# AN OPTIMIZATION MODEL FOR ECO-DRIVING AT SIGNALIZED INTERSECTION

A Thesis by ZHI CHEN

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## MASTER OF SCIENCE

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#### ABSTRACT

This research develops an optimization model for eco-driving at signalized intersection. In urban areas, signalized intersections are the "hot spots" of air emissions and have significant negative environmental and health impacts. Eco-driving is a strategy which aims to reduce exclusive fuel consumption and emissions by modifying or optimizing drivers' behaviors. With the help of vehicle-to-vehicle (V2V) communication and vehicle-to-infrastructure communication (V2I), eco-driving could utilize the signal phase and the queue-discharging time information to optimize the speed trajectories for the vehicles approaching an intersection in order to reduce fuel consumption and emissions. A few research studies have been conducted on the development of algorithms that utilize traffic signal information to reduce fuel consumption and emissions.

Hence, the goal of this research is to develop an optimization model to determine the optimal eco-driving trajectory (the speed profile) at a signalized intersection, which aims to achieve the minimization of a linear combination of emissions and travel time. Then enumeration method, simplex optimization and genetic algorithm are investigated to determine a practicable and efficient method to solve the proposed optimization problem. As various scenarios of distance from the vehicle to the intersection, queue discharging time and weights of emission/travel time will lead to different optimal trajectories and different emissions and travel times. A sensitivity study is conducted to analyze and compare the performance of the optimal solution in various scenarios of

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different such parameters. In addition, a baseline study is conducted to investigate the benefits of eco-driving when drivers only decelerate in advance but not apply the recommended speed trajectory. The results of case study show that genetic algorithm is a preferred method to solve the proposed optimization problem; Eco-driving could achieve satisfied reduction in emissions without significantly increasing travel time and emissions is more sensitive to various scenarios than travel time; Eco-driving still could achieve reduction in emissions as long as the drivers decelerate earlier even though the they would not apply the recommended speed trajectory under certain conditions.

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#### **CHAPTER I**

#### INTRODUCTION

In the United Sates, the transportation sector is the second largest atmospheric carbon emitter (EPA, 2010). About 71% of the total petroleum consumption of the United States is the transportation sector's share (Davis et al., 2010). As a result, on road traffic contributes 59.6% of Carbon Monoxide (CO), 33.1% of Nitrogen Oxides (NOx), and 26% of Volatile Organic Compounds (VOCs) to the total emissions (EPA, 2012). Such a large amount of emissions has created significant environmental stress on society. In urban areas, the signalized intersections typically involve the highest traffic density, the longest vehicle queuing and idling time, and the most deceleration and acceleration operations, which make the signalized intersections the "hot spots" of air emissions and have significant negative environmental and health impacts (Lv, 2012). Hence, many strategies have been promoted to mitigate environmental problems caused by on road traffic. "Eco-driving" is one such strategy, aiming to reduce exclusive fuel consumption and emissions by modifying or optimizing drivers' behaviors. Recently, with help of the intelligent transportation system (ITS) technology, the dynamic eco-driving advice attracts more and more attentions and can be implemented using real-time traffic sensing and telematics, allowing for a traffic management center to communicate in real-time with equipped vehicles. The overall goal of dynamic eco-driving is to smooth the traffic flow (and thereby decreasing fuel consumption) by dynamically advising vehicles to travel at specific speeds.

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As aforementioned, it is important to highlight the signalized intersections that involve the most deceleration and acceleration operations and thus the most production of air pollutants. With the help of vehicle-to-vehicle (V2V) communication and vehicleto-infrastructure communication (V2I), eco-driving could utilize the signal phase and the queue-discharging time information to optimize the speed trajectories for the vehicles approaching an intersection in order to reduce fuel consumption and emissions at signalized intersections. One frequent scenario around the signalized intersection is shown in Figure 1. It illustrates a scenario when a vehicle is approaching to a signalized intersection, if it remains its normal operation speed, it has to decelerate and fully stop at the rear of the queue and then accelerate to its normal speed from the zero speed after the signal turns green and the front queue is discharged, the corresponding normal driving trajectory is shown as the black dash line. While the eco-driving suggests another driving strategy, which advises the driver to decelerate actively, thus make it that when the vehicle approaches the intersection with a specific speed (larger than zero), the queue is rightly cleared, the corresponding eco-driving trajectory is shown as the blue solid line. So the vehicle doesn't need to stop at the intersection nor accelerate from the zero speed. Some studies have demonstrated that such an eco-driving strategy will effectively reduce the consumption and emissions (Barth et al., 2011; Rakha and Kamalanathsharma, 2011).

