In inattentive case, one driver is set to be aggressive (75%).

Table 2.3. Occurrence of crash

Crash type	The number of crashes
Crash (Attentive)	1
Near Crash (Attentive)	12
Crash (inattentive)	2
Near Crash (inattentive)	26

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<sup>11</sup> Note that the aggressiveness is used to reflect driver's inattention or distraction as well as willful aggressiveness in this paper.

To compare the rate of crash, the occurrence of crash is converted to rate per MVMT (Million Vehicle Miles Traveled). The result of the model cannot be directly compared with the field because the levels of aggressiveness and Near Crash are based on our own discretion. However, the comparison shows a certain level of effectiveness of the model in representing the *traffic unsafety*, as shown in Figure 2.14. The results of the model have a similar level of overapproximation to the field data.

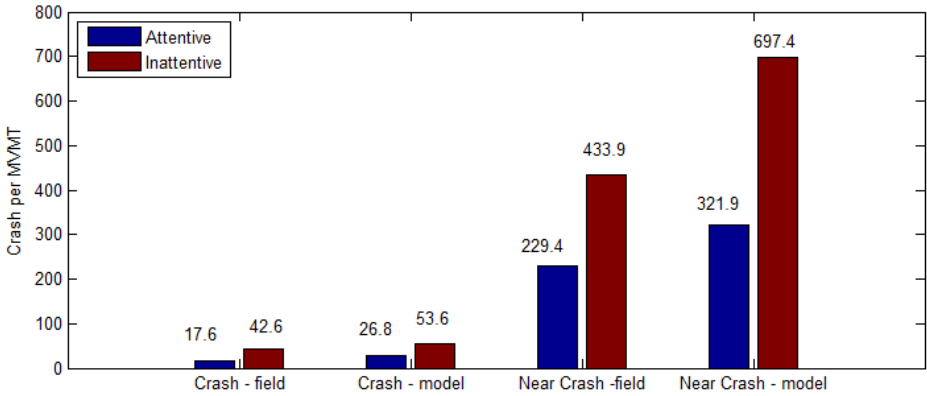


Figure 2.14. Comparison of crash rate

The cumulative collision possibility results are presented in Figure 2.15. To evaluate the effects of aggressive drivers, the ratio of aggressive drivers is set to 50% and 100% in the test cases of Aggr./Timid and Aggr./Aggr. in Figure 2.15. 5, 50, 500 runs of the

simulation with the density of 6 veh./section yield 30,300, and 3000 vehicles simulation. 40, 400 and 4000 vehicles resulted from 8 veh/section. Thus, the left and right figures show the different density settings. Bars in each figure show the difference caused by different drivers' aggressiveness. It can be seen that aggressiveness combinations influence the accumulated collision possibility regardless of the number of vehicles. Similar to the previous result, the collision possibility shows the growing tendency as the ratio of aggressive drivers increases. Also, when the density of the vehicle is higher in Figure 2.15 (b) compared with (a), the collision possibility increases in every aggressiveness combination, which means shorter relative distances escalate the collision possibility as described in the previous ANFIS model. Finally, our ANFIS result and the application to the traffic flow simulation of the Stakelberg game based driver model imply a natural result that aggressiveness of drivers and traffic densities impact traffic safety, observed by other researchers [33, 60], which shows the effectiveness of our more realistic driver model. This validation step is intended to facilitate the use of this model in assessing the impact of implementation of naturalistic driving models for autonomous vehicles. Other uses of this model may include driver education campaigns and policy analysis for transportation systems.

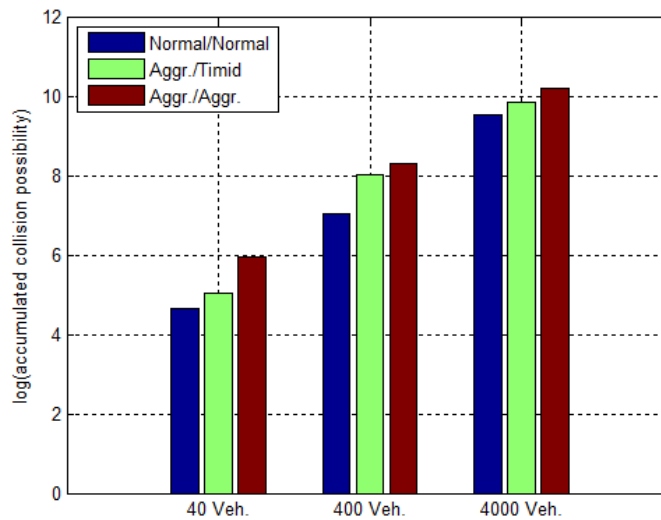
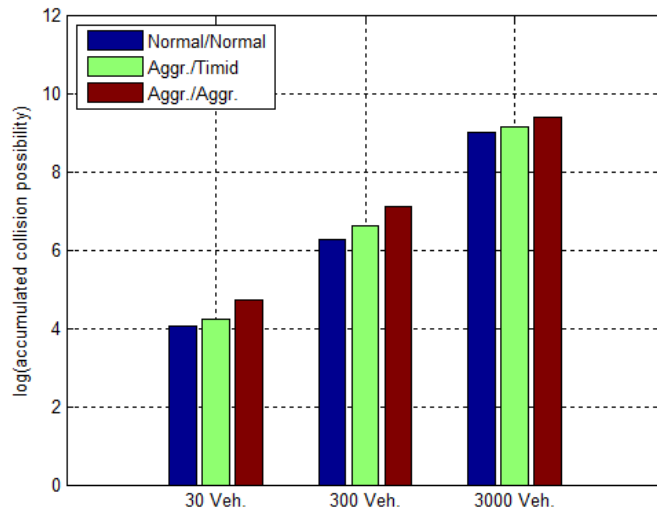


Figure 2.15. Distribution of cumulative collision possibilities

### 2.2.3. Summary

In Section 2.2, we developed a Stackelberg game based driver reasoning model of highway driving with the consideration of the driver's uncertainties. Since

Stackelberg game theory can be used to demonstrate the human decision process when the event has the structure of a hierarchy, a game theoretic traffic simulation based on 3-person Stackelberg game theory and ground vehicle dynamics has been presented in order to simulate human behavior in certain driving situations. We assumed that every vehicle in the simulation constituted a 3-person game, where the given vehicle is the first leader of the game. This setting was intended to add the uncertainties that occur in real driving since drivers can only predict the other drivers' behaviors and cannot control them. In order to simulate realistic characteristics of a variety of drivers, we presented utility designs that originated from the drivers' intentions and are influenced by their dispositions. The driver's manipulation was also adjusted by their dispositions. From the simulation results, we showed that Stackelberg game theory played an effective role in choosing a certain action among the possible action sets, much like human reasoning. Second, we presented a collision possibility model in terms of the driver's aggressiveness. There is no doubt of the fact that road violence leads to more accidents and finally more deaths and injuries. We assumed that an aggressive driver has irrational logical grounds although the driver makes decisions through a rational method. We deteriorated or restricted the logical grounds for the drivers to consider when they choose lanes. With the unit tests, we showed that the above game theoretic approaches to the traffic simulation can provide sufficiently explainable demonstrations. According to the different drivers' dispositions, the simulation showed plausible results about the interactions of the vehicles. Based on this, the collision possibility model was studied. Through a Monte Carlo simulation, we first demonstrated a general result that the

aggressiveness combination can be the main reason for high collision possibilities. Using the data from the Monte Carlo simulation, we built an ANFIS model of collision possibility according to the relative distance and aggressiveness combinations. It was shown that there exists a direct link between drivers' aggressiveness and possibility of collision. The model was validated for the crash and near crash events that are defined by using collision possibility index. Our driver decision model effectively represented the *traffic unsafety*, with a certain level of overapproximation to the field data. The result of the ANFIS model was also evaluated in traffic simulation based on the developed Stackelberg game-based highway driver model. Consequently, it was shown that our Stackelberg game based driver decision model can produce an observation-based natural result that traffic safety is influenced by the driver's aggressiveness and traffic densities in the traffic simulations. This also supported the validity of the tendency of our ANFIS model. The model showed *traffic unsafety* when the aggressive drivers interact with other drivers although the model needs to consider more desires and restrictions. Based on these results, the model would be extended to assess the impact of naturalistic driver models in future autonomous vehicle-mixed traffic or conversely utilize it in designing an autonomous vehicle's responses to the naturalistic drivers who may be irrational. This model may also be used in driver education campaigns and policy analysis in terms of aggressive driving and traffic safety.

### 2.3. Highway Merging

Merge operation is also one of the important issues in studying roadway traffic. Merging disturbs the mainline of traffic, which reduces the efficiency or capacity of the highway system. In this section, we develop a driver merging model with Stackelberg game theory in the same context as the previous section. The driver merging model is also developed with the considerations on the utilities that originated from the drivers' intentions, which will determine the instant to merge and acceleration/deceleration with respect to the driver's aggressiveness in the game theoretic framework. This is intended to simulate realistic characteristics of a variety of drivers with different sensible levels in the merging situations. Accordingly, the interaction among multiple vehicles, especially among drivers that have different aggressiveness will be studied. To this end, we will combine the driver merging model with the highway driver model that was developed and analyze the interaction between them. The merging behaviors impact the mainline vehicles, which may lead to a variety of influences, such as collisions or reduced roadway throughput. Especially, as pointed out in [14] the disturbance from merging to the mainline of traffic can be amplified. A small delay at one point can lead to severe congestion downstream, which is known as slinky effect (or string instability) [58, 117, 118]. Consequently, these impacts, both longitudinal and lateral disturbances in the mainline due to their interaction, depend on the level of aggressiveness of the driver who merges in and those in the mainline. This can be another step to extend the driver decision model to the qualitative traffic research, such as understanding the effects of

driver's aggressiveness to road congestion as well as traffic safety that we have investigated earlier.

### *2.3.1. Design of a Game for Highway Merging*

#### **2.3.1.1. Game Definition**

We establish a Stackelberg game between a merging-in vehicle and the vehicle in the adjacent mainline. It is required for the merging-in vehicle to decide whether to merge in and whether to change its speed. Thus, we formulate two different games for the respective purposes. A Stackelberg game, i.e. the *merging game*, determines the merging point of the vehicle based on the current information concerning the vehicles and of the merging lane. Another Stackelberg game, i.e. the *acceleration game*, is used to decide whether the given vehicle accelerates, decelerates, or maintains its present speed. The predicted future positions of the vehicles determine which vehicles participate in the merging game. We shall further clarify the inter-relationship between these two games and the impact of their interconnection shortly. With this in mind, we configure a straight road with one merge lane. The assumption is that the merge lane has an entrance/acceleration area for merging, and is regarded as an additional lane where only the merging-in vehicle can occupy. On the main road, there are two kinds of vehicles: vehicles with decision makers and vehicles that just move with constant speed and without changing lanes. The latter form the boundary of the simulation and enable us to concentrate on the merging interaction of the two vehicles that are the focus of the



study. For example, merging-in vehicle,  $P1$ , performs its action based on a 2-person Stackelberg game with  $P2$ , the vehicle in the mainline, which may follow a 3-person Stackelberg game that was introduced in [119]<sup>12</sup>. We assume that the effects of the interaction between the game playing vehicles and other vehicles are indirectly considered in the payoff calculation.

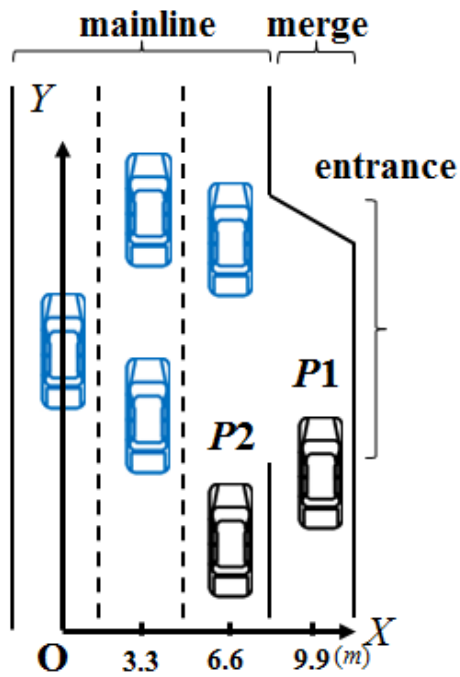


Figure 2.16. Configuration for the merging situation

<sup>12</sup> Every vehicle on the road may be involved a 3-person game with other 2 vehicles in two adjacent lanes at every instant, where the vehicles have three strategies: going straight and changing their lane to the left or right. The payoffs used in the game depend on the driver's aggressiveness to reflect the realistic characteristics of driving.

For instance,  $P2$  drives, based on the decision from the game with its surrounding vehicles. In turn,  $P1$  makes a game theoretic decision in the game with  $P2$ . Moreover, the players' payoffs are not calculated by using only themselves' information but also the surrounding vehicles' information. Therefore, the assumption does not lose much of generality. The merging-in vehicle,  $P1$ , executes a game that has two players:  $P1$  and  $P2$ , in the adjacent mainline as shown in Figure 2.16. Note that the merging-in vehicle is always regarded to be the leader in this 2-person game. In the acceleration game if  $P2$  were located ahead of  $P1$ ,  $P1$  may accelerate to move ahead of  $P2$  prior to merging in. If it does not, then  $P1$  would be effectively engaging with the vehicle immediately behind itself in the mainline, which will be named  $P2$ . In other words, in both cases  $P1$  will be ahead of its counterpart in the mainline. Therefore, the games are simply defined as

*Players :  $P1$  and  $P2$*

*Strategy space :  $\Gamma_1 \times \Gamma_2$*  (2.18)

$\Gamma_{1,2} = \{l, s\}$

where  $P1$  designates the leader;  $P2$ , the follower, and  $\Gamma_{1,2}$  the action of going left  $l$  and going straight  $s$ . Going left or going straight can be understood as merging in or staying in the merge lane respectively for the leader and as yielding the right of way for the follower. Note that as the vehicles keep moving, each player possesses a different payoff matrix, which changes continuously. Hence, the solution of the game may change at every instant. As stated earlier in this paper, merging procedures are assumed to have

two simultaneous decision processes: merging-in and acceleration/deceleration. The decision to merge-in and the related payoffs are based on present information of vehicles. For instance, if the solution of the game is to merge in, the vehicle directly starts to change its lane. However, the decision to accelerate or decelerate and the associated payoffs are based on the predicted positions of  $P1$  and the corresponding new follower (if it exists),  $P2'$ , as shown in Figure 2.17.

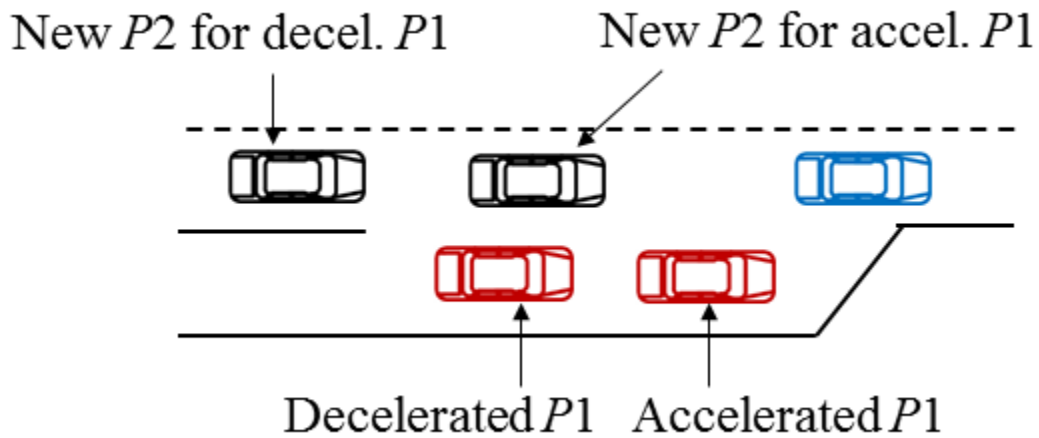


Figure 2.17. Position assumptions for the acceleration decision

The relative distances, headways, and the distance to the end of the merge lane are based on the choice of acceleration or deceleration and involve the predicted as opposed to the

current positions of the respective vehicles. This *acceleration game* has the same solution candidates as the prior game for the decision to merge in: merging in and staying in the merge lane.

