DYNAMIC ECONOMY WITH HETEROGENEOUS AGENTS

A Dissertation

by

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ABSTRACT

This dissertation consists of three essays about heterogeneous agents in the dynamic economy and how to deal with the asymmetric information arose by heterogeneity.

Firstly, I consider the optimal taxation issue in a dynamic endogenous growth model with considering human capital accumulation, and agents ability is heterogeneous and private information. Moreover, the agents with higher ability have positive external effects on others. By using the two-sector endogenous model, I show that it is optimal to impose different income and capital income taxes on people with different abilities. Specifically, positive marginal income tax is adopted for people with lower ability while no tax is imposed for people with higher ability; marginal capital income tax is zero whatever the agent's is low or high. As for people using the capital and labor for human capital accumulation, the government should subsidize them whatever their ability is.

Secondly, I study the optimal monetary and fiscal policy with heterogeneous agents based on the search-theoretical environment where money is essential and consider the private information. I first solve the households' problem in the centralized and decentralized market, and find out the optimal conditions. Then, in this section, I describe the problem that social planner faces by involving uncertainty and agents whose types are continuous. By comparing the optimal conditions in this generous setting, I show that the Friedman rule is no longer optimal when jointed with nonlinear taxation of income. Moreover, the capital income taxation is not zero.

Moreover, I constructs a general theoretical model to consider two kinds of financial frictions in the economy with financial intermediaries. By quantitative analysis the model with three separate shocks which are a negative collateral shock, a negative productivity shock and a positive shock to bankers' divert rate, I find that a negative collateral shock which tightens firms' financing constraints on investment can generate an equity price boom which is different from what is observed in recessions. Therefore, the collateral shock is not the main reason for the business cycle, while the negative productivity shock and bankers' moral hazard problem are more important aspects to explain current economy.

DEDICATION

To My Parents

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Firstly, I am deeply indebted to my advisor, Dennis W. Jansen. Three years ago, I decided to choose macroeconomics as my field. Starting from that time, he has given me the most valuable advice I can expect. During the five years in our department, I have always learned from him as a graduate student, as a research assistant, as an instructor. This dissertation has been benefited remarkably from his detailed comments.

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1. INTRODUCTION

Lucas [66] points out that for all practical purposes, in fact, the central problem of depression prevention for macroeconomics has already been solved for many decades. However, right now, better policies provide people with more incentive to work which leads to great gains in welfare. By introducing heterogeneity in the dynamic economy, my research has focused on studying various issues in optimal fiscal and monetary policy using the dynamic Mirrlees framework, building on Golosov, Kocherlakota, and Tsyvinski [44], and Golosov, Tsyvinski and Werning[47]. An important idea articulated in this approach is that as heterogeneity matters the economy, it is reasonable to consider optimal monetary policy and fiscal policy together in the environment with heterogeneous agents instead of the one with homogeneous representative agents. The main objective of my research is trying to find out the optimal trade-off between incentive and insurance. On one hand, we need to consider how to supply the right insurance against bad shocks for those agents with low skills, on the other hand, we need to provide incentives for those agents with high skills to tell the truth.

In Section 2, I consider the optimal non-linear taxation issues in a limited horizon dynamic economy with human capital accumulation, where agents' ability is heterogeneous and private information. Moreover, the agents with higher ability have positive external effects on others. By using a two-sector endogenous growth model, this paper finds out that it is optimal to impose different income and capital income taxes on people with different abilities. Specifically, positive marginal income tax is adopted for people with lower ability while no tax is imposed for people with higher ability; marginal capital income tax is zero whatever the agent is low ability person or not. Furthermore, if people use the capital and labor for human capital accumulation, the government should subsidize both them whatever is the agent's ability, which is consistent with Mankiw, Weinzierl and Yagan [69]'s statement.

Based on this, I have gone one step further. Since the fiscal pressure can have important consequences for monetary policy, specifically, the source of fiscal pressure on monetary policy is the consolidated fiscal-monetary budget constraint, it is necessary to consider the monetary policy and fiscal policy together. The Section 3 seeks to build models that contain many of the feature commonly used in the macroeconomic models and supply the micro-foundations of money demand. Previous papers consider optimal fiscal policy in an economy without involving money, or discuss optimal monetary policy with homogeneous agents in reduced-form approaches such as money-in-utility model and cash-in-advance model. Different from them, in this section, I study the optimal monetary and fiscal policy with heterogeneous agents other than homogeneous agents. In the economy, as there exists friction to value money, making money essential can expand the set of feasible trades. Based on the search-theoretical environment such as Lagos and Wright [64] and Williamson and Wright [87], I first solve the households' problem in the centralized and decentralized market, and find out the optimal conditions. Then, the section describes the problem that social planner faces by involving uncertainty and heterogeneous agents whose types are continuous. While the households in the economy are heterogeneity in skills which are private information and always changing over time, this section develops a complete description of the optimal allocations in this economy. In order to classify different kinds of agents, we need to consider a large number of incentive constraints, which would lead us to reconsider the optimal monetary and fiscal policy under these settings. By comparing the optimal conditions in this generous setting, I show that the Friedman rule is not optimal when jointed with nonlinear taxation of income. Moreover, the capital income taxation is not zero.

Since financial sectors are playing more and more important role in modern economies, following the Great Depression, economists such as Fisher [35] and Keynes [55] have pointed out that the failure of financial markets would result in the economic downturn. In many standard macroeconomic models, identical households can invest in non-financial firms directly, without using financial sectors. As stated in Brunnermeier and Sannikov [16], this approach can only yield realistic macroeconomic predictions if, in reality, there are no frictions in the economy. However, The current financial crisis starting in August 2007 has underscored and reminded us once again of the importance of financial intermediaries and financial market frictions for the business cycles.

In the last section, I construct a general theoretical model to consider two kinds of financial frictions in the economy with financial intermediaries. One is the classical principal-agency problem between financial intermediaries and households, which will limit the bankers' ability to obtain deposits without constraints by borrowing extra funds from households. The other financial friction is a collateral constraint when firms borrow external funds from banks. By formulating households, bankers and entrepreneurs' decisions in a monetary DSGE framework with nominal rigidities and determining the competitive equilibrium, this model supplies the environment that we can simulate the production shocks and liquidity shocks which can be used to analysis the effects of monetary policies. By calibrating the model with three separate shocks which are a negative collateral shock, a negative productivity shock and a positive shock to bankers' divert rate, this paper finds that a negative collateral shock which tightens firms' financing constraints on investment can generate an equity price boom which is different from what is observed in recessions. Therefore, a collateral shock is not the primary driving power of these business cycles, while a negative productivity shock and bankers' moral hazard problem are more important aspects to explain current economy. Moreover, it provides a caution for policy makers: they should find the reasons of the shortfall in liquidity other than simply pumping liquidity into the market.

2. HUMAN CAPITAL ACCUMULATION, POSITIVE EXTERNALITY AND OPTIMAL TAXATION

Lucas [67] has pointed out that the sources of the present economic world can be treated partially as a result of accumulation in physical and human capital, especially during the process of transiting from a agricultural society to a society of industrialization. Retrospecting the history of economic development, we could found out the following facts: the process of industrialization is accompanied with urbanization. During the proceeding, the rate of population living in cities is increasing and the share of the workforce in agriculture is decreasing dramatically. In 1850, the farmers made up 64% of labor force in United States, and the proportion fell to 38% in 1900, to 12.2% in 1950, and then to 3.6% in 1980, while the similar story also happened in Britain.

As we could see from the Table A.1, when the economy took off during the periods, accordingly, the share of rural population in that country declined significantly. In South Korea, as a typical example of Newly Industrial Economics (NIEs), the growth rate maintained exceptionally high between the early 1960s and 1990s, meanwhile, the share of rural population fell from 72.3% in 1960 to 20.4% in 2000. During the last twenty years of the twentieth century, some Asian countries have achieved great progress in economic development, such as China, Indonesia, Malaysia and Philippines, the shares of rural population also decreased remarkably. Since China started the Reform and Open policy in the end of 1970s, the rate of rural population has been declined from 80.4% in 1980 to 56.9% in 2008, as a country with over one billion people, there are millions of people crowd into the cities and adapt to the new life there. Just shown in the Table A.1, Argentina and Brazil have been undergoing the similar transitions between 1960 and 2008.

Without double, physical capital accumulation is the crucial element to determine the growth rate of the economy especially during the period of industrial revolution. As for human capital, as Galor and Moav [38] point out human capital accumulation has replaced the physical accumulation as a driving force of economic development during the transition from the Industrial Revolution to current growth. Since labor's ability is the vital point which influences the speed of transition, the labor forces from rural areas have the incentive to invest their resources to improve their personal ability which will increase the intensity of competition in the cities. Accordingly, the original skilled workers, in the urban areas, need to increase their investment in order to enhance their skill levels, which would creates benefit for other agents from the rural areas [1].

Since human capital's effect on aggregate output is extremely important not only for policymakers but also for economists, and while taxation policies are important instruments for government to improve the economic efficiency and social equity, especially when individuals ability is heterogeneous and private information, it is meaningful to consider how to constitute proper public policies to accelerate human capital and substance capital accumulation with regarding the distortion.

There are two different approaches in considering the optimal taxation. Starting from Ramsey [76], it studied the problem of choosing an optimal linear tax system in an economy with homogeneous agents when lump-sum taxation is not available. By giving the set of tax instruments, this approach has the conclusion that as for the taxation on commodities, the proportionate taxes should lead the production of each taxed commodity to decrease the same proportion. Following the Ramsey's

[1] Acemoglu [1] has emphasized the importance of this kind of human capital accumulation empirically.

work, Atkinson and Stiglitz [8] points out that if the utility function is separable and homothetic, the taxation on commodity should be uniform, which means that the optimal taxes across consumption goods are the same. Moreover, Atkinson and Stiglitz [9] and [7] states that tax rates depend on income elasticities, with necessities taxed more than luxuries. The familiar intuition from partial equilibrium that goods with low price elasticities should be taxed heavily does not necessarily apply in a general equilibrium setting. As for the intermediate goods, the taxes on them should be zero, and there are taxes only on final goods (Chari and Keheo[20]). In these papers, the only inputs for goods production is labor and do not need capital and technology.

As considering the taxing capital income, in a neoclassical model with individuals who are living infinitely, Judd [60] and Chamley [18] state that the capital income could tax at initially high rates, while in the long run, the taxing rates should drop to zero. Here, in view of the capital's significance for goods production, they made the above conclusion and did not consider the capital was also important for human capital production. Based on Chamley-Judd result, Jones, Manuelli and Rossi [50] show that in the long run, the human capital's return ought not to be taxed. However, in this model, it did not consider the situation with heterogeneous agents, and made the conclusion about the optimal capital income tax when their ability was uncertain in the future.

Different from Ramsey approach, the Mirrlees approach of optimal taxation is set up on a totally different background. The central problem of the Mirrlees approach is trying to find the optimal trade-off between incentive and insurance. On one hand, we need to consider how to supply the right insurance against bad shocks for those agents with low skills, on the other hand, we need to provide incentives for the agents with high skills to tell the truth. The vital conclusion is that the margin between consumption and leisure for agents with the highest skill is not distorted (Mirrlees [71],Diamond and Mirrlees[28] [29]).

In recent years, beginning with Golosov, Kocherlakota and Tsyvinski [44] and Werning [86], lots of literature applies the Mirrlees [71] set-up to dynamic environments which is more perplexed than implementation of either Ramsey models or static Mirrlees models(Golosov, Tsyvinski and Werning [47]). In Golosov, Kocherlakota and Tsyvinski [44] and Kocherlakota [61], they study the optimal taxation problems in the economy where individual skills are their own private information and change randomly over time. If the preferences are additive separability between labor and consumption, they have the different conclusion that generally, even though the uniform commodity taxation theorem still holds, the capital income taxation is not zero. Depending on the above study, Golosov and Tsyvinski [45] suggest an asset-tested disability program to implement the optimum allocation. In this program, the disability insurance can be achieved, and meanwhile, it is also accomplished through a direct mechanism that joins with the wealth and income tax system. Moreover, this paper also proved that when agents are heterogeneous and market is incomplete, the linear taxation in Ramsey model is not the Pareto tax.

In the above series of papers, they assumed that the agents' ability was heterogeneous and unobservable, while they did not consider the knowledge spillovers between different kinds of people. Obviously, the people with high ability have positive external effect on the people with low ability. Meanwhile, they did not take the human capital accumulation into account, or just considered the time was the only input for human capital accumulation, without taking into account the physical capital's role in the process of human capital accumulation.

In this section, I consider the optimal taxation with the dynamic Mirrlees models in the neoclassical economics. The basic assumptions are as follows: there exist heterogeneous agents with private information about their personal abilities. But differ from the above dynamic model with regarding the human capital accumulation. In the paper Golosov, Kocherlakota and Tsyvinski [44], they assume the agents's skill levels which could be regarded as human capital are heterogeneous and independent to each other, and agents could use their capital and time to improve their human capital. Besides, the agents' skill level are interaction other than independent. As we know, production can be given a spatial dimension by postulation a production externality that makes any individual more productive if other productive people are nearby. Eaton and Eckstein [31] point out that the externality affects the technology used for human capital accumulation other than the technology used for producing final goods. Thus, I use the two-sector economy in Rebelo [78] and Jones, Maneulli and Rossi [51][50] with regarding knowledge spillovers.

The rest of the section is organized as follows: In the section 2, I state the assumptions in the paper. Then, I adopt the three-period model to work out the conditions for first best allocation when the information is complete. In the section 4, when the information is incomplete, I get the conditions for second best allocation. In the section 5, I have the conclusions about optimal taxation in the discrete economy.

2.1 The Model

The basic environment is similar to Golosov, Tsyvinski and Werning [47], Costa and Maestri[24] and Bohacek and Kapicka[14]. The main difference is that there exists knowledge spillover effect between different kinds of agents, and in order to improve the personal ability, they also need to input physical capital besides the time. The reason of these assumptions is to emphases the externality of the human capital accumulation and the physical capital's importance in the process of producing the final goods and increasing their skill levels.

In the economy, there exists two kinds of people and the only difference between them is their personal ability denoted by θ_i, $\theta_H > \theta_h$, $\theta_i \in R_{++}$, here, the θ_i is exogenous, unobservable and unchangeable. Furthermore, we could regard the difference of ability is decided by nature such as the intelligence.

Just as stated in Rebelo [78] and Jones, Maneulli and Rossi [51][50] , throughout the paper I consider two-sector economy: one sector is specialized in producing the final production and the other sector only accumulates the human capital. Although they have different production function, both of them need input physical capital and labor. Moreover, the consumption market is perfect competitive.

Everyone has one unit endowment of time, which uses for leisure L_t, producing physical goods l_t and human capital accumulation N_t, obviously, there exists $L_t = 1 - N_t - l_t$ and $l_t, N_t \in (0, 1)$.

2.1.1 Preferences

Each agent lives for three periods, which just like life cycle, in the first period, both types of agents have the same human capital level; in the second period, as agetns' learning ability is different across agents, they will have the different human capital level, and the externality will happen during this period; in the last period, they will allocate any resource in physical and human capital accumulation since they would like to use all their income for consumption. Meanwhile, their preferences are the same which can be represented as follows:

$$\sum_{t=0}^{3} \beta^t U(c_t, L_t), \tag{2.1}$$

where $\beta \in (0,1)$, the utility function U is continuous differential and satisfies $U_c > 0$, $U_L > 0$, $U_{cc} < 0$, $U_{LL} < 0$ and Inada conditions. According to Atkinson and Stiglitz [8][7], it is normal to assume the utility function is separable between consumption and leisure, which means $U_{cL} = 0$, and we can rewrite the instantaneous utility function as $U(c_t, L_t) = u(c_t) + v(1 - N_t - l_t)$.

2.1.2 Production Function

Before we describe the productive function, we need to know the concept of efficient labor. Just as in Golosov, Kocherlakota and Tsyvinski[44]Kapicka[54] and Kocherlakota[61] [62], the efficient labor is:

$$y = HN,$$

where H denotes the human capital and N is the agent's real labor input which are both unobservable, which Efficient labor y can be observed.

In period t, the human capital accumulation depends on the agent's personal ability, time spending and physical capital. The law of motion equation for high ability agents' human capital can be written as:

$$H_{H,t+1} = G(\theta_H, x_{Ht}, y_{Ht}^H) + (1 - \delta_H)H_{Ht}, \tag{2.2}$$

where effective labor $y_{Ht}^H = H_{Ht}N_{Ht}$, $N_{Ht} \in (0,1)$ is the amount of agent's real labor used in accumulating human capital, x_{Ht} is the physical capital input, $\delta_H \in (0,1)$ is the discount rate of human capital. As in traditional, the production function $G(\theta_H, x_{Ht}, y_{Ht}^H)$ is strictly increasing and concave.

With regarding knowledge spillovers, just as in Eaton and Eckstein [31], Lucas [67], in the proceeding of human capital accumulation, as a result of the difference among agents, the levels of human capital would be diversified. More importantly, the agents with high ability will have positive external effect on people with low ability, and the law of motion equation for low ability agents' human capital can be written as:

$$H_{h,t+1} = (\frac{H_{Ht}}{H_{ht}})^{\delta} G(\theta_h, x_{ht}, y_{ht}^H) + (1 - \delta_H) H_{ht}, \tag{2.3}$$

here, $\delta > 0$, x_{ht} denotes the low ability people's physical capital input, y_{ht}^H is the effective labor input.

In period t, the agent i's productive function of final goods is:

$$Y_i(t) = F(K_{it}, y_{it}^l), \tag{2.4}$$

where effective labor $y_{it}^l = H_{it} l_{it}$, l_{it} is the agent's labor input in producing final goods; K_{it} is the agent's physical capital and the law of motion is:

$$K_{i,t+1} = I_{it} + (1 - \delta_k) K_{it}, \tag{2.5}$$

here I_{it} is agent i's physical capital investment, δ_k is the capital discount rate.

Especially, just as in Barro and Sala-i-Martin [11], there exists different density in using physical capital and labor between the productive functions of final goods and human capital. Obviously, the productive function of final goods is relative density in using physical capital while the function of human capital is relative density in using labor.

2.1.3 Taxation Policy

As in the Mirrlees private information framework, the taxation policy is the function of observed variables, which means that taxation rate depends on the individual's labor income and capital income[2], thus the taxation:

$$T_t = T(y_t^H, y_t^l, x_t, I_t) \tag{2.6}$$

In every period, the government uses the tax income to keep balance, which can be indicated by the resource constraints and the individuals' budget constraints.

2.2 First Best Allocation

The economy lasts for 3 periods. Initially, the agents with different ability have the same human capital and physical capital, according to their difference in ability, their human capital will diversify in the next period.

Firstly, we discuss the optimal allocation when information is perfect and government knows their personal ability. The object of this section is to supply a benchmark so that it can be used to compared with the results in other sections.

The social planner characterizes optimal allocation $\{c_{it}, N_{it}, l_{it}, x_{it}, I_{it}\}_{i=H,h}$ by solving the following problem:

$$\max_{\{c_{it},N_{it},l_{it},x_{it},I_{it}\}} \sum_{i=H,h} (\sum_{t=1}^{2} \beta^{t-1} U(c_{it}, 1 - N_{it} - ll_{it}) + \beta^2 U(c_{i3}, 1 - l_{i3}). \tag{2.7}$$

In the first best allocation, the constraints for social planner are just the feasible conditions in every period. In period one, the feasible condition for government is

$$\sum_{i=H,h} (c_{i1} + x_{i1} + I_{i1}) + g_1 = \sum_{i=H,h} F(K_{i1,}, H_{i1} l_{i1}). \tag{2.8}$$

[2]Since the labor and capital market are competitive, wage and rental rates are given. Therefore, we can rate the taxation policy as a function depends on efficient labor and capital level.

In period two,

$$\sum_{i=H,h}(c_{i2}+x_{i2}+I_{i2})+g_2=\sum_{i=H,h}F(K_{i2},H_{i2}l_{i2}). \tag{2.9}$$

Here, $H_{i2}=G(\theta_i,x_{i1}+H_{i1}H_{i1})+(1-\delta_i)H_{i1}$, and $K_{i2}=(1-\delta_k)K_{i1}+I_{i1}$, where $i=H,h$. In period three,

$$\sum_{i=H,h}c_{i3}+g_3=\sum_{i=H,h}F(K_{i3},H_{i3}l_{i3}). \tag{2.10}$$

Here, the physical capital $K_{i3}=(1-\delta_k)K_{i2}+I_{i2}$; the human capital of agent with high ability is $H_{H3}=G(\theta_H,x_{H2}N_{H2})+(1-\delta_H)H_{H2}$.

With considering the knowledge spillovers, the human capital of agent with low ability is $H_{h3}=(\frac{H_{H2}}{H_{h2}})^{\delta}G(\theta_h,x_{h2},H_{h2}N_{h2})+(1-\delta_h)H_{h2}$.

It is easy to solve the problem (2.7) under the constraints of (2.8) (2.9) (2.10), and get the first order conditions as follows:

As for the consumptions in each period,

$$\frac{\partial U(c_{Ht},L_{Ht})}{\partial L_{Ht}}=\frac{\partial U(c_{ht},L_{ht})}{\partial L_{ht}} \tag{2.11}$$

Here, $t=1,2,3$. If $U_{cL}=0$, equation (2.11) means different agents should have the same consumption levels when the information is perfect.