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Figure 1 Time-space diagram representing different vehicle trajectories approaching an intersection

#### **Problem statement**

This study will focus on the scenario described in Figure 1. There are infinite eco-driving trajectories that allow the approaching vehicle pass the intersection without stop such like the blue solid line shown in Figure 1, and different trajectories will lead to different emissions and travel time, thus it is meaningful and necessary to find out the optimal one. So this study will develop an optimization model to determine the optimal eco-driving trajectory at a signalized intersection. The objective function of this optimization problem would be a linear combination of emissions and travel time. The decision variables will be the coefficients of deceleration ad acceleration functions (refer to the Methodology part), and based on decision variables the second-by-second speeds (the speed profile) will be estimated, which are used to calculate the emissions and the travel time. The important parameters include normal operation speed  $(v_0)$ , the distance of the vehicle from the intersection when the driver decide to decelerate (S), the time (T) needed when the signal turns green and the queue discharges clearly, and the weights  $(w_i)$  of emissions and travel time.

In addition, various scenarios of distance from the vehicle to the intersection, queue discharging time, normal operation speed and weights of emission/travel time will lead to different optimal trajectories and different emissions and travel times. A sensitivity study will be conducted to analyze and compare the performance of the optimal solution in various scenarios of different  $v_0$ , S and T. The impact of different  $v_0$ , S, T and  $w_i$  on the emissions and travel time will be discussed based on the sensitivity study.

## **Research objectives**

The goal of this research is to develop an optimization model to determine the optimal eco-driving trajectory (the speed profile) at a signalized intersection, which aims to achieve the minimization of a linear combination of emissions and travel time. In addition, this research will analyze the impact of parameters of  $v_0$ , S, T and  $w_i$  on the performance (emissions and travel time) of the optimal solution. The research objectives are:

• To develop an optimization problem to model the eco-driving strategy at signalized intersection with objective of minimizing a linear combination of emissions and travel time.

- To formulate a reasonable linear model to consider both emissions and travel time.
- To find a practicable and efficient method to solve the developed optimization problem.
- To find out the impact of different v<sub>0</sub>, S, T and w<sub>i</sub> on the performance (emissions and travel time) of the optimal solution.

#### **Research benefits**

This research work is being done to develop and improve the eco-driving strategy at the signalized intersection. An optimization problem is developed to model the eco-driving strategy at signalized intersection, and a practicable and efficient method is proposed to solve this optimization problem. Then the optimal speed profile could be estimated based on the solution. This speed profile will achieve the minimization of a linear combination of emissions and travel time, which could effectively reduce the emissions without increase the travel time significantly. In addition, the impact of different  $v_0$ , S, T and  $w_i$  on the emissions and travel time will be analyzed, which will provide insights into how the various scenarios would affect the performance and limit the benefits of the eco-driving strategy.

#### **Thesis organization**

This thesis is composed of four chapters. The first section of the thesis addresses the background including the problem statement and research objectives. The second part of the thesis provides a review of the state-of-the-art concerning on the concepts and applications of eco-driving strategy including previous methodology on the dynamic eco-driving especially its application at the signalized intersection. In addition, the researcher will review literature dealing with the mostly used traffic emission models. The third section presents the development of models including equations governing upstream portion and downstream portion, traffic emissions, travel time and objective model. The fourth section describes some case studies to demonstrate the application of the developed model. Three solution methods are compared to find a practicable and efficient method to solve the proposed optimization problem. In addition, travel time and emissions will be evaluated in various scenarios with different S, T and w<sub>i</sub>. Lastly, the fifth section states the executive summary of this research including findings, limitations and the needs for future work.

#### **CHAPTER II**

#### LITERATURE REVIEW

This section of the proposal provides information on the introduction and benefit of ecodriving, and the previous research gone into the modeling of eco-driving at the signalized intersection. This section also provides background information of traffic emission models.