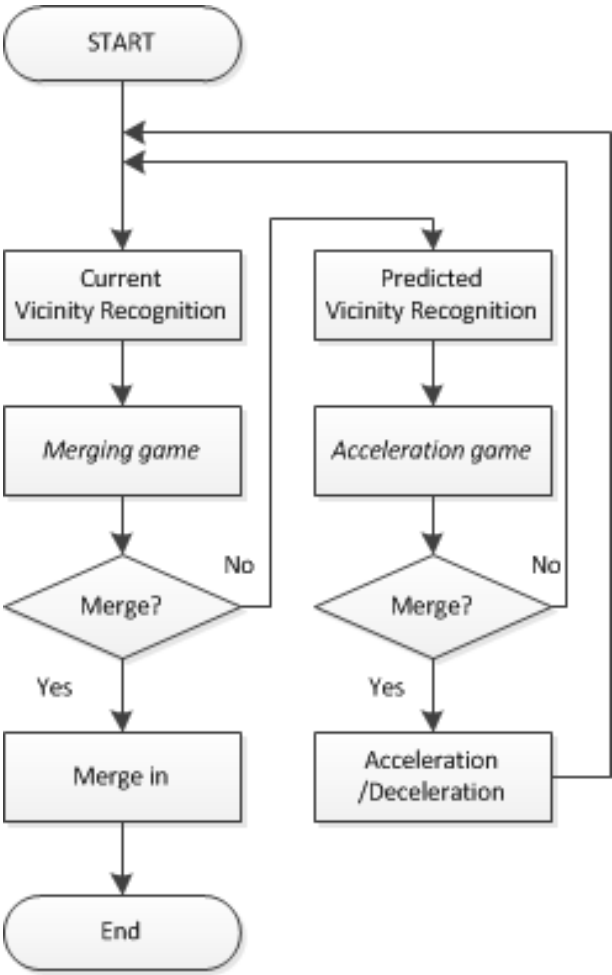


Figure 2.18. Relationship between the two Stackelberg games

Yet they do not literally mean immediate merging in or staying. For example, the solution, merging in, of the game with an assumption of deceleration means that if the vehicle decelerates, it may join the mainline. Notice that joining the mainline cannot be guaranteed but predicted to be possible. In the example shown in Figure 2.17, we have two decisions: going straight against  $P2$  and merging in against  $P2'$ . Thus,  $P1$  does not merge in immediately but will accelerate to cut in later. Consequently,  $P2'$  takes the place of  $P2$  in the *merging game*. Figure 2.18 shows the order of the two games. One can easily recognize that *acceleration game* is meaningful only when the merge operation is not believed to be feasible through *merging game*.

### **2.3.1.2. Utility Design**

We define two utility functions: *positive* utility and *negative* utility. Both the *merging* and *acceleration* games use the same utility functions. However, as stated previously, inputs for the functions are different from each other. They use current and predicted information respectively. These utility functions are designed to be adjusted by the drivers' aggressiveness, which denotes an index that reflects how aggressive a given driver is. The aggressiveness does not reflect only willfully aggressive driving but also the human driver's recklessness or distraction due to a number of reasons. We define the index to be 100% if the driver is completely aggressive. As before, if the driver is normally attentive, the index is set to 50%, and, if the driver is completely timid, the index is set to 0%. A prerequisite for the subsequent detailed utility design is the recognition of lane number in which lane the vehicle is located. The recognition point

that produces the lane number varies in accordance with the aggressiveness of the driver, in consideration of the driver's response time. The driver's response time includes both the observation and physical manipulation delay. For instance, an inattentive driver is modeled to notice the lane occupation of the other vehicle after the vehicle passes over the lane line. Attentive drivers are designed to respond to the other vehicles' lane-changes more quickly; before the vehicles cross the line. When it comes to the detailed utility design, we mainly have two utilities, *positive* and *negative* utilities, that are related to the two factors among multiple factors that Gipps [11] considered; *speed advantage* and *unacceptable collision risk*. Moreover, in contrast to the lane-change within the mainline, the merge lane has the particular limitation that the road is closed; i.e. the vehicle in the merge lane must join the mainline before the end of the road. The limitation works as a main factor in designing the payoffs of the merging model in addition to the previous work.

#### 2.3.1.2.1. *Positive Utility*

A basic assumption is that every driver wants to have more headway ahead of them. Thus, *positive* utility is defined by Eq. (1) and (2), the free distance in front of the vehicle. The headway is limited by the human driver's visibility assumption [107]. For example, when the strategy is to merge in, *positive* utility, combined with *negative* utility in later section, represents a free space in the mainline. The large space enough to fit in should be ensured for merging in general.

$$U_{pos} = \begin{cases} \min(d_r, d_{vr}), & \text{if there exists a vehicle ahead} \\ d_{vr} & , \text{ otherwise} \end{cases} \quad (2.19)$$

$$d_{vr} = \alpha_{visibility}(q_a) \cdot d_v \quad (2.20)$$

where  $d_r$  denotes the relative distance between the vehicle ahead and the player,  $d_{vr}$  the restricted visibility distance,  $\alpha_{visibility}$  the degree of restriction,  $q_a$  the index of the driver's aggressiveness with range of 0 to 100%, and  $U_{pos}$  the *positive* utility.

### 2.3.1.2.2. Negative Utility

There are only two choices for the merging-in vehicle: merging-in and staying in the merge lane. There exists reluctance in either choice; even though the reluctance of staying in the merge lane eventually dominates. If the driver has determined to merge in, a vehicle approaching in the adjacent lane threatens the given vehicle's merging-in action. Otherwise, the possibility of failure to merge in increases as the given vehicle approaches the end of the merge lane. In contrast to the lane-change within the mainline, the presence of *negative* utility in maintaining the same lane plays a pivotal role in deciding the solutions of both the *merging* and *acceleration* games. The *negative* utilities for the two strategies ( $U_{neg,L}$  for merging or going left and  $U_{neg,S}$  for staying in the same lane) are respectively defined by

$$U_{neg,L} = d_r - v_r \cdot T(q_a) - D_{suf} \quad (2.21)$$

$$U_{neg,S} = \begin{cases} d_e - v \cdot T(q_a) - D_{suf} , & \text{for the leader} \\ 0 & , \text{otherwise} \end{cases} \quad (2.22)$$

where  $d_r$  and  $v_r$  denote the relative distance and velocity between the given vehicle and the vehicle behind in the adjacent lane, respectively. Also,  $T$  designates the prediction time,  $d_e$  the distance to the end of the merge lane, and  $D_{suf}$  the distance essential to change lanes: e.g. the diagonal length of the vehicle. Time headway of highway drivers [109] is used for the prediction time. The prediction time varies according to driver's aggressiveness. An aggressive driver has the minimum value among the time headway variations of highway drivers and vice versa. In general, aggressive drivers tend to change lanes or merge in even if the headway between the given vehicle and the target is comparatively small and, conversely, the prerequisite to merge in is sufficiently larger for unaccustomed or timid drivers.

#### 2.3.1.2.3. Utility Modification

We run two decisions with the Stackelberg games for the merging-in and acceleration decisions. We devise some utility modifications to characterize each game. First, we need to consider a merge-prohibited area before the merge entrance. We set the area before the merge entrance as the collision area. Thus, the strategy of merging-in will have the least payoff in that area. However, there is no such a restriction for the decision to accelerate/decelerate because the *acceleration game* does not determine whether or not to merge in but simply checks the possibility to merge in from the



perspective of relative position to the new follower. In other words, since the purpose of the *acceleration game* is to attain the position possible to merge in the future, it is unnecessary to consider the current obstacles like the aforementioned area. The predicted positions for the *acceleration game* are calculated with the assumptions of acceleration/deceleration. The acceleration less than 0.1  $g$  does not cause passengers to feel discomfort and can be maximally 0.3  $g$  for normal operation [120], where  $g$  denotes the gravitational acceleration. Thus, the acceleration or deceleration value is designed to vary from 0.1  $g$  to 0.3  $g$  according to the driver's aggressiveness. Every component in the payoff calculation (relative distances, headways, or the distance to the end of the merge) and which vehicle the follower is, are reevaluated based on the accelerated and decelerated positions. Therefore, it is possible that an aggressive driver accelerates to overtake the vehicle in the mainline but a timid driver does not in the same situation.

### 2.3.1.3. Solution of the Merging and Acceleration Games

Since the merging vehicle follows the 2-person Stackelberg game, the solution  $(\gamma^{1*}, \gamma^{2*})$  of the game is obtained by Eq. (5) and (6) with the sum of the above *positive* and *negative* utilities,

$$\gamma^{1*} = \arg \max_{\gamma^1} (\min_{\gamma^2 \in S^2(\gamma^1)} U^1(\gamma^1, \gamma^2)) \quad (2.23)$$

$$S^2(\gamma^1) \triangleq \{\xi \in \Gamma^2 : U^2(\gamma^1, \xi) \geq U^2(\gamma^1, \gamma^2), \forall \gamma^2 \in \Gamma^2\} \quad (2.24)$$

$$U = U_{pos} + U_{neg} \quad (2.25)$$

where  $U^1$  denotes the utility of the leader,  $U^2$  the utility of the follower,  $\gamma^1$  is the possible action of the leader.,  $\gamma^2$  is an optimal action candidate of the follower for given  $\gamma^1$  among  $\xi$ , the possible action of the follower, and  $S^2$  is the set of  $\gamma^2$  values. Based on the set of  $\gamma^2$  values, the leader chooses its action that maximizes its worst utility, which is  $\gamma^{1*}$ , an optimal action of the leader. The solution is the strategy (merging in or staying in the merge lane) that maximizes lower limit of the payoff from the viewpoint of the leader with the consideration of the follower's reacting strategy (going straight or yieding the right of way). In simple terms, the solution indicates the pair that maximizes their payoffs in the worst case in order. However, since the corresponding strategy  $\gamma^{1*}$  is not necessarily unique although the payoff of the optimal solution,  $U^{1*} \triangleq \min_{\gamma^2 \in S^2(\gamma^{1*})} U^1(\gamma^{1*}, \gamma^2)$ , is unique and every two-person finite Stackelberg game admits a strategy for the leader [72], we set up a heuristic priority among the strategies that have the same payoffs to guarantee the uniqueness of the solution: for example, the driver chooses “staying in the lane” if the payoffs of “staying in the lane” and “merging in” are the same. After that, the follower's optimal strategy  $\gamma^{2*} \in S^2(\gamma^{1*})$  is determined in the same manner. As mentioned above, we run two games (*merging* and *acceleration* games) at every instant for the two respective decisions; an instant to merge and acceleration/deceleration. However, although these games are run at the same time, as shown in Figure 2.18, the *acceleration game* is meaningful only when the solution of the *merging game* is to stay in the merge lane regardless of whatever the reason is (i.e. until the vehicle merges in).

## 2.3.2. Simulations

### 2.3.2.1. Unit Test Scenarios

With the driver model for the merging situation, we need to verify 1) if the merging-in vehicle can merge successfully with the designed driver merging model, 2) if acceleration or deceleration produces a beneficial results, and 3) how different the action of the vehicle is according to the given driver aggressiveness. For these purposes, we test two different scenarios where there are a vehicle in the merging lane and five vehicles in the mainline. The merging vehicle has the proposed decision maker and the other five vehicles in the mainline move with constant speed and without changing lanes. Each scenario is prepared to provide the merging-in vehicle with the environment, where the merge is possible when the merging-in vehicle accelerates or decelerates respectively. We impose three different aggressiveness settings (timid, normal, and aggressive) on the merging-in vehicle in each scenario. We suppose that the merging-in vehicle accelerates to a certain speed such that merging to the highway is possible. The merge entrance starts from 50 *m* in longitudinal direction and the length of the entrance is 100 *m*. Lane 1, 2, and 3 and merge lane are set to 0, 3.3, 6.6, and 9.9 *m* in lateral direction as shown in Figure 2.16 and Table 2.4 and 2.5.

Table 2.4. Test Scenario 1

	Initial Conditions		
	$x_0$ (m)	$y_0$ (m)	$v_0$ (km/h)
Vehicle 1 in the mainline	0	10	80
Vehicle 2 in the mainline	3.3	10	80
Vehicle 3 in the mainline	6.6	10	80
Vehicle 4 in the mainline	6.6	5	80
Vehicle 5 in the mainline	6.6	-10	80
Merging Vehicle	9.9	0	70

Table 2.5. Test Scenario 2

	Initial Conditions		
	$x_0$ (m)	$y_0$ (m)	$v_0$ (km/h)
Vehicle 1 in the mainline	0	10	80
Vehicle 2 in the mainline	3.3	10	80
Vehicle 3 in the mainline	6.6	10	80
Vehicle 4 in the mainline	6.6	5	80
Vehicle 5 in the mainline	6.6	-10	80
Merging Vehicle	9.9	0	70

### 2.3.2.2. Unit Test Results

For above two scenarios, the simulation results are shown in Figure 2.19 and 2.20. These figures are intended to show the two respective decisions (a merging instant and an acceleration) of the proposed driver model for the merging operation in consideration of the driver's aggressiveness. Bottom figures show the relative longitudinal distances of the vehicles with respect to the most rear mainline vehicle.