As for the agent's labor input l_t in producing final goods, there exists following relation.

$$\frac{\partial U(c_{it},L_{it})}{\partial L_{it}}=\frac{\partial U(c_{it},L_{it})}{\partial c_{it}}\frac{\partial F(K_{it},y^l_{it})}{\partial y^l_{it}}H_{it}, \tag{2.12}$$

where $i=H,h$ and $t=1,2,3$. This equation reflects the marginal substitution between the labor input and consumption in the first best condition.

In period one, the labor N_{H1} and capital x_{H1} inputs in producing human capital for high ability should satisfy,

$$U_{L_{H1}} = \beta U_{c_{H2}} F_{y_{H2}} l_{H2} \frac{\partial H_{H2}}{\partial N_{H1}} + \beta^2 U_{c_{H3}} F_{y_{H3}} l_{H3} l_{H3} \frac{\partial H_{H3}}{\partial H_{H2}} \frac{\partial H_{H2}}{\partial N_{H1}} + \beta^2 U_{c_{h3}} F_{y_{h3}} l_{h3} \frac{\partial H_{h3}}{\partial H_{H2}} \frac{\partial H_{H2}}{\partial N_{H1}}, \tag{2.13}$$

$$U_{c_{H1}} = \beta U_{c_{H2}} F_{y_{H2}} l_{H2} \frac{\partial H_{H2}}{\partial x_{H1}} + \beta^2 U_{c_{H3}} F_{y_{H3}} l_{H3} \frac{\partial H_{H3}}{\partial H_{H2}} \frac{\partial H_{H2}}{\partial x_{H1}} + \beta^2 U_{c_{h3}} F_{y_{h3}} l_{h3} \frac{\partial H_{h3}}{\partial H_{H2}} \frac{\partial H_{H2}}{\partial x_{H1}}. \tag{2.14}$$

They are the Euler equations of N_{H1} and x_{H1}. The last parts of (2.13) (2.14) show the external effect when high ability agent uses his resources in producing human capital.

Theorem 1 *When consider the knowledge spillovers, the agent with high ability should invest more resources in producing human capital.*

If we do not consider the external effect, the last parts of (2.13) (2.14) would not exist in the optimal condition. While

$$\beta^2 U_{c_{h3}} F_{y_{h3}} l_{h3} \frac{\partial H_{h3}}{\partial H_{H2}} \frac{\partial H_{H2}}{\partial N_{H1}} > 0$$

and

$$\beta^2 U_{c_{h3}} F_{y_{h3}} l_{h3} \frac{\partial H_{h3}}{\partial H_{H2}} \frac{\partial H_{H2}}{\partial x_{H1}} > 0,$$

then, $U_{L_{H1}}$ and $U_{c_{H1}}$ would be higher when consider the knowledge spillovers. As $U_{LL} < 0$ and $U_{cc} < 0$, the agent has to spend less time in leisure and consume less goods. In other words, the agent should use more labor and capital in producing human capital.

In period one, the labor N_{h1} and capital x_{h1} inputs in producing human capital for low ability should satisfy, which are:

$$U_{L_{h1}} = \beta U_{c_{h2}} F_{y_{h2}} l_{h2} \frac{\partial H_{h2}}{\partial N_{h1}} + \beta^2 U_{c_{h3}} F_{y_{h3}} l_{h3} \frac{\partial H_{h3}}{\partial H_{h2}} \frac{\partial H_{h2}}{\partial N_{h1}}, \tag{2.15}$$

$$U_{c_{h1}} = \beta U_{c_{h2}} F_{y_{h2}} l_{h2} \frac{\partial H_{h2}}{\partial x_{h1}} + \beta^2 U_{c_{h3}} F_{y_{h3}} l_{h3} \frac{\partial H_{h3}}{\partial H_{h2}} \frac{\partial H_{h2}}{\partial x_{h1}}. \tag{2.16}$$

They are N_{h1} and x_{h1}'s Euler equations which are the same as the conditions without considering the external effect.

In period two, the labor N_{i2} and capital x_{i2} inputs in producing human capital should satisfy,

$$U_{L_{i2}} = \beta U_{c_{i3}} F_{y_{i3}} l_{i3} \frac{\partial H_{i3}}{\partial N_{i2}}, \tag{2.17}$$

$$U_{c_{i2}} = \beta U_{c_{i3}} F_{y_{i3}} l_{i3} \frac{\partial H_{i3}}{\partial x_{i2}}. \tag{2.18}$$

Here $i = H, h$. As individuals just live for three period, in the last period, the agents will not invest any resources in producing human capital, then the inputs in producing human capital in period two would just have effects on producing the final goods. Therefore, these Euler equations are the same as the conditions without considering the external effects.

In period one, the physical capital I_{i1} input in producing final goods, we get

$$U_{c_{i1}} = \beta U_{c_{i2}} F_{k_{i2}} + \beta^2 U_{c_{i3}} F_{k_{i3}} (1 - \delta_k), \tag{2.19}$$

$$U_{c_{i2}} = \beta U_{c_{i3}} F_{k_{i3}}. \tag{2.20}$$

Here $i = H, h$.

From equation (2.11),we could find out that in every period, each type has the same consumption. However, from equation (2.12), the agent with high ability has to invest more resources in human capital accumulation. Under the condition with private information, the agent with high ability would like to hide his ability and pretend to be low ability; therefore, the allocation will not be Pareto optimal.

2.3 Second Best Allocation

In this part, I consider the economy with private information, and the social planner could just observe the set $(c, y^H, y^l, x, I) = (c_{it}, y^H_{it}, y^l_{it}, x_{it}, I_{it})$, here $i = H, h$ and $t = 1, 2, 3$. Besides the feasible conditions in every period just as the equation (2.8), (2.9), (2.10), the social planner uses the incentive-compatibility conditions to distinguish agents with different types, and the government recognizes a given agent's ability at the beginning of period.

In third period, the incentive-compatibility condition for high ability agent:

$$U(c_{H3}, L_{H3}) = u(c_{H3}) + v(1 - \frac{y^l_{H3}}{H_{H3}}) \geq u(c_{h3}) + v(1 - \frac{y^l_{H3}}{H_{H3}}). \tag{2.21}$$

Here $H_{H3} = G(\theta_H, x_{H2}, y^H_{H2}) + (1 - \delta_H)H_{H2}$.

In second period, the incentive-compatibility condition for high ability agent:

$$U(c_{H2}, 1-\frac{y^H_{H3}}{H_{H2}}-\frac{y^l_{H3}}{H_{H2}})+\beta(U(c_{H3}, 1-\frac{y^l_{H3}}{H_{H3}})) \geq U(c_{h2}, 1-\frac{y^H_{h3}}{H_{H2}}-\frac{y^l_{h3}}{H_{H2}})+\beta U(c_{h3}, 1-\frac{y^l_{h3}}{H_{H3}}) \tag{2.22}$$

Here $H_{H2} = G(\theta_H, x_{H2}, y^H_{H2}) + (1 - \delta_H)H_{H1}$ and $H_{H3} = G(\theta_H, x_{h2}, y^H_{h2}) + (1 - \delta_H)H_{H2}$.

In period one, the incentive-compatibility condition for high ability agent,

$$\sum_{t=1}^{2} \beta^{t-1} U(c_{Ht}, 1 - \frac{y^H_{Ht}}{H_{Ht}} - \frac{y^l_{Ht}}{H_{Ht}}) + \beta^2 U(c_{H3}, 1 - \frac{y^l_{H3}}{H_{H3}})$$
$$\geq U(c_{h1}, 1 - \frac{y^H_{h1}}{H_{H1}} - \frac{y^l_{h1}}{H_{H1}}) + \beta U(c_{h2}, 1 - \frac{y^H_{h2}}{H'_{H2}} - \frac{y^l_{h2}}{H'_{H2}}) + \beta^2 U(c_{h3}, 1 - \frac{y^l_{h3}}{H"_{H3}}). \tag{2.23}$$

Here $H'_{H2} = G(\theta_H, x_{hl}, y^H_{hl}) + (1-\delta_H)H_{H1}$ and $H"_{H3} = G(\theta_H, x_{h2}, y^H_{h2}) + (1-\delta_H)H'_{H2}$.

In period 1, the social planner considers the optimal problem under the constraints of feasible conditions, equation (2.8), (2.9), (2.10) and incentive-compatibility , equation (2.21), (2.22), (2.23). In this case, at the optimal condition, the agent is indifferent between truthful telling and applying the strategy relevant to the above binding incentive-compatibility constraints. However, once the social planner recognizes the agent's real type and the series of allocation would be decided, then, among the three incentive-compatibility constraints, only the (2.23) incentive-compatibility constraint will be valid.

The first order conditions with respect to $(c, y^H, y^l, x, I) = (c_{it}, y^H_{it}, y^l_{it}, x_{it}, I_{it})$ are as follow, and see the detail calculation process in Appendix A.

The consumption of high ability agent c_{Ht},

$$\beta^{t-1}(1+\mu)\frac{\partial U(c_{Ht}, L_{Ht})}{\partial c_{Ht}} = \lambda_t. \tag{2.24}$$

The consumption of low ability agent c_{ht},

$$\beta^{t-1}(1+\mu)\frac{\partial U(c_{ht}, L_{ht})}{\partial c_{ht}} = \lambda_t. \tag{2.25}$$

Here λ_t for $t = 1, 2, 3$ denotes the Lagrange multipliers of the feasible conditions (2.8) (2.9) (2.10), and μ denotes the Lagrange multiplier of the incentive-compatibility constraint (2.23), which means the shadow price of the variables.

Theorem 2 *When the agents' ability is heterogeneous and private information, the optimal allocation for consumption should be $c_{Ht} > c_{ht}$, which means we need to pay the information rent to high ability agent.*

From equation (2.24) and (2.25), we have $(1+\mu)U_{c_{Ht}} = (1-\mu)U_{c_{ht}}$, while the

IC condition (2.23) is binding, the Lagrange multiplier $\mu > 0$, then $U_{c_{Ht}} < U_{c_{ht}}$, as $U_{cc} < 0$, then we have $c_{Ht} > c_{ht}$.

The high ability agent's effective labor y^l_{Ht} which is used for producing final goods,

$$\frac{\partial U(c_{Ht}, L_{Ht})}{\partial L_{Ht}} \frac{1}{H_{Ht}} = \frac{\partial U(c_{Ht}, L_{Ht})}{\partial c_{Ht}} \frac{\partial F(K_{Ht}, y^l_{Ht})}{\partial y^l_{Ht}}, \tag{2.26}$$

here $t = 1, 2, 3$, which is the same as the conditions in the first best allocation.

The low ability agent's effective labor y^l_{ht} which is used for producing final goods,

$$\frac{\partial U(c_{ht}, L_{ht})}{\partial L_{ht}} \frac{1}{H_{ht}} < \frac{\partial U(c_{ht}, L_{ht})}{\partial c_{ht}} \frac{\partial F(K_{ht}, y^l_{ht})}{\partial y^l_{ht}}. \tag{2.27}$$

Here $t = 2, 3$, except the initial period, the optimal condition of y^l_{ht} is different from the first best allocation.

The high ability agent's effective labor y^H_{Ht} which is used for producing human capital, in period one:

$$\begin{aligned} &-\frac{U_{L_{H1}}}{H_{H1}} + \beta U_{L_{H2}} \frac{y^H_{H2} + y^l_{H2}}{(H_{H2})^2} \frac{\partial H_{H2}}{\partial y^H_{H1}} + \beta^2 U_{L_{H3}} \frac{y^l_{H3}}{(H_{H3})^2} \frac{\partial H_{H3}}{\partial H_{H2}} \frac{\partial H_{H2}}{\partial y^H_{H1}} \\ &+ \beta^2 U_{L_{h3}} \frac{y^l_{h3}}{(H_{h3})^2} \frac{\partial H_{h3}}{\partial H_{H2}} \frac{\partial H_{H2}}{\partial y^H_{H1}} \\ &= \frac{\mu}{1+\mu} \beta^2 U_{L_{h3}} U_{L_{h3}} \frac{y^l_{h3}}{(H_{h3})^2} \frac{\partial H_{h3}}{\partial H_{H2}} \frac{\partial H_{H2}}{\partial y^H_{H1}}. \end{aligned} \tag{2.28}$$

In period two:

$$\frac{U_{L_{H2}}}{H_{H2}} = \beta U_{L_{H3}} \frac{y^l_{H3}}{(H_{H3})^2} \frac{\partial H_{H3}}{\partial y^H_{H2}}. \tag{2.29}$$

Obviously , the equation (2.28) is different from the first best allocation while (2.29) is the same.

The low ability agent's effective labor y^H_{ht} which is used for producing human capital, in period one:

$$\begin{aligned}&-\frac{U_{L_{h1}}}{H_{h1}}+\beta U_{L_{h2}}\frac{y_{h2}^{H}+y_{h2}^{l}}{(H_{h2})^2}\frac{\partial H_{h2}}{\partial y_{h1}^{H}}+\beta^2 U_{L_{h3}}\frac{y_{h3}^{l}}{(H_{h3})^2}\frac{\partial H_{h3}}{\partial H_{h2}}\frac{\partial H_{h2}}{\partial y_{h1}^{H}}\\&=\mu(-\frac{U_{L_{H1}}^{*}}{H_{H1}}+\beta U_{L_{H2}}^{*}\frac{y_{h2}^{H}+y_{h2}^{l}}{(H'_{H2})^2}\frac{\partial H'_{H2}}{\partial y_{h1}^{H}}+\beta^2 U_{L_{H3}}^{*}\frac{y_{h3}^{l}}{(H_{h3}^{*})^2}\frac{\partial H_{H3}^{*}}{\partial H'_{H2}}\frac{\partial H'_{h2}}{\partial y_{h1}^{H}})\end{aligned}\tag{2.30}$$

In period two:

$$-\frac{U_{L_{h2}}}{H_{h2}}+\beta U_{L_{h3}}\frac{y_{h3}^{l}}{(H_{h3})^2}\frac{\partial H_{h3}}{\partial y_{h2}^{H}}=\mu(-\frac{U_{L_{H2}}^{*}}{H'_{H2}}+\beta U_{L_{H3}}^{*}\frac{y_{h3}^{l}}{(H"_{H3})^2}\frac{\partial H"_{H3}}{\partial y_{h2}^{H}})\tag{2.31}$$

Here U^* denotes the utility when high ability agent pretends to be low ability agent.

The high ability agent's physical capital x_{Ht} which is used for human capital accumulation, in period one:

$$\begin{aligned}U_{c_{H1}}&=\beta U_{L_{H2}}\frac{y_{H2}^{H}+y_{H2}^{l}}{(H_{H2})^2}\frac{\partial H_{H2}}{\partial x_{H1}}+\beta^2 U_{L_{H3}}\frac{y_{H3}^{l}}{(H_{H3})^2}\frac{\partial H_{H3}}{\partial H_{H2}}\frac{\partial H_{H2}}{\partial x_{H1}}\\&+\frac{1}{1+\mu}(\beta^2 U_{L_{h3}}\frac{y_{h3}^{l}}{(H_{h3})^2}\frac{\partial H_{h3}}{\partial H_{H2}}\frac{\partial H_{H2}}{\partial x_{H2}})\end{aligned}\tag{2.32}$$

In period two:

$$U_{c_{H2}}=\beta U_{L_{H3}}\frac{y_{H3}^{l}}{(H_{H3})^2}\frac{\partial H_{H3}}{\partial x_{H2}}\tag{2.33}$$

Compared with equation (2.18) in the first-best allocation, the right hand side of equation (2.32) is less than $\frac{\mu}{1+\mu}(\beta^2 U_{L_{h3}}\frac{y_{h3}^{l}}{(H_{h3})^2}\frac{\partial H_{h3}}{\partial H_{H2}}\frac{\partial H_{H2}}{\partial x_{H2}})$, which means the value of $U_{c_{H1}}$ is less than that in the first best allocation and the high ability agent would consume more in period one when the agents' ability is private information.

The low ability agents' physical capital x_{ht}, which is used for human capital accumulation, in period one:

$$U_{c_{hl}}>\beta U_{L_{h2}}\frac{y_{h2}^{H}+y_{h2}^{l}}{(H_{h2})^2}\frac{\partial H_{h2}}{\partial x_{h1}}+\beta^2 U_{L_{h3}}\frac{y_{h3}^{l}}{(H_{h3})^2}\frac{\partial H_{h3}}{\partial H_{h2}}\frac{\partial H_{h2}}{\partial x_{h1}}.\tag{2.34}$$

In period two:

$$U_{c_{h2}} > \beta U_{L_{h3}} \frac{y_{h3}^l}{(H_{h3})^2} \frac{\partial H_{h3}}{\partial x_{h2}}. \tag{2.35}$$

The first order condition of I_{it} is consistent with the first best allocation, here $i = H, h$ and $t = 1, 2$.

2.4 Optimal Taxation

When agents are heterogeneous, the linear taxation is not the Pareto optimal taxation Golosov and Tsyvinski [46]. Here, I discuss the non-linear taxation which depends on the observed information $\{c, y^H, y^l, x, I\} = \{c_{it}, y_{it}^H, y_{it}^l, x_{it}, I_{it}\}$.

In the discrete economy, the agent i faces the following problem:

$$\max_{\{c_{it}, y_{it}^H, y_{it}^l, x_{it}, I_{it}\}} \sum_{t=1}^{2} \beta^{t-1} U(c_{it}, 1 - \frac{y_{it}^H}{H_{it}} - \frac{y_{it}^l}{H_{it}}) + \beta^2 U(c_{i3}, 1 - \frac{y_{i3}^l}{H_{i3}}) \tag{2.36}$$

The feasible conditions in every period are the constraints, in the perfect competitive market, as the firms are owned by agents; we have the feasible condition in period one:

$$c_{i1} + x_{i1} + I_{i1} + T(y_{i1}^H, y_{i1}^l, x_{i1}, I_{i1}) = F(K_{i1}, y_{i1}^l). \tag{2.37}$$

In period two:

$$c_{i2} + x_{i2} + I_{i2} + T(y_{i2}^H, y_{i2}^l, x_{i2}, I_{i2}) = F(K_{i2}, y_{i2}^l). \tag{2.38}$$

In period three:

$$c_{i2} + T(y_{i3}^l) = F(K_{i3}, y_{i3}^l). \tag{2.39}$$

The first order conditions for the effective labor y_{it}^l used for producing final goods:

$$\frac{U_{L_{it}}}{H_{it}} = U_{c_{it}} * (F_{y_{it}^l} - \frac{\partial T(y_{it}^H, y_{it}^l, x_{it}, I_{it})}{\partial y_{it}^l}). \tag{2.40}$$

The effective labor y_{it}^H used for producing human capital in period one:

$$\begin{aligned} -\frac{U_{L_{i1}}}{H_{i1}} + \beta U_{L_{i2}} \frac{y_{i2}^H + y_{H2}^l}{(H_{i2})^2} \frac{\partial H_{i2}}{\partial y_{i1}^H} + \beta^2 U_{L_{i3}} \frac{y_{i3}^l}{(H_{i3})^2} \frac{\partial H_{i3}}{\partial H_{i2}} \frac{\partial H_{i2}}{\partial y_{i1}^H} \\ = U_{c_{it}} * \frac{\partial T(y_{i1}^H, y_{i1}^l, x_{i1}, I_{i1})}{\partial y_{i1}^H}. \end{aligned} \tag{2.41}$$

In period two:

$$-\frac{U_{L_{i2}}}{H_{i2}} + \beta U_{L_{i3}} \frac{y_{i3}^l}{(H_{i3})^2} \frac{\partial H_{i3}}{\partial y_{i2}^H} = U_{c_{i2}} * \frac{\partial T(y_{i2}^H, y_{i2}^l, x_{i2}, I_{i2})}{\partial y_{i2}^H}. \tag{2.42}$$

The physical capital x_{it} used for human capital accumulation in period one:

$$\begin{aligned} U_{c_{i1}}(1 + \frac{\partial T(y_{i1}^H, y_{i1}^l, x_{i1}, I_{i1})}{\partial x_{i1}}) \\ = \beta U_{L_{i2}} \frac{y_{i2}^H + y_{H2}^l}{(H_{i2})^2} \frac{\partial H_{i2}}{\partial x_{i1}} + \beta^2 U_{L_{i3}} \frac{y_{i3}^l}{(H_{i3})^2} \frac{\partial H_{i3}}{\partial H_{i2}} \frac{\partial H_{i2}}{\partial x_{i1}}. \end{aligned} \tag{2.43}$$

In period two:

$$U_{c_{i2}}(1 + \frac{\partial T(y_{i2}^H, y_{i2}^l, x_{i2}, I_{i2})}{\partial x_{i2}}) = \beta U_{L_{i3}} \frac{y_{i3}^l}{(H_{i3})^2} \frac{\partial H_{i3}}{\partial H_{i2}}. \tag{2.44}$$

The physical capital I_{i1} used for producing final goods, in period one:

$$U_{c_{i1}}(1 + \frac{\partial T(y_{i1}^H, y_{i1}^l, x_{i1}, I_{i1})}{\partial I_{i1}}) = \beta U_{c_{i2}} F_{k_{i2}} + \beta^2 U_{c_{i3}} F_{k_{i3}} (1 - \delta_k). \tag{2.45}$$

In period two:

$$U_{c_{i2}}(1+\frac{\partial T(y_{i1}^{H},y_{i1}^{l},x_{i1},I_{i1})}{\partial I_{i2}}=\beta U_{c_{i3}}F_{k_{i3}}. \tag{2.46}$$

Here $i=H,h;t=1,2,3$

Compared the above conditions with the conditions in second best allocation, we have the following conclusions:

Theorem 3 *When the information is incomplete, other than the initial period, the marginal labor income tax on high ability agent should be zero, while on low ability agent should be positive.*

According to the equation of (2.26), we could find out in equation (2.40),

$$\frac{\partial T(y_{Ht}^{H},y_{Ht}^{l},x_{Ht},I_{Ht})}{\partial y_{Ht}^{l}}=0,$$

while for low ability agent, we have the inequality of (2.27), it is easy to have $\frac{\partial T(y_{ht}^{H},y_{ht}^{l},x_{ht},I_{ht})}{\partial y_{ht}^{l}}>0$. Then, we have the above conclusion.

