MILK ADVERTISING, VENDING MACHINE PURCHASES, AND THEIR HEALTH IMPLICATIONS

A Dissertation

by

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ABSTRACT

The United States of America is facing a disease – obesity. In order to combat or attempt to combat this problem, studying specific food groups and who consumes them and why is of upmost importance. This series of papers addresses the 'why' and 'who' of consumption for two industries: fluid milk and vending machine. Each industry will be analyzed using demand analysis methods to answer questions regarding what influences consumption of goods within these two industries.

The fluid milk industry has four fluid milk types, differentiated by milk fat percentage. Advertising strategies focus on generic fluid milk consumption rather than specific fluid milk types. Understanding how generic milk advertising affects specific milk type consumption is necessary to see if advertising improvements can be made. The fluid milk industry data are in time series format and complete (QUAIDS and Barten Synthetic) and incomplete demand systems are used to understand various relationships among prices, income, seasonality, and generic advertising. The incorporation of a polynomial distributed lag advertising variable in each demand model specification shows that generic milk advertising affects fluid milk type consumption differently. Compensated elasticities show that low-fat milk and skim milk and whole milk and skim milk are substitutes. Income elasticities show that each fluid milk type is a normal good. Catering advertising efforts towards specific milk type consumption may result in higher sales as long-term advertising affects milk type consumption differently. Further, Government programs separate milk types in regards to what qualifies for specific types of food assistance programs. If the fluid milk industry caters to such separation, fluid milk consumption, particularly whole milk, may increase.

The vending machine industry is an easy access provider of snacks and sodas. The vending machine industry is analyzed with cross-sectional data over a four year period from 2009 to 2012. With these data we analyze household characteristics that influence the decision to purchase from a vending machine through the use of a Tobit model and a probit model. We examine how socio-demographic characteristics and

other purchasing habits affect vending machine purchases both through conditional and unconditional effects and likelihoods. Results indicate that socio-demographic characteristics significantly affect whether or not a purchase is made from a vending machine. Further, other purchasing habits, such as food away from home, chips and colas for at home consumption, and tobacco products positively and significantly affect a household's vending machine purchases. Perhaps offering a larger variety of goods will attract a larger consumer base.

With the combination of the industries and methods, we are able to answer several questions and provide policy recommendations in regard to marketing strategies that target consumption habits. Further, we add to the current literature through both theoretical and applicable contributions.

DEDICATION

This dissertation is dedicated to persons who think they do not have the strength and devotion to accomplish anything they want. To quote Ayn Rand: “The question isn’t who is going to let me; it’s who is going to stop me.”

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ABSTRACT

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CHAPTER I
INTRODUCTION

It is no shock to hear or read about America's weight problem whenever a television is turned on or a news source is opened. Evans *et al.* (2005) pointed out that news coverage for obesity and related topics has grown at least as quickly as the epidemic itself. While the finger is pointed at many things as the cause of America's obesity problem (notably eating habits, lack of physical activity, genetics, and environment), there seems to be no agreement upon direct causes of obesity. Perhaps instead of trying to pinpoint particular causes, one could analyze the consumer who is making food choices that may contribute to poor or healthy lifestyles. Research that explores the type of consumer who purchases specific foods and why could contribute to the pool of existing research on America's weight and health problems.

The existing literature on causes and effects of eating habits and how they relate to a person's health is extensive. There appears to be a void when it comes to addressing the economic side of specific item consumption and the types of consumers who purchase such goods. The purpose of this set of papers is to address such a void. By exploring specific food channels, we plan to learn more about the consumer who is purchasing particular food items. Whether or not a person is overweight or obese is likely related to his/her eating habits; by understanding the consumer who prefers specific foods, we may be able to provide better public health policy advice to officials. However, providing some background on how serious the health issues are in the United States is necessary to fully understand the goals we intend to fulfill.

The Centers of Disease Control and Prevention (CDC) defines being overweight and obese as having a body mass index (BMI) of 25.0 – 29.9 and 30 or higher, respectively, where BMI is a measurement of an adult's weight in relation to height (CDCa 2013). In the 1980s, the percentage of obese adults in the United States started climbing. In 1999, obesity was declared an epidemic in the United States (Health Tidbits 1999). About six years later, in 2005, the *New York Times* reported that for the first time, the CDC sent a team of specialists to study the obesity problem (Kolata 2005). By 2012, no state in the United States had a prevalence of adult obesity less than 20%; forty-one states had prevalence equal to or greater than 25%, with 13 of those states having an obesity prevalence of 30% or higher (CDCa 2013).

Adult health and weight issues are not the only concern. Childhood weight problems also are a national problem. Approximately 17% of children and adolescents (ages 2 – 19) are obese[1] (CDCb 2013). In 2010, childhood obesity was declared a national epidemic (Barnes 2010). In 2011, only four states had percentages of obese high school children less than 10% (ranging from 7 – 9%) while twelve states had 15-19% of high school students being obese compared to only three states in 2003 (CDCc 2013).

An article in *The Economist* (Howard 2012) gives a good overview of the health issues related to weight that not only America is facing but also the world is facing. The article mentions that more than two-thirds of Americans are overweight and brings to light the 'stereotype' food Americans are known for eating. Books such as *Salt, Sugar, Fat* by Michael Moss (2013) and *Cooked* by Michael Pollan (2013) also highlight the infamous American diet, reminding readers that processed foods and sugary beverages account for a large percentage of the diet. Another contribution to the obesity epidemic in America is the increase in portion size over the past three decades (Wang and Beydoun 2007).

Instead of just changes in eating habits, less physical activity also is contributing to the obesity epidemic (Jacobs 2006). Though the health benefits of physical activity are pretty well known, physical activity levels in the United States are declining (Harvard School of Public Health(a) 2013). The CDC (2012) reported that less than half of all adults meet the suggested physical activity guidelines, while less than 30% of high school students participate in at least 60 minutes of physical activity every day.

[1] Where obese here is defined as students who were ≥ 95th percentile for body mass index, based on gender- and age-specific reference data from the 2000 CDC growth charts; see http://www.cdc.gov/healthyyouth/yrbs/pdf/ us_obesity_combo.pdf

Another popular topic in regards to obesity issues and consequences is cost. The Harvard School of Public Health separates costs of obesity into two groups – direct and indirect costs. The direct costs include out/inpatient health services, laboratory, and radiological tests, as well as drug therapy. Indirect costs include value of lost work, insurance, and wages. Finkelstein *et al.* (2009) reported that in 1998, the medical costs of obesity were as high as $78.5 billion; they estimated the costs to increase to $147 billion per year by 2008. They also reported that obese persons have $1,429 (42%) higher per capita medical spending than medical spending for normal persons. Not only is obesity costly when it comes to dollars, but it can have some life threatening side effects as well. Some of the risk factors associated with being overweight and obese include heart disease, various cancers, type II diabetes, stroke, hypertension, and other side effects (CDC 2012). During an address to Congress, Dr. Carmona, the U.S. Surgeon General in 2003, stated that one out of every eight deaths in America is caused by an illness directly related to being overweight or obese. By 2010, the leading cause of death was heart disease, which was followed by cancer, lower respiratory diseases, and stroke (CDCd 2013).

While some of the causes and side effects of being overweight and obese are known, little information is available as to what types of persons purchase *specific* types of foods and why. To better address this issue, we propose to explore two industries: (1) the fluid milk industry and (2) the vending machine industry. Specifically, this contribution will differ from previous literature in that we are analyzing the economic side of how generic advertising affects consumer purchases of fluid milk types and the characteristics of persons who purchase from vending machines. Though previous literature exists on generic advertising's effect on various products, including milk types, ours differs because we include an advertising variable that allows us to examine carry over effects of advertising. Further, we analyze how generic advertising affects fluid milk type consumption. Though some previous literature exists on vending machine product consumption such as experiments, no literature exists which examines what affects whether or not a consumer purchases from a vending machine. If we can understand the type of consumer who prefers specific foods or reacts to advertising signals, we may be able to provide better advice to help guide health officials when making policy decisions in regards to public health issues.

Objectives

This research has various objectives to meet with each industry, both theoretical and applicable, as well as overall goals for literature contributions. In regards to the dairy industry, the objectives are to:

(1) determine if generic advertising affects specific milk types differently;
(2) incorporate advertising data into demand systems, both incomplete and complete, using polynomial distributed lags, capturing dynamic effects of advertising;
(3) estimate budget elasticities (complete system) and income elasticities (incomplete system), and own/cross price elasticities (both systems), and
(4) provide results to dairy advertising campaigns, possibly resulting in a more appropriate use of funds;

The vending machine industry is an industry where various beverages and snacks can be purchased. Little research has been conducted on who exactly spends at these machines and why. Using data from the Bureau of Labor Statistics' (BLS) Consumer Expenditure Survey (CES) from 2009 to 2012, we intend to:

(1) develop profiles of households who purchase from vending machines;
(2) calculate conditional and unconditional elasticities for such purchases, and
(3) provide public health officials with results that may be used to target specific segments of consumers to induce more healthy/less unhealthy eating habits.

Both research topics contribute to the current literature pool in several ways. The dairy industry study provides a new method to analyze advertising carry over effects – within a demand system. The vending machine piece adds to economic literature because it is the first paper (to our knowledge) that analyzes specific factors that affect vending machine purchases. Thus, we will be providing fresh results about an industry with which many are quite familiar.

Once the results of these studies are compiled, we will be able to make some comparisons. For instance, the fluid milk industry has four products, each with different fat percentages; thus, purchasers have an option to choose a milk type with fewer or more calories. The vending machine has no advertising scheme as that of the dairy industry. Little is known about its target audience and consumption strategies. Thus, we are able to contrast and compare these two industries, each with vastly different product choices.

The Dairy Sector: Fluid Milk

Per capita consumption of fluid milk has been on a decline in the United States for more than a decade (ERS 2013; USCB 2012 2010 2001). In the year 2000, per capita consumption of fluid milk was approximately 21.0 gallons per year. By 2011, that total dropped to 17.8 gallons (a 14.9% drop) a year (Nielsen Scantrak 2012; USCB 2012 2010 2001). During the same period, milk advertising funds decreased from $321 million to $240 million (a 25.2% decline) per year (Dairy Management Inc 2013; MilkPeP 2013; Qualified Programs 2013). There are four types of fluid milk which are differentiated by their fat content; those include whole milk (3.25% milk fat), two-percent milk, one-percent milk, and skim milk (less than 0.5% milk fat) (Agricultural Marketing Service 1995).

There seems to be domestic concern with the correlation of the rise in obesity and milk consumption, particularly whole milk. The Surgeon General's 2010 report stated: "by age 2, children should be drinking low-fat or non-fat milk," which is reemphasized by WIC's requirements (2012) of only allowing parents whose child(ren) is(are)less than two years as being eligible to use WIC to purchase whole milk. This recommendation likely has an effect on the consumption of liquid milk products here in the United States, particularly the 'fatter' milk products.

Multiple studies have been conducted reflecting the potential relationship between milk consumption and weight gain, particularly among adolescents. Berkey *et al.* (2005) conducted a study on 12,829 United States children aged 9 – 14 years through the use of a survey. This survey collected information on height, weight, and food frequency. They found that children who drank more than three servings a day of milk gained more in BMI than those who drank smaller amounts. Skim and 1% milk appeared more strongly linked to weight gain than whole or 2% milk. However, this research provided little information about specific relationships among milk type consumption and no information about milk prices.

Popkin *et al.* (2006) formed a beverage panel whose purpose was to systematically review the literature on beverages and health and provide guidance to the consumer. Though the paper was mainly a review of available literature on beverages, the panel suggested that a person with a 2200-kcalorie daily energy requirement can drink up to 16 ounces of low-fat milk (out of 98 fluid ounces of total fluid consumption). They also suggested that whole-fat dairy products are a notable source of saturated fat in the American diet with whole-fat milk contributing to that intake. While this paper provides useful information on previously conducted studies, little is contributed to the literature on how fluid milk consumption can affect a person's weight.

Wiley (2010) used data from the National Health and Nutrition Examination Survey (1999 – 2004) to test hypotheses that milk is associated with higher BMI of various ethnicities among children. Summary statistics suggested that Black children consumed significantly less milk than white children or Mexican-American children. There was not much variation among BMIs of children aged 5 – 10. However, results indicated that milk intake reported from a 24 – hour recall was positively associated with BMI among children of 2 – 4 years of age and children of 5 – 10 years of age after controlling for ethnicity and birth weight.

Chen *et al.* (2012) identified the effects of dairy consumption on body weight and fat mass from randomized controlled trials in hopes of assessing the effect dairy products may have on body weight. They found that consumption of dairy products did not result in a significant reduction in weight. Likewise, a study by Mozaffarian *et al.* (2011) conducted on 120,877 US men and women found that changes in the consumption of all liquids except milk were positively associated with weight gain. They also found that no significant differences were evident for consumption of low-fat and skim milks verses whole-fat milk.

Scharf *et al.* (2013) evaluated relationships between type of milk consumed and weight status among preschoolers. They concluded that consumption of 1% milk and skim milk was more common among overweight/obese preschoolers. However, they did note that this situation may be related to the fact that parents choose to give their overweight/obese children low-fat milk to drink. They concluded, however, that low fat-milk did not appear to restrain body weight gain for children aged 2 – 4 years, emphasizing the need for weight-targeted recommendations.

Though this literature does not begin to cover all of the studies pertaining to milk consumption and its effects on weight, it is evident that findings are mixed. Because we are interested in knowing the impact of generic advertising on milk type consumption, it is necessary to have knowledge about the potential health effects of consuming milk. If advertising does affect milk consumption this research will provide insight in how to modify advertising tactics to increase fluid milk sales. While these studies provide some economic insight about the types of persons consuming milk and the effects of milk consumption on particular age types, ethnicities, or genders, none of the studies tie the effects of generic milk advertising to the consumption of the four milk types. Consequently, it is useful to analyze how advertising affects fluid milk intake. However, typically fluid milk is advertised as a commodity – not differentiating among the four types of milks.

We present studies in the extant literature that pertain to the effects of advertising on various beverages and products. Briefly, most of the previous work in the extant literature in regard to milk types and advertising effects groups milk types together (Kinnucan and Forker 1986; Kaiser and Reberte 1996; Gould 1996; Zheng and Kaiser 2008; Kinnucan *et al.* 2001). Some previous works focused on data from particular regions, namely New York, rather than the entire United States (Kaiser and Reberte 1996; Kinnucan and Forker 1986). Additional works such as Kinnucan *et al.* (2001) and Zheng and Kaiser (2008) looked at advertising for non-alcoholic beverages, including milk, across the United States using annual time-series data. Our research is differentiated from previous works in several ways. First, we will be analyzing per capita milk consumption across the entire United States. Also, we plan to differentiate among all three types of milk (i.e., not combining consumption among all milk types). Finally, we look at how generic milk advertising affects consumption by milk type.

The data used to analyze the effects of generic advertising on the four milk types comes from multiple sources. First, we use Nielsen Scantrak data for milk prices and quantities. Per capita consumption will be calculated using US Census Bureau population data. Advertising expenditures are provided from multiple dairy sector organizations. The data we will be using is monthly, another key difference among previous works. The monthly data span from January 2000 to December 2011.

We will be using two types of models to analyze generic advertising's effect on per capita fluid milk consumption. First, we will employ complete demand systems followed by an incomplete demand system. Each model will be modified so as to include advertising expenditures using polynomial distributed lags.[2] With this analysis, we will be able to model generic advertising effects on the four types of fluid milk. If we were to know how generic advertising separately affects consumption of the four types of milks, we may be able to provide better guidance to health and policy officials when making decisions in regards to fluid milk advertising; also, we will be able to better explain consumer purchasing habits with regard to fluid milk advertising. This information may be particularly useful for agencies setting guidelines for WIC programs.

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[2] All data analysis will be conducted using STATA v12.

The Vending Machine Channel

Sales of food and beverages through vending machines reached $19.3 billion in 2012 (VMW 2013). Of these sales, the top three categories were cold beverages ($6 billion), candy, snacks, and confections ($4.1 billion), and manual food service[3] ($3.3 billion). The three most popular locations of vending machines in 2012 were manufacturing sites (22.5%), offices (20.1%), and other sites (11.5%). While only 19% of households reported purchasing from vending machines in a two-week period in 2012, of those who did purchase, approximately $6.83 was spent, contributing to the growing percentage of money spent on food away from home (BLS 2013). An examination of the products with the highest expenditures for vending machine operators reveals that it is not an understatement to assume most purchases from vending machines are snacks and bottled beverages.

A good deal of literature exists on vending machine purchases and the potential setbacks towards public health because of such purchases. For instance, Chriqui *et al.* (2008) looked at how sales tax varied by product and retail location (vending machines versus grocery stores). They found that sales taxes were applied to soft drinks sold through vending machines in 39 states and to snack products to 32-38 states, depending on the item; they also found that sales taxes are higher for soft drinks than for snack products for vended items as compared to grocery items, suggesting a ‘disfavor’ towards sales of these products. Their paper did not, however, examine the effects of these taxes towards the purchases of foods.

French *et al.* (1997) examined the role of price on purchases of low-fat snacks from vending machines. Here, they monitored nine vending machines and dropped prices by up to 50%, determining that lower prices of low-fat foods were effective in increasing choices of low-fat foods. Details about the types of consumers purchasing the products were not included. In a similar study by French *et al.* (2001), the effects of pricing and promotion strategies on purchases of low-fat snacks from vending machines at 12 schools and 12 worksites in Minneapolis-St. Paul, Minnesota were examined.

[3] Manual food service refers to cafeteria and lunchroom sales.

They found that price reduction was associated significantly with the percentage of low-fat snack sales. Specifically, price reductions of 50% 25%, and 10% were associated with increases in low-fat snack purchases of 93%, 39%, and 9%, respectively. However, this paper lacks in providing *detailed* characteristics of the consumers who were making the vending machine purchases.

Researchers also addressed the impacts and side effects of having vending machines at secondary educational institutions (Pasch *et al.* 2011; Park *et al.* 2010; Evans, *et al.* 2005; Kubik *et al.* 2003; French *et al.* 2003; Wechsler *et al.* 2001) which is likely related to the fact that many secondary educational sites have vending machine access. In 2012, *Vending Market Watch* reported that 8.2% of vending machines were on secondary educational sites (VMW 2013) – a decrease from 2011. Readily available vending machine access probably raises concern because the products that have the highest sales for vending machines are 'unhealthy' choices. Coupled with the number of overweight and obese children having increased drastically in the past twenty years (CDCc 2013), it should not be surprising that the Food and Drug Administration's final rule regarding calorie disclosure required by Obamacare and the US Department of Agriculture is predicted to constrict vending machine operator profits (VMW 2013).

Wechsler *et al.* (2001) used data from the School Health Policies and Programs Study (SHPPS) 2000 to describe state and district level policies and practices related to various school food service issues. The results show that students at nearly all senior high schools, most middle and junior high schools, and more than one-fourth of elementary schools have access to foods and beverages at school through vending machines. While more than half of the schools sell bottled water and 100% fruit or vegetable juice, more schools sell items that are high in fat, sodium, and added sugars. Though the survey was representative of the entire United States, including the District of Columbia, details about the demographics of children at these schools were not included.

French *et al.* (2003) conducted a study on the food environment of 20 Minnesota secondary schools by mailing surveys to school principals and food service directors.

They found that the median number of vending machines in schools was 12 with 88% of snack machines and 37% of soft drink machines turned on at all hours. More than two-thirds of schools had soft drink vending machines. Because à la carte and soft drink sales contributed to the revenue stream for the school districts, they suggested that revenue sources should be used to replace the potential revenue reductions that may result from policies inducing a healthier school food environment. However, we are left with little demographic information about the persons who are purchasing the products from the vending machines.

Kubik *et al.* (2003) conducted a similar study but included some demographic information. Their study examined the association between young adolescents' dietary behaviors using a cross-sectional study of seventh and eighth graders in schools in St.Paul – Minneapolis. They found that the number of snack vending machines was negatively correlated with fruit consumption for seventh graders. They also found that students with access to vending machines were choosing low-nutrient snacks instead of fruit. Though percentages of students (i.e., 63% white, 51% male etc.) were included in the survey, little detail was provided as to what types of students purchase what types of products from vending machines.

A study by Park *et al.* (2010) examined the prevalence and behavioral predictors of students in middle school who purchase items from school vending machines instead of purchasing a traditional cafeteria lunch. The population for this survey was all Florida regular public school students in sixth, seventh, and eighth grades. Ten percent of respondents reported substituting a school lunch with a vending machine purchase once in a five-day period. They also provided some demographic information on those who were purchasing from vending machines. However, we are limited here in that the study was conducted towards a specific age group, and no information was provided about the respondents' parents (income, education, etc…).

A recent study by Pasch *et al.* (2011) described which vending machines at 106 schools located in the St. Paul – Minneapolis area meet criteria for beverages, calories, and fat based on selected criteria offered by the Institute of Medicine. Among the 106 schools surveyed, a total of 829 vending machines were counted. Middle schools offered the highest proportion of sugar-sweetened beverages and sports drinks and only 18% of beverages met the established criteria for healthy beverages. This study provided some insight as to differences between private and public schools. For instance, private schools had significantly fewer beverages meeting the criteria than public schools. In regard to food vending, salty snacks were the most commonly offered food item across all school types except private schools (candy bars held a higher percentage), and fruits and vegetables only accounted for 2% of offerings in all schools. We are limited in the availability of how demographic characteristics affect vending machine purchases from this study.

Though there is clearly an abundance of vending machine research, few studies offer insight as to the type of person who is purchasing from the vending machines. Also, hardly any information is provided as to how much persons spend at vending machines on a regular basis. Our contribution will address such issues. We will be analyzing the characteristics of persons who purchase from vending machines as well as their likelihood of purchasing from vending machines. To do so, we plan to use data from the Consumer Expenditure Survey (CES) that the Bureau of Labor Statistics (BLS) conducts on a yearly basis. The Diary portion of this Survey provides detailed information on consumer households, including their purchasing habits, over a consecutive two-week period. The data will allow us to study characteristics associated with households who make vending machine expenditures based on a probit model.

To analyze characteristics of households that purchase from vending machines, we will first compare general statistical measures (means, medians, minimums, and maximums) among those households who purchase from vending machines and those households who do not. Elasticities also are of interest. We will use a Tobit model (censored regression) to analyze what influences a household's decision to purchase from vending machines. With this model, we will be able to calculate conditional and unconditional elasticities for household income and other chosen demographic factors.

Endogeneity of selected explanatory variables may be a problem. Hence, we incorporated instrumental variables and two-stage least squares methods.[4]

Conclusion

The United States is facing an epidemic – obesity, and it is weighing heavily on the shoulder of health researchers. To better address and potentially help solve this problem, more information about the consumer who chooses to eat specific items is necessary. Likewise, information regarding how advertising affects specific consumption habits is needed as typically advertising helps fuel consumption habits. To contribute to the current literature, we will analyze two very different industries, each with its special contribution.

Fluid milk has both 'healthier' options and 'fattier' options, yet just one generic advertising campaign. By incorporating a generic advertising variable into each model specification, we will be able to understand how this advertising campaign affects specific milk type consumption. Further, we will analyze how income and prices affect fluid milk type consumption as well.

Vending machines offer mainly snack food choices and have convenient locations. Understanding why consumers choose to purchase particular food items is necessary to help address this country's weight problem. We will estimate how specific demographic characteristics and how selected at home consumption expenditures affect a household's spending and probability of spending at a vending machine. This research not only will contribute to the current health/obesity related literature but also may further be used as a basis to analyze other food industries.

[4] All data analysis will be conducted using STATA v12.

CHAPTER II

DYNAMICS OF ADVERTISING AND DEMAND FOR MILK IN THE UNITED STATES: THE COMPLETE DEMAND SYSTEM

Introduction

Per capita fluid milk consumption has been on a decline in the United States for more than a decade (ERS 2013; USCB 2012 2010 2001, see figures 1 and 2). However, per capita consumption of all dairy products and milk supply has been on a rise (ERS 2013). In 2000, per capita consumption of fluid milk was approximately 20.96 gallons per year. By 2011, that total dropped to 17.8 gallons – a 14.9% drop (Nielsen Scantrak 2012; USCB 2012, 2010, 2001). During the same period, generic advertising funds for milk decreased from $321 million to $240 million per year – a 25.2% decline (Dairy Management Inc. 2013; MilkPeP 2013; Qualified Programs 2013; see figure 3). The decline in fluid milk consumption may be solely attributed to the decrease in advertising funds, there are likely other factors affecting consumption such as prices of fluid milk, availability of other dairy products, and health implications associated with milk consumption.

Figure 1: United States Monthly Per Capita Milk Consumption, Gallons January 2000 – December 2011

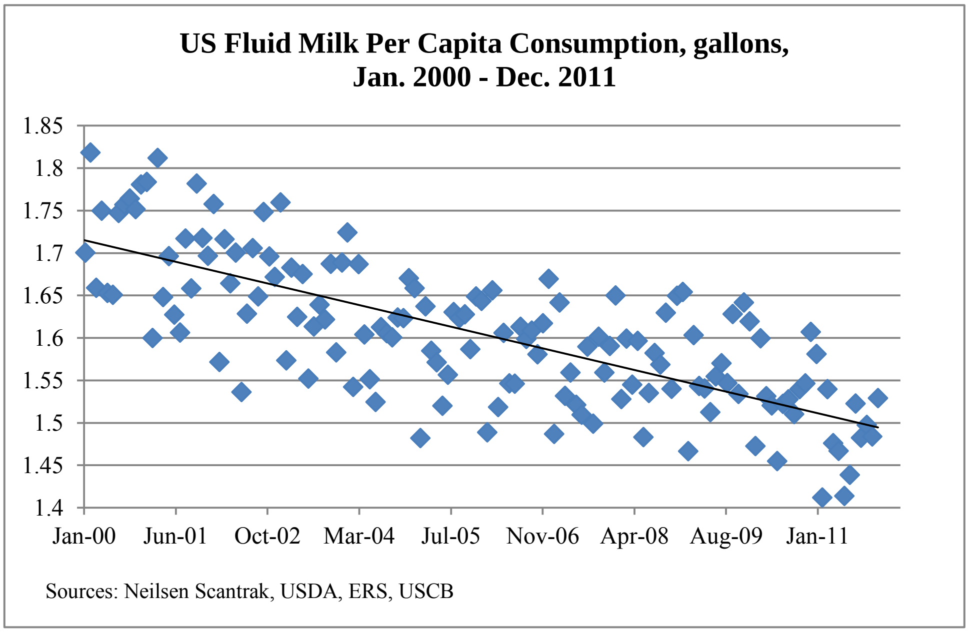


Figure 2: United States Monthly Per Capita Milk Consumption by Milk Fat Type, January 2000 – December 2011

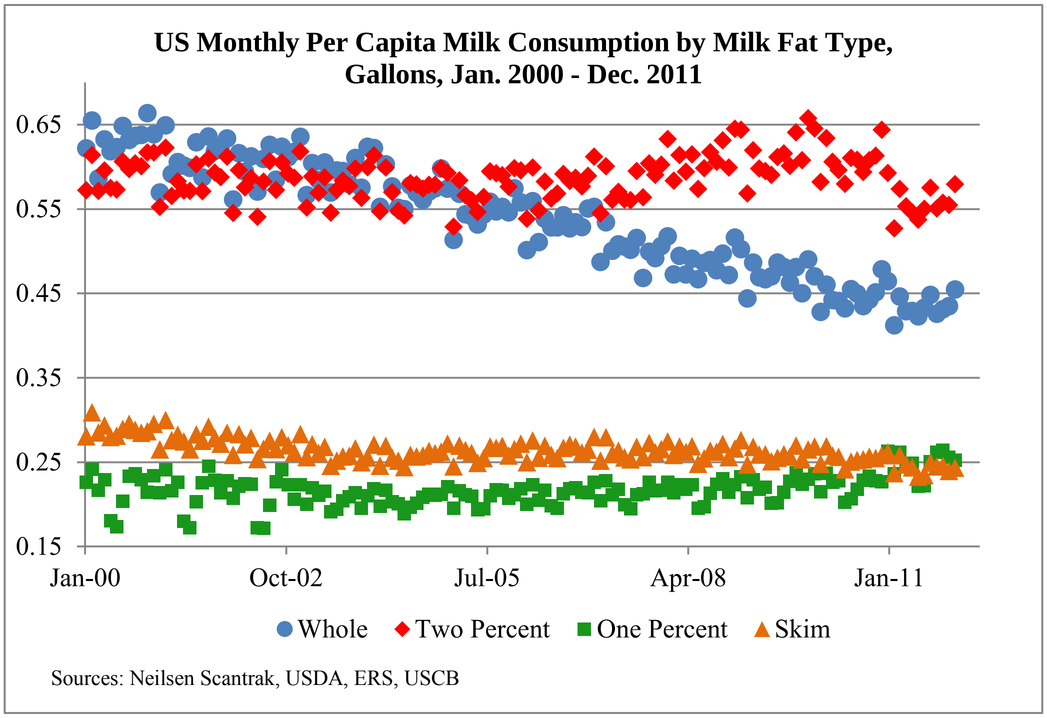
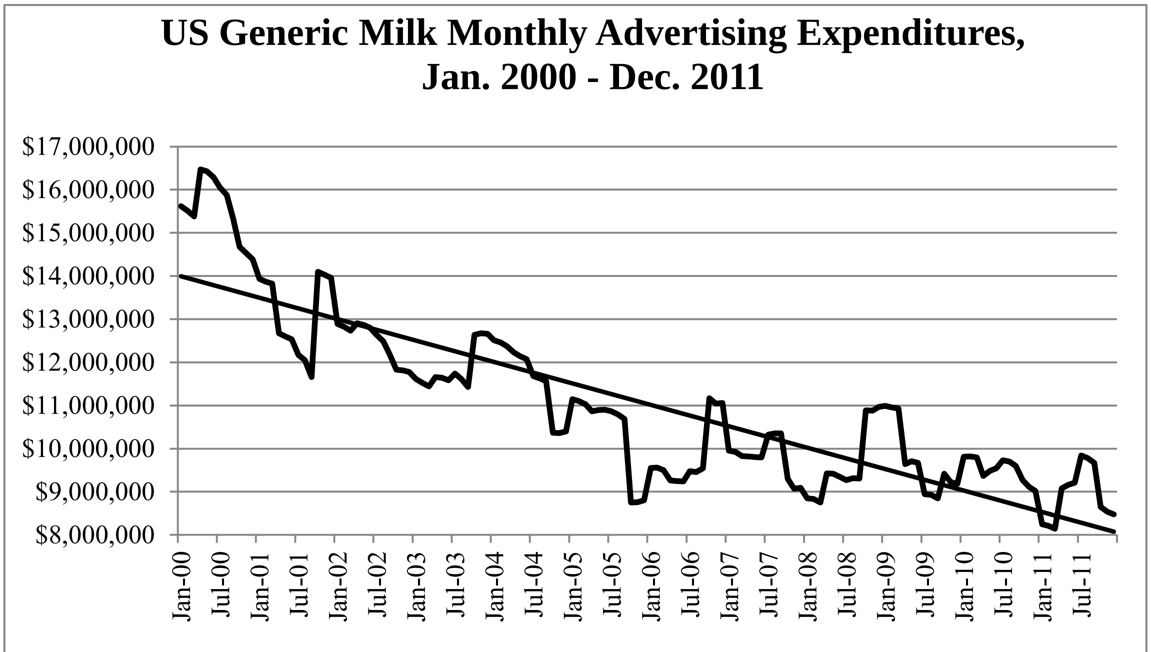


Figure 3: United States Generic Milk Monthly Advertising Expenditures, January 2000 – December 2011



Sources: MilkPep, Dairy Management Inc, Qualified Programs

There are four 'main' types of fluid milk which are differentiated by their fat content; those types include whole (3.25% milk fat), two-percent, one-percent, and skim (< 0.5% milk fat) milks (Agricultural Marketing Service 1995). In recent years, there seems to be developing concern with the correlation of the rise in obesity and milk consumption, particularly whole milk (Berkey *et al.* 2005; Wiley 2010). The Surgeon General's 2010 report stated: "by age 2, children should be drinking low-fat or non-fat milk", which was reemphasized by Women, Infants, and Children's (WIC) requirements (2012) of only allowing parents whose child(ren) is(are)less than two years as being eligible to use WIC to purchase whole milk. This restriction likely has an effect on the consumption of fluid milk products in the United States, particularly milk products with a higher fat content, such as whole milk. However, advertising schemes for fluid milk are generic, not differentiating among milk fat types. Understanding how generic advertising affects specific fluid milk type consumption is needed. If advertising in fact affects consumption of fluid milk differently, there may be need to modify advertising schemes to align with consumer perspectives about milk types.

