

APPLICATION OF THE ENSEMBLE KALMAN FILTER TO ESTIMATE
FRACTURE PARAMETERS IN UNCONVENTIONAL HORIZONTAL WELLS BY
DOWNHOLE TEMPERATURE MEASUREMENTS

A Thesis

by

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Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

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August 2013

Major Subject: Petroleum Engineering

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ABSTRACT

The increase in energy demand throughout the world has forced the oil industry to develop and expand on current technologies to optimize well productivity. Distributed temperature sensing has become a current and fairly inexpensive way to monitor performance in hydraulic fractured wells in real time by the aid of fiber optic. However, no applications have yet been attempted to describe or estimate the fracture parameters using distributed temperature sensing as the observation parameter. The Ensemble Kalman Filter, a recursive filter, has proved to be an effective tool in the application of inverse problems to determine parameters of non-linear models. Even though large amounts of data are acquired as the information used to apply an estimation, the Ensemble Kalman Filter effectively minimizes the time of operation by only using “snapshots” of the ensembles collected by various simulations where the estimation is updated continuously to be calibrated by comparing it to a reference model.

A reservoir model using ECLIPSE is constructed that measures temperature throughout the wellbore. This model is a hybrid representation of what distributed temperature sensing measures in real-time throughout the wellbore. Reservoir and fracture parameters are selected in this model with similar properties and values to an unconventional well. However, certain parameters such as fracture width are manipulated to significantly diminish the computation time.

A sensitivity study is performed for all the reservoir and fracture parameters in order to

understand which parameters require more or less data to allow the Ensemble Kalman Filter to arrive to an acceptable estimation. Two fracture parameters are selected based on their low sensitivity and importance in fracture design to perform the Ensemble Kalman Filter on various simulations.

Fracture permeability has very low sensitivity. However, when applying the estimation the Ensemble Kalman Filter arrives to an acceptable estimation. Similarly fracture half-length, with medium sensitivity, arrives to an acceptable estimation around the same number of integration steps. The true effectiveness of the Ensemble Kalman Filter is presented when both parameters are estimated jointly and arrive to an acceptable estimation without being computationally expensive. The effectiveness of the Ensemble Kalman Filter is directly connected to the quantity of data acquired. The more data available to run simulations, the better and faster the filter performs.

DEDICATION

To God, my parents, Luis and Miriam, my sister, Erika, my brothers Roberto, Mauricio and Michael, and my grandmothers Rosa and Elisa for their support and encouragement.

ACKNOWLEDGEMENTS

I would like to thank my committee chair, Dr. Gildin, and my committee members, Dr. Zhu, and Dr. Efendiev, for their guidance and support throughout the course of this research. Thanks also go to my friends and colleagues and the department faculty and staff for making my time at Texas A&M University a great experience.

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1. INTRODUCTION

1.1 BACKGROUND

Unconventional resources have taken an important role in the current energy demand scenario. However, specifically in the area of oil shales, in order to design more efficient, accurate and cost-effective hydraulic fracture jobs, there must be a better understanding of the relationships between reservoir and fracture parameters, and how they affect the performance throughout the life of a well. All unconventional reservoirs bring unique technical difficulties that must be addressed as part of the reservoir management plan at an early stage, but these are also closely related to the optimization aspect that is controlled during the life of the well (Miskimins 2009). The oil industry is quickly evolving into a field where technology innovation must meet the highest possible operation efficiency by minimizing capital investments and operation cost to maximize economic recovery of hydrocarbons from a reservoir. Hydraulic fracturing is an area in the oil industry where an in-depth understanding of flow behavior can significantly impact the life of a well and consequently increase its productivity. Speaking particularly in regards to hydraulic fracturing in horizontal wells, the focus of this study is based upon the fact that each individual hydraulic fracturing job can cost millions of dollars of capital investment. Therefore, in order to maximize economic recovery it is imperative to find new techniques to improve performance and obtain a better understanding of the behavior of hydraulically fractured horizontal wells.

A potential bridge for understanding horizontal wells at a deeper level is the use and application of fiber optic in the wellbore. Recent technologies, such as fiber-optic Distributed Temperature Sensing (DTS) have been implemented in the oil industry, providing an accurate real-time temperature reading from a hydraulic fractured well (Nath et al. 2008). Currently, DTS is used for fracture depth estimation, number of fractures detection and undesired flow monitoring behind casing (Sierra, J., et al 2008). Not only does DTS have the capability of working effectively in high-temperature scenarios but also it is highly sensitive to small temperature changes. Temperature measurements can be used to improve estimations in situations where lack of sufficient pressure or flow-rate data makes parameter estimation difficult or impossible (Duru and Horne 2011). Temperature data is more reliable than pressure data since the accuracy of the measurements is very high and it is not dependent on wellbore flow conditions. This provides a great aide for horizontal well analysis since the fluctuation of temperature throughout the horizontal section does not vary greatly; DTS accurately detects the small changes of temperature caused by inflow or injection and can display this information in real-time.

Due to various completion techniques used in well construction, an important aspect for DTS to work effectively is the installation setup. DTS can be installed inside the flow path or cemented behind the casing. Ideally cemented behind the casing, the recognition of well-defined temperature patterns is obtained and can be used to compare between the flow inside and behind the casing. This setup provides an out-of-zone fracture

assessment (Sierra, J., et al 2008). To this end, DTS could potentially be used to estimate accurately fracture parameters by means of inverse problem techniques as a way to history-match a well or the entire field.

Recently in the oil industry, inversion methods have been successfully applied in the interpretation of pressure, temperature, flowrate profiles (Duru and Horne 2011) using Bayesian methods and permeability fields (Li, Chen et al. 2012) using specifically the Ensemble Kalman Filter (EnKF). The addition of hydraulic fractures to a horizontal well increases the number of parameters (fracture half-length, fracture permeability and fracture height), which must be taken into account when applying an inverse method. The advantage of the EnKF is, being a Monte Carlo method where the covariance matrix is updated from a finite number of ensembles and where these ensembles are sets of parameters or observations from the model (Evensen 2007), it does not require analyzing the entire space of data; in this investigation the aforementioned parameters are related to the hydraulic fracture such as fracture permeability, fracture half-length and fracture width. Instead, it selects values from each ensemble created making it computationally cheaper compared to other methods. The EnKF is selected as the filter to be used for the inverse problem in this investigation. This filter is capable of handling large amounts of data and moderate nonlinearities (Aanonsen, Naevdal et al. 2009). The ability to estimate accurately hydraulic fracture parameters can provide petroleum engineers important information to improve the economic life of a hydraulically fractured horizontal well. The fast-paced increase in computational capabilities in the present day allows various

inverse problem techniques to be feasible for application in production engineering. While the EnKF might be a feasible route to run an inversion problem, the main challenges still remain, when dealing with strong nonlinearities in the forward model, application to large fields and the model errors that must be corrected by effectively updating the reservoir model when new information is provided throughout the life of the simulation and field (Aanonsen, Naevdal et al. 2009).

1.2 LITERATURE REVIEW

1.2.1 UNCONVENTIONAL RESOURCES

There are various descriptions of unconventional resources. However, in order to bridge the differences and provide an all-encompassing definition the “Society of Petroleum Engineers – Petroleum Resources Management System” (SPE-PRMS) defines unconventional resources as follows:

Unconventional resources exist in petroleum accumulations that are pervasive throughout a large area and that are not significantly affected by hydrodynamic influences (also called “continuous-type deposits”). Examples include coalbed methane (CBM), basin-centered gas, shale gas, gas hydrates, natural bitumen, and oil shale deposits. Typically, such accumulations require specialized extraction technology (e.g., dewatering of CBM, massive fracturing programs for shale gas, steam and/or solvent

to mobilize bitumen for in-situ recovery, and, in some cases, mining activities) Moreover; the extracted petroleum may require significant processing prior to sale (e.g., bitumen upgraders).

The focus of this study is to understand oil-shale formations and their productivity. Oil-shale formations are primarily made of carbonate rock, which has a very high content of organic sedimentary material known as kerogen. Oil-shales are also younger in geologic age than crude-oil bearing formations (Knaus, Killen et al. 2009). The permeability of a field varies throughout and in an unconventional oil-shale reservoir the values are very small; using the Bakken shale oil system as an example, the permeability ranges from 0.05 md to 0.5 md with very long horizontal hydraulically stimulated completions that range from 3,000 ft to 4,000 ft (Miskimins 2009) in order to make it economically viable to exploit. After horizontal wells are drilled, the ultimate goal maintaining the wellbore in the pay-zone is to create longitudinal hydraulic fractures that will increase the conductivity between the fracture and wellbore by maximizing the number of intersections. The stress states of the field must be well understood in order to effectively design the direction of the well and ultimately fractures as Figure 1.1 shows.

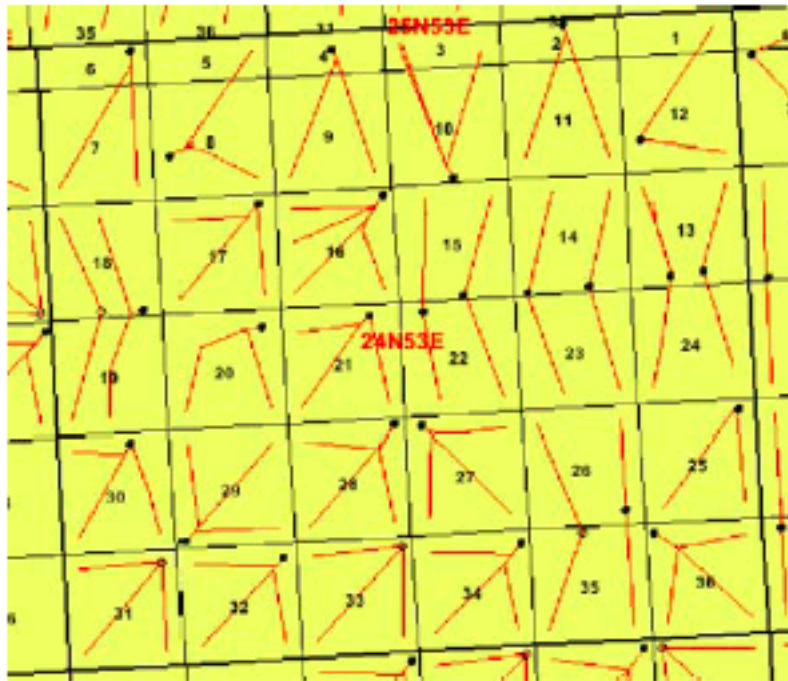


Figure 1.1 – Horizontal Wellbore Azimuths in the Bakken Field. Source:

“Miskimins, J. L. 2009”

Shale oil production in the U.S. is estimated to reach 500,000 Bbl/d by 2020 and it is expected to remain at that steady rate through 2035 (Biglarbigi, Killen et al. 2009). The current high demand for the development of domestic oil shale resources calls the attention for the need of improvement of recovery technologies. In order to increase further understanding of this type of unconventional resources the gap between production data analysis and geo-statistics will be bridged to gain a deeper understanding of the relationship between fracture parameters and wellbore temperature.

1.2.2 DISTRIBUTED TEMPERATURE SENSING

Horizontal wells with hydraulic-fracture treatments have proved to be an effective method for developing unconventional oil and gas reservoirs (Tabatabaei and Zhu 2012). Recently, the oil and gas industry has begun deploying extensively the use of DTS technology particularly in horizontal wells with hydraulic-fracture treatments because this technology allows real-time observation of the temperature profile throughout the wellbore. This allows the diagnosis of the estimation of fracture-initiation points, number of fractures created and distribution of stimulation fluid along each isolated zone (Tabatabaei and Zhu 2012), which will ultimately allow the operator to optimize the field and improve economics. DTS measures temperature distribution along the wellbore by the use of an optical fiber. A pulse of light is sent down the optic fiber, the returning light's properties are analyzed. The returning light, known as backscatter, contains three spectral components: Brillouin, Raman and Rayleigh bands. In order to obtain temperature information the Raman band is analyzed.