Since positive marginal tax rates have negative effects on the agents' efforts, and compared with the low ability agent, the individual with high ability has higher elasticity of labor supply, then if the revenue collected by tax is the same, the zero marginal tax on earning beyond some level could be optimal.

As Mankiw, Weinzierl and Yagan [69] state the optimal marginal taxation depends on the distribution of ability, while in practice, the top marginal income tax rate in US has declined from 70% in 1979 to 35% in 2010, which is partially consistent with the above results.

Theorem 4 *When the information is incomplete and the high ability agents' inputs* *of producing human capital have external effects, it is optimal for government to subsidy them.*

At period one, the effective labor y^H_{H1} have the equation of (2.41), with considering the equation (2.27), we have,

$$\frac{\partial T(y^H_{Ht}, y^l_{Ht}, x_{Ht}, I_{Ht})}{\partial y^H_{Ht}} = -\frac{\beta^2 U_{L_{h3}}}{(1+\mu)U_{c_{H1}}} \frac{y^l_{h3}}{(H_{h3})^2} \frac{\partial H_{h3}}{\partial H_{H2}} \frac{\partial H_{H2}}{\partial y^H_{H1}}. \tag{2.47}$$

Obviously, we have $\frac{\partial T(y^H_{Ht}, y^l_{Ht}, x_{Ht}, I_{Ht})}{\partial y^H_{Ht}} < 0$; as for the physical capital x_{H1}, we have equation (2.43), compared with equation (2.32), we get,

$$\frac{\partial T(y^H_{H1}, y^l_{H1}, x_{H1}, I_{H1})}{\partial x_{H1}} = -(\frac{\beta^2 U_{L_{h3}}}{(1+\mu)U_{c_{H1}}} \frac{y^l_{h3}}{(H_{h3})^2} \frac{\partial H_{h3}}{\partial H_{H2}} \frac{\partial H_{H2}}{\partial y^H_{H1}}). \tag{2.48}$$

thus, $\frac{\partial T(y^H_{H1}, y^l_{H1}, x_{H1}, I_{H1})}{\partial x_{H1}} < 0$.

Rauch [77] concludes that if the average education increases one more year in a metropolitan area, wages would increase 3%. In modern economics, the emergence of high-trained labor force is normally treated as resource of economic growth and their emigration is constantly disputed as the barrier to the rapid growth of lots of developing countries. Since the high ability agent's investment in human capital has the social scale effects, the subsidy will obviously improve the social welfare.

Theorem 5 *When the information is incomplete, the government should subsidy the low ability agent's physical capital which is used for human capital accumulation.*

In the second best economy, the conditions for physical capital input are inequation (2.34) (2.35), compared with (2.43) (2.44), we have $\frac{\partial T(y^H_{ht}, y^l_{ht}, x_{ht}, I_{ht})}{\partial x_{ht}} < 0, t = 1, 2$.

Even though the low-skilled agents' increasing of human capital accumulation does not have the external effect, considering the catching-up effect, it is still worth for government to subsidy the low-skilled agents, which could narrow the gap between the rich and poor so that the social welfare could be improved.

Theorem 6 *When the information is incomplete, the marginal tax rate of physical capital which is used for producing final goods is zero.*

With the equation (2.45) and (2.46) and (2.19) (2.20) in the second best economy, we have $\frac{\partial T(y^H_{ht}, y^l_{ht}, x_{ht}, I_{ht})}{\partial I_{it}} = 0, i = H, h.$

Just as Chamley [18] and Judd [60] show that at least in people's expectation, the optimal capital income should be zero, the intuition behind the zero capital taxes is that the taxation on capital could lead large distortions to intertemporal consumption, and consequently decrease the amount saving, which would lower the aggregate output in the future.

2.4.1 Conclusion

In this section, we explored the optimal taxation issue in the neoclassical economy with considering the positive externality between different kinds of agents. The agent heterogeneity will be result in different taxation distortion across individuals. Based on the assumption that both physical capital and time are necessary, not only to produce the final goods, but also to improve the skill levels, it gave the theoretical support for some public policies, such as subsidizing the education.

Several assumptions are made in this section, and now I would conjecture on their role in the main conclusions. Firstly, in the dynamic environment, I just ignore any aggregate or idiosyncratic uncertainty, which allow us to simplify the optimal dynamic taxation issues to a static subproblem. Conversely, the sign of marginal income taxation would be influenced by the uncertainty of the economics,especially the marginal capital income taxation.

Secondly, the model just considers the issue about how to minimize the distortion when the government need to collect the given amount of revenue, but we ignore the important function caused by the government expenditure, such as the supplying the public goods and the redistribution effects. Barro [10] supplies the endogenous model with considering the contribution of government spending in utility and production, which we could use for analyzing the optimal taxations.

3. OPTIMAL FISCAL AND MONETARY POLICY WITH HETEROGENEOUS AGENTS AND NONLINEAR INCOME TAXATION

Previous papers consider optimal fiscal policy in an economy without involving money, or discuss optimal monetary policy with homogeneous agents in reduced-form approaches such as money in the utility model and cash-in-advance model. Different from them, in this section, I study the optimal monetary and fiscal policy with heterogeneous agents other than homogeneous agents. In the economy, as there exists friction to value money, making money essential can expand the set of feasible trades. Based on the search-theoretical environment, I first solve the households' problem in the centralized and decentralized market, and find out the optimal conditions. Then, we describes the problem that social planner faces by involving uncertainty and agents whose types are continuous. By comparing the optimal conditions in this generous setting, I show that the Friedman rule is no longer optimal when jointed with nonlinear taxation of income. Moreover, the capital income taxation is not zero.

3.1 Introduction

Friedman [36] states that in order to attain the first-best allocation, the nominal interest rate need to be set to zero since positive nominal interest rates indicate a distorting tax on real money balances, which is known as the Friedman rule. Without double, it is a critical conclusion in monetary economics. Chari,Christiano and Kehoe [19] show that in three different kinds of homogeneous agent models of money demand such as the putting money in utility function model (Sidrauski [74] and [73]), the cash-credit model (Lucas and Stokey [68]), and the shopping-time model (Kimbrough [56]), when preferences are homotheticity and weakly separability, the Friedman rule is valid. However, if we consider the heterogeneity, other than representative agents in the economy, whether the Friedman rule still holds? If not, what is the optimal monetary policy ?

Since a disproportionate amount of money holding, monetary policy would have different effects across agents . For example, for those families with lower income, the amount of cash in hand has always occupied a higher proportion of their total wealth. Erosa and Gervais [32], based on US data, show that compared with high income households, low income families use cash for a higher fraction of their total consumptions. By using the cash-in-advance model, Wen[85] shows that if the annual inflation rate is 10%, households would like to recrease up to 15% of average consumption an annual year in order to obtain the Friedman rule inflation rate, which is significantly bigger than that of with considering homogeneous agents (Lucas [66]). Therefore, the heterogeneity could lead to dramatically different results of monetary policies from those under the representative-agent assumption.

In recent years, some studies have explored the optimal monetary policy by involving the heterogeneity in the economy, and achieved different results. Costa and Werning [26] consider an economy where individuals have different labor productivities and government stipulates the nominal interest rate and nonlinear labor income taxes. They use the money-in-utility model and assume individual labor ability are their own information, based on Mirrlees [71]. The paper gains the conclusion that the Friedman rule still holds if money and leisure in utility are complements. However, as Albanesi [4] points out the empirical works is inconsistent with their assumptions which would cause a cross-sectional distribution of money holdings. Moreover, their model considers the money in a reduced-form model and has not supplied the micro-foundation of money in the economy.

Bhattacharya, Haslag and Martin [13] discuss several alternative monetary economies in which agents have heterogeneous money holding, and conclude that Friedman rule does not maximize ex post steady-state welfare with considering the redistribution effects. However, their results are not robust as they ignore the asset market in their economy. If the asset market is incomplete, as noted by Aiyagari [2], agents would have more incentives to prepare for self-insure against individual shocks by precautionary savings, which motivate agents to hold extra cash in hand to prevent severe budget constraints in the future. Therefore, in this condition, money serves not solely as a medium of exchange, but also as a store of value.

This section reconsiders the optimal fiscal and monetary policy problem with search-theoretic framework. Different from the classical search-theoretical papers such as Lagos and Wright [64] and Williamson and Wright [87], an important assumption of this article is that individuals also need to accumulate physical capital to produce in the economy. Just as Kocherlakota [62] states, we need to answer the following questions: what are the micro-foundations for the difference in returns between other assets and money ? How are these micro-foundations to affect the feature of optimal monetary policy and fiscal policy?

Since in this section, each period is assigned into two subperiods, and there exists two kinds of markets, opening separately in the economy which are centralized market and decentralized market. While in the centralized market, economy is operating as in the classical growth models. Households supply their labors and capitals, receive their payment and adjust their asset levels in the frictionless market. In the decentralized market, some parts of households could trade with some one by anonymous matching, which supplies the microfoundations of monetary theory.

While the households in the economy are heterogeneity in skills which are private information and always changing over time, the section develops a full characterization of the socially optimal allocation in this set-up. In order to classify different kinds of agents, we need to consider a large number of incentive constraints, which would lead us to reconsider the optimal policy under these settings. The section finds out that Friedman rule is not optimal when jointed with nonlinear taxation. In addition, the capital income taxation is not zero and marginal labor income taxation is negative correlated with skills.

By investigating different kinds of assets and heterogeneity in the standard search-theoretical model, the main goal of this section is to find out the solutions of the above questions. Section 2 introduces the model and the basic problems for households. Section 3 derives the optimal conditions for social planner's problem and find out the optimal monetary policy. Section 4 presents the conclusions.

3.2 The Model

The model extends the framework in Lagos and Wright [64], and is close to the baseline model in Aruoba and Chugh [5], Aruoba, Waller and Wright [6] and Williamson and Wright [87], by allowing capital accumulation. The main difference is that in this economy, there exists a continuum of infinitely lived individuals with differences in productivity other than homogeneous agents. The purpose of this assumption is to introduce the heterogeneity in a tractable way with considering the micro-foundation of monetary economics.

The model considers an economy with infinite periods where time is discrete $t = 0, 1, ...$ There exists a continuum unit measure of agents, indexed by $i \in [0, 1]$ with identical preferences, whose skills differ across households and over time. Let $\Theta = [\overline{\theta}, \underline{\theta}]$ be the set of ex-ante types and the distribution is stable over time by applying the law of large number. At the starting of period t, agent i privately notifies his history $\theta_i^t = (\theta_{i0}, ..., \theta_{it}) \in \Theta^t$ of past and current skill types but not his future skills which are governed by Markov process. At each period t, the probability measure of type θ^t is denoted by $\mu(\theta^t) \geq 0$, with $\int\limits_{\theta^t \in \Theta^t} d\mu(\theta^t) = 1$ [1]. This implies that the individual's choices in period t just depends on his history. Moreover, the skill processes are independent across households and there is no aggregate uncertainty in the environment, as argued in Aruoba and Chugh [5], this is without loss of generality since we always gain the conclusions in the deterministic steady states.

As in the typical search model, each period is separated into two subperiods, say day and night. A frictionless centralized market(CM) opens during the day, and a decentralized market (DM) opens at night. During the CM, households rent their previously accumulated capital and supply labor in the competitive market, and they also choose their consumption level in the goods market. Meanwhile, they also adjust their holdings of money, capital and government bond[2].

When entering the DM, each family receives an individual shock which governs his trading status and is independent with households' skills. A given household is a seller in the DM with a fixed probability σ, a buyer with fixed probability σ, neither a seller nor a buyer with probability $1 - 2\sigma$ which means that with probability with $1 - 2\sigma$, a family can not trade at all in the DM, here $\sigma \in [0, 1/2]$ [3]. As there is no credit or record keeping, households interact with anonymous bilateral matching. The buyer uses the money to purchases goods from the seller who produces goods by using his own labor and capital while the terms of trade is determined trough bargaining just like the standard search models[4].

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[1] The heterogeneity in productivity may have the same effect, in some sense, as the preference shocks in the model, as in Wen [85].

[2] Different from Walsh[84]), the asset market opens first before individuals face the cash-in-advance constraint, which is consistent with Lucas and Stockey [68].

[3] This structure is similar to Trejos and Wright (1995).

[4] They also could trade by directed search and price posting, such as Mortensen and Wright [72], other than random matching and bargaining, but it will increase the complexity of the model considerably with the heterogeneous agents. If not having additional constraints about production function, the agents with highest ability may monopoly the DM, which will violate the original intention of considering the behaviors in DM.

In what follows, I provide the information about production, household behavior and government in more details, and define the monetary equilibrium.

3.2.1 Production

Both in the CM and DM, the inputs of production are capital and effective labor. A household with skill θ_i uses his real labor input l_i ot generate effective labor h_i according to the function $h_i = \theta_i l_i$. Here, both actual labor l_i and skill θ_i are unobservable while only effective labor h_i can be observed.

In the CM, just like in the standard neoclassical growth theory, there is a final good that can be used not only for consumption but also for investment, produced by the constant-returns technology $Y_t = F(K_t, H_t)$, here, the function $F(.)$ is strictly increasing and strictly concave, $K_t = \int k(\theta^t) d\mu(\theta^t)$ denotes the aggregate capital, $H_t = \int h(\theta^t) d\mu(\theta^t)$ denotes aggregate effective labor. In competitive markets, firms hire labor and capital from household, and profit maximization implies the real wage $w_t = F_H(K_t, H_t)$ and real rental rate $r_t = F_K(K_t, H_t))$.

In the DM, although firms do not operate, the sellers' own effective labor e and capital k, carried from the CM in this period, can be used to produce with the technology $q = f(e, k)$, which implies that the cost of production is $e = c(q, k)$ and disutility is $c(q, k)/\theta$. Since the production function $f(.)$ is strictly increasing and strictly concave, it is easy to show that $c_q, c_{qq}, c_{kk} > 0, c_k, c_{qk} < 0$.

3.2.2 Government

As in Aruoba and Chugh [5], government consumption takes place in the CM while the expenditure could be financed by taxes, money creation and debt issuance. As for the form of taxation, the model is based on the dynamic Mirrlees literature which is different from those of Ramsey approach. At period t, the taxation is a nonlinear function of agent's effective labor supply and capital stock in that period, which could be denoted as $T(h_t, k_t)$.[5] The government's budget constraint in period t is:

$$M_t + B_t + P_t \int T(h_t, k_t) d\mu(\theta^t) = P_t G_t + R_{t-1} B_{t-1} + M_{t-1}.$$

The government also face the standard no-Ponzi constraint, which is

$$\sum_{t=0}^{\infty} [\frac{P_t}{P_0} \Pi_{s=0}^{t-1} \frac{1}{R_{s-1}} (G_t - \frac{M_t - M_{t-1}}{P_t} - \int T(h_t, k_t) d\mu(\theta^t))] \leq 0.$$

3.2.3 Households

At period t, CM opens first and I consider the household's problem in CM and DM, respectively.

3.2.3.1 Household's Problem in CM

In the beginning of period t, a household joins the CM with nominal government bond b_{t-1}, money holdings m_{t-1} and capital k_t, and knows his own skill type at time t denoted by θ_t, which means that the history of his skill $\theta^t = (\theta_0, ..., \theta_t)$. Instantaneous utility for everyone in the period t CM is $U(x_t) - l_t$, where x_t is consumption level and l_t is labor[6]. Assume that U is twice continuously differentiable with $U' > 0$, $U'' < 0$, $\lim_{x \to 0} U'(x) = \infty$, and there exist $x^* \in (0, \infty)$ so that $U'(x^*) = 1$ with $U(x^*) > x^*$.

Let $W(\theta^t; m_{t-1}, b_{t-1}, k_t)$ denote the value function for a household with type θ^t at the beginning of CM, and $V(\theta^t; m_t, b_t, k_{t+1})$ be the value function in the DM. The discount rate between the DM and CM is $\beta \in (0, 1)$ while there is no discount

[5] Since the agents face the skill shock over time and there is no record keeping, which is similar to the basic environment in Albanesi and Sleet [3], the taxation just depends on agents' behaviors in each period, other than Kocherlakota [62] which depends on the all history of agents' labor supplies.

[6] With the quasi-linear utility, we could derive may results analytically, while with general preferences, the model requires numerical methods, just like Chiu and Molico [21]. Rocheteau, Shell and Wright [79] show how to get the same simplification with general preferences.

between the CM and DM. Then, the household's problem in CM can be writen as:

$$W(\theta^t; m_{t-1}, b_{t-1}, k_t) = \max_{\{x_t, l_t, m_t, b_t, k_{t+1}\}} U(x_t) - l_t + V(\theta^t; m_t, b_t, k_{t+1})$$

subject to

$$b_t + m_t + P_t x_t + P_t(k_{t+1} - (1-\delta)k_t) = P_t w_t h_t + m_{t-1} + P_t r_t k_t + R_{t-1} b_{t-1} - P_t T(h_t, k_t).$$

Here, P_t means the consumption goods' price level in the CM, δ is the discount rate of capital, r_t is the capital's real rent rate, R_{t-1} is the gross nominal return of the one-period government bond which is purchased in period $t-1$. While the effective labor $h_t = \theta_t l_t$, replace it in the above equitation and we can find out the optimal conditions of this problem as follows:

$$U^{'}(x_t) = \frac{1}{\theta_t(w_t - T_h(h_t, k_t))}, \tag{3.1}$$

$$W_m(\theta^t; m_{t-1}, b_{t-1}, k_t) = V_m(\theta^t; m_t, b_t, k_{t+1}) = \frac{1}{P_t\theta_t(w_t - T_h(h_t, k_t))}, \tag{3.2}$$

$$W_b(\theta^t; m_{t-1}, b_{t-1}, k_t) = R_{t-1} V_b(\theta^t; m_t, b_t, k_{t+1}) = \frac{R_{t-1}}{P_t\theta_t(w_t - T_h(h_t, k_t))}, \tag{3.3}$$

$$W_k(\theta^t; m_{t-1}, b_{t-1}, k_t) = (1+r_t-\delta-T_k(h_t,k_t))V_k(\theta^t; m_t, b_t, k_{t+1}) = \frac{1+r_t-\delta-T_k(h_t,k_t)}{\theta_t(w_t - T_h(h_t,k_t))}. \tag{3.4}$$

Define $\lambda_{\theta_t} = \frac{1}{P_t\theta_t(w_t - T_h(h_t,k_t))}$ as the agent with skill θ_t' marginal value of entering period t with an additional unit of money. Based on equation (3.1), we know that the consumption good demand x_t in the CM is an increasing function of agent's skill θ_t. The first-order conditions (3.2)-(3.4) tell us that the value functions $W(.)$ and $V(.)$ are linear, and a specific household's marginal utility of wealth is not related with his trading status in the previous DM, but depends on the skill type. Different from Lagos and Wright (2005), the holdings of money, bond and capital are not identical any more.

3.2.3.2 Household's Problem in DM

After the CM closes, agents draw the shocks deciding whether they are buyer or seller. When the DM opens, households know their types and could match bilaterally. While the capital cannot be used for DM payment, as Williamson and Wright [87] state, the reason is that physical capital is installed in place and it is not convenient for consumers to travel around by taking it with them [7]. Therefore, even though in the DM, households can use their accumulated capital level as an input to produce the general goods, households can not use the holdings of capital as payment when

[7] By introducing the financial intermediation in the economy, we could consider the capital and credit as the payment method just as Woodford [89].

they are buyers. The household's problem in DM can be written as follows:

$$
\begin{aligned}
V(\theta^t; m_t, b_t, k_{t+1}) = +\sigma[-c(q_t^s, k_{t+1})/\theta_t + \beta E_t(W(\theta^{t+1}; m_t + d_t^s, b_t, k_{t+1}))] \\
\sigma[u(q_t^b) + \beta E_t(W(\theta^{t+1}; m_t - d_t^b, b_t, k_{t+1}))] \\
+(1-2\sigma)\beta E_t(W(\theta^{t+1}; m_t, b_t, k_{t+1})).
\end{aligned} \tag{3.5}
$$

Here, $E_t(.) = E(.|\theta^t)$ means the conditional expectation relied on the information until time t, and (q_t^b, d_t^b) and (q_t^s, d_t^s) mean the amount of goods and dollars exchanged when the agent is buyer and seller respectively. The first item on the right hand side of equation (3.5) represents the expected payoff if the household is a seller, the second item represents the expected payoff if the household is a seller, and the last one represents the expected payoff if the household neither a seller nor a buyer in the DM.