Multiple studies have been conducted examining the potential relationship between milk consumption and weight gain, particularly among adolescents. Some studies, such as Berkey *et al*. (2005) and Wiley (2010), linked milk consumption to an increased body mass index (BMI) among children and adolescents. However, Chen *et al*. (2012) and Mozaffarian *et al*. (2011) were unable to connect drinking milk to gaining weight. Though there are numerous studies on this topic, there is little consensus on how milk consumption affects BMI. Regardless, America's obesity problem is no secret – and fluid milk is unique two ways: it provides four products all at different fat (calorie) levels, and it is generically advertised with no unique milk type targeted.[5]

Milk advertising typically does not differentiate among the four. Suppliers and processors of fluid milk products contribute to the pool of advertising funds. Instead of appropriating specific advertising amounts to types of milk, in general, milk is generically advertised. If generic advertising was known to separately affect consumption of the four types of milk, we may be able to advise advertising firms to cater to specific types of advertising indigenous to milk type. In turn, public officials may be able to use this information to increase the overall health status for individuals in the United States.

One key difference among milk advertising and other products is its generic structure. Rather than advertising for select types, such as whole milk, advertising funds are used to push for increased milk consumption overall. While there are some brand advertising strategies within milk products, the brands are advertised separately by the specific company. If generic advertising for milk affects milk type consumption separately, there may be need to restructure advertising methods. For instance,

[5] Some milk companies advertise their specific brand or milk type. This study analyzes the impact of generic fluid milk advertising funds.

McDonald’s does not only advertise for its total consumption; rather, the company targets specific products within its advertising strategies as well as its brand, thus likely affecting consumption of individual products as well as total consumption of all McDonald’s products.

Previous work has been done on examining the effects of advertising on numerous products (Funk *et al*. 1977; Brester and Schroeder 1995; Schmit *et al*. 2002) including beverages and fluid milk (Gould 1996; Kaiser and Reberte 1996; Kinnucan *et al*. 2001; Zheng and Kaiser 2008). Numerous models have been used, and findings for advertising effects are not uniform. Funk *et al*. (1977) used a simple demand model which included competitors’ (grocery chains) advertising and found that an increase in beef advertising is associated with a relatively small increase in beef sales. Brester and Schroeder (1995) used a Rotterdam model (Theil 1965) to measure the impacts of brand and generic advertising on meat demand, finding mixed results for the effect advertising has on various meat products. Schmit *et al*. (2002) used a probit model and incorporated a polynomial distributed lag advertising (Almon 1965) variable. Again, mixed results were found for advertising effects (elasticities in this example).

Measuring the effects generic advertising for milk types is not a new concept. However, most of the previous work in extant literature aggregates milk types (Kinnucan and Forker 1986; Capps and Schmitz 1991; Kaiser and Reberte 1996; Gould 1996; Zheng and Kaiser 2008; Kinnucan *et al*. 2001). Some previous works focused on data from particular regions, namely New York, rather than the entire US (Kaiser and Reberte 1996; Kinnucan and Forker 1986) and Texas (Capps and Schmitz 1991). Additional works such as Kinnucan *et al*. (2001) and Zheng and Kaiser (2008) looked at advertising for non-alcoholic beverages, including milk, across the US using annual time-series data. To the best of our knowledge, no paper has analyzed monthly time series data and per capita milk consumption representative of the entire US when modeling generic advertising effects on milk type consumption.

Optimal advertising lags for dairy milk consumption vary among previous works. Capps and Schmitz (1991) and Ward and McDonald (1986) found an optimal lag of 12 (months); Kaiser and Reberte (1996) modeled advertising using 11 lags; Kinnucan (1986) had carry over effects of advertising for six months with the maximum effect at four months; Kinnucan and Forker (1986) set their expenditure contributions to `goodwill' at six months as well. Clarke (1976) wrote that "90% of the cumulative effect of advertising on sales of mature, frequently purchased, low-priced products occurs within three to nine months of the advertisement." While not all of these studies fall within Clarke's category, there is evidence that optimal advertising lengths for fluid milk likely fall within a three – twelve month range.

Methodology

The overall purpose of this study is to measure the impact of generic milk advertising on per capita consumption of four major types of fluid milk which are delineated by milk fat type. To do so, we use complete demand systems where the sum of expenditures of separable categories of milk fat types is equal to the total expenditure of the system. A complete system of demand equations describes the allocation of expenditure among some exhaustive set of consumption categories (Pollack and Wales 1978). A complete demand system is theoretically plausible if it is derivable from a well-behaved utility function, or equivalently if the demand equations are homogeneous of degree zero in prices and expenditure, and the implied Slutsky matrix is symmetric and negative semi-definite (Pollack and Wales 1978), or in other words, the integrability and rationality conditions are satisfied.

We assume fluid milk products are weakly separable from all other goods. By doing so, we reduce the number of goods in the model thus reducing the number of parameters to be estimated. Weak separability, a necessary and sufficient condition for the second stage of two stage budgeting, allows us to break the problem into two stages. First, total expenditure is allocated to the group of interest (fluid milk), based on the price of that commodity group. Then, in the second stage, expenditure on that commodity group is allocated to each good in the group based on the prices of the said goods (Strotz 1957; Cranfield 2012).

Data

Previous papers examining milk consumption have used varieties of data including yearly, monthly, and area specific (Kaiser and Reberte 1996; Capps and Schmitz 1991; Ward and McDonald 1986; Kinnucan 1986; Kinnucan and Forker 1986). For this analysis, we use monthly price data of the four milk types, per capita consumption, as well as advertising expenditure for fluid milk products. Prices are obtained from the Nielsen Scantrack reports on refrigerated milk for the four milk types from January 2000 through December 2011. They are averaged across 52 Scantrack markets (U.S. cities and regions) defined by Nielsen.[6] In the Scantrack data, monthly quantities were reported in terms of millions of pounds and represent the consumption of the entire United States. Per capita quantity values are calculated using the monthly quantity data described above, population estimates (United States Census Bureau 2012; 2010 2001), and a pounds of milk to gallons of milk conversion of 8.6 (Dairy Facts 2008).

Advertising data are gathered from Dairy Management Inc, MilkPeP, and Qualified Programs and are reported quarterly. Monthly advertising data for all milk types are imputed using these quarterly advertising expenditures using the `Proc IM' command in SAS. All milk prices and advertising were deflated using the Consumer Price Index (CPI). Table 1 reports the summary statistics for the variables used in the model.

[6] We found strong correlations between the Nielsen price data and that of the BLS (2013) for whole milk.

Table 1: Complete Demand System Summary Statistics

Variable	Units of Measurement	Mean	Std. Dev.	Minimum	Maximum
Advertising*	US$	11,034,110	1,992,288	8,144,160	16,467,014
Base CPI	Price Index	198.25	17.29	168.80	226.89
Income/	US$	52,876	1,322	50,004	54,841
Whole Price*	US$/gal	1.53	0.12	1.25	1.86
2% Price*	US$/gal	1.47	0.12	1.21	1.79
1% Price*	US$/gal	1.46	0.12	1.20	1.76
Skim Price*	US$/gal	1.44	0.11	1.17	1.71
Fluid Milk^	Gallons/month	1.61	0.09	1.41	1.87
Whole Milk^	Gallons/month	0.54	0.07	0.41	0.67
2% Milk^	Gallons/month	0.59	0.03	0.53	0.66
1% Milk^	Gallons/month	0.22	0.02	0.17	0.26
Skim Milk^	Gallons/month	0.26	0.01	0.23	0.32

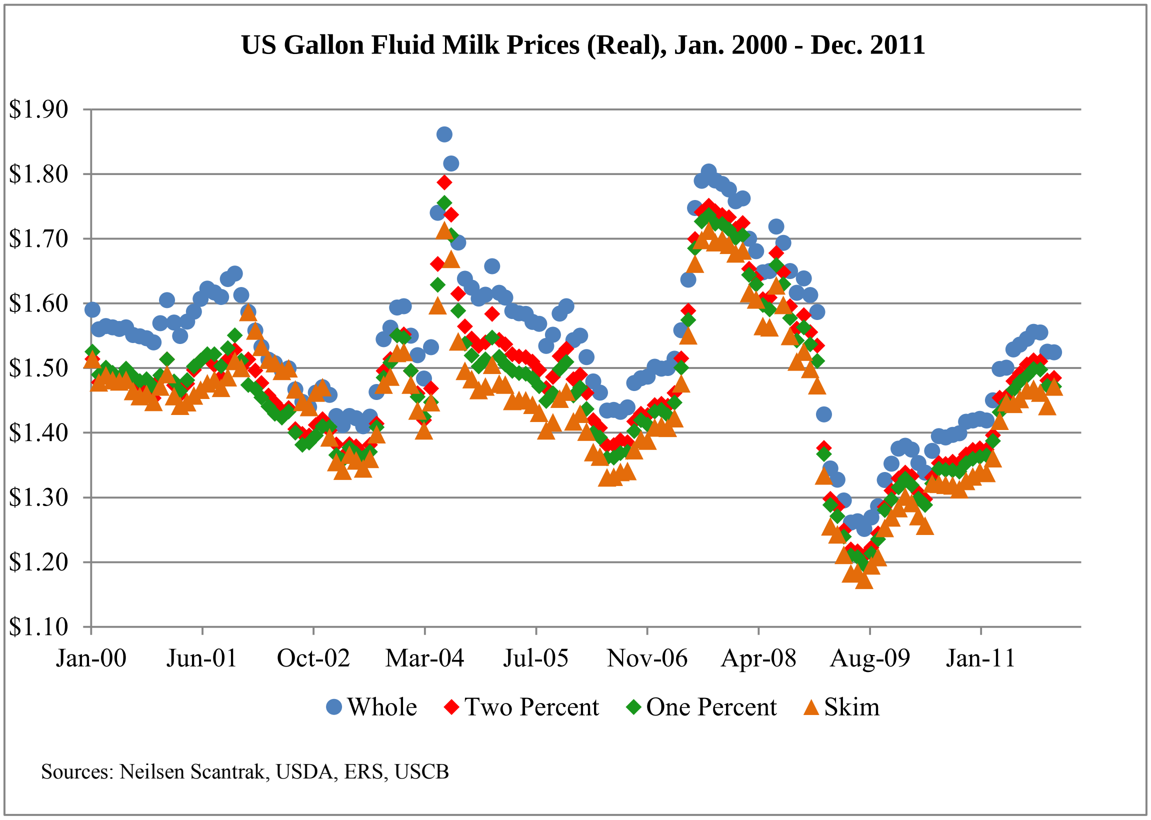
/: Per capita median income in 2011 dollars; * Adjusted for inflation using CPI; ^: per capita consumption

Sources: Calculated by author.

The main take-away from this table is the price proximity among all four milk type prices. Though these prices are typically related to fat content, there has been a tighter price range among the milk types in recent months (see figure 4). We see that two percent milk and whole milk have the highest per capita consumption, with skim and one-percent milks having the least. Though not clear in the table, per capita consumption of these four milk types has been on a decline. This decline is hypothesized to be related to milk advertising funds decreasing as well as more dairy options becoming available on the market, and health concerns discussed previously.

Though there are four fluid milk types of interest, a low-fat milk category was created to group two percent and one percent milks. This grouping is plausible since whole milk is the most high in fat fluid milk while skim milk is the least in fat (or no fat). By grouping the milk types together, we are able to better analyze milk fat types and their relationships among each other as well as how advertising affects consumption of different milk fat types.

Figure 4: United States Fluid Milk Real Prices, Jan 2000 - Dec. 2011



Quadratic Almost Ideal Demand System

Various models have been executed in measuring the effect of advertising on fluid milk such as a double logarithmic model (Kinnucan and Forker 1986; Kaiser and Reberte 1996), a Rotterdam model (Kinnucan *et al*. 2001), and an Almost Ideal Demand System (AIDS) model (Zheng and Kaiser 2008). Following Zheng and Kaiser (2008), we begin with an AIDS model (Deaton and Muellbauer 1980) which takes the following form:

$$w_{it} = \alpha_i + \beta_i ln\left(\frac{m_t}{P}\right) + \sum_j \gamma_{ij} ln(p_{jt}) + e_{it} \tag{1}$$

where w_{it} is the budget share for good i at time t, α_i is a constant for milk product i, m_t is total expenditures at time t, P is a price index which will be further defined below, p_{jt} is the price of good j at time t, β_i and γ_{ij} are parameters to be estimated and e_{it} is the error term. The following restrictions on the AIDS models are imposed.

$$\begin{aligned} &\sum_i \alpha_i = 1,\ \ \sum_i \beta_i = 0,\ \ \sum_i \gamma_{ij} = 0 && \text{(Adding Up)} \\ &\sum_j \gamma_{ji} = 0 && \text{(Homogeneity)} \\ &\gamma_{ij} = \gamma_{ji} && \text{(Symmetry)} \end{aligned} \tag{2}$$

Homogeneity restrictions imply that the budget shares will not change if all prices and expenditures are multiplied by the same positive constant while symmetry restrictions require compensated demand effects to be symmetric (Hahn 1994). In general, two price indexes are used – the Stone Price Index (SPI) and the Translog Price index (TPI). The SPI takes the following form:

$$lnP \approx \sum_i w_i ln(p_i). \tag{3}$$

As suggested by Deaton and Muellbauer (1980) which is further emphasized by Hahn (1994), the 'true' AIDS model utilizes the TPI which takes the following form:

$$lnP = \alpha_0 + \sum_k \alpha_k ln(p_k) + \frac{1}{2}\left(\sum_k \sum_l \gamma_{kl} ln(p_k) ln(p_l)\right). \tag{4}$$

As Banks *et al*. (1997) noted, incomes vary considerably across individuals, and income elasticities vary across goods; therefore, the income effect for individuals at different points in the income distribution must be fully captured in order for a demand model to predict responses to tax reform (or other policy areas) usefully. To meet these criteria, Banks *et al*. (1997) developed a new demand system that has log income as the leading term in an expenditure share model and additional higher order income terms. In other words, the model allows for Engel curves that are potentially non-linear in the log of expenditure (Cranfield 2012). This model is referred to as the Quadratic Almost Ideal Demand System (QUAIDS) and takes the following form:

$$w_{it} = \alpha_i + \sum_j \gamma_{ij} lnp_j + \beta_i ln\left(\frac{m}{a(p)}\right) + \frac{\lambda_i}{b(p)}\left\{ln\left[\frac{m}{a(p)}\right]\right\}^2 + e_{it} \tag{5}$$

where variables are defined above as before, the *ln*(*a*(*p*)) is the TPI, and *b*(*p*) is the simple Cobb-Douglas price aggregator defined as:

$$b(p) = \prod_{i=1}^{n} p_i^{\beta_i} \tag{6}$$

and λ is defined as:

$$\lambda(p) = \sum_{i=1}^{n} \lambda_i ln(p_i), \qquad \text{where} \sum_i \lambda_i = 0. \tag{7}$$

However, λ(p) is assumed to be independent of prices, which makes the underlying indirect utility function where QUAIDS is derived to be observationally equivalent to PIGLOG class (price independent generalized logarithmic; see Banks *et al. 1*997). The demands generated are rank three (maximum possible rank for any demand system that is linear in functions of income (see Gorman 1981), exactly aggregable, are derived from utility maximization, and permit goods to be luxuries at some income levels and necessities at others (Banks *et al*. 1997). The QUAIDS model is advantageous because it embodies very flexible price and income effects (Cranfield 2012). Note, when $\lambda_i = 0$ for all *i*, QUAIDS collapses to the AIDS model; also, QUAIDS only has local monotonicity and curvature properties (Cranfield 2012).

Polynomial Distributed Lag Advertising

In order to model the effects for advertising, we incorporate a polynomial distributed lag model (Almon 1965) into the QUAIDS model as follows:

$$w_{it} = \alpha_i + \sum_j \gamma_{ij} lnp_j + \beta_i ln\left(\frac{m_t}{a(p)}\right) + \frac{\lambda_i}{b(p)}\left\{ln\left[\frac{m_t}{a(p)}\right]\right\}^2 + \sum_{k=0} \theta_{ik} lnA_{t-k} + e_t \tag{8}$$

where A represents advertising expenditures at time t-k and e is an error term, and other variables are described above. It is assumed that θ_{ik} can be represented with a polynomial of degree m, where m = 0 1 2 … , m such that for a particular commodity i:

$$\theta_{ik} = \varphi_0 + \varphi_1 k + \varphi_2 k^2 + \varphi_3 k^3 + ... + \varphi_m k^m \tag{9}$$

Suppose that a lag length of five is chosen for the advertising variable. This selection would imply that we have t-1, … t-5. Now, assuming a second degree polynomial for θ_{ik}, (i.e., m = 2, k = 1, … , 5), we obtain the following:

$$\theta_{ik} = \varphi_0 + \varphi_1 k + \varphi_2 k^2, \text{for k} = 0, 1, ..., \ 5 \tag{10}$$

By imposing head and tail restrictions of no effects before k = 0 and after k = 5, we have the following:

$$\begin{aligned} \theta_{i,-1} &= \varphi_0 - \varphi_1 + \varphi_2 = 0, \text{for k} = \text{-}1 \\ \theta_{i,6} &= \varphi_0 + 6\varphi_1 + 36\varphi_2 = 0, \text{for k} = 6 \end{aligned} \tag{11}$$

Combining like terms, we reach:

$$\begin{aligned} &\varphi_0 - \varphi_1 + \varphi_2 = 0 \quad \text{or} \quad \varphi_0 = \varphi_1 - \varphi_2 \\ &\text{Now, substituting in } \varphi_1 - \varphi_2 \text{ for } \varphi_0 : \\ &\varphi_1 - \varphi_2 + 6\varphi_1 + 36\varphi_2 = 0 \Rightarrow 7\varphi_1 + 35\varphi_2 = 0 \text{ or } \varphi_1 = -5\varphi_2 \\ &\text{Consequently, } \varphi_0 = -6\varphi_2 \quad \therefore \text{we need only to estimate } \varphi_2 \end{aligned} \tag{12}$$

However, since the lag length is not generally known in advance, we must estimate the distribution using varying numbers of periods, then choose the best among them (Almon 1965). In this sense, the carryover effects of advertising can be captured in a dynamic setting (Dharmasena *et al*. 2012). However, because a QUAIDS model is a complete demand system, we must use a common advertising lag among the three milk categories due to recovering the third equations using the adding up restriction. To determine the optimal PDL advertising lag, we ran the QUAIDS model multiple times, with varying PDL advertising lengths from k = 1 to k = 14.

While using demand systems to model the effects advertising has on various products is not a new idea, we model the effects of long term and short term advertising expenditures using a quadratic AIDS (QUAIDS) model. We modify the quadratic almost ideal system to include a PDL specification for advertising, incorporating generic advertising's effect on milk type consumption.

QUAIDS Model

The first complete demand system implemented here is a modified quadratic AIDS (QUAIDS) model. By construction, the model exhibits endogeneity issues with both expenditure shares and total expenditure. To adjust for total expenditure endogeneity, an instrumental variable (IV) approach was used. Following Attfield (1985), Capps *et al*. (1994), and Dharmasena and Capps (2012), the following IV regression was estimated:

$$\ln y_t = \tau_0 + \sum_{i=1}^{3} \tau_i \ln p_{it} + \tau_5 \ln inc_t + v_t \tag{13}$$

where y_t is the expenditure at time t, τ_0 is a constant, p_{it} is the price of good i at time t, inc is per capita income at time t, and v_t is the error term. Estimates are provided below in table 2:

Table 2: Expenditure Endogeneity Parameter Estimates (Equation 13)

	Coefficient	Standard Error
Price (whole)	2.071***	0.344
Price (Low – Fat)[7]	-2.026***	0.381
Price (skim)	0.921***	0.159
Income	0.885***	0.155
Constant	-9.198***	1.673
Observations	144	---
R-squared	0.854	---
Durbin Watson	1.97	

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

[7] 2% and 1% milks were modeled jointly to represent a low-fat category.

No correction for serial correlation was necessary in equation 13. From equation 13, y_t-hat was calculated which replaced *m* in equation 8. Seasonality and previous consumption were also accounted for in the QUAIDS model. Seasonality was incorporated to capture monthly consumption affects while the month's previous consumption was incorporated to capture habit formation; thus, the following model was estimated:

$$w_{it} = \alpha_i + \sum_j \gamma_{ij} lnp_j + \beta_i ln\left(\frac{\hat{m}}{a(p)}\right) + \frac{\lambda_i}{b(p)}\left\{ln\left[\frac{\hat{m}}{a(p)}\right]\right\}^2 + \sum_{k=0}^{k} \theta_{ik} lnA_{t-k} + \sum_{m=1}^{11} \pi_{im} D_m + \eta_i q_{i,t\text{-}1} + e_{it} \tag{14}$$

where parameters and variables remain as discussed above, D_q is a monthly dummy, π is the parameter associated with the monthly dummy, *m-hat* is the estimated total expenditure from the IV regression represented by equation 13, and $q_{i,t-1}$ is the previous month's consumption of good *i,* or the quantity lag of milk type *i*. In addition to the conditions stated in equation 2, because of the modifications, the following additional adding up restrictions must be met for the QUAIDS model in order to recover the dropped equation:

$$\begin{aligned} &\sum_{i=1}^{3} \theta_{ik} = 0 \\ &\sum_{i=1}^{3} \pi_{im} = 0 \\ &\sum_{qlagi=1}^{3} \eta_i = 0 \end{aligned} \tag{15}$$

One issue considered was the value to set the intercept of the Translog Price Index (TPI). As Deaton and Muellbauer (1980) noted, practical identification of alpha-not is likely to be problematic, and since the parameter can be interpreted as the outlay required for a minimal standard of living (Deaton and Muellbauer 1980), we chose to use one. Serial correlation of the errors for equation 14 was another concern with the model specification. After running the model specified in equation 14, the system errors were calculated, hypothesizing that no serial correlation of the error terms existed. This hypothesis was rejected.[8] Equation 14 was further modified for serial correlation using an AR(p) process. Following Hatanaka (1974), the model is corrected for serial correlation using a lagged difference and its rho estimator. Thus, the final model includes a serial correlation coefficient, ρ:[9]

$$\begin{aligned} w_{it}* = {} & \alpha_i + \sum_j \gamma_{ij} lnp_j + \beta_i ln\left(\frac{\hat{m}}{a(p)}\right) + \frac{\lambda_i}{b(p)}\left\{ln\left[\frac{\hat{m}}{a(p)}\right]\right\}^2 + \sum_{k=0}^{k} \theta_{ik} lnA_{t-k} + \sum_{q=1}^{3} \pi_{im} D_q + \eta_i \mathrm{q}_{i,t\text{-}1} \\ & - \rho_1 \Bigg(\alpha_i + \sum_j \gamma_{ij} lnp_j + \beta_i ln\left(\frac{\hat{m}}{a(p)}\right) + \frac{\lambda_i}{b(p)}\left\{ln\left[\frac{\hat{m}}{a(p)}\right]\right\}^2 + \sum_{k=0}^{k} \theta_{ik} lnA_{t-k} + \sum_{q=1}^{3} \pi_{im} D_q + \eta_i \mathrm{q}_{i,t\text{-}1}\Bigg)_{t\text{-}1} \\ & + \rho_1 (w_{it})_{t-1} + e_t \end{aligned} \tag{16}$$

where w_{it}^* is a modified budget share calculated using the predicted expenditure shares. Thus, we are able to estimate all three equations due to the summation of all budget shares at time t not summing to one.

QUAIDS Results

A modified quadratic AIDS model was used to estimate the demand relationships among three categories of milk types. Endogeneity of total expenditure and serial correlation[10] were taken into account and corrected for as discussed previously. The estimation results for a QUAIDS modified with an advertising PDL of degree two, lag five are presented below. Though several models were examined, the results of the model that incorporated an advertising lag of five were selected based on information criteria and model fit. The results are presented in table 3.

---

[8] An AR(p) model was needed to adjust for serial correlation.
[9] All data analysis was conducted in STATA v12.1
[10] This model was corrected for using an AR(3) coefficient.

Table 3: QUAIDS Parameter Estimates (Equation 16)

	Whole	Low-Fat	Skim
Alpha	0.006	0.007	0.000
	(0.004)	(0.007)	(0.002)
Beta	0.000	0.001	-0.000
	(0.001)	(0.002)	(0.001)
Gamma $_{i1}$	0.178***	-0.103*	-0.076***
	(0.050)	(0.054)	(0.016)
Gamma $_{i2}$	-0.103*	0.116*	-0.013
	(0.054)	(0.063)	(0.019)
Gamma $_{i3}$	-0.076***	-0.013	0.089***
	(0.016)	(0.019)	(0.009)
Lambda	-0.000	-0.000	-0.000
	(0.000)	(0.000)	(0.000)
Phi i2	-0.067*	-0.032	-0.026
	(0.038)	(0.064)	(0.016)
Quantity Lag	-0.006	-0.055	-0.012***
	(0.384)	(0.384)	(0.003)
January	-0.028	0.017	-0.010
	(0.384)	(0.651)	(0.162)
February	-0.040	-0.019	0.003
	(0.384)	(0.651)	(0.162)
March	-0.026***	-0.042***	-0.007***
	(0.003)	(0.004)	(0.001)
April	-0.024	0.004	-0.016
	(0.384)	(0.651)	(0.162)
May	-0.033	-0.014	0.002
	(0.384)	(0.651)	(0.162)
June	-0.024***	-0.013***	-0.000
	(0.002)	(0.004)	(0.001)
July	-0.016	0.040	-0.007
	(0.384)	(0.651)	(0.162)
August	-0.035	-0.016	0.000
	(0.384)	(0.651)	(0.162)
September	0.051	-0.002	-0.014*
	(0.651)	(0.162)	(0.008)
October	-0.040	-0.004	-0.005
	(0.651)	(0.162)	(0.014)
November	0.003	0.005***	-0.000
	(0.004)	(0.001)	(0.003)
December	0.213	0.044	0.054
	(2.953)	(3.623)	(0.842)
R-Squared	0.775	0.759	0.783
Observations	136	136	136

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

The equations for whole milk, low-fat milk and skim were estimated. We were able to estimate all three equations due to calculating the expenditure shares using the predicted expenditure values to compute each budget share, relaxing the summation of all budget shares equaling one while simultaneously enforcing homogeneity. Thus, our error covariance matrix is not singular even though all three goods in the system are estimated simultaneously.

Remembering that the dependent variable is expenditure share, we should be aware that the interpretation of these parameter estimates is not necessarily straightforward. Results indicate that quantity lag only significantly affects skim milk expenditure, and it affects it negatively. This result was not expected as it was hypothesized that previous month's consumption would positively affect the next month's expenditure. Seasonality informs us that budget shares are highest in December and lowest in March. December having significantly higher budget shares compared to the rest of the year is likely related to the holiday season and cooking more at home.

Though parameter estimates provide information regarding relationships, we are more interested in the resulting elasticities. Budget and price elasticities can be calculated by differentiating the QUAIDS model with respect to ln$m$ and lnp_j, respectively, to obtain the following[11]:

$$
\begin{aligned}
&\mu_i \equiv \frac{\partial w_i}{\partial \ln m} = \beta_i + \frac{2\lambda_i}{b(p)}\left\{\ln\left[\frac{m}{a(p)}\right]\right\} \\
&\mu_{ij} \equiv \frac{\partial w_i}{\partial \ln p_j} = \gamma_{ij} - \mu_i\left(\alpha_j + \sum_k \gamma_{jk} \ln P_k\right) - \frac{\lambda_i \beta_j}{b(p)}\left\{\ln\left[\frac{m}{a(p)}\right]\right\}^2 \\
&e_i = \frac{\mu_i}{w_i} + 1, \qquad e_{ij}^u = \frac{\mu_{ij}}{w_i} - \delta_{ij}, \qquad e_{ij}^c = e_{ij}^u + e_i w_j \\
&A_{ik} = \frac{\theta_{ik}}{w_i}
\end{aligned}
\tag{17}
$$

[11] To see the derivation of the elasticities, please see the appendix.

where μ_i and μ_{ij} are intermediate steps for the calculations, e_i is income elasticity with respect to good i, e_{ij}^u is uncompensated own price ($i = j$, $\delta_{ij} = 1$ if $i = j$, 0 otherwise) or cross price elasticity ($i \neq j$) where , δ_{ij} is the Kronecker delta, and e_{ij}^c is compensated own price ($i = j$) or cross price elasticity ($i \neq j$) (Banks *et al*. 1997), and A_{ik} is the resulting advertising elasticity for good i at PDL lag k. Some commodities have the characteristics of luxuries at low levels of total expenditure and necessities at high levels of expenditure. This result can be seen by examining the signs on β and λ. For instance, if β is positive and λ is negative, the budget elasticity will be greater than unity at low levels of expenditure and less than unity as total expenditure increases (Banks *et al*. 1997). Uncompensated and budget elasticities are shown below in table 4.