#### Introduction and benefits of eco-driving

To reduce the air emissions, particularly green gas emission, from the transportation sector, the policy makers in the U.S. are promoting more strategies to mitigate environmental problems caused by on road traffic. Eco-driving is one such strategy that has recently become an important research interest worldwide due to its advantage of cost effective and easy to be applied to all kinds of vehicles on the road immediately (Gense, 2000). Eco-driving primarily consists of a variety of driving techniques that save fuel and lower emissions. Representative eco-driving involves various driving behaviors, such as maintaining a steady speed, avoiding heavy acceleration and deceleration, well anticipating the traffic flow ahead, and minimizing idling time. These behaviors will tend to smooth vehicle movements and avoid unnecessary fuel consumption, thereby reducing greenhouse emissions.

Martin et al. (2012) conducted a study to assess the effectiveness of static, webbased information on eco-driving. They designed a controlled study in which respondents were divided equally into an experimental and a control group. Then the experimental group was then asked to visit the EcoDrivingUSA website. The results of a longitudinal survey showed that 57% of experimental group respondents increased their eco-driving score. They also concluded that the eco-driving followers were more likely to be female; drive a newer, more-efficient vehicle; and live in a smaller household.

Symmons and Rose (2009)conducted a field test in which a small group of heavy vehicle drivers underwent an eco-drive training course. The training particularly focused on progressive gear shifting and progressive braking, "flowing" the vehicle and forward scanning of the road ahead. The result demonstrated that the eco-driving training could achieve a 27% reduction in fuel consumption by heavy-vehicle drivers.

Boriboonsomsin et al. (2010) evaluated how an on-board eco-driving device that provides instantaneous fuel economy feedback affects drivers' behaviors, and consequently fuel economy of gasoline-engine vehicle drivers in the U.S. under realworld driving conditions. The study result showed that on average the fuel economy on city streets improves by 6% while the fuel economy on highways improves by 1%.

Ando and Nishihori (2011) conducted a analysis on how many cars kept in following and how many cars gave up the following and then overtook the eco-driving car. They found that the percentage of car following time behind the eco-driving car over all running time is about 76% which demonstrates that the eco-driving car may affect the following cars to drive economically and ecologically even the drivers may not be active.

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Hallihan et al. (2011) examined the effects of a hybrid-interface on eco-driving behavior and driver distraction. Measures of accelerations and eye movements were collected during simulated drives to test these potential impacts. Their study showed that while using the hybrid-interface, significant reductions in acceleration from a stop were observed compared to when driving without using the hybrid-interface.

Ahn et al. (2011) pointed out that the roadway grades have a significant impact on vehicle fuel consumption and CO2 emission rates. They developed an eco-cruise control system allows the vehicle to travel faster along downgrades and slower along upgrades. A vehicle powertrain model was applied to maintain and adjust the vehicle speed. The results of field test showed that the proposed eco-cruise control system can averagely save 10.33 percent in fuel consumption and CO2 emissions compared to the traditional cruise control operations at hilly road.

Park et al. (2013) compared the performance of manual driving, conventional cruise control (CCC) driving, and Eco-cruise control (ECC) driving with regard to fuel consumption. They conducted the field experiment on five test vehicles along a 24-km section of Interstate 81 in Virginia that was comprised of  $\pm 4\%$  uphill and downhill grade sections. The instantaneous fuel consumption rates and other driving parameters were collected using an Onboard Diagnostic II reader. The results showed that the average fuel economy enhancement across all the field tests was 3.3% with and without the CCC system enabled. Additionally, this test demonstrated that an ECC system would achieve fuel savings ranging between 8 and 16 percent with increases in travel times ranging between 3 and 6 percent.

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It is important to note that nearly all eco-driving-related research in the early time has been on providing static advice to drivers and measuring before-and-after differences. However, when coupled with Intelligent transportation system (ITS) technology, applying of real-time signal information to forecast the external factors to the vehicle and predict a fuel-optimal strategy was the focus of newer eco-driving research. Such dynamic eco-driving advice can be implemented using real-time traffic sensing and telematics, allowing for a traffic management center to communicate in real-time with equipped vehicles. The overall goal of dynamic eco-driving is to smooth the traffic flow (and thereby decreasing fuel consumption) by dynamically advising vehicles to travel at specific speeds.