Since the value function $W(.)$ is linearity, we could rewrite the equation (3.5) as follow:

$$
\begin{aligned}
V(\theta^t; m_t, b_t, k_{t+1}) = \sigma[u(q_t^b) - c(q_t^s, k_{t+1})/\theta_t - \beta E_t(\lambda_{\theta_{t+1}}(d_t^b - d_t^s))] \\
+\beta E_t(W(\theta^{t+1}; m_t, b_t, k_{t+1})).
\end{aligned} \tag{3.6}
$$

Therefore, in order to find out the solution of the household's problem, all we have to do is to find out how q_t, d_t is determined under this pricing schemes.

3.2.4 Bargaining

Under the search-theoretical framework, in the DM, the quantity of items used for trades are most generally decided by bargaining. Let $\phi \in [0,1]$ represent the bargaining power of buyers, while $\phi = 0$ means the seller make a take-it-or-leave-it offer to the buyer, and $\phi = 1$ means the power reverses. Suppose individual i with skill type θ_i^t is a buyer in DM, who will meet the seller with skill type θ_j^t. The generalized Nash bargaining problem can be written as follows:

$$\begin{aligned}
&\max_{\{q_{it},d_{it}\}}[u(q_{it}) + \beta E_t(W(\theta_i^{t+1}; m_{it} - d_{it}, b_{it}, k_{i,t+1}) \\
&\quad - \beta E_t(W(\theta_i^{t+1}; m_{it}, b_{it}, k_{i,t+1}))]^{\phi}[-\frac{c(q_{it}, k_{j,t+1})}{\theta_{jt}} + \beta E_t(W(\theta_j^{t+1}; m_{jt} + d_{it}, b_{jt}, k_{j,t+1}) \\
&\quad - \beta E_t(W(\theta_j^{t+1}; m_{jt}, b_{jt}, k_{j,t+1}))]^{1-\phi}
\end{aligned} \tag{3.7}$$

subject to

$$d_{it} \leq m_{it}. \tag{3.8}$$

Here, q_{it} denotes the quantity of goods that the seller i gives to the buyer in exchange of the amount of money d_{it}. The constraint (3.8) is the feasibility condition for the buyer, which is just like the cash-in-advance constraint in Stokey and Lucas [68]. A question naturally arises: Whether the constraint (3.8) always binds as in the representative-agent models? As Walsh [84] states, even though in the next period, the CM opens first, the constraint may rarely bind because the household can almost fully self-insure itself against random liquidity-demand shocks by working harder and accumulating more cash in hand in the CM, especially the households with high skill type in period t. On the other hand, as households ability is uncertainty in next period, the household has incentive to reduce the consumption in DM and take the money to the next period to smooth the future consumption.

As $W(.)$ is a linear function, we can rewrite the bargaining problem as follows:

$$\max_{\{q_{it},d_{it}\}}[u(q_{it}) - \beta E_t(\lambda_{\theta_{i,t+1}} d_{it})]^{\phi}[-c(q_{it}, k_{j,t+1})/\theta_{jt} + \beta E_t(\lambda_{\theta_{j,t+1}} d_{it})]^{1-\phi}$$

subject to (3.8).

Let χ_{it} denotes the Lagrange multiplier of the constraint (3.8). The first-oder conditions are:

$$\begin{aligned}\phi u'(q_{it})[-\frac{c(q_{it},k_{j,t+1})}{\theta_{jt}}+\beta E_t(\lambda_{\theta_{j,t+1}}d_{it})]\\ -(1-\phi)\frac{c_q(q_{it},k_{j,t+1})}{\theta_{jt}}[u(q_{it})-\beta E_t(\lambda_{\theta_{i,t+1}}d_{it})]=0,\end{aligned}\tag{3.9}$$

$$\begin{aligned}\phi\beta E_t\lambda_{\theta_{i,t+1}}[-\frac{c(q_{it},k_{j,t+1})}{\theta_{jt}}+\beta E_t(\lambda_{\theta_{j,t+1}}d_{it})]\\ -(1-\phi)\beta E_t\lambda_{\theta_{j,t+1}}[u(q_{it})-\beta E_t(\lambda_{\theta_{i,t+1}}d_{it})]=\\ -\chi_{it}[u(q_{it})-\beta E_t(\lambda_{\theta_{i,t+1}}d_{it})]^{1-\phi}[-c(q_{it},k_{j,t+1})/\theta_{jt}+\beta E_t(\lambda_{\theta_{j,t+1}}d_{it})]^{\phi},\end{aligned}\tag{3.10}$$

$$\chi_{it}(m_{it}-d_{it})=0,\chi_{it}\geq 0,\tag{3.11}$$

From the equation (4.1), the relation between d_{it} and q_{it} is:

$$d_{it}=\frac{\phi u'(q_{it})c(q_{it},k_{j,t+1})+(1-\phi)c_q(q_{it},k_{j,t+1})u(q_{it})}{\phi\theta_{jt}u'(q_{it})\beta E_t\lambda_{\theta_{j,t+1}}+(1-\phi)c_q(q_{it},k_{j,t+1})\beta E_t\lambda_{\theta_{i,t+1}}}.\tag{3.12}$$

Define the left hand side of (3.12)=$z(\theta_{it},\theta_{jt},q_{it},k_{j,t+1})$, and here, $z_q>0$ and $z_k<0$. Different from the previous work with identical individuals, the bargaining game not only depends on buyer and seller's asset holding, but also related with their skill types. By using the Jensen's Inequality, the amount d_{it} is lower than the amount under the situation without uncertainty. As the uncertainty in the future, the buyer would like to provide less money to exchange the goods in the DM.

Let's go back to the previous households' problem in the DM, and find out the first-order conditions as follows:

$$V_m(\theta_t;m_t,b_t,k_{t+1})=\sigma[u_q\frac{\partial q_t^b}{\partial m_t}-\beta E_t(\lambda_{\theta_{t+1}})\frac{\partial d_t^b}{\partial m_t}]+\beta E_t(\lambda_{\theta_{t+1}}),\tag{3.13}$$

$$V_k(\theta^t; m_t, b_t, k_{t+1}) = \sigma[\frac{c_q}{\theta_t}\frac{z_k}{z_q} - \frac{c_k}{\theta_t} + \beta E_t(\lambda_{\theta_{t+1}})z_k] + \beta E_t(W_k(\theta^{t+1}; m_t, b_t, k_{t+1})). \quad (3.14)$$

Define $\gamma(\theta_t, q_t, k_{t+1}) = \frac{c_q}{\theta_t}\frac{z_k}{z_q} - \frac{c_k}{\theta_t} + \beta E_t(\lambda_{\theta_{t+1}})z_k$, we have intertemporal optimal condition:

$$\begin{aligned} U_x(t) = \beta E_t[U_x(t+1)((1 + F_K(t+1) - \delta - T_k(t+1)] \\ +\sigma\gamma(\theta_t, q_t, k_{t+1}). \end{aligned} \quad (3.15)$$

Beside the expression $\sigma\gamma(\theta_t, q_t, k_{t+1})$, equation (3.15) is the same as a normal intertemporal optimal condition for investment. While in this section, physical capital is used both in the CM and in the DM, the decisions for capital investment need to reflect this character. Suppose all else are the same, the item $\sigma\gamma(\theta_t, q_t, k_{t+1})$ means the capital's return in the DM, which captures the argument that the cost of producing a certain amount of output in DM is lower if sellers has already accumulated more physical capital.

3.2.5 Monetary Equilibrium

Given the initial conditions $\{M_0, B_0, K_0\}$, policy processes $\{T(.), R_t\}$ and the government spending process $\{G_t\}$, an monetary equilibrium is a collection of $\{x_t, q_t, k_t, K_t, l_t, P_t, m_t, M_t, b_t, B_t, r_t, w_t\}$ such that:

i. Households optimize $\{x_t, l_t, k_t, m_t, q_t\}$ to maximize their welfare with considering the budget constraint, taking the price$\{P_t, r_t, w_t\}$ and the policy processes as given;

ii. Government budget constraints hold every period;

iii. Market clear:

$$\int m_t d\mu(\theta^t) = M_t; \int b_t d\mu(\theta^t) = B_t;$$
$$\int k_t d\mu(\theta^t) = K_t; \int h_t d\mu(\theta^t) = H_t$$

iv.Resource constraint:

$$\int x_t d\mu(\theta^t) + K_{t+1} + G_t = (1-\delta)K_t + F(K_t, H_t),$$

$$\int q_t d\mu(\theta^t) = f(\int e_t d\mu(\theta^t), K_{t+1}).$$

3.3 Optimal Policy

Different from the traditional optimal taxation approach since Ramsey [76], the households are heterogeneous and have the private information about their skills, and I use the Mirrlees approach to formulate the social planner's problem that chooses the specific allocations as a monetary equilibrium (Golosov et al.[44], [46], Costa and Werning [26]).

Since the individual's skill history θ^t is unobservable, the objective of the government act as the social planner is to find the optimal incentive-insurance trade-off [8], which means the allocation must respect incentive-compatibility conditions. A reporting stategy ξ is a mapping from Θ^t into Θ^t. Let

$$\widetilde{W}(\xi; x, h, q, e) = E_t \sum_{i=0}^{\infty} \beta^i \{U(x_{t+i}(\xi)) - \frac{h_{t+i}(\xi)}{\theta_{t+i}} + \sigma(u(q_{t+i}(\xi)) - \frac{e_{t+i}(\xi)}{\theta_{t+i}})\},$$

which is the payoff from reporting strategy ξ for agent with skill type θ^t.

[8]The government have the full commitment power such that we could abandon the issues brought by the time inconsistent.

Definition 1 An allocation (x, h, K, q, e) is incentive-compatible if

$$\widetilde{W}(\xi^*; x, h, q, e) \geq \widetilde{W}(\xi; x, h, q, e) \tag{3.16}$$

for any $\xi \in \Theta^t$, while $\xi^*(\theta^t) = \theta^t$ for all θ^t is the truth-telling strategy.

Besides the incentive-compatible constraint, the series of allocations (x, h, K, q, e) also need to face the resource constraints which are the feasible conditions.

Definition 2 Define an allocation (x, h, K, q, e) to be feasible if

$$\int x_t d\mu(\theta^t) + K_{t+1} + G_t \leq (1-\delta)K_t + F(K_t, \int h_t d\mu(\theta^t)), \tag{3.17}$$

$$\int q_t d\mu(\theta^t) \leq f(\int e_t d\mu(\theta^t), K_{t+1}), \tag{3.18}$$

for all period t

Here, I only consider direct mechanisms. Based on the revelation principle, the households' output and consumption just rely on their own announcements. The government or social planner's problem $SP(K_t)$ is:

$$TV(K_t) = \max_{x,h,K,q,e} \sum_{i=0}^{\infty} \beta^i \int U(x_{t+i}) - \frac{h_{t+i}}{\theta_{t+i}} + \sigma(u(q_{t+i}) - \frac{e_{t+i}}{\theta_{t+i}}) d\mu(\theta^t)$$

subject to two feasible conditions (C.2) (C.3) and incentive-compatible condition (3.16).

In the economy, the planner maximizes the social welfare by given the initially K_t^* units of capital. Different from Acemoglu et al.(2010), the ex ante objective weights agents are the same as the distribution of the skill levels.

Proposition 1 *Suppose that any* $(x^*, h^*, K^*, q^*, e^*)$ *that solves the social planner's problem* $SP(.)$*, then the function* $TV(.)$ *is strictly increasing, which means* $TV(K_t) < TV(K_t^*)$ *for all* $K_t < K_t^*$.

Proof. In Appendix B.

The basic idea of this proposition can be explained as below. Suppose that $TV(.)$ is not strictly increasing, which means that the benevolent planner has not allocated all the given capital level. Therefore, the social planner could increase $\epsilon/U'(x_{it}^*(\theta_i^t))$ for all θ_i^t, which guarantees that all households' utility level increase by ϵ. Since all types' utility are going up with the same value, this improvement will not affect incentive-compatibility. Meanwhile, ϵ is small enough to ensure the feasible condition will not be violated. Thus, we could have the conclusion that the aggregate value function is strictly increasing.

Proposition 2 *There exists* $(x^*, h^*, K^*, q^*, e^*)$ *solve the social planner's problem, which satisfies:*

$$\begin{aligned} &\frac{1 - \delta + F_K(K_{t+1}^*, \int h_{t+1}^* d\mu(\theta^{t+1}))}{U'(x_t^*) - \sigma u'(q_t^*) f_K(\int e_t^* d\mu(\theta^t), K_{t+1}^*)} \\ &= E_t \frac{1}{\beta U'(x_{t+1}^*)}. \end{aligned} \tag{3.19}$$

Proof. In Appendix B.

Here is the basic idea of the proof. As the distribution of individuals' ability is continuous, the difficulty of solving the social problem's problem $SP(K_t)$ is to articulate the incentive-compatible conditions for all individuals. In order to overcome the difficulty, we transfer the maximization problem into the minimization problem which uses the least resource but delivers the same utility. Since the social planner maximizes the social welfare with considering the feasible constraints, according, the optimum allocation $(x^*, h^*, K^*, q^*, e^*)$ is also the solution of the minimization problem. I define the new allocation $(\widetilde{x}, h^*, \widetilde{K}, \widetilde{q}, e^*)$ as follows:

$$\widetilde{x} = x^*, \widetilde{K} = K^*, \widetilde{q} = q^*$$

for all the other periods, except that

$$\begin{aligned} \widetilde{x}_t &= x_t^* + \psi_t, \\ \widetilde{x}_{t+1} &= x_{t+1}^* + \psi_{t+1}, \\ \widetilde{K}_{t+1} &= K_{t+1}^* + \varsigma_t, \\ \widetilde{q}_t &= q_t^* + \varphi_t, \end{aligned}$$

The main point is that on one hand, this variation will not influence the countless incentive constraints. On the other hand, it just have affected the resource constraint. Then, we need to prove the new allocation satisfies the conditions of feasible and incentive-compatible. By applying the monotonicity of the aggregate value function, we can get the result that the maximization problem and minimization problem are equal. Therefore, as the former allocation is optimal, it should be that the changes in the new allocation are zero which means that $(\psi_t, \varsigma_t, \varphi_t) = 0$ is the solution of the minimization problem.

Actually, in Golosov et al. [46], the above intertemporal optimal condition is a typical Inverse Euler Equation. However, if there is no uncertainty in the economy, we could rewrite the condition as:

$$U^{'}(x_t) = \sigma u^{'}(q_t) f_K(t) + \beta U^{'}(x_{t+1})(1 + F_K(t+1) - \delta), \tag{3.20}$$

which is just the standard Euler Equation.

Proposition 3 *The optimal taxation on capital income is positive, and the optimal nominal interest rate is greater than 1, which means the Friedman rule is not optimal.*

By using the Jensen's Inequality, we can rewrite the equation (3.19) as follows:

$$U'(x_t^*) \leq \sigma u'(q_t^*) f_K(\int e_t^* d\mu(\theta^t), K_{t+1}^*) + \beta E_t U'(x_{t+1}^*)(1 - \delta + F_K(K_{t+1}^*, \int h_{t+1}^* d\mu(\theta^{t+1}))) \tag{3.21}$$

Comparing the result with the intertemporal optimal conditions, such as equation (15), we know that $T_k(k_{t+1}, h_{t+1}) > 0$. Intuitively, if an agent, especially those with higher income in this period, prefers not to tell the truth, he has incentive to save more in this period. The positive taxation on their capital income will prevent such kinds of deviation.

The optimal real interest rate r_{t+1} satisfies

$$\frac{1}{U'(x_t)} = \frac{1}{\beta(1 + r_{t+1})} E_t \frac{1}{U'(x_{t+1})}. \tag{3.22}$$

Combining with equation (3.19), we can get:

$$1 + r_{t+1} = \frac{U'(x_t)(1 + F_K(t+1) - \delta)}{U'(x_t) - \sigma u'(q_t) f_K(t)} > 1 + F_K(t+1) - \delta. \tag{3.23}$$

Therefore, the gross real interest rate is higher than 1 so is the nominal interest rate. On one hand, although the interest rate is greater than 1, individuals still have incentive to holding money as money acts as the instrument to exchange goods in the night. On the other hand, as showed in the equation (3.12), individuals hold extra money that they do not use in the DM. In some sense, the extra money plays the role as insurance to against the uncertainty in the future, which also affects the incentive to work. The positive real interest rate will decrease the amount of extra money.

3.4 Conclusion

Based on search-theoretical environment, this section studies the optimal policy issues in the economy with heterogeneous agents, supplying the microfoundation of holding money and introducing money essential other than previous literature which treat money in the reduce form. In the spirit of Mirrlees's private information framework, I prove that the capital income taxation is no longer zero and the Friedman rule is not optimal when combined with nonlinear taxation of income.

Woodford [89] points out the financial intermediaries are playing more and more important role in the economy, especially during the financial crisis. Since in the model, money is an exchange instrument. If we consider other payment methods, such as credit and collateral in the economy, we need to add the financial intermediation, which will help us explain the the phenomenons in the asset market during the financial crisis. I will leave this as the further job.

4. LIQUIDITY, COLLATERAL CONSTRAINT AND FINANCIAL INTERMEDIATION

4.1 Introduction

In many standard macroeconomic models, identical households can invest in non-financial firms directly, without using a financial sector. As stated in Brunnermeier [16], this approach can only yield realistic macroeconomic predictions if, in reality, there are no financial frictions. Following the Great Depression, economists such as Fisher [35] and Keynes [55] pointed out that failure of financial markets would result in an economic downturn. Financial crises happen every now and then, spilling over into the real economy, and Kindleberger[57] documents that financial crises have occurred at roughly ten-year intervals in Western Europe over the past four centuries. This means they are relatively common in history. The current financial crisis starting in August 2007 has underscored and reminded us once again of the importance of financial intermediaries for the business cycle.

During the last decade, asset prices have been cyclical and have led the business cycle. Figure C.1 and C.2 in the Appendix C describe the time series of two broad stock price indexes and select macro variables for the US from 1997Q1 to 2012Q2. All series are percentage deviations of quarterly data from trend, and the percentage deviations of investment, GDP and bond prices are rescaled as indicated in the figures.[1] Two phenomena are apparent in these figures. First, investment and real GDP move closely with stock prices. Second, stock prices lead the business cycle.

[1] There are two kinds of stock price indexes. One is the Wilshire 5000 total market full cap index which is an index of the market value of all stocks actively traded in the US weighted by market capitalization. The other index is the S&P 500 index which contains the top 500 companies in main industries of the U.S. economy. The data for investment is real private nonresidential fixed investment. All the data is available at the Federal Reserve Bank of St.Louis.

For example, in the 2001-2002 recession, stock prices reached the summit in 2000Q1 and dropped to the trough in 2002Q3, while the investment peaked in 2000Q4 and reached the trough in 2003Q1. Similarly, in the 2008-2009 recession, stock prices peaked in 2007Q3 before falling to the trough in 2009Q1. Investment and output did not reach the peak until 2008Q2 and did not reach the trough until 2009Q2.

One intuitive explanation for these patterns is that shocks to the asset market are an important cause of the business cycle instead of merely a response to it. A popular hypothesis along this line is as follows. Sudden drops in equity market liquidity leads equity prices to fall and the price of liquid assets, such as bonds, to rise. When non-financial firms face financing constraints on investment, this reduction in equity price reduces these firms' ability to finance investment, which will leads to loss of liquidity in the equity markets and a fall in investment and output, thus this is a recession. Kiyotaki and Moore [59] and Shi [80] have formulated this hypothesis with a model that places two equity-market frictions at the center. One is the difficulty to issue new equity: a firm can issue new equity to finance at most a fraction of intended investment. The other friction is the lack of ability to resell existing equity. However, issuing new equity is just one channel for a limit number of firms to finance their investment, while a large number of firms borrow the funds they need for intended investment from financial intermediaries. Meanwhile, these firms face a borrowing constraints during the process, as non-financial firms use their market value of capital as collateral.

In this paper, households do not hold capital and do not supply funds directly to non-financial firms. Instead, they provide funds to banks which act as the liquidity provider, and non-financial firms borrow from banks to satisfy their liquidity demand. In addition to introducing financial intermediaries in the economy, this paper introduces two kinds of financial frictions. The first one is the friction between financial intermediaries and households. As Hellmann[49] state, moral hazard in the banking system plays a critical role in a financial crisis, and abolishing formal deposit insurance systems does not itself solve this agency problem. Since bankers use funds obtained from depositors to make the lending decision, bankers have an incentive to select a more risky asset portfolio which earns high profits if the investment succeeds, while depositors suffer from losses when the investment fails (Kane[53]; Cole[23]). Therefore, households prefer to limit bankers' ability to obtain funds.