Table 4: QUAIDS Uncompensated Own- and Cross- Price Elasticities, and Expenditure Elasticities

	Whole	Low-Fat	Skim
Expenditure	1.001***	1.001***	0.999***
	(0.004)	(0.005)	(0.004)
Whole	-0.482***	-0.207*	-0.474***
	(0.146)	(0.108)	(0.098)
Low-Fat	-0.298*	-0.766***	-0.084
	(0.156)	(0.127)	(0.118)
Skim	-0.220***	-0.027	-0.442***
	(0.045)	(0.038)	(0.057)

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

All expenditure elasticities are positive, significant, and near one. Because this metric is not income elasticity, it cannot be fully interpreted that milk is a ‘necessity’ or normal good. Rather, within this budget set, as total expenditure increases, each milk type’s expenditure increases as well. Another interpretation is that expenditure elasticity reveals the percentage change in the consumption of a given milk type given a one percent change in the expenditure on the set of all milk types (Dharmasena 2010). Own-price elasticities for all milks are negative and are structurally different from zero, conforming to demand theory that as the price of a good increases, its quantity demand decreases.

Cross-price compensated elasticities inform us about the relationships among milk types. For instance, if the cross-price elasticity between two goods is positive – as the price of good one increases, the quantity consumed of good two also increases – this relationship indicates the two goods are (gross) substitutes. Alternatively, if the cross-price elasticity is negative, the two goods are (gross) complements. Though it is hypothesized that most milk types would be substitutes to one another, the results indicate milk types are complements to one another. Compensated (net) elasticities are presented below in table 5.

Table 5: QUAIDS Compensated Elasticities

	Whole	Low-Fat	Skim
Whole	-0.138	0.138	-0.130
	(0.146)	(0.109)	(0.098)
Low-Fat	0.198	-0.270**	0.412***
	(0.156)	(0.127)	(0.118)
Skim	-0.060	0.133***	-0.283***
	(0.045)	(0.038)	(0.057)

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Compensated elasticities (net relationships) allow inference about the substitution effect among milk types. We see again that all own price elasticities are negative, though whole milk lost its significance. However, we see a net substitution effect between low-fat and skim milk. Further results show that whole and low fat milk are substitutes, though not results are not significant.

The Slutsky equation $e_{ij}^{c} = e_{ij}^{u} + e_i w_j$ was used to calculate the compensated elasticities. Further, we can assess the symmetry and negativity conditions by examining the Slutsky matrix elements, calculated as $S_{ij} = w_i[e_{ij}^{c}]$ (Banks *et al*. 1997).

Table 6: QUAIDS Slutsky Symmetry Matrix

	Whole	Low-Fat	Skim
S_{i1}	-0.047	0.068	-0.021
	(0.050)	(0.054)	(0.016)
S_{i2}	0.068	-0.134**	0.066***
	(0.054)	(0.063)	(0.019)
S_{i3}	-0.021	0.066***	-0.045***
	(0.016)	(0.019)	(0.009)

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 6 results indicate that the Slutsky matrix is negative semidefinite and symmetric, satisfying theoretical regulatory conditions of demand theory.

The main purpose of this research was to examine the effects of generic advertising on individual milk types. To capture advertising effects, we incorporated a PDL. The estimated parameter, φ_2, is the only parameter needed to be estimated in order to recover the theta values (because of end point restrictions). For a lag length of five, we recover six thetas corresponding to time lags one – five as well as the contemporaneous theta, or theta-not. Each theta is calculated such as:

$$\theta_k = \varphi_0 + k * \varphi_1 + k^2 * \varphi_2, \qquad k = 0,...,5 \tag{18}$$

The thetas are presented below for each milk type in table 7:

Table 7: QUAIDS Recovered PDL Advertising Thetas

	Whole	Low-Fat	Skim
Theta 0	0.403*	0.194	0.154
	(0.228)	(0.387)	(0.097)
Theta 1	0.671*	0.323	0.257
	(0.381)	(0.645)	(0.161)
Theta 2	0.806*	0.388	0.308
	(0.457)	(0.774)	(0.193)
Theta 3	0.806*	0.388	0.308
	(0.457)	(0.774)	(0.193)
Theta 4	0.671*	0.323	0.257
	(0.381)	(0.645)	(0.161)
Theta 5	0.403*	0.194	0.154
	(0.228)	(0.387)	(0.097)

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Though all positive, which is expected, the values are much larger than expected. Values this large will result in large long – run effects and corresponding elasticity values. For instance, the long run effect, which is simply the summation of all thetas for each milk type, is larger than three for whole milk, indicating that for each increase in advertising funds, the budget share for whole milk increases by three fold. These results are not logical nor do they resemble previous literature that examines advertising effects on fluid milk (Kaiser and Reberte 1996; Capps and Schmitz 1991). Nevertheless, the advertising elasticities were calculated and are presented below in table 8.

Table 8: QUAIDS Advertising Elasticities

	Whole	Low-Fat	Skim
Advertising Short Run	1.169*	0.391	0.966
	(0.663)	(0.780)	(0.605)
Advertising Long Run	10.914*	3.652	9.012
	(6.191)	(7.280)	(5.645)

As expected, based on the high theta values, the advertising elasticities are far too large. Results indicate, in the long run, that a 1% increase in advertising expenditures leads to an 11% increase in whole milk consumption. Due to the unappealing results of the multiple QUAIDS model runs, we concluded that the model selected was not consistent with a priori beliefs about advertising elasticities. Subsequently, it was decided to implement the Barten Synthetic Model.

Barten Synthetic Nested Demand Model

Barten (1993) developed a demand system that incorporated four demand models. The four models that compose the Barten Synthetic Demand Model (BSM) include the Rotterdam (Theil 1965 and Barten 1966), the AIDS model (linearized approximation, LA/AIDS),[12] (Deaton and Muellbauer 1980a), the (Dutch) Central Bureau of Statistics (CBS) (Keller and van Driel 1985), and the NBR model (Neves 1987). The equations are as follows:

[12] This is the linear approximation of the AIDS in differential form where the Translog Price Index is replaced by the Stone Price Index (Matsuda 2005).

$$
\begin{aligned}
&w_i d\log q_i = b_i d\log Q + \sum_{j=1}^{N} s_{ij} d\log p_j && \text{(Rotterdam)} \\
&w_i (d\log q_i - d\log \overline{m}) = c_i d\log Q + \sum_{j=1}^{N} s_{ij} d\log p_j && \text{(CBS)} \\
&dw_i = c_i d\log Q + \sum_{j=1}^{N} r_{ij} d\log p_j && \text{(LA/AIDS)} \\
&dw_i + w_i d\log q_i = b_i d\log Q + \sum_{j=1}^{N} r_{ij} d\log p_j && \text{(NBR)}
\end{aligned}
\qquad (19)
$$

where Q is the divisia price index, $d\log Q \equiv \sum_i w_i d\log q_i$, $s_{ij} = \left(\frac{p_i p_j}{m}\right)\left(\frac{\partial h_i}{\partial p_j}\right)$ which is the Slutsky ij^{th} term of the Slutsky matrix, $c_i \equiv b_i - w_i$, and $r_{ij} \equiv s_{ij} + w_i(\delta_{ij} - w_j)$. Notice that the CBS, the LA/AIDS, and the NBR equations do not have the same left hand side as that of the Rotterdam, but with simple algebra, each equation can have the same left hand side formation; the following modified equations are nested within the BSM:

$$
\begin{aligned}
&w_i d\log q_i = b_i d\log Q + \sum_{j=1}^{N} s_{ij} d\log p_j && \text{(Rotterdam)} \\
&w_i d\log q_i = (c_i + w_i) d\log Q + \sum_{j=1}^{N} s_{ij} d\log p_j && \text{(CBS`)} \\
&w_i \log q_i = (c_i + w_i) d\log Q + \sum_{j=1}^{N} [r_{ij} - w_i(\delta_{ij} - w_j)] d\log p_j && \text{(LA/AIDS`)} \\
&w_i d\log q_i = b_i d\log Q + \sum_{j=1}^{N} [r_{ij} - w_i(\delta_{ij} - w_j)] d\log p_j && \text{(NBR`)}
\end{aligned}
\qquad (20)
$$

Barten (1993) showed that although none of the above equations had another nested within it, a synthetic model of relatively simple form that nested these four differential demand systems could be constructed as (Matsuda 2005):

$$
w_i d\log q_i = (\beta_i + \lambda w_i) d\log \overline{m} + \sum_{j=1}^{N} [\gamma_{ij} - \mu w_i(\delta_{ij} - w_j)] d\log p_j \qquad (21)
$$

where $\beta_i \equiv (1-\lambda_i)b_i + \lambda c_i$ and $\gamma_{ij} \equiv (1-\mu)s_{ij} + \mu r_{ij}$ and other variables are as described as before. Equation 21 is reduced to the Rotterdam when $(\lambda, \mu) = (0, 0)$, to the CBS when

(λ, μ) = (1, 0), to the NBR when (λ, μ) = (0 1), and to the LA/AIDS when (λ, μ) = (1 1) (Matsuda 2005). One of the very attractive side effects of using a BSM is the ability to test which model within the synthetic model is the best fit to the data set, based on the values of λ and μ. Though it is obvious that this synthetic model serves well for the purpose of specifying functional forms of differential demand systems, it is in a rather mechanical manner that λ and μ are involved in linear combinations of the coefficients of the nested models; however, their economic implications seem unclear, particularly when they take values other than zero and unity (Matsuda 2005).

There is an abundance of previous literature on the uses of the BSM and various other synthetic models and their applications. Specific applications include cannibalization (Yuan *et al*. 2009), US demand structure for rice and its close substitutes (Gao *et al* 1994), consumer demand for alcoholic beverages (Gao *et al*. 1995), and advertising effects for alcoholic beverage consumption in the United Kingdom (Duffy 2001). None of the literature read discussed a BSM incorporated with generic advertising for milk products only. While the incorporation of distributed lags has been modeled before within a Rotterdam model (Capps and Schmitz 1991), emphasis was on nutritional information.

Many demand models have been used to model milk consumption, the relationships among the four milk types, and generic advertising effects. The idea of incorporating generic advertising into a BSM strictly modeling fluid milk demand is new. Further, the data is representative of the entire United States, a unique trait missing from most milk demand studies (cited previously). The importance of modeling milk demand correctly cannot be taken lightly. Recalling the milk consumption graph, fluid milk consumption, particularly whole milk, is on a decline. Properly understanding the relationships among the milk types as well advertising effects is imperative in order to advise advertising campaigns and producers how best to combat this declination.

To examine the effects of generic advertising on milk type consumption, we estimate a Barten Synthetic Model, incorporated with a polynomial distributed lag advertising variable (Almon 1965). Our BSM takes the following form:

$$w_{it} d \ln q_{it} = (\beta_i + \lambda w_i) d \ln Q + \sum_{j=1}^{N} [\gamma_{ij} - \mu w_{it} (\delta_{ij} - w_{jt})] d \ln p_{jt} + \sum_{k=0} \alpha_{ik} \ln Adv_{t-k} \tag{22}$$

where w_{it} is the i^{th} budget share for time period t, q_{it} is the quantity of the i^{th} good at time period t, p_{it} is the i^{th} price at time period t, Adv_{t-k} is the PDL representation of advertising at time t with lag k, and ln refers to the natural log. The remaining variables are as follows:

$$\begin{aligned} d \ln q_{it} &= \ln\left(\frac{q_{it}}{q_{i,t-1}}\right) \\ d \ln p_{it} &= \ln\left(\frac{p_{it}}{p_{i,t-1}}\right) \\ d \ln Q &= \sum_i w_i * d \ln q_i \end{aligned} \tag{23}$$

As with other models mentioned, restrictions of the BSM are imposed to satisfy demand theory. The restrictions are as follows:

$$\begin{aligned} &\sum_i \beta_i + \lambda = 1, \quad \sum_i \gamma_{ij} = 0 \quad \forall j && \text{(adding up)} \\ &\sum_j \gamma_{ij} = 0 \quad \forall j && \text{(homogeneity)} \\ &\gamma_{ij} = \gamma_{ji} \quad \forall i, j && \text{(Slutsky Symmetry)} \end{aligned} \tag{24}$$

Barten Results

A Barten Synthetic Model modified with advertising effects is estimated using the `nlsur' command in Stata v12.1. Estimated equations include a free form, symmetry imposed, and symmetry and homogeneity imposed. Advertising is incorporated using a PDL of degree two, lag five.[13] All three equations were estimated. This was possible because when calculating budget shares, we replaced the denominator with a predicted total expenditure, thus avoiding singularity of the error covariance matrix. For instance, our expenditure shares take the following form:

[13] Multiple model specifications were estimated using various lag length values from k = 1, … 14. A lag of five was selected based on information criteria, previous research, and the following chapter.

$$w_{it}^{*} = \frac{p_i * q_i}{\widehat{te}} \tag{25}$$

where *te-hat* was estimated using the method described before to rid the QUAIDS system of expenditure endogeneity. Serial correlation of the error terms was tested with the null hypothesis being that no serial correlation exists. We failed to reject the hypothesis, thus no serial correlation adjustment of the Barten Synthetic Model was necessary. Parameter estimates for the estimation with both homogeneity and symmetry imposed are presented in table 9.[14]

Table 9: Barten Synthetic Model Estimates (Equation 22)

	Whole	Low-Fat	Skim
Beta	0.330***	0.493***	0.143***
	(0.016)	(0.020)	(0.007)
Gamma i1	-0.116	0.094	0.022
	(0.111)	(0.097)	(0.039)
Gamma i2	0.094	-0.103	0.009
	(0.097)	(0.100)	(0.036)
Gamma i3	0.022	0.009	-0.031
	(0.039)	(0.036)	(0.020)
lambda	0.034	0.034	0.034
	(0.039)	(0.039)	(0.039)
mu	0.022	0.022	0.022
	(0.054)	(0.054)	(0.054)
Advertising	5.58e-7	-6.03e-7	9.54e-8
	(0.000)	(0.000)	(0.000)

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

[14] When comparing across the three estimation procedures, results did not alter much in regard to signs and significance.

As presented previously, various functional forms of specific demand functions can be tested using the results of the Barten Synthetic Model, specifically the *lambda* and *mu* parameters. Below in table 10 are the test results of each functional form along with their p-values and chi-squared values:

Table 10: Barten Synthetic Model Functional Form Tests

Test	Lambda	Mu	Chi-Square	P-Value
Rotterdam	0	0	0.92	0.63
LA/AIDS	1	1	921.90	0.00
CBS	1	0	607.93	0.00
NBR	0	1	324.24	0.00

From the above table, it is clear that all functional forms are rejected except the Rotterdam model. When looking at the parameter results of the estimated model, both *lambda* and *mu* are not statistically different from zero. Therefore, it is likely that the only functional form that is not rejected is the Rotterdam model, which this table supports.

Elasticities for expenditure and price (compensated own price and cross price) were calculated as:

$$e_i = \frac{\beta_i}{w_i} + \lambda, \qquad e_{ij}^C = \frac{\gamma_{ij}}{w_i} - \mu(\delta_{ij} - w_j), \qquad e_{ij}^U = e_{ij}^C - e_i w_j \tag{26}$$

and are presented in table 11.

Table 11: Barten Synthetic Model, Uncompensated Own-Price and Cross Price Elasticities and Expenditure Elasticities

	Whole	Low-Fat	Skim
Expenditure	0.991***	1.028***	0.928***
	(0.025)	(0.014)	(0.021)
Whole	-0.694**	-0.172	-0.190
	(0.315)	(0.200)	(0.255)
Low-Fat	-0.229	-0.730***	-0.417*
	(0.287)	(0.205)	(0.216)
Skim	-0.098	-0.150**	-0.360***
	(0.118)	(0.069)	(0.131)

Standard errors in parentheses
*** $p<0.01$, ** $p<0.05$, * $p<0.1$

All expenditure elasticities are near one and statistically significant. As total expenditure increases, each milk type's expenditure increases as well. All own price elasticities are negative and significant. The own price elasticity results indicate that all milk types are inelastic with skim milk being the most inelastic. In regards to cross price elasticities, these results suggest that all milk types are complements; however, the only significant relationships are for low-fat and skim milks. We see that low-fat milk is a complement for skim milk and vice versa. Perhaps within households, there is a demand for both low-fat and skim milks. For instance, persons within the household could have different preferences, or higher in fat milks may be preferred for cooking.

Table 12: Barten Synthetic Model, Compensated Elasticities

	Whole	Low-Fat	Skim
Whole	-0.352	0.183	0.130
	(0.314)	(0.199)	(0.255)
Low-Fat	0.263	-0.219	0.044
	(0.287)	(0.205)	(0.216)
Skim	0.060	0.014	-0.211
	(0.118)	(0.069)	(0.131)

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

The compensated elasticities in table 12, which include substitution effects, do not provide as clear of a picture. Though all of the own price elasticities are negative, none of them are significant. Further, no significant relationships exist among the cross-price elasticities. As with the QUAIDS model modification, a PDL was incorporated into the Barten synthetic model. Table 13 presents the recovered theta estimates.

Table 13: Barten Synthetic Model, PDL Advertising Recovered Theta Estimates

	Whole	Low-Fat	Skim
Theta i0	-3.35e-06	3.62e-06*	-5.73e-07
	(2.99e-06)	(2.44e-06)	(1.20e-06)
Theta i1	-5.58e-06	6.03e-06*	-9.54e-07
	(4.98e-06)	(4.07e-06)	(1.99e-06)
Theta i2	-6.70e-06	7.24e-06*	-1.15e-06
	(5.97e-06)	(4.89e-06)	(2.39e-06)
Theta i3	-6.70e-06	7.24e-06*	-1.15e-06
	(5.97e-06)	(4.89e-06)	(2.39e-06)
Theta i4	-5.58e-06	6.03e-06*	-9.54e-07
	(4.98e-06)	(4.07e-06)	(1.99e-06)
Theta i5	-3.35e-06	3.62e-06*	-5.73e-07
	(2.99e-06)	(2.44e-06)	(1.20e-06)

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

We can see that significant advertising effects exist only for low-fat milk. Though the signs are negative for both whole and skim, the magnitudes are not statistically significant. However, we see positive advertising relationships for low-fat milk. Low-fat's theta values are near zero as well, but it is evident that a positive advertising effect exists for low-fat milk consumption. Long term and short term advertising effects can be calculated based on the theta parameters. However, even when summed for each milk type, total effects are still zero. In table 14, the advertising elasticities are calculated based on the recovered theta estimates.

Table 14: Barten Synthetic Model, Advertising Elasticities

	Whole	Low-Fat	Skim
Short Run Advertising	-9.71e-6	7.30e-6*	-3.58e-6
	(8.66e-6)	(4.92e-6)	(07.48e-6)
Long Run Advertising	-0.00009	0.00007*	-0.00003
	(0.00008)	(0.00005)	(0.00007)

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

We see positive, significant effects for low-fat milk; however, the estimates are still close to zero. Though negative for both whole and skim milks, the elasticities are not statistically significant.

Conclusion

Milk consumption has been on a decline for many years. Understanding how advertising affects milk consumption is essential to the fluid milk industry. This research suggests that generic advertising affects milk type consumption differently. Advertising effects were estimated by incorporating a polynomial distributed lag into both a quadratic AIDS and Barten Synthetic model. Results from the quadratic AIDS model show positive advertising affects for the three milk types, though the magnitudes were far too large. The Barten Synthetic Model results show positive advertising effects only for low-fat milk, with magnitudes near zero. While the advertising results were not similar between the two models, budget, own-, and cross-price elasticities were. Both models report budget elasticities near one for all milk types and show negative own price elasticities, compensated and uncompensated. Though originally hypothesized to be substitutes, uncompensated elasticities show that milk types, in general, are complements to each other suggesting consumers prefer a variety of milk types in the home.

One reason the QUAIDS model reported the large advertising elasticities may be related to the model being highly quadratic or prices being extremely collinear. The restrictions built into a complete demand system should mitigate collinearity, but perhaps the high correlations among milk prices are creating estimation issues for the PDL. Using an estimated expenditure rather than the actual expenditure may also be creating estimation issues with the advertising elasticities. Though the remaining elasticities were similar to the Barten results, more work needs to be focused on how to best incorporate a PDL advertising variable into a QUAIDS model. Other modeling considerations included first differencing the PDL variable in the Barten Synthetic model. The math to incorporate a first difference PDL was clear. The implementation was conducted showing that both low-fat milks and skim milk had positive advertising effects, though none were statistically significant. More time needs to be spent here ensuring the recovering of the parameters is implemented correctly. However, the Barten Sytenthic model provided results which most closely aligned to hypothesized advertising effects.

Milk is advertised generically, but the effects are not generic. If the milk industry's goal is to increase the consumption of milk, developing campaigns that cater to specific milk types may be of interest. Further with health issues being a public concern, particularly obesity, milk campaigns may be able to exploit consumers and cater to trends focusing ads on healthier milk options. In the case of whole milk, whose declination is extreme, campaigns may target mothers with young babies, athletes, and women in general (who are known to develop osteoporosis more so than men). Regardless, it is clear that advertising has different effects across the separate milk types.

CHAPTER III

DYNAMICS OF ADVERTISING AND DEMAND FOR MILK IN THE UNITED STATES: THE INCOMPLETE DEMAND SYSTEM

Introduction

By invoking separability, demand systems allow us to examine subsets of goods, or goods whose consumption is not affected by the prices of other sets of goods. To complete demand analysis, two key factors are required – prices and quantities. Some examples of demand system analysis include analyzing non-alcoholic beverages (Kinnucan *et al*. 2001; Zheng and Kaiser 2008), milk (Kaiser and Reberte 1996), non-durables (Deaton and Muellbauer 1980), and meat (Nayga and Capps 1994; Brester and Schroeder 1995). The demand analysis literature focuses on two classes of demand systems – complete and incomplete, or single equation. The former is classified as such due to its system's income equaling the total sum of each good's expenditure. Some of the more well-known examples of complete demand systems include the Rotterdam system (Theil 1965), the Almost Ideal Demand System[15] (AIDS) (Deaton and Muellbauer 1980), and the Barten Synthetic Model (BSM) (Barten 1993).

One commodity that is of much interest is fluid milk. For multiple years, per capita fluid milk consumption has been on a decline, though total dairy product consumption has been on a rise (ERS 2013). While milk leaves the farm to become part of a plethora of dairy products, fluid milk processors are likely curious as to why total fluid milk consumption is falling. One reason may be related to health concerns. Multiple studies have been conducted in recent years relating the correlation between the rise in obesity and milk consumption (Berkey *et al.* 2005; Wiley 2010; Mozaffarian *et al*. 2011; Chen *et al*. 2012), though findings are mixed. Further, Women, Infants, and Children's (WIC) requirements (2012) were recently modified to only allow parents whose child(ren) is(are)less than two years as being eligible to use WIC to purchase *whole* milk.

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[15] Including its variations such as the linearized (LA/AIDS) and quadratic (QUAIDS) (Banks *et al*. 1997).

Milk is a unique commodity because it offers four types of milk to analyze, all with different milk fat contents, similar prices, and a generic advertising campaign which is not designed to target specific milk advertising strategies. Previous studies have analyzed generic advertising's effect on milk consumption. While different methods of modeling advertising have been used, results are rather similar. Kaiser and Reberte (1996) used a log-log model with eleven monthly advertising lags. Though results were confined to New York City, long term advertising elasticities were positive and significant ranging from 0.16 (whole milk) to 0.19 (low-fat milk). In a similar model specification, Kinnucan and Forker (1986) examined monthly interactions with advertising strategies finding that the cumulative effect of milk advertising on sales was the greatest in months when consumers have the strongest preference for milk. These results, too, were confined to New York City. Another study, conducted by Capps and Schmitz (1991), used a log-log model modified by a polynomial distributed lag to capture advertising effects of milk in Texas, finding a long run advertising elasticity effect of 0.0075 for fluid milk.

Though these findings have similar results, implications can only be utilized in the specific areas each particular study catered to or for all fluid milk. In addition, consumption behavior within the United States (US) population has changed, particularly where milk is concerned. To better address this issue, using recent data representative of the entire US may be more appropriate and applicable. Further, addressing the key issue of whether each milk type has the same advertising lag and effect is necessary to model advertising effects appropriately for each milk type.

Methodology

Though complete demand systems have been used to model the effects of advertising on various products, one key issue that exists within the complete demand system framework is singularity of the variance-covariance matrix of error terms. Due to the sum of expenditures equaling the income of the system, estimation can only occur for N-1 equations, using adding up to recover the final equation. Thus, for relationships that may seem transparent, such as advertising effects, the system's framework may force at least one negative relationship for the goods within the system. For instance, Kinnucan *et al*. (2001) utilized a Rotterdam model to analyze the effects of advertising on non-alcoholic beverages, finding negative advertising effects and elasticities for a few of the beverage categories. Likewise, Zheng and Kaiser (2008), who examined similar goods as Kinnucan *et al*. (2001) but with an AIDS model, also had a negative own advertising elasticity for juice. Simply stated, complete demand systems force a negative relationship with at least one variable for one good within the system.[16]

Another issue, with less concern it seems, is the correlation of variables within the system, particularly prices. Take fluid milk for instance. While whole, two-percent, one-percent, and skim prices will not necessarily be equal to one another, it is likely all four prices are related. Within the complete demand system framework, we cannot drop a price from an equation to adjust for correlation issues. Because of the mentioned concerns, another class of demand systems is appealing.

Incomplete demand systems allow a more general class of functional forms than complete demand models (LaFrance and Hanemann 1989). The added generality is due to the adding-up condition not being an equality restriction but rather an inequality restriction on the total expenditure for the goods of interest (LaFrance and Hanemann 1989). An incomplete system can be linear in the prices of the goods of interest and in total expenditure and can satisfy the conditions for integrability (LaFrance 1985).

There are many forms of incomplete demand systems.[17] As von Haefen (2002) shows, the dependent variable form is flexible and may be an expenditure share, expenditure, or the actual quantity. Other than having flexibility where the dependent variable is concerned, incomplete demand system framework allows us to include income as a variable, as opposed to system expenditure. This alleviates one concern and provides a desired outcome. Endogeneity of expenditure is not an issue as income rather than system expenditure is used. Hence, we can generate income elasticities as

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[16] There are various `tricks' that can be done to fool the adding up constraint. See the previous chapter.
[17] To see multiple variations of incomplete demand models, see Appendix I.

compared to expenditure elasticities. Incomplete demand systems typically incorporate quantity, price, and income. A simple example of an equation is (Lafrance 1985):

$$Q_i = \alpha_i(q) + \sum_{k=1}^{n} \beta_{ik} p_{ik} + \gamma_i y, \quad \forall\, i \tag{27}$$

where Q is the Marshaillian demand for good i, q is the quantity of good i, p the price of good i, and y is income. This formula can be in log-log form, providing the elasticities as the estimates themselves. This general model was selected to analyze three fluid milk types, delineated by fat content including whole milk, low-fat (two-percent and one-percent) milk, and skim milk. Further, we utilize Zellner's (1962) seemingly unrelated regression technique (SUR) by estimating all of the equations together. This estimation technique is appealing due to possible correlation across the errors in different equations which can provide links that can be exploited during estimation (Wooldridge 2010).

Due to milk's decrease in per capita consumption of milk (ERS 2013), we are interested in the effects generic advertising has on milk. Specifically, we intend to capture advertising effects by incorporating a polynomial distributed lag (PDL) (Almon 1965) advertising variable, whose lag corresponds to each milk type. As was shown in the previous chapter, a polynomial of degree two can be used to recover time specific advertising effects. These effects then can be summed to provide long run advertising effects. The implementation of the PDL will differ from that specified previously. Rather than the entire system having the same lag length, each equation may possibly have a separate optimal lag length.

Optimal advertising lag lengths have been explored for many products including milk (Capps and Schmitz 1991; Kinnucan 1986; Kinnucan and Forker 1986; Kaiser and Reberte 1996). The number of optimal advertising lags for each of these papers differs slightly ranging from six months (Kinnucan 1986) to one year (Capps and Schmitz 1991). Clarke (1976) noted that 90% of the cumulative advertising effect of advertising on sales of mature, frequently purchased, low-priced items occurs within three to nine months of the advertisement. Following previous research and Clarke's (1976) assessment, lag lengths were searched varying from one month to 14 months. The optimal lag length for each milk type was chosen based on the Schwarz's Bayesian Information Criterion (SBIC) and overall model fit. Our modified incomplete demand model now takes this form[18]:

$$\ln q_{it} = \beta_{i1} \ln q_{i,t-1} + \beta_{i2} \ln inc_t + \sum_{k=0} \theta_{ik} \ln Adv_{t-k} + \sum_j \alpha_{ij} \ln p_{jt} + \sum_m \pi_i m_{mt} \tag{28}$$

where q_{it} is the per capita consumption (in gallons) of milk type `*i*' during month `*t*', *p* is the price for each milk type *i* at time *t*, *inc* is household income, *Adv* is the polynomial distributed lag advertising variable, and *m* is a dummy variable corresponding to the month of the observation. The optimal PDL advertising lag was five for total fluid milk and for each individual milk type. The lagged quantity was also included to capture how the previous month's consumption affects current consumption.

Because of such a large decrease in the quantity of milk consumed, particularly whole milk, there may be improved opportunities for milk advertising agencies to advertise specifically for each milk type, rather than for fluid milk in general. If the same pool of advertising funds in fact affects milk types differently, advertising strategies may be adjusted to compensate for milk types whose generic campaign does not affect consumption as much as other milk types. Further, there may be separate advertising time (lag) effects for each milk type, intensifying the need for campaign adjustments.

Data

The data used for this study correspond to milk consumption and prices from January 2000 to December 2011 and is representative of the entire US. Milk prices, from Neilsen Scantrak data, are deflated using the Consumer Price Index (CPI) (BLS 2014). Likewise, advertising expenditures were deflated by the CPI, and gathered from Dairy Management Inc, MilkPeP, and Qualified Programs. The advertising expenditures are reported quarterly; thus, we create monthly advertising expenditures, and then adjust those for seasonality using SAS v9.3. Quantities were reported in millions of pounds

[18] A moving average was considered as well

(ERS 2013; Neilsen 2013) and then converted to per capita consumption using Census population estimates (USCB 2012 2010 2001) and a conversion factor of 8.6 pounds of milk per gallon (Dairy Facts 2008). Median per capita income was retrieved from the Census (USCB 2013). Table 15 provides the summary statistics of the variables used.

Table 15: Incomplete Demand System Summary Statistics

Variable	Units of Measurement	Mean	Std. Dev.	Minimum	Maximum
Advertising*	US$	11,034,110	1,992,288	8,144,160	16,467,014
Base CPI	Price Index	198.25	17.29	168.8	226.89
Income/	US$	52,876.75	1,322.82	50,054.00	54,841.00
Whole Price*	US$/gal	1.53	0.12	1.25	1.86
2% Price*	US$/gal	1.47	0.12	1.21	1.79
1% Price*	US$/gal	1.46	0.12	1.2	1.76
Skim Price*	US$/gal	1.44	0.11	1.17	1.71
Fluid Milk^	Gallons/month	1.61	0.09	1.41	1.87
Whole Milk^	Gallons/month	0.54	0.07	0.41	0.67
2% Milk^	Gallons/month	0.59	0.03	0.53	0.66
1% Milk^	Gallons/month	0.22	0.02	0.17	0.26
Skim Milk^	Gallons/month	0.26	0.01	0.23	0.32

/: in 2011 dollars ; * Adjusted for inflation using CPI; ^: per capita consumption;

The summary statistics provide us with some useful information. We see that average monthly advertising expenditures exceed 11 million dollars. Milk prices (deflated) range from \$1.44 – \$1.53, with the higher prices corresponding to the higher milk fat content products. We can also see that total per capita fluid milk consumption is about one and a half gallons per month. Further, two percent milk is consumed the most, with whole milk being a close second.