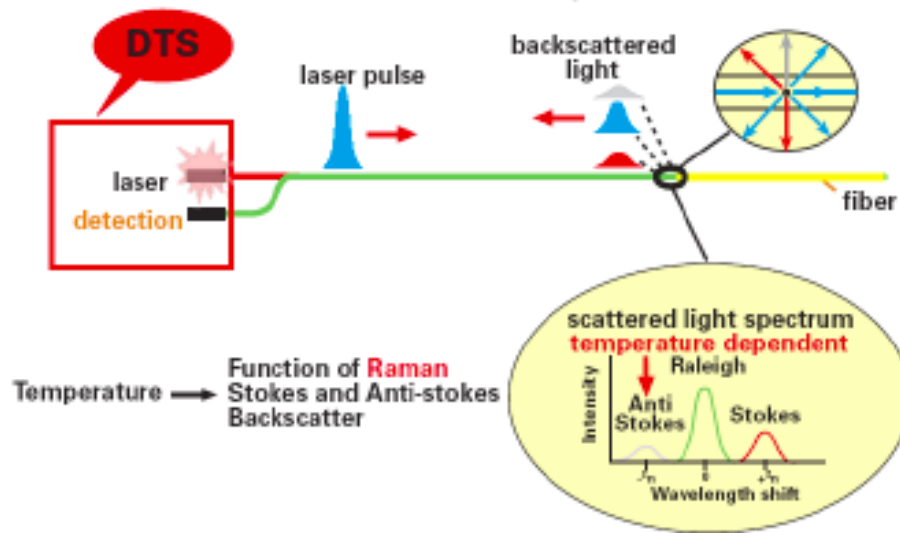


Figure 1.2 – Distributed Temperature Sensing Concept (Effect of Raman Bands).

Source: “Sierra, J., et al (2008)”

The Raman band itself has two components; Stokes and Anti-Stokes backscatter, Figure 1.2. Temperature is heavily influenced by Anti-Stokes backscatter and lightly influenced by Stokes backscatter (Sierra, J., et al 2008). Since the interest of this investigation is to use production data along with DTS temperature readings for an estimation of fracture parameters, the deployment of optic fiber behind the pipe fits the purpose because a longer period of production data is required to be analyzed and also the proximity of the DTS cable between the formation and wellbore provides the ideal location for real-time temperature reading.

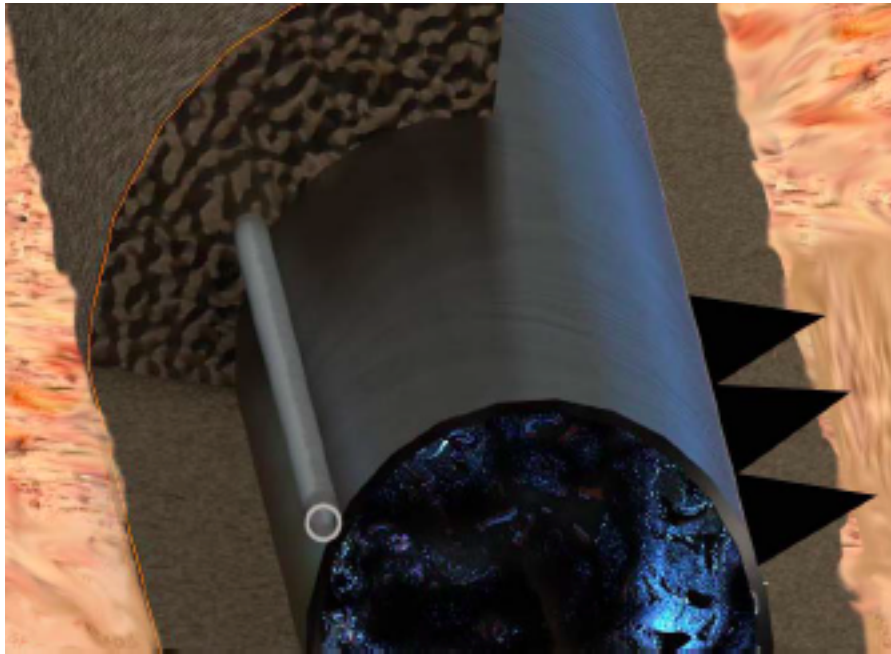


Figure 1.3 – Behind Casing DTS Cable. Source: “Sierra, J., et al (2008)”

Permanently installed behind-casing DTS cable is important for reservoir management throughout the life of the well, making DTS an essential tool for well optimization as seen in Figure 1.3.

1.2.3 KALMAN FILTER

The Kalman filter is a recurrent filter that estimates the state of a linear system from a series of noisy measurements. The key element for the filter to operate successfully is to have a model equation where the current state of the system is associated with an uncertainty and an observation equation that relates a linear combination of the states to measurements (Aanonsen, Naevdal et al. 2009). A key step in the Kalman Filter is the update of the propagated states of the system through a linear operation in which the

difference between field measurement and model output is multiplied by the so called Kalman gain of the system. The Kalman gain in turn, is computed by a series of matrix computations involving the covariance of the states and measurements. However, for large-scale systems, such as a reservoir field, the covariance matrix of the model uncertainty is too large and makes it computationally expensive and inefficient to apply inverse problem estimations. To this end, a variant of the Kalman Filter, called the Ensemble Kalman Filter (EnKF), as described in the next section, will be used for the large-scale computation in the reservoir simulations.

1.2.4 ENSEMBLE KALMAN FILTER

The goal is to approach the inverse problem using a stochastic method, which will avoid local minima problems while searching for an answer (Tarantola 2005). For this type of inverse problem the EnKF fits the ideal recursive filter to estimate the state of a non-linear dynamical system, such as the horizontal hydraulically fractured well, from a series of noisy measurements (Aanonsen, Naevdal et al. 2009) being the temperature and flowrates. The EnKF is ideal for forecasting since it is a simplification of a smoother solution for the non-linear problem with Gaussian statistics, which does not project information backwards in time (Evensen 2007). The application of large-scale systems to history-match reservoir production and temperature is the focus of this study. The EnKF can handle covariance matrices of uncertainty that are only partially propagated, significantly decreasing the computation time and improving in accuracy. Also, by increasing the quantity of quality data inputted into the system the EnKF performs more

efficiently (Tarantola 2005). The way to sample the various realizations obtained from running the model for a certain time is to select the first ensemble from singular vectors associated with the largest eigenvalues of the covariances (Aanonsen, Naevdal et al. 2009), however, the prior geological model may not be consistent with the property realizations. In previous research, EnKF has been successfully applied to estimate three-phase relative permeability from production data, however, the endpoint of permeability values must be known in order to arrive at the approximations within a reasonable time (Li, Chen et al. 2012), this emphasizes the importance of the a priori data obtained prior to running the inversion and the quality of data. All numerical models of physical phenomena contain errors; an improvement in the solution can be achieved by including error in the model and the analysis, which allows the estimated parameters to become more stable (Aanonsen 2009) when the EnKF is applied. Figure 1.4 depicts the flowchart for the EnKF update, where when observations are present the Kalman Gain is computed to obtain a new forecast. The forecasted values are compared to the model values and the process is iterated until the relative error between both is significantly minimal.

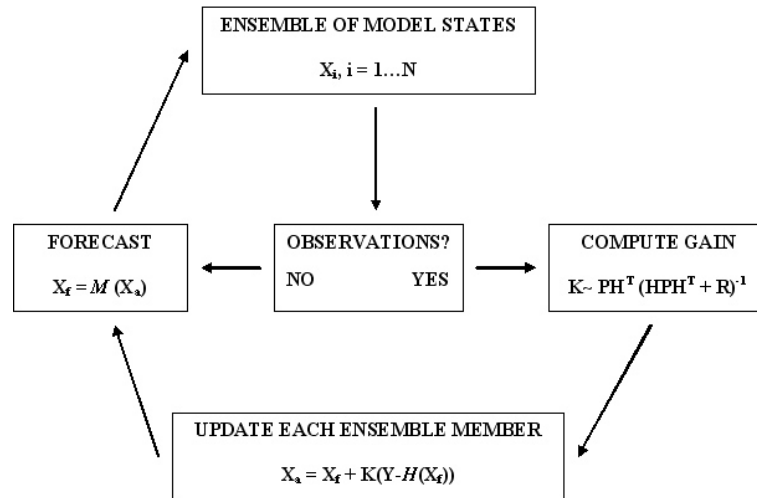


Figure 1.4 – EnKF Update Flowchart

1.3 OBJECTIVES AND APPROACH

The main objective of this study is to obtain an accurate estimation of the fracture parameters: fracture half-length, fracture permeability and fracture width from a hydraulically fractured oil well by the coupling of the EnKF and a forward model which calculates oil, gas, and water flowrates along with temperature profile readings throughout the wellbore mimicking DTS technology thus creating a hybrid model. The use of hybrid and actual DTS readings will aid in the implementation of the Ensemble Kalman Filter by using the aforementioned temperature readings as the observation of the efficient recursive filter.

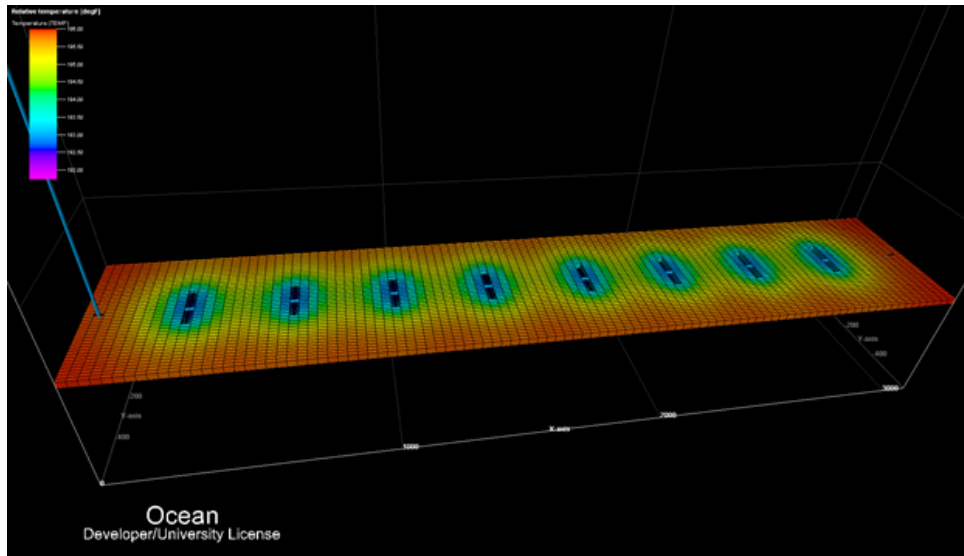


Figure 1.5 – Simulation of DTS in a Hydraulically Fractured Well

ECLIPSE software by Schlumberger is used to construct the forward simulator (reservoir model) that will have temperature readings as outputs. The reservoir has non-isothermal properties. The number of simulations to obtain an accurate approximation of the fracture parameters using the EnKF will be directly dependent on the accuracy of the temperature readings obtained from the DTS readings and from the flowrates obtained in the simulator. A sensitivity analysis will be performed to determine which parameters are most directly affected by the creation of fractures and how it is related to the temperature readings. An example is shown in Figure 1.5 of the DTS hybrid simulation.

2. TEMPERATURE MODEL

2.1 FORWARD MODEL OVERVIEW

The forward model, in this case the reservoir simulator, is constructed to model temperature readings throughout the entire reservoir based on pressure, rock and fluid properties. The implementation of the temperature model is used as a hybrid example to a real-life DTS reading. This hybrid model mimics DTS technology by approximating the temperature readings along the wellbore and passing through the hydraulic fractures. The hydraulic fractures themselves are locally refined to improve resolution and accuracy. In order to include the temperature approximation to the model, ECLIPSE reservoir simulation software by Schlumberger, offers the possibility to measure the temperature at each grid cellblock throughout time by the use of the temperature option. The model is executed with both, ECLIPSE 100 as a blackoil model and ECLIPSE 300 as a compositional model. Accuracy is significantly improved by running the simulation in ECLIPSE 300 where both, flow and energy conservation equations are calculated and solved simultaneously, however it takes more time to run the simulations. The specific heat capacity of the rock and the fluids are required for the temperature option. The rock and fluid properties within each grid block are assumed to be at the same temperature to facilitate the updated computations throughout the simulation. A simple rectangular shaped reservoir is used to perform the simulation for the sake of computational simplicity.