The other friction is between financial intermediaries and non-financial firms. In the beginning of each period, non-financial firms need to pay wages and get investment funds before they receive the revenue of production. In order to pay for these operating expenses, entrepreneurs who own firms can use the internal funds accumulated in previous periods, and external funds borrowed from banks by using the market value of the firms as collateral. Geanakoplos[39] points out, in time of crisis, compared with the interest rate, the collateral rate is playing more important role, and a shock to the collateral rate will indeed influence the real economy. Gilchrist[43] empirically show that credit market shocks, which result from deterioration in the supply of credit due to weak balance sheets of firms or the disruptions in the health of banks that supply credit, have been an important factor for U.S. business cycle fluctuation during the 1990-2008 period and account for more than 30% of the variation in economic activity as measured by industrial production. Mimir[70] finds that the leverage ratio is acyclical, while liabilities and equity are procyclical and the credit spread is countercyclical. Moreover, drops in stock prices will reduce the non-financial firms' ability to finance even though they are not receiving the external funds via issuing new equity.

The paper is part of a recent and growing literature which attemps to model the financial intermediaries as an active agent in the economy. Most closely related to my work are Curdia and Woodford[25], Gertler and Karadi[40], Gertler and Kiyotaki [41] and Gertler et.al[42]. Based on the traditional New Keynesian Model, Curdia and Woodford[25] allow for the existence of interest gap between savers and borrowers. However, they do not consider the agency problem which is arose by introducing financial intermediaries in the economy. Gertler and Karadi[40] develop a monetary dynamic stochastic general equilibrium model with involving financial intermediaries which need to confront endogenously determined balance sheet constraints. Based on Gertler and Karadi[40], Gertler and Kiyotaki[41] and Gertler et.al[42] consider the financial friction between the bankers and households similar to mine, and explain why banks would use so risky balance sheet originally. They assume that non-financial firms can only issue new equity to finance investment and they do not consider the financial friction between non-financial firms and banks. In my paper, I regard this existing financial friction which limits non-financial firms ability to borrow, just as facing a liquidity constraint.

The rationale for liquidity constraints in my model is related to the one in Kiyotaki and Moore[58] who have supplied a theory to understand how shocks to credit-constrained firms are amplified. Kiyotaki and Moore[59] extend the model of a monetary economy where there are differences in liquidity across assets and investigate how aggregate activity and asset prices fluctuate with shocks to productivity and liquidity. Based on this, Shi[80] finds that for equity price to fall as it typically does in a recession, a negative liquidity shock must be accompanied or caused by other shocks that relax firms' financing constraint on investment. The main characteristic between the above models and mine is that the assumption that firms' financing constraints are exogenous variables and they did not consider the role of financial intermediaries in the economy.

Gorton and Winton[48] and Brunnermeier and Eisenbach[15] state that financial intermediaries act as liquidity providers in the economy, building upon the Bernanke and Gertler[12]. They find that the financial frictions between banks and non-financial firms lead to persistence and non-linear amplification effects. However, they did not consider the problem that can be brought by banks themselves, which is the classical principal-agency problem between bankers and households. This moral hazard problem will limit financial intermediaries' ability to obtain deposits from households, which has effect on the supply side of the market for loanable funds, thereby affect the investment and output.

The basic logic in this paper is as follow: banks are playing an even more important role in the modern economy, especially as liquidity providers for non-financial firms. While non-financial firms need to borrow from banks to pay the operating expense such as wages and investments, they can use their firms' market value as collateral to obtain the external funds which can explain the phenomenon between stock prices and investment level. On the other hand, the existence of banks in the economy will give rise to agency problems. The main contribution of this section is to construct a theoretical model to consider frictions between financial intermediaries and other individuals in the economy. This model formulates households, bankers and entrepreneurs' decisions in the monetary dynamic stochastic general equilibrium framework with nominal rigidities, and determines the competitive equilibriums. By calibrating the model with a negative collateral shock, a negative productivity shock and a positive shock to bankers' divert rate, this paper finds that a negative collateral shock which tightens firms' financing constraints on investment can generate an equity price boom which is different from what is observed in recessions. Therefore, a collateral shock is not the primary driving force of the business cycle, while a negative productivity shock and bankers' moral hazard problem are important aspects to explain current economy. This section proceeds as follows. In section 2, I outline the environment and describe the equilibrium. Then, I drive the optimal conditions for households, bankers and entrepreneurs, and get the propositions. In section 3, I calibrate the model with three different shocks. In section 4, I conclude.

4.2 The Model

In the economy, there exists three kinds of individuals: workers, bankers and entrepreneurs. The basic framework is the monetary dynamic stochastic general equilibrium (DSGE) model with nominal rigidities developed by Christiano et.al[22], Sveen and Weinke[82] and Smets and Wouters[81]. Basis on these, I introduce financial intermediaries into the economy and the principal-agent problem between bankers and households limits the financial intermediaries' ability to acquire funds from households, which is close to the models in Gertler and Kiyotaki[41] and Gertler et.al[42]. Households do not have investment opportunities besides saving in financial intermediations, and cannot borrow against their future labor incomes, which means that households face a liquidity constraint each period. As in the traditional monopolistic competitive economy, there exist one unit of firms. Each firm produces goods by using classical Cobb-Douglas production function. Similar to Bernanke and Gertler[12] and Fiore et.al[34], entrepreneurs, who are risk neutral, need to pay the wage and make investment decision in advance to get the revenue from production. In order to finance the liquidity, the entrepreneurs use the accumulated net worth from previous periods and the market value of firms' capital levels as collateral to borrow the needed funds from financial intermediaries.

If we do not consider financial intermediaries and there were no financial frictions in the economy, the equilibrium is the same as a solution of the social planner's problem in the standard DSGE model which chooses the series of $\{C_t, L_t, I_t\}$ as a function of state variables $\{K_t, A_t\}$ in oder to maximize the total welfare subject to the resource constraints. In the section, we use this frictionless DSGE economy as a benchmark to which can be used to compare with the economy which exists financial frictions.

To be more specific about the timing of events, we decompose each period into three subperiods. Aggregate shocks and the idiosyncratic collateral ratio shock revealed in the first subperiod. Non-financial firms know whether they can re-optimize their price level or not, hire labor supplied by workers and pay their wage bills and collect the funds for investment. They use internal funds brought in from the previous period, and external funds borrowed from financial intermediaries. In the second subperiod, non-financial firms enter into production and sell the outcome of production and buy the goods for investment. Entrepreneurs need to pay tax for their consumptions while households purchase consumption goods and pay lump sum taxes. Entrepreneurs pay debts back to banks and workers make the deposits in the last subperiod.

In the following, I will introduce the households, financial intermediaries and non-financial firms in details and study the consequences for their activities.

4.2.1 Households

Assume that households can not supply the funds for non-financial firms directly, which means they only lend to firms via financial intermediaries. Based on Gertler and Karadi[40], in period t, a representative household with a continuum of members of unit mass, who lives for infinite time periods, chooses aggregate consumption C_t and labor supply L_t in the competitive market, and make deposits in the financial intermediaries. As in [30], the household seeks to maximize the total expected utility

$$E_t \sum_{j=0}^{\infty} \beta^j U(C_{t+j}, L_{t+j}). \tag{4.1}$$

Here, $U_C > 0, U_L, U_{CC}, U_{LL} < 0$, $U_{CL} = 0$, the discount rate $\beta \in (0,1)$, and C_t denotes the aggregate consumption level at time t, which is

$$C_t = (\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{4.2}$$

where $\varepsilon \geq 1$ is the rate of substitution elasticity between different kinds of goods $C_t(i)$.

Households take the aggregate price level P_t and the composite goods prices $P_t(i)$ as given and beyond their control. The household need to deal with the following minimizing problem:

$$min \int_0^1 P_t(i) C_t(i) di \tag{4.3}$$

subject to

$$(\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di)^{\frac{\varepsilon}{\varepsilon-1}} \geq C_t, \tag{4.4}$$

where $P_t(i)$ is the price of good i. After solving the problem, the household's demand for good i can then be written as:

$$C_t(i) = (\frac{P_t(i)}{P_t})^{-\varepsilon} C_t, \tag{4.5}$$

here the aggregate price index $P_t = (\int_0^1 P_t(i)^{1-\varepsilon} di)^{\frac{1}{1-\varepsilon}}$[2].

In particular, there exists two kinds of members in each household, which are workers and bankers. The proportion $1-f$ of the members are workers and the other proportion f are bankers.[3]While each worker supplies labor in the competitive labor

[2]The price elasticity of demand for good i is equal to ε. If $\varepsilon \to \infty$, there would exist more substitutes for each specific goods, and as a result, it is similar to the competitive economy.

[3]The number of workers in a household is far more than the number of bankers, which implies $f < 1 - f$.

market and receives the payment which belongs to the whole household, bankers administer a specific financial intermediation and return any nonnegative earnings to the big family. Therefore, the household effectively owns the intermediation that its bankers administer. However, the household's deposits can not be put in the financial intermediation that it owns. This assumption guarantees the bank can make decision independently and the banker maximizes its own net worth other than their utility since the depositors and the owners of the bank are two different group of individuals.

At any moment of time t, the members in a representative household can switch casually between the two occupations. If the member in this period is A banker, he can still be a banker in the next period with a fixed possibility θ. Hence, on average, a banker can live for $\frac{1}{1-\theta}$ in any given period, which is finite but may be quite long[4]. In each period, the fraction of $(1-\theta)f$ bankers quite and turn into workers, meanwhile the same amount of workers are randomly picked and become bankers so that the relative amount of bankers and workers are stable over time. Bankers who exit from the financial intermediation turn their accumulated net worth to their household that they belong to, while the household supplies certain limited start-up funds for the new bankers.

While households supply funds to banks rather than hold capital by their own nor do they provide capital to non-financial firms directly, they just simply make the deposits in the banks. Since both deposits and government debt's maturity is just one period bonds which are both riskless, the deposits and government debt are perfect substitutes in our economy. At period t, the household chooses $\{C_t, L_t, D_t\}$ to maximize the total expected welfare (4.1) subject to the following sequence of

[4]In order to make sure that the bankers do not reach certain period when they can finance all funds from their own net worth other than deposits, the bankers are not living with infinitely periods.

budget constraints:

$$P_tC_t + D_t = W_tL_t + R_t^dD_{t-1} + \Pi_t - T_t. \tag{4.6}$$

Here W_t denotes the wage rate, R_t^d is the nominal interest rate, Π_t is the net distributions from ownership of banks, and T_t is the lump sum taxes. In each period, since households can not finance their present consumptions from their future labor incomes, they also face the borrowing constraint in each period, which means that:

$$D_t \geq 0. \tag{4.7}$$

The household's optimal conditions are as follows:

$$\frac{W_t}{P_t}U_{Ct} = -U_{Lt}, \tag{4.8}$$

$$U_{Ct} \geq \beta E_t \frac{R_{t+1}^d U_{C,t+1}}{1+\pi_{t+1}} = \beta E_t(1+r_{t+1}^d)U_{C,t+1}. \tag{4.9}$$

Here, $\pi_{t+1} = \frac{P_{t+1}}{P_t} - 1$ is the inflation rate in period $t+1$, $1 + r_{t+1}^d = \frac{R_{t+1}^d}{1+\pi_{t+1}}$ is the real interest rate, U_{Ct} denotes the marginal utility of consumption and U_{Lt} is the marginal dis-utility of labor. Equation (4.8) states the intratemporal optimal condition, and equation (4.9) is the Euler equation which can only be equalized if the borrowing constraint (4.7)is not binding.

Assumption 7 *For any t, $\beta < \frac{1}{1+r_t^d}$.*

Let $1 + r_{0,t}^d = \Pi_{i=0}^t(1 + r_t^d)$, within equation (4.9), for any t, we know that $\beta^t(1 + r_{0,t}^d)U_{Ct}$ is a bounded supermartingale. If $\beta(1 + r_t^d) < 1$, which denotes that the household is relatively impatient given the interest rate, then $\beta^t(1 + r_{0,t}^d)U_{Ct}$will convergence based on Doob's convergence theorem, and it means that consumption

C_t and deposits D_t will not diverge. Actually, $\frac{1}{1+r_t^d}$ is entrepreneurs' discount rate, if there is no uncertainty in the economy, in the steady state, households will consume their labor income and do not save at all since they are impatient.

4.2.2 Financial Intermediaries

In each period, the financial intermediaries obtain funds from households and their own net worth which is treated as inside equity. At beginning of period t, as all the funds can be used as loans to non-financial firms, the balance sheet identity of a financial intermediation is:

$$d_t^s = D_t + n_t, \tag{4.10}$$

where d_t^s denotes the amount of loanable funds supplied by a typical bank and n_t is the banker's net worth.

The household saves his deposits into bank at the end of previous period and obtains the nominal interest rate R_t^d at the end of period t, which means D_t is the liability of the bank and n_t is its equity. Respectively, at the end of period t, the bank receives the nominal return rate R_t^l from its lending. Since $R_t^l \geq R_t^d$, the bankers always have the incentive to engage in financial intermediaries. The law of motion for the bank's bet worth is :

$$n_{t+1} = R_t^l d_t^s - R_t^d D_t. \tag{4.11}$$

Using the balance sheet of banks given by equation (4.10), we can re-write equation (4.11) as follows:

$$n_{t+1} = (R_t^l - R_t^d) d_t^s + R_t^d n_t. \tag{4.12}$$

The bank's net worth relies on the premium $R_t^l - R_t^d$ that earns on the amounts supplied in the loanable funds market.

In period $t+j$, if the following inequality holds, the bank will always have incentive to lend out:

$$E_t \beta^j \Lambda_{t,t+j}(R_{t+j}^l - R_{t+j}^d) \geq 0, i \geq 0, \tag{4.13}$$

where $\Lambda_{t,t+j} = \frac{U_{C,t+j}}{U_{Ct}}$. In a perfect financial market, the return on loans is always equal to the cost of deposits, which means the interest rate spread is zero. However, if the financial market is imperfect, the interest rate gap is positive since bankers can not obtain funds as much as their want. Justiniano et.al [52] supplies the evidence that this premium is highly countercyclical and was widely increased during the recent recession.

At the beginning of period t, the bank's objective is to maximize the expected discounted terminal net worth, which is[5]

$$N_t = E_t \sum_{j=0}^{\infty} (1-\theta)\theta^j \beta^j \Lambda_{t,t+j} n_{t+j}. \tag{4.14}$$

Since the premium $\beta^j \Lambda_{t,t+j}(R_{t+j}^l - R_{t+j}^d)$ is positive, bankers will always have the incentive to receive more deposits from households. In order to set a limit on its ability to do so, the economy involves the the agency problem. At the end of the period, after obtaining funds from households, the banker can choose to divert φ fraction of assets to satisfy his family's consumption. If the banker chooses to do so, he can not be a banker any more. Thus, the cost that the banker need to face is that the depositors can drive the bank into bankruptcy and obtain the remaining

[5] As bankers face financing constraints, they have incentive to keep all earning in the banks' account until they can not be banker any more.

proportion $1 - \varphi$ of assets.

Since the banker can divert the fund in each period and if the banker does not have incentive to do so, the following incentive constraint must be satisfied:

$$N_t \geq \varphi d_t^s. \tag{4.15}$$

The left hand side of the above equation is the value of operating for the bank, which would lose if the banker diverts a fraction of assets. The right hand side denotes the gain from doing so.

Accordingly, the bank chooses d_t^s to maximize the expected discounted terminal net worth (4.14) subject to the law of motion for net worth (4.12) and the incentive constraint (4.15).

Proposition 4 *The expected discounted terminal net worth can be expressed as the discounted total return to its loan to firms and the expected discounted total return to its existing net worth, which means that*

$$N_t = \nu_t d_t^s + \eta_t n_t, \tag{4.16}$$

where $x_{t,t+j} = \frac{d_{t+j}^s}{d_t^s}$ *and* $z_{t,t+j} = \frac{n_{t+j}}{n_t}$

$$\begin{aligned} \nu_t &= (1-\theta)(R_t^l - R_t^d) + E_t[\theta\beta\Lambda_{t,t+1}x_{t,t+1}\upsilon_{t+1}], \\ \eta_t &= (1-\theta)R_t^d + E_t[\theta\beta\Lambda_{t,t+1}z_{t,t+1}\eta_{t+1}]. \end{aligned} \tag{4.17}$$

The proof of the above proposition is contained in Appendix C. In equation (4.16), ν_t denotes the expected discounted marginal gain to the bank of supplying one more unit of loan, while η_t is regarded as the expected discounted marginal benefit of having one extra amount of net worth. As shown in Appendix C, the incentive constraint (4.15) is always binding which limits the leverage of the bank, thus:

$$d_t^s = \frac{\eta_t n_t}{\varphi - \nu_t}. \tag{4.18}$$

Combing with equation (4.10), the bank's leverage ratio is as follows:

$$\frac{D_t}{n_t} = \frac{\eta_t}{\varphi - \nu_t} - 1. \tag{4.19}$$

Based on the above equation, we can find that the leverage ratio is an increasing function of the expected marginal return of receiving one more amount of deposits ν_t and having one extra amount of net worth η_t. Meanwhile, it is also a decreasing function of diverting fraction rate φ. Intuitively, if the marginal returns increases, bankers have less incentive to divert funds today, which will give households more willings to entrust the bankers, and the decreasing of diverting fraction rate will also have the same effect.

At the beginning of the next period, as only θ of financial inter mediations still survive, the intermediaries' aggregate net worth $\bar{n}_t$ at period $t+1$ is constructed by two parts. One part is the total amount of existing financial intermediaries' net worth, while the other part is the sum of the net worth of newly entering financial intermediaries. Households supply start-up funds for the related financial intermediaries, which are supposed to be $\frac{\epsilon}{1-\theta}$. Combing with equation (4.12) ,(4.18) and (4.19), we have intermediaries' aggregate net worth $\bar{n}_{t+1}$:

$$\bar{n}_{t+1} = \theta[(R_t^l - R_t^d)\frac{\eta_t}{\varphi - \nu_t} + R_t^d]\bar{n}_t + \epsilon\bar{n}_t. \tag{4.20}$$

4.2.3 Non-financial Firms

The assumptions on the entrepreneurs are as in Bernanke and Gertler [12] and Fiore et.al [34]. There is a continuum one unit of monopolistically competitive firms which are owned by specific entrepreneurs. They are risk neutral and have linear preferences over consumption with time preference $\beta^e = \frac{1}{1+r^d}$. Their total expected utility function is:

$$E_t \sum_{j=0}^{\infty} (\beta^e)^j C^e_{t+j}. \tag{4.21}$$

Here C^e_{t+j} is the entrepreneur's aggregate consumption. In each period t, entrepreneurs can choose whether use the accumulated internal funds M_t to buy the consumption goods or not. One more unit of consumption at t, means that there will be less $1+\tau$ units of real internal funds to be available for production at next period. These funds earn expected return $\frac{R^l_{t+1}}{(1+\pi_{t+1})(1+\tau)}$ units of consumption at $t+1$. Since $R^l_{t+1} > R^d_{t+1}$ and $\beta^e = \frac{1}{1+r^d}$, we have:

$$(1+\tau)\beta^e E_t \frac{R^l_{t+1}}{(1+\pi_{t+1})(1+\tau)} > 1, \tag{4.22}$$

then it is better to postpone consumption as they are more patient compared with households. Moreover, the decision does not depend on the consumption tax rate [6]. Therefore, the entrepreneur's object is to maximize the accumulated internal funds which is the market value of the firm.

[6]Here, I just consider a special case in which entrepreneurs' consumptions are fully taxed. The extremely high tax rate will lead the consumption of entrepreneurs to approach zero. Actually, even though I introduce entrepreneurs in the economy, I do not treat them as actual individuals, just as a method to introduce the financial friction between non-financial firms and financial intermediation, which is the main objective of this section.

Each firm i's production function is:

$$Y_t(i) = A_t(i)(K_t(i))^{\alpha}(L_t(i))^{1-\alpha}, \tag{4.23}$$

here $\alpha \in (0,1)$ is the capital share in the production function, $A_t(i)$ follows a stationary stochastic process which is common knowledge to all entrepreneurs and realized in the beginning of each period, $K_t(i)$ and $L_t(i)$ represent firm i's accumulated capital level and labor input used in producting $Y_t(i)$.

Following Woodford [88], I assume that there are two kinds of restrictions on capital accumulation. Firstly, the investment in capital can be used to produce in the next period other than this period. Secondly, the capital adjustment cost function is convex. Thus, we can write the investment function as follow:

$$I_t(i) = f(\frac{K_{t+1}(i)}{K_t(i)})K_t(i), \tag{4.24}$$

here $I_t(i)$ is the quantity of investment goods purchased by firm i [7]. The function $f(.)$ satisfies the following condition: $f(1) = \delta \in (0,1)$, $f^{'}(1) = 1$, and $f^{''}(1) = \xi > 0$. Parameter δ is capital's depreciation rate, and ξ is the elasticity of the investment-to-capital ratio which is strictly larger than zero.