Park et al. (2012) developed a predictive eco-cruise control system that generates vehicle control plans for fuel-consumption reduction by utilizing the given topographic information. The proposed system consists of three building blocks: a powertrain module, a fuel consumption module, and an optimization module. The fuel consumption model applies the Virginia Tech Comprehensive Power-based Fuel Model which utilizes instantaneous power as an input variable to estimate the fuel consumption. In the optimization module, three parameters: the unit distance, the optimization lookahead distance, are use to find the optimal vehicle control set. Finally, a field test demonstrated fuel savings up to 15 percent with the proposed system. Specifically, the test results showed that the largest fuel savings are achieved along hilly terrain sections.

Ahn et al. (2013) developed an eco-drive system that combines eco-cruise control logic with car-following models. The Van Aerde steady-state car-following

model, the collision avoidance model and the vehicle dynamics Model were developed in the proposed car-following algorithm. The field tests showed 27% reduction in fuel consumption with an average spacing of 47 m. Moreover, This study concluded the carfollowing threshold setting significantly affects the fuel economy and the spacing between vehicles.

Barth and Boriboonsomsin (2009) investigated the concept of dynamic ecodriving and proposed a dynamic strategy which takes advantage of real-time traffic sensing and telematics to monitor traffic speed, density, and flow, and then communicates advice in real-time back to the vehicles. They found that by providing dynamic advice to drivers, approximately 10–20% in fuel savings and lower CO2 emissions are achieved without a significant increase in travel time.

Boriboonsomsin et al. (2011) evaluated how an on-board eco-driving device (Eco-Way unit produced by Earthrise Technology) that provides instantaneous fuel economy feedback affects driving behavior. The results from 20 driver samples show that the group of participating drivers were willing to adopt eco-driving practices in the near future, and the eco-driving adoption rate could go up to 95% if the gasoline price increased to \$4.4 per gallon.

Ando and Nishihori (2012)analyzed the factors affecting drivers' improvement of eco-driving based on the data collected from a social experiment undertaken during October 2009 and January 2010 in Toyota City. In the social experiment, all monitors were requested to behave as usual during the first week, then were requested to behave by referring the information provided if they like in the second week. The evaluation indicator of eco-driving is defined as the summation of points including starting indicator, travel indicator, idling indicator and emission indicator. Then based on the change of evaluation indicator of eco-driving, they analyzed the relations between change of eco-driving effects and influence factors and the influence of information frequencies. They concluded that when providing the information with middle frequency (several time a week), the information users may keep in the eco-driving status for a long term.

Qian and Chung (2011) evaluated the effects of eco-driving on the basis of traffic flow by using traffic micro-simulation model. They pointed out that the traffic condition has significant impact on the performance of eco-driving and it's found that eco-driving will produce negative effects when the traffic was congested. They also concluded that a moderate and smooth acceleration has great potentials in fuel saving without major increase in travel time under normal traffic condition, demonstrated by the result that an 11% fuel saving was achieved by adopting active eco-driving while the increase of travel time was only 3%.

Xia et al. (2011)evaluated the indirect network-wide emission benefits of the dynamic eco-driving in the Paramics traffic micro-simulation environment. Simulation runs were performed for different levels of congestion and different market penetration rates of the dynamic eco-driving technology. Their study result showed that there is indeed additional network-wide fuel savings and emission reductions, due to the fact that the normal vehicles are forced to follow the eco-driving trajectories if they are following a dynamic eco-driving vehicle. Additionally, they concluded that the maximum fuel

saving and emission reduction occurs during medium congestion (corresponding to traffic volume of 300 vehicles/lane/hour) and with low penetration rates ( $5\% \sim 20\%$ ).

Mensing et al. (2013) pointed out that to provide the driver with the realistically optimal velocity trajectory for a given trip, road and traffic constraints have to be taken into account. Considering the relative speed of the preceding vehicle, speed limits, safe braking distances and the time-to-collision, an trajectory optimization for eco-driving was developed which integrates traffic constraints in the form of a vehicle following situation. The results of field test showed that when considering the traffic constraints such as the safety distances, the optimal fuel consumption will increase 16-54% compared to the situation without the traffic constraints.

Eco-driving also garners increased interest from automobile manufacturers. For example, Nissan has developed on-board eco-driving support service which composed of navigation system and telematics center in order to promote and deploy eco-driving habits on the road. It's found that by using the system effectively, it resulted in an average of 18% fuel consumption improvement (Satou et al., 2010).