2.2 TEMPERATURE MODEL

The temperature model is created using a reservoir simulator with the temperature option in ECLIPSE, which allows the modeling of temperature effects based on rock and fluid properties as a function of pressure and time. The reservoir simulator is a numerical model that is divided into several discrete grid blocks in three dimensions and where the fluid and reservoir properties are solved through space and time in a series of discrete steps.

2.2.1 RESERVOIR AND WELLBORE MODEL

The following sections present how the simulator calculates and models the temperature throughout the wellbore.

Reservoir Flow Model

Each cell within the reservoir contains two equations that are solved at each time step.

The first equation is the derived form of Darcy's Law flow through a porous medium

$$q = -\frac{k}{\mu} \nabla P \dots\dots\dots(2.1)$$

where q is the flowrate, k is the permeability, μ is the fluid viscosity and P is pressure.

The second equation is the material balance, which takes into account the total accumulation and injection/production in relation to the total mass flux

$$-\nabla \cdot M = \frac{\partial}{\partial t}(\phi\rho) + q \dots\dots\dots(2.2)$$

where M is the mobility factor, ϕ is the porosity, ρ is the fluid density and q is the flowrate.

Combining Equation 2.1 and Equation 2.2 for each grid-block in the reservoir provides the simulator flow equations with gravity included as follows

$$\nabla \cdot [\lambda(\nabla P - \gamma\nabla Z)] = \frac{\partial}{\partial t}\left(\frac{q}{\rho}\right) \dots\dots\dots(2.3)$$

where λ is the aforementioned mobility term defined by Equation 2.4, P is pressure, γ is an average that takes into account the change of pressure over a height ∇Z for compressible fluids defined by Equation 2.5.

$$M = \lambda = \frac{k}{\mu\beta} \dots\dots\dots(2.4)$$

$$\gamma = \gamma_c \rho g \dots\dots\dots(2.5)$$

Well Model

The next part required to connect all the grid-blocks in the reservoir is the well model. The flowrate at any grid-block of the reservoir is determined by three aspects, the transmissibility, mobility and pressure change as described below in Equation 2.6.

$$q = TM(P_j - P_{wf} - P_{datum}) \dots \dots \dots (2.6)$$

Transmissibility is described by Equation 2.7.

$$T = \frac{c\theta kh}{\ln(r_o/r_w)+s} \dots \dots \dots (2.7)$$

The previous Equation 2.4 describes mobility.

A head pressure connection in the well is selected as the datum, in this case described by P_{datum} , P_j becomes the pressure at the selected grid-block and P_{wf} is the bottom hole pressure. ECLIPSE solves Equation 2.3 for every cell by the finite difference approach where the equation is discretized on a fixed grid.

Reservoir Thermal Model

Energy is transported from the reservoir in two ways, by the way of conduction and convection, when heat is transferred from the reservoir to the wellbore or vice versa. The energy conservation equation states that for each grid block the accumulation of energy is equal to the addition of rate of energy production plus the total rate of energy transported into the grid block Equation 2.8.

$$\left[\begin{array}{c} \text{Accumulation} \\ \text{rate of energy} \\ \text{in grid block} \end{array} \right] = \left[\begin{array}{c} \text{Net rate of} \\ \text{energy transport} \\ \text{into grid block} \end{array} \right] + \left[\begin{array}{c} \text{Rate of} \\ \text{energy production} \\ \text{in grid block} \end{array} \right] \dots \dots \dots (2.8)$$

Following previous work (Yoshida 2013), the next Equation 2.9 is the final energy balance that is developed in relation to the reservoir temperature for each grid block.

$$\begin{aligned}
 & - \left[\sum_{j=1}^{N_p} (\phi \rho_j S_j C_{p,j}) + (1 - \phi) \rho_s C_{p,s} \right] \frac{\partial T}{\partial t} + \sum_{j=1}^{N_p} \left(\phi S_j \beta_j T \frac{\partial p_j}{\partial t} \right) = \sum_{j=1}^{N_p} \rho_j \mathbf{u}_j \cdot C_{p,j} \nabla T - \\
 & \nabla \cdot (K_{Tt} \nabla T) + \sum_{j=1}^{N_p} \mathbf{u}_j \cdot \nabla p_j - \sum_{j=1}^{N_p} \beta_j T (\mathbf{u}_j \cdot \nabla p_j) + \sum_{j=1}^{N_p} \rho_j \mathbf{u}_j \cdot \nabla (gD) \dots \dots \dots (2.9)
 \end{aligned}$$

In Equation 2.9, H is the enthalpy of fluid, K_{Tt} is the total heat conductivity, A is the surface area of the grid block and $C_{p,j}$ is the heat capacity of the phase j , T_l is the reservoir temperature at the contact between the reservoir and wellbore and T_w is the wellbore flowing temperature. The first term describes the accumulation as temperature changes throughout time, the second term describes the thermal expansion as pressure changes throughout time, the third term is the convection transmitted, the fourth term is the conduction transmitted, the fifth term is the viscous dissipation heating, the sixth term is the thermal expansion and the last term is the potential energy.

The model was implemented and executed in two versions, ECLIPSE Blackoil and ECLIPSE Compositional. The ECLIPSE Blackoil model makes the general assumption where oil and gas phase are combined and represented as one component as the simulation moves throughout time. The properties are free to change as pressure and temperature vary; however the composition stays constant throughout time. The ECLIPSE Compositional model specifically takes into account the change of each

component in the oil and gas phases throughout time, making it more computationally expensive than the Blackoil model but more accurate if the fluids are near critical points where changes in pressure or temperature will affect the composition of the fluid.

The model is constructed as a horizontal well with 8 transverse fractures and temperature readings along the wellbore. The model parameters are displayed in Table 2.1 below.

Table 2.1 – Reference Model Parameters

Reservoir Configuration	100x40x30
Reservoir Size	3100(ft) x 600(ft) x 150(ft)
Initial Reservoir Pressure	6000 (psi)
Fracture Height	45 (ft)
Number of Fractures	8
Fracture Half Length	90 (ft)
Reservoir Permeability	0.2 (md)
Fracture Width	1 (ft)
Fracture Permeability	1000 (md)
Well Flowing Pressure	2000 (psi)

Fracture width contains a significantly larger value compared to what is usually measured or estimated, which is about less than an inch, the reason for this is to prevent very long simulations for each run due to the low permeability from the unconventional

reservoir. In regards to the accuracy of the calculations in the fracture, the use of Local Grid Refinement (LGR) is implemented as seen in Figure 2.1. A higher resolution in the hydraulic fracture is needed to accurately record significant changes in the variables of interest throughout time; in this case, the temperature change at the hydraulic fracture.

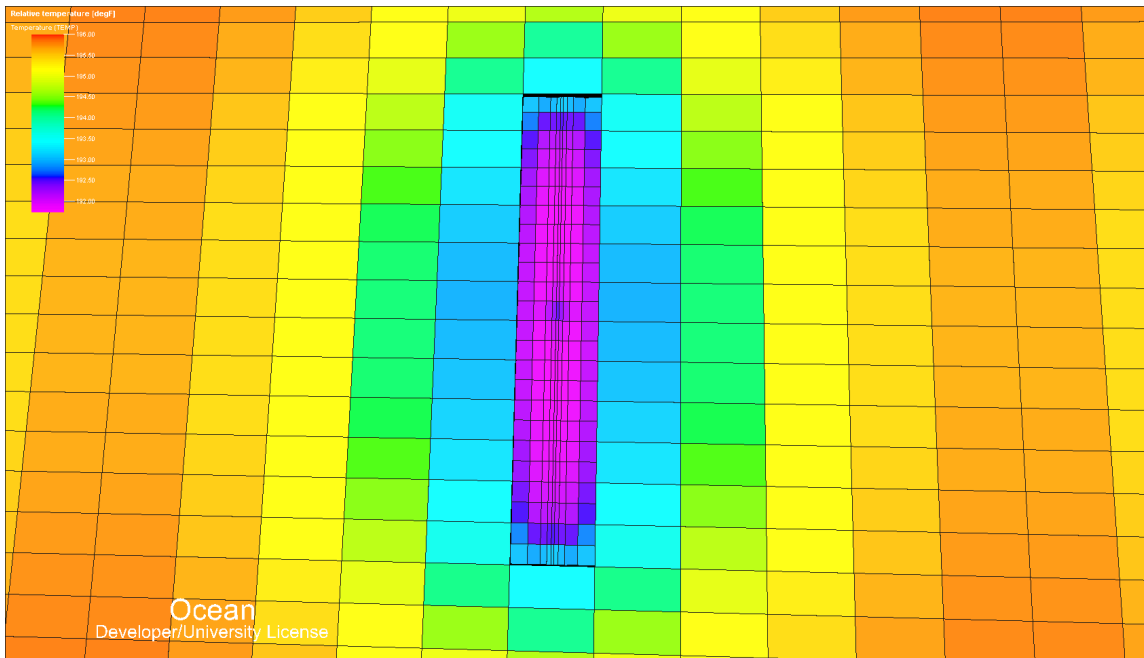


Figure 2.1 – Local Grid Refinement in Hydraulic Fracture for Temperature Model

The initial reservoir temperature begins at 196 Fahrenheit and the simulation runs for approximately 3 years. Rock compressibility, specific heat of rock and liquids and rock and fluid thermal conductivity are inputs for the “Temperature Option” to run in ECLIPSE as seen in Table 2.2. The rock and fluids thermal conductivity is included as a porosity weighted average of the phase and rock conductivities.

Table 2.2 – Rock and Fluid Properties

Rock Specific Heat	35	Btu/ft ³ /R ^o
Oil Specific Heat	0.5	Btu/lb/R ^o
Water Specific Heat	1.5	Btu/lb/R ^o
Gas Specific Heat	0.5	Btu/lb/R ^o
Rock Compressibility	5x10 ⁻⁶	psi ⁻¹
Rock & Fluid Thermal Conductivity	24	Btu/ft/day/R ^o

2.3 RESULTS

Figure 2.2 shows the change of temperature readings at the specific fracture locations and throughout the wellbore for seven integration steps that range from a few hours after production to almost three years after production. At the end of 1000 days the temperature stabilizes to an average of 194 Fahrenheit. As expected the temperature in the fractures drops suddenly and quickly at the beginning before stabilizing. This transient period of temperature change is important for the application of the EnKF.