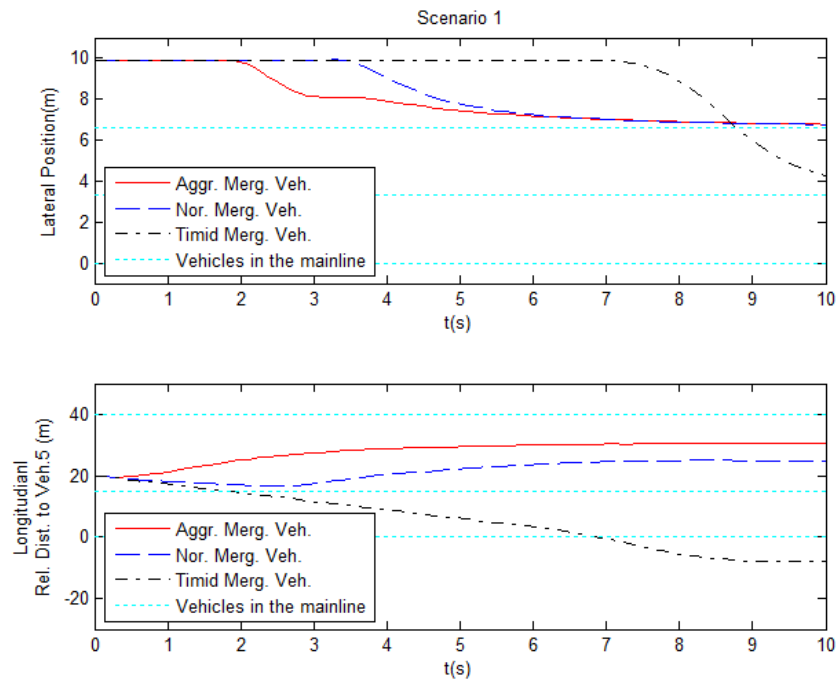


Figure 2.19. Scenario 1 simulation results

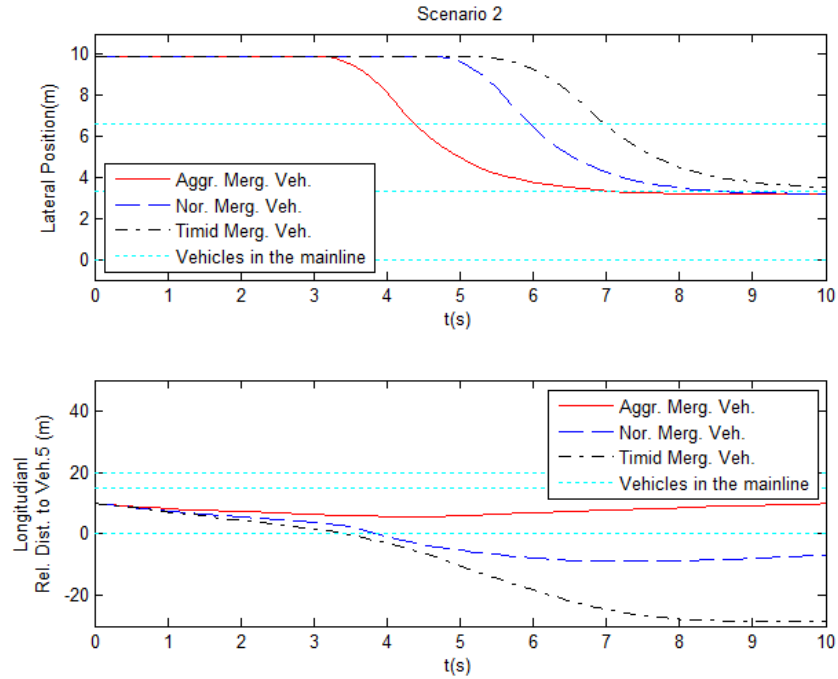


Figure 2.20. Scenario 2 simulation results

The mainline vehicles that are depicted as light blue don't have decision model, which means they have constant speed and lane. Scenario 2 is intended to give the merging-in vehicle smaller cut-in gap, compared to Scenario 1. Also, merging-in vehicle has slightly lower initial velocity when they start to merge-in as in a normal highway, which is the cause that relative longitudinal distances initially decrease, nevertheless aggressive (or normal) vehicle accelerates to cut in. In Scenario 1, although the merging instants are different, the aggressive driver and the normal driver merge in while overtaking the vehicle in the adjacent mainline, whereas the timid driver slows down even if there is a

large front space enough to merge in. This is because the timid driver is assumed to operate within a comparatively small acceleration limits. In the second scenario, overtaking did not happen because of the tight gap in front. However, it is shown that the normal and timid drivers select the larger space rearward. In Scenario 2, an aggressive driver does not accelerate as in Scenario 1. However, the driver cut in into comparatively smaller gap that a normal and a timid driver gave up. The mainline vehicles are mainly located in the lane adjacent to the merging lane and in the downstream. Thus, the vehicles continuously change two lanes in Scenario 2. Note that this is because the driver model follows the highway driving model after merging in the mainline. Except the lane adjacent to the merging lane (i.e. 3rd lane in a highway), there are only leading vehicles as shown in Figure 2.21. Thus, when the vehicle merged in front gap, they stay in the lane. However, when the vehicle slowed down, the vehicle continuously changes lane in a highway. It is intended to show a change from merging decision to highway driving decision in a vehicle by vacating the first and second lanes. Even though the vehicle motion looks continuous, the discrete decisions occur distinctively.

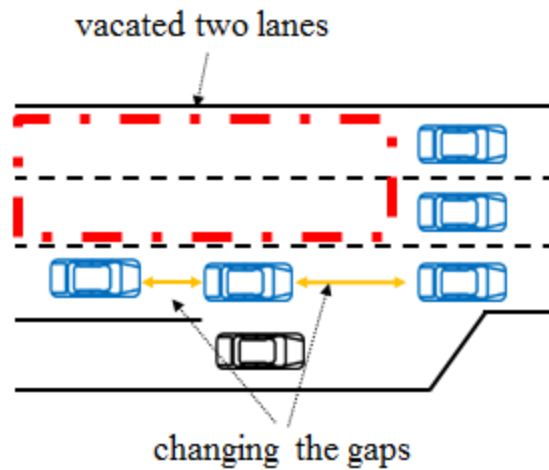


Figure 2.21. Highway section for the simulation

### 2.3.2.3. Interaction between the Driver Merging Model and Highway Driving

#### Model

In this section, we analyze the interaction between the driver merging model and the highway driving model, which can be interpreted as the disturbances that the merge leads to in the mainline. To this end, in addition to the previous simulation to check the effectiveness of the merging model, we place a vehicle with the driver decision model for highway driving in the adjacent mainline. Specifically, since the merging-in vehicle follows the highway driving model after joining the mainline, we devote our consideration to analyze the initial disturbances according to the aggressiveness combinations of both vehicles: the merging-in vehicle and the vehicle in the adjacent mainline.



### 2.3.2.3.1. Disturbances

We define two disturbances from the viewpoint of the mainline in both longitudinal and lateral directions for the vehicle in the adjacent mainline. First, the longitudinal disturbance,  $D_{long}$ , is defined as the integration of the decelerated velocity of the vehicle in the adjacent mainline, which is brought about while the merging-in vehicle is entering the mainline.

$$D_{long} = \frac{1}{N} \sum \int \max(v_0 - v, 0) \cdot dt \quad (2.26)$$

$$D_{lat} = \frac{1}{N} \sum \max(x_0 - x, 0) \quad (2.27)$$

Even though the vehicle slowed down can speed up and follow the front vehicle, the amount of pullback causes rearward vehicles to slow down as well adversely affecting the traffic in the mainline. Next, the lateral disturbance is defined as the laterally moving distance due to the lane-changes of the vehicle in the adjacent mainline. Although it does not make the following vehicles in its original lane slow down, it results in lane-changes or slowing down of the following vehicles in the lateral lane.

### 2.3.2.3.2. Longitudinal and Lateral Disturbances of Mainline

Figure 2.22 and 2.23 show the longitudinal and lateral initial disturbances of the vehicle in the mainline respectively, which are caused by the merge. If the vehicle in the

mainline is timid (0% aggressiveness), the disturbances are minimized in both directions when the merging-in driver has normal aggressiveness and, conversely, maximized when the merging-in driver is at both extremes; timid and aggressive. Thus, the disturbances are determined by the merging-in driver's aggressiveness when the vehicle in the mainline is timid. When the vehicle in the mainline is normal (50% aggressiveness), the lateral disturbance shows the same result of being minimized when the merging-in vehicle is normal, whereas, the longitudinal disturbance stays at similar level regardless of the aggressiveness of the merging-in vehicle. The results of the lateral disturbances imply that timid merging drivers cause the trouble as well as aggressive merging drivers. In addition, when the vehicle in the mainline is aggressive (100% aggressiveness), the longitudinal disturbance on the successive<sup>13</sup> aggressive vehicle is not so big compared with the other aggressiveness settings<sup>14</sup>. Consequently, the mainline is hardly influenced by the aggressiveness of the merging vehicle in longitudinal direction when its aggressiveness is greater than normal while only normal merging vehicle does not affect the mainline in the lateral direction. This is because the higher aggressiveness of the highway driving model works as an intense inducement to have a certain level of headway rather than obstructing the merging driver on purpose, which mitigates the effects of the merge in the longitudinal direction. In sum, both extremes in aggressiveness of the merging-in driver distinctly affect the mainline except the

---

<sup>13</sup> Since the vehicle in the mainline passes by the merging-in vehicle when it is aggressive (100% aggressiveness), we investigate the successive vehicle in the mainline.

<sup>14</sup> This is because the aggressive vehicle tends to maintain a comparatively small headway.

longitudinal disturbances when the aggressiveness of the mainline vehicle is higher than normal.

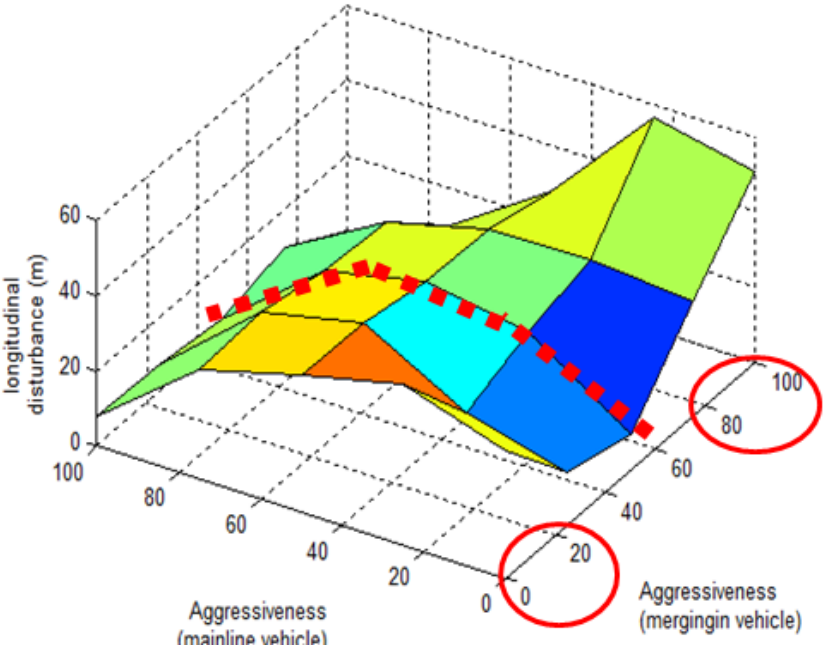


Figure 2.22. The longitudinal disturbance in the mainline caused by the merge

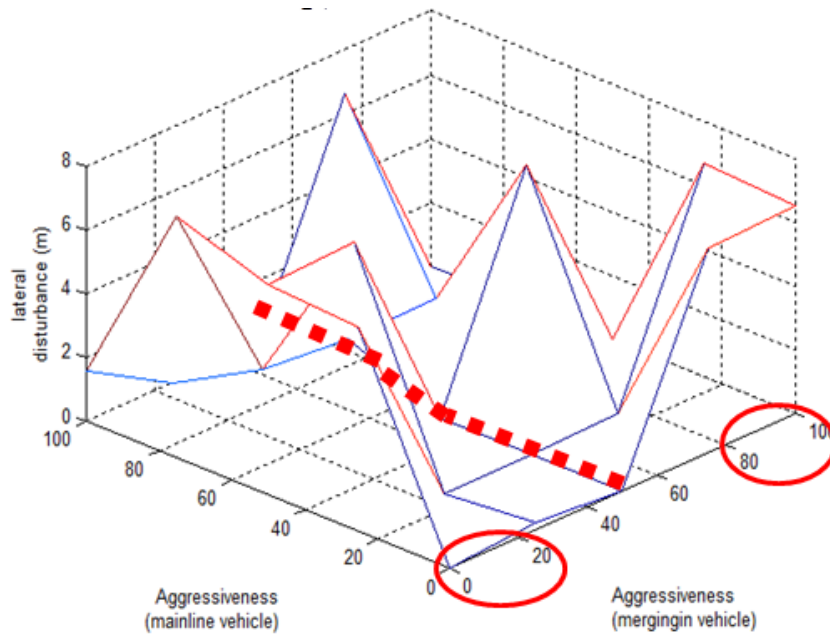


Figure 2.23. The lateral disturbance in the mainline caused by the merge

### 2.3.3. Summary

The driver merging model based on the 2-person Stackelberg game was presented. We designed two different games for the merging vehicle to determine the merging point and acceleration. Payoffs are designed to reflect the driver's aggressiveness, restrictions of the ramp, and the driver's prediction for the acceleration/deceleration. The merging vehicle with the proposed model showed the successful merges with reasonable overtaking and slowing down according to the given aggressiveness settings in the two scenarios. We also combined the proposed merging model with the highway driving model to further investigate the effects of the merge to

the mainline. It was shown that both extremes in aggressiveness of the merging-in driver distinctly affect the mainline except the longitudinal disturbances when the mainline is normal or aggressive. This is because the vehicle in the mainline was designed to maximize his payoff according to his aggressiveness level instead of minimizing the counterpart's payoff. The driver model would also be utilized for the purpose of developing a driving safety controller against the various drivers including such willful malicious driving.

### 3. DRIVER DRIVING MODEL

The focus of the previous section was development of decision making that has a wide range of rational levels so that the driver model can reproduce human driver behaviors that may be somewhat illogical. Game theory was utilized to resolve conflicts among a driver's discrete multiple strategies, such as lane selection or whether to merge, while they progress in continuous vehicle dynamics. It provided viable methodology to produce reasonable microscopic results and enable an extension of the model to a qualitative traffic study. Meanwhile, one of the challenges for autonomous driving, presented in Campbell's report [21], is to predict the actions of the other car as human drivers do. As human drivers can distinguish dangerous drivers from reasonable drivers and use the information in the decision making process, autonomous vehicles must be able to cope with even non-collaborative driving from neighboring cars. In sum, it is necessary to develop a framework for modeling driver behaviors in view of human prediction ability and pursuit of safety based on the prediction when we consider an upcoming mixed situation of autonomous vehicles and human drivers because of increasingly improving autonomous driving technologies. It is also stated that the individual driver's perception of driving safety can be interpreted through objective risk, subjective risk estimate, and feeling of risk as shown in Figure 3.1 [61]. All of this suggest that it is necessary to build a stratified collision risk model so as to encompass diversity from aggressive drivers (assumed to be illogical) to autonomous vehicles (pursuing maximum safety).

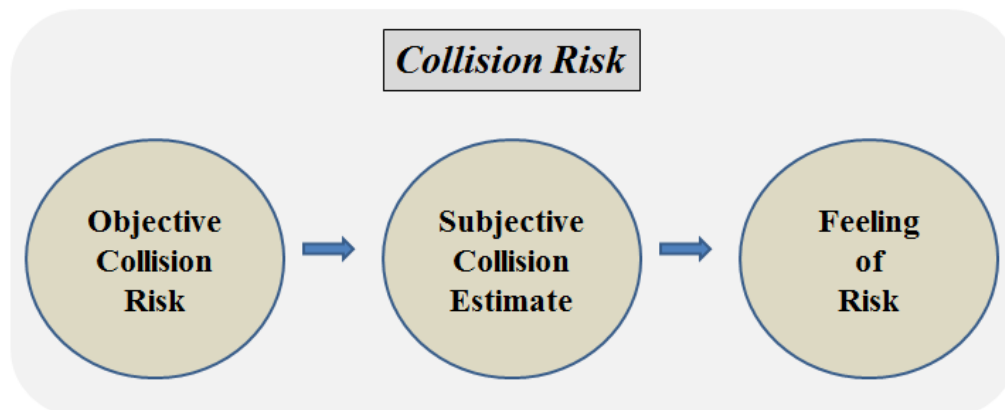


Figure 3.1. Collision risks in cognitive process

With this, we intend to model the upcoming situation where there exist autonomous vehicles and manually operated vehicles on the road, and this model can be used to better understand and analyze intelligent transportation systems.

### 3.1. Model Configuration

The driving model mainly consists of three procedures: the prediction of the adjacent collision, subjective perception of the predictive collision probability, and a driver's manipulation to avoid the subjectively anticipated collision. We have looked into what various drivers who have different aggressiveness are likely to do. This includes irrational decision making (i.e. non-collaborative driving) as well as reasonably understandable decisions. In this section, the model is extended to have a predictive characteristic for the other vehicles and corresponding control so that it can guarantee

the objective driving safety of the subject vehicle. This would be the first step of the stratified collision risk estimation in the driver driving model. Since the objective risk represents the objective probability for the collision, it will be designed to have probabilistic information such that the vehicle can be involved in a collision with the other vehicle during a certain prediction horizon. Toward this end, a reachable set computation based on the hybrid dynamical system will be utilized and, in turn, the reachable set of the collision area will be associated with the optimal probability distribution attributed to the mixed strategy Nash game. In particular, the focus needs to be on human predictive behavior designs with no guarantee of other drivers' sensible driving. It will enrich understanding the role of human drivers and interactions among drivers. Subsequently, the subjective collision estimate is proposed, based on the objective collision prediction and the driver's aggressiveness that stands for the driver's awareness and judgment for the objective collision probability. Next, a Model Predictive Control (MPC) is designed to avoid the predicted collision for the prediction time horizon, founded on the subjective collision estimate that varies for every individual driver who has different aggressiveness. The MPC is a suitable control scheme to design optimal behaviors locally in time when we need to consider a wide variety of constraints, such as unsafe region to avoid. We call the controller Subjectively Predictive Safety Controller (SPSC) in the following discussion.