Collinearity and Combining Milk Types

As was mentioned previously, collinearity among prices is a concern. Independent variables are perfectly collinear if one variable is an exact linear combination of the remaining variables, or in other words, if the data matrix does not have full rank. When this relationship occurs, obtaining regression results using ordinary least squares cannot be done. We can still obtain estimation results when there exists high collinearity among variables; however, estimates may have the wrong signs, be sensitive to slight changes in the data or model specification, or may not yield statistically significant results for theoretically important explanatory variables (Hill and Adkins 2001).

Due to the data we are modeling, it should not be surprising that high collinearity exists among the four milk prices. Table 16 provides the correlation estimates among the four milk prices.

Table 16: Fluid Milk Price Correlations

	Whole	2%	1%	Skim
Whole	1	--	--	--
2%	0.986	1	--	--
1%	0.989	0.993	1	--
Skim	0.942	0.950	0.955	1

As we can see, all four milk prices are highly correlated. Previous work on dairy milk demand by Kaiser and Reberte (1996) mentioned this problem. One method for adjusting a highly collinear problem is to drop specific prices from each equation. However, we instead create a price ratio. For each milk type `*i*', we use the following:

$$\text{Price Ratio}_i = \frac{p_i}{\left(\dfrac{\sum_{j\neq i} p_j q_j}{\sum_{j\neq i} q_j}\right)} \equiv \frac{p_i * \sum_{j\neq i} q_j}{\sum_{j\neq i} p_j q_j} \tag{29}$$

Using this specification of price, we are able to mitigate the issue of high price collinearity.

Several pieces of literature within the fluid milk sector have combined milk types to represent 'low-fat' milk (see literature above). In other words, two-percent and one-percent milks are combined to create a low-fat category. Quantities are simply added while price is an index calculated such as:

$$p_{2\%,1\%} = \frac{p_{2\%} * q_{2\%} + p_{1\%} * q_{1\%}}{q_{2\%} + q_{1\%}} \tag{30}$$

Due to combining two percent and one percent, we have three milk types and three equations to estimate. The three equations, though each has its own set of parameter estimates, are likely related through prices and consumption. Employing a seemingly unrelated regression method will help address this issue. Serial correlation of the error terms for each individual equation was of concern. The final equation for each milk type was corrected for serial correlation using the specific equation's serial correlation coefficient such as:

$$\begin{aligned} \ln q_{it} &= \rho_i * \ln q_{i,t-1} + \beta_{i1} \ln q_{i,t-1} + \beta_{i2} \ln inc_t + \sum_{k=0} \theta_{ik} \ln Adv_{t-k} + \ln pr_{it} + \sum_m \pi_i m_{mt} \\ &- \rho_i * \left(\beta_{i1} \ln q_{i,t-2} + \beta_{i2} \ln inc_{t-1} + \sum_{k=0} \theta_{ik} \ln Adv_{t-k-1} + \ln pr_{i,t-1} + \sum_m \pi_i m_{m,t-1} \right) + \varepsilon_{it} \end{aligned} \tag{31}$$

where q_{it} is the per capita consumption (in gallons) of milk type `*i*' during month `*t*', *pr* is the price ratio for each milk type *i* at time *t*, *inc* is household income, *Adv* is the polynomial distributed lag advertising variable with lag five, *m* is a dummy variable corresponding to the month of the observation, and ρ represents each equations' AR(p) serial correlation term.

Single Equation Estimation

First, we estimate all fluid milk and the milk types separately. Then, with SUR estimation in Stata v12.1, we estimate three equations for whole, low-fat, and skim milks. Results for single equation estimations are presented first. A single parameter estimate for advertising is the only parameter estimated in regards to the effect of advertising. This is possible because of the polynomial distributed lag of degree two with end point restrictions. Table 17 presents the single equation estimation results.

Table 17: Fluid Milk Single Equation Parameter Results

	All Fluid Milk	Whole	Low-Fat	Skim
AR(p)	AR(0)	AR(1,3)	AR(1)	AR(1,3)
Constant	-1.579	-1.503	-1.299	-4.092***
	(1.170)	(1.290)	(1.461)	(0.999)
Quantity Lag	0.059	0.946***	0.134	0.667***
	(0.091)	(0.031)	(0.091)	(0.064)
Price Ratio/	-0.069***	-0.043	-0.064*	-0.009
	(0.021)	(0.130)	(0.035)	(0.073)
Income	0.138	0.071	0.038	0.298***
	(0.092)	(0.092)	(0.162)	(0.089)
Advertising	-0.001*	-0.001*	-0.001*	-0.001**
	(0.000)	(0.000)	(0.001)	(0.000)
January	0.014*	-0.059***	0.020**	0.033***
	(0.008)	(0.011)	(0.008)	(0.010)
February	-0.095***	-0.158***	-0.091***	-0.087***
	(0.008)	(0.009)	(0.010)	(0.009)
March	0.001	0.036***	0.012	0.062***
	(0.009)	(0.008)	(0.011)	(0.007)
April	-0.054***	-0.099***	-0.051***	-0.044***
	(0.008)	(0.010)	(0.010)	(0.010)
May	-0.038***	-0.037***	-0.032***	0.001
	(0.007)	(0.009)	(0.010)	(0.009)
June	-0.082***	-0.073***	-0.092***	-0.059***

Table 17 Continued

AR(p)	All Fluid Milk AR(0)	Whole AR(1,3)	Low-Fat AR(1)	Skim AR(1,3)
	(0.007)	(0.009)	(0.010)	(0.008)
July	-0.055***	-0.021**	-0.060***	-0.008
	(0.008)	(0.009)	(0.011)	(0.009)
August	-0.021***	-0.033***	-0.016	0.017*
	(0.007)	(0.010)	(0.010)	(0.009)
September	-0.040***	-0.089***	-0.027***	-0.016**
	(0.007)	(0.008)	(0.009)	(0.007)
October	-0.006	-0.014	0.006	0.016*
	(0.007)	(0.009)	(0.009)	(0.008)
November	-0.033***	-0.068***	-0.029***	-0.037***
	(0.007)	(0.011)	(0.008)	(0.010)
Trend	-0.001***	--	0.001***	--
	(0.000)	--	(0.000)	--
R-Squared	0.9038	0.9818	0.7811	0.8742
Observations	139	136	138	136

Standard errors in parentheses; *** p<0.01, ** p<0.05, * p<0.1
/: For low-fat milk, the ratio is as described in equation (30); for the other types, the price ratio is as described in equation (29).

Results for all fluid milk indicate that over time, total fluid milk consumption has been on a significant decline (see trend). Seasonality is captured using monthly dummies, and we see that most months have lower total milk consumption when compared to December. Income, though not significant, indicates that milk is a necessity good; this coincides with Capps and Schmitz's (1991) analysis of fluid milk consumption in Texas, though their income coefficient was significant. Our optimal advertising lag length was a five lag polynomial distributed lag of degree two. Compared to Capps and Schmitz (1991) and Kaiser and Reberte (1996), our advertising lag length is short. However, our advertising lag length is similar to that of Clarke (1976) and Kinnucan (1986).

For the individual milk type equations, results resemble that of total fluid milk. A trend variable was included in the low-fat milk equation to capture the increase in purchases of low-fat milk over this time span. In fact, it is positive and significant, suggesting that even though total fluid milk consumption is on a decline, low-fat milk consumption has been increasing. We see that a quantity lag positively and significantly affects both whole and skim milks, indicating significant habitual purchasing behavior. Though the price ratios were negative for all milk types, only the low-fat milk ratio had a significant effect.

The estimated coefficient for advertising was negative; however, this coefficient is the estimated phi resulting from imposing no effect restrictions (heads and tails) on the PDL. To recover the total value of each advertising lag, or theta, we use simple algebra and substitution. The following formula was used in calculating each theta:

$$\theta_{ij} = \varphi_0 + i * \varphi_1 + i^2 * \varphi_2 \qquad \text{for i} = 0, \ldots 5, \forall \text{ j} \tag{32}$$

This formula builds a symmetric relationship, which is supported by the recovered values below provided in table 18.

Table 18: Fluid Milk Single Equation Recovered Thetas

	All Fluid Milk	Whole	Low-Fat	Skim
Theta 0	0.004**	0.004**	0.008**	0.003**
	(0.002)	(0.002)	(0.004)	(0.002)
Theta 1	0.007**	0.006**	0.014**	0.005**
	(0.004)	(0.004)	(0.007)	(0.003)
Theta 2	0.008**	0.008**	0.016**	0.006**
	(0.005)	(0.004)	(0.009)	(0.003)
Theta 3	0.008**	0.008**	0.016**	0.006**
	(0.005)	(0.004)	(0.009)	(0.003)
Theta 4	0.007**	0.006**	0.014**	0.005**
	(0.004)	(0.004)	(0.007)	(0.003)
Theta 5	0.004**	0.004**	0.008**	0.003**
	(0.002)	(0.002)	(0.004)	(0.002)

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Total fluid milk and individual milk types have positive, significant total advertising effects for each time period from current to five lags. Low-fat milk has the largest magnitude with skim having the highest significance. Summing all of the thetas for each milk types allows us to see the long run effects of advertising. Those results are presented below in Table 19.

Table 19: Fluid Milk Single Equation Advertising Effects

	All Fluid Milk	Whole	Low-Fat	Skim
Short Run Advertising	0.004**	0.004**	0.008**	0.003**
	(0.002)	(0.002)	(0.004)	(0.002)
Long Run Advertising	0.039**	0.036**	0.077**	0.030**
	(0.022)	(0.020)	(0.041)	(0.014)

The short run and long run advertising effects tell us the contemporaneous and total advertising effect. Long term effects are greatest for low-fat milk consumption. Though advertising is generic, it affects milk types differently; there is a two-fold increase in effects for low-fat milks when compared to both whole and skim milks. Because the equation was in log-log form, these resulting values are also elasticities. For a 10% increase in advertising expenditures, we see a 0.39% increase in total milk consumption, over a five month lag.

Seemingly Unrelated Regression Estimation

After estimating each equation individually, we estimated the equations using the `sureg' command in Stata which is the Seemingly Unrelated Regression technique (Zellner, 1962). Each equation was adjusted for serial correlation, specific to that equation (not the system). Parameter results for equation 31 are presented below in table 20:

Table 20: Fluid Milk SUR Estimation Parameter Results

	Whole	Low-Fat	Skim
AR(p)	AR(1,2)	AR(1)	AR(1,2)
Constant	1.383	-1.696*	-3.226***
	(0.898)	(1.018)	(0.756)
Quantity Lag	1.012***	0.739***	0.882***
	(0.009)	(0.040)	(0.019)
Price Ratio/	-0.047	-0.009	-0.027
	(0.041)	(0.010)	(0.019)
Income	-0.051	0.058	0.123***
	(0.035)	(0.067)	(0.035)
Advertising	-0.000	-0.001**	-0.000*
	(0.000)	(0.000)	(0.000)
January	-0.063***	0.005	0.028***
	(0.012)	(0.010)	(0.009)
February	-0.162***	-0.119***	-0.100***
	(0.010)	(0.008)	(0.008)
March	0.040***	0.050***	0.069***
	(0.008)	(0.009)	(0.007)
April	-0.101***	-0.067***	-0.055***
	(0.011)	(0.009)	(0.008)
May	-0.035***	-0.018**	-0.000
	(0.010)	(0.009)	(0.008)
June	-0.072***	-0.085***	-0.063***
	(0.009)	(0.009)	(0.008)
July	-0.019*	-0.016*	-0.001
	(0.010)	(0.009)	(0.008)
August	-0.033***	0.011	0.020**
	(0.011)	(0.009)	(0.008)
September	-0.090***	-0.028***	-0.020***
	(0.008)	(0.009)	(0.007)
October	-0.013	0.007	0.014*
	(0.010)	(0.008)	(0.008)
November	-0.068***	-0.047***	-0.045***
	(0.012)	(0.010)	(0.009)
Trend	---	0.000***	---
	---	(0.000)	---
R-Squared	0.996	0.848	0.963
Observations	137	137	137

Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1
/: For low-fat milk, the ratio is as described in equation (30); for the other types, the price ratio is as described in equation (29).

The standard errors of the SUR equation results are smaller than that of the single equation estimation, and coefficient estimates are asymptotically more efficient (Zellner 1962). Parameter estimates from SUR estimation will not be the same as those from the single equation estimation due to each equation having different explanatory variables and the error terms being correlated.

Contrary to previous results, income has a negative sign for whole milk consumption, though it is not statistically different from zero. All equations have negative price ratios, though none are significant. Advertising effects are significant for both low-fat and skim milks. Habit formation, captured by the quantity lag, increased for all three milk categories when compared to the single equation estimations. Milk consumption for the previous month significantly affects the quantity consumed for the next month.

Seasonality, captured by months, is similar when compared to the single equation estimation. In general, consumption during most months is lower when compared to December. Interestingly, whole milk consumption significantly drops during the month of January while skim milk consumption significantly increases when compared to December, perhaps relating to New Year's resolutions and persons trying to reduce calorie consumption. Milk consumption is high in March relative to all other months. This could be due to spring break vacations within schools and parents providing more milk at home for children during that time. Thetas are recovered from the advertising variable as discussed previously. Results are presented below in table 21.

Table 21: Fluid Milk SUR Estimation Recovered Thetas

	Whole	Low-Fat	Skim
Theta 0	0.000	0.004***	0.001**
	(0.001)	(0.002)	(0.001)
Theta 1	0.000	0.007***	0.002**
	(0.001)	(0.003)	(0.001)
Theta 2	0.000	0.008***	0.002**
	(0.001)	(0.003)	(0.001)
Theta 3	0.000	0.008***	0.002**
	(0.001)	(0.003)	(0.001)
Theta 4	0.000	0.007***	0.002**
	(0.001)	(0.003)	(0.001)
Theta 5	0.000	0.004***	0.001**
	(0.001)	(0.002)	(0.001)

Standard errors in parentheses
*** $p<0.01$, ** $p<0.05$, * $p<0.1$

The single equation estimation resulted in significant advertising effects for all milk types; with the SUR equation estimation, we see there are no significant effects of advertising for whole milk. This further supports the idea that though advertising is generic for all fluid milk, it has different effects for specific milk types. As with the single equation estimation, low-fat milk's advertising effects are higher in magnitude than that of skim milk, and in this case, are also more significant. The results are presented in table 22.

Table 22: Fluid Milk SUR Estimation Advertising Effects

	Whole	Low-Fat	Skim
Short Run Advertising	0.000	0.004***	0.001**
	(0.001)	(0.002)	(0.001)
Long Run Advertising	0.001	0.036***	0.009**
	(0.007)	(0.014)	(0.005)

Standard errors in parentheses
*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Long run advertising effects are smaller in magnitude for the SUR estimation compared to the single equation estimation. If advertising expenditures were to increase by 10%, we see that low-fat milk consumption would increase by about 0.36%, compared to more than 0.7% for the single equation estimation. Further, there is little effect for whole milk, and its advertising effects are not different from zero.

Conclusion

This paper measures the effects of a polynomial distributed lag advertising variable on fluid milk types using an incomplete demand system approach. We analyze advertising effects on per capita consumption for three milk types including whole milk, low-fat milk (two-percent and one-percent milk), and skim milk, mitigating collinearity issues among prices by using a price index. Single equation estimation for total fluid milk and each milk type is conducted followed by seemingly unrelated regression equation estimation for the three milk types. The optimal advertising lag length for all milk types is five months.

When estimating using a SUR, results suggest that long run advertising effects vary across milk types. For low-fat milk, advertising effects are the largest in magnitude as well as the most significant. Generic advertising had no effect on whole milk consumption. Advertising elasticities suggest that if advertising expenditures increase, both low-fat and skim milk consumption will increase, skim milk increasing only moderately. Both low-fat and skim milks are necessities while no income effect was found for whole milk. Seasonality suggests that milk consumption peaks during March and December for all milk types.

Due to the different advertising effects for whole, low-fat, and skim milks, advertising expenditures may be spent accordingly to cater to those consumption differences. For instance, since generic advertising does not affect whole milk consumption, perhaps a different strategy could focus on whole milk while the generic advertising (or low-fat advertising) caters to two-percent, one-percent, and skim milks. Further, such campaigns may be able to increase consumption of specific milk types.

While there are no separate advertising expenditures for milk types, future research should examine other similar products/commodities and their advertising effects. For instance, Pima cotton and `regular' cotton have separate advertising campaigns; perhaps examining if any variation exists in advertising effects between the two products would provide insight for other commodity advertising campaigns. Finding ways to accommodate high collinearity among prices is also of interest. Though complete demand systems' structures mitigate price collinearity through specific restrictions, such restrictions are not implemented in the incomplete demand systems approach. First differencing can reduce multicollinearity, but Burt (1987) points out that first differencing used in this manner is a `fallacy.' Though price indexes rid the issue of multicollinearity, it comes at a cost as we are no longer able to interpret cross – price effects.

CHAPTER IV

ECONOMIC AND SOCIODEMOGRAPHIC DRIVERS ASSOCIATED WITH VENDING MACHINE PURCHASERS IN THE UNITED STATES

Introduction

In recent years, the number of overweight and obese persons in the United States has increased drastically – so much that America is now facing an obesity epidemic (Health Tidbits 1999). Though dietary recommendations are available through the Government, poor eating choices are still made, making dietary improvement an important public priority (Guthrie *et al*. 2013). One industry that typically caters to poor or unhealthy eating behaviors is the vending machine industry. In 2012, the top selling product categories in vending machines were cold beverages ($5.97 billion) and candy, snack, and confections ($4.13 billion) (VWM 2013). Policies promoting interest in healthy foods and ways to accurately identify them may increase consumers' demand for these foods, improving diets and health (Guthrie *et al*. 2013). If we understood what influences the consumer who frequently purchases products from vending machines, health officials may be better equipped to target such consumers in hopes of steering their eating behaviors towards more nutritious and lower calorie items.

The vending machine industry is a multi-billion dollar industry (VMW 2013) which should come as little surprise since vending machines are now found in manufacturing sites, offices, various retail sites, educational sites, and many other areas. In 2012, vending machine sales reached $19.31 billion (VMW 2013). Though approximately only 19% of US households reported purchasing from vending machines in a two-week consecutive period in 2012, of those who did purchase, approximately $6.68 was spent during the two week period (BLS 2013). Approximately every two weeks, households who do purchase from vending machines consume more than $6 worth of items with about half of that amount being spent on beverages and candy, snacks, and confections, based on the top selling products provided by Vending Market Watch (VMW 2013). The capability to adequately ascertain historical, current, and future patterns of food consumption is of extreme importance (Capps and Schmitz 1991). It is clear as to why more understanding of this purchasing behavior is needed.

An abundance of literature exists on vending machine product purchases and the potential health concerns related to such purchases. For instance, Pasch *et al*. (2011), French *et al*. (2003), and Kubik *et al*. (2003) analyzed vending machines in schools and their product effects on eating behavior. In fact, vending machines were deemed as a 'common source of competitive foods' by Pasch's (2011) study indicating that school students were purchasing items from the machines instead of purchasing a school lunch. Because of public health concerns, the FDA set forth a final rule regarding calorie disclosure required by Obamacare, and the USDA proposed a rule regarding the items allowed in school vending machines (VWM 2013). In fact, in 2011 13.7% of vending machines were located in elementary, middle, and high schools while in 2012 that percentage decreased to 8.2% (VMW 2013). Though steps have been made towards decreasing overall calorie intake from vending machine purchases, especially for children, there is room for improvement.

Due to the FDA's final rule mentioned above, vending machine operators may be seeking other sources or means of generating income, which may be achieved in several ways. First, vending machines in schools can cater to those specifications deemed by the FDA. A study by Evans *et al*. (2005) found that 74% of its survey respondents favored restricting the availability of *unhealthy* foods in vending; perhaps including more healthy options would encourage parents to allow their children to purchase snacks from vending machines. However, restricting the items allotted in a vending machine may affect total vending machine sales.

Second, vending machine operators may be able to extract higher margins by targeting specific audiences or increasing the number of persons purchasing from a vending machine. A study by French *et al*. (2001), with machines placed both at secondary schools and various worksites, showed that when low fat snacks and 'regular' snacks have the same price, low-fat snacks are sold. Not surprisingly, as prices fall for the low-fat snacks, sales increase; additionally, average monthly profits did not significantly differ by price reduction strategies (French *et al*. 2001).

Some companies are seeing this 'health push' as a niche market in which to escape a slump in profits. For instance, Fresh Healthy Vending Company operates 800 vending machines in the United States, with machine locations other than public schools (Bayles 2011). A vending machine program, backed by Iowa's state health department with locations at various rest stops, had a higher than expected success rate with sales (Baker 2012). If more were known about current and potential vending machine purchasers, the vending machine industry could make better choices about what products to place where. Perhaps by catering to specific groups of current or potential customers, vending machine operators could expand product selection and comply with suggested government regulations while not decreasing profit margins. Further, the general public may have favorable outcomes on health, particularly in regards to weight issues. In other words, if we were to know what affects a person's decision of purchasing from a vending machine and what factors influence this purchasing behavior, we may be able to provide information to public health officials who are trying to push for a healthier America.

Food Away From Home Studies

Although dietary guidelines have been issued and updated since 1980, Americans still make poor eating choices, consuming too much saturated fat and sodium and too little fruits and vegetables (Guthrie *et al*. 2013). Several studies have examined what affects food away from home consumption, which is more of a nutritional concern than food consumed at home (Guthrie *et al*. 2013). For instance, McCracken and Brandt (1987) examined FAFH consumption and three subsets including expenditures at restaurants, fast-food facilities, and other commercial facilities, using a Tobit model. They hypothesized that FAFH differs by the type of eating establishment, and results support the hypothesis. Byrne *et al*. (1996) conducted a probit analysis of FAFH expenditures spanning from 1982 – 1989 finding that many socioeconomic and demographic characteristics significantly affect the decision to eat away from home. Logit analysis conducted on FAFH by Nayga and Capps (1992) found that employed individuals are more likely to consume FAFH than unemployed individuals; they also found that age decreases the probability of FAFH consumption, while income increases the probability of FAFH consumption.

Studies examining FAFH consumption also have incorporated nutritional information (Capps and Schmitz 1991; Kinnucan *et al*. 1997; Binkley 2006). These studies all find some statistical significance for the impact of nutritional information on food consumption. Though no nutritional information is included within this study, it is important to note that the Food and Drug Administration (FDA 2013) recently required calorie labeling on menus in chain restaurants, retail food establishments, and vending machines (with 20 or more locations/machines)[19]. This research does not include any of this effect as our data time period does not overlap with this regulation.

Though all of these studies are concerned with food away from home consumption, none directly examine expenditures at vending machines. One frequently cited reason for persistently poor diets is today's food environment which offers many opportunities to make unhealthy food choices (Mancino *et al*. 2009). Vending falls into this unhealthy food choice within the food environment. Using previous studies' methods on examining food away from home analysis will direct us to the proper analytical steps to take for examining what affects vending machine item consumption.

We want to form hypotheses about what affects a household's decision to purchase vending machine products including: (1) increased hours spent at work will significantly increase vending machine expenditures; (2) having school aged children/adolescents will increase a household's vending machine expenditures; (3) households who consume more salty/sugary snacks and beverages on average will positively affect spending at vending machines.

[19] Not all of the implementation has taken effect; further, vending machine operators who own less than 20 machines will not be required to post calorie information for food products.

There are several objectives of this research. First, we want to develop a profile of vending machine consumers. Second, we want to analyze consumers who purchase fresh fruits and fresh vegetables, noting any common patterns among vending machine purchasing households and fresh fruit and vegetable purchasing households. Ideally, we will be able to see if there is any potential for vending machines to cater to persons who do not yet purchase from vending machines due to the lack of fresh fruits or vegetables provided. Further, we want to determine what affects the probability of a household being above the limit (zero expenditures on vending machine purchases) as well as if already above the limit (positive expenditures on vending purchases) (McDonald and Moffitt 1980).

Methodology

Food away from home (FAFH) expenditures include purchases at restaurants, bars, hotels, motels, recreational places, vending machines, schools, and colleges (ERS 2014). As McCracken and Brandt (1987) pointed out, the food away from home market is most appropriately analyzed within the theoretical context of household production economics. Since vending machine purchases are considered to be 'away from home' food purchases, household production theory is most fitting to analyze this study.[20] In household production theory, which was developed by Lancaster (1966), consumption is an activity in which goods are inputs, and output is a collection of characteristics. In other words, goods purchased in the marketplace are used as inputs into the production of commodities within the household (McCracken and Brandt 1987).

Instead of looking at the individual level of utility, household production theory looks at the household itself; thus, we assume the household maximizes utility constrained by its budget set, as well as additional production and time (McCracken and Brandt 1987). Further, market good demand can be derived as a function of multiple characteristics including the household's income, price of the good, opportunity cost,

[20] More information regarding household production theory is provided in the appendix.

and other environmental variables (Lancaster 1966 1971; Michael 1972; McCracken and Brandt 1987). This relationship can be represented as:

$$C_{ij} = C_i(P_j, Y_j, W_j, E_j),\ i = 1,...,n \tag{33}$$

where C_{ij} is the j^{th} household's consumption of the i^{th} market good, P_j is the vector of market prices faced by the j^{th} household, Y_j is the j^{th} household's measure of income, W_j is the j^{th} household's value of time, and E_j is a vector of variables reflecting the environment in which production for the j^{th} household occurs (McCracken and Brandt 1987).

Following McCracken and Brandt (1987), we modify equation 33 to disaggregate the dependent variable from total expenditures on food to only expenditures at vending machines. With this disaggregation, we will be able to test the hypotheses listed previously.

For this study, we are analyzing purchases at vending machines over a consecutive two-week period. Thus, our variable of interest only takes on non-negative values. There are many studies and suggested methods as to how to best model data sets that have a limited range of values. This class of models is often referred to as discrete models; as the name suggests, the dependent variables have a 'limit' as to what values they can hold. The more common models are logit and probit models. Both manage binary dependent variables very well, and can be further extended to an 'order' or ranking of the dependent variable, such as a preference ranking of a product (i.e., 0 => would not buy, … 5 => would definitely purchase).

These discrete models have been applied to many studies concerning various topics, particularly food choices. For instance, probit models are commonly used to categorize and predict willingness to pay or likeliness to purchase a product based on specific product attributes (Gvillo *et al*. 2013; Cranfield and Magnusson 2003; Loureiro and Hine 2002; Jekanowski *et al*. 2000). Logit models, too, are commonly used to measure and predict consumer preferences for specific foods (Loureiro and Umberger 2006; Alfnes 2004). Both the logit and probit models have variations such as multinomial, mixed, Heckman two-step, and others.

We choose to focus on a Tobit model first. The key difference is that though our dependent variable is truncated at the lower level (zero in this case), there is no set limit or truncation or categorical value on the upper boundary as there would be on a logit or probit model. Thus, instead of categorizing expenditures, we are able to leave the dependent values as they are and censor on the lower boundary, which is naturally at zero. We then used a probit model to estimate what affects a household's probability of purchasing from a vending machine. With this model framework, we a binary outcome or zero or one, indicating zero expenditures or positive expenditures at a vending machine.

Tobit models were first introduced by Tobin (1958), where the model's dependent variables have a number of its values clustered at a limiting value, usually zero (McDonald and Moffitt 1980). The Tobit technique uses all of the observations in the data set both at the limit and above to estimate a regression line. It is thus preferred in general over alternative techniques that estimate a regression line only above the limit (McDonald and Moffitt 1980).

As Tobin (1958) explained, account should be taken of the observations concentrated at the limiting value when estimating the relationship of a limited variable to other variables. A censored variable is similar to a truncated variable; however, the distributions differ. For a censored variable, we begin with a normal distribution, and we assume the censoring point is zero. A truncated distribution would only consider the distribution above zero in making computations (Greene 2008). When data are censored, the distribution that applies to the sample data is a mixture of discrete and continuous distributions (Greene 2008). What Tobin (1958) deemed a 'hybrid of probit analysis and multiple regression' is now known as a Tobit model and was developed to account for concentrations at the limiting value. The Tobit technique is preferred, in general, over alternative techniques that estimate a line only with the observations above the limit (McDonald and Moffitt 1980).

The Tobit technique uses all observations, both those at the limit and those above it, to estimate a regression line (McDonald and Moffitt 1980) and measures effects of the explanatory variables on the participation decision and the level decision from a single parameter estimate (Byrne *et al*. 1996). The Tobit model is particularly useful in this analysis because it can be used to determine both changes in the probability of being above the limit and changes in the value of the dependent variable if it is already above the limit (McDonald and Moffitt 1980).

Tobit analysis is a theoretically preferred technique that uses information about all households in estimating the regression function (McCracken and Brandt 1987). With Tobit analysis, in a cross sectional analysis, it is possible to estimate both the quantity responses of households actively consuming (conditional quantity elasticities) and the participation adjustments of exit-entry households (market participation elasticities) (McCracken and Brandt 1987).

Multiple studies have used household production theory framework and then employed a Tobit model for analysis of various expenditures, particularly food (Lee and Lin 2013; Mancino and Newman 2007; McCracken and Brandt 1987). McCracken and Brandt (1987) used Tobit analysis to identify and measure the influence of factors affecting food away from home consumption behavior by facility type, including total FAFH expenditures, restaurant expenditures, fast-food expenditures, and other commercial expenditures. Mancino and Newman (2007) used a Tobit model to look at how certain factors influence a family's food preparation time. Lee and Lin (2013) also used a Tobit analysis to study the demand for convenience food. The data set we are employing has been used in studies previously. While some expenditures may have a recorded value of zero, it is of less concern than Yen (1993) who did not collapse the CES data (i.e., had one week recordings of expenditures, two observations for each CU); thus, the number of zeroes in this study are likely to be more representative of not purchasing at all rather than not purchasing in a particular week.