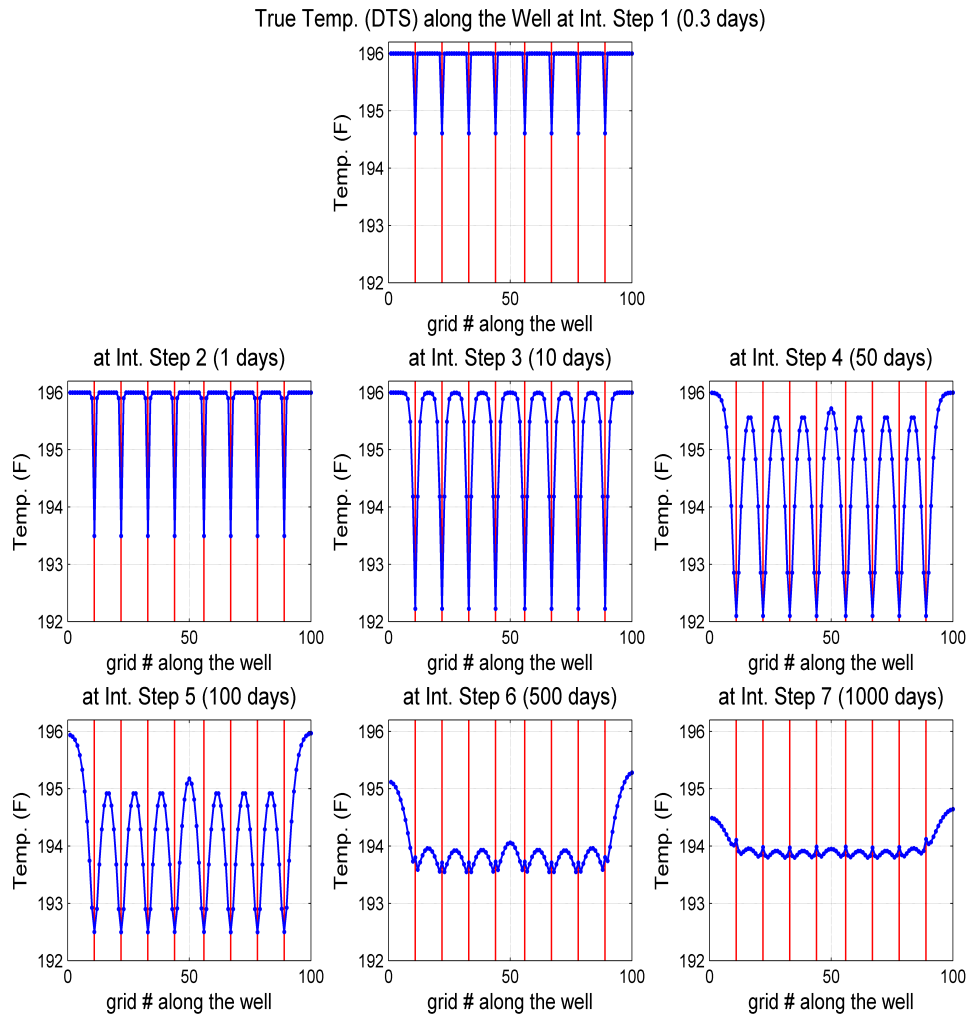


Figure 2.2 – Temperature Reading at Hydraulic Fractures for 7 Integration Steps

Another way to compare the change of temperature throughout the wellbore and at the fractures for 7 integration steps is displayed in Figure 2.3. Temperature stabilizes at a value in between the highest and lowest temperature readings recorded as expected.

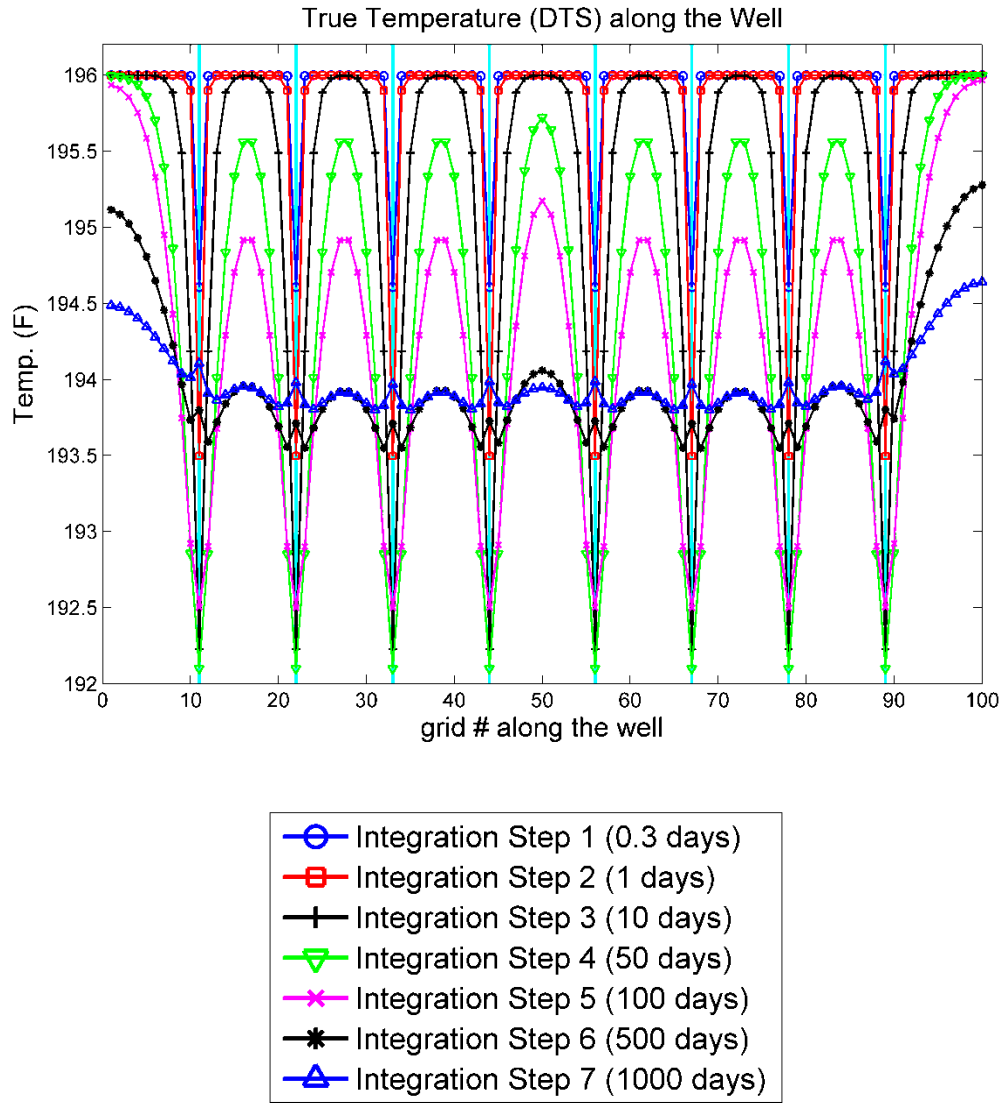


Figure 2.3 – Superimposed Temperature Reading at Hydraulic Fractures

The following Figure 2.4 shows the temperature modeling in a slice of the reservoir.

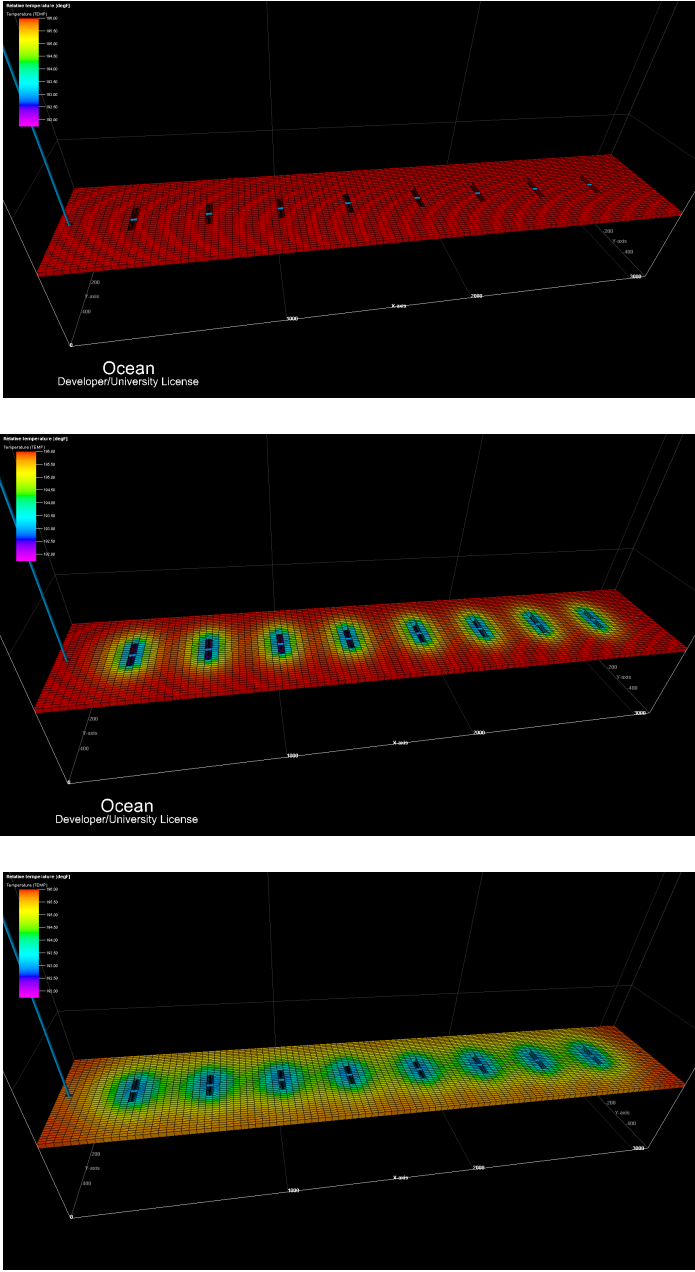


Figure 2.4 – Temperature Model During Production

3. INVERSE PROBLEM & ENSEMBLE KALMAN FILTER

3.1 INTRODUCTION TO THE INVERSE PROBLEM

In the petroleum industry, specifically talking about reservoir simulation, it is seldom known the actual values of the reservoir parameters throughout the field that aid in the characterization of the fluid flow and its behavior. Consequently, only direct observations can be obtained from seismic, production rates and pressures just to name a few. Since most of these observations are obtained from an oil well, the idea of history matching is applied to determine the variables that describe the reservoir system, thus creating the inverse problem. In this study, the application of DTS allows a direct and sensitive connection between an observation, such as temperature readings throughout the wellbore, to aid in the approximation of fracture parameters to better understand well performance in a hydraulically fractured horizontal well. One of the major differences between a forward and inverse model is that inverse problems are ill posed, meaning there are many if not infinite number of possible solutions that would arrive at the same output value. Taking this into account, one of the most important factors for estimating fracture parameters is to have an educated idea of the range of values each parameter of interest would fall under, this is usually deduced by measured/field observations. The EnKF is selected as the ideal tool to solve this specific inverse problem because of its agile performance and a priori data induced effectiveness, meaning that as more quality data is used it will perform better and come to the solution faster.

3.2 ENSEMBLE KALMAN FILTER

The EnKF is an updated version of the recursive Bayesian filter, in other words a Monte-Carlo implementation of the Bayesian update, where the probability of the a priori information and the likelihood are multiplied to obtain an improved posterior probability to sample from that does not constraint the state and allows the update of the information throughout time by incorporating new data. Equation 3.1 shows the original Bayesian filter:

$$P(H/D) = \frac{P(H) \times P(D/H)}{P(D)} \dots\dots\dots(3.1)$$

Where the probability of the hypothesis given the data is represented by $P(H/D)$, the probability of the hypothesis is given by $P(H)$, the probability of the data given the hypothesis is given by $P(D/H)$ and the probability of the data is given by $P(D)$. The EnKF allows the movement of the state by updating the posterior $P(H/D)$ in Equation 3.1 and comparing the hypothesis of the posterior to the previous hypothesis posterior recorded given the data D as seen in Equation 3.2.

$$P(H_t/H_{t-1}, D_t) \dots\dots\dots(3.2)$$

The goal of the EnKF is to obtain a state prediction for a model, in the case of the reservoir model two assumptions will be taken to obtain the state prediction values; the

estimates will be linear as seen in Equation 3.3 and the distribution for the noise will be assumed to be Gaussian. Also, the distribution of the state vector is represented by a sample, which in the EnKF is called an ensemble. A matrix of the state vector x is created where each individual column is a sample from the prior distribution.

$$x_t = x_{t-1} + K(Observation_{Actual} - Observation_{Predicted}) \dots \dots \dots (3.3)$$

The updated estimate in Equation 3.3 contains a linear combination with the Kalman Gain K and the observation of the system being multiplied. The actual observation in Equation 3.3 consists of the temperature readings organized in a vector with the observation temperature predicted that is selected randomly from the normal distribution. The EnKF is calculated by replacing the state covariance by just the sample covariance, which is computed from the ensemble; this ensemble is called the ensemble covariance. The final estimate x_t in Equation 3.3 becomes the posterior ensemble and it is used by perturbing the data matrix which in this case is the actual observation. In other words, the posterior ensemble consists of linear combinations of the members created by the prior ensemble x_{t-1} .