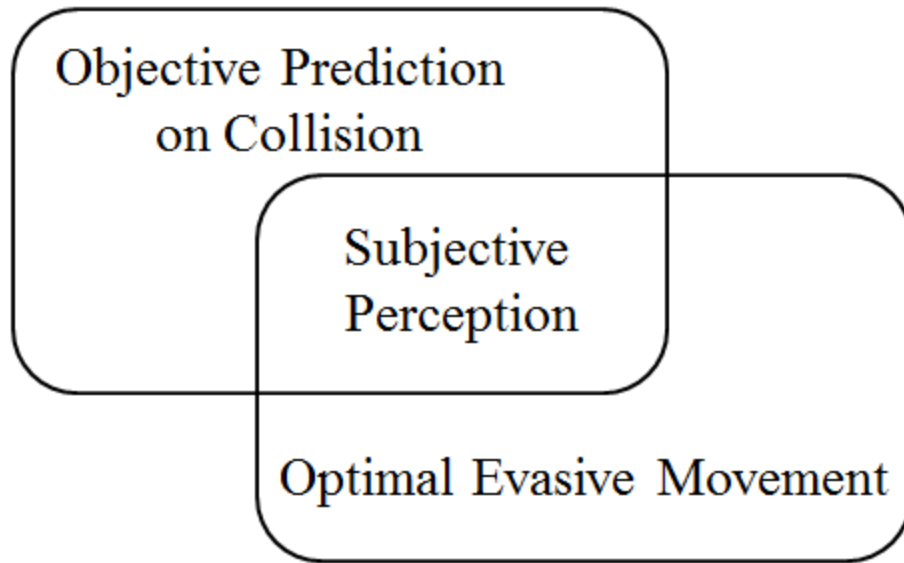


Figure 3.2. Concept behind the driver driving model

Note that a series of procedures in the driver driving model shown in Figure 3.2 also exhibit the proposition stated earlier based on theories of the irrational behaviors [35-38], which is that an irrational driving behavior can be modeled by a logical way together with unreasonable perceptions. The final driving behavior is determined by an optimal control method, MPC, on the horizon. However, the collision estimate can be illogical with respect to the driver's aggressiveness. In other words, if the drivers' perceptions on the surroundings or situation are not ideal, it can induce undesirable traffic situation although they are expected to try their best to avoid collisions. Note that if one places an autonomous vehicle on a road, the model would behave reasonably and

pursue maximum driving safety based on the objective collision prediction as an autonomous vehicle.

### **3.2. Collision Risk Estimation**

As we can easily recognize, two interacting vehicles have uncertainties relating to the counterpart's merging (or lane-changing) instant and lane change maneuver, respectively if we cannot say that all the vehicle are autonomous vehicles, sense all of physical information, and share their decisions. Thus, I intend to model these uncertainties in relation to the driver's aggressiveness as an index to represent the driver's rationality degree, and derive a human prediction characteristic with the use of mixed strategy Nash game and the reachable set computation.

#### *3.2.1. Multi-Agent Hybrid System*

The model that I propose subsumes a multi-agent hybrid system that results from discrete decisions and continuous dynamics of multiple drivers. To illustrate the basic system configuration that I propose to develop, I suppose that we have only two agents with two strategies as shown in Figure 3.3. Every agent makes a decision based on the driver decision model with the use of the information from other players. The decision is realized by the continuous dynamics that evolve according to the agent's chosen discrete strategies.

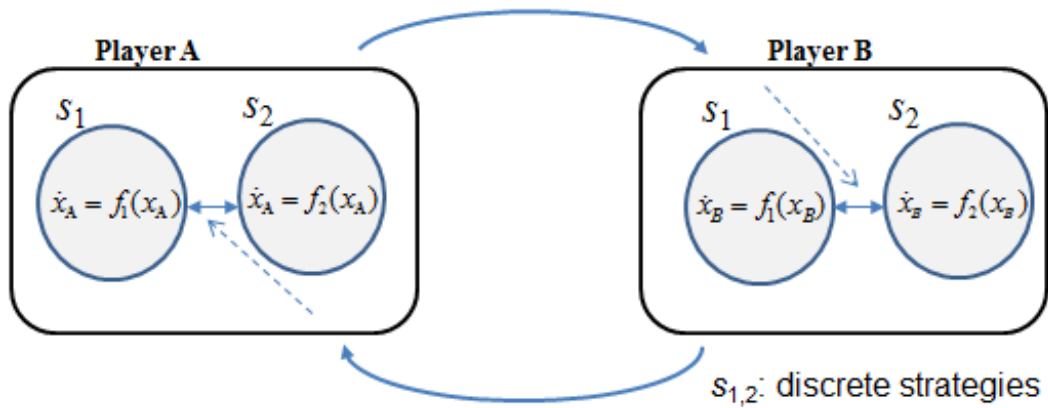


Figure 3.3. Multi-agent hybrid system

To illustrate the basic system configuration in practice, I take an example of merging operations. When we assume a situation where two vehicles compete to merge in from a merging lane and out from a mainline, respectively, we can construct the combinations of the discrete strategies as shown in Figure 3.4. Note that the same conflict may occur between two vehicles that change their lanes to each other's lane in normal highway driving.

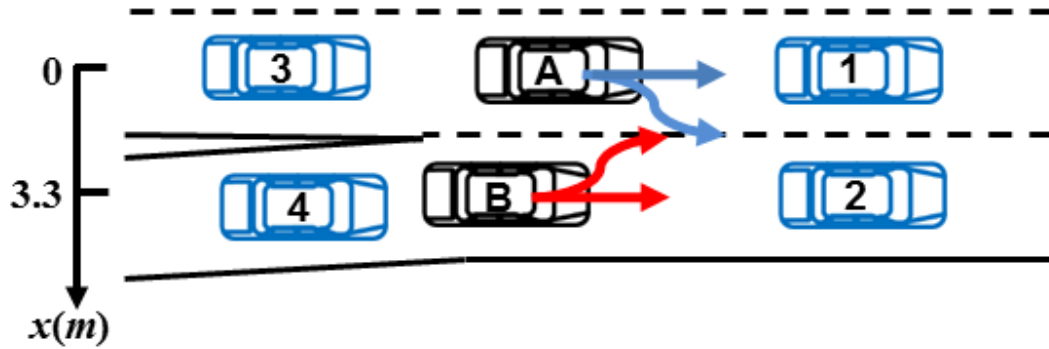


Figure 3.4. An example of conflict situations

Moreover, the discrete strategies and continuous movements of the vehicle will not be considered separately in this work. Instead, we regard the interaction between the two players as one system. Since each vehicle has two discrete strategies, we can say that the whole system has 4 different combinational modes as shown in Figure 3.5: going straight/going straight, going straight/merging in, merging out/going straight, and merging in/merging out for the players. By using the continuous differential equations in the next section, we can define the relative dynamics in the local coordinate attached to the vehicle of interest.

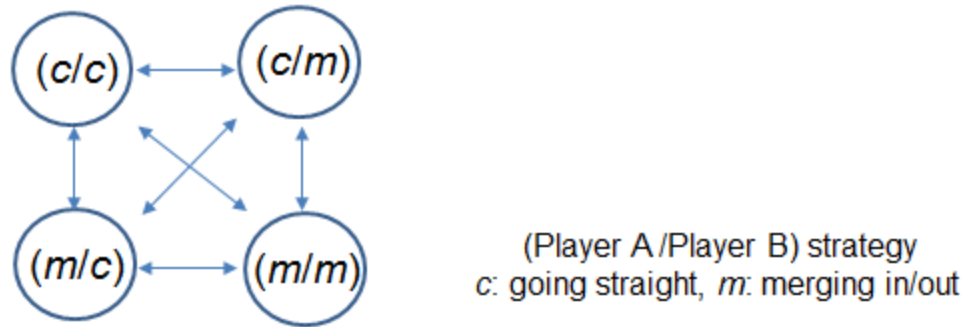


Figure 3.5. Combination of the strategies of two agents in merging situation

### 3.2.2. Hybrid Model for Lane-Changes (HMLC)

The driver driving model is aimed to model both human-like imperfection and robot-like perfection by adjusting the uncertainty of a driver's rationality. Thus, the differential equations for a lane change trajectory must be designed in consideration of the driver's rationality, i.e. aggressiveness. We design a differential equation to represent different kinematic trajectories of lane-changes with respect to the driver's aggressiveness. However, after careful study of literature, I found that there is no work that deals with the kinematic differential equation of a lane-change, whereas a number of papers provide the representation of the vehicle motions based on vehicle dynamics and steering controllers [85, 121, 122] as well as static trajectory equations [86, 123, 124].

## Hybrid Model for Lane Change

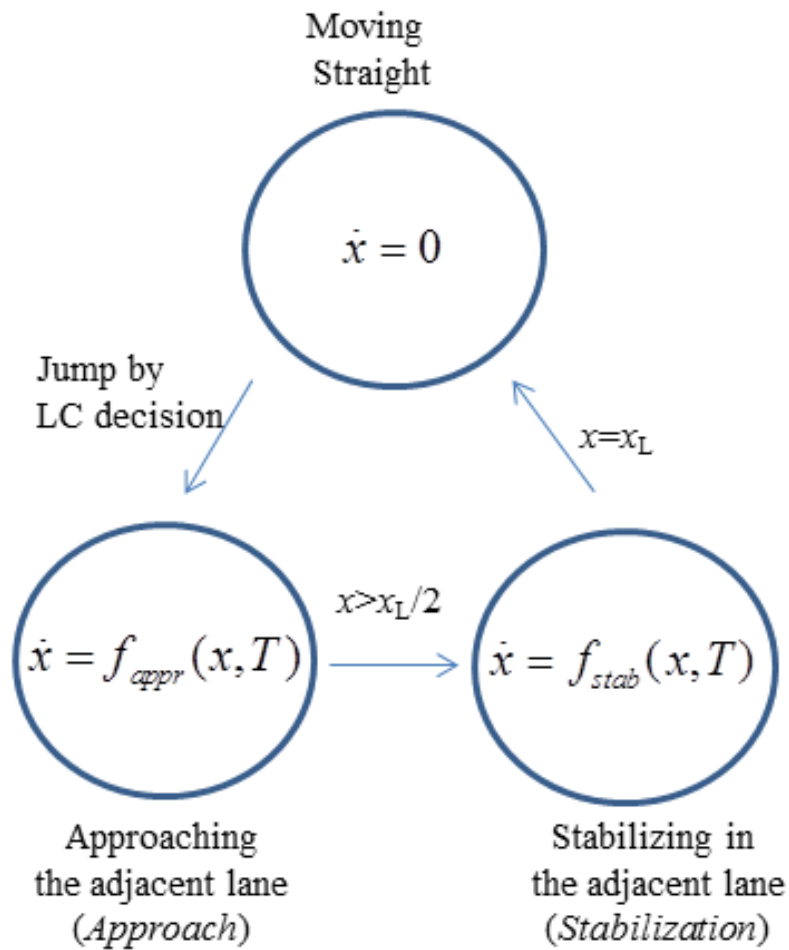


Figure 3.6. Hybrid model for lane-changes (HMLC)

Thus, I propose a hybrid model of the vehicle lateral position for the lane-change trajectory to similarly reproduce the time history of the lateral position observed from the human steering pattern [125, 126] in a lane change maneuver. Van Winsum et al.

investigated the sequential phases of a lane-change maneuver from the viewpoint of steering wheel angle [127]. During the first phase, the steering wheel is turned to its maximum<sup>15</sup>. The second phase is from the maximum angle to zero. The vehicle obtains maximum heading angle at the end of the second phase. Finally, the steering wheel is turned to the opposite maximum to stabilize the vehicle in the adjacent lane. With this in mind, we divide a lane-change maneuver into two discrete modes, which are the *approach* mode and the *stabilization* mode. The *approach* mode is the conjunction of the first phase and second phase in the Van Winsum's work [127] such that the subject vehicle moves into the adjacent lane. The *stabilization* mode is nothing but a restatement in the opposite direction so that vehicle tracks the adjacent lane. However, they are modeled in the time domain from the perspective of a direct kinematic representation rather than steering wheel angle. The hybrid model and the corresponding continuous lateral dynamics are shown in Figure 3.6 and (3.1) and (3.2).

$$f_{appr} = \frac{K}{T}(x + a) \quad (3.1)$$

$$f_{stab} = -\frac{K}{T}(x - b) \quad (3.2)$$

where  $T$  denotes the period of trajectory,  $x_L$  lane width.  $K$ ,  $a$ , and  $b$  are model parameters: 7.0528, 0.05, and 3.35 are respectively used for the parameters in this work.

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<sup>15</sup> This does not denote a mechanical maximum angle but the maximum in the time history of the steering angle.

Every driver has a different estimated time of lane change completion with respect to his aggressiveness. Since  $T$  determines severity of rapid lane-change, we utilize the period of trajectory  $T$  to reflect a driver's aggressiveness. An aggressive driver will have a shorter completion time and a rapid change of the motion. For example, the lateral displacement of a lane change that is finished within a range of 1 s to 4 s is shown in Figure 3.7. As the period is small, the lane change trajectory shows a rapid change of the lateral displacement. For the sake of simplicity, we use the same  $T$  in the two discrete states; *approach* and *stabilization*.

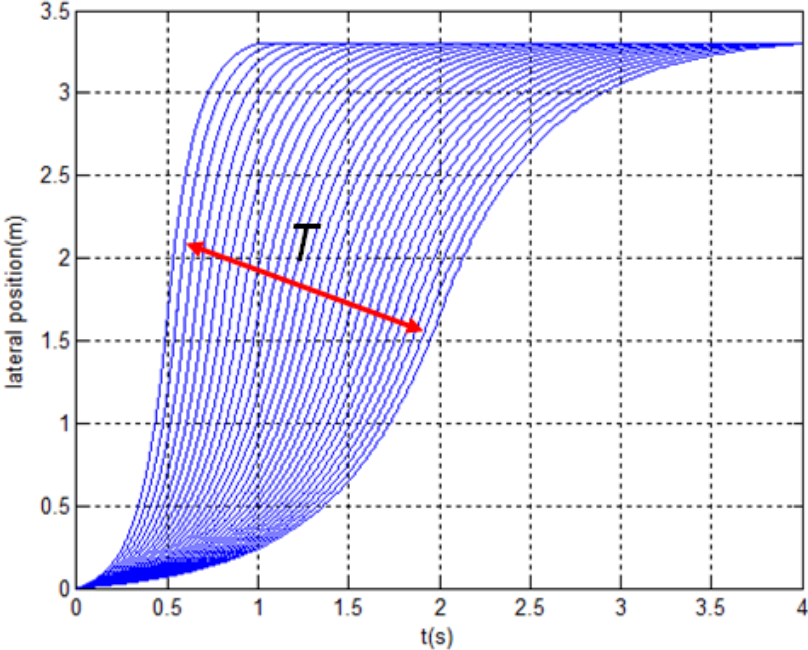


Figure 3.7. Lateral displacement of a lane change



### 3.2.3. Discrete Reachable Set

The goal of this section is to develop a predictive framework for human driver's behaviors and a controller to achieve a certain level of safety based on the prediction. Hence, to determine the level of safety, I define an unsafe set (or collision area) as shown in Figure 3.8 when the two vehicles A and B compete to merge in and out or change lanes in opposite direction each other, from the perspective of hybrid systems. For the sake of the simplicity of the problem, we do not deal with the effects of vehicle rotations in this part.