To examine vending machine purchases, a Tobit model will be used. A Tobit model takes the following form (McDonald and Moffitt 1980):

$$\begin{aligned} y_i &= X_i'\beta + e_i \qquad \text{if } X_i'\beta + e_i > 0 \\ y_i &= 0 \qquad\qquad\quad \text{if } X_i'\beta + e_i \leq 0, \quad i = 1,\ldots,N, \end{aligned} \tag{34}$$

where N is the number of observations, y_i is the dependent variable, X_i is a vector of independent variables, β is a vector of unknown coefficients, and e_i is an independently distributed error term with zero mean and constant variance, σ^2. Thus, the model assumes there is an underlying, stochastic index equal to ($X_i\beta + e_i$) which is observed only when it is positive, and hence qualifies as an unobserved, latent variable (McDonald and Moffitt 1980). Tobin (1958) showed that the expected value of y from equation 34 is:

$$Ey = X_i'\beta F(z) + \sigma f(z) \tag{35}$$

where $z = X`\beta/\sigma$, $f(z)$ is the unit normal density, and F(z) is the cumulative normal distribution function (McDonald and Moffitt 1980). The expected value of y^* for observations above the limit is:

$$\begin{aligned} Ey^* &= E(y \mid y > 0) \\ &= E(y \mid e > X'\beta) \\ &= X'\beta + \frac{\sigma f(z)}{F(z)} \end{aligned} \tag{36}$$

The basic relationship between the expected value of all observations, Ey, the expected value conditional upon being the limit, Ey^*, and the probability of being above the limit, $F(z)$ is (McDonald and Moffitt 1980):

$$Ey = F(z)Ey^* \tag{37}$$

McDonald and Moffitt (1980) found a useful decomposition of the Tobit model by considering the effect of a change in the i^{th} variable of X on y:

$$\frac{\partial Ey}{\partial X_i} = F(z)\left(\frac{\partial Ey^*}{\partial X_i}\right) + Ey^*\left(\frac{\partial F(z)}{\partial X_i}\right) \tag{38}$$

Thus, the total change in y can be disaggregated into two parts: (1) the change in y of those above the limit, weighted by the probability of being above the limit; and (2) the change in the probability of being above the limit, weighted by the expected value of y if above. With the estimates of β and σ, each of the terms in the above equation can be evaluated at some value of $X'\beta$, typically the mean of the X's, or X-bar (McDonald and Moffitt 1980). Then, the value of Ey^* can be calculated from equation 37, and $F(z)$ is easily obtainable. The partial derivatives are calculated as (McDonald and Moffitt 1980):

$$\frac{\partial F(z)}{\partial X_i} = \frac{f(z)\beta_i}{\sigma} \tag{39}$$

and

$$\begin{aligned}\frac{\partial Ey^*}{\partial X_i} &= \beta_i + \left(\frac{\sigma}{F(z)}\right)\frac{\partial f(z)}{\partial X_i} - \left(\frac{\partial f(z)}{F(z)^2}\right)\frac{\partial F(z)}{\partial X_i} \\ &= \beta_i\left[1 - \frac{zf(z)}{F(z)} - \frac{f(z)^2}{F(z)^2}\right]\end{aligned} \tag{40}$$

where $F`(z) = f(z)$, and $f`(z) = -z*f(z)$ for a unit normal density (McDonald and Moffitt 1980). Though a common mistake, the effect of a change in X_i on $y*$ is not β_i for this would be true only when X equals infinity, then $F(z) = 1, f(z) = 0$; however, this will obviously not hold for any X (McDonald and Moffitt 1980). Further, McDonald and Moffitt (1980) note that when equations 39) and 40 are substituted into equation 38, the total effect $\frac{\partial Ey}{\partial X_i}$ can be seen to equal $F(z)\beta_i$, and by dividing both sides of equation 40 by $F(z)\beta_i$, it is seen that the fraction of the total effect due to the effect above the limit, $\frac{\partial Ey^*}{\partial X_i}$ is $\left[1 - \frac{zf(z)}{F(z)} - \frac{zf(z)^2}{F(z)^2}\right]$. Hence, the information in the decomposition can be obtained by calculating the above fraction which is also the fraction by which the β_i coefficients must be adjusted to obtain correct regression effects for observations above the limit (McDonald and Moffitt 1980).

Our model takes the following form:

$$\begin{aligned}y_i &= X_i'\beta + e_i \qquad \text{if } X_i'\beta + e_i > 0 \\ y_i &= 0 \qquad\qquad\quad \text{if } X_i'\beta + e_i \leq 0,\quad i = 1,\ldots,N,\end{aligned} \tag{41}$$

where N is the number of households, y_i is the total amount of vending machine expenditures for household i (within a two-week period), β is a vector of unknown coefficients, e_i is an independently distributed error term with zero mean and constant variance, σ^2, and X_i is a vector of independent variables that has the following composition[21]:

$$X'\beta = \text{age, fam\_size, fincaftx, fincaftx}^2\text{, hhhours, male, smoksupp, urban, married, white, black, hispanic, college, midwest, south, west, northeast, foodaway, frshfrut, frshveg, sweets, bottled, coke, chips, nuts, q1, q2, q3, q4, 2009, 2010, 2011, 2012.} \tag{42}$$

Through the use of a Tobit model (Tobin 1958) we will be able to examine each of the above discussed objectives. Several of the above variables could be potentially endogenous. Therefore, several models will be estimated including a model which assumes no endogeneity and a model with all potential endogenous variables replaced with instrumental variables (IV).

Data

To provide thorough analysis to address the issues stated above, a detailed data set is needed. The source of data for this analysis comes from the Bureau of Labor Statistics (BLS). The BLS conducts a yearly survey deemed the Consumer Expenditure Survey (CES).[22] This survey includes two separate surveys – the Interview Survey and the Diary Survey. While both surveys provide information on the buying habits of American consumers (BLS 2013), the Diary Survey is of interest for this analysis.

The Diary Survey is comprised of several data files. For this study, the expenditure (EXPN) files and the family (FMLY) files are used. The expenditure files consist of a 'diary' of expenditures which the respondent records for two consecutive one week periods and is designed to track data on frequently purchased items. The family files contain demographic information and characteristics of the respondents (households) (BLS 2013). Each household has two observation periods, corresponding to each week. We merged the two weeks for each household; thus, each household only

[21] Variables are formally defined in a future table.
[22] The BLS refers to the Consumer Expenditure Survey as 'CEX'.

has one observation, which includes spending over a two week period. Though the CE survey spans over many years, the time period we chose to focus on is 2009 – 2012. This period is chosen based on the large drop in vending machine sales from 2008 to 2009 (VMW 2013) and continues to the most recent data available. The data set refers to households as consumer units (CUs). The BLS (2008) defines a CU as comprising either: (1) all members of a particular household who are related by blood, marriage, adoption, or other legal arrangements; (2) a person living alone or sharing a household with others or living as a roomer in a private home or lodging house or in permanent living quarters in a hotel or motel, but who is financially independent; or (3) two or more persons living together who use their income to make joint expenditure decisions.[23]

The expenditure files do not contain quantity or price information. Rather, the files contain a recording of a household's expenditures for a consecutive two week time period. Because of this, we are able to extract how much each household spends at a vending machine during the time frame. There are several vending machine expenditures recorded for the survey including breakfast, lunch, dinner, and snacks purchased from vending machines, as well as tobacco or alcohol purchased from vending machines. Here, we utilize food and non-alcoholic beverage purchases only at vending machines. Using this information, we can categorize households into groups, or by total expenditures allowing us to compare various other expenditures across households/groups. Further, we can analyze specific characteristics of each group by incorporating the household's characteristics by using the family (FMLY) files.

Sample Statistics

Each year, the BLS' targeted sample size for its CES responses is 7,050. To reach this target, the total work load is approximately 12,200 sample units (BLS 2013). In 2012, the response rate was 67.8%, with other years having similar results (BLS 2013).[24] After compiling four years of data from 2009 – 2012 and dropping households

[23] For other formal definitions, see http://www.bls.gov/cex/csxgloss.htm.
[24] All available previous years' surveys are available from the BLS (2013) reference.

with insufficient information and outliers for income and food expenditures[25], our data set includes 20,504 observations, a significant decrease from the original sample size of 27,225. Originally, the data set had two observations for each household, representing week one and week two of the diary. For each household, the two weeks are merged, representing two weeks' worth of spending per household of the selected expenditure categories. Table 23 below summarizes the means and standard deviations for the entire sample and the subset of the sample that has positive vending expenditures. The definitions of each variable are also provided below in table 23.

Looking at the tables below, we see that of the 20,518 households, 3,373 have positive vending expenditures within a two week period, or about 16% of the sample.[26] The average age of the entire sample is about five years older than the subset of the sample that purchases from vending machines. Household hours worked is almost nine hours more for the subset. About 61% of the entire sample has at least some college education while 68% of the subset has at least some college education. Family size is slightly higher for the subset of the sample, and average income (fincaftx) is almost $10,000 higher as well.

Comparing expenditures across the two sets of sample statistics provides some more information. In general, the subset of the sample spends more on food and beverages at home including chips, cola, fresh fruit, fresh vegetables, and candy. The subset also spends more on tobacco products. In general, the households who spend at vending machines appear to spend more on total food away from home consumption as well (which does not include vending machine expenditures).

[25] Households were dropped from the data set if income or expenditures exceeded the mean value of the variable ± 3*standard deviation.
[26] The number of households contained within the data set was 27,225 prior to data cleaning.

Table 23: Variable Means & Standard Deviations (SD) for Equation 42

	Sample Average		Positive Vending	
	Mean	Std. Dev.	Mean	Std. Dev.
Observations	20,518	---	3.373	---
Totvend	0.63	1.92	3.82	3.21
Age_ref	50.04	17.66	44.96	15.24
Fam_size	2.32	1.30	2.54	1.34
Fincaftx	42,102.95	43,161.35	51,843.97	45,133.65
Hhhours	38.66	30.93	47.49	28.59
Male	0.46	0.50	0.47	0.50
Smoksupp	5.06	15.18	7.38	17.72
Urban	0.95	0.22	0.94	0.23
Married	0.49	0.50	0.53	0.50
White	0.81	0.39	0.81	0.39
Black	0.13	0.34	0.13	0.33
Hispanic	0.13	0.33	0.14	0.34
College	0.61	0.49	0.68	0.47
Midwest	0.24	0.43	0.27	0.44
South	0.36	0.48	0.36	0.48
West	0.20	0.40	0.18	0.39
Northeast	0.19	0.40	0.18	0.38
Urban	0.95	0.22	0.94	0.23
Frshfrut	7.80	9.20	8.32	9.72
Frshveg	7.05	7.87	7.39	8.29
Candy	2.09	3.51	2.36	3.82
Foodaway_	72.48	82.17	111.20	90.39
Bottled	2.05	3.67	2.35	3.92
Cola	2.24	3.54	2.78	3.96
Chips	3.02	4.12	3.78	4.65
Nuts	0.92	2.21	0.85	2.14
January	0.09	0.29	0.08	0.27
February	0.08	0.27	0.07	0.26
March	0.09	0.28	0.09	0.29
April	0.09	0.29	0.09	0.29
May	0.09	0.28	0.09	0.29
June	0.09	0.28	0.10	0.29
July	0.08	0.27	0.08	0.27
August	0.08	0.27	0.08	0.27
September	0.08	0.28	0.09	0.29
October	0.09	0.28	0.09	0.29
November	0.08	0.27	0.07	0.26
December	0.06	0.24	0.06	0.24
Quarter 1	0.26	0.44	0.24	0.43
Quarter 2	0.27	0.44	0.28	0.45

Table 23 Continued

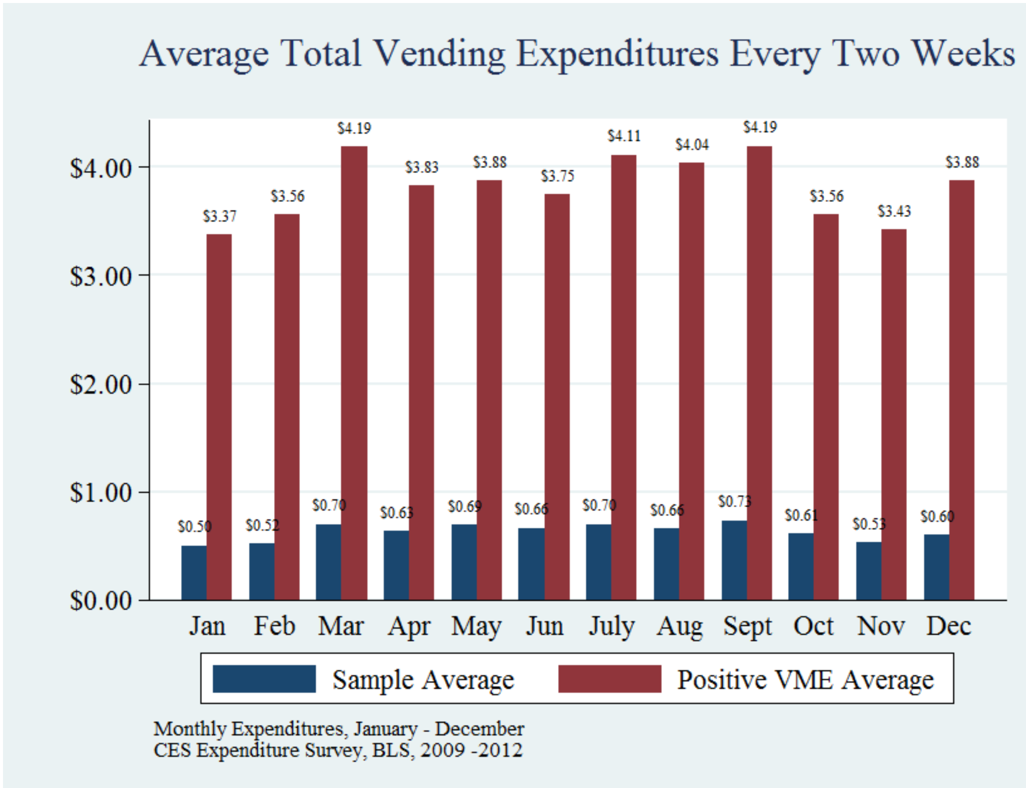
	Sample Average		Positive Vending	
	Mean	Std. Dev.	Mean	Std. Dev.
Quarter 3	0.25	0.43	0.25	0.43
Quarter 4	0.23	0.42	0.22	0.42
2009	0.26	0.44	0.29	0.45
2010	0.26	0.44	0.25	0.43
2011	0.24	0.43	0.22	0.42
2012	0.25	0.43	0.25	0.43

Months, quarters, and years represent when consumption (diary entries) occurred for the household. We can see that there is an approximate equal distribution of survey response across months, quarters, and years. When categorizing over months, vending machine expenditures (VME) are highest in March, September, and July while VME are lowest in January (see figure 5). January has the lowest vending machine expenditures average likely due to 'New Year's resolutions'. March and the summer months likely have higher expenditures because of spring break, summer travel, and back-to-school rush.

Table 24: Vending Variable Definitions

Variable	Definition
Age_ref	Age of reference person
Black	1 if reference race is Black, 0 otherwise
Bottled	Two week expenditure on sports drinks and bottled water
Candy	Two week expenditure on candy
Chips	Two week expenditure on potato chips and other snacks
Coke	Two week expenditure on cola drinks
College	1 if reference has at least some college education, 0 otherwise
Fam_size	Number of members in CU
Fincaftx	Amount of CU income after taxes in past 12 months
Foodaway_	Two week expenditure on food away from home minus monies spent at a vending machine
Frshfrut	Two week expenditure on fresh fruits
Frshveg	Two week expenditure on fresh vegetables
HHhours	Total number of hours usually worked per week by reference person and spouse
Hispanic	1 if reference race is Hispanic, 0 otherwise
Male	1 if reference is a male, 0 otherwise
Married	1 if reference is married, 0 otherwise
Midwest	1 if CU resides in Midwest, 0 otherwise
Northeast	1 if CU resides in Northeast 0 otherwise
Nuts	Two week expenditure on nuts
Quarter i	1 for recorded quarter i of CU consumption, 0 otherwise
Smoksupp	Two week expenditure on tobacco products
South	1 if CU resides in South, 0 otherwise
Urban	1 if CU resides in urban area, 0 otherwise
West	1 if CU resides in West, 0 otherwise
White	1 if reference race is White 0 otherwise
$Year_i$	1 for recorded year *i* of CU consumption, 0 otherwise

Figure 5: Average Two Week Vending Expenditures, Categorized Monthly



Model Specifications

Understanding what affects a household's decision to purchase from a vending machine is likely related to what affects a household's decision to purchase food away from home. However, including food away from home as an explanatory may lead to inconsistent and/or biased results due to endogeneity issues[27] resulting from possibly omitted variables which can lead to correlation between an explanatory variable and the error term. While food away from home likely helps explain what a household consumes from a vending machine, including it in the model specification may produce inconsistent results.

To address the above stated concern, several models are estimated to examine what affects a household's decision to purchase from a vending machine. Model I assumes all potentially endogenous variables are exogenous. Model II uses two-stages of Tobit estimation; the first stage is used to create instrumental variables (IV) for all expenditure categories and the second stage includes the predicted values of the IV within the model. We also examine a probit model. All three models use the above described data. All estimations were conducted in Stata v.12.1; for the Tobit model, we used the Tobit command, setting a lower limit (censored) at zero with no upper limit. For the probit model, we used the `probit' command. Further, heteroskedasticity[28] was taken into account using the `vce(robust)' option[29] within Stata. Though heteroskedasticity does not cause biasedness or inconsistency with the estimators, it does cause the variance of the estimators to be biased, thus causing the standard errors to be biased.

All models include the following variables (or an IV of the following variables):

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[27] Endogeneity is usually present because of omitted variables, measurement error, or simultaneity (Wooldridge 2010).
[28] Formally defined as when the variance of the error term is not constant, given the explanatory variables.
[29] This robust option computes the White/Huber/sandwich estimator which is robust as long as the observations are independent (see Stata Manual, vce_options, Variance Estimators, http://www.stata.com/manuals13/xtvce_options.pdf)

$$X'\beta = \text{age, fam\_size, fincaftx, fincaftx}^2\text{, hhhours, male, urban, married, white, black, hispanic, college, midwest, south, west, northeast, frshfrut, smoksupp, frshveg, candy, bottled, coke, chips, nuts, foodaway, q1, q2, q3, q4, 2009, 2010, 2011, 2012.}$$

For each model, a second measure of r-squared was calculated by measuring the correlation of total vending purchases and the expected values of total vending purchases, squared. For the Tobit specification, and following McDonald and Moffitt (1980), we estimated predicted values of vending expenditures for each model using the following equation:

$$Ey = X_i'\beta F(z) + \sigma f(z) \tag{43}$$

and calculated the correlation coefficient as :

$$p = corr[totvend, E(totalvend)]\text{^}2 \tag{44}$$

Though the parameter estimates can provide some useful information, marginal effects and elasticities are of more interest. With Tobit analysis, in a cross-sectional data set, it is possible to estimate both the quantity responses of households actively consuming (conditional quantity elasticities) and the participation adjustments of exit-entry households (market participation/unconditional elasticities) (McCracken and Brandt 1987). The following formulas (see McCracken and Brandt 1987, following McDonald and Moffitt 1980) provide the relationships:

$$E(Y_i \mid X_i) = F\left(\frac{\beta' X_i}{\sigma}\right)(\beta' X_i) + \sigma f\left(\frac{\beta' X_i}{\sigma}\right)$$

$$E(Y_i^* \mid X_i) = E(Y_i \mid X_i, Y_i > 0) \tag{45}$$

$$= (\beta' X_i) + \frac{\sigma f\left(\frac{\beta' X_i}{\sigma}\right)}{F\left(\frac{\beta' X_i}{\sigma}\right)}$$

where $F(\cdot)$ and $f(\cdot)$ are the standard normal density and distribution functions, respectively. Then the effect of a change in an independent variable, X_i, on $E(Y_i|X_i)$ in elasticity form can be decomposed as:

$$E_i = \frac{\partial F\left(\frac{\beta' X_i}{\sigma}\right)}{\partial X_i} \frac{X_i}{F\left(\frac{\beta' X_i}{\sigma}\right)} + \frac{\partial E(Y_i^* \mid X_i)}{\partial X_i} \frac{X_i}{E(Y_i^* \mid X_i)} \tag{46}$$

with the first component being the elasticity of the probability of consumption and the second being the elasticity of expected consumption of presently consuming households (McCracken and Brandt 1987). To predict the conditional marginal effects, we use the `margins, dydx(*)' command in Stata v.12.1. Likewise, to predict the elasticities, we use the `margins, eyex(*)' command where specified values are given to ensure we are predicting the conditional and unconditional elasticities. All marginal effects and elasticities were calculated at the means. To ensure coding was implemented correctly, direct computations of the marginal effects and elasticities were calculated as well.

Model I: Tobit Analysis, Assuming All Expenditures are Exogenous

Vending machine expenditures fall into the food away from home consumption category. Hypothesizing that a household spending more on food away from home positively affects a household spending at a vending machine is logical. We present results assuming all other explanatory food expenditure variables are exogenous within the model. Table 25 presents the results:

Table 25: Tobit Estimation, Exogenous Model Parameter Estimates

	Parameters	Standard Errors
Age_ref	-0.052***	(0.005)
Fam_size	0.041	(0.067)
Fincaftx	0.000***	(0.000)
Fincaftx2	-0.000***	(0.000)
Hhhours	0.016***	(0.003)
Male	-0.273*	(0.143)
Urban	-1.160***	(0.309)
Married	-0.563***	(0.189)
Black	0.463**	(0.215)
Hispanic	0.732***	(0.224)
College	0.350**	(0.159)
Midwest	0.741***	(0.213)
South	0.035	(0.201)
West	-0.392*	(0.232)
Foodaway_	0.020***	(0.001)
Frshfrut	-0.004	(0.009)
Frshveg	0.001	(0.011)
Candy	0.009	(0.021)
Bottled	-0.017	(0.020)
Cola	0.094***	(0.020)
Smoksupp	0.030***	(0.004)
Chips	0.073***	(0.017)
Nuts	-0.134***	(0.035)
January	-0.432	(0.369)
February	-0.505	(0.381)
March	0.018	(0.369)
April	-0.011	(0.364)
May	0.255	(0.365)
June	0.174	(0.366)
July	0.332	(0.380)
August	0.013	(0.375)
September	0.418	(0.371)
October	0.178	(0.365)
November	-0.190	(0.377)
2009	0.479**	(0.193)
2010	0.013	(0.197)

Table 25 Continued

	Parameters	Standard Errors
2011	-0.281	(0.203)
Constant	-6.569***	(0.592)
Sigma	6.698***	(0.096)
Observations	20,518	---

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

The above results assume all explanatory variables are exogenous. Age affects vending machine spending in a negative way. Expenditures that positively and significantly affect a household's expenditure at a vending machine include food away from home, coke, tobacco, and chips. Several demographic characteristics affect purchases including gender, race, income, hours worked, college education, and being married.

Table 26 compares the actual vending expenditures to those predicted from this model. Further, an additional R-squared is calculated as well as described previously; its value is 0.166.

Table 26: Comparison of Actual Vending Expenditures to Predicted Expenditures Exogenous Model

	Mean	Std. Dev.	Min.	Max
Actual	0.625	1.915	0	14.32
Predicted	0.507	0.382	0	2.27

Assuming FAFH is exogenous results in predicted expenditures at vending machines to be $0.51 per household every two weeks. Table 27 presents the Tobit marginal effect results followed by table 28 with the elasticity results.

Table 27: Tobit Estimation, Marginal Effects, All Exogenous Variables

	Conditional		Unconditional	
	Marginal Effect	Standard Error	Marginal Effect	Standard Error
Age_ref	-0.010***	(0.001)	-0.008***	(0.001)
Fam_size	0.008	(0.013)	0.006	(0.010)
Fincaftx	0.000***	(0.000)	0.000***	(0.000)
Fincaftx2	-0.000***	(0.000)	-0.000***	(0.000)
Hhhours	0.003***	(0.001)	0.002***	(0.000)
Male	-0.053*	(0.028)	-0.040*	(0.021)
Urban	-0.224***	(0.060)	-0.170***	(0.045)
Married	-0.109***	(0.036)	-0.083***	(0.028)
Black	0.090**	(0.042)	0.068**	(0.032)
Hispanic	0.142***	(0.043)	0.108***	(0.033)
College	0.068**	(0.031)	0.051**	(0.023)
Midwest	0.143***	(0.041)	0.109***	(0.031)
South	0.007	(0.039)	0.005	(0.030)
West	-0.076*	(0.045)	-0.058*	(0.034)
Foodaway_	0.004***	(0.000)	0.003***	(0.000)
Frshfrut	-0.001	(0.002)	-0.001	(0.001)
Frshveg	0.000	(0.002)	0.000	(0.002)
Candy	0.002	(0.004)	0.001	(0.003)
Bottled	-0.003	(0.004)	-0.003	(0.003)
Cola	0.018***	(0.004)	0.014***	(0.003)
Smoksupp	0.006***	(0.001)	0.004***	(0.001)
Chips	0.014***	(0.003)	0.011***	(0.003)
Nuts	-0.026***	(0.007)	-0.020***	(0.005)
Observations	20,518			

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

The estimated mean conditional effect of food away from home is 0.004, indicating an increase on vending purchases if the household purchases food away from home. This is only slightly higher than the unconditional marginal effect of 0.003. Further, each extra hour a household works increases vending expenditures by $0.003

(conditional) and $0.002 (unconditional), respectively. Below, elasticities are presented for the model.

Table 28: Tobit Estimation, Elasticities, All Exogenous Variables

	Conditional		Unconditional	
	Elasticity	Standard Error	Elasticity	Standard Error
Age_ref	-0.145***	(0.014)	-0.752***	(0.071)
Fam_size	0.005	(0.009)	0.028	(0.045)
Fincaftx	0.054***	(0.010)	0.280***	(0.052)
Fincaftx2	-0.025***	(0.005)	-0.128***	(0.028)
Hhhours	0.034***	(0.007)	0.175***	(0.034)
Foodaway_	0.080***	(0.003)	0.413***	(0.017)
Frshfrut	-0.002	(0.004)	-0.009	(0.021)
Frshveg	0.001	(0.004)	0.003	(0.023)
Candy	0.001	(0.002)	0.005	(0.013)
Bottled	-0.002	(0.002)	-0.010	(0.012)
Cola	0.012***	(0.002)	0.061***	(0.013)
Smoksupp	0.009***	(0.001)	0.044***	(0.006)
Chips	0.012***	(0.003)	0.064***	(0.015)
Nuts	-0.007***	(0.002)	-0.036***	(0.009)
Observations	20,518		20,518	

From the elasticities, we are able to see how much income (*fincaxft*) affects vending machine expenditure. Here, we see that a 10% increase in income results in a $2.80 increase in vending expenditures, unconditionally. However, the total income effect will increase at a decreasing rate, as the signs of income squared are negative.

Model II: Tobit Analysis and Modeling Expenditures with IVs

Returning to our previous specification which includes all expenditure variables as exogenous explanatory variables, we have a slightly modified model than what was specified above. The second model chosen to analyze what factors influence a household's decision to purchase foods from a vending machine includes the same explanatory variables specified previously with all expenditure variables replaced with instrumental variables (frshfrut, smoksupp, frshveg, sweets, bottled, coke, chips, nuts, foodaway).

One problem we are concerned with the exogenous model are potential endogeneity issues. Suppose total household food away from home (FAFH) expenditures is an independent variable for our Tobit model. In other words, we have the following:

$$VME_i = \alpha_0 + \beta_1 FAFH_i + \sum_{j=2} \beta_j x_j + e_i \tag{47}$$

The resulting parameter estimates are inconsistent if *FAFH* for household *i* is correlated with the error term (i.e., *Cov* (*FAFH*, *e*) $\neq 0$). In order to obtain consistent estimates of the above equation, we need to introduce an instrumental variable (IV). For an instrument to be valid, two conditions must be satisfied. These conditions are:

$$\begin{aligned} &(1): Cov(\text{IV}, \text{error}) = 0 \\ &(2): Cov(\text{IV}, FAFH) \neq 0 \end{aligned} \tag{48}$$

where IV is the instrumental variable (or vector), error is the error term associated with equation 47, and *FAFH* is the variable whose correlation with the error term is not zero. According to Wooldridge (2010), a linear projection of the endogenous variable onto all of the exogenous variables where the coefficient on the instrumental variable is not zero, the summation of the expected error is zero ,and the error and all other variables are uncorrelated is a more precise statement of condition two above.

Finding a valid IV is not an easy task. Looking at previous research models for variables that may be potentially endogenous in this model, such as *FAFH*, is a good basis on which to start. For instance, McCracken and Brandt (1987) and Byrne *et al*. (1996) analyzed FAFH expenditure patterns for the US. Based on Byrne *et al*. (1996), our *FAFH* equation takes the following form with variable descriptions provided in the table 29:

$$
\begin{aligned}
FAFH_i = {} & \alpha_0 + \beta_0 Fincaftx_i + \beta_1 Fincaftx_i^2 + \beta_2 Fincaftx_i * Fam\_Size_i + \\
& \beta_3 HHhours_i + \beta_4 Midwest_i + \beta_5 South_i + \beta_6 West_i + \beta_7 Urban + \\
& \beta_8 Black_i + \beta_9 Hispanic_i + \beta_{10} OtherRace_i + \\
& \beta_{11} Fam\_Size_i + \beta_{12} Fam\_Size_i^2 + \beta_{13} College_i + \beta_{14} Male_i + \\
& \beta_{15} Married_i + \beta_{16} Q2_i + \beta_{17} Q3_i + \beta_{18} Q4_i + \varepsilon_i
\end{aligned} \tag{49}
$$

where FAFH is total expenditures of food away from home per household minus expenditures at a vending machine.

Table 29: Variable Definitions for FAFH Instrumental Variable

Variable	Definition
Fincaftx	Amount of CU income after taxes in past 12 months
$Fincaftx^2$	Fincaftx squared
HHhours	Total # of hours usually worked per week by reference person and spouse
Midwest	1 if CU resides in Midwest, 0 otherwise
South	1 if CU resides in South, 0 otherwise
West	1 if CU resides in West, 0 otherwise
Urban	1 if CU resides in an urban area; 0 otherwise
Black	1 if reference race is Black, 0 otherwise
Hispanic	1 if reference race is Hispanic, 0 otherwise
Otherrace	1 for other races, 0 otherwise
Fam_size	Number of members in CU
Fam_size^2	Fam_Size squared
College	1 if reference has at least some college education, 0 otherwise
Male	1 if reference is a male, 0 otherwise
Married	1 if reference is married, 0 otherwise
Q_i	1 for recorded quarter *i* of CU consumption, 0 otherwise

The key variables that are excluded in the second stage Tobit specification is the interaction term between household size and income and the squared term of family size. In order for this parameter to be a valid IV, there can be no significant effect of the interaction term and family size squared on total vending (in the second stage). In other words, household size * income and family size squared only affect total vending expenditures through food away from home.[30] Equation 47 was estimated using the `tobit' command in Stata v12.1 with the lower limit being set at zero (no upper limit censoring). Results from equation 47 are provided in table 30.