The Kalman gain is a measurement of information in a way, which will be explained shortly. The equation to calculate the Kalman gain is described as follows by Equation 3.4.

$$K_t = \bar{\Sigma}_t C^T (C \bar{\Sigma}_t C^T + E_{Measurement})^{-1} \dots \dots \dots (3.4)$$

Also, the covariance of the state vector Σ_t is calculated by the use of the prior covariance matrix plus the expected variance of the state E_x as presented in Equation 3.5. The other two parameters and matrices in the Kalman Gain in Equation 3.4 are the linear relationship between measurements and the state converted to a matrix C and the variance from the measurement described by the matrix $E_{Measurement}$.

$$\bar{\Sigma}_t = \Sigma_{t-1} + E_x \dots \dots \dots (3.5)$$

The variance from the measurement in Equation 3.4 carries the most important information for the Kalman Gain. In other words, the larger the variance from the measurement the smaller the Kalman Gain will be which in turn makes the data less informative. The more informative the Kalman gain the tighter the variance will be, providing a better estimation of the state. Finally, the covariance of the state vector is updated throughout the procedure by Equation 3.6 where I is the identity matrix and the updated Kalman gain is used along with the relationship matrix C and the covariance Σ_t .

$$\Sigma_t = (I - K_t C) \bar{\Sigma}_t \dots \dots \dots (3.6)$$

For the reservoir model that will be used to perform the EnKF to estimate fracture parameters, the observation parameter will be DTS readings throughout the length of the wellbore. Specifically, it will be one hundred grid blocks that cover the wellbore and pass through each of the fractures, which have been locally refined to obtain accurate temperature readings.

4. APPLICATION OF THE ENSEMBLE KALMAN FILTER TO TEMPERATURE INVERSION

4.1 MODEL AND SIMULATIONS

The temperature readings obtained throughout the wellbore by the aid of the temperature option in Schlumberger's reservoir simulation software ECLIPSE are the key element in the application of the EnKF as presented in chapter 2. The length of the lateral section of the wellbore is equal to the length of the reservoir and it is discretized to 100 grid blocks. Specifically, the 100 grid block temperature readings, which include the wellbore, become the hybrid model of DTS readings. As mentioned in chapter 3, the observation parameter in the EnKF model becomes the temperature readings obtained by the hybrid DTS.

Two parameters, fracture half-length and fracture permeability, are applied to the EnKF estimation individually and later as a joint estimation. Fracture half-length is selected because in low permeability wells the length of the fracture is the driving parameter. Fracture permeability is selected as the other parameter because of its low sensitivity. A similar procedure is performed for both parameters when they are estimated individually and jointly. The reference model, as seen in Table 2.1, is used for the comparison of the true values of both parameters, 90 ft for fracture half-length and 1000 md for fracture permeability. The following step is to generate 100 samples or realizations from the specific intervals of (5 ft-3000 ft) for fracture half-length and (100 md-5000 md). The

expected values of the parameters of interest must fall within this range as observed if compared with Table 2.1, else a solution will never be able to be achieved by the EnKF. The next step is to run the temperature model with the previous 100 fracture half-length and 100 fracture permeability samples, in other words, the temperature model runs 100 times for each parameter. This is also known as the Monte Carlo simulation or the propagation step. The temperature model will output 200 different DTS responses that correspond to 100 fracture half-length samples and 100 fracture permeability samples. The EnKF then makes use of inferred correlations between 100 fracture half-length samples and its corresponding DTS response and 100 fracture permeability samples and its corresponding DTS response by the creation of the matrix C to produce the Kalman Gain as previously presented in Chapter 3. The Kalman Gain is used in the analysis or update equation where the EnKF updates the old fracture half-length and old fracture permeability samples based on the difference of its DTS responses and the true reference response as seen in Equation 4.1 and Equation 4.2 below.

$$x_{half-length,estimate} = x_{half-length,predicted} + K(DTS_{Actual} - DTS_{Predicted}) \dots \dots \dots (4.1)$$

$$k_{fracture, estimate} = k_{fracture, predicted} + K(DTS_{Actual} - DTS_{Predicted}) \dots \dots \dots (4.2)$$

This process is performed for all 200 samples, 100 samples for each parameters, to obtain new and improved approximations of the respective parameters. The true or reference temperature reading is the perturbed value used to quantify the mismatch

through each step. As the simulation advances, the parameter under analysis is updated and improved until it arrives to an acceptable estimated value. The higher the quantity and quality of the data, the more effective and efficient the EnKF performs. The results will be discussed in Section 4.3; however, a sensitivity study must be performed first.

The sampling performed of the fracture parameter is the initial ensemble that is randomly drawn from a uniform probability distribution function, known as *PDF*, with an interval that is selected for each parameter under investigation. One hundred random samples are selected from the uniform *PDF* giving the size of the ensemble/realizations to be one hundred. Also, observed data is not accumulated, instead in each integration step an assimilation or match is performed on the current available data making the process significantly cheaper in computation time.

4.2 SENSITIVITY STUDY

4.2.1 METHODOLOGY OF SENSITIVITY STUDY

In order to effectively understand the performance and behavior of the EnKF with temperature readings from DTS, the parameters of the reservoir model must be quantified. In this study, fracture width, fracture half-length, fracture height, fracture permeability, reservoir permeability and number of fractures, are analyzed individually over a wide range of values while maintaining the remaining parameters constant. This intensive analysis will demonstrate which parameters are closely related to temperature readings and how they affect each other.

4.2.2 SENSITIVITY STUDY OF RESERVOIR MODEL

The following scenarios present the sensitivity study performed for the various reservoir and fracture parameters in relation to the total absolute misfit of temperature along the wellbore.

Sensitivity Of Fracture Width

Wide ranges of fracture width are simulated for the sake of visual observation. The simulations range from a value of 2 inches up to 84 inches. The reference model value for fracture width is 12 inches. The sensitivity in regards to this parameter is very noticeable for number of fractures below the reference value and significantly less noticeable for number of fracture width above the reference value as seen in Figure 4.1. Although most of the fracture width values do not portray realistic measurements, they are chosen in order for the ECLIPSE temperature model to operate efficiently and obtain solutions in adequate time.

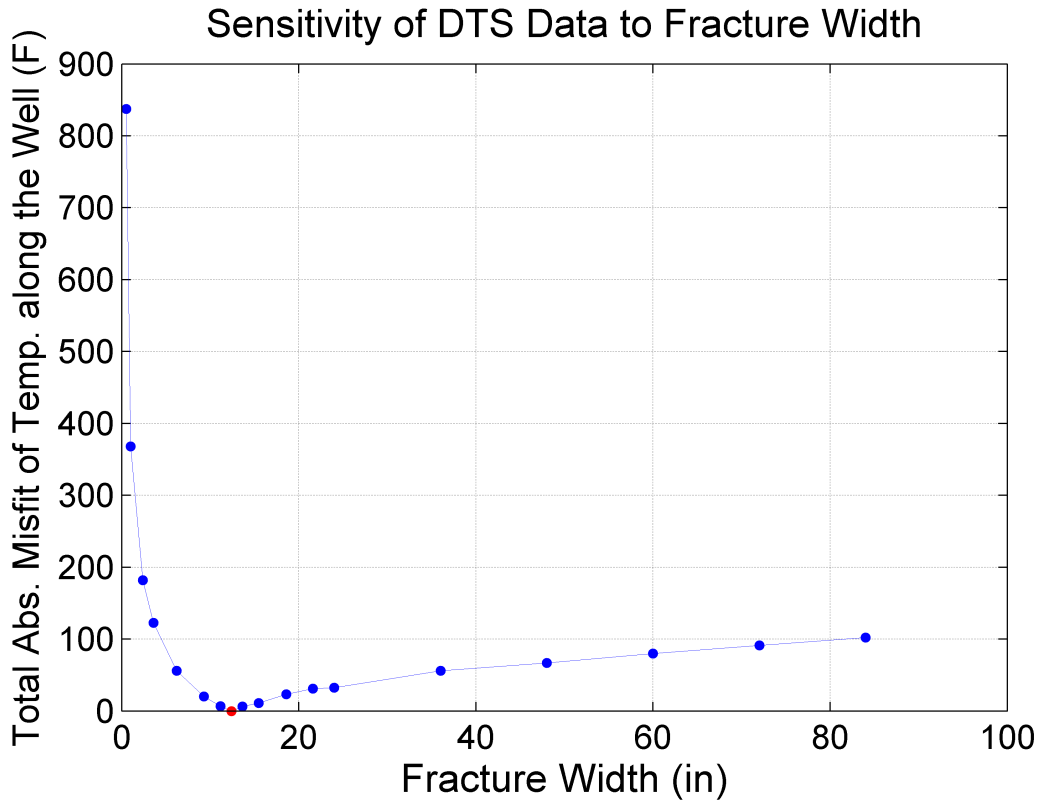


Figure 4.1 – Sensitivity Analysis of DTS for Fracture Width

Sensitivity Of Fracture Half-Length

Fracture half-length is one the most important fracture design parameters for low permeability reservoirs since connectivity with the reservoir is more important than the size of each fracture. Ideally, in a low permeability field similar to the reference model the goal is to obtain long fractures to penetrate as much as the formation as possible. The simulations range from a value of 20 feet up to 300 feet. The reference model value for fracture half-length is 150 feet as seen in Figure 4.2. The sensitivity in regards to this parameter is noticeable for fracture half-length below the reference value and significantly less noticeable for fracture half-length above the reference value.

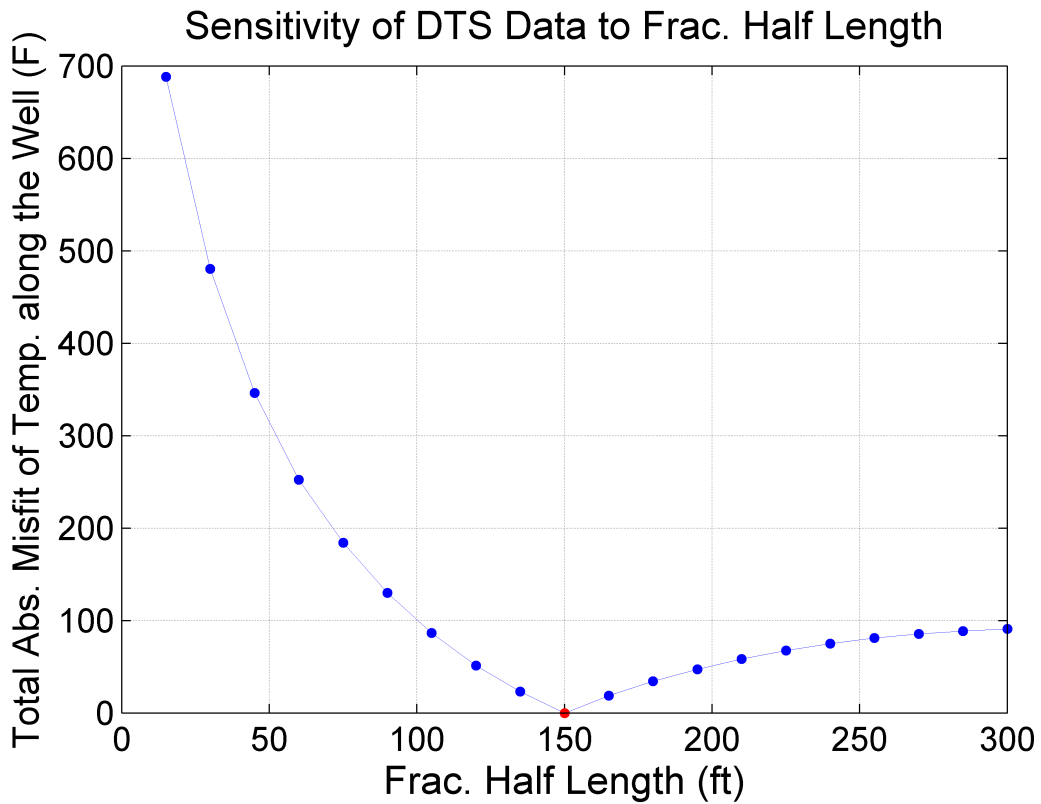


Figure 4.2 – Sensitivity Analysis of DTS for Fracture Half-Length

Sensitivity Of Fracture Height

Fracture height is directly related to fracture half-length and fracture width, parameters that describe the geometry of the fracture. The simulations range from a value of 20 feet up to 115 feet as seen in Figure 4.3. The reference model value for fracture half-length is 45 feet. The sensitivity is more noticeable for fracture height below the reference value since it decreases quickly and more less noticeable for fracture height above the reference value as it increases only 200 units in the total misfit of temperature along the well.