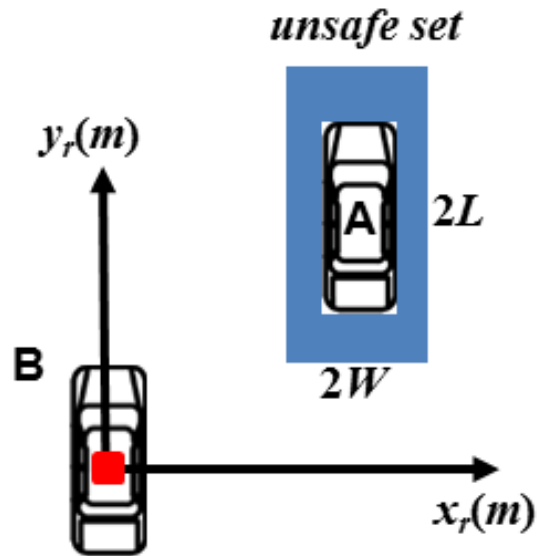


Figure 3.8. Unsafe set in the relative coordinate

Thus, the unsafe set (i.e. the collision region of both vehicles) is defined as a rectangle with  $2W$  and  $2L$  in the coordinates of the both vehicles' relative positions with an assumption that the physical sizes of the vehicles are identical.  $W$  and  $L$  are the width and the length of the vehicle, respectively. The coordinate is assumed to be attached to the vehicle of interest (i.e. the subject vehicle). Note that it is also possible to set up a multilayered unsafe set with several collision severities. If we leave aside the concerns on the combinational modes<sup>16</sup> for a moment, there is no doubting the fact that the unsafe set will propagate along the HMLC as time goes on given the strategies of the two vehicles.

$$\dot{x}_r = f_{\text{mode},A}(x_A, T_A) - f_{\text{mode},B}(x_A - x_r, T_B) \quad (3.3)$$

$$\dot{y}_r = v_A - v_B \quad (3.4)$$

where  $x_r = x_A - x_B$  and  $y_r = y_A - y_B$ . For example, if we specify the discrete modes of Vehicle A and B to be *stabilization* and *approach*, respectively, the unsafe set flows along the corresponding relative dynamics,

$$\dot{x}_r = f_{\text{stab}}(x_A, T_A) - f_{\text{appr}}(x_A - x_r, T_B) \quad (3.5)$$

---

<sup>16</sup> The combinational mode of the vehicles is assumed to be not identified because we have uncertainties on the behaviors that occur in the future.

where  $f_{stab}$  and  $f_{appr}$  designate the continuous dynamics of each state of HMLC. Note that the discrete state may jump to another state during the propagation and the corresponding continuous dynamics may also change. With these, we cope with the uncertainty of the counterpart's aggressiveness. As stated earlier, the estimated time of the lane-change completion is different among drivers and a shorter completion time results in a rapid change of the lateral displacement, which must threaten other vehicles. Thus, we model the period  $T$  as a function of a driver's unknown aggressiveness. This provides uncertainty on the collision prediction. In addition, separate from choosing the discrete strategy, the subject driver can accelerate or decelerate to cope with the anticipated collision according to the situations in continuous time domain. Therefore, we regard the counterpart's indirect aggressiveness factor  $T_A$  and the subject's velocity  $v_B$  as a disturbance  $d$  and an input  $u$  for the subject vehicle in the multi-agent hybrid system shown in Figure 3.8. Vehicle B is the subject vehicle against the counterpart Vehicle A in Figure 3.8. It is assumed that the subject driver's aggressiveness  $T_B$  and the counterpart's velocity  $v_A$  do not change during the propagation of the unsafe set, which does not lose much of generality since a driver predicts the future based on the current physical properties of the vehicles and mental status of oneself. What the driver does not know is the rationality degree of the counterpart. Even though the counterpart turns on a turn signal, it is not guaranteed exactly when the vehicle will change lanes or how aggressive the motion of the lane-change will be. Accordingly, we have the relative dynamics of the vehicles in the multi-agent hybrid system as follows,

$$\dot{x}_r = f_{stab}(x_A, d) - f_{appr}(x_A - x_r, T_B) \quad (3.6)$$

$$\dot{y}_r = v_A - u \quad (3.7)$$

for the exemplar case. To compute a reachable set of this sort of dynamics, several methods have been presented. The representative method is to solve Hamilton-Jacobi-Isaacs (HJI) equation, based on pursuit-evasion game [83]. Mitchell provided a toolbox [128] that can be used to solve HJI equations by using level set methods [129]. Dang proposed an approximation algorithm for the reachable sets of polynomial systems [130]. Althoff et al. presented reachability analysis for linear systems and nonlinear systems with uncertain parameters [131, 132]. Here, we recall the purpose that the model serves, which is to develop a predictive framework that driver's perception will be based on. We need to notice that it is not the goal to accomplish absolute safety by computing and escaping a backward reachable set of an explicit final unsafe set. Thus, we calculate the forward reachable set along the hybrid dynamics of the system so that the model can foresee the time evolution of the unsafe set for the finite prediction horizon, which can be called a prediction literally. Since the continuous dynamics used in each mode of the HMLC are linear, the relative dynamics of the system shown in Figure 3.8 also become linear, which is

$$\dot{z} = A(d, u)z + B(d, u) \quad (3.8)$$

$$\dot{z} = \begin{bmatrix} x_r \\ y_r \end{bmatrix} \quad (3.9)$$

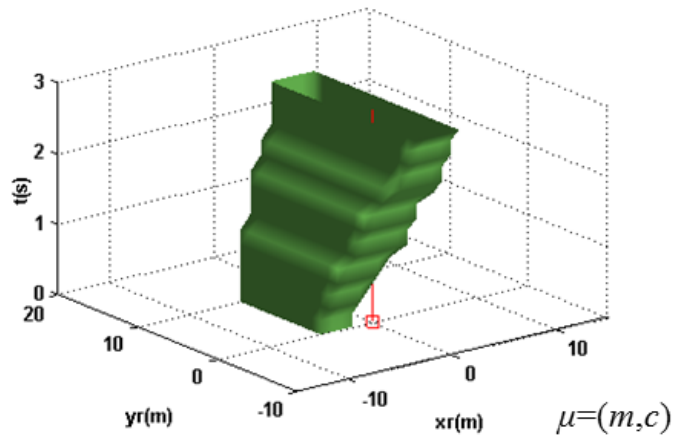
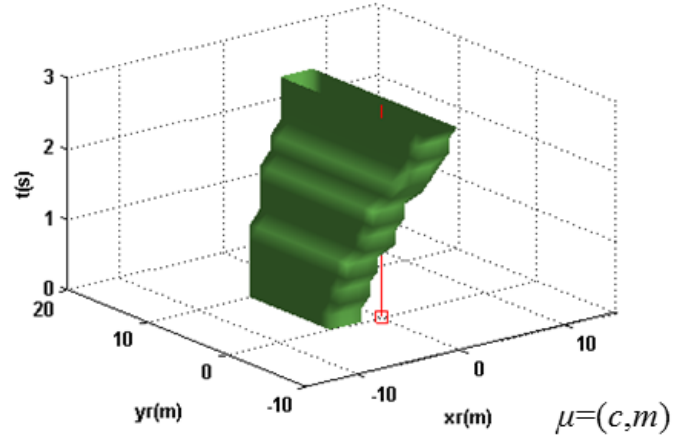
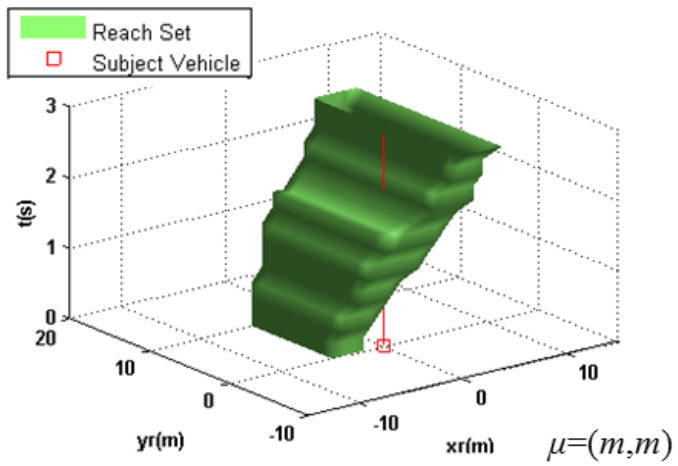
where, according to the discrete states of both vehicles, the matrices  $A$  and  $B$  of the linear system  $z$  are obtained by equation (3.1) to (3.7). Thus, given the dynamics of the system (3.8), the exact reachable set  $R(\mu, u, d, t)$  of the system at time horizon  $h$  is defined by

$$R(\mu, u, d, t) = \left\{ z(u, d, t) \mid z(u, d, t) = z_0 + \int_0^t f_\mu(z(\tau), u(\tau), d(\tau)) d\tau \right. \quad (3.10) \\ \left. , u(\tau) \in U, d(\tau) \in I \right\}$$

where  $U$  and  $D$  are bounded convex sets for input  $u$  and disturbance  $d$ , and  $\mu$  is a combinational mode defined in Figure 3.5. Note that the system dynamics  $f_\mu$  is linear although it switches with respect to the combinational mode of the system, which enables us to compute the exact reachable set. Next, since all reachable sets do not intersect with the subject vehicle<sup>17</sup>, we need a backward checking step to figure out inputs such that the reachable set of the unsafe set meets the coordinate origin regardless for time and disturbance.

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<sup>17</sup> The origin of the coordinate designates the location of the subject vehicle because we use a relative coordinate that is attached to the subject vehicle.



I.C.:  $x_A=0m$ ;  $x_I=-3.3m$ ;  $y_I=5m$   
 $v_A=80kph$

Figure 3.9. Discrete forward reachable sets of unsafe set in the relative coordinate

The set of inputs that resulted in the collision,  $U_{unsafe}$ , is defined as

$$U_{unsafe} = \{u \mid \exists d \in D \text{ and } t \geq 0 \text{ such that } (0, 0)_r \in \text{Reach}(\mu, u, d, t)\} \quad (3.11)$$

Consequently, the final unsafe forward reachable set with respect to the combinational mode,  $\text{Reach}(\mu, t)$  is defined as

$$\text{Reach}(\mu, t) \triangleq \bigcup_{u \in U_{unsafe}, d \in D} R(\mu, u, d, t) \quad (3.12)$$

and shown in the following Figure 3.9.

#### 3.2.4. Objective Collision Prediction

Let us consider the concerns on the discrete mode that we put aside in the previous section. For this, we utilize game theory again to estimate the counterpart's strategy in response to the subject's strategy and corresponding combinational mode of the system. However, the sharing point with the driver decision model is not the game structure but the utility designs to represent human driver intentions. It is assumed that the priority between the game participants does not exist for a conservative judgment. That is to say, the subject vehicle cannot assert its priority on information against the preceding counterpart. Thus, we design a noncooperative mixed strategy Nash game [64]

with the utility functions designed in the driver decision model<sup>18</sup>. Although the utilities are modified in consideration of physical limitations, they are briefly an extension of the following two intentions: a desire to have a longer headway and a reluctance to change lanes due to the vehicle in the proximity of the subject vehicle. In the noncooperative mixed strategy Nash game, the saddle point  $(p^*, q^*)$  to a bimatrix game  $(U_A, U_B)$  is determined by the following inequalities for all  $p \in P$  and  $q \in Q$  [73, 133]:

$$p^{*T}U_Aq^* \geq p^T U_A q^* \quad (3.13)$$

$$p^{*T}U_Bq^* \geq p^{*T}U_Bq \quad (3.14)$$

$p$  and  $q$  are respective probability distributions on the pure strategy spaces of both players A and B, which satisfies

$$P = \{p \in \mathbb{R}^m : \sum_{i=1}^m p_i = 1, p_i \geq 0\} \quad (3.15)$$

$$Q = \{q \in \mathbb{R}^n : \sum_{i=1}^n q_i = 1, q_i \geq 0\} \quad (3.16)$$

This implies that the mixed strategy saddle point is based on the deduction of every participant, which is that every participant can expect that the counterpart will not want

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<sup>18</sup> The driver model utilizes two different games. The Stackelberg game is used for the counterpart whom the subject can assert his/her information priority to. Meanwhile, Nash game is used when the subject do not have information priority against the counterpart: for instance, in the case that the counterpart place ahead of the subject driver.



the situation where the subject has the possibility to increase the subject's outcome. The counterpart will choose his strategies with the probability distributions such that the subject's outcomes are equal regardless of his strategies. Thus, the partial derivation of the counterpart's outcome with respect to his probability distribution leads to the subject's solution [73] in our two strategy game; two strategies are *going straight* and *changing a lane*.

$$\frac{\partial \sum_i^2 \sum_j^2 p_i q_j u_{ij,B}}{\partial q_j} = 0 \rightarrow p_i \quad (3.17)$$

$$\frac{\partial \sum_i^2 \sum_j^2 p_i q_j u_{ij,A}}{\partial p_j} = 0 \rightarrow q_j \quad (3.18)$$

It has been proved, in the mixed strategy Nash game, that an optimal solution exists as a set of probability distributions on the pure strategy space [64], where the pure strategy space denotes the discrete modes of the vehicle; *going straight* and *changing a lane* (or *merging*). Thus, we have an optimal probability distribution on the 4 combinational modes from the pure strategies (discrete state of each vehicle). We say that the final reachable set of the unsafe set is a probabilistic combination of the discrete unsafe reachable sets with respect to the optimal probability distribution. Therefore, the probabilistic collision prediction on  $\mathbb{R}^2 \times \Omega$ ,  $\text{Reach}(t)$ , is defined as

$$\text{Reach}(t) = \sum_{i=1}^2 \sum_{j=1}^2 p_i q_j \text{Reach}_{ij}(\mu_{ij}, t) \quad (3.19)$$

where  $\Omega$  is a bounded convex set for collision probability,  $\omega$ ; i.e.  $[0, 1]$ . For instance, let us assume the probabilities of Vehicle A and B for *moving straight*,  $p_1$  and  $q_1$ <sup>19</sup>, are 0.3. Then, the combinations of modes (*m/m*, *m/c*, *c/m*, and *c/c*) will have the probabilities of 0.49, 0.21, 0.21, and 0.09, respectively. Since we already have presented the discrete reachable sets of the collision area with respect to the combinational modes of the vehicles in the preceding section, we obtain the probabilistic reachable set,  $\text{Reach}(t)$  as shown in Figure 3.10.  $\text{Reach}(t)$  at time,  $t = 0, 1, 2$ , and 3 sec are shown. Collision probability,  $\omega$  is set as 1 (i.e. collision occurs.) within the unsafe set that we defined earlier at  $t = 0$  sec. It is shown that, along the time evolution, the region with the highest collision probability moves through the subject vehicle, and unsafe region spreads due to the disparate modes that are possible in the future although the collision probability decreases.

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<sup>19</sup> We define that  $p_1$  and  $q_1$  are the probabilities of Vehicle A and B for *moving straight* and  $p_2$  and  $q_2$  are the probabilities of Vehicle A and B for *changing lanes*.