Table 30: First Stage Tobit Estimation, Results from Equation 47 - Food Away From Home IV

	Parameters	Std. Errors
Fincaftx	0.001***	(0.000)
Fincaftx2	-0.000**	(0.000)
Inchhsize^	-0.000*	(0.000)
Hhhours	0.385***	(0.026)
Midwest	-8.070***	(1.947)
South	-0.287	(1.834)
West	7.469***	(2.111)
Urban	11.788***	(2.498)
Black	-19.225***	(1.874)
Otherrace	-0.039	(2.981)
Hispanic	-3.768*	(2.084)
Fam_size	16.932***	(2.341)
Fam_size2	-2.005***	(0.383)
College	24.920***	(1.373)
Male	12.108***	(1.302)
Married	2.917*	(1.700)
Q2	4.063**	(1.754)
Q3	1.230	(1.775)
Q4	-4.685***	(1.795)
Constant	-31.731***	(4.066)
Sigma	87.979***	(0.697)
Observations	20,504	

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1, ^: P-value was 0.101

[30] The second stage Tobit was estimated including the interaction term and family size squared showing no effect.

By introducing the IV, our previous model, equation 47, is now modified. The final model chosen to analyze what factors influence a household's decision to purchase foods from a vending machine includes the following explanatory variables:

$$
\begin{aligned}
X'\beta = {} & \text{age, fam\_size, fincaftx, fincaftx}^2\text{, hhhours, male,} \\
& \text{smoksupp, urban, married, white, black, hispanic, college,} \\
& \text{midwest, south, west, northeast, q1, q2, q3, q4,} \\
& \text{foodaway*, totfastfood, meat, frshfrut, frshveg, sweets,} \\
& \text{bottled, coke, chips, nuts, 2009, 2010, 2011, 2012.}
\end{aligned}
$$

where *q4*, *northeast*, *white*, and *2012* were omitted. There are multiple ways to calculate the values of the estimated IV, or y-hats. Due to the Tobit specification, and following McDonald and Moffitt (1980), we estimated the values of the IV (foodaway*) using the following equation with results presented in table 31:

$$Ey = X_i'\beta F(z) + \sigma f(z) \tag{50}$$

Table 31: Comparisons of Actual FAFH and E(FAFH) IV

	Mean	Std. Dev.	Min.	Max
Actual	72.57	82.28	0	442
Predicted	73.77	30.70	0	184.13

After the expected values of `foodaway' were calculated, those values were used as an explanatory variable in the second stage Tobit model. Similar steps were taken for the remaining expenditure variables. The parameter results for equation 41 including the IVs are provided below in table 32.

Table 32: Tobit Estimation, Parameter Results, IV

	Parameters	Standard Errors
Age_ref	-0.060***	(0.005)
Fam_size	-0.010	(0.092)
Fincaftx	0.000***	(0.000)
Fincaftx2	-0.000***	(0.000)
Hhhours	0.025***	(0.008)
Male	-0.118	(0.259)
Urban	-0.007	(0.425)
Married	-0.497**	(0.228)
Black	0.407	(0.404)
Hispanic	1.325***	(0.348)
College	1.154**	(0.495)
Midwest	-0.252	(0.316)
South	-0.789***	(0.271)
West	-0.121	(0.275)
Ey_fafh	-0.017	(0.025)
Ey_frshfrut	0.001	(0.021)
Ey_frshveg	-0.016	(0.025)
Ey_candy	-0.022	(0.046)
Ey_bottled	-0.485***	(0.117)
Ey_coke	1.271***	(0.252)
Ey_smoksupp	0.051*	(0.029)
Ey_chips	0.169*	(0.101)
January	-0.234	(0.388)
February	-0.254	(0.398)
March	0.457	(0.386)
April	0.435	(0.406)
May	0.893**	(0.409)
June	0.724*	(0.405)
July	0.768*	(0.405)
August	0.557	(0.406)
September	0.947**	(0.401)
October	0.307	(0.374)
November	-0.195	(0.385)
2009	0.478**	(0.198)
2010	-0.037	(0.202)
2011	-0.259	(0.207)
Constant	-2.470***	(0.921)
Sigma	6.917***	(0.098)
Observations	20,518	

Robust standard errors in parentheses
*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Compared to the results in table 25, we do not see any drastic changes. Parameters still hold the same signs, with significance changing slightly for some estimates. However, food away from home is not different from zero.[31] Comparing the actual vending expenditures to the estimated expenditures allows us to see if the model is specified well. The following table includes actual vending expenditures and predicted expenditures. The predicted expenditures are calculated as discussed previously.

Table 33: Actual and Predicted Vending Expenditures, IV

	Mean	Std. Dev.	Min.	Max
Actual	0.625	1.915	0	14.32
Predicted	0.552	0.326	0	2.39

Compared to the previous model, the IV model has predicted vending expenditures slightly larger. Comparing the calculated R-squared as before will provide more information about this model specification. The calculated R-square is 0.13, which is a slightly lower correlation coefficient as the model that assumed all explanatory variables were exogenous. The marginal effects of the IV model, which are calculated at the mean, are presented in table 34:

[31] Correlation among all explanatory variables and the error term was calculated in two forms: (1) the linear predictions of the error term, and (2) the expected value of total vending subtracted from total vending, generating the error. Correlations were larger for method (1) compared to very small correlations from method (2); both calculation's correlations were *at most* moderately correlated.

Table 34: Tobit Estimation, Marginal Effects, IV Model

	Conditional		Unconditional	
	Parameters	Std. Error	Parameters	Std. Error
Age_ref	-0.012***	(0.001)	-0.009***	(0.001)
Fam_size	-0.002	(0.018)	-0.002	(0.014)
Fincaftx	0.000***	(0.000)	0.000***	(0.000)
Fincaftx2	-0.000***	(0.000)	-0.000***	(0.000)
Hhhours	0.005***	(0.002)	0.004***	(0.001)
Male	-0.023	(0.051)	-0.018	(0.040)
Urban	-0.001	(0.083)	-0.001	(0.065)
Married	-0.098**	(0.045)	-0.076**	(0.035)
Black	0.080	(0.079)	0.062	(0.062)
Hispanic	0.260***	(0.068)	0.203***	(0.053)
College	0.227**	(0.097)	0.177**	(0.076)
Midwest	-0.049	(0.062)	-0.039	(0.049)
South	-0.155***	(0.053)	-0.121***	(0.042)
West	-0.024	(0.054)	-0.019	(0.042)
Ey_fafh	-0.003	(0.005)	-0.003	(0.004)
Ey_frshfrut	0.000	(0.004)	0.000	(0.003)
Ey_frshveg	-0.003	(0.005)	-0.003	(0.004)
Ey_candy	-0.004	(0.009)	-0.003	(0.007)
Ey_bottled	-0.095***	(0.023)	-0.074***	(0.018)
Ey_coke	0.250***	(0.049)	0.195***	(0.039)
Ey_smoksupp	0.010*	(0.006)	0.008*	(0.004)
Ey_chips	0.033*	(0.020)	0.026*	(0.015)
January	-0.046	(0.076)	-0.036	(0.059)
February	-0.050	(0.078)	-0.039	(0.061)
March	0.090	(0.076)	0.070	(0.059)
April	0.086	(0.080)	0.067	(0.062)
May	0.176**	(0.080)	0.137**	(0.063)
June	0.142*	(0.080)	0.111*	(0.062)
July	0.151*	(0.080)	0.118*	(0.062)
August	0.110	(0.080)	0.085	(0.062)
September	0.186**	(0.079)	0.145**	(0.062)
October	0.060	(0.073)	0.047	(0.057)
November	-0.038	(0.076)	-0.030	(0.059)
2009	0.094**	(0.039)	0.073**	(0.030)
2010	-0.007	(0.040)	-0.006	(0.031)

Table 34 Continued

	Conditional		Unconditional	
	Parameters	Std. Error	Parameters	Std. Error
2011	-0.051	(0.041)	-0.040	(0.032)
Observations	20,518		20,518	

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

We see similar results as with the previous model in regards to both the conditional and unconditional marginal effects. The elasticities from the IV model are presented in table 35 below, and too, are calculated at the means:

Table 35: Tobit Estimation, Elasticities, IV Model

	Conditional		Unconditional	
	Elasticities	Std. Error	Elasticities	Std. Error
Age_ref	-0.163***	(0.013)	-0.831***	(0.069)
Fam_size	-0.001	(0.012)	-0.007	(0.059)
Fincaftx	0.082***	(0.027)	0.419***	(0.140)
Fincaftx2	-0.018***	(0.007)	-0.091***	(0.033)
Hhhours	0.053***	(0.016)	0.271***	(0.083)
Ey_fafh	-0.067	(0.101)	-0.338	(0.512)
Ey_frshfrut	0.000	(0.008)	0.002	(0.042)
Ey_frshveg	-0.006	(0.009)	-0.029	(0.045)
Ey_candy	-0.001	(0.003)	-0.007	(0.015)
Ey_bottled	-0.022***	(0.005)	-0.111***	(0.027)
Ey_coke	-0.149***	(0.030)	-0.759***	(0.151)
Ey_smoksupp	-0.021*	(0.012)	-0.105*	(0.060)
Ey_chips	0.023*	(0.014)	0.117*	(0.070)
Observations	20,518			

Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

Comparing our results using the IV method to McCracken and Brandt's examination of FAFH expenditures, we see again that our household size elasticity is much smaller than theirs. While the conditional income elasticity is similar to their estimate, the unconditional elasticity presented here is much higher compared to their `other' commercial expenditures' category. In general, these elasticity results do not differ much than the previous model's specification results.

Though hypothesized to affect vending machine expenditures, the food away from home IV is not different from zero. The implementation of the IV did not alter the results of the parameter estimates, marginal effects, nor elasticities much when compared to the original specification with no food away from home. Perhaps the IV was misspecified, or we are excluding an important variable that would capture the effect of food away from home. Alternatively, this specification could be an improvement, and the previous specification resulted in significant results for food away from home due to endogeneity.

Model III: Probit Model Incorporating IVs

Though a Tobit model can provide analysis on the conditional and unconditional effects of purchasing from a vending machine, a probit model examines what influences the probability or likelihood of purchasing from a vending machine. Our probit model, a binary model, takes on two values – zero (zero vending expenditures) and one (positive vending expenditures). Following the notation of the Tobit model, our probit model takes the following form:

$$\begin{aligned} y_i^* &= X_i'\beta + e_i \\ y_i &= 1 \qquad \text{if } y_i^* > 0 \\ y_i &= 0 \qquad \text{otherwise} \end{aligned} \tag{51}$$

where

$$\Pr(y = 1 \mid x) = \Phi(X'\beta) \tag{52}$$

where Φ is the cumulative distribution function (CDF) of the standard normal distribution. The model has the same variables as described before including the instrumental variables (assumed that expenditures were endogenous). Table 36 reports the results followed by the marginal effects in table 37:

Marginal effects provide insight as to how the variables affect the probability of purchasing from a vending machine. For instance, consumers who purchase chips or sodas are more likely to purchase from a vending machine. As age increases, household are less likely to purchase from a vending machine. Income positively affects the probability as well. Again, these results, though interpreted differently, are similar to that of the Tobit specification.

Table 36: Probit Estimation, Parameter Estimates, IV

	Parameters	Standard Errors
Age_ref	-0.009***	(0.001)
Fam_size	-0.008	(0.014)
Fincaftx	0.000***	(0.000)
Fincaftx2	-0.000***	(0.000)
Hhhours	0.004***	(0.001)
Male	-0.030	(0.039)
Urban	-0.000	(0.064)
Married	-0.064*	(0.034)
Black	0.076	(0.061)
Hispanic	0.198***	(0.052)
College	0.184**	(0.075)
Midwest	-0.025	(0.048)
South	-0.107***	(0.041)
West	-0.025	(0.041)
Ey_fafh	-0.002	(0.004)
Ey_frshfrut	-0.001	(0.003)
Ey_frshveg	-0.002	(0.004)
Ey_candy	-0.005	(0.007)
Ey_bottled	-0.073***	(0.018)
Ey_coke	0.192***	(0.038)
Ey_smoksupp	0.009**	(0.004)
Ey_chips	0.027*	(0.015)
January	-0.009	(0.059)
February	-0.024	(0.060)
March	0.054	(0.058)
April	0.069	(0.061)
May	0.137**	(0.061)
June	0.119*	(0.061)
July	0.104*	(0.061)
August	0.078	(0.061)
September	0.128**	(0.060)
October	0.066	(0.057)
November	-0.007	(0.058)
2009	0.075**	(0.030)
2010	-0.015	(0.030)
2011	-0.056*	(0.031)
Constant	-0.326**	(0.138)
Observations	20,518	

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Probit results inform us of how variables affect the likelihood or probability of a household purchasing from a vending machine. Significance and direction of the variables are similar to that of the Tobit specifications. While parameter results provide relationships a probit model is not linear. Therefore, we instead focus on interpreting the marginal effects, which are presented below in table 37.

Table 37: Probit Estimation, Marginal Effects, IV

	Parameters	Standard Errors
Age_ref	-0.002***	(0.000)
Fam_size	-0.002	(0.003)
Fincaftx	0.000***	(0.000)
Fincaftx2	-0.000***	(0.000)
Hhhours	0.001***	(0.000)
Male	-0.007	(0.009)
Urban	-0.000	(0.015)
Married	-0.015*	(0.008)
Black	0.018	(0.014)
Hispanic	0.047***	(0.012)
College	0.044**	(0.018)
Midwest	-0.006	(0.011)
South	-0.025***	(0.010)
West	-0.006	(0.010)
Ey_fafh	-0.001	(0.001)
Ey_frshfrut	-0.000	(0.001)
Ey_frshveg	-0.000	(0.001)
Ey_candy	-0.001	(0.002)
Ey_bottled	-0.017***	(0.004)
Ey_coke	0.045***	(0.009)
Ey_smoksupp	0.002**	(0.001)
Ey_chips	0.006*	(0.004)
January	-0.002	(0.014)
February	-0.006	(0.014)
March	0.013	(0.014)
April	0.016	(0.015)

Table 37 Continued

	Parameters	Standard Errors
May	0.032**	(0.015)
June	0.028*	(0.015)
July	0.025*	(0.014)
August	0.018	(0.014)
September	0.030**	(0.014)
October	0.016	(0.013)
November	-0.002	(0.014)
2009	0.018**	(0.007)
2010	-0.004	(0.007)
2011	-0.013*	(0.007)
Observations	20,518	

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Conclusion

The purpose of this chapter was to examine socio-demographic factors that influence a household's decision to purchase from a vending machine. Data used was extracted from the Diary files from the BLS CE survey. Several models were selected to address this issue including a Tobit model and a probit model, incorporating instrumental variables. Results across all models were similar. This research issue is important because vending machines are an outlet where unhealthy food is readily available; though public health advocates have urged restaurants and fast food places to make more healthy options available (Guthrie *et al*. 2013), there is little regulation in regards to healthy choices in vending machines. Understanding who purchases from these machines and what other foods households purchase allows some inference on `what if fresh fruits and vegetables' were placed in a vending machine, how would persons react based on what they currently purchase. Further, understanding the profile of a vending machine purchaser allows us to see what demographic factors affect a person's purchasing from a vending machine.

Fresh fruit and vegetable expenditures had no positive, significant effect on whether or not a household purchases from a vending machine with all model specifications. Perhaps incorporating fresh fruits and vegetables into vending machines may attract another consumer base. Due to the lack of significance, we cannot infer whether or not current vending machine purchasers who also purchase fresh fruit and vegetables would purchase such items if available. Households who purchase nuts are less likely to purchase from a vending machine. Perhaps if machines offered more variety of nut packages, households would purchase from a vending machine more often. Households who purchase items that are readily available in vending machines, such as chips and cokes, spend more at vending machines as well. However, sweets, bottled sports drinks, and food away from home (IV method) purchases had no effect on vending machine expenditures.

Interestingly, households who purchase tobacco have higher expenditures at vending machines. Tobacco consumption, a well-known, `bad for you' habit can result in cancer and possibly death. Likewise, poor eating habits can contribute to poor health outcomes and possible death. While there are numerous advertisements explaining the health risks associated with tobacco use, there is little in comparison in advertisement numbers for eating unhealthily or being severely overweight. While education (and income) is sometimes to blame for persons' poor health choices, being more educated resulted in higher expenditures at a vending machine. Likewise, higher incomes resulted in more spending as well. Education may not be the issue here. We may be seeing that more educated persons work long hours in offices with easy access to vending machines, therefore are spending more at these outlets. Perhaps occupations where tobacco consumption is more accessible (construction, non-office jobs, jobs with breaks) there is also easy access to vending machines.

Though family size had no significant effect on vending machine purchases, age did. In fact, for the unconditional marginal effects, for each year increase in age, a person's expenditure at vending machine decreases a good bit. We can infer a great deal from the income elasticities, particularly in regards to the entire sample. For instance, for the whole sample, in both model specifications, a small increase in income leads to a significant increase in vending purchases.

This research provides information about the profile of a vending machine consumer. Future research should incorporate experiments, seeing if having more healthy options available for purchase in a vending machine leads to more healthy items actually being purchased. Understanding why more educated people are choosing to purchase from a vending machine even though the items for sale are more or less unhealthy is another area to explore.

There are several limitations[32] to this study. First, we do not know what specific items are purchased from the vending machine, only what meal the items were purchased for such as breakfast, lunch, dinner, or a snack. Second, we do not have prices. To address the price issue, there is a vending consumer price index available that could be incorporated into the model in the future. Another limitation is that the data is self-reported. There could be measurement error while reporting. One of the major limitations of the Tobit model is that both the conditional and unconditional marginal effects will have the same sign. Exploring alternative modeling methods is necessary to fully understand the consumer base who purchases from vending machines. Other model options include the Cragg's model (Cragg 1971) and Heckman two-step model both which incorporate a probit model. While there are limitations to this study, we have answered a question that was not addressed previously – what socio-demographic factors affect a household purchases from a vending machine.

[32] I would like to thank Dr. Geoffery Paulin at the BLS (CE) for providing useful feedback on this empirical analysis.

CHAPTER V

CONCLUDING COMMENTS AND SUGGESTIONS FOR FUTURE RESEARCH

This research answered several questions that had not been addressed previously. First, we addressed whether or not generic advertising has different effects on fluid milk type consumption. Further, we examined consumption relationships among milk types, how income affects milk type consumption, and how prices affect milk type consumption. This was done so using multiple model specifications. Particularly, each model was modified to include a polynomial distributed lag advertising variable to capture both long run and short run advertising effects. The results indicate that generic advertising has different effects for milk type consumption. We also answered a question regarding vending machine consumption – what demographic factors affect expenditures at vending machines. By using both Tobit and probit model specifications, we were able to assess how specific demographic and at home consumption spending affects vending machine purchases. While each study has its limitations, contributions to the literature have been added.

Many things affect a person's decision of whether or not to consume a good. Though advertising is one of the factors, the results are not immediate. Optimal advertising lags for fluid milk suggested that total advertising effects are highest at a lag of five months. These results are confirmed through both complete and incomplete demand analysis. Specifically, we estimated a quadratic AIDS model and Barten Synthetic model, both complete demand systems approaches, to address this issue. This was followed by an incomplete demand systems approach, where advertising results resembled the previous two models. Since we know advertising does affect consumption, there could be benefits to both the consumer and firm. Milk campaigns can focus advertising on specific types of milk; two-percent milk consumption has increased in recent years while whole milk has been on a decline. The Government has modified conditions for WIC recipients. Perhaps milk campaigns can focus advertising efforts on two percent, or even one percent and skim milks. The downfall is – what happens to whole milk? Whole milk is the highest in fat milk of the four discussed here; whole milk consumption has been on a major decline the past several years. How much higher in fat is whole milk?[33] In fact, there is little difference in milk fat percentages between whole and two percent. Recall the mixed findings of various milk consumption studies and its effect on BMI. Milk campaigns may be better off saving whole milk through education and the `natural' or `foodie' movements and focusing generic advertising to two, one, and skim milks.

In regards to modeling advertising effects, it is evident that employing a complete demand system gives us results that may not make sense. Recall results for the quadratic AIDS model indicated that advertising had a tenfold effect for consumption. This rationale does not make much sense. We must realize that while complete demand systems offer many theoretical properties that can be tested, general application may not be best recommended based on the results of a specific complete demand system. Perhaps modifying the polynomial distributed variable some, such as a first difference, would mitigate the issue, particularly since advertising expenditures may have stationary issues.

Another problem faced within a demand system is price correlations among the goods in the system. Though the goods are all `different', we have assumed separability; thus there are some common characteristics within the group of goods being modeled. Perhaps milk is a rare case where prices are nearly perfectly collinear. Regardless, this is a characteristic that should not be taken lightly. Though this issue can be adjusted in other model types, due to symmetry and adding up within a complete demand system, we are forced to have all of the prices of each good in the system, collinear or not.

This characterization led us to model an incomplete demand system. Here, we adjusted for collinearity by using price ratios instead of each good's price. Though we are able to attain logical advertising results as well as income elasticities, it comes at a cost for we are not able to analyze own and cross price relationships among the four milk types. Among the models selected to analyze milk consumption, the incomplete

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[33] The difference in fat percentages between whole and 2% milk is about 1.25.

Seemingly Unrelated Regression model was the model that best predicted hypothesized results in regards to advertising. Though the complete demand models provided insight as to how milk prices affect substitution among milk types, the incomplete model specification resulted in advertising results most similar to previous literature.

What about industries that do not have advertising campaigns? What affects consumption in such industries? The vending machine industry does not have a direct advertising campaign such as milk or cotton or other commodities. Further, a lot of vending machines are privately owned, particularly the ones with food items (i.e., a machine selling Coke products is owned by Coke; a machine selling Snickers and Lay's Potato Chips is likely individually owned). So, if direct advertising is not done by these vending machine owners, what is driving consumption of these goods? Though the Government stepped in where whole milk was concerned (with WIC purchases), there seems to be little enforcement done where vending machines are concerned even though unhealthy food snacks are sold within a vending machine.[34] In fact, a Snickers bar has 12 grams of fat[35] in one serving while one serving of whole milk has eight. Of course, WIC cannot be used to purchase Snickers bars, but the point is, if the Government wants to modify junk food consumption, particularly from a vending machine, in hopes of decreasing America's BMIs, perhaps it needs to advertise or promote (1) not eating entire meals from a machine or (2) selecting more healthy options. In order to do this, we need to know who eats from a vending machine and why, which was the motivation for chapter four.

However, if vending machine owners opt to advertise for specific product consumption, it is likely results will not occur for months (as was evident by the milk studies). Further, to see actual health improvements, it may take years. Though steps have been taken to modify vending machine product selections within public schools, information provided here does not indicate that vending machine sales have dropped significantly. Students may also be supplementing lunches with gas station purchases

[34] Recently, the Government issued legislation in regards to posting calorie information for food purchases in vending machines (see previous chapter). Posting has not been completed yet.
[35] http://www.snickers.com/Nutritional-Info

such as sodas and candies. Here again, perhaps advertising for healthy eating and education is the best option.

Having a more in depth understanding of who purchases from a vending machine, what affects the decision to purchase, and what items are consumed at what prices would lead to better recommendations as to how best to address the issue of consuming high in calorie snacks. Using different modeling techniques will answer such questions. Further, conducting experiments on what would happen if `item x' were available for purchase in a vending machine would provide key insights to alternative food options, state of mind when purchasing, and price analysis for the items offered.

When the effects of smoking were better understood and found to be potentially deadly (lung cancer), many promotional ads were used to deter Americans from smoking. Further, tobacco has different taxes associated with its purchases, as does alcohol. Perhaps obesity should not be treated differently. There are serious side effects of being overweight, many of which lead to death, and several of which are in the top leading causes of death in America (in fact, the leading cause is heart disease). It is clear that advertising can be used to affect consumption. Perhaps it is time to address the need of advertising for more healthy consumption habits.

More research is needed in regards to both the dairy and vending industries. The Government has separated whole milk from low-fat and skim milks for WIC purchasers. This may suggest to consumers that the other milk types are better than whole milk. Understanding how consumers perceive fluid milk types and their nutritional value is necessary. Perhaps survey data asking questions targeting what consumers believe about fluid milk and each type of fluid milk is the first step in understanding how consumers perceive fluid milk. Further, there are many substitutes now for fluid milk including cheese, yogurt, other types of milk (soy, almond, etc). Incorporating these products into a demand system may provide more understanding in how consumers substitute among various dairy products.

There is ample need for more research focusing on the vending machine industry. Knowing how consumers react to price changes among various products in vending machines will provide more information about how sensitive consumers are to prices. Understanding the consumer's state of mind when making the decision to purchase from a vending machine is needed as well. Perhaps consumers are really hungry and forgot to pack a snack, are just passing by a machine and saw a product that initiated a craving, have habitual tendencies, or are having an emotional day and want a snack. There are numerous reasons as to why a consumer makes the initial step towards purchasing at a vending machine. Some consumers may make the first decision to go to the vending machine, see the product selection, then opt to not purchase perhaps based on price or lack of selection. Having more understanding as to why consumers purchase from a vending machine and what they purchase for what prices is needed.

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APPENDIX I

A Brief Overview of Demand Theory and Demand Systems

In this section, derivations of demand functions, as well as resulting properties of such demand functions are provided. Further, details about how to use and interpret demand functions will be provided as well. This treatment by no means covers the entire theory behind demand; rather, it is a brief overview of some of the key concepts related to demand functions and their use. For much more detailed explanations about demand theory, see Varian (1992), Mas-Colell *et al.* (1995), and Deaton and Muellbauer (1980).

Brief Overview of Demand Functions

There are two general ways to derive demand functions. One is to maximize utility subject to a budget constraint. The second is to minimize expenditures with respect to a specific level of utility. The demand functions are referred to as Marshallian demand and Hicksian demand, respectfully. Marshallain demand functions are expressed as a function of prices and income, thus it is observable. Hicksian demand functions depend on utility, which is not directly observable (Varian 1992). Below, there is a brief discussion on these two demand functions as well as some general properties of demand functions.

Marshallian (Uncompensated, Walrasian) Demand Derivation

Suppose we maximize a utility function, $U(x_1, \ldots, x_n)$, subject to a budget constraint, $p`x = m$. Then, we have the following problem:

$$\begin{aligned} &v(p,m^*) = \max \;\; u(x) \\ &\text{such that} \quad px \leq m^* \\ &x_i^0 = x_i(p,m), \qquad \forall i \end{aligned} \tag{53}$$

where $v(p,m^*)$ is an indirect utility function, $u(x)$ is the utility level achieved by the vector of x commodities, x is an n x 1 vector of commodities whose *ith* component corresponds to the quantity of the *ith* commodity consumed[36], p is the corresponding commodity price vector, m is total expenditure, and the zero superscript represents the optimal quantities from solving the maximization problem. This indirect utility function results in demand equations represented by equation 51 being homogeneous of degree zero in prices and total expenditure. Further, the Slutsky substitution matrix is symmetric and negative semi-definite, as long as the demand function is derived from maximization of rational preferences. We can also apply WARP to this setting. Suppose we derived the demand function, $x(p, m)$. So, at prices p and wealth m, the consumer chooses x. Formally stated, $x(p, m)$ satisfies WARP if for any two wealth situations, (p, m), (p', m') (Mas-Colell *et al.* 1995):

$$p * x(p', m') \leq w \quad \text{and} \quad x(p', m') \neq x(p, m), \quad \text{then,} \quad p' * x(p, w) > w'. \tag{54}$$

Hicksian (Compensated) Demand Derivation

A Hicksian demand function, often referred to as compensated demand function, is called such because the demand function is viewed as being constructed by varying prices and income so as to keep the consumer at a fixed level of utility. Thus the income changes are arranged to compensate for the price changes (Varian 1992). Suppose we minimize expenditures (expenditure function), p`x, subject to a minimum utility requirement, u^* such as:

$$\begin{aligned} &e(p, u^*) = \min \; px \\ &\text{such that } \; u(x) \; \geq \; u^* \end{aligned} \tag{55}$$

Hence, we arrive at a consumption bundle minimizing total expenditure and achieving a target level of utility (Varian 1992). Hicksian demand functions are often deemed as compensating such because we are able to see how much the consumer must be compensated to achieve a set level of utility. For instance, if the price of good x rises, Hicksian demand functions can tell us how much to compensate the consumer to offset the ‘harm’ caused by the increase in a price so as to keep the consumer just as satisfied

[36] At times, q may be used in place of x; both refer to the quantity of the good specified.

as she was before the price change. This concept is referred to as compensating variation (Hicks 1942).

Relationship between Compensated and Uncompensated Demand

These demand functions are tied together through several identities. First, the minimum expenditure necessary to reach utility *v(p,m)* is *m*. The Marshallian demand at income *m* is the same as the Hicksian demand at utility *v(p,m)*. The Hicksian demand at utility *u* is the same as the Marshallian demand at income *e(p,u)*. This last property shows that the Hicksian demand function is equal to the Marshallian demand function at an appropriate level of income. Thus, any demanded bundle can be expressed either as the solution to the utility maximization problem or the expenditure minimization problem (Varian 1992). One application of these identities, Roy's Identity, states the following:

$$x_i(p,m) = -\frac{\dfrac{\partial v(p,m)}{\partial p_i}}{\dfrac{\partial v(p,m)}{\partial m}} \tag{56}$$

The key relationship between these two methods of deriving demand is that utility maximization implies expenditure minimization and expenditure minimization implies utility maximization (Varian 1992). Hence, from our identities, we know that $x(p^*, m^*) \equiv h(p^*, u^*)$. Another useful property from the duality of these demand functions is Shepard's Lemma. It states the following:

$$\begin{aligned} &h_i(p,u) = \frac{\partial e(p,u)}{\partial p_i} \\ &\textit{and} \\ &x_i(w,y) = \frac{\partial c(w,y)}{\partial w_i} \end{aligned} \tag{57}$$

Here, we see that the demand for a particular good, *i*, for a given level of utility, *u* and prices *p*, equals the derivative of the expenditure function with respect to that good's price. This application is used in the sense of the firm as well where conditional factor demand for the input good, *i*, is equal to the derivative of the cost function with respect to that good's cost, *w*.

Various Demand Properties

Here, I move into a brief overview of various demand function properties along with insight about each property. This list is not meant to be exhaustive; rather, it helps set up the framework of using demand functions to build demand systems.

Homogeneous of Degree Zero

For Marshallian demand functions, homogeneous of degree zero implies that if both prices and expenditure change in the same proportion, that individual's consumption choice does not change. To test if a demand function is homogeneous of degree zero, one can multiply all prices (p) and expenditure (m) by a constant, $\alpha > 0$; once simplifying, the answer should arrive at the original demand function. In other words, there is no change in the consumer's set of feasible consumption bundles (Mas-Colell *et al.* 1995). Intuitively, suppose person A's income increased by 10%, but prices for the goods A consumed increased by 10% as well. Then, there is no overall change in the feasible consumption bundle; hence, the consumption bundle (choice) does not alter.