The entrepreneur i joins into period t with capital level $K_t(i)$ and accumulated internal funds $M_t(i)$. As in [58] and [75], the entrepreneur need to pay the wage and make investment decision in advance to receive the revenue from production, and he or she need to use the market value of capital as collateral to borrow external funds from banks. The firm i have the internal funds $M_t(i)$ and the amount it need to

[7] The investment goods is the same as the aggregate consumption bundles.

borrow is $d_t(i) = W_tL_t(i) + P_tI_t(i) - M_t(i)$, and the collateral constraint:

$$R^l_t(W_tL_t(i) + P_tI_t(i) - M_t(i)) \leq \psi_t(i)E_t[Q_{t+1}(i)K_{t+1}(i)]. \tag{4.25}$$

Here, R^l_t is the lending interest rate, $Q_{t+1}(i)$ is the market price of capital in currency units, and $\psi_t(i)$ is a collateral rate shock which is a way to denote the tightness in the credit market. Following Liu et.al[65], if the owners of non-financial firms are unable to pay the debt back, this bank can seize the accumulated capital and own the non-financial firm. Meanwhile, it is costly to sell the capital stock, the banker at most would like to lend a fraction $\psi_t(i)$ of the total value of collateral assets. The $\psi_t(i)$ follows the stochastic process:

$$log\psi_t(i) = (1-\rho)log\bar{\psi}(i) + \rho log\psi_{t-1}(i) + \epsilon_{\psi it}, \tag{4.26}$$

where $\bar{\psi}(i)$ is the average value of $\psi_t(i)$, $\rho \in (0,1)$ denotes the persistence, and $\epsilon_{\psi it}$ is an $i.i.d$ white noise process with mean zero and variance $\sigma^2_{\psi i}$.

Assume the staggered price setting as in Calvo [17] and Yun [90]: in the beginning of each period, every firm can re-optimize its price level with probability $1-\omega$ which is exogenous and constant over. Thus, at time t, firm i's nominal price $P_t(i)$ is either $P^*_{t-1}(i)$ with probability ω or the price $P^*_t(i)$ with probability $1-\omega$.

As discussed in the household's cost minimization problem, the demand for each individual goods i in period t is:

$$Y^d_t(i) = (\frac{P_t(i)}{P_t})^{-\varepsilon}Y^d_t, \tag{4.27}$$

where Y_t^d is the aggregate demand at time t, which is:

$$Y_t^d = C_t + I_t + G_t,$$

where G_t is the government expenditure and $I_t \equiv \int_0^1 I_t(i)di$ means the aggregate investment level.

Define $Q_t K_t(i) = V(K_t(i), M_t(i), P_{t-1}(i))$, which is the nominal value function of firm i joining into period t. Since in the beginning of period t, the entrepreneur i has already known whether he can re-optimize or not before borrow the external funds for banks, he will make different choice when he can control the price level. Let $L_t^*(i)$ and $I_t^*(i)$ respectively be the labor demand and investment level when the price level is $P_t^*(i)$. The entrepreneur chooses $\{L_t(i), K_{t+1}(i), L_t^*(i), K_{t+1}^*(i), P_t^*(i)\}$ to maximize:

$$\begin{aligned}\frac{V(K_t(i), M_t(i), P_{t-1}(i))}{P_t} &= \omega[\frac{P_{t-1}(i)Y_t(i)}{P_t} - \frac{R_t^l d_t(i)}{P_t} + \beta^e E_t \frac{V(K_{t+1}(i), M_{t+1}(i), P_{t-1}(i))}{P_{t+1}}] \\ &+(1-\omega)[\frac{P_t^*(i)Y_t^*(i)}{P_t} - \frac{R_t^l d_t^*(i)}{P_t} + \beta^e E_t \frac{V(K_{t+1}^*(i), M_{t+1}^*(i), P_t^*(i))}{P_{t+1}}],\end{aligned} \tag{4.28}$$

subject to production function (4.23), capital motion equation (4.24), collateral constraint (4.25) and demand function (4.27). In equation (4.28), $V(K_{t+1}(i), M_{t+1}(i), P_{t-1}(i))$ denotes the value function in next period if the firm can not re-optimize its price level, while $V(K_{t+1}^*(i), M_{t+1}^*(i), P_t^*(i))$ is the value function if the firm can optimally choose price level. The $M_{t+1}(i)$ and $M_{t+1}^*(i)$ are expressed as follows:

$$\begin{aligned}M_{t+1}(i) &= P_{t-1}(i)Y_t(i) - R_t^l d_t(i), \\ M_{t+1}^*(i) &= P_t^*(i)Y_t^*(i) - R_t^l d_t^*(i).\end{aligned} \tag{4.29}$$

The detailed steps in solving the firm's problem are in Appendix C.

Lemma 1 *When the collateral constraint is binding, the labor demand level is lower than in the frictionless economy.*

The existence of collateral constraint will lead the entrepreneur to face more constraints when he makes the decision, and we have:

$$\begin{aligned} \frac{P_{t-1}(i)Y_{Lt}(i)}{P_t} &\geq \frac{R_t^l W_t}{P_t}, \\ \frac{(1-\frac{1}{\varepsilon})P_t^*(i)Y_{Lt}^*(i)}{P_t} &\geq \frac{R_t^l W_t}{P_t}. \end{aligned} \tag{4.30}$$

From the above two inequalities, even though the marginal benefit of adding one extra unit of labor is no less than the marginal cost, which means that the firm can gain extra profit from increasing the labor input, the firm can not do so because of facing the collateral constraint.

If the collateral constraint is not binding, we can have the first-order condition for price setting as follows:

$$\sum_{j=0}^{\infty}(\omega\beta^e)^j E_t\{\frac{Y_{t+j}(i)}{P_{t+j}}[P_t^*(i) - \frac{\varepsilon}{\varepsilon-1}MC_{t+j}(i)]\} = 0, \tag{4.31}$$

where $MC_t(i)$ means the nominal marginal cost of firm i at period t. By solving firm i's cost minimization problem, the expression of $MC_t(i)$ is :

$$MC_t(i) = \frac{W_t * R_t^l}{Y_{Lt}(i)}, \tag{4.32}$$

where $Y_{Lt}(i)$ is the marginal production of labor. Since the chosen price can be still used as the firms' future price level, they not only need to consider the current expected marginal cost but also need to take into account the future's which is included in the equation (4.31).

4.2.4 Equilibrium

In the economy, the government's budget constraint is

$$P_t G_t = M_{t+1} - M_t + T_t, \tag{4.33}$$

where M_t denotes the money supply at time t.

For all $i \in (0,1)$, given the exogenous process $\{A_t, \psi_t(i)\}_{t=0}^{\infty}$, the policy processes $\{T_t\}_{t=0}^{\infty}$, the government spending $\{G_t\}_{t=0}^{\infty}$, and initial conditions $\{M_0(i), K_0(i)\}$, a competitive equilibrium of this economy is a collection of $\{C_t(i), L_t, L_t(i), K_t(i), R_t^d, R_t^l, D_t, d_t^s, d_t, I_t(i), M_t(i), M_t, P_t, P_t(i), W_t\}_{t=0}^{\infty}$ such that:

(i) households choose $\{C_t, L_t, D_t\}$ to maximize the total expected discounted utility subject to the budget constraint and borrowing constraint, taking the prices $\{P_t(i), P_t, R_t^d, W_t\}$ and the policy processes as given; bankers choose $\{d_t^s\}$ to maximize the expected discounted terminal net worth subject to the incentive constraint, and entrepreneurs optimize $\{L_t(i), I_t(i), L_t^*(i), I_t^*(i), P_t^*(i)\}$ to maximize total discount value subject to the collateral constraints by taking $\{R_t^d, R_t^l\}$ as given.

(ii) All the markets are clear: the labor market $\int_0^1 L_t(i)di = L_t$; the goods market; the money $\int_0^1 M_t(i)di = M_t$; the loanable funds $d_t^s = \int_0^1 d_t(i)di$.

(iii) Government budget constraints hold every period.

(iv) Resource constraint: $Y_t = C_t + I_t + G_t$.

Since in each period, the fraction $1-\omega$ of firms re-optimize their price level and the remaining firms still use the same price level as previous period, the aggregate price is decided by these two kinds of price levels. However, since firms were randomly chosen from the pool of all firms, the average price of firms which are not selected to adjust their price is the average price of all firms that prevailed in previous period.

Thus, aggregate index $P_t = (\int_0^1 P_t(i)^{1-\varepsilon} di)^{\frac{1}{1-\varepsilon}}$ can also be written as:

$$P_t^{1-\varepsilon} = (1-\omega)(P_t^*(i))^{1-\varepsilon} + \omega P_{t-1}^{1-\varepsilon}. \tag{4.34}$$

By approximating equation (4.31) and (4.34) around steady-state equilibrium which the inflation is zero, we can have the expression for inflation as follow:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \widehat{mc_t}, \tag{4.35}$$

where $\pi_t = log(P_t) - log(P_{t-1})$, κ is a decreasing function of the fraction of firms which can not adjust price levels each period, nominal marginal cost $mc_t \equiv \int_0^1 \frac{MC_t(i)}{P_t} di$, and $\widehat{mc_t}$ means the real marginal cost's percentage deviation from the steady-state [8]. We can treat equation (4.35) as the *New Keynesian Phillips Curve*, which implies that real marginal cost is the right driving force for the inflation and the inflation process is forward-looking as a firm cares more about the inflation in the future when it sets its price level.

4.3 Model Analysis

In order to examine how the economy responds collateral shocks and technology shocks, I calibrate the model based on Gertler and Karadi [40], Kiyotaki and Moore [59] and Sveen and Weinke [83].

4.3.1 Calibration

The utility function and capital adjustment function used for quantitative analysis are:

$$U(C, L) = log(C) + \Psi log(1-L), \tag{4.36}$$

[8] See Sveen and Weinke [82] and [83] for the detailed calculation process.

$$f(\frac{K_{t+1}(i)}{K_t(i)}) = \frac{K_{t+1}(i)}{K_t(i)} - 1 + \frac{\xi}{2}(\frac{K_{t+1}(i)}{K_t(i)} - 1)^2 + \delta. \quad (4.37)$$

You can find the choice of parameter in Table C.2. Since I consider the quarterly data, the discount factor β is chosen as 0.99 in order to match the U.S. avergage annual real interest rate which is 4%. The relative utility weight of labor is determined so that the steady-state value of L is 0.33. As estimated in Gali et.al [37], $\varepsilon = 11$ implies a frictionless makeup of 10%. The fraction of fund φ which can be diverted, the percentage tansfer to new borned bankers ϵ and the survival probability θ, are chose to match the specific targets, just as stated in Gertler and Karadi [40][9]. Turning to non-financial firms, the price stickiness parameter $\omega = 0.779$ is often considered to be empirically plausible.[10] The parameter premultiplying the marginal cost in equation (4.35) is $((1 - \beta^e\omega)(1 - \omega)/\omega)(1 - \alpha)(1 - \alpha + \alpha\varepsilon)$.

In the model, there exists productivity shocks, collateral shocks and bankers' diverting shocks which I consider separately. The technology A_t follows the stochastic process:

$$logA_t = \rho_a log(A_{t-1}) + \epsilon_t^a, \quad (4.38)$$

where $\epsilon_t^a \sim N(0, \sigma_a)$. I set the quarterly autoregressive factor ρ_a to 0.96 which is commonly used in the literature, and the standard deviation of shocks is 0.0038. The stochastic process of collateral shock $\psi_t(i)$ is treated as equation (4.26). Following Liu et.al [65], the average value of collateral ratio is 0.75, the persistence of collateral ratio shocks is 0.979 and the standard deviation is 0.0126 which are estimated by using the Bayesian method to match their model to quarterly U.S data. Moreover,

[9]In the steady-state, the interest rate spread is 100 basis points, and the average suivival time of bank is eight years.

[10]Sveen and Weinke [83] points out that the micro evidence on price adjustment is mixed.

the bankers' diverting rate φ shock is an $i.i.d$ shock:

$$log(\varphi_t) = \epsilon_t^{\varphi}, \tag{4.39}$$

where $\epsilon_t^{\varphi} \sim N(0, \sigma_{\varphi})$. I choose the standard deviation of the shock $\sigma_{\varphi} = 0.001713$ so that the value of diverting rate will increase 10% when one unit standard deviation shock happens.

Furthermore, all these shocks are uncorrelated in the model such that we could find out their specific effects.

4.3.2 Experiments

In order to classify the effects arose by the different shocks, I introduce the three kinds of shocks to the model separately. All the figures show the deviations of these variables from the steady state values. Figure C.3 shows the impulse responses to a one-time, one-standard deviation negative shock to collateral ratio φ. The negative collateral shock increases the investment cost and the interest rate spread, which lead the stock price to rise. On one hand, compared with the steady state, because of the negative collateral shock, the external funds that firms can borrow from banks is lower. On the other hand, as the stock price is higher, it allows the firms' capital to be more valuable which mitigates the negative effect and the collateral constraint becomes less tight. Furthermore, entrepreneurs spend more expenditure in investment which lowers households' labor income, even though their labor supply is higher.

Figure C.4 presents the impulse responses to a negative shock to productivity. Since the negative technology shock reduces the marginal productivity of capital, the stock price is lower which makes investment to be less profitable for entrepreneurs. As we can observed from Figure C.4, during the first 4 years, stock price, investment and capital level's deviations from steady state are increasing. Meanwhile, as households can not borrow from the banks, the negative productivity shock lowers the marginal production of labor and real wage, which in turn, decreases the consumption significantly and induces households to supply more labor. Moreover, banks have difficulty in obtaining deposits from households since marginal utility of consumption is higher. It is result in the decreasing in their leverage ratio, which means the productivity shocks generates a procyclical leverage ratio.

Figure C.5 shows the impulse response to a positive shock to bankers' divert rate. If the divert fraction φ increases, bankers can steal more proportion of money which means that there exists more severe moral hazard problem. In order to convince households to save, bankers have to pay higher interest rate for deposits, which dominates the other effects and rises the leverage ratio. However, since bankers divert funds to their specific households which increases their income, with regarding the income effect, households' consumption is higher and labor supply is lower temporarily, which reduce the amount of loanable funds in the market and also the investment level. As a result, the production is lower. Without double, the situation with higher consumption level and lower labor supply can not last for a long time which we can find out from the figure.

These responses are broadly consistent with historical data depicted in Figure C.1 and Figure C.2, although some of the responses do not match in the magnitude. The consistency suggests that productivity shocks are critical aspects to explain the cyclical behavior of stock prices with other macro variables. If productivity shocks are the only shocks, then stock prices would fall and banks' financing ability becomes worse in response to a negative productivity shock. Meanwhile, positive shocks to bankers' divert rate can also be used to explain the phenomenons. Even though the reducing in stock prices and outputs is along with higher consumptions, it is mainly result from the bankers' diverting. By taking into account this effect, bankers' moral hazard problem is also important in explaining the cyclical behaviors. Since I have not considered the wage rigidity in the model, the absence of it may imply that labor and output do not fall enough as wage is flexible and can be adjusted without any cost, which may be the reason why labor and output level are increasing after the negative collateral shock.

4.4 Conclusion

In order to analyze the financial crisis of 2007-2010, Kocherlakota [63] points out DSGE models need to incorporate both stickiness and financial market frictions. In this paper, I introduce financial intermediaries in the DSGE framework with regarding the nominal rigidities. Accordingly, I consider two kinds of financial frictions between financial intermediaries and other individuals. The first one is the moral hazard problem between financial intermediaries and households as bankers are not using their own funds to make the investment. The other one is the collateral constraint when firms need to borrow external funds from banks. By calibrating the model with three separate shocks which are a negative collateral shock, a negative productivity shock and a positive shock to bankers' divert rate, I find that compared with the negative collateral shock, the negative productivity shock and bankers' moral hazard problem are more important aspects. Moreover, it provides a note of caution for policymakers: they should find the reasons of the shortfall in liquidity other than simply pumping liquidity into the market. On one hand, if firms' shortfall in liquidity is not generated by the fundamental events, it is a good policy for government to supply liquidity to them. If firms' shortfall in liquidity is generated by productivity, it is not reasonable for government to do so. Moreover, for the purpose of mitigating bankers' moral hazard problem, government need to strengthen the supervision of the financial intermediaries, especially in the recession because bankers have more incentive to divert assets during this period.

This paper could be extended into two directions. Firstly, we could use the framework of this paper to have the policy analysis. In the financial crisis, the government normally provide the liquidity for banks. Within considering the moral hazard problem, this policy seems a bad idea. Secondly, as financial intermediaries act as liquidity providers in the economy, supplying the microfoundations of the agency problem between bankers, households and entrepreneurs, such as that in Diamond and Dybvig [27] and Farhi and Tirole [33], represents a promising avenue for future research.

5. CONCLUSION

In short, on one hand, my work focuses on the optimal conduct of fiscal and monetary policy in the dynamic economy by introducing heterogeneity and private information, and it uses the basic framework of the new dynamic public finance literature which builds on the tradition of Mirrlees [71]. On the other hand and more importantly, my work mainly try to explain some phenomenons happened in the recent financial crisis by emphasizing the financial intermediaries' role in the economy, and studies financial frictions arise between financial sectors and other individuals. I could use the framework in Section 4 to have the policy analysis. In the financial crisis, the government normally provide the liquidity for banks. Within considering the moral hazard problem, this policy seems a bad idea. Moreover, as financial intermediaries act as liquidity providers in the economy, supplying the microfoundations of the agency problem between bankers, households and entrepreneurs, such as that in Diamond and Dybvig [27] and Farhi and Tirole [33], represents a promising avenue for future research, which is also my research goals during period of affiliation.

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APPENDIX A

APPENDIX OF SECTION 2

Country	1960	1970	1980	1990	2000	2008
Argentina	26.4	21.1	17.1	13	9.9	8
Brazil	55.1	44.2	32.6	25.2	18.8	14.42
China	84	82.6	80.4	72.6	64.2	56.9
Indonesia	85.4	82.9	77.9	69.4	58	48.54
Malaysia	73.4	66.5	58	50.2	38	29.64
Mexico	49.2	41	33.7	28.6	25.3	22.8
Philippines	69.7	67	62.5	51.2	41.5	35.08
South Korea	72.3	59.3	43.3	26.2	20.4	18.54
Thailand	80.3	79.1	73.2	70.6	68.9	66.68

Table A.1: Share of Rural Population (%)

In the economy with private information,the government as the social planner solves the optimal program:

$$\max_{(c_{it},N_{it},l_{it},x_{it},I_{it})} \sum_{i=H,h} (\sum_{t=1}^{2} \beta^{t-1} U(c_{it}, 1-N_{it}-l_{it}) + \beta^2 U(c_{i3}, 1-l_{i3})), \tag{A.1}$$

s.t. equation(2.8) (2.9) (2.10) and (2.23).

Let $\lambda_t, t = 1, 2.3$ denote the Lagrange multipliers of the feasible conditions (2.8) (2.9) (2.10), and μ denotes the Lagrange multiplier of the IC constraint (2.23),U^* denotes the utility function when high ability agent pretends to be low ability agent.