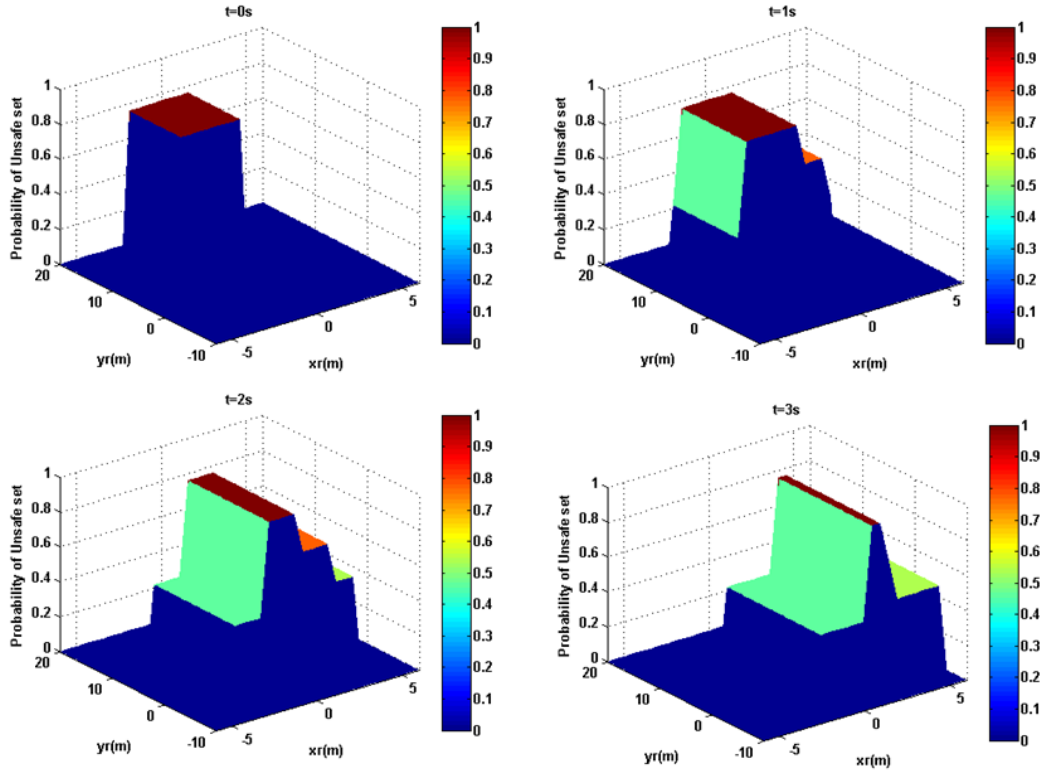


Figure 3.10. Collision probability in the relative coordinate at time,  $t = 0, 1, 2,$  and  $3\text{sec}$

Note that “*Objective risk may be defined as the objective probability of being involved in an accident*” [61] from the psychological point of view. Thus, we define an objective collision prediction,  $C_o$ , as  $\text{Reach}(t)$  on the given time horizon  $[0, h]$ , where  $h$  refers to a driver’s prediction time horizon.

$$C_o \triangleq \{(x_r, y_r, \omega, t) \mid \text{Reach}(t) \text{ for } t \in [0, h]\} \quad (3.20)$$

We hereby have constructed an objective prediction model on adjacent collisions by analytically calculating the evolutions of collision area in the relative coordinate and combining them according to the game theoretic probability distribution.

### 3.2.5. Subjective Collision Estimate

Subjective collision estimate refers to a driver's own cognitive process on the objective probability of collision [61]. Thus, we propose a driver's safety assurance level to judge whether the given collision probability can intimidate the driver. The *safety assurance level*,  $s_a$ , is defined to be the complement of the aggressiveness,  $q_a$ , that we used in the driver's decision model. It may be a straightforward conversion of the indicator that represents a driver's aggressiveness or inattentiveness in opposite direction.

$$s_a = 1 - q_a \quad (3.21)$$

Subsequently, the subjective estimate of the collision risks is technically translated to the region that has less *safety assurance level* than the individual driver's expectation. Thus, we say that a subjective perception of the adjacent collision,  $C_s$ , is defined as

$$C_s \triangleq \{(x_r, y_r, t) \mid \text{Reach}(t) \text{ such that } 1 - \omega \leq s_a^e, \text{ for } t \in [0, h]\} \quad (3.22)$$

where  $s_a^e \in [0,1]$  is a driver's safety expectation and  $\omega$  refers to the collision probability of the objective collision prediction. Note that the subjective perception,  $C_S$ , excludes the probabilistic information conversely. This implies that the subjective perception model acts as a deterministic model for an individual driver. The following figures show subjective perception of the collision with different levels of *safety assurance*. It is shown that higher the *safety assurance level* is, larger the region that the driver should avoid<sup>20</sup> is as shown in Figure 3.11a and Figure 3.11b.

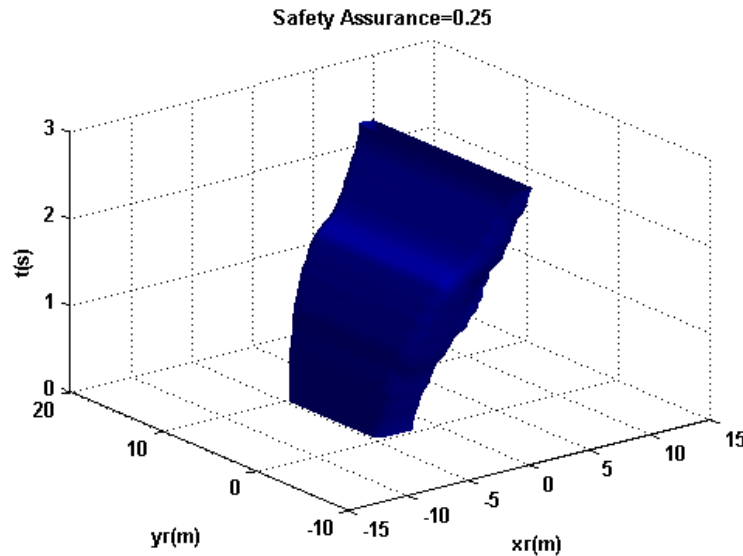


Figure 3.11a. Subjective collision estimate with safety assurance level of 0.25

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<sup>20</sup> Since the perception model excluded the probabilistic information, the region from the subjective perception model is regarded as to have the collision probability of 1. Thus, as we can easily expect, an individual driver will try to remain outside of the unsafe region that is anticipated, which will be considered in the section that deals with driving control.

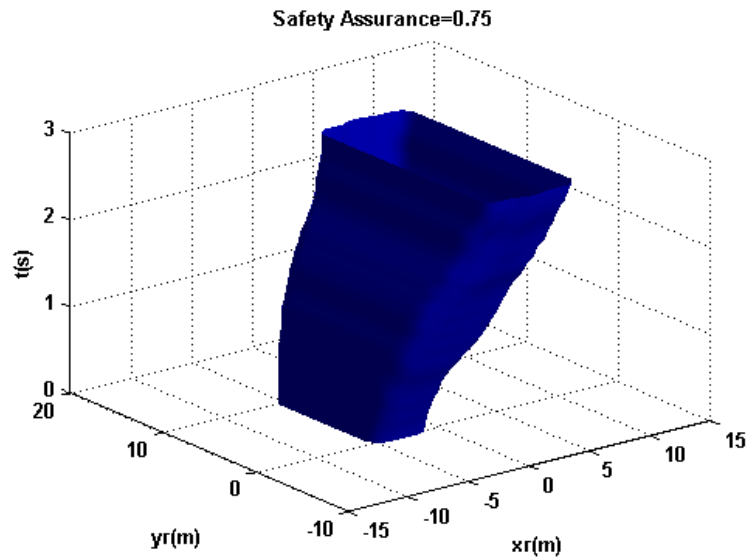


Figure 3.11b. Subjective collision estimate with safety assurance level of 0.75

### 3.2.6. Summary

In sum, the collision risk estimation model was presented for individual drivers who have different aggressiveness. Specifically, the collision estimation has been designed to have a stratified structure based on the psychological argument that an interpretation of the collision risk should be identified differently between objective prospects and subjective recognitions. First, the objective prediction of the collision was developed in a framework of the multi-agent hybrid system. The objective prediction of the collision computed the forward reachable set within the driver's prediction horizon along the proposing hybrid model for lane-changes. The hybrid model for lane-changes was designed to express the kinematic trajectory of the vehicle with differential

equations that vary according to the lane-change phases. Moreover, my focus is on the *traffic unsafety* based on human dispositions (The human dispositions may be irrational). Thus, the hybrid model for lane-changes was designed to have various driving trajectories with respect to the drivers' disposition indicators (i.e. aggressiveness). The corresponding reachable set computation was considered in a situation where we cannot guarantee the other drivers' sensible driving. In particular, the probabilistic property inherent in the objective collision prediction could be attained by means of the mixed strategy Nash game. The mixed strategy Nash game provided an optimal probability distribution among the discrete states that are defined within the multi-agent hybrid system. In the sequel, we have investigated a subjective risk estimate that reflects a driver's own cognitive process. Since drivers have different safety requirements, the subjective estimate of the collision risk was designed as a region that has less safety than the driver's own safety requirement in the objective probabilistic collision prediction. The subjective estimate of the collision risk is regarded as a deterministic unsafe region for the driver himself (or herself). That is to say, the subjective perception acts as a collision area with the collision probability of 1 such that the driver should avoid while driving.

### **3.3. Driving Control**

The collision risk model has various collision probabilities from the objective viewpoint and subject estimate differ among drivers. However, the subject risk estimate is assumed to be a critical risk for the driver directly involved. Thus, an individual driver

will try to remain outside of the critical risk (i.e. collision) that is anticipated<sup>21</sup>. To reflect this aspect, I design a controller that drives the vehicle to stay outside the subjectively anticipated collision. I name the controller as Subjectively Predictive Safety Controller (SPSC). The idea behind this work to appreciate irrationality that occurs in real world driving is that the irrational driving behavior can be modeled by a rational (optimal) method together with illogical perceptions. Thus, SPSC needs to be designed to have optimality locally within the driver's prediction horizon although the target to track may not be objectively safe. For the optimal control that guarantees the safety of the vehicle, we design a control scheme based on Model Predictive Control (MPC). This is because MPC enables us to take the constraints into account in time domain, as stated in Section 1. The constraints include the safety conditions, such as the unsafe region that the vehicle should stay outside, as well as the physical constraints, such as acceleration limit or speed limit. Moreover, local optimality in time domain is one of the distinctive characteristics that MPC provides. The prediction time horizon of the drivers is not infinite. The MPC designed would work as a driver's driving control in order to reduce the provisional collision risk that varies according to the driver's *safety assurance level*. In addition, the model would behave to pursue maximum driving safety based on the objective collision prediction for autonomous vehicles<sup>22</sup>.

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<sup>21</sup> However, if the unsafety that results from surroundings or situation is underestimated, it can bring about undesirable outcome although the vehicle is optimally controlled by the driver.

<sup>22</sup> When we say that the autonomous vehicle needs the highest *safety assurance level*, the subjective collision estimate is the same as the objective collision prediction.



### 3.3.1. Control Objective

First, the main objective that the SPSC must achieve is to let the vehicle stay outside the unsafe region. For this, we need to identify the cross section of the subjective perception region in  $\mathbb{R}^3$ . This is because we design the driver driving model so that the driver is allowed to control its longitudinal velocity to avoid the collision. The lateral motion of the subject vehicle is determined by the HMLC based on the driver's the lane-change completion time, which is assumed to be known and constant<sup>23</sup>. Note that, in the case of the counterpart, we considered the evolution of the unsafe set along all possible lane-change trajectories in the HMLC since the aggressiveness of the counterpart is unknown and the subject vehicle cannot guarantee the sensible driving of the counterpart. Thus, an additional lateral motion is not considered in obtaining the cross section of interest. Since the subject vehicle is located at origin in the relative coordinate, we derive the cross section  $C_v$  that contains the origin.

$$C_v \triangleq \{(y_r, t) \mid C_s \text{ such that } (0, 0, t)_r \in C_s \wedge x_r = 0, \text{ for } t \in [0, h]\} \quad (3.23)$$

The SPSC must be designed to produce velocity profiles to avoid  $C_v$ . However, as shown in Figure 3.12, the driver may avoid  $C_v$  in two ways; 1) over the upper surface (acceleration), and 2) under the lower surface (deceleration). In such a case, it is

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<sup>23</sup> This does not lose much of generality since a driver estimates the future based on the current status.

necessary to choose a control scheme that has less cost between the acceleration and the deceleration.

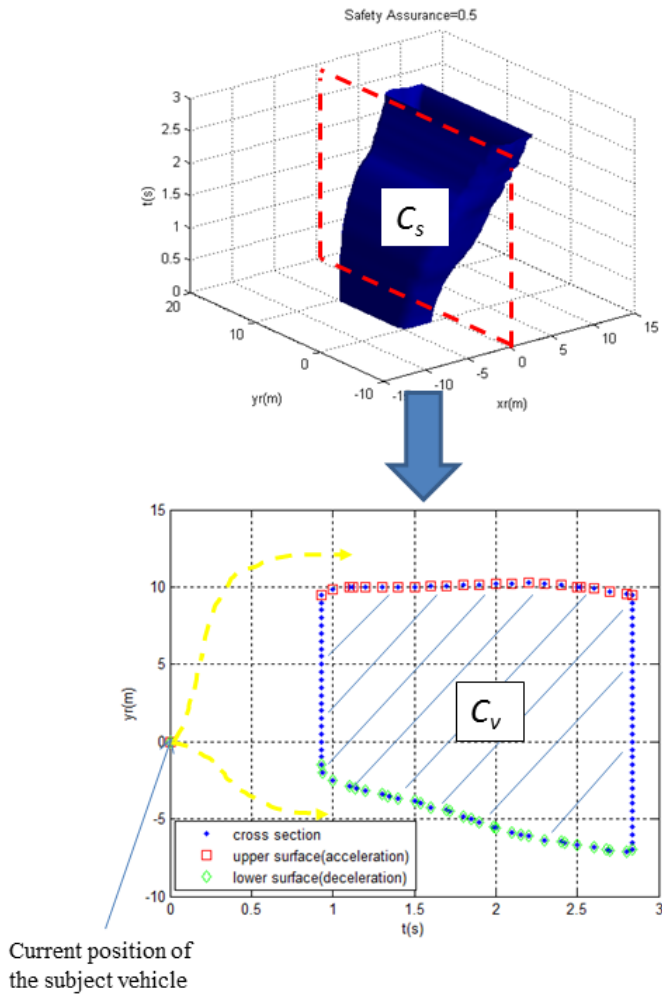


Figure 3.12. Control objective

### 3.3.2. Design of MPC in SPSC

We consider a simple kinematic model in longitudinal direction<sup>24</sup>. Let  $y_r$  be the state variable  $x_1$ . Corresponding longitudinal velocity and the control input are denoted as the state variable  $x_2$  and acceleration input  $u$ , respectively, we have a state equation

$\dot{x} = f(x, u)$  with a state vector  $x = [x_1 \ x_2]^T$  as the following

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ u \end{bmatrix} \quad (3.24)$$

Thus, the MPC is formulated as an argument that satisfies the following objective: i.e. minimizing the cost function  $J$  that is defined on  $[t_0 \ t_f]$ , where  $t_f = t_0 + h$ .

$$\begin{aligned} \min_{u(\bullet)} \quad & J(x, u) \\ \text{subject to} \quad & \dot{x} = f(x, u), \ x_1(t_0) = 0 \\ & x_1 \geq C_{v, \text{upper}} \vee x_1 \leq C_{v, \text{lower}} \\ & x_2(t_f) = 0 \\ & a_{\min} \leq \dot{u} \leq a_{\max} \end{aligned} \quad (3.25)$$

where  $a_{\min}$  and  $a_{\max}$  denote lower and upper bounds of the input acceleration. In MPC, one can choose any cost function  $J$  to suit the purpose. Here, we simply define  $J$  as the

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<sup>24</sup> Trajectory parameterization that we will use in designing MPC can handle nonlinear cases without additional efforts. Thus, other dynamics models can be easily transplanted when we need to consider complex dynamics of the vehicle that we don't deal with in this work.

following quadratic cost function to minimize excessive state deviation from the origin<sup>25</sup> and control efforts in a finite time:

$$J = \int_{t_0}^{t_f} (\mathbf{x}^T Q \mathbf{x} + u^T R u) d\tau \quad (3.26)$$

with weighting matrices,  $Q$  and  $R$ . The main control purpose is accomplished by satisfying the trajectory constraint  $x_1 \geq C_{v,\text{upper}} \vee x_1 \leq C_{v,\text{lower}}$  and thus, steering the state to outside the boundary of  $C_v$ .