Example (Mas-Colell *et al.* 1995): suppose we have the following demand function:

$$x_i(p,m) = \frac{p_j}{p_i + p_j + p_k} * \frac{m}{p_i} \tag{58}$$

Now, we multiply all prices and expenditures (p, m) by a constant, $\alpha > 0$:

$$x_i(\alpha p, \alpha m) = \frac{\alpha p_j}{\alpha p_i + \alpha p_j + \alpha p_k} * \frac{\alpha m}{\alpha p_i} \tag{59}$$

Notice that all α cancel out, resulting in the original demand function. Thus, this demand function is homogeneous of degree zero in prices and expenditure. Hicksian demand functions are homogeneous of degree zero in prices. This is because Hicksian demands are the derivatives of a function homogeneous of degree one (Deaton and Muellbauer 1980).

Slutsky (Substitution) Matrix

The Slutsky matrix is referred to as a substitution matrix because its elements are known as substitution effects. The Slutsky matrix takes the following form:

$$S_{lk}(p,m) = \frac{\partial x_l(p,m)}{\partial p_k} + \frac{\partial x_l(p,m)}{\partial m} * x_k(p,m) \qquad (60)$$

$S_{lk}(p,m)$ measures the differential change in the consumption of commodity l (substitution to or from other commodities) due to a differential change in the price of commodity k when wealth is adjusted so that the consumer can still just afford her original consumption bundle (due solely to a change in relative prices) (Mas-Colell *et al.* 1995). If the Slutsky matrix is negative semi-definite, this implies all S_{ll} entries are ≤ 0. Thus, the substitution effect of good l with respect to its own price is always non-positive. If the Slutsky matrix is symmetric, this implies that $S_{lk} = S_{kl}$. Generating demand from the maximization of rational preferences ensures a symmetric Slutsky matrix. Though the above equation does not directly state it, the Hicksian derivative can be calculated from the derivative of the Marshallian demand with respect to price and income (Varian 1992). Another way to state the Slutsky equation is:

$$\frac{\partial x_l(p,m)}{\partial p_k} = \frac{\partial h_l(p,v(p,m))}{\partial p_k} - \frac{\partial x_l(p,m)}{\partial m} * x_k(p,m) \qquad (61)$$

This decomposes the demand change induced by a price change in two separate effects, the substitution effect and the income effect (Varian 1992).

Though briefly mentioned above in regards to Giffen goods, there is a possibility that a S_{ll} though negative, may be outweighed by a positive income effect. This is only possible if the good is highly inferior and if purchased in large quantities (Deaton and Muellbauer 1980). One more nice application of the Slutsky matrix is that we can infer if goods are complements or substitutes. As before, goods are compliments if the S_{lk} ($l \neq k$) entries are negative and substitutes if the entry is positive.

Symmetry

The cross-price derivatives of the Hicksian demand are symmetric; hence, for all $i \neq j$,

$$\frac{\partial h_i(u,p)}{\partial h_j} = \frac{\partial h_j(u,p)}{\partial h_i} \tag{62}$$

Recalling the above about the Slutsky matrix, we can see why, for the maximization of preferences, the Slutsky matrix is symmetric. Without symmetry, inconsistent choices are made (Deaton and Muellbauer 1980).

Adding Up

Adding up implies that the sum of each function's expenditure adds up to the total of the complete system's expenditure. Noting the properties of each type of demand function, the following is true (Deaton and Muellbauer 1980):

$$\sum p_k h_k(p,u) = \sum p_k x_k(p,m) = m \tag{63}$$

Separability

The underlying idea behind separability is that consumers can separate commodities into groups so that preferences within each group are independent of quantities in other groups. For instance, three large 'groups' a consumer may be able to identify are food, shelter, and entertainment. The consumer's *preferences* within the food 'group' should not be affected by her shelter consumption, or anything else outside of the food group. Thus, we can have subutility functions for each group that when combined, give rise to that consumer's total utility (Deaton and Muellbauer 1980). This property is essential when working with demand systems, and will be emphasized later.

Translating

Pollack and Wales (1978) are credited as being the first to thoroughly discuss translating for demand systems. Translating is a general method for incorporating demographic variables into complete systems of demand equations (Pollack and Wales 1978). Using translating, the original demand systems is replaced by a new system containing parameters suitable for introducing such variables; then, it is assumed these new parameters are the only ones which depend on the demographic variables. This process is completed by specifying the functional form which related those parameters to the demographic variables; hence, the original demand system is replaced by a modified system (Pollack and Wales 1978). Translating postulates that demographic effects operate through a particular subset of *n* independent parameters which enter the demand system in a simple way (Pollack and Wales 1978).

Demand Systems and Their Applications

Before exploring two classes of demand systems, the purpose and use of such demand functions and systems should be discussed. There are several applications of demand functions once solved including, but not limited to, Engel curves, elasticities, and welfare analysis. All three are related in regards to understanding how behavior changes when say, a price changes.

Engel Curves

Engel curves, named after Ernst Engel (1895), relate household expenditure on particular goods and household income. We can arrive to the following two relationships when the additivity restriction is imposed:

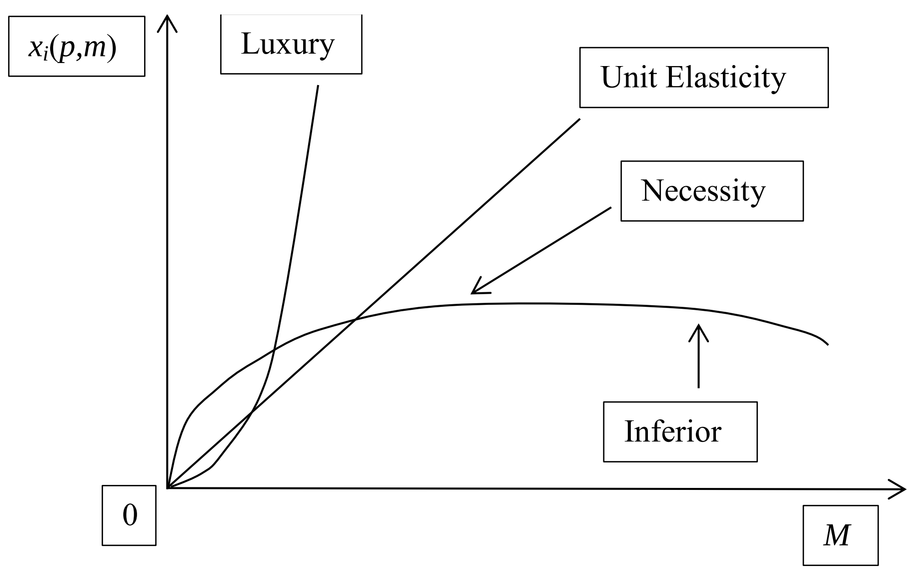
$$\sum_k p_k \frac{\partial x_i(p,m)}{\partial p_k} = 1 \;; \qquad \sum_k p_k \frac{\partial x_i(p,m)}{\partial p_k} + x_i = 0 \tag{64}$$

These relationships are referred to as Engel aggregation and Cournot aggregation. Through homogeneity, we have:

$$\sum_k p_k \frac{\partial x_i(p,m)}{\partial p_k} + m\frac{\partial x_i(p,m)}{\partial m} = 0 \tag{65}$$

This property says that a proportionate change in *p* and *m* will leave the purchases of good *i* unchanged (Deaton and Muellbauer 1980). Engel curves can be used to classify goods as luxuries, necessities, and inferior goods, just as income elasticities can do (see below). Luxury goods take up a larger share of a household's budget when households are better off (traveling) whereas necessities take up a larger share of a household's budget when the household is not as well off (or the vice-versa of luxuries; example: housing); inferior goods are those whose purchase absolutely declines as total household income increases (hotdogs, ramen noodles) (Deaton and Muellbauer 1980). Figure 6 below depicts the three different scenarios:

Figure 6: Engel Curves



Elasticities

Elasticity is a relationship between the quantity of a good and its price (own price elasticity), the purchaser's income (income elasticity), or the price of another good whose price may affect the quantity consumed of the original good (cross-price elasticity). Elasticities take the following form:

$$\frac{\partial x_i(p,m)}{\partial p_j} * \frac{p_j}{x_i(p,m)} \qquad \text{for } i = j, \text{ own price elasticity; for } i \neq j, \text{cross price elasticity}$$
$$\frac{\partial x_i(p,m)}{\partial M} * \frac{M}{x_i(p,m)} \qquad \text{Income elasticity;} \quad \text{M = income or total expenditure} \tag{66}$$

Own price elasticity is theoretically negative (or at most, zero)[37]; hence, the slope of the demand curve and the *Law of Demand*. Income elasticities allow the researcher to determine if a good is necessary, normal, or inferior, while cross-price elasticities give inference about two goods' relationship – whether the goods are consumed together (complements) or are substitutes. Own price and cross price elasticities are particularly useful to persons setting or changing the prices of goods (i.e., business owner). In general, elasticities are very useful and have many applications.

Elasticity formulas can also be expressed in budget share form. Budget shares take the following form:

$$w_i = \frac{p_i x_i}{m} \tag{67}$$

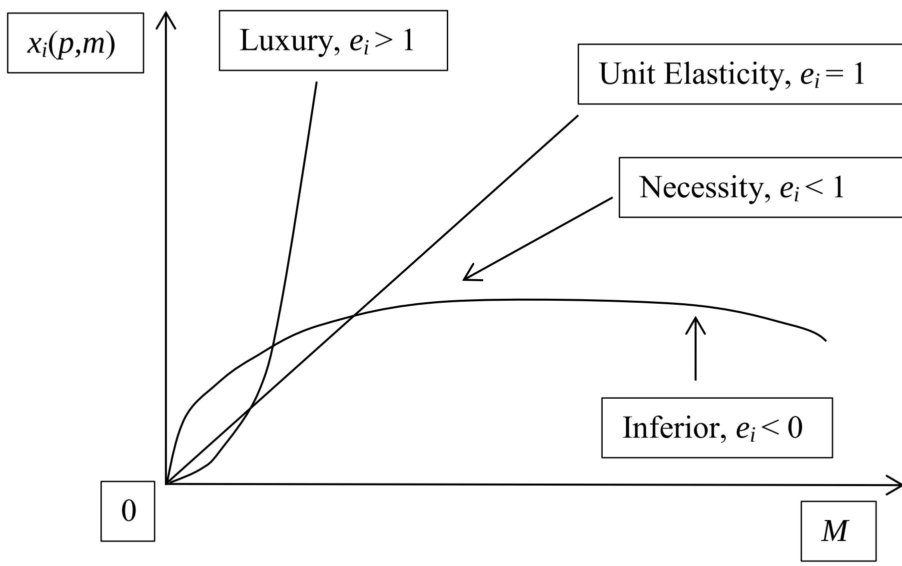
We see that budget shares are the proportion of income going to a particular good, for instance 15% of one's income going to apartment rent. The logarithmic derivatives of the Marshallian demands are the total expenditure and price elasticities, or gross/uncompensated elasticities (Deaton and Muellbauer 1980). Total expenditure and price elasticities can be represented by the following:

$$e_i = \frac{\partial \ln x_i(p,m)}{\partial \ln m} \qquad e_{ij} = \frac{\partial \ln x_i(p,m)}{\partial \ln p_j} \tag{68}$$

[37] Giffen goods (Sir Robert Giffen) are a violation of this theory; however, there is much debate on their true existence. The idea of a price rising and thus, quantity of that good rising does not conform to the *Law of Demand*.

If $e_i > 1$, the good i is said to be a luxury; If $e_i < 1$, the good i is said to be a necessity. If $e_i < 0$, the good is inferior. Below, we see figure 7 which is figure 6 modified to include the values of elasticities for each scenario:

Figure 7: Engel Curves with Elasticity Values



There is a more convenient way to understand equations 66 and 68. Expressing these equations in budget share forms, we can calculate the following:

Beginning with equation 66:

$$(1): \quad \frac{\partial \ln x_i(p,m)}{\partial \ln m} = \frac{\partial x_i(p,m)}{\partial m} * \frac{m}{x_i(p,m)}$$

(2): Remembering our budget share formulation:

$$w_i = \frac{p_i x_i(p,m)}{m} \qquad \text{and substituting into (1), we have:}$$

$$\sum_i \frac{p_i x_i(p,m)}{m} \frac{\partial x_i(p,m)}{\partial m} * \frac{m}{x_i(p,m)} = 1 \tag{69}$$

(3): Simplifying, we arrive to the original:

$$\sum_i p_i \frac{\partial x_i(p,m)}{\partial m} = 1$$

Similar steps can be taken to show the other relationships. Thus, we arrive to a more condensed formulation of equations 66 and 68:

$$\sum_k w_k e_k = 1 \quad ; \quad \sum_k w_k e_{ki} + w_i = 0 \tag{70}$$

and

$$\sum_k e_{ik} + e_i = 0 \tag{71}$$

which are referred to as Engel and Cournot aggregation, and finally homogeneity.

Some More Relationships

Here, I present the reader with a convenient way to 'move' from function to function within demand analysis, particularly between cost minimization and utility maximization in figure 8. This is taken from Deaton and Muellbauer (1980).

Figure 8: Duality and Inversion Relationships between Utility Maximization

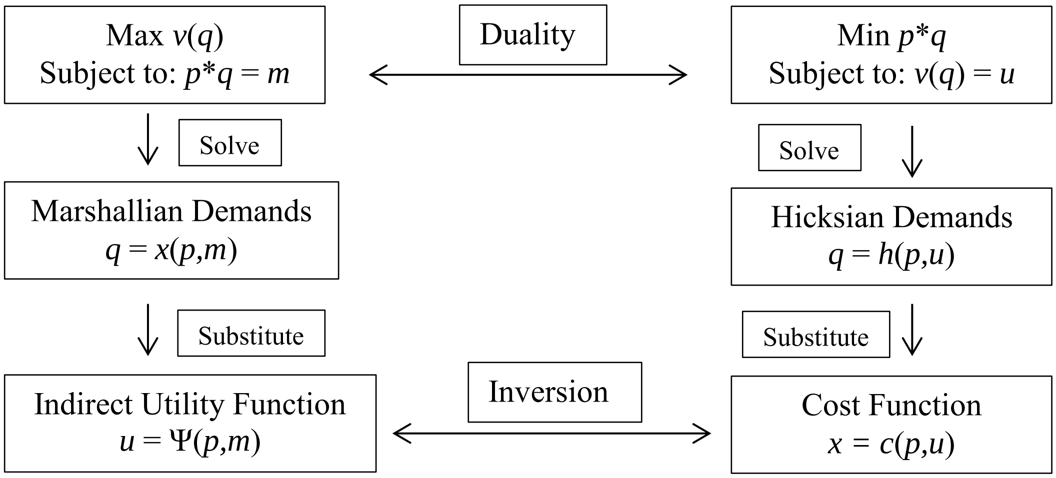
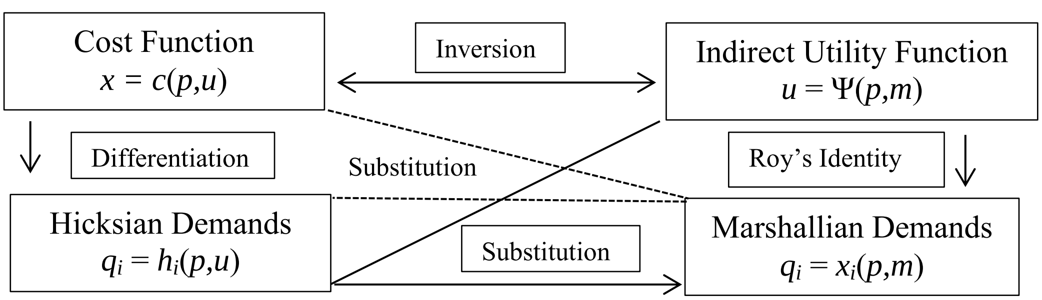


Figure 9 is another chart that summarizes the way we can transform the functions discussed earlier (Deaton and Muellbauer 1980)

Figure 9: Demand, Cost, and Indirect Utility Functions



Complete Demand Systems

Complete demand systems are deemed as such because of their 'completeness' when modeling. A complete system of demand equations describes the household's allocation of expenditure among some exhaustive set of consumption categories (Pollack and Wales 1978). For instance, suppose we are interested in looking at total at home food consumption. We can break that into categories (say N categories, meat, fruit, dairy, etc…), each with its own equation. Thus, we have 'N' expenditure values; the sum of the N expenditure values is total expenditure for that system. Hence, we have modeled the complete expenditure for the system. Notice that we will have to drop one equation in the system to avoid singularity in the matrix as the left hand side variables will sum to one if expenditure shares are used as the left hand side variable. The dropped equation's parameters are recovered using the additivity property that characterizes a complete demand system. Thus, in a complete demand system, we estimate (N-1) equations and recover the dropped equation.

A complete system of demand equations is said to be theoretically plausible if it is derivable from a well behaved utility function; equivalently, the demand equations are homogeneous of degree zero in prices and total expenditure, and the Slutsky matrix is negative semi-definite and symmetric (Pollack and Wales 1978). Complete demand systems can be used to test theory, such as symmetry and homogeneity, if specific restrictions are not enforced. Alternatively, the restrictions can be forced, thus resulting in a 'theoretically conforming' demand system.

Linear Expenditure System

One of the first recognized demand systems was developed by Richard Stone (1954). His linear expenditure system (LES), as the name describes, is linear because expenditures on individual commodities are expressed as linear functions of total expenditure and prices. The linear expenditure system is based on a utility function:

$$U = \prod_{i=1}^{N} (x_i - \gamma_i)^{\beta_i} \tag{72}$$

The general form is as follows:

$$p_i x_i = \gamma_i p_i + \beta_i (m - \sum_j \gamma_j p_j) . \tag{73}$$

Here, p_i and x_i are prices and quantities of good i, γ_i is the parameter associated with p_i, m is total expenditure, $p_j\gamma_j$ is total committed expenditure, and b_i is the parameter associated with total expenditure minus committed expenditure (Parks 1969). Stone notes that this is the most general linear expenditure system which is compatible with three of the most frequently imposed conditions on demand systems including additivity, homogeneity, and symmetry of the substitution (Slutsky) matrix. There are some limitations to the *LES* developed by Stone. Because of the restrictions of the Slutsky matrix, mainly negative diagonal elements, b_i must be between zero and one. Thus, the model cannot capture inferior goods. If inferior goods are ruled out, cross relationships can only be positive; thus, complementary goods cannot be modeled either (Stone 1954; Parks 1969).

Quadratic Expenditure System

Howe (1974) along with Pollack and Wales (1978) are credited with the derivation of the quadratic expenditure system (QES). The following indirect utility function is used to generate the *QES*.

$$\psi(P,u) = -\frac{\prod p_k^{a_k}}{\mu - \sum p_k b_k} + \lambda \frac{\prod p_k^{a_k}}{\prod p_k^{c_k}} \tag{74}$$

The share form of the *QES* is:

$$w_i = \frac{p_i b_i}{\mu} + a_i \left[1 - \frac{\sum p_k b_k}{\mu} \right] + (c_i - a_i)\lambda\pi \left(\frac{p_k}{\mu} \right)^{-c_k} \left[1 - \frac{\sum p_k b_k}{\mu} \right]^2 \tag{75}$$

where in both equation 72 and 73, the sum of c_k and a_k is equal to 1. If $c_i = a_i$ for all i, then the quadratic terms vanish and the *QES* reduces to the *LES* (Pollack and Wales 1978). Unlike the *LES*, the relationship between the underlying parameters and the marginal budget shares is not a simple one, and no parameters of the system are identified by a single budget study; if the marginal budget shares are independent of expenditure, the system reduces to the *LES* and the *a*'s are identified by a single budget study (Pollack and Wales 1978).

Rotterdam Demand System

Theil (1965) and Barten (1966) are credited with developing the Rotterdam model, a differential demand model (Parks 1969). The Rotterdam model arises from utility maximization subject to a budget constraint, and can be written in terms of prices and a measure of real income. In other words, a first-order approximation to the demand functions themselves is used (Deaton and Muellbauer 1980). It takes the following form:

$$w_i d \log x_i = b_i d \log \overline{m} + \sum_{j=1}^{N} c_{ij} d \log p_j \tag{76}$$

where

$$\begin{aligned} d \log \overline{m} &= d \log m - \sum w_k d \log p_k = \sum w_k d \log x_k \\ b_i &= w_i e_i = p_i \frac{\partial x_i}{\partial m} \\ c_{ij} &= w_i e_{ij}^* = \frac{p_i p_j s_{ij}}{m} \end{aligned} \tag{77}$$

and $w_i = p_i * x_i / m$, or the expenditure share for good *i*, b_i is the i^{th} income elasticity weighted by the expenditure share or, equivalently, the derivative of expenditure on the i^{th} good with respect to income, c_{ij} is the compensated cross price elasticity, weighted by the corresponding expenditure share, e_{ij}^* is the compensated cross price elasticity, and s_{ij} is the ij^{th} entry of the Slutsky matrix. The classical demand restrictions require the following:

$$\begin{aligned} &\sum_k b_k = 1; \quad \sum_k c_{kj} = 0 \quad \text{(adding up)} \\ &\sum_k c_{jk} = 0 \quad \text{(homogeneity)} \\ &c_{ij} = c_{ji} \quad \text{(symmetry)} \end{aligned} \tag{78}$$

Translog Models

Christensen *et al*., (1975) developed translog functions based on direct or indirect utility. The direct translog utility function is based on maximizing utility subject to a budget constraint. The following represents the demand equation derived from the direct translog utility function:

$$w_i = \frac{\alpha_j + \sum \beta_{ji} \ln X_i}{\alpha_m + \sum \beta_{mi} \ln X_i} \qquad \forall \ \mathrm{j} \tag{79}$$

The indirect utility function leads to the following derivation of demand equations:

$$w_i = \frac{\alpha_j + \sum \beta_{ji} \ln \frac{p_i}{m}}{\alpha_m + \sum \beta_{mi} \ln \frac{p_i}{m}} \qquad \forall \ \mathrm{j} \tag{80}$$

Here, X_i represents consumption of good *i* and for symmetry to hold, $B_{ji} = B_{ij}$ for all *i, j*. A basic translog system (BTL) does not include translation parameters; therefore, the introduction of demographic characteristics must be preceded by the introduction of a translation parameter for each good. Incorporating demographic variables through demographic translating requires estimation of additional parameters (Pollack and Wales 1980). When $\Sigma_j \beta_{ij} = 0$ for all i, the *BTL* transforms into a special case, or the 'homogeneous translog' (*HTL*) (Pollack and Wales 1980).

Pollack and Wales (1980) introduced a 'generalized translog' model (GTL). This demand system includes three indirect translog forms. It is derived from the following indirect utility function:

$$\psi(P,u) = -\sum a_k \log\left[\frac{p_k}{(\mu - \sum_t p_t b_t}\right] - \frac{1}{2}\sum\sum_j \beta_{kj} \log\left[\frac{p_k}{(\mu - \sum_t p_t b_t}\right] \log\left[\frac{p_j}{(\mu - \sum_t p_t b_t}\right] \tag{81}$$

where

$\beta_{ij} = \beta_{ji} \quad \forall$ i, j;

$\sum \alpha_{\mathrm{k}} + \sum\sum_j \beta_{kj} = 1$

From equation 79, the following demand function is derived:

$$w_i = \frac{b_i p_i}{\mu} + \left[1 - \frac{\sum p_k b_k}{\mu}\right] * \left[\frac{\alpha_i + \sum_j \beta_{ij} \ln\left[\frac{p_j}{\left(\mu - \sum p_k b_k\right)}\right]}{\sum \alpha_k + \sum \sum_j \beta_{kj} \ln\left[\frac{p_j}{\left(\mu - \sum p_k b_k\right)}\right]} \right] \tag{82}$$

where

$\beta_{ij} = \beta_{ji} \quad \forall$ i, j;

$\sum \alpha_{\mathrm{k}} + \sum \sum_j \beta_{kj} = 1$

Pollack and Wales (1980) suggested that the GTL functional form was a significant improvement over the BTL. Likewise, their research showed that the QES yielded a higher likelihood value than the BTL.

Almost Ideal Demand System (AIDS)

In 1980, Deaton and Muellbauer introduced the Almost Ideal Demand System, or *AIDS*. This model, though comparable to the Rotterdam and Translog models, has considerable advantages over both. This model gives an arbitrary first-order approximation to any demand system, satisfies the axioms of choice exactly, aggregates perfectly over consumers without invoking parallel linear Engel curves, has a functional form, largely avoids the need for non-linear estimation, and can be used to test the restrictions of homogeneity and symmetry through linear restrictions on fixed parameters (Deaton and Muellbauer 1980a). This model is 'ideal' because, though the Rotterdam and translog models possess many of the listed properties, neither possess all of the properties at the same time (Deaton and Muellbauer 1980a).

Rather than start from an arbitrary preference ordering, the AIDS model begins from a specific class of preferences known as PIGLOG preferences (Muellbauer 1975 1976). These preferences are represented via the cost or expenditure function and are denoted as $c(u,p)$. The PIGLOG class is defined by:

$$\log c(u, p) = (1-u)\log\{a(p)\} + u\log\{b(p)\} \tag{83}$$

where u lies between 0 (subsistence) and 1 (bliss)[38] so that the positive linearly homogeneous functions $a(p)$ and $b(p)$ can be regarded as the costs of subsistence and bliss, respectively (Deaton and Muellbauer 1980a). Following this, the functional forms for *log* $a(p)$ and *log* $b(p)$ are derived:

$$\begin{aligned} \log a(p) &= \alpha_0 + \sum_k \alpha_k \log p_k + \frac{1}{2}\sum_k \sum_j \gamma^*_{kj} \log p_k \log p_j \\ \log b(p) &= \log a(p) + \beta_0 \prod_k p_k^{\beta_k} \end{aligned} \tag{84}$$

Now the AIDS cost function is written as:

$$\log c(u,p) = \alpha_0 + \sum_k \alpha_k \log p_k + \frac{1}{2}\sum_k \sum_j \gamma^*_{kj} \log p_k \log p_j + u\beta_0 \prod_k p_k^{\beta_k} \tag{85}$$

where α_i, β_i, and γ_{ij}^* are parameters. It can be checked that $c(u,p)$ is linearly homogeneous in p, as it must be to be a valid representation of preferences, provided that $\Sigma_i\alpha_i = 1$, $\Sigma_k\gamma_{kj}^* = \Sigma_j\gamma_{kj}^* = \Sigma_j\beta_j = 0$. A fundamental property of the cost function is that its price derivatives are the quantities demanded (see Shepard's Lemma above): $\partial c(u,p)/\partial p_i = q_i$. By multiplying both sides by $p_i/c(u,p)$, we arrive at the following (Deaton and Muellbauer 1980a):

$$\frac{\partial \log c(u,p)}{\partial \log p_i} = \frac{p_i q_i}{c(u,p)} = w_i \tag{86}$$

where w_i is the budget share of good i. The logarithmic differentiation of equation 83 yields the budget shares as a function of prices and utility:

$$w_i = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i u \beta_0 \prod p_k^{\beta_k} \tag{87}$$

where

$$\gamma_{ij} = \frac{1}{2}(\gamma^*_{ij} = \gamma^*_{ji}) \tag{88}$$

Now, for a utility maximizing consumer, total expenditure, x, is equal to $c(u,p)$; this equality can be inverted to give u as a function of p and x, the indirect utility function. Doing this for equation 85 and substituting into equation 87, we have the budget shares

[38] There are some exceptions to this. See Deaton and Muellbauer 1980a, Appendix.

as functions of p and x. Thus, we arrive at the AIDS demand functions in budget share form (Deaton and Muellbauer 1980a):

$$w_i = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log\left\{\frac{x}{P}\right\} \tag{89}$$

where P is a price log index. There are two popular price indexes – the Stone Price Index (SPI) and the Translog Price Index (TPI). The SPI takes the following form:

$$lnP \approx \sum_i w_i\, ln(p_i)\,. \tag{90}$$

As suggested by (Hahn 1994), the 'true' AIDS model utilizes the TPI which takes the following form:

$$lnP = \alpha_0 + \sum_k \alpha_k ln(p_k) + \frac{1}{2}\left(\sum_k \sum_l \gamma_{kl} ln(p_k) ln(p_l)\right). \tag{91}$$

The restrictions on the parameters of equation 83 along with equation 86 imply restrictions on the AIDS equation. The following restrictions are imposed:

$$\begin{aligned}
&\sum_i \alpha_i = 1,\ \ \sum_i \beta_i = 0,\ \ \sum_i \gamma_{ij} = 0 && \text{(Adding Up)}\\
&\sum_i \gamma_{ji} = 0 && \text{(Homogeneity)}\\
&\gamma_{ij} = \gamma_{ji} && \text{(Symmetry)}
\end{aligned} \tag{92}$$

Provided the conditions above hold, the AIDS equation represents a system of demand functions which add up to total expenditure ($\Sigma w_i = 1$), are homogeneous of degree zero in prices and total expenditure, and satisfy Slutsky symmetry. AIDS is therefore interpreted as: in the absence of changes in relative prices and real expenditure, the budget shares are constant, and this is the natural starting point for predictions using the model (Deaton and Muellbauer 1980a). Changes in relative prices work through the terms γ_{ij}; each γ_{ij} represents 10^2 times the effect on the ith budget share of a 1% increase in the jth price with (x/P) held constant. Changes in real expenditure operate through β_i coefficients which add up to zero. Notice that when β_i is positive, it denotes a luxury while when it is negative, it denotes a necessity (Deaton and Muellbauer 1980a). The uncompensated price elasticity formula for the AIDS model is:

$$-\delta_{ij} + \frac{\gamma_{ij}}{w_i} - \frac{\beta_i \alpha_j}{w_i} - \frac{\beta_i}{w_i} \sum_k \gamma_{kj} \ln P_k \tag{93}$$

where δ_{ij} is the Kronecker delta (if $i = j$, $\delta_{ij} = 1$; if $i \neq j$, $\delta_{ij} = 0$).