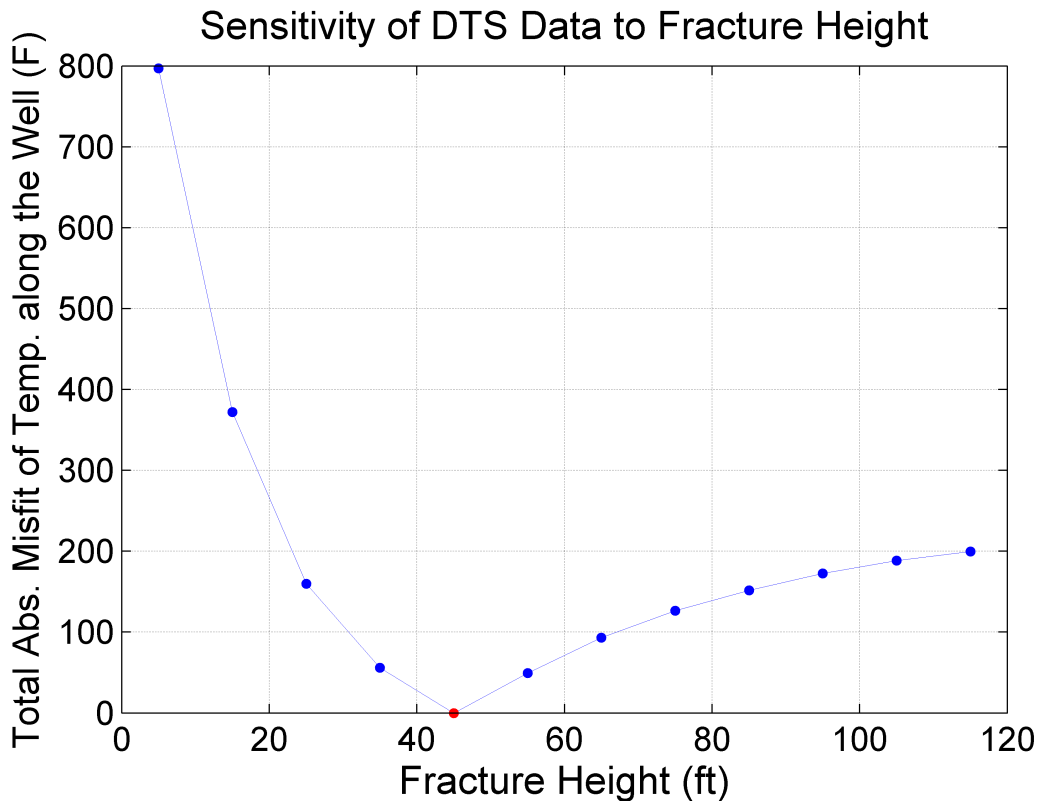


Figure 4.3 – Sensitivity Analysis of DTS for Fracture Height

Sensitivity Of Fracture Permeability

Fracture permeability is one of the main parameters of interest in this study. The simulations range from a value of 10 md up to 10,000 md. The reference model value for fracture permeability is 1,000 md. The sensitivity decreases drastically for fracture permeability below the reference value and almost no change is observed for fracture permeability above the reference value as seen in Figure 4.4.

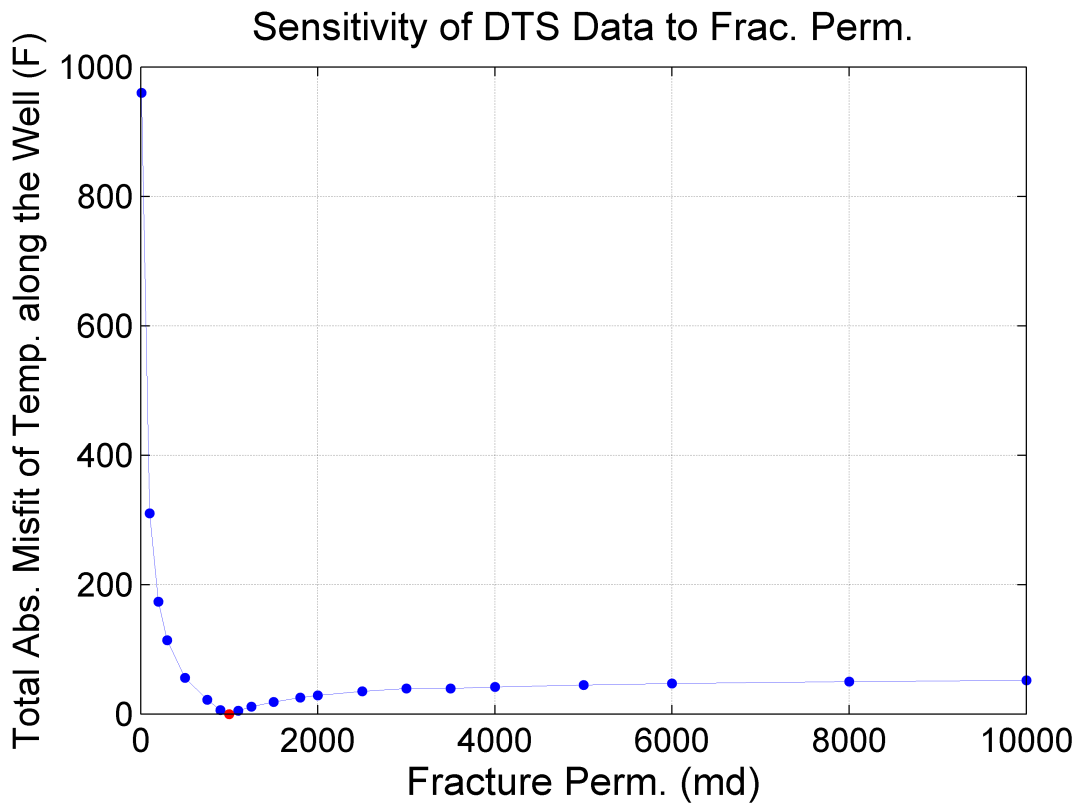


Figure 4.4 – Sensitivity Analysis of DTS for Fracture Permeability

Sensitivity Of Reservoir Permeability

Reservoir permeability is usually a known parameter within the field. The simulations range from a value of 0.002 md up to 100 md for 24 values as seen in Figure 4.5. The reference model value for fracture permeability is 0.2 md. The sensitivity is more noticeable for reservoir permeability below the reference value and significantly less noticeable for reservoir permeability above the reference value.

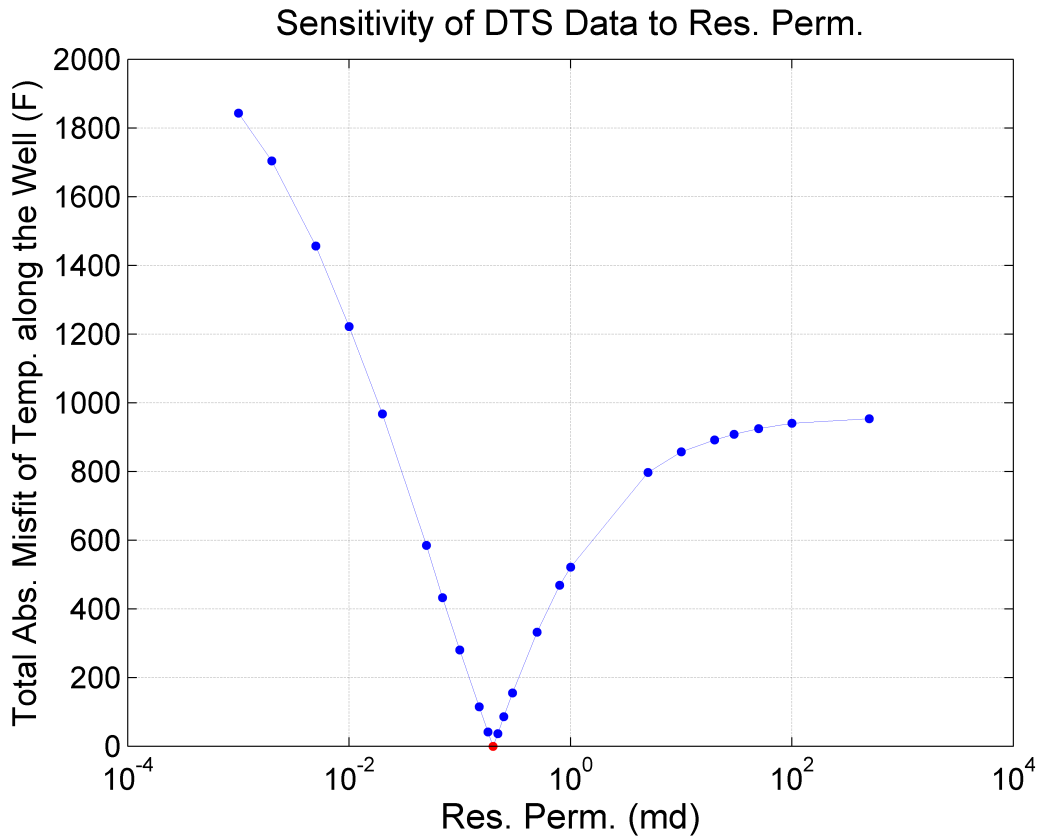


Figure 4.5 – Sensitivity Analysis of DTS for Reservoir Permeability

Sensitivity Of Number Of Fractures

In order to visualize the sensitivity of number of fractures based on the wellbore temperature a wide range of fractures are simulated. The first simulation includes only one fracture and each subsequent simulation increases to 40 fractures in increments of one. The reference model value for number of fractures is 10. The sensitivity in regards to this parameter is more noticeable for number of fractures below the reference value and less noticeable, although it increases with a smaller slope, for number of fractures above the reference value as seen in Figure 4.6. Even though the number of fractures is an important parameter, it is not an unknown factor for this investigation. It is only analyzed for the sake of comparison with the other reservoir and fracture parameters.