#### Previous research on modeling of eco-driving at signalized intersection

The traffic intersections are the places where some of the drivers have to stop and wait for the right of way. The procedure of the stop-and-go involves numerous acceleration operations. It is found that acceleration operations have a significant effect on emissions and strong acceleration tends to generate high instantaneous emission rates and produce high levels of pollution. So, automobiles will have more possibility to contribute to excessive fuel consumption and emissions near traffic intersections. Overall, this situation implies that eco-driving with a moderate acceleration has great potential to reduce fuel consumption and emissions at traffic intersections. A few research efforts have been conducted aiming at developing algorithms that utilize traffic signal information to reduce fuel consumption and emissions.

Mandava et al. (2010) developed an eco-driving strategy which provided arterial velocity advisory to the drivers regarding the most fuel optimal using the upcoming signal information. The objectives of the proposed strategy are to maximize the probability of having a green light for a vehicle when approaching signalized intersections given the traffic signal information, and to minimize the acceleration rate. By applying an engine power constraint, the proposed algorithm took into consideration that a vehicle could have a larger acceleration rate with a lower velocity. Then a case study was conducted using a stochastic simulation technique. The results showed that the energy/emission savings for vehicles with velocity planning are found to be 12-14% compared to those without velocity planning.

Asadi and Vahidi (2010) developed a cruise control system which utilized constrained optimization to minimize the probability of approaching a stop line during a red phase by varying the speeds within an interval and achieved 47 percent consumption deduction.

Barth et al. (2011) have developed a dynamic eco-driving system by using the signal phase and timing information for signalized corridors that consists of an arterial velocity planning algorithm that attempts to minimize vehicle fuel consumption and

emissions. They proposed that the control logic for the optimal velocity tries to minimize the fuel consumption is minimizing the total tractive power demand and the idling time while ensuring that the optimal velocity is less than or equal to speed limit. They have chosen a family of velocity profiles with a trigonometric increase in velocity in order to minimizes fuel consumption/emissions and is still comfortable to the passengers. The simulation results of their velocity planning algorithms show approximately a 10% to15% fuel economy improvement over a standard baseline case without the velocity planning.

Sun et al. (2013) developed a dynamic eco-driving speed guidance strategy (DESGS) with application of real-time signal timing and vehicle positioning information. In their study, an optimization-based rolling horizon and a dynamic programming approach were put forward to track the optimal guided velocity for individual vehicles along the traveling segment. A piecewise model showing the relationship between the fuel rate and vehicle specific power (VSP) was regressed to estimate the fuel consumption and emissions. A case study was conducted in which 15 drivers attended the speed guidance experiments using multi-vehicle driving simulators, the test result showed that the number of stops is significantly reduced and fuel consumption and CO2 emissions can be reduced by approximately 25% for the vehicles with DESGS as compared to the vehicles without speed guidance. The aforementioned literature shows that there has been research in developing dynamic eco-driving logic at intersection using V2I communication. However, none of these approaches used an explicit optimization objective of reducing emissions.

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Rakha and Kamalanathsharma (2011) developed a eco-driving framework which yields the most fuel-optimal speed profile for a vehicle approaching a signalized intersection using V2I communication capabilities. The VT-Micro model was used to estimate fuel consumption for various alternative speed profiles and determines which is the optimum. They divided the vehicle trajectory into the trajectory upstream and downstream of the traffic signal stop-line and a combine optimum is calculated using mode-specific fuel consumption and emission levels for vehicle deceleration, cruising/idling, and acceleration modes.

However, this did not provide the speed profile for the downstream of the stopline but only offer the throttle information during the acceleration. In addition, The aforementioned studies only focused on the fuel consumption and emissions but not considered the travel time which is also a important issue. Since different speeds approaching the intersection and different accelerating strategies will lead to various emissions and travel times, there might be tradeoff between travel time and emissions, while few research efforts have been conducted on this point. So it is it is necessary to further develop an optimization problem to determine the optimal eco-driving trajectory at a signalized intersection considering both emissions and travel time.