Barten Synthetic Model

Barten (1993) developed a demand system that incorporated four demand models. The four models which comprise the Barten Synthetic Demand Model (BSM) include two that were mentioned previously, the Rotterdam (Theil 1965 and Barten 1966) and the AIDS model (linearized approximation, LA/AIDS)[39] (Deaton and Muellbauer 1980a). The other two models Barten nested within his model were the (Dutch) Central Bureau of Statistics (CBS) (Keller and van Driel 1985) and the NBR model (Neves 1987). The equations are as follows:

$$\begin{aligned}
&w_i d \log x_i = b_i d \log Q + \sum_{j=1}^{N} s_{ij} d \log p_j && \text{(Rotterdam)} \\
&w_i (d \log x_i - d \log \overline{m}) = c_i d \log Q + \sum_{j=1}^{N} s_{ij} d \log p_j && \text{(CBS)} \\
&dw_i = c_i d \log Q + \sum_{j=1}^{N} r_{ij} d \log p_j && \text{(LA/AIDS)} \\
&dw_i + w_i d \log x_i = b_i d \log Q + \sum_{j=1}^{N} r_{ij} d \log p_j && \text{(NBR)}
\end{aligned} \tag{94}$$

where some slight modifications have been made to the two demand systems previously defined including: $d \log Q \equiv \sum_i w_i d \log x_i$, $s_{ij} = \left(\frac{p_i p_j}{m} \right) \left(\frac{\partial h_i}{\partial p_j} \right)$ which is the Slutsky ij^{th} term of the Slutsky matrix, $c_i \equiv b_i - w_i$, and $r_{ij} \equiv s_{ij} + w_i (\delta_{ij} - w_j)$. Notice that the CBS, the LA/AIDS, and the NBR equations do not have the same left hand side as that of the Rotterdam; however, it is clear that slight modifications can be made so that all four

[39] This is the linear approximation of the AIDS in differential form where the Translog Price Index is replaced by the Stone Price Index (Matsuda 2005).

equations have the same dependent variable. The following modified equations are nested within the BSM:

$$
\begin{aligned}
&w_i d\log x_i = b_i d\log Q + \sum_{j=1}^{N} s_{ij} d\log p_j && \text{(Rotterdam)} \\
&w_i d\log x_i = (c_i + w_i) d\log Q + \sum_{j=1}^{N} s_{ij} d\log p_j && \text{(CBS`)} \\
&w_i \log x_i = (c_i + w_i) d\log Q + \sum_{j=1}^{N} [r_{ij} - w_i(\delta_{ij} - w_j)] d\log p_j && \text{(LA/AIDS`)} \\
&w_i d\log x_i = b_i d\log Q + \sum_{j=1}^{N} [r_{ij} - w_i(\delta_{ij} - w_j)] d\log p_j && \text{(NBR`)}
\end{aligned} \tag{95}
$$

Barten (1993) showed that although none of the above equations had another nested within it, a synthetic model of relatively simple form that nested these four differential demand systems could be constructed as (Matsuda 2005):

$$
w_i d\log x_i = (\beta_i + \lambda w_i) d\log \overline{m} + \sum_{j=1}^{N} [\gamma_{ij} - \mu w_i(\delta_{ij} - w_j)] d\log p_j \tag{96}
$$

where $\beta_i \equiv (1-\lambda_i)b_i + \lambda c_i$ and $\gamma_{ij} \equiv (1-\mu)s_{ij} + \mu r_{ij}$. Equation (above) is reduced to the Rotterdam when (λ, μ) = (0, 0), to the CBS when (λ, μ) = (1, 0), to the NBR when (λ, μ) = (0 1), and to the LA/AIDS when (λ, μ) = (1 1) (Matsuda 2005). Though it is obvious that this synthetic model serves well for the purpose of specifying functional forms of differential demand systems, it is in a rather mechanical manner that λ and μ are involved in linear combinations of the coefficients of the nested models; however their economic implications seem unclear particularly when they take values other than zero and unity (Matsuda 2005).

As with other models mentioned, restrictions of the BSM are imposed to satisfy demand theory. The restrictions are as follows:

$$
\begin{aligned}
&\sum_i \beta_i + \lambda = 1, \quad \sum_i \gamma_{ij} = 0 \quad \forall j && \text{(adding up)} \\
&\sum_j \gamma_{ij} = 0 \quad \forall j && \text{(homogeneity)} \\
&\gamma_{ij} = \gamma_{ji} \quad \forall i, j && \text{(Slutsky Symmetry)}
\end{aligned} \tag{97}
$$

Elasticities for expenditure and price (compensated own price and cross price) are as follows:

$$e_i = \frac{\beta_i}{w_i} + \lambda, \qquad e_{ij}^C = \frac{\gamma_{iij}}{w_i} - \mu(\delta_{ij} - w_j), \qquad e_{ij}^U = e_{ij}^C - e_i w_j \tag{98}$$

Matsuda (2005) shows that the BSM is a model in its own right showing that the BSM has the same marginal budget shares as generated by the Box-Cox transformed Engle curves. In other words, the expenditure elasticities in an arbitrary differential demand system correspond to those in a specific form of Engle curve.

Quadratic Almost Ideal Demand System

Perhaps the most recently developed complete demand system is that of Banks *et al.* (1997). Their demand system is referred to as the Quadratic Almost Ideal Demand System (QUAIDS). The basis of developing the model was that for many commodities, standard empirical demand models do not provide an accurate picture of observed behavior across income groups. Their extension to the AIDS model contains a new class of demand systems that have log income as the leading term in an expenditure share model and additional higher order income terms (Banks *et al.* 1997). The QUAIDS model takes the following form:

$$w_{it} = \alpha_i + \sum_j \gamma_{ij} lnp_j + \beta_i ln\left(\frac{m}{a(p)}\right) + \frac{\lambda_i}{b(p)}\left\{ln\left[\frac{m}{a(p)}\right]\right\}^2 + e_{it} \tag{99}$$

where variables are defined above as before, the $Ln(a(p))$ is the TPI, and $b(p)$ is the simple Cobb-Douglas price aggregator defined as:

$$b(p) = \prod_{i=1}^{n} p_i^{\beta_i} \tag{100}$$

and λ is defined as:

$$\lambda(p) = \sum_{i=1}^{n} \lambda_i ln(p_i), \qquad \text{where} \sum_i \lambda_i = 0. \tag{101}$$

However, λ(p) is assumed to be independent of prices, thus λ is not a function of p. The demands generated are rank three (maximum possible rank for any demand system that is linear in functions of income (see Gorman 1981)), exactly aggregable, are derived from utility maximization, and permit goods to be luxuries at some income levels and necessities at others (Banks *et al.* 1997). The QUAIDS model is advantageous because it embodies very flexible price and income effects (Cranfield 2012). Note, when $\lambda_i = 0$ for all *i*, QUAIDS collapses to the previously mentioned AIDS model; also, QUAIDS only has local monotonicity and curvature properties (Cranfield 2012).

The elasticities for a QUAIDS model are rather complex. They are listed below:

$$\mu_i \equiv \frac{\partial w_i}{\partial \ln m} = \beta_i + \frac{2\lambda_i}{b(p)}\left\{\ln\left[\frac{m}{a(p)}\right]\right\}$$

$$\mu_{ij} \equiv \frac{\partial w_i}{\partial \ln p_j} = \gamma_{ij} - \mu_i\left(\alpha_j + \sum_k \gamma_{jk} \ln P_k\right) - \frac{\lambda_i \beta_j}{b(p)}\left\{\ln\left[\frac{m}{a(p)}\right]\right\}^2 \tag{102}$$

$$e_i = \frac{\mu_i}{w_i} + 1, \qquad e_{ij}^u = \frac{\mu_{ij}}{w_i} - \delta_{ij}, \qquad e_{ij}^c = e_{ij}^u + e_i w_j$$

where e_i is income elasticity with respect to good *i*, e_{ij}^u is uncompensated own price ($i = j$, $\delta_{ij} = 1$; if $i \neq j$, $\delta_{ij} = 0$) or cross price elasticity ($i \neq j$) where δ_{ij} is the Kronecker delta, and e_{ij}^c is compensated own price ($i = j$) or cross price elasticity ($i \neq j$) (Banks *et al.* 1997). To arrive at the elasticities, a few simple steps can be taken provided in figure 10:

Figure 10: QUAIDS Budget Elasticity Derivation

(1) : Recall the QUAIDS model :

$$w_i = \alpha_i + \sum_j \gamma_{ij} lnp_j + \beta_i ln\left(\frac{m}{a(p)}\right) + \frac{\lambda_i}{b(p)}\left\{ln\left[\frac{m}{a(p)}\right]\right\}^2$$

(2) : To arrive at the budget elasticity, we must differentiate the above with respect to $\ln m$:

$$\frac{\partial w_i}{\partial \ln m} = \beta_i + \frac{2\lambda_i}{b(p)}\left\{ln\left[\frac{m}{a(p)}\right]\right\}$$

(3) : The budget/expenditure elasticity formula is (Banks et al., 1997) :

$$e_i = \frac{\partial \ln x_i}{\partial \ln m} = \frac{\partial \ln w_i}{\partial \ln m} * \frac{1}{w_i} + 1$$

(4) : Remembering that QUAIDS is in budget share formulation :

$$w_i = \frac{p_i x_i(p,m)}{m}$$

Figure 10 Continued:

(5) : We need to use a derivative property:

$$\frac{\partial x_i}{\partial m} * \frac{m}{x_i} \equiv \frac{\partial \ln x_i}{\partial \ln m}$$

(6) : Taking the natural log of step (4) we arrive at :

$$\ln w_i = \ln p_i + \ln x_i \text{ - } \ln m$$

(7) : Divide through by $\ln m$:

$$\frac{\ln w_i}{\ln m} = \frac{\ln p_i}{\ln m} + \frac{\ln x_i}{\ln m} \text{ - } \frac{\ln m}{\ln m}$$

(8) : Solve for x_i; take derivative & use property(5) :

$$\frac{\partial \ln x_i}{\partial \ln m} = \frac{\partial \ln w_i}{\partial \ln m} - \frac{\partial \ln p_i}{\partial \ln m} + \frac{\partial \ln m}{\partial \ln m} \equiv \frac{\partial w_i}{\partial \ln m} * \frac{1}{w_i} - \frac{\partial \ln p_i}{\partial \ln m} + \frac{\partial \ln m}{\partial \ln m}$$

(9) : This simplifies to the original budget elasticity formulation (1) :

$$e_i = \frac{\partial \ln w_i}{\partial \ln m} * \frac{1}{w_i} + 1$$

(10) : Substituting (2) into (9) we arrive to the formula provided by Banks et al. (1997) :

$$e_i = \left(\beta_i + \frac{2\lambda_i}{b(p)} \left\{ \ln \frac{m}{a(p)} \right\} \right) * \frac{1}{w_i} + 1$$

The uncompensated price elasticity derivation is not quite as straight forward. We will take similar steps, but the price functions complicate things a bit. By taking the derivative with respect to the natural log of the specified price, we have an easier derivation than trying to derive with respect to the specified price alone. Derivation is provided in figure 11:

Figure 11: QUAIDS Uncompensated Price Elasticity Derivation

We need to arrive at the Banks et al. (1997) price elasticity formula:

$$\frac{\partial \ln x_i}{\partial \ln p_j} = \gamma_{ij} - \left(\beta_i + \frac{2\lambda_i}{b(p)}\left\{\ln\left[\frac{m}{a(p)}\right]\right\}\right) * \left(\alpha_j + \sum_k \gamma_{jk} \ln P_k\right) - \frac{\lambda_i \beta_j}{b(p)}\left\{\ln\left[\frac{m}{a(p)}\right]\right\}^2 * \frac{1}{w_i} - \delta_{ij}$$

(1): Recalling the QUAIDS formula, and plugging in the respectable price functions, we have:

$$w_i = \alpha_i + \sum_j \gamma_{ij} ln p_j + \beta_i ln\left(\frac{m}{\left(\alpha_0 + \sum_i \alpha_i ln(p_i) + \frac{1}{2}\sum_i \sum_j \gamma_{ij} ln(p_i) ln(p_j)\right)}\right) +$$

$$\frac{\lambda_i}{\prod_{i=1}^{n} p_i^{\beta_i}}\left\{ln\left[\frac{m}{\left(\alpha_0 + \sum_i \alpha_i ln(p_i) + \frac{1}{2}\sum_i \sum_j \gamma_{ij} ln(p_i) ln(p_j)\right)}\right]\right\}^2$$

(2): After simplification, we arrive at:

$$w_i = \alpha_i + \sum_j \gamma_{ij} ln p_j + \beta_i\left[\ln m - \left(\alpha_0 + \sum_i \alpha_i ln(p_i) + \frac{1}{2}\sum_i \sum_j \gamma_{ij} ln(p_i) ln(p_j)\right)\right] +$$

$$\lambda_i\left(e^{-\sum_i \beta_i \ln p_i}\right)\left[\ln m - \left(\alpha_0 + \sum_i \alpha_i ln(p_i) + \frac{1}{2}\sum_i \sum_j \gamma_{ij} ln(p_i) ln(p_j)\right)\right]^2$$

(3): Differentiating (2) with respect to $\ln p_j$, we arrive at:

$$\frac{\partial w_i}{\partial \ln p_j} = \gamma_{ij} - \beta_i \alpha_j - \beta_i \sum_i \gamma_{ij} \ln p_i + 2\lambda_i\left(\sum_i \beta_i \ln p_i\right)^{-1}\left[\ln m - \left(\alpha_0 + \sum_i \alpha_i ln(p_i) + \frac{1}{2}\sum_i \sum_j \gamma_{ij} ln(p_i) ln(p_j)\right)\right] *$$

$$\left(-\alpha_j - \sum_i \gamma_{ij} \ln p_i\right) - \lambda_i \beta_j\left(e^{-\sum_i \beta_i \ln p_i}\right)\left[\ln m - \left(\alpha_0 + \sum_i \alpha_i ln(p_i) + \frac{1}{2}\sum_i \sum_j \gamma_{ij} ln(p_i) ln(p_j)\right)\right]^2$$

(4): We need to simplify and replace the price functions with their original notation:

$$\frac{\partial w_i}{\partial \ln p_j} = \gamma_{ij} - \beta_i\left(\alpha_j + \sum_i \gamma_{ij} \ln p_i\right) + \frac{2\lambda_i}{b(p)}(\ln m - \ln a(p)) * \left(-\alpha_j - \sum_i \gamma_{ij} \ln p_i\right) - \frac{\lambda_i \beta_j}{b(p)}(\ln m - \ln a(p))^2$$

(5): From here, we see we can simplify even further:

$$\frac{\partial w_i}{\partial \ln p_j} = \gamma_{ij} - \left(\alpha_j + \sum_i \gamma_{ij} \ln p_i\right)\left(\beta_i + \frac{2\lambda_i}{b(p)}(\ln m - \ln a(p))\right) - \frac{\lambda_i \beta_j}{b(p)}(\ln m - \ln a(p))^2$$

Figure 11 Continued:

(6) : Taking similar steps as we did with the budget elasticity :

$$\frac{\partial \ln x_i}{\partial \ln p_j} = \frac{\partial \ln w_i}{\partial \ln p_j} - \frac{\partial \ln p_i}{\partial \ln p_j} + \frac{\partial \ln m}{\partial \ln p_j} \equiv \frac{\partial w_i}{\partial \ln p_j} * \frac{1}{w_i} - \frac{\partial \ln p_i}{\partial \ln p_j} + \frac{\partial \ln m}{\partial \ln p_j}$$

(7) : Now, substituting (6) into (5) :

$$e_{ij}^{u} = \left\{ \gamma_{ij} - \left(\alpha_j + \sum_i \gamma_{ij} \ln p_i \right) \left(\beta_i + \frac{2\lambda_i}{b(p)} \left(\ln m - \ln a(p) \right) \right) - \frac{\lambda_i \beta_j}{b(p)^2} \left(\ln m - \ln a(p) \right)^2 \right\} * \frac{1}{w_i} - \delta_{ij}$$

where δ_{ij} is the Kronecker delta ($\delta_{ij} = 1$ if $i = j$; $\delta_{ij} = 0$ if $i \neq j$).

We arrive at the uncompensated price elasticity for a QUAIDS model. As was seen in Chapter 2, we modified a QUAIDS model to include an advertising variable; specifically, we incorporated a polynomial distributed lag advertising variable (Almon 1965). As a result, the modified QUAIDS model takes this form:

$$w_{it} = \alpha_i + \sum_j \gamma_{ij} lnp_j + \beta_i ln\left(\frac{m}{a(p)} \right) + \frac{\lambda_i}{b(p)} \left\{ ln\left[\frac{m}{a(p)} \right] \right\}^2 + \sum_{k=0}^{k} \theta_{ik} lnA_{t-k} + e_{it} \qquad (103)$$

Much the same as deriving the elasticities for both budgets and prices, we will have to derive the elasticity for advertising. However, to much simplicity, the advertising variable stands alone and is not incorporated within other variables. Its derivation is more simple and provided in figure 12.

Figure 12: QUAIDS Advertising Elasticity Derivation

(1) : In order to derive the advertising elasticity, we start with the general formula :

$$\frac{\partial x_i}{\partial adv} * \frac{adv}{x_i} \equiv \frac{\partial \ln x_i}{\partial \ln adv}$$

(2) : Differentiating our modified QUAIDS equation above with respect to ln Adv, we have :

$$\frac{\partial w_i}{\partial \ln adv} = \theta_i$$

(3) : Again, this model is in budget share form; thus, we follow similar steps above by taking the natural log of the budget share equation and arrive at :

$$\ln w_i = \ln p_i + \ln x_i - \ln m$$

(4) : Divide through by ln adv; take its derivative, and solve for x_i :

$$\frac{\partial \ln x_i}{\partial \ln adv} = \frac{\partial \ln w_i}{\partial \ln adv} - \frac{\partial \ln p_i}{\partial \ln adv} + \frac{\partial \ln m}{\partial \ln adv} \equiv \frac{\partial w_i}{\partial \ln adv} * \frac{1}{w_i} - \frac{\partial \ln p_i}{\partial \ln adv} + \frac{\partial \ln m}{\partial \ln adv}$$

(5) : Thus, we see the advertising elasticity is :

$$\frac{\partial \ln x_i}{\partial \ln adv} = \frac{\partial w_i}{\partial \ln adv} * \frac{1}{w_i} = \frac{\theta_{ik}}{w_i}$$

Incomplete Demand Systems

Another class of demand systems is referred to as incomplete demand systems. Though Epstein (1982) is credited with the initial idea, LaFrance and Hanemann (various) have thoroughly examined and analyzed the incomplete demand system framework. The need for incomplete demand systems can be explained rather easily. As LaFrance (1990) suggests, incomplete information is the standard/normal scenario. Though unlikely then due to computing space, one may argue that today, there is enough space to compute a ‘complete’ demand system. However, yet again, we would likely arrive at a dimensionality problem, and likely, would not be able to count all of the goods in a market. Lafrance (1990) offers three potential solutions to this problem: (1) aggregate across commodities; however, this is restrictive and information is lost; (2) assuming preferences are separable; this may result in simultaneous equations bias in conditional demand models, and this only reveals the structure of a subutility function; (3) specify an incomplete demand system; though challenged, LaFrance (1990) suggests that his and Haneman's (1989) work provides answers to the challenges.

As was shown previously, one way to derive demand functions is by stating a direct or indirect utility function, then deriving the demand functions. Conversely, one can simply specify a demand function directly (LaFrance and Hanemann 1989). The problem with directly specifying a demand function is its ability (or inability) of being integrated. Without integration, welfare analysis cannot be conducted and recovering properties cannot be completed.

Unlike complete demand systems where we assume separability for the particular subset of goods we are analyzing, incomplete demand systems accept the fact that not all consumed goods are modeled within the system. Hence, LaFrance and Hanemann (1989) suggest that in applied research, incomplete demand models are the rule rather than the exception. Incomplete demand systems allow a more general class of functional forms than complete demand models (LaFrance and Hanemann 1989). The added generality is due to the adding-up condition not being an equality restriction but rather an inequality restriction on the total expenditure for the goods of interest (LaFrance and Hanemann 1989). A complete demand system cannot be linear in the prices and goods of interest, but an incomplete system can be linear in the prices of the goods of interest and in total expenditure and satisfy the conditions for integrability (LaFrance and Hanemann 1989; see LaFrance 1985).

LaFrance and Hanemann's (1989) paper offers the step-by-step derivation of the integrability of an incomplete demand system. Weak integrability of incomplete demand systems shows: (1) the dual relationships between recoverable parts of the expenditure, indirect, and direct utility functions are analogous to the dual relationships for complete demand systems, (2) exact welfare measures can be calculated, and (3) the conditional preference structure for the central commodities can be recovered (LaFrance and Hanemann 1989). Further, incomplete demand models that are the result of utility maximization subject to a linear budget constraint have four properties (LaFrance 1990): (1) the demands are positive valued; (2) the demands are homogeneous of degree zero in all prices and income; (3) the substitution effects matrix for each subset of goods is symmetric, negative semi-definite; and (4) income is greater than the total expenditure on any proper subset of goods consumed. If the demand model satisfies those four conditions, then the following is also true (LaFrance 1990): (1) the conditional preference structure for the goods under study can be recovered from the demand equations; (2) the dual structures for the recoverable parts of the utility expenditure and indirect utility functions are analogous to the dual structures for complete demand equations; and (3) exact welfare measures can be derived from the incomplete demand system. Thus, a coherently specified incomplete demand model contains all of the necessary information to complete any of the usual tasks of applied economic analysis (LaFrance and Hanemann 1989; LaFrance 1990).

There are many variations of incomplete demand systems since there is less of an exact structure to follow. Variations include linear in all quantities, prices, and income (LaFrance 1985), logarithmic in all quantities, prices, and income, and semilogarithmic demand models (LaFrance 1990) including linear or logarithmic in quantities, prices, and income but are neither linear nor logarithmic for all three sets of variables. Roger von Haefen (2002) summarizes and presents many variations of incomplete demand systems. Variations include simply 'x' as the dependent variable, expenditure as the dependent variable, and perhaps the more familiar expenditure share as the dependent variable. He also includes Slutsky symmetry restrictions for each model presented. The following figures below are taken from von Haefen (2002); multiple variations of incomplete demand system models are provided below in figures 13, 14, and 15:

Figure 13: Incomplete Demand System Models (von Haefen 2002); previously specified by LaFrance (1985; 1986; 1990)

$$(x1):\ \ x_i = \alpha_i(q) + \sum_{k=1}^{n} \beta_{ik} p_k + \gamma_i y, \quad \forall\, i$$

$$(x2):\ \ x_i = \alpha_i(q) + \sum_{k=1}^{n} \beta_{ik} p_k + \gamma_i \ln(y), \quad \forall\, i$$

$$(x3):\ \ x_i = \alpha_i(q) + \sum_{k=1}^{n} \beta_{ik} \ln(p_k) + \gamma_i y, \quad \forall\, i$$

$$(x4):\ \ x_i = \alpha_i(q) + \sum_{k=1}^{n} \beta_{ik} \ln(p_k) + \gamma_i \ln(y), \quad \forall\, i$$

$$(x5):\ \ x_i = \alpha_i(q) \exp\left\{ \sum_{k=1}^{n} \beta_{ik} p_k + \gamma_i y, \right\}, \quad \forall\, i$$

$$(x6):\ \ x_i = \alpha_i(q) \exp\left\{ \sum_{k=1}^{n} \beta_{ik} p_k \right\} y^{\gamma_i}, \quad \forall\, i$$

$$(x7):\ \ x_i = \alpha_i(q) \exp\left\{ \prod_{k=1}^{n} p_k^{\beta_{ik}} \right\} \exp(\gamma_i y), \quad \forall\, i$$

$$(x8):\ \ x_i = \alpha_i(q) \exp\left\{ \prod_{k=1}^{n} p_k^{\beta_{ik}} \right\} y^{\gamma_i}, \quad \forall\, i$$

Figure 14: Incomplete Expenditure System Models (von Haefen 2002)

$$(e1):\ \ x_i = \alpha_i(q) + \sum_{k=1}^{n} \beta_{ik} p_k + \gamma_i y, \quad \forall\, i$$

$$(e2):\ \ x_i = \alpha_i(q) + \sum_{k=1}^{n} \beta_{ik} p_k + \gamma_i \ln(y), \quad \forall\, i$$

$$(e3):\ \ x_i = \alpha_i(q) + \sum_{k=1}^{n} \beta_{ik} \ln(p_k) + \gamma_i y, \quad \forall\, i$$

$$(e4):\ \ x_i = \alpha_i(q) + \sum_{k=1}^{n} \beta_{ik} \ln(p_k) + \gamma_i \ln(y), \quad \forall\, i$$

$$(e5):\ \ x_i = \alpha_i(q) \exp\left\{ \sum_{k=1}^{n} \beta_{ik} p_k + \gamma_i y, \right\}, \quad \forall\, i$$

$$(e6):\ \ x_i = \alpha_i(q) \exp\left\{ \sum_{k=1}^{n} \beta_{ik} p_k \right\} y^{\gamma_i}, \quad \forall\, i$$

$$(e7)^a:\ \ x_i = \alpha_i(q) \exp\left\{ \prod_{k=1}^{n} p_k^{\beta_{ik}} \right\} \exp(\gamma_i y), \quad \forall\, i$$

$$(e8)^b:\ \ x_i = \alpha_i(q) \exp\left\{ \prod_{k=1}^{n} p_k^{\beta_{ik}} \right\} y^{\gamma_i}, \quad \forall\, i$$

[a]: Note the equivalence between this specification and (x7) (figure 13) if the following parametric transformations are made: $\beta_{ii}^{(e7)} = \beta_{ii}^{(x7)} + 1, \ \ \forall i$.

[b]: Note the equivalence between this specification and (x7) (figure 13) if the following parametric transformations are made: $\beta_{ii}^{(e8)} = \beta_{ii}^{(x8)} + 1, \ \ \forall i$.

Figure 15: Incomplete Expenditure Share System Models (von Haefen 2002)

$(s1):\ x_i = \alpha_i(q) + \sum_{k=1}^{n} \beta_{ik} p_k + \gamma_i y, \quad \forall\, i$	$(s5):\ x_i = \alpha_i(q) \exp\left\{ \sum_{k=1}^{n} \beta_{ik} p_k + \gamma_i y, \quad \right\}, \quad \forall\, i$
$(s2):\ x_i = \alpha_i(q) + \sum_{k=1}^{n} \beta_{ik} p_k + \gamma_i \ln(y), \quad \forall\, i$	$(s6)^a:\ x_i = \alpha_i(q) \exp\left\{ \sum_{k=1}^{n} \beta_{ik} p_k \right\} y^{\gamma_i}, \quad \forall\, i$
$(s3):\ x_i = \alpha_i(q) + \sum_{k=1}^{n} \beta_{ik} \ln(p_k) + \gamma_i y, \quad \forall\, i$	$(s7):\ x_i = \alpha_i(q) \exp\left\{ \prod_{k=1}^{n} p_k^{\beta_{ik}} \right\} \exp(\gamma_i y), \quad \forall\, i$
$(s4):\ x_i = \alpha_i(q) + \sum_{k=1}^{n} \beta_{ik} \ln(p_k) + \gamma_i \ln(y), \quad \forall\, i$	$(s8)^b:\ x_i = \alpha_i(q) \exp\left\{ \prod_{k=1}^{n} p_k^{\beta_{ik}} \right\} y^{\gamma_i}, \quad \forall\, i$

[a]: Note the equivalence between this specification and (e6) (figure 14) if the following parametric transformations are made: $\gamma_i^{(s6)} = \gamma_i^{(e6)} - 1, \quad \forall i$.

[b]: Note the equivalence between this specification and (x7) (figure 13) if the following parametric transformations are made: $\gamma_i^{(s8)} = \gamma_i^{(e8)} - 1, \quad \forall i$; also note the equivalence between this specification and (x8) (figure16) if the following parametric transformations are made: $\gamma_i^{(s8)} = \gamma_i^{(x8)} - 1; \quad \beta_{ii}^{(s8)} = \beta_{ii}^{(x8)} + 1, \quad \forall i$.

Polynomial Distributed Lags

Almon (1965) presented the polynomial distributed lag (PDL) as very flexible and easy to estimate. The name PDL provides us with a lot of information. First, we have a polynomial form of a specified degree (two, three, four, etc). We have a distribution in the polynomial degree form (symmetric), and finally, we have a lag of the variable of interest. Almon provided several steps to take in order to find the optimal lag length; however, computations are much easier now. In general, we determine two points of the lag. These are referred to as end points, or head/tail restrictions and are specified to have a value of zero. Thus, the polynomial will always pass through the number of lags plus two points. These points can be specified as zero because we are assuming that past a cutoff, say $t - k$, there are no significant effects of the lagged variable. This same interpretation is used for various lag length; the lagged variable past a selected time period will have no significant effects on the variable of interest. Once the lag length k is determined (can determine using Schwarz/Bayesian Information

Criteria (S/BIC), Akiake Information Criteria (AIC), or others), we set the end points. Then, the coefficients of the distributed lag fit on a polynomial of degree specified.

This PDL technique is often used to find optimal lag lengths and to estimate advertising or expenditure effects on various outcomes such as products purchase, and research and development (Almon 1965; Falk and Miller 1977; Sougiannis 1994). In this application, we used a polynomial distributed lag for advertising expenditures and measured its effect on fluid milk consumption.

Suppose we have a simple equation:

$$Sales_{it} = \alpha_0 + \theta_{i0} A_{it} + \theta_{i1} A_{it-1} + \theta_{i2} A_{it-2} + ... + \theta_{ik} A_{it-k} + \varepsilon_{it} \tag{104}$$

where $Sales_{it}$ represents sales of product *i* at time *t*, *A* represents advertising expenditures at time *t-k* for the i^{th} product, and θ_{ik} is its corresponding coefficient, α_0 is an intercept, and ε_{it} is an error term. It is assumed that θ_{ik} can be represented with a polynomial of degree *m*, where *m* = 0 1 2 … , *m* such that:

$$\theta_{ik} = \varphi_0 + \varphi_1 k + \varphi_2 k^2 + \varphi_3 k^3 + ... + \varphi_m k^m \tag{105}$$

Suppose that a lag length of four is chosen for the advertising variable. This would imply that we have *t*-1, … *t*-4. Now, assuming a second degree polynomial for θ_{ik}, (i.e., *m* = 2, *k* = 1 2, 3, 4), we reach the following:

$$\theta_{ik} = \varphi_0 + \varphi_1 k + \varphi_2 k^2, \text{for k} = 0, 1, 2, 3, 4 \tag{106}$$

By imposing head and tail restrictions of no effects before k = 0 and after k = 4, we have the following:

$$\begin{aligned} \theta_{i,-1} &= \varphi_0 - \varphi_1 + \varphi_2 = 0, \text{for k} = \text{-}1 \\ \theta_{i,5} &= \varphi_0 + 5\varphi_1 + 25\varphi_2 = 0, \text{for k} = 5 \end{aligned} \tag{107}$$

Combining like terms, we reach:

$$\begin{aligned} &\varphi_0 = \varphi_1 - \varphi_2 = 0 \\ &\text{Now, substituting in } \varphi_1 - \varphi_2 \text{ for } \varphi_0: \\ &\varphi_1 - \varphi_2 + 5\varphi_1 + 25\varphi_2 = 0 \Rightarrow 6\varphi_1 + 24\varphi_2 \text{ or } \varphi_1 = -4\varphi_2 \\ &\text{Consequently, } \varphi_0 = -5\varphi_2 \quad \therefore \text{we need only to estimate } \varphi_2 \end{aligned} \tag{108}$$

However, since the lag length is not generally known in advance, we must estimate the distribution using varying numbers of periods, then choose the best among them (Almon 1965). This process was employed in the preceding chapter. Though we chose a polynomial of degree two, we are not limited to such a choice. Likewise, a lag of four is not a limitation. We could have shown a polynomial of degree four with seven lags; the steps are the same.

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