The first order conditions are as follows:

$$\begin{aligned}
c_{Ht} :& \beta^{t-1}(1+\mu)\frac{\partial U(c_{Ht}, L_{Ht})}{\partial c_{Ht}} = \lambda_t, \\
c_{ht} :& \beta^{t-1}(1-\mu)\frac{\partial U(c_{ht}, L_{ht})}{\partial c_{ht}} = \lambda_t, \\
y^l_{Ht} :& \frac{\partial U(c_{Ht}, L_{Ht})}{\partial L_{Ht}} \frac{1+\mu}{H_{Ht}} = \lambda_t \frac{\partial F(K_{Ht}, y^l_{Ht})}{\partial y^l_{Ht}} \\
y^l_{ht} :& \frac{\partial U(c_{ht}, L_{ht})}{\partial L_{ht}} \frac{\beta^{t-1}}{H_{Ht}} = \lambda_t \frac{\partial F(K_{ht}, y^l_{ht})}{\partial y^l_{ht}} + \mu \frac{\beta^{t-1}}{H_{Ht}} \frac{\partial U^*(c_{ht}, L_{ht})}{\partial L_{ht}} \\
y^H_{H1} :& (-\frac{U_{L_{H1}}}{H_{H1}} + \beta U_{L_{H2}} \frac{y^H_{H2} + y^l_{H2}}{(H_{i2})^2} \frac{\partial H_{H2}}{\partial y^H_{H1}} + \beta^2 U_{L_{H3}} \frac{y^l_{H3}}{(H_{H3})^2} \frac{\partial H_{H3}}{\partial H_{H2}} \frac{\partial H_{H2}}{\partial y^H_{H1}} (1+\mu) \\
&= -\frac{1}{1+\mu} \beta^2 U_{L_{h3}} \frac{\partial y^l_{h3}}{\partial y^H_{i1}} \\
y^H_{H2} :& (-\frac{U_{L_{H2}}}{H_{H2}} + \beta U_{L_{H3}} \frac{y^l_{H3}}{(H_{H3})^2} \frac{\partial H_{H3}}{\partial y^H_{H2}} (1+\mu) = 0, \\
y^H_{h1} :& (-\frac{U_{L_{h1}}}{H_{h1}} + \beta U_{L_{h2}} \frac{y^H_{h2} + y^l_{h2}}{(H_{h2})^2} \frac{\partial H_{h2}}{\partial y^H_{h1}} \\
&+ \beta^2 U_{L_{h3}} \frac{y^l_{h3}}{(H_{h3})^2} \frac{\partial H_{h3}}{\partial H_{h2}} \frac{\partial H_{h2}}{\partial y^H_{h1}} - \mu(-\frac{U^*_{L_{H1}}}{H_{H1}} + \\
&\beta U^*_{L_{H2}} \frac{y^H_{h2} + y^l_{h2}}{(H_{H2})^2} \frac{\partial H_{H2}}{\partial y^H_{h1}} + \beta^2 U^*_{L_{H3}} \frac{y^l_{h3}}{(H^*_{H3})^2} \frac{\partial H^*_{H3}}{\partial H_{H2}} \frac{\partial H_{h2}}{\partial y^H_{h1}} = 0,
\end{aligned}$$

$$
\begin{aligned}
y_{h2}^{H} :& (-\frac{U_{L_{h2}}}{H_{h2}} + \beta U_{L_{h3}} \frac{y_{h3}^{l}}{(H_{h3})^2} \frac{\partial H_{h3}}{\partial y_{h2}^{H}} - \mu(-\frac{U_{L_{H2}}^{*}}{H_{H3}} + \beta U_{L_{H3}}^{*} \frac{y_{H3}^{l}}{(H_{H3}^{*})^2} \frac{\partial H_{H3}^{*}}{\partial y_{h2}^{H}} = 0 \\
x_{H1} :& \lambda_1 = (\beta U_{L_{H2}} - \frac{y_{H_2}^{H} + y_{H_2}^{l}}{H_{H2}^{2}} \frac{\partial H_{H2}}{\partial x_{H1}} + \beta^2 U_{L_{H3}} \frac{y_{H3}^{l}}{(H_{H3})^2} \frac{\partial H_{H3}}{\partial H_{H2}} \frac{\partial H_{H2}}{\partial x_{H1}})(1+\mu) \\
& + \beta^2 U_{L_{h3}} \frac{y_{h3}^{l}}{(H_{h3})^2} \frac{\partial H_{h3}}{\partial H_{H2}} \frac{\partial H_{H2}}{\partial x_{H2}} \\
x_{H2} :& \lambda_2 = \beta^2 U_{L_{H3}} \frac{y_{H_3}^{l}}{H_{H3}^{2}} \frac{\partial H_{H3}}{\partial x_{H2}} (1+\mu) \\
x_{h1} :& \lambda_1 = \beta U_{L_{h2}} - \frac{y_{h_2}^{H} + y_{h_2}^{l}}{H_{h2}^{2}} \frac{\partial H_{h2}}{\partial x_{h1}} + \beta^2 U_{L_{h3}} \frac{y_{h3}^{l}}{(H_{h3})^2} \frac{\partial H_{h3}}{\partial H_{h2}} \frac{\partial H_{h2}}{\partial x_{h1}} - \\
& \mu(\beta U_{L_{h3}}^{*} \frac{y_{h2}^{H} + y_{h2}^{l}}{(H_{h2})^2} \frac{\partial H_{h2}}{\partial x_{h1}} + \beta^2 U_{L_{h3}}^{*} \frac{y_{h3}^{l}}{(H_{h3})^2} \frac{\partial H_{h3}^{*}}{\partial H_{h2}} \frac{\partial H_{h2}}{\partial x_{h1}}) \\
x_{h2} :& \lambda_2 = \beta^2 U_{L_{h3}} \frac{y_{h_3}^{l}}{(H_{h3})^2} \frac{\partial H_{h3}}{\partial x_{h2}} - \mu \beta^2 U_{L_{h3}}^{*} \frac{y_{h_3}^{l}}{(H_{h3}^{*})^2} \frac{\partial H_{h3}^{*}}{\partial x_{h2}} \\
I_{i1} :& \lambda_1 = \lambda_2 F_{k_{i2}} + \lambda_3 F_{k_{i3}} (1-\delta_k) \\
I_{i2} :& \lambda_2 = \lambda_3 F_{k_{i3}}
\end{aligned}
$$

As for inequation (2.27), the optimal condition of y_{ht}^{l}:

$$
\frac{\partial U(c_{ht}, L_{ht})}{\partial L_{ht}} \frac{1}{H_{ht}} = \lambda_t \frac{\partial F(K_{ht}, y_{ht}^{l})}{\partial y_{ht}^{l}} + \frac{\partial U^{*}(c_{ht}, L_{ht})}{\partial L_{ht}} \frac{\mu}{H_{Ht}}. \tag{A.2}
$$

Other than initial period, the human capital of high ability agent is higher than low ability agent, obviously, facing the same effective labor, high ability will work less and enjoy more leisure, then we get $\frac{\partial U(c_{ht}, L_{ht}}{\partial L_{ht}} \frac{1-\mu}{H_{ht}} < \lambda_t \frac{\partial F(K_{ht}, y_{ht}^{l}}{\partial y_{ht}^{l}}$.According to the optimal condition of c_{ht}, we have the (2.27).

With considering the optimal condition of x_{h1}, while $U_{L_{h2}}^{*} < U_{L_{h2}}, U_{L_{h3}}^{*} < U_{L_{h3}}$, and as for human capital accumulation, with the same inputs, the low ability agent's marginal production of physical capital and labor is higher than high ability agent, it is easy to get (2.31) and (2.32).

APPENDIX B

APPENDIX OF SECTION 3

Proof of Proposition 1

Suppose there exists some $K_t < K_t^*$ such that $TV(K_t) = TV(K_t^*)$.

Let $(x^*, h^*, K^*, q^*, e^*)$ be the solution of social planner's problem $SP(K_t)$, which means that it satisfies the feasible conditions and the incentive-compatible condition. Since $K_t < K_t^*$ and $TV(K_t) = TV(K_t^*)$, $(x^*, h^*, K^*, q^*, e^*)$ also satisfies the constraints of social planner's problem $SP(K_t^*)$, then it is also the solution of $SP(K_t^*)$.

While $(x^*, h^*, K^*, q^*, e^*)$ satisfies the feasible conditions, then for all $\theta^t \in \Theta^t$, we have:

$$\int x_t^* d\mu(\theta^t) + K_{t+1}^* + G_t^* \leq F(K_t, \int h_t^* d\mu(\theta^t)) + (1-\delta)K_t < F(K_t^*, \int h_t^* d\mu(\theta^t)) + (1-\delta)K_t^*.$$

For any $\epsilon > 0$, let $\widetilde{x}_t = x_t^* + \frac{\epsilon}{U'(x_t^*)}$, we have:

$$\int \widetilde{x}_t d\mu(\theta^t) + K_{t+1}^* + G_t^* < F(K_t^*, \int h_t^* d\mu(\theta^t)) + (1-\delta)K_t^*.$$

which means that the feasible condition will always hold as long as ϵ is small enough. Therefore, $(\widetilde{x}_t, h^*, K^*, q^*, e^*)$ satisfies the feasible conditions of $SP(K_t^*)$.

Meanwhile, for all $\theta^t \in \Theta^t$, we get:

$$U(\widetilde{x}_t) - \frac{h_t^*}{\theta_t} + \sigma(u(q_t^*) - \frac{e_t^*}{\theta_t}) = U(x_t^*) + \epsilon - \frac{h_t^*}{\theta_t} + \sigma(u(q_t^*) - \frac{e_t^*}{\theta_t}) > U(x_t^*) - \frac{h_t^*}{\theta_t} + \sigma(u(q_t^*) - \frac{e_t^*}{\theta_t}),$$

and

$$\widetilde{W}(\xi^*(\theta^t); , h^*, q^*, e^*) > \widetilde{W}(\xi^*(\theta^t); x^*, h^*, q^*, e^*).$$

While $(x^*, h^*, K^*, q^*, e^*)$ satisfies the incentive-compatible condition of $SP(K_t^*)$, then $\widetilde{W}(\xi^*(\theta^t); x^*, h^*, q^*, e^*) \geq \widetilde{W}(\xi; x^*, h^*, q^*, e^*)$ for any $\xi \in \Theta^t$, hence, $(\widetilde{x}_t, h^*, K^*, q^*, e^*)$ also satisfies the incentive-compatible condition of $SP(K_t^*)$.

Thus,$(x^*, h^*, K^*, q^*, e^*)$ could not be the solution of social planner's problem $SP(K_t^*)$, which means that the assumption could not be true. So, $TV(K_t) < TV(K_t^*)$ for all $K_t < K_t^*$.

Proof of Proposition 2

Based on Proposition 1, as the function $TV(K_t)$ is strictly increasing, we are able to rewrite the maximization problem as minimization problem. In other words, we need to solve the problem by minimizing the resources used in period t, which means that the solution $(x^*, h^*, K^*, q^*, e^*)$ of the problem should provide the same objective value and use less initial resource than any other packages.

Suppose that there is a two-period deviation from $(x^*, h^*, K^*, q^*, e^*)$ which guarantees that all individuals have the same utility level as does $(x^*, h^*, K^*, q^*, e^*)$. Moreover, the deviation all satisfies the feasibility condition and is incentive-compatible. We can express the problem as follows:

$$\min_{\psi_t, \varsigma_t, \varphi_t, \psi_{t+1}} \int \psi_t d\mu(\theta^t) + \varsigma_t$$

subject to the feasible conditions in the night of period t:

$$\int (q_t^* + \varphi_t) d\mu(\theta^t) = f(\int e_t^* d\mu(\theta^t), K_{t+1}^* + \varsigma_t), \tag{B.1}$$

and in the day of period $t+1$ is:

$$\int (x^*_{t+1} + \psi_{t+1}) d\mu(\theta^{t+1}) + K^*_{t+2} + G_{t+1} = F(K^*_{t+1} + \varsigma_t, \int h_t d\mu(\theta^{t+1})) + (1-\delta)(K^*_{t+1} + \varsigma_t). \tag{B.2}$$

and all individuals have the same utility level, which means:

$$\begin{aligned} U(x^*_t + \psi_t) - \frac{h^*_t}{\theta_t} + \sigma(u(q^*_t + \varphi_t) - \frac{e^*_t}{\theta_t}) + \beta[U(x^*_{t+1} + \psi_{t+1}) - \frac{h^*_{t+1}}{\theta_{t+1}} \\ + \sigma(u(q^*_{t+1}) - \frac{e^*_{t+1}}{\theta_{t+1}})] = U(x^*_t) - \frac{h^*_t}{\theta_t} + \sigma(u(q^*_t) - \frac{e^*_t}{\theta_t}) \\ + \beta[U(x^*_{t+1}) - \frac{h^*_{t+1}}{\theta_{t+1}} + \sigma(u(q^*_{t+1}) - \frac{e^*_{t+1}}{\theta_{t+1}})]. \end{aligned} \tag{B.3}$$

Since the feasible conditions for $(x^*, h^*, K^*, q^*, e^*)$ in the night of period t:

$$\int q^*_t d\mu(\theta^t) = f(\int e^*_t d\mu(\theta^t), K^*_{t+1}),$$

and in the day of period $t+1$

$$\int x^*_{t+1} d\mu(\theta^{t+1}) + K^*_{t+2} + G_{t+1} = F(K^*_{t+1}, \int h_t d\mu(\theta^{t+1})) + (1-\delta)(K^*_{t+1}),$$

combing with the equation (B.1) and (B.2), we can rewrite the feasible conditions as:

$$\int \varphi_t d\mu(\theta^t) = f(\int e^*_t d\mu(\theta^t), K^*_{t+1} + \varsigma_t) - f(\int e^*_t d\mu(\theta^t), K^*_{t+1}), \tag{B.4}$$

$$\int \psi_{t+1} d\mu(\theta^{t+1}) = F(K^*_{t+1} + \varsigma_t, \int h_t d\mu(\theta^{t+1})) - F(K^*_{t+1}, \int h_t d\mu(\theta^{t+1})) + (1-\delta)\varsigma_t. \tag{B.5}$$

Since $(x^*, h^*, K^*, q^*, e^*)$ is the solution of social planner's problem, we conjecture that $\psi_t = 0, \varsigma_t = 0, \varphi_t = 0, \psi_{t+1} = 0$ is the solution of the above problem.

Suppose not, and the element $\psi_t, \varsigma_t, \varphi_t, \psi_{t+1}$ satisfy the constraints and can lead a negative value for the objective of the minimizing problem. Define the new allocation $(\widetilde{x}, h^*, \widetilde{K}, \widetilde{q}, e^*)$ as follows:

$$\widetilde{x} = x^*, \widetilde{K} = K^*, \widetilde{q} = q^*$$

for all the other periods, except that

$$\begin{aligned}
\widetilde{x}_t &= x_t^* + \psi_t, \\
\widetilde{x}_{t+1} &= x_{t+1}^* + \psi_{t+1}, \\
\widetilde{K}_{t+1} &= K_{t+1}^* + \varsigma_t, \\
\widetilde{q}_t &= q_t^* + \varphi_t,
\end{aligned}$$

If the new allocation $(\widetilde{x}, h^*, \widetilde{K}, \widetilde{q}, e^*)$ satisfies the feasible conditions and incentive-compatible condition, and can guarantee the same utility level, but uses less resources, we can find out the contradiction. In the following, we need to show the allocation is feasible and incentive-compatible separately.

At period t, in the CM, as $\int \psi_t d\mu(\theta^t) + \varsigma_t < 0$, we can have

$$\begin{aligned}
\int \widetilde{x}_t d\mu(\theta^t) + G_t + \widetilde{K}_{t+1} &= \int (x_t^* + \psi_t) d\mu(\theta^t) + G_t + K_{t+1}^* + \varsigma_t \\
&< \int x_t^* d\mu(\theta^t) + G_t + K_{t+1}^*.
\end{aligned} \tag{B.6}$$

Obviously, the allocation $(\widetilde{x}, h^*, \widetilde{K}, \widetilde{q}, e^*)$ is feasible.

Moreover, based on the equation (B.3), for all individuals and any period t, the total utility is:

$$
\begin{aligned}
\widetilde{W}(\xi;x,h,q,e) &= E_t\sum_{i=0}^{\infty}\beta^i\{U(x_{t+i}(\xi)) - \frac{h_{t+i}(\xi)}{\theta_{t+i}} + \sigma(u(q_{t+i}(\xi)) - \frac{e_{t+i}(\xi)}{\theta_{t+i}})\} \\
&= E_t[U(\widetilde{x}_t) - \frac{h_t^*}{\theta_t} + \sigma(u(\widetilde{q}_t) - \frac{e_t^*}{\theta_t}) + \beta[U(\widetilde{x}_{t+1}) - \frac{h_{t+1}^*}{\theta_{t+1}} \\
&+ \sigma(u(q_{t+1}^*) - \frac{e_{t+1}^*}{\theta_{t+1}})] + \sum_{i=2}^{\infty}\beta^i\{U(x_{t+i}^*(\xi)) - \frac{h_{t+i}^*(\xi)}{\theta_{t+i}} \\
&+ \sigma(u(q_{t+i}^*(\xi)) - \frac{e_{t+i}^*(\xi)}{\theta_{t+i}})\}] \\
&= E_t[U(x_t^*) - \frac{h_t^*}{\theta_t} + \sigma(u(q_t^*) - \frac{e_t^*}{\theta_t}) + \beta[U(x_{t+1}^*) - \frac{h_{t+1}^*}{\theta_{t+1}} \\
&+ \sigma(u(q_{t+1}^*) - \frac{e_{t+1}^*}{\theta_{t+1}})] + \sum_{i=2}^{\infty}\beta^i\{U(x_{t+i}^*(\xi)) - \frac{h_{t+i}^*(\xi)}{\theta_{t+i}} \\
&+ \sigma(u(q_{t+i}^*(\xi)) - \frac{e_{t+i}^*(\xi)}{\theta_{t+i}})\}] \\
&= E_t[\sum_{i=0}^{\infty}\beta^i\{U(x_{t+i}^*(\xi)) - \frac{h_{t+i}^*(\xi)}{\theta_{t+i}} + \sigma(u(q_{t+i}^*(\xi)) - \frac{e_{t+i}^*(\xi)}{\theta_{t+i}})\}].
\end{aligned} \tag{B.7}
$$

All individuals will have the same utility with the new allocation $(\widetilde{x}, h^*, \widetilde{K}, \widetilde{q}, e^*)$. Therefore, it also satisfies incentive-compatible since $(x^*, h^*, K^*, q^*, e^*)$ does.

The new allocation $(\widetilde{x}, h^*, \widetilde{K}, \widetilde{q}, e^*)$ is feasible and incentive-compatible, and can achieve the same utility. More importantly, it uses less resource. However, it violates the Proposition 1. Hence, $\psi_t = 0, \varsigma_t = 0, \varphi_t = 0, \psi_{t+1} = 0$ must be the solution of the minimization problem.

Right now, other than solving the social problem problem $SP(K_t)$, we can get the first-order condition by solving the above minimization problem. Respectively, let Ψ_t and Ξ_t be the Lagrangian multiplier of equation (B.4) and (B.5). We can write the Lagrangian function as follows:

$$\begin{aligned} L(\psi_t, \varsigma_t, \varphi_t) = & \int \psi_t d\mu(\theta^t) + \varsigma_t \\ & + \Psi_t[\int \varphi_t d\mu(\theta^t) - f(\int e_t^* d\mu(\theta^t), K_{t+1}^* + \varsigma_t) + f(\int e_t^* d\mu(\theta^t), K_{t+1}^*)] \\ & + \Xi_t[\int E_t(U^{-1}(\frac{U(x_t^*) + \sigma u(q_t^*) + \beta U(x_{t+1}^*) - U(x_t^* + \psi_t) - \sigma u(q_t^* + \varphi_t)}{\beta}) \\ & - x_{t+1}^*) d\mu(\theta^t) - F(K_{t+1}^* + \varsigma_t, \int h_{t+1} d\mu(\theta^{t+1})) + F(K_{t+1}^*, \int h_{t+1} d\mu(\theta^{t+1})) \\ & - (1-\delta)\varsigma_t]. \end{aligned} \tag{B.8}$$

Since $(\psi_t, \varsigma_t, \varphi_t) = 0$ is the solution, we can have the following conditions:

$$1 = \Psi_t^* f_K(\int e_t^* d\mu(\theta^t), K_{t+1}^*) + \Xi_t^*[F_K(K_{t+1}^*, \int h_{t+1} d\mu(\theta^{t+1})) + (1-\delta)], \tag{B.9}$$

$$\Psi_t^* = \Xi_t^* E_t \frac{\sigma u'(q_t^*)}{\beta U'(x_{t+1}^*)}, \tag{B.10}$$

$$1 = \Xi_t^* E_t \frac{U'(x_t^*)}{\beta U'(x_{t+1}^*)}. \tag{B.11}$$

Combing the above three equations, we could have:

$$E_t \frac{1}{\beta U'(x_{t+1}^*)} = \frac{F_K(t+1) + 1 - \delta}{U'(x_t^*) - \sigma u'(q_t^*) f_K(t)}.$$

APPENDIX C

APPENDIX OF SECTION 4

Proof of Proposition 4

Replacing the law of motion for net worth (4.12) into equation (4.14), the profit maximization problem by a representative bank is given by

$$N_t = E_t \sum_{j=0}^{\infty} (1-\theta)\theta^j \beta^j \Lambda_{t,t+j}[(R^l_{t+j} - R^d_{t+j})d^s_{t+j} + R^d_{t+j} n_{t+j}] \tag{C.1}$$

$$s.t. N_t \geq \varphi d^s_t,$$

where μ_t is the Lagrange multiplier associated with the incentive compatibility constraint. By using the Lagrangian, the first oder conditions are given by

$$(1-\theta)\theta\beta\Lambda_{t,t+1}(R^l_{t+j} - R^d_{t+j}) - \mu_t \varphi = 0. \tag{C.2}$$

$$\mu_t(N_t - \varphi d^s_t) = 0, \mu_t \geq 0. \tag{C.3}$$

Since, by assumption, the premium $\beta^j \Lambda_{t,t+j}(R^l_{t+j} - R^d_{t+j})$ is positive in any period, from equation (C.2), we know that $\mu_t > 0$, thus the incentive compatibility constraint will hold with equality, which means

$$N_t = \varphi d^s_t. \tag{C.4}$$

Now, let's write the N_t in a recursive form and define:

$$\begin{aligned}\nu_t d_t^s &= E_t \sum_{j=0}^{\infty} (1-\theta)\theta^j \beta^j \Lambda_{t,t+j}[(R_{t+j}^l - R_{t+j}^d) d_{t+j}^s], \\ \eta_t n_t &= E_t \sum_{j=0}^{\infty} (1-\theta)\theta^j \beta^j \Lambda_{t,t+j}[R_{t+j}^d n_{t+j}].\end{aligned} \tag{C.5}$$

Then,

$$\begin{aligned}\nu_t &= (1-\theta)(R_t^l - R_t^d) + E_t \sum_{j=1}^{\infty} (1-\theta)\theta^j \beta^j \Lambda_{t,t+j}[(R_{t+j}^l - R_{t+j}^d) \frac{d_{t+j}^s}{d_t^s}], \\ \eta_t &= (1-\theta) R_t^d + E_t \sum_{j=1}^{\infty} (1-\theta)\theta^j \beta^j \Lambda_{t,t+j}[R_{t+j}^d \frac{n_{t+j}}{n_t}].\end{aligned} \tag{C.6}$$

Update the above equation one period further, we could have:

$$\begin{aligned}\nu_t &= (1-\theta)(R_t^l - R_t^d) + E_t[\theta \beta \Lambda_{t,t+1} x_{t,t+1} \upsilon_{t+1}], \\ \eta_t &= (1-\theta) R_t^d + E_t[\theta \beta \Lambda_{t,t+1} z_{t,t+1} \eta_{t+1}],\end{aligned} \tag{C.7}$$

where $x_{t,t+j} = \frac{d_{t+j}^s}{d_t^s}$ and $z_{t,t+j} = \frac{n_{t+j}}{n_t}$.