#### Introduction of traffic emission models

Traffic emission models are important for the estimation of air emissions emitted by on-road traffic. There have been three mainstream traffic emission models used over the years in U.S.: MOBILE, CMEM and MOVES. MOBILE is one of earliest traffic emission models and the newest version is MOBILE6.2. It is a macroscopic model and estimates emissions based on only one parameter of traffic dynamics that is average speeds, so the emission estimations neglect the impact of individual vehicle stops and accelerations (EPA, 2003). Accordingly, such estimations lose accuracy in microscopic scenarios, such as individual vehicle's going through the intersection.

CMEM was developed by the University of California at Riverside (UC-Riverside). CMEM was considered microscopic because it can provide emission estimations for individual vehicles second-by-second (Barth et al., 2000). CMEM classified vehicles into 26 categories. In each category, the emission rate was determined by the vehicle speed and acceleration.

MOVES is the newest microscopic model. It is based on model activity, which represents a fundamental shift in the methodology used to estimate on-road vehicle emissions. The MOVES develops running emission rates associated with vehicle operating modes. The emission rates are dependent on second-by-second vehicle specific power (VSP) and speed. Accordingly, MOVES classifies 23 vehicle operating modes and pairs travel activities with these modal-based emission rates, allocated in units of time (EPA, 2009).

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#### **CHAPTER III**

#### MODEL DEVELOPMENT

The section presents the development of models including equations governing upstream portion and downstream portion, traffic emissions, travel time and objective model.

#### Model for speed profile

The speed profile for proposed eco-driving strategy at the signalized intersection can be divided into two portions:

i. Upstream of the intersection, to incorporate delay maintaining the distance to intersection, time to green and if any, time to clear queues in front of the vehicle, and

ii. Downstream of the intersection, where the vehicle accelerates back to its original speed.

Downstream of the intersection is considered because the emissions in that portion depends on the speed of the vehicle passing the stop-line. The lower this speed, the longer the accelerating time and the larger the fuel consumed to accelerate back and thus the more emissions.

1) Equations Governing Upstream Portion

As shown in Figure 1, it's assumed that the normal operation speed  $(v_0)$ , the distance of the vehicle from the intersection when the driver decides to decelerate (S),

the time period (T) needed when the signal turns green and the queue discharges clearly are available.

In upstream portion, the uniform acceleration model is applied, in which the deceleration rate  $(a_D)$  is constant until the vehicle decelerates to a specific value  $(v_s)$ , then the deceleration rate becomes zero and the vehicle pass the intersection with speed of  $v_s$ . Since the total distance S is equal to the sum of decelerating distance and the cruise distance  $(S_s)$  with speed of  $v_s$ , So we have:

$$\mathbf{S} = \frac{\mathbf{v_0^2 - v_s^2}}{2\mathbf{a_D}} + \mathbf{S_s}$$
 Eq. 1

$$S_s = S - \frac{v_0^2 - v_s^2}{2a_D}$$
 Eq. 2

Also, the total time T is equal to the sum of decelerating time  $(t_D)$  and the cruise time:

$$\mathbf{T} = \frac{\mathbf{v_0} - \mathbf{v_s}}{\mathbf{a_D}} + \frac{\mathbf{s_s}}{\mathbf{v_s}}$$
 Eq. 3

$$\mathbf{T} = \frac{\mathbf{v}_0 - \mathbf{v}_s}{\mathbf{a}_D} + \frac{1}{\mathbf{v}_s} \left( \mathbf{S} - \frac{\mathbf{v}_0^2 - \mathbf{v}_s^2}{2\mathbf{a}_D} \right)$$
 Eq. 4

So  $v_s$  is the positive solution of the above equation:

$$\mathbf{v}_{s} = \mathbf{v}_{0} - \mathbf{a}_{D} * \mathbf{T} + \sqrt{\mathbf{a}_{D}(\mathbf{a}_{D} * \mathbf{T}^{2} - 2\mathbf{v}_{0} * \mathbf{T} + 2 * \mathbf{S})}$$
 Eq. 5

Once  $v_s$  is computed, the decelerating time  $(t_D)$  could be calculated by:

$$\mathbf{t}_{\mathbf{D}} = \frac{\mathbf{v}_0 - \mathbf{v}_s}{\mathbf{a}_{\mathbf{D}}}$$
 Eq. 6