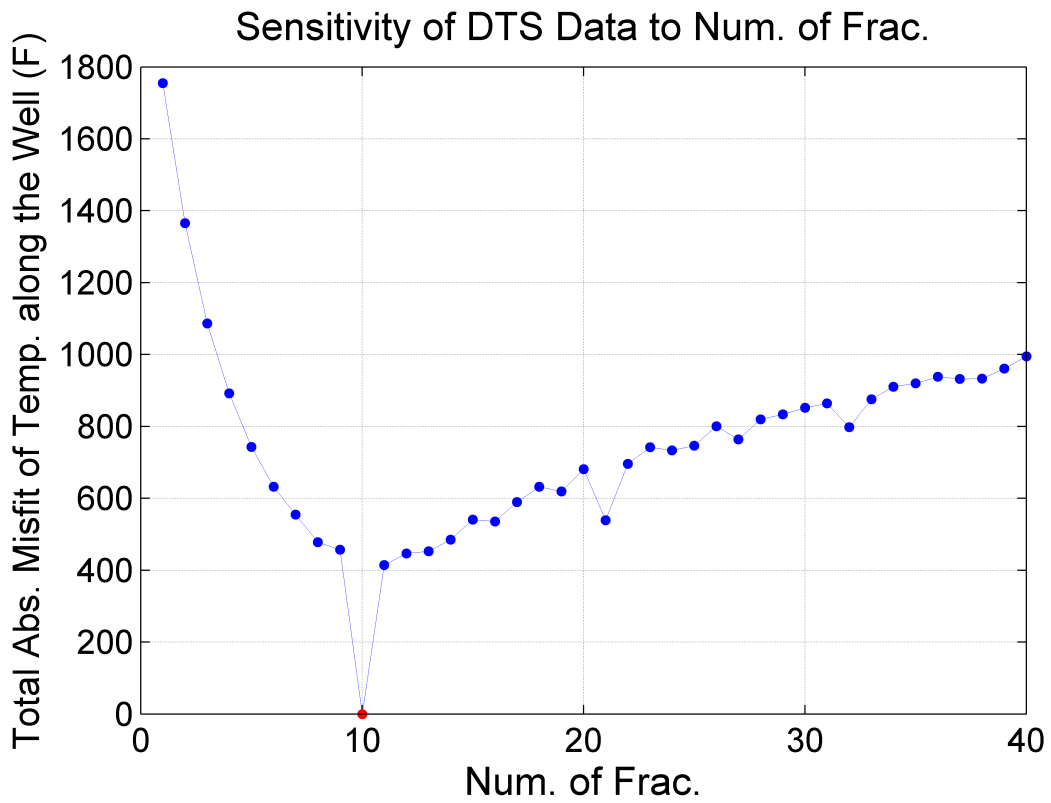


Figure 4.6 – Sensitivity Analysis of DTS for Number of Fractures

Sensitivity Of Number Of Refined Grids

A sensitivity analysis is performed for the number of refined grids. The number of grid blocks is refined in the fracture grid perpendicular to the fracture plane, along the horizontal well. The fracture width is kept constant. The width of the refined grids changes linearly from the center to the sides; the width of the refined grid increases as it moves to the sides. The reference model has 9 refined grids. The refinement number is an odd number in order for the fracture placement in the refined grid to be at the center. The results display after 17 refined grids in the model there is small changes in the total absolute misfit of the temperature along the wellbore in the model.

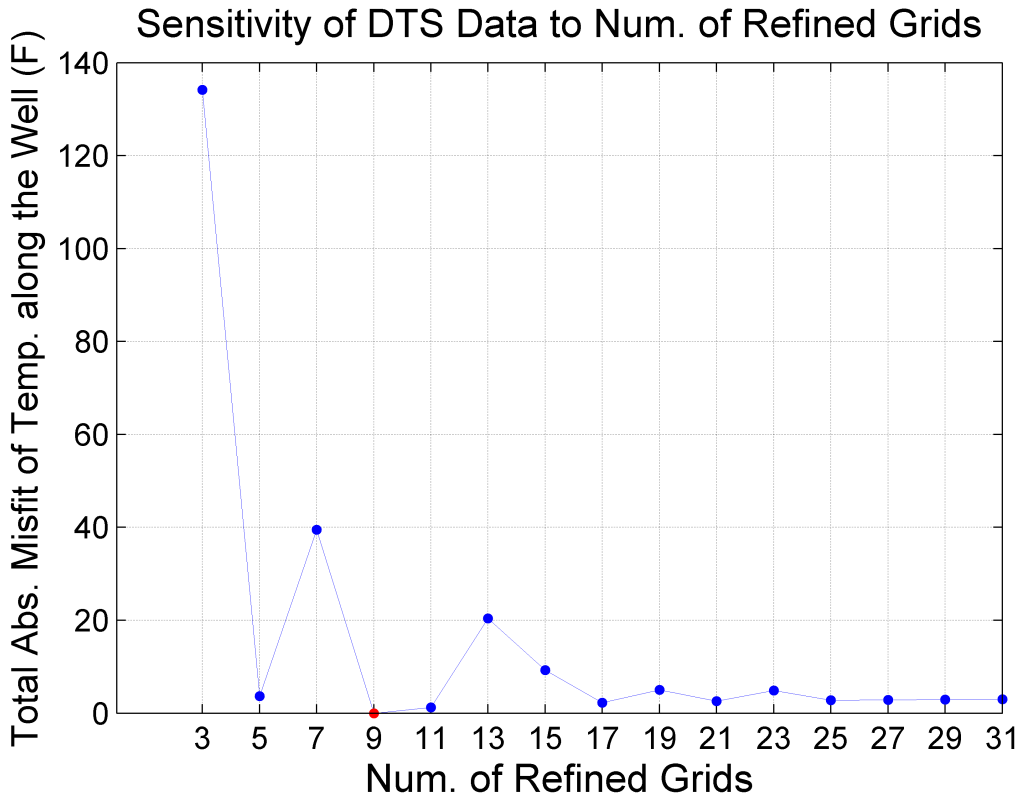


Figure 4.7 - Sensitivity Analysis of DTS for Number of Refined Grids

4.2.3 SUMMARY OF SENSITIVITY STUDY

Parameters are sorted out based on their effectiveness on the model response from the most effective to least effective. After a careful analysis of how each parameter behaves, the main purpose of a sensitivity analysis is to compare the relative importance of each variable in relation to the other variables. The most sensitive parameters observed are the number of fractures and reservoir permeability as seen in Figure 4.7.

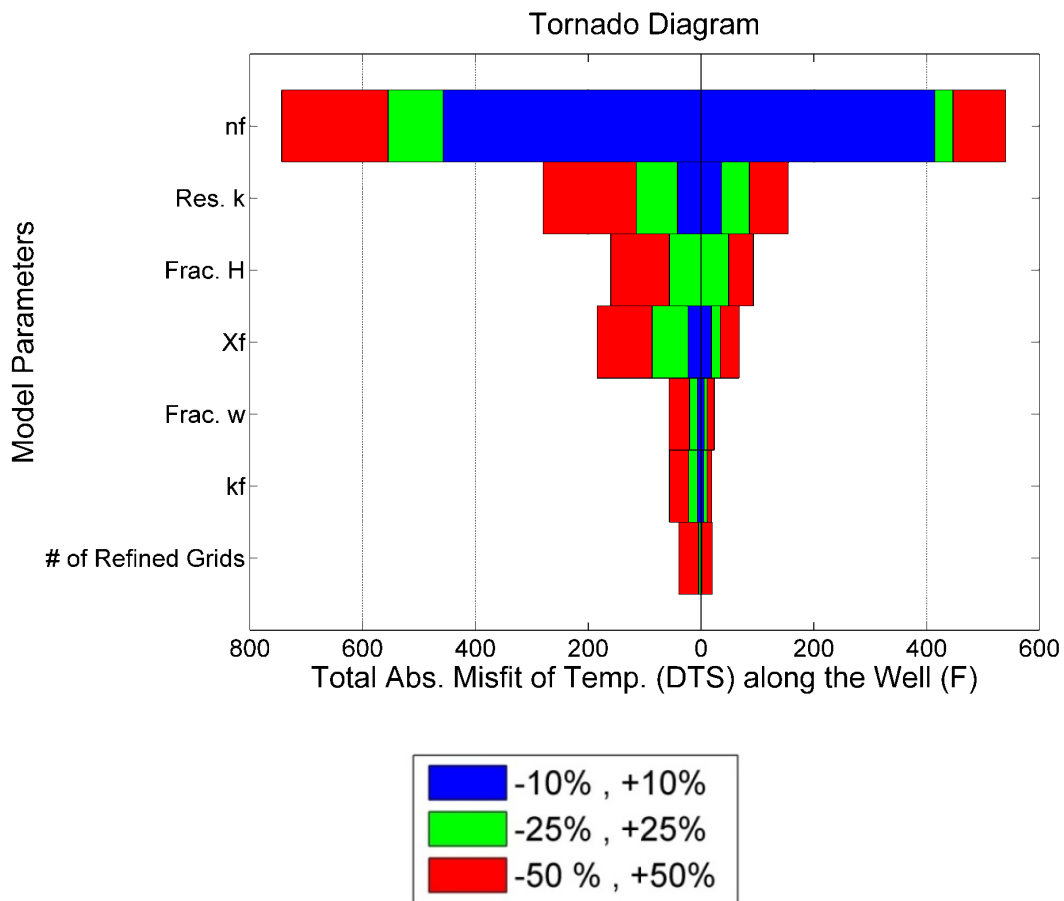


Figure 4.8 – Tornado Diagram for Reservoir and Fracture Parameters

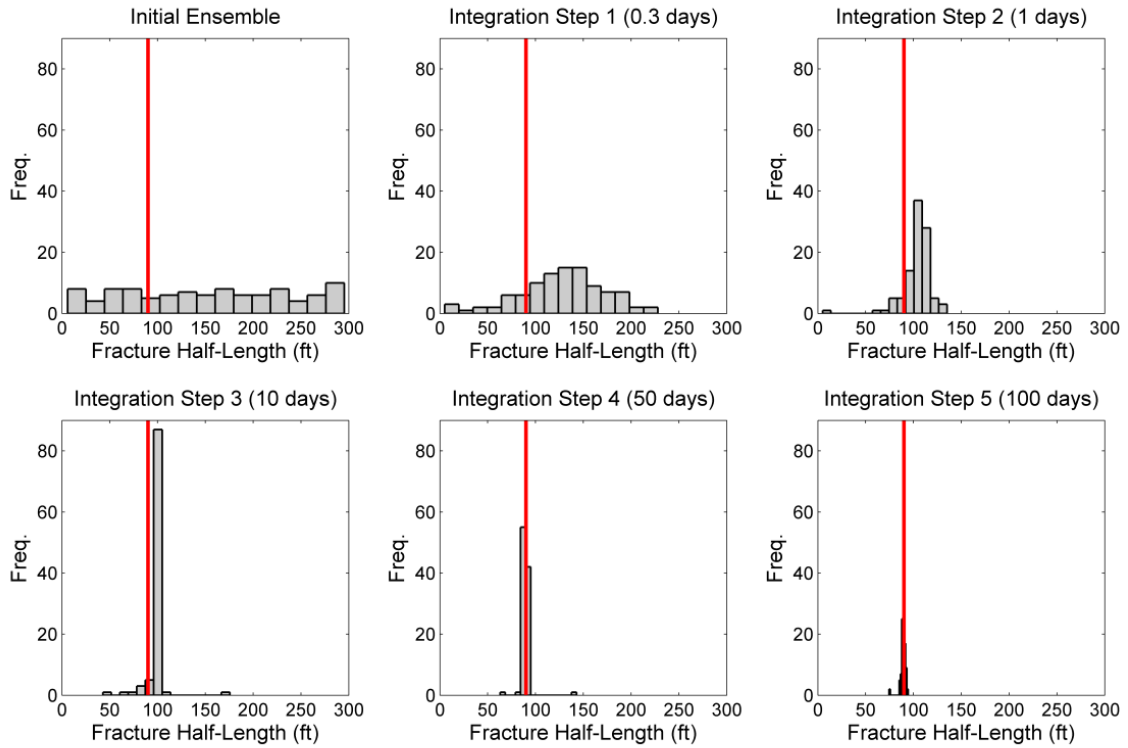
However, the number of fractures and reservoir permeability are not unknown parameters when designing a fracture job and therefore not a focus of this investigation. Also, the number of refined grids is analyzed for sensitivity solely to display that there is not an effect on the model. Consequently, the most important and least sensitive parameters for the EnKF study are the fracture parameters; fracture height, fracture half-length, fracture width and fracture permeability. The application of observations by DTS ensures the EnKF will perform effectively for fracture parameters, however, the quantity of observations recorded are of utmost importance; the more data provided improves EnKF performance.

4.3 ENKF APPLICATION FOR FRACTURE PARAMETERS

The time step updates for the EnKF are selected intuitively based on the amount of time it takes to compute the estimation. In this case, for the sake of comparison between two parameters, the integration steps were between five and seven. As the integration steps increase, the EnKF takes longer to arrive at the solution. Also, since the change in DTS data is more rapid in the beginning of the production the first integration steps are closer than at the end.