The firm's optimization problem

Replace the detail expressions of related variables into the entrepreneur i's value function (4.28). Let $\chi_t(i)$ be the Lagrange multiplier of collateral constraint when the firm can not re-optimize the price level, and $\chi_t^*(i)$ is the Lagrange multiplier associated with collateral constraint when the firm can re-optimize the price level. The first-order conditions are

$$L_t(i) : [\frac{P_{t-1}(i) Y_{Lt}(i)}{P_t} - \frac{R_t^l W_t}{P_t}][1 + \beta^e E_t \frac{V_M(t+1)}{1+\pi_{t+1}}] = \chi_t(i) R_t^l W_t, \tag{C.8}$$

$$L_t^*(i) : [\frac{(1-\frac{1}{\varepsilon})P_t^*(i)Y_{Lt}^*(i)}{P_t} - \frac{R_t^l W_t}{P_t}][1+\beta^e E_t \frac{V_M^*(t+1)}{1+\pi_{t+1}}] = \chi_t^*(i) R_t^l W_t, \tag{C.9}$$

$$\begin{aligned} K_{t+1}(i) : -R_t^l \frac{\partial I_t(i)}{\partial K_{t+1}(i)} + \beta^e E_t [\frac{V_K(t+1)}{P_{t+1}} - \frac{V_M(t+1)R_t^l}{1+\pi_{t+1}} \frac{\partial I_t(i)}{\partial K_{t+1}(i)}] \\ = \chi_t(i)[R_t^l \frac{\partial I_t(i)}{\partial K_{t+1}(i)} - \psi_t(i) E_t Q_{t+1}(i)], \end{aligned} \tag{C.10}$$

$$\begin{aligned} K_{t+1}^*(i) : -R_t^l \frac{\partial I_t^*(i)}{\partial K_{t+1}^*(i)} + \beta^e E_t [\frac{V_K^*(t+1)}{P_{t+1}} - \frac{V_M^*(t+1)R_t^l}{1+\pi_{t+1}} \frac{\partial I_t^*(i)}{\partial K_{t+1}^*(i)}] \\ = \chi_t^*(i)[R_t^l \frac{\partial I_t^*(i)}{\partial K_{t+1}^*(i)} - \psi_t(i) E_t Q_{t+1}^*(i)], \end{aligned} \tag{C.11}$$

$$P_t^*(i) : \frac{(1-\epsilon)Y_t^*(i)}{P_t}[1+\beta^e E_t \frac{V_M^*(t+1)}{P_{t+1}}] + \beta^e E_t \frac{V_P(t+1)}{P_{t+1}} = 0. \tag{C.12}$$

By using the Envelop Theorem, we have:

$$\begin{aligned} K_t(i) : & \frac{V_K(t)}{P_t} = \omega[(\frac{P_{t-1}(i)Y_{Kt}(i)}{P_t} - -\frac{\partial I_t(i)}{\partial K_t(i)} R_t^l)(1+\beta^e E_t \frac{V_M(t+1)}{1+\pi_{t+1}}) \\ & + \beta^e E_t \frac{V_K(t+1)(1-\delta)}{P_{t+1}}] + (1-\omega)[(\frac{(1-\varepsilon)P_t^*(i)Y_{Kt}^*(i)}{P_t} \\ & - \frac{\partial I_t*(i)}{\partial K_t(i)} R_t^l)(1+\beta^e E_t \frac{V_M^*(t+1)}{1+\pi_{t+1}}) + \beta^e E_t \frac{V_K^*(t+1)(1-\delta)}{P_{t+1}}] \\ & - [\omega \chi_t(i) \psi_t(i) \frac{\partial I_t(i)}{\partial K_t(i)} R_t^l + (1-\omega)\chi_t(i)^* \psi_t^*(i) \frac{\partial I_t*(i)}{\partial K_t(i)} R_t^l], \end{aligned} \tag{C.13}$$

$$\begin{aligned} M_t(i) : \frac{V_M(t)}{P_t} = \omega[\frac{R_t^l}{P_t} + \beta^e E_t \frac{V_M(t+1)R_t^l}{P_{t+1}} + \frac{\chi_t(i)R_t^l}{P_t}] \\ +(1-\omega)[\frac{R_t^l}{P_t} + \beta^e E_t \frac{V_M^*(t+1)R_t^l}{P_{t+1}} + \frac{\chi_t^*(i)R_t^l}{P_t}], \end{aligned} \tag{C.14}$$

$$P_{t-1}(i) : \frac{V_P(t)}{P_t} = \omega[\frac{Y_t(i)}{P_t} + \beta^e E_t \frac{V_M(t+1)Y_t(i) + V_P(t+1)}{P_{t+1}}]. \tag{C.15}$$

Here, $V_x(t)$ denotes the partial deviation of the value function with respect to x at period t, $V_x^*(t)$ denotes the partial deviation of the value function when the firm can optimally choose the price level, $Y_{Lt}(i)$ is the marginal labor production, $Y_{Kt}(i)$ is the marginal capital production.

Since $\chi_t(i) \geq 0$ and $\chi_t^*(i) \geq 0$, within equation (C.8) and (C.9), we could have

$$\begin{aligned} \frac{P_{t-1}(i)Y_{Lt}(i)}{P_t} &\geq \frac{R_t^l W_t}{P_t}, \\ \frac{(1-\frac{1}{\varepsilon})P_t^*(i)Y_{Lt}^*(i)}{P_t} &\geq \frac{R_t^l W_t}{P_t}. \end{aligned} \tag{C.16}$$

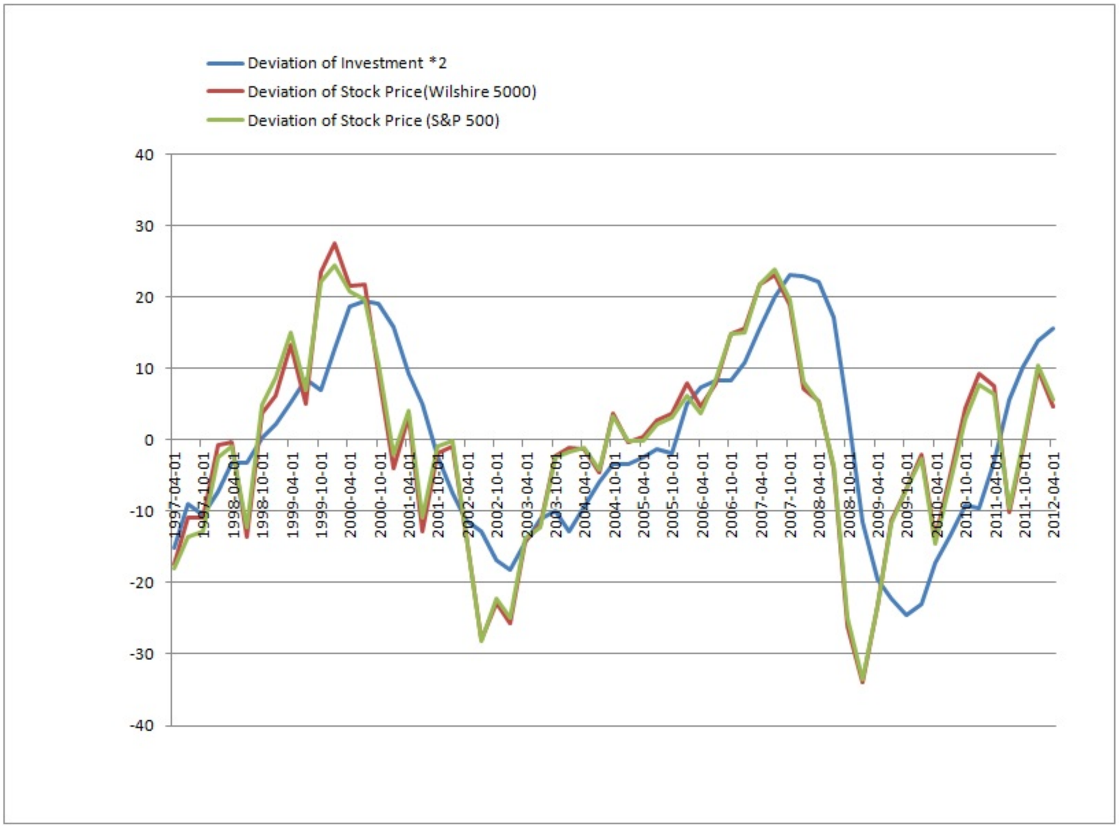

Figure C.1: Stock Price and Investment's Deviation from Trend (%).

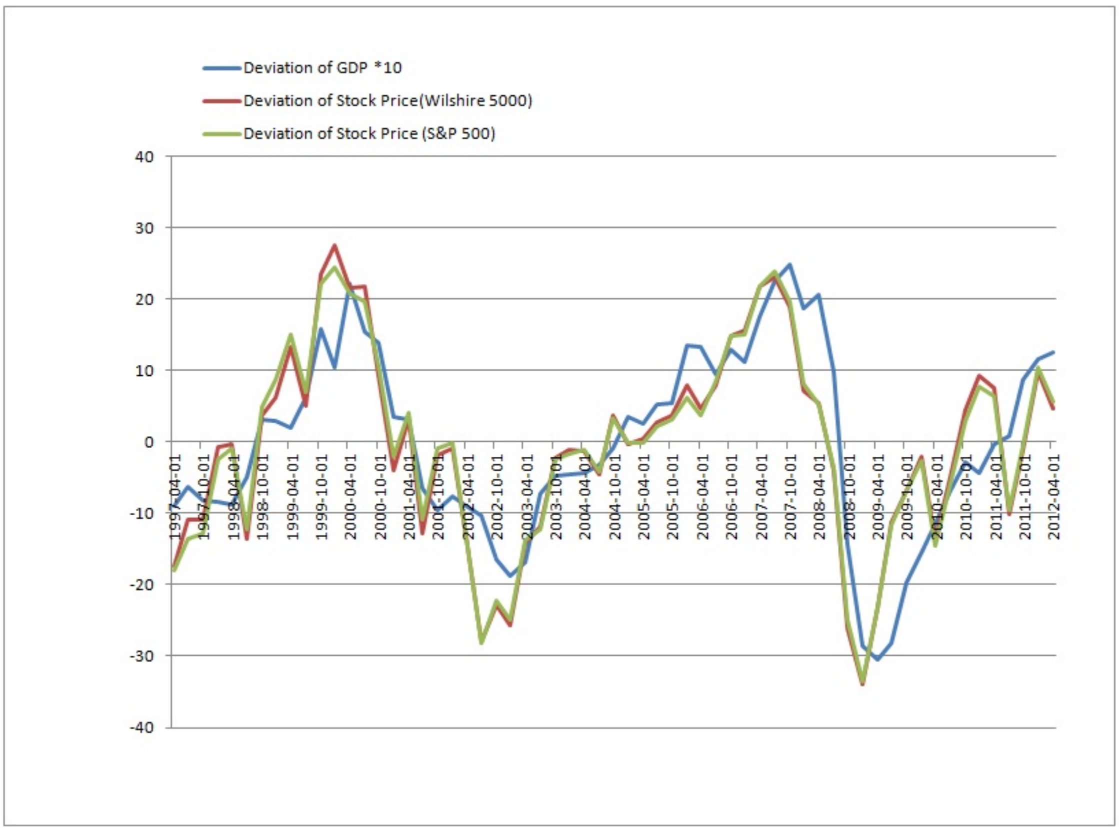

Figure C.2: Stock Price and GDP's Deviation from Trend (%).

	Preferences	
β	0.99	Discount rate
Ψ	1.63	Relative utility weight of labor
ε	11	Elasticity of substitution
	Financial Intermediaries	
φ	0.381	Fraction of capital that can be diverted
θ	0.972	Survival rate of bankers
ϵ	0.002	Proportional transfer to the new entering bankers
	Non-financial Firms	
α	0.36	Capital share in production function
ω	0.779	Probability of keeping prices fixed
δ	0.025	Depreciation rate
ξ	3	Elasticity of the investment to capital ratio
	Exogenous Process	
ρ	0.979	Quarterly persistence of log collateral ratio shocks
$\bar{\psi}$	0.75	The average value of collateral ratio
σ_ψ	0.0126	Standard deviation of log collateral ratio shocks
ρ_a	0.95	Quarterly persistence of log TFP process
σ_a	0.0038	Standard deviation of log TFP shock
σ_φ	0.001713	Standard deviation of bankers' diverting ratio shock

Table C.1: Parameters

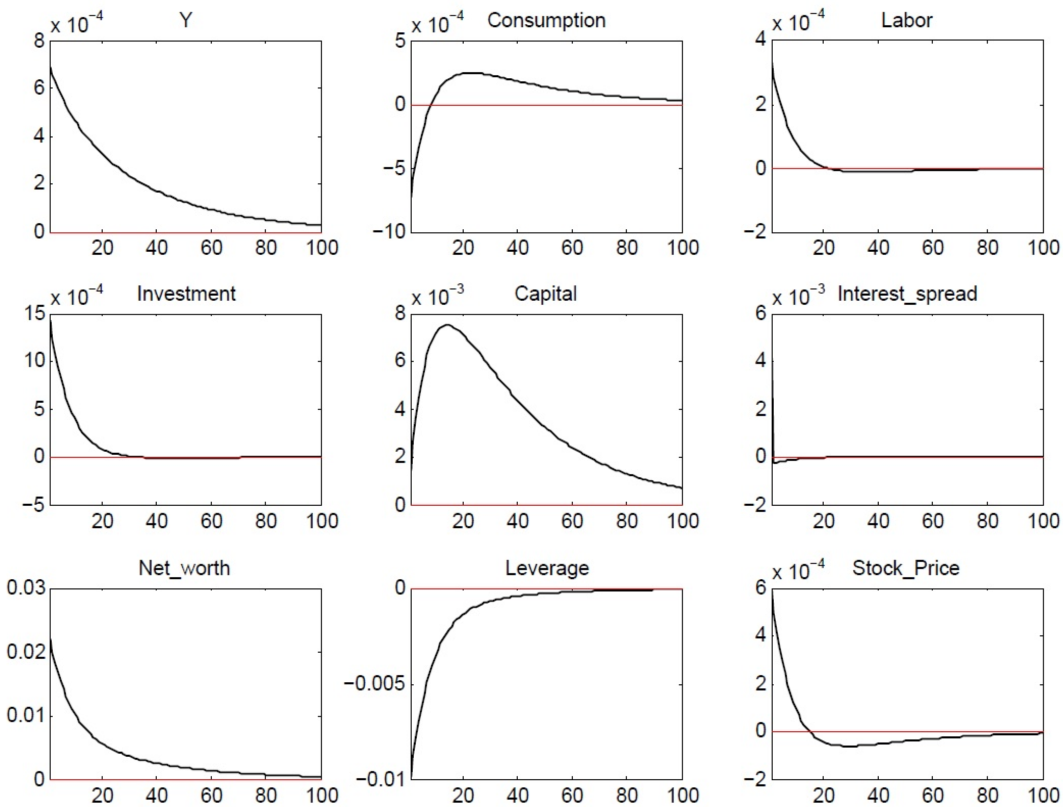

Figure C.3: Impulse Responses to a Negative Collateral Shock: Deviation from Steady-state.

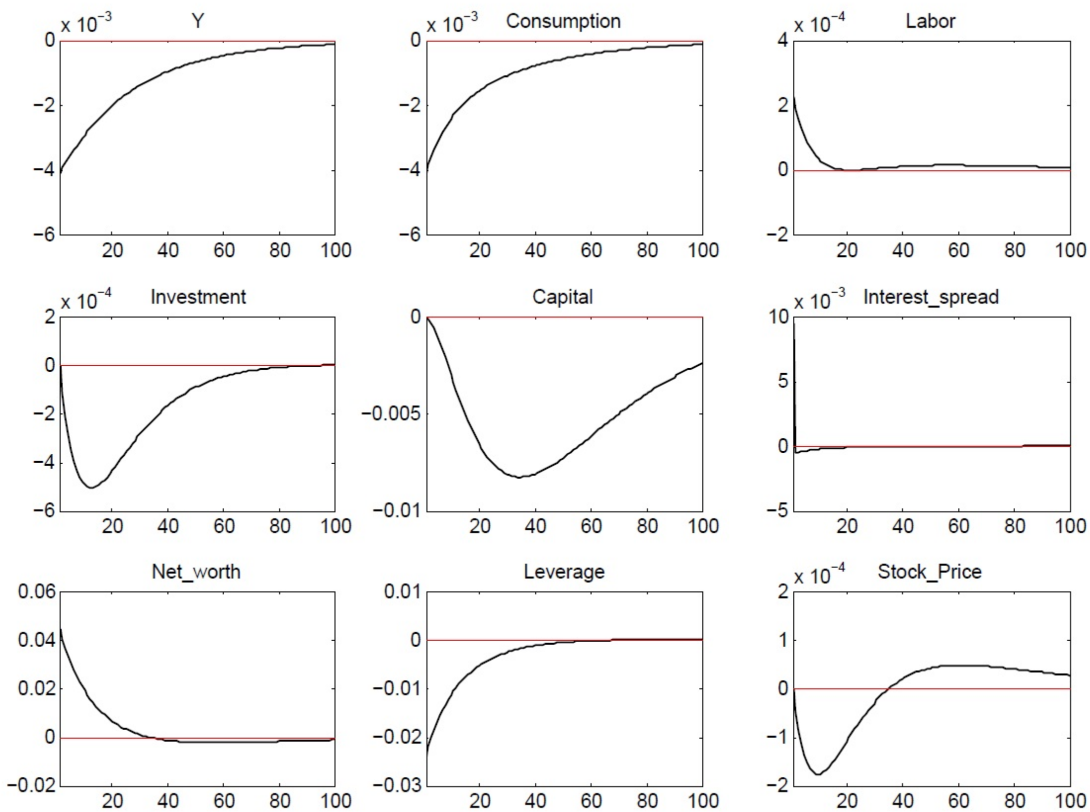

Figure C.4: Impulse Responses to a Negative Technology Shock: Deviation from Steady-state.

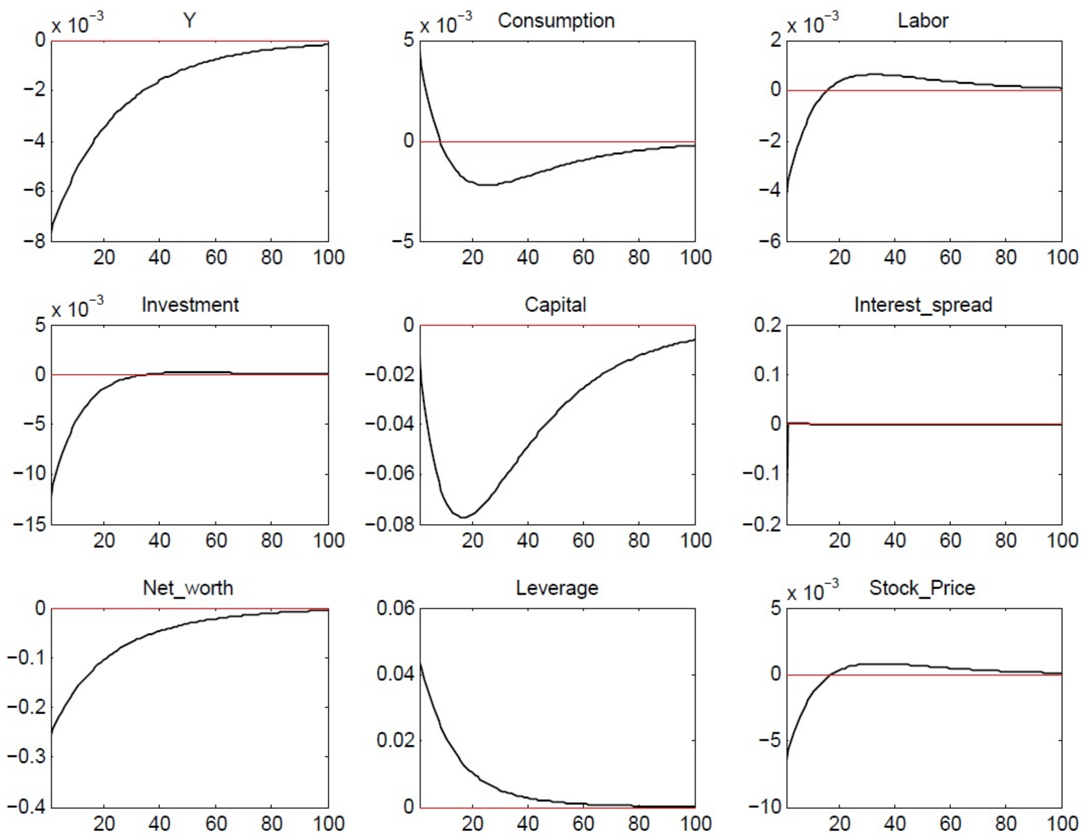

Figure C.5: Impulse Responses to a Positive Diverting Shock: Deviation from Steady-state.