So the speed profile at the upstream of the intersection is:

$$\mathbf{v}_{Ut} = \begin{cases} \mathbf{v}_0 - \mathbf{a}_D * \mathbf{t} & \mathbf{0} \le \mathbf{t} \le \mathbf{t}_D \\ \mathbf{v}_s & \mathbf{t}_D \le \mathbf{t} \le \mathbf{T} \end{cases}$$
Eq. 7

$$\mathbf{a}_{\text{Ut}} = \begin{cases} \mathbf{a}_{\text{D}} & , \ \mathbf{0} \leq \mathbf{t} \leq \mathbf{t}_{\text{D}} \\ \mathbf{0} & , \ \mathbf{t}_{\text{D}} \leq \mathbf{t} \leq \mathbf{T} \end{cases}$$
 Eq. 8

#### 2) Equations Governing Downstream Portion

This part will consider two acceleration models: uniform acceleration model and non-uniform acceleration model. In the task of case study, both two models will be examined and the best one will be chosen.

If applying the non-uniform acceleration model, the accelerating rate  $(a_t)$  has a linearly decreasing relationship with speed, which is described as:

$$\mathbf{a}_{\mathbf{t}} = \mathbf{\beta}_{\mathbf{0}} - \mathbf{\beta}_{\mathbf{1}} * \mathbf{v}_{\mathbf{t}}$$
 Eq. 9

where  $v_t$  represents vehicle speed;  $\beta_0$  and  $\beta_1$  are two coefficients.

According to  $a_t = \frac{\partial v_t}{\partial t}$ , the speed during the acceleration process can be derived

$$\mathbf{v}_{\mathbf{t}} = \frac{\beta_0}{-\beta_1} \left( \mathbf{e}^{-\beta_1 * \mathbf{t}} - \mathbf{1} \right)$$
 Eq. 10

$$\mathbf{t} = \frac{1}{-\beta_1} * \log\left(\frac{-\beta_1 * \mathbf{v}_t}{\beta_0} + \mathbf{1}\right)$$
Eq. 11

So the accelerating time  $(t_A)$  for the vehicle to accelerate form  $v_s$  to  $v_0$  is:

$$\mathbf{t}_{A} = \frac{1}{-\beta_{1}} * \log\left(\frac{-\beta_{1} * \mathbf{v}_{0}}{\beta_{0}} + 1\right) - \frac{1}{-\beta_{1}} * \log\left(\frac{-\beta_{1} * \mathbf{v}_{s}}{\beta_{0}} + 1\right)$$
 Eq. 12

let: 
$$t_{v_0} = \frac{1}{-\beta_1} * \log\left(\frac{-\beta_1 * v_0}{\beta_0} + 1\right)$$
 and  $t_{v_s} = \frac{1}{-\beta_1} * \log\left(\frac{-\beta_1 * v_s}{\beta_0} + 1\right)$ , then:  
 $\mathbf{t}_{\mathbf{A}} = \mathbf{t}_{v_0} - \mathbf{t}_{v_s}$  Eq. 13

The acceleration distance (SA) is:

$$\mathbf{S}_{A} = \frac{\beta_{0} * t_{A}}{-\beta_{1}} - \frac{\beta_{0}}{\beta_{1}^{2}} \left( \mathbf{1} - \mathbf{e}^{-\beta_{1} * t_{A}} \right) + \frac{v_{s}}{-\beta_{1}} \left( \mathbf{1} - \mathbf{e}^{-\beta_{1} * t_{A}} \right)$$
Eq. 14

Denoting the total distance in consideration at downstream as D (D $\ge$  S<sub>A</sub>), so the cruise distance (DS):

$$\mathbf{D}_{\mathbf{S}} = \mathbf{D} - \mathbf{S}_{\mathbf{A}}$$
 Eq. 15

So the total time (TD) needed to drive through D is:

So the speed profile at the downstream of the intersection is:

$$\mathbf{v}_{Dt} = \begin{cases} \frac{\beta_0}{-\beta_1} \left( e^{-\beta_1 * \left( t + t_{v_s} \right)} - 1 \right) & 0 \le t \le t_A \\ \mathbf{v}_0 & \mathbf{t}_A \le t \le T_D \end{cases}$$
Eq. 17

$$\mathbf{a}_{\mathrm{Dt}} = \begin{cases} \boldsymbol{\beta}_0 - \boldsymbol{\beta}_1 * \mathbf{v}_{\mathrm{Dt}} & \mathbf{0} \le \mathbf{t} \le \mathbf{t}_{\mathrm{A}} \\ \mathbf{0} & \mathbf{t}_{\mathrm{A}} \le \mathbf{t} \le \mathbf{T}_{\mathrm{D}} \end{cases}$$
Eq. 18