4.3.1 ENKF RESULTS USING FRACTURE HALF-LENGTH

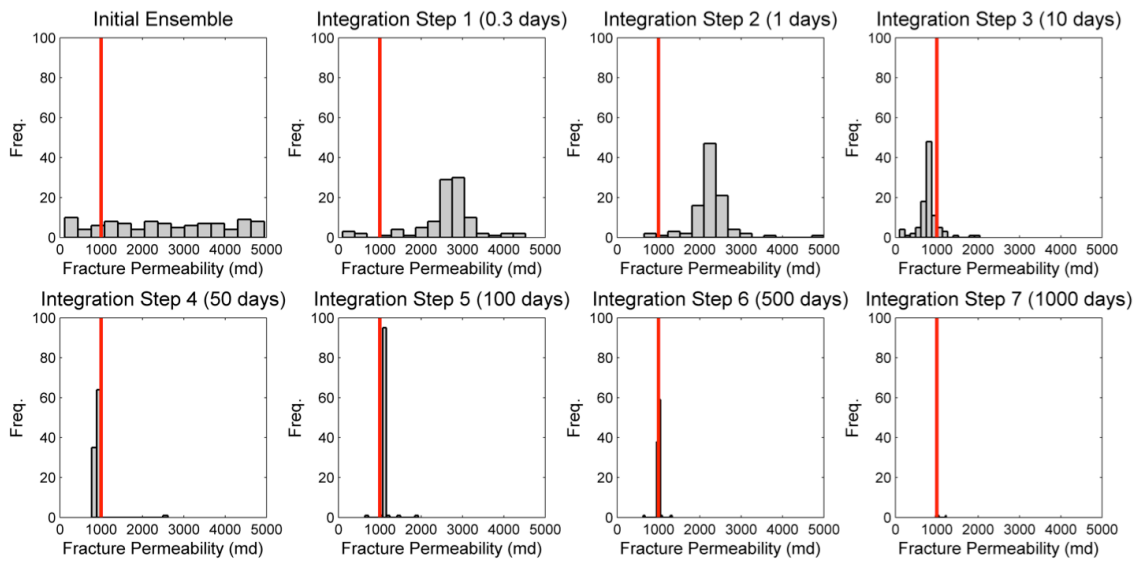
Fracture half-length has substantial sensitivity to the reference model as it can be seen in Figure 4.7 when the sensitivity analysis is performed, and it is also one of the most important fracture parameters based in low permeability reservoirs as observed in unconventional oil wells, Economides (2013). The EnKF arrives at an acceptable estimation of fracture half-length of 90 ft in 5 integration steps or 100 days of temperature simulation. As expected, when the EnKF moves throughout the integration steps the ensemble becomes tighter, approaching the reference half-length as expected. The agility of the EnKF to arrive to an acceptable estimate is directly related to the sensitivity of the parameter under investigation. In the case of fracture half-length, by the analysis of Figure 4.2 various ranges of temperature can be obtained by running the selected range of fracture half length, between 20 ft up to 300 ft, therefore aiding the EnKF in creating the inferred correlations that create the Kalman Gain used to update/improve the parameter estimation as more samples are tested.



**Figure 4.9 – Estimation of Fracture Half-Length Using EnKF
True Half-Length = 90 ft (Red Line)**

4.3.2 ENKF RESULTS USING FRACTURE PERMEABILITY

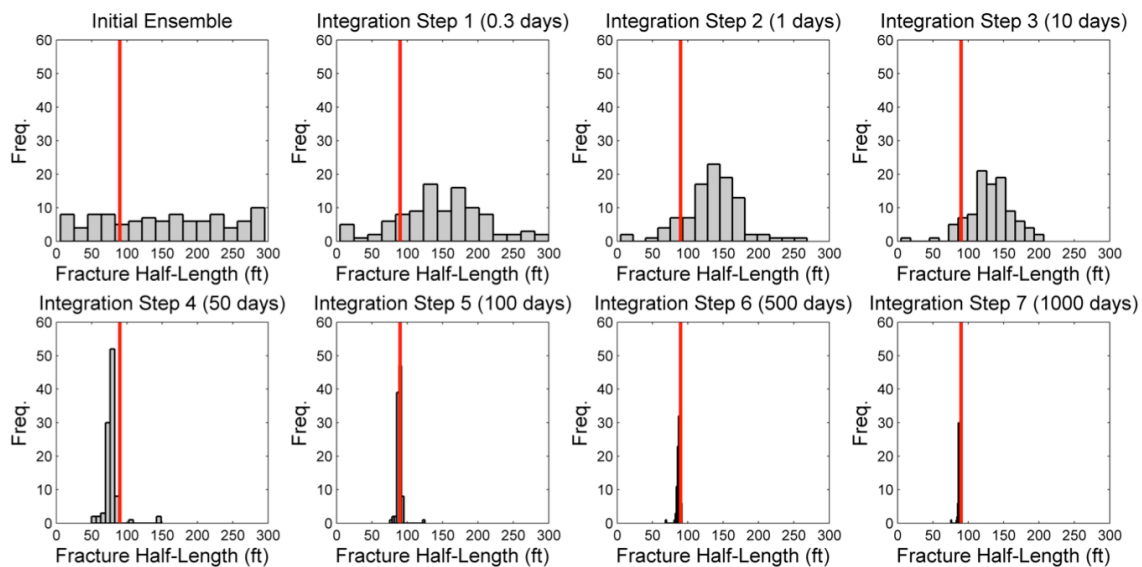
Fracture permeability is the least sensitive and one of the two most important fracture parameters based in fracture optimization, along with fracture half-length being the other one; for this reason it is selected for the EnKF implementation in the homogeneous permeability field. As seen in Figure 4.9, the EnKF takes seven integration steps, which constitute a total of 1000 days of temperature simulation to arrive to a desirable estimation value of 1000 md. Nevertheless, once again in Figure 4.9 it can be observed that by integration step 6 the EnKF has arrived to an acceptable and expected solution of 1000 md portrayed by the red line in each graph. The number of integration steps that takes the EnKF to arrive to an acceptable approximation will increase as the sensitivity towards the observation of temperature decreases. However, it will take substantially less time than other methods that either fail to arrive to the solution and/or become too computationally expensive to model.



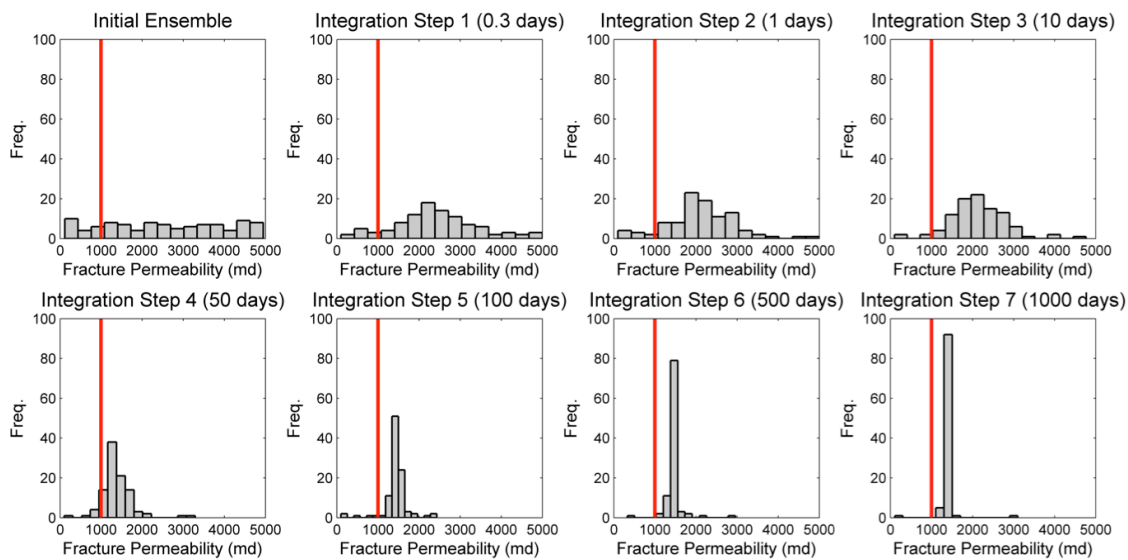
**Figure 4.10 – Estimation of Fracture Permeability Using EnKF
True Fracture Permeability = 1000 md (Red Line)**

4.3.3 ENKF RESULTS USING JOINT ESTIMATION

One of the main advantages of the EnKF in history-matching inverse problems is its ability to solve the inverse problem for more than one parameter in an effective and timely manner, substantially diminishing the computation time and effectively using the ensembles created for the parameters estimation. Methods such as MCMC (Markov chain Monte Carlo) are feasible to solve the inverse problem, however, it can be computationally expensive if it can even arrive at the solution due to the non-uniqueness nature of the inverse problem. The estimation for two parameters, fracture half-length and fracture permeability, is presented in Figures 4.10 and 4.11. The estimation is computed simultaneously. Even though 5 integration steps or 100 days can accurately estimate fracture half-length close enough to 90 ft as it was done when fracture half-length was estimated individually, it takes at least 7 integration steps or 1000 days to approach relatively close to a tight histogram distribution to the expected solution of fracture permeability of 1000 md in the joint estimation. The fracture permeability's low sensitivity gives an increase in computation time compared to the previous individual runs obtained in Figure 4.8 and Figure 4.9. This occurs because in order for the model to notice a variation in the temperature values the model has to run an extensive number and range of fracture permeability values. However, both parameters arrive at an acceptable estimation to the solution in a timely fashion.



**Figure 4.11 – Joint Estimation of Fracture Half-Length Using EnKF
True Fracture Half-Length = 90 ft (Red Line)**



**Figure 4.12 – Joint Estimation of Fracture Permeability Using EnKF
True Fracture Permeability = 1000 md (Red Line)**

5. CONCLUSIONS

The EnKF proves to have significant potential in parameter estimation for hydraulically fractured horizontal oil wells with the aid of DTS readings. First, the sensitivity of each reservoir parameter was quantified to understand which parameters have the strongest effect on the reservoir model and the best response to temperature. This was accomplished by running a wide range of values for each reservoir parameter while maintaining the remaining parameters constant. Consequently, individual parameters were tested to show how quickly EnKF arrives at the solution. In average, it took 6 integration steps to arrive to an acceptable value. The next step was to apply a joint estimation of two parameters, selecting a parameter with small sensitivity and a parameter with medium sensitivity. This step was performed to prove the agility of the EnKF to arrive at the solution effectively and in a timely fashion when more than one parameter is under investigation. The joint estimation has the potential to be easily expanded to more than two parameters; however, it quickly becomes computationally expensive if the required hardware is not available as the number of parameters under analysis increases the required data processing needs substantially. Also, as expected the probability density functions closely mimic the expected normal distribution assumed for this study.

This investigation is the groundwork for future research in which it can be expanded and improved on for a better understanding of the EnKF in history matching unconventional

wells. The first and most important recommendation is to obtain field DTS data from an oil/gas well and compare the results with a hybrid model to see how quickly both arrive to an acceptable estimation value. The second recommendation is to integrate simultaneously DTS and production (oil/gas/water) rates in the EnKF as the observations to improve the estimation results; one thing to take into consideration will be how the increase of observation data changes the EnKF's performance. The third recommendation is to use a heterogeneous permeability field and once again gauge how the EnKF's performance is affected. Ideally, obtain estimated fracture parameters from the field and use them for the reference model for which the EnKF is based of. This will provide more realistic results and allow the comparison of actual values based on field data. Ultimately, the goal is to obtain acceptable estimation values for hydraulic fractures in order to better understand and optimize the life of current and future horizontal wells. The EnKF provides a new efficient way in inverse problems for estimating reservoir parameters and also potentially becoming a new method for history matching in forward models in petroleum-related forecasting.

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