### 3.3.3. Practical Suboptimal Approaches

To solve the Optimal Control Problem (OCP) in receding predictive horizon, Linear Quadratic Regulator(LQR) can be used when the system is linear [84], and Hamilton-Jacobi-Bellman (HJB) formulation can be used for the nonlinear case [128], although it is hard to analytically solve. In this work, we solve the OCP, based on the suboptimal approximation methods (finite element approach and collocation approach [134]) for practical purposes. By using both methods, we can obtain the MPC solution efficiently without integrations (including ODE integration to compute feasible trajectories) [135, 136]. First, the finite element approach is to estimate the system state and control trajectories by parameterizing the trajectories as the form of

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<sup>25</sup> State is defined in relative coordinates. Thus, the state  $x_1$  mean relative position deviation with the counterpart.

$$\hat{x} = \sum \phi_i(t) \cdot \alpha_i \quad (3.27)$$

by using basis functions  $\phi_i(t)$ , where  $\alpha_i$  are the scalar coefficients that correspond to respective basis functions. However, for the sake of the accuracy of the parameterization as in (3.27), the integration of the projection error is necessary.

$$\min \int_{t_0}^{t_f} (x - \hat{x}) \cdot \phi_i(t) d\tau \quad (3.28)$$

We also have another integration to solve the OCP, integration of the cost function on the finite time horizon. Thus, we utilize a numerical approach called collocation method to parameterize the trajectories in time and solve the OCP without analytic integrations, which enables us to compute them with vector or matrix calculations [136]. By discretizing the basis functions in time, the MPC is transformed into NonLinear Programming (NLP) problem. To be precise, our MPC is solved by using a nonlinear constrained optimization. First, the B-spline basis functions are constructed with the following B-spline parameters in Table 3.1. States and control of the system are defined on  $[t_0 \ t_f]$  using B-splines shown in Figure 3.13. With these basis functions, we parameterize states and control input as

$$\hat{x}_1 = \Phi \cdot \alpha \quad (3.29)$$

$$\hat{x}_2 = \Phi \cdot \beta \quad (3.30)$$

$$\hat{u} = \Phi \cdot \gamma \quad (3.31)$$

where the basis function matrix  $\Phi$  and coefficient vectors,  $\alpha$ ,  $\beta$ , and  $\gamma$  are defined as

$$\Phi = [\phi_1 \cdots \phi_N], \phi_i = [\phi_i(0) \cdots \phi_i(h)]^T \quad (3.32)$$

$$\alpha = [\alpha_1 \cdots \alpha_N]^T \quad (3.33)$$

$$\beta = [\beta_1 \cdots \beta_N]^T \quad (3.34)$$

$$\gamma = [\gamma_1 \cdots \gamma_N]^T \quad (3.35)$$

by using collocation method.

Table 3.1. B-spline parameters

Parameter	Value
Number of polynomial pieces	10
Number of continuous derivatives	2
Order of each polynomial piece	3
Multiplicity of knots	1

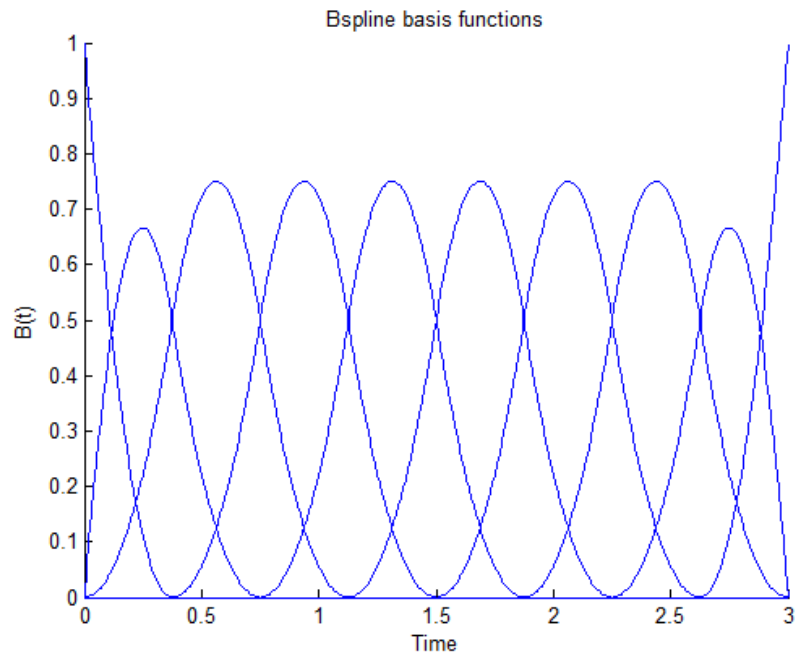


Figure 3.13. B-spline basis functions  $\phi_i(t)$

Let  $\rho$  be the coefficient vector of  $\alpha$ ,  $\beta$ , and  $\gamma$ ,

$$\rho = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} \tag{3.36}$$

The, the initial MPC scheme is transformed into

$$\begin{aligned}
\min_{\rho} \quad & NI(\hat{x}^T Q x + u^T R u) \\
\text{s.t.} \quad & (d\Phi \cdot \alpha - \hat{x}_2) \cdot \phi_i = 0, \quad i=1, \dots, N \\
& (d\Phi \cdot \beta - \hat{u}) \cdot \phi_i = 0, \quad i=1, \dots, N \\
& \hat{x}_1 \geq C_{v,\text{upper}} \vee x_1 \leq C_{v,\text{lower}} \\
& \hat{x}_1(t_0)=0, \quad x_2(t_f) = 0 \\
& a_{\min} \leq \hat{u} \leq a_{\max}
\end{aligned} \tag{3.37}$$

where  $d\Phi$  is the discretized collocation matrix of the derivative of the basis function matrix  $\Phi$ .  $NI$  represents any numerical method for approximating the definite integral, for instance, the trapezoidal rule [137]. Note that the OCP has changed to an optimization problem of finding a coefficient vector  $\rho$ , which can be solved using a MATLAB command *fmincon* for the nonlinear constrained optimization. The MPC results at  $t_0$  are shown in Figure 3.14.

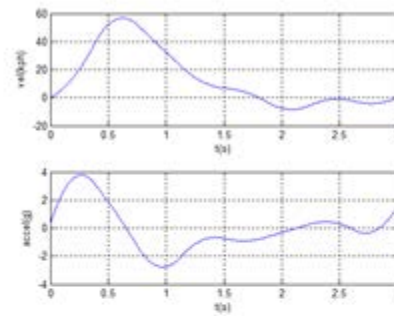
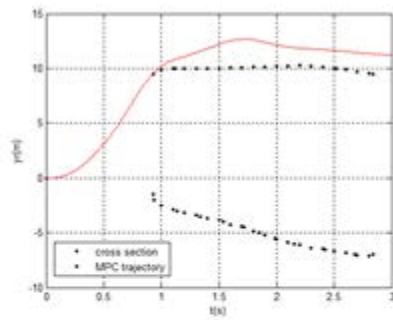
#### 3.3.4. Simulations

The final driving control does not depend on only SPSC stated in the previous section. In real word, we have to consider the situation where there are no threats from the front vehicles. For example, the evolution of the unsafe set can never meet the subject vehicle within the prediction time. Moreover, it is also possible that there are not vehicles ahead. Thus, SPSC is combined with PD controllers introduced in the driver decision model [119] as the final driving controller. Only when we can detect the subjective collision risk, the SPSC will engage to adjust the speed of the vehicle additionally with a loop period of 1sec. Otherwise, SPSC will limit the PD controls so as



not to override the anticipated collision area. Also, although we considered only one side lane in the previous control scheme formulation, it is necessary to look out both right and left lanes to estimate the collision risk in reality. Thus, it is also required to combine the reachable sets from both lanes in the simulation.

– Acceleration



– Deceleration

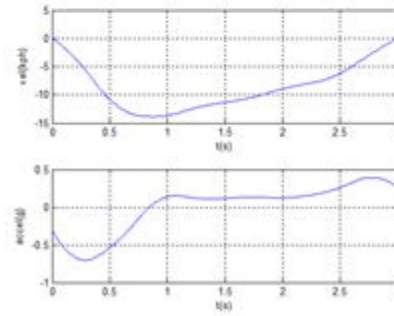
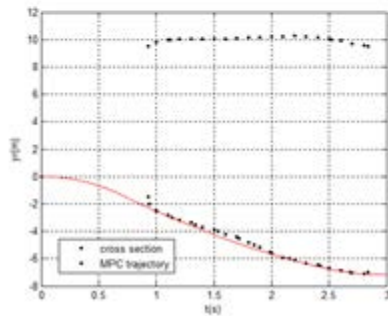


Figure 3.14. Exemplar MPC control outcome at  $t_0$

### 3.3.4.1 Scenarios

In order to investigate both side lanes for the vehicle of interest, we set 3-lane road, which will be an extension of the road shown in Figure 3.4. Since we set the width of the lanes to be  $3.3m$ , the center line of the first lane is laterally located at  $0m$ , the second lane,  $3.3m$ , and the third lane,  $6.6m$ . At least 1 vehicle is located at both side lanes. Vehicle 1 and 2 are located close to the subject vehicle longitudinally to derive meaningful unsafe reachable set for the normal driver.

Table 3.2. The initial positions of the vehicles

	Initial conditions		
	lat. pos. (m)	long. pos. (m)	velocity (km/h)
Subject Vehicle	3.3	0	90
Vehicle 1	6.6	12	90
Vehicle 2	0	13	90
Vehicle 3	6.6	30	90

When we run a simulation with only driver decision model [119] without SPSC, the subject vehicle accelerates its speed to the speed limit and keeps its speed because it does not have any vehicle ahead. This scenario is intended to show the engagement of the SPSC and its effects according to the driver's aggressiveness.

### 3.3.4.2 Simulation Results

First, we simulate a driver who has a normal *safety assurance level*, 0.5 for 3 seconds. The subjective risk estimation at initial time is shown in Figure 3.15. The subject vehicle follows the SPSC control initially. Thus, the subject vehicle slows down as shown in Figure 3.16, which enables Vehicle 1 to cut in ahead of the subject vehicle as shown in Figure 3.17. In sequel, the subject vehicle changes its lane to the first lane due to the Vehicle 1. After that, the subjective risk estimation set does not appear because the unsafe reachable set does not intersect with the subject vehicle any longer. Thus, the vehicle follows the basic driver model<sup>26</sup> for [1 3]sec.

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<sup>26</sup> Basically, Vehicle 1 to 3 follows the driver decision model developed in our previous works. Thus, it can also track a vehicle ahead and changes its lane as an intelligent agent.

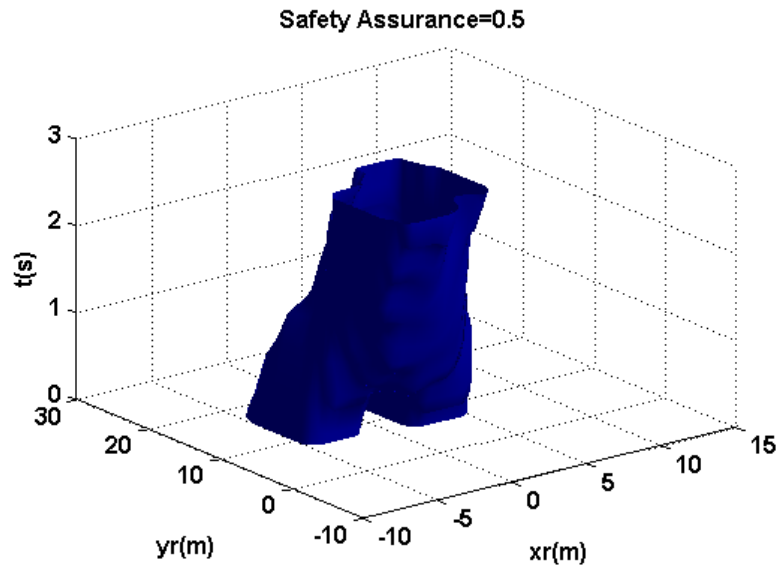


Figure 3.15. Subjective risk estimation for a normal driver at  $t=0$ sec

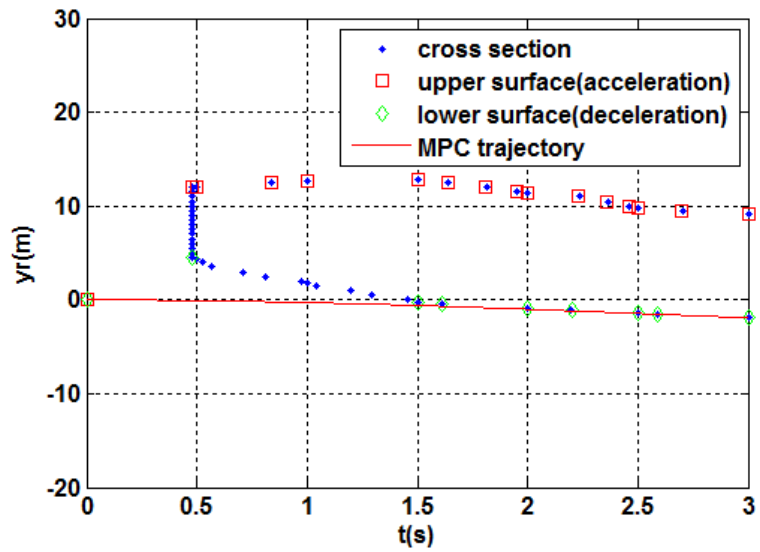


Figure 3.16. MPC trajectory for a normal driver at  $t=0$ sec

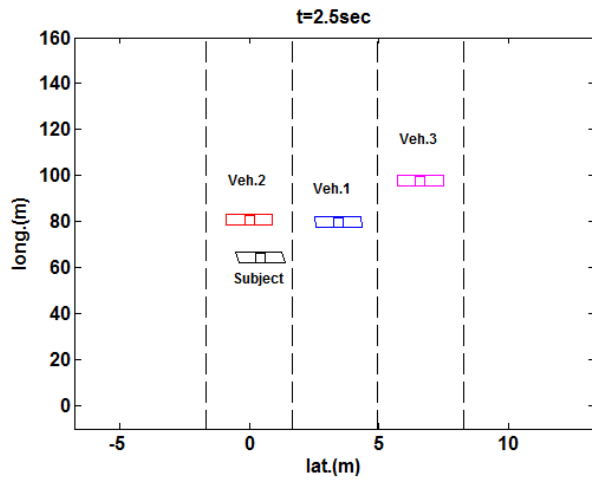
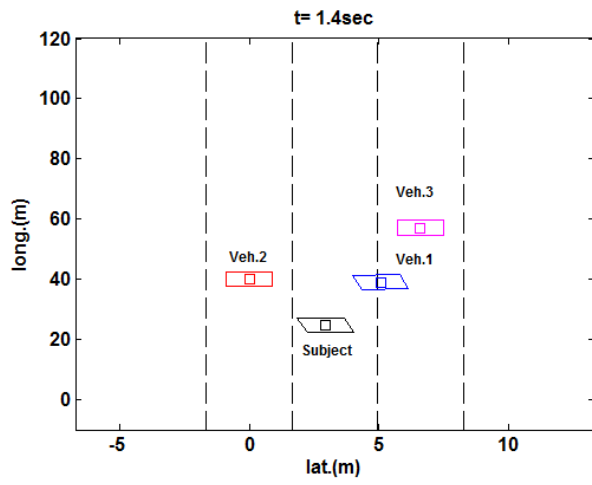
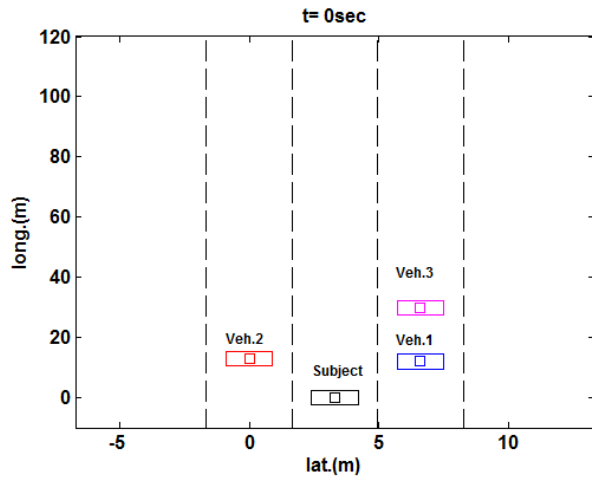


Figure 3.17. Driving simulation results for a normal driver

Next, we simulate the same scenario with different *safety assurance level*, 0 and 1. When *safety assurance level* is 0 (i.e. the driver is aggressive.), it is considered that there is no the subjective risk estimation of collision at [0 1]sec as shown in Figure 3.18 and Figure 3.19. Thus, the subject vehicle does not slow down as shown in Figure 3.20. This is because aggressiveness is reflected on the utilities used in Nash game. It weakened the probability for the discrete reachable sets where Vehicle 1 cuts in. Correspondingly, the probability of the final objective collision is calculated with low collision probabilities, which is not regarded as subjective collision risk with the lower *safety assurance level*. Thus, in case of *assurance level*, 0, SPSC does not need to engage at [0 1]sec. However, the probability for Vehicle 1 cannot be ignored as the relative distance decreases after 1sec, and thus, the subject vehicle does not accelerate any longer. In this case, SPSC engages after 1sec.

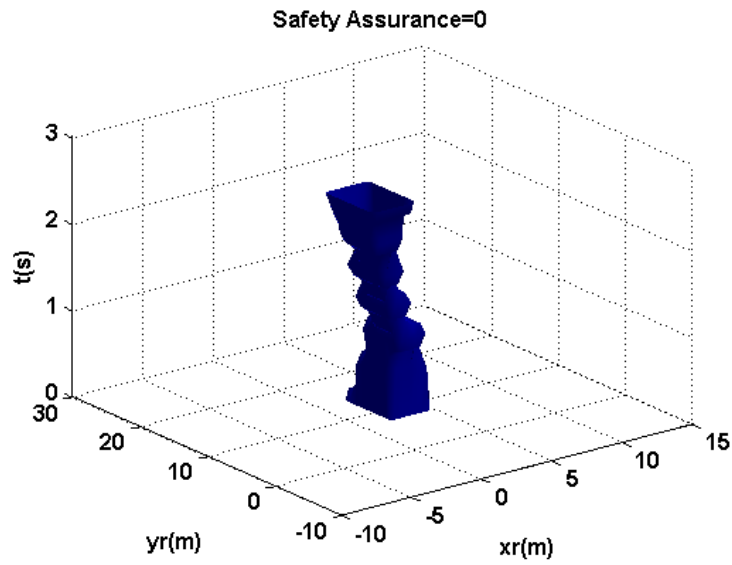


Figure 3.18. Subjective risk estimation for an aggressive driver at t=0sec

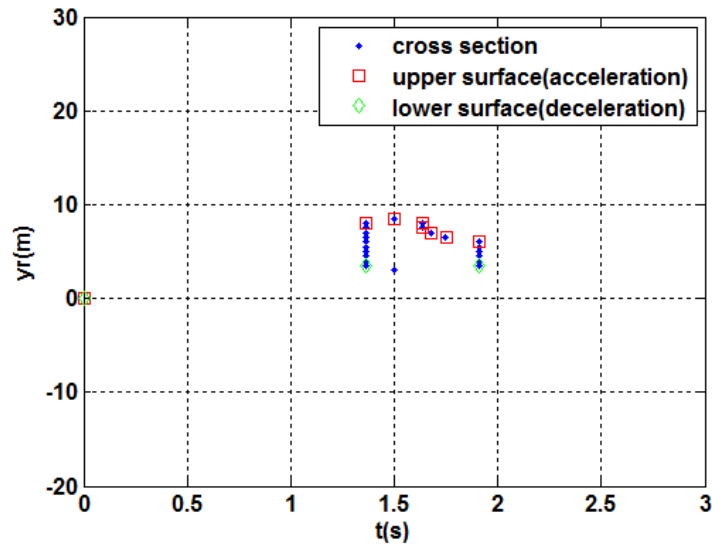


Figure 3.19. MPC trajectory for an aggressive driver at t=0sec

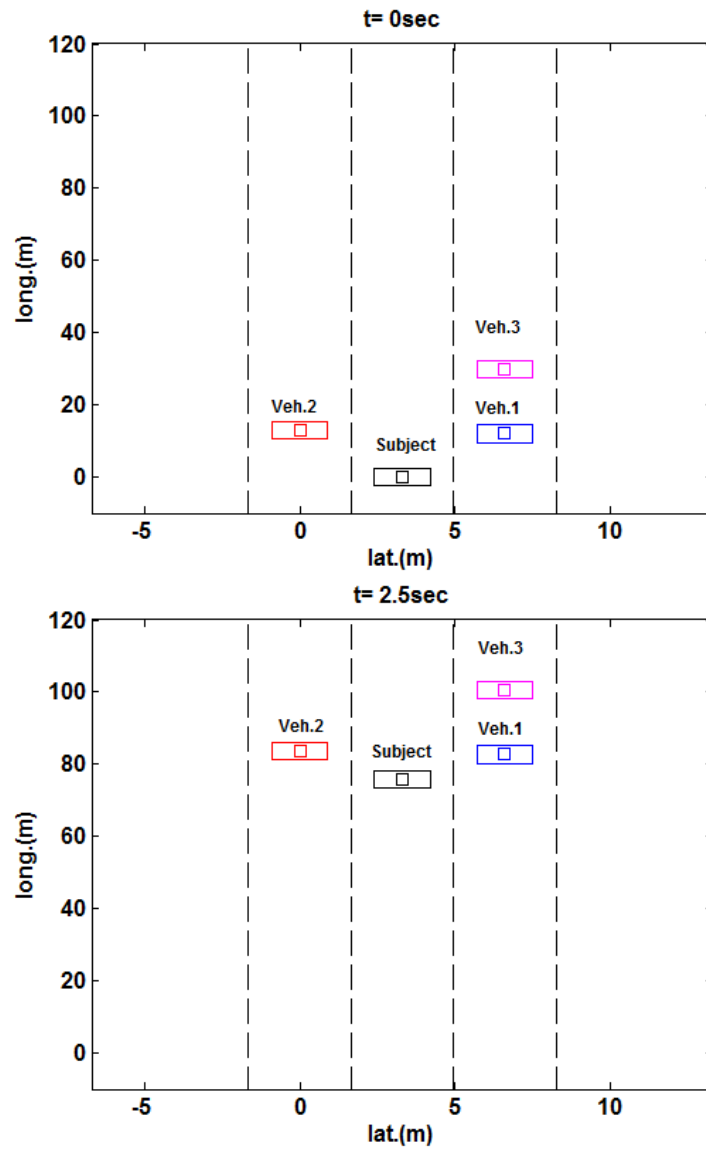


Figure 3.20. Driving simulation results for an aggressive driver

Finally, when we simulate the same scenario with different *safety assurance level*, 1 (i.e. the driver is timid.), the subjective risk estimation of collision is maximized at [0



1]sec as shown in Figure 3.20 and Figure 3.21. Thus, the subject vehicle slows down like the case of *safety assurance level*, 0.5. The difference is that the subject vehicle slows down with higher deceleration because of bigger subjective risk estimation.

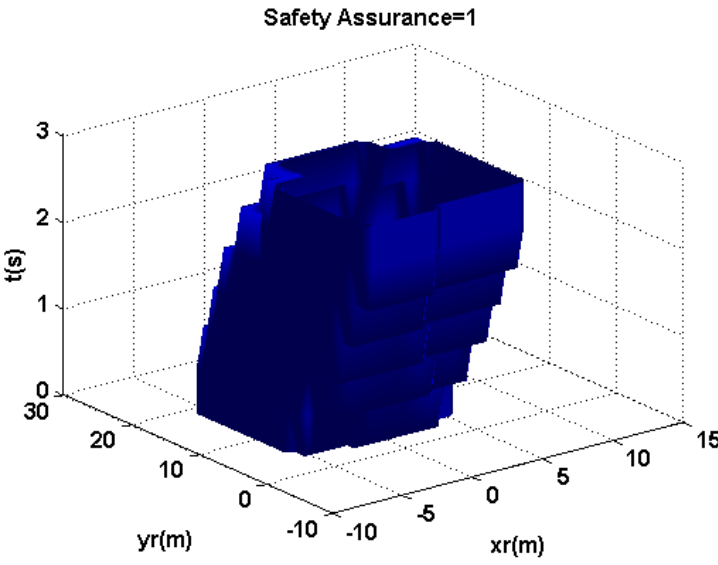


Figure 3.21. Subjective risk estimation for a timid driver at t=0sec

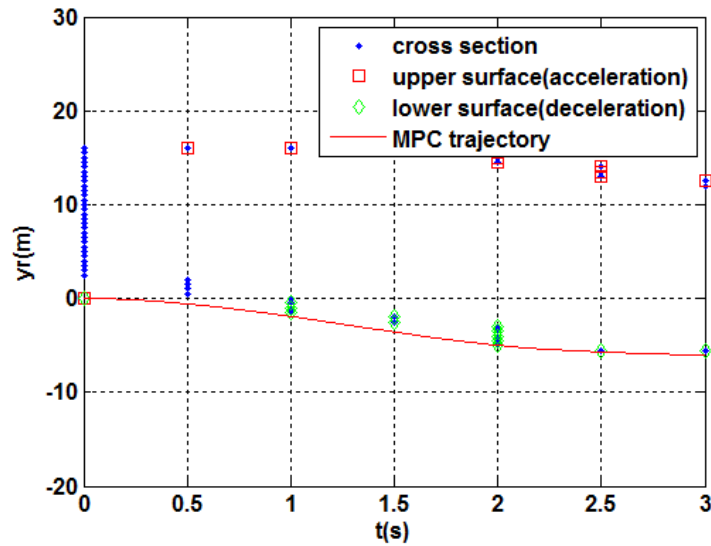


Figure 3.22. MPC trajectory for a timid driver at t=0sec

In sum, we could see that the normal and timid drivers with higher safety assurance level than aggressive driver slowed down with respect to their subjective risk estimation. This enabled another vehicle (Vehicle 1) to cut in ahead of the subject vehicle. Conversely, the aggressive driver with *safety assurance level* of 0 accelerated and kept comparatively smaller relative distance with another vehicle (Vehicle 1), which prevents another vehicle from changing lanes. With this, this driver model showed that different *driver courtesy* [12] can be addressed analytically according to the driver's aggressiveness.

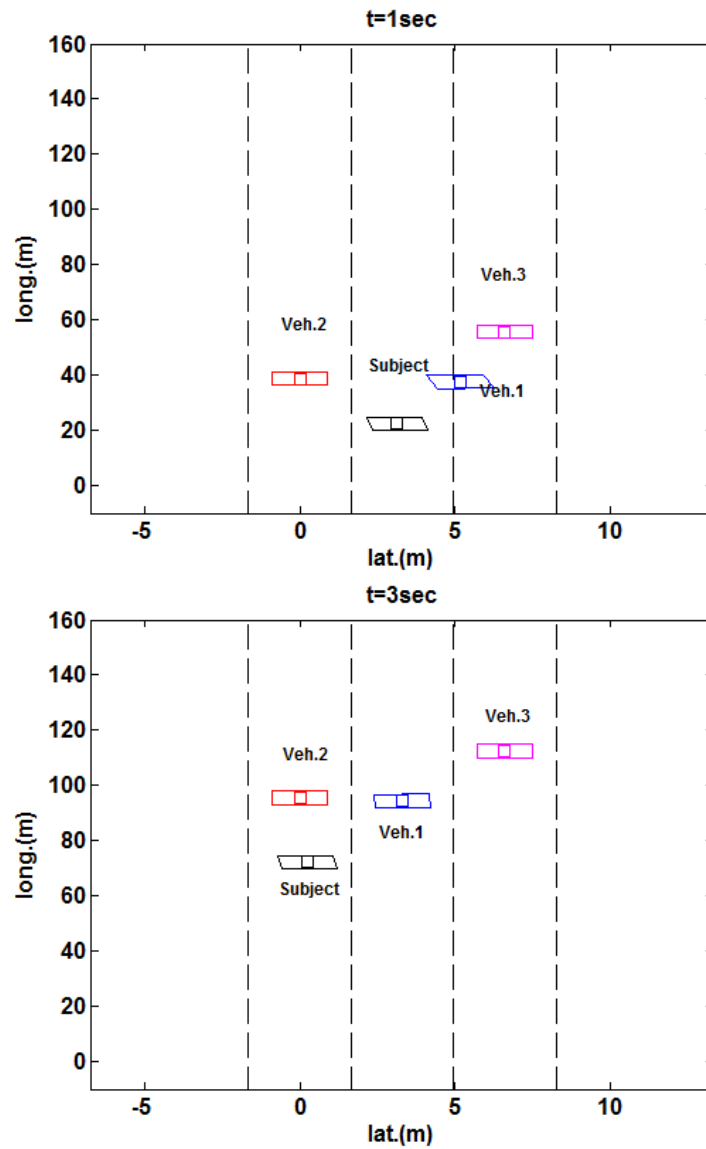


Figure 3.23. Driving simulation results for an aggressive driver

### 3.3.5. Summary

Based on the subjective perception of the collision that differs among drivers, we designed a controller, SPSC, that drives the vehicle to stay outside the subjectively

anticipated collision. The controller is designed with the consideration of a rational (optimal) method and illogical perceptions. For the local optimality on the pursuit of the safety within the driver's prediction horizon, we designed an MPC controller on the prediction horizon with the use of trajectory parameterization and collocation. The developed controller, based on the risk estimation, was combined with the previous driver decision model and showed additional predictive responses to the threats from the adjacent front vehicles. The predictive responses to the adjacent front vehicles made different interactions of the vehicles in the viewpoint of yielding. The acceptable gap for the other vehicles was possible without any request from them. The upcoming mixed situation of autonomous vehicles and human drivers is of considerable significance. In this regard, it was shown that the model I propose can be applied widely, from an aggressive human driver model that has a certain level of uncertainty to an autonomous vehicle that pursues maximum driving safety.

## 4. CONCLUSIONS

In this work, I developed a driver model for better formal understanding of human drivers. Toward the end, I developed two different models in a driver model, a driver decision model and a driver driving model.

In the first part, I discussed the application of game theory to an individual driver's reasoning process. In fact, the driver decision model is based on Stackelberg game theory. I focused on undesirable traffic situations that may occur due to the participants' irrational decisions. In this regard, drivers' insensible payoffs are designed. To validate the effectiveness of the model, I performed Monte Carlo simulations, ANFIS modelling, and traffic simulations. These reproduced meaningful relations with the driver's inattentiveness and *traffic unsafety* which are observed in the real world.

Next, the driving model performed an active prediction of the other drivers' behaviors. A subjective collision risk estimation that differs among individuals was presented based on reachable set computation in the multi-agent hybrid system. Then, the SPSC computes a trajectory to stay outside the subjective collision estimate through suboptimal approaches. The model showed adequate predictive responses against the other vehicles.

In sum, I have presented an effective model that corresponds to and predicts traffic situations according to a human driver's irrationality factor. This model showed a meaningful similarity to the *traffic unsafety* of the real world and accounted for predictive yielding behaviors according to the driver's rationality.

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