## Model for traffic emissions

After second-by-second speed and acceleration data are produced according to acceleration models, MOVES is used to estimate vehicle emissions during the acceleration (including deceleration) process at an intersection. In addition to second-by-

second speed and acceleration, MOVES needs VSP to determine operating modes and to estimate emissions. VSP shall be calculated as (EPA, 2004):

$$VSP_t = 0.3227 * v_t * a_t + 0.0954 * v_t + 0.0000272 * v_t^3$$
 Eq. 19

where  $v_t$  is the instantaneous speed in mph, and  $a_t$  is the instantaneous acceleration in ft/s2. In this study, the second-by-second emission calculation and comparison adopt a set of emission rates of Carbon Monoxide (CO) for the evaluation year 2010 (EPA, 2009), which are shown in Table 65.

$$\mathbf{E}_{\mathbf{t}} = \mathbf{f} \left( \mathbf{VSP}_{\mathbf{t}}, \, \mathbf{v}_{\mathbf{t}}, \mathbf{a}_{\mathbf{t}} \right)$$
 Eq. 20

Then the total emissions (E) equals the sum of emissions of all seconds including both upstream portion and downstream portion of the intersection.

$$\mathbf{E} = \sum \mathbf{E}_{\mathbf{Ut}} + \sum \mathbf{E}_{\mathbf{Dt}}$$
 Eq. 21

#### Model for travel time

Since for upstream portion of the intersection, the approaching vehicle has to wait for time of T to pass the intersection, the travel time for upstream portion of the intersection is a constant value of T. For downstream portion of the intersection, the travel time is the time needed for the driver to drive distance of D, it equals the sum of the accelerating time ( $t_A$ ) for the vehicle to accelerate form  $v_s$  to  $v_0$ , and the cruise time. So in this research the travel time (TT) can be represented as the travel time during the downstream portion of the intersection:

$$TT = T_D = t_A + \frac{D_S}{v_0}$$
 Eq. 22

Optimization model

The objective of this optimization problem is minimization of a linear combination of emissions and travel time:

O.B. Min: 
$$w * \frac{E}{E_B} + (1 - w) * \frac{TT}{TT_B}$$
 Eq. 23

where w is the weight of emissions,  $E_B$  is the base value for emissions under the strategy when only emissions is minimized and  $TT_B$  is the base value for travel time under the strategy when only travel time is minimized.

The decision variables will be  $a_D$ ,  $\beta_0$  and  $\beta_1$ .

# CHAPTER IV

## CASE STUDY

In this section, several case studies are conducted to demonstrate the application of the developed model. Three solution methods are applied and investigated aiming to find a practicable and efficient method to solve the proposed optimization problem. In addition, sensitivity analysis and baseline study are conducted to evaluate the performance of ecodriving in various scenarios with different S, T and w.

#### **Investigation of solution methods**

There are three decision variables in the proposed optimization problem, what's more, the nonlinear and piecewise nature of traffic emission model makes the proposed problem more complicated, and makes it a challenge to solve this problem. This part aims to find a practicable and efficient method to solve the proposed optimization problem. Three methods are applied to solve the proposed problem and their results are compared, they are: enumeration method, simplex optimization, and genetic algorithm.

Following are the introduction of these three methods and their application in two cases. The two cases used to compare these three methods are:

Case 1: T=10 s, S=400 ft, D=1400 ft, vo=58.667 ft/s (40 mile/h) Case 2: T=20 s, S=400 ft, D=1400 ft, vo=58.667 ft/s (40 mile/h)
#### 1) Enumeration Method (EM)

Enumeration method (Venkataraman, 2001) is the simplest of the combinatorial optimization techniques. The principle of this method is to evaluate all combinations of the discrete variables. The total number of evaluation  $(n_e)$  is:

$$n_e = \prod_i^{n_d} pi$$

 $n_d$  is the number of discrete variables, pi is the pre-established set of discrete values. The optimal solution obtained is thus the minimum value by scanning the list of feasible solutions. The interval of the pre-established set of discrete values has a significant impact on the solution. If the interval is too large, the enumeration method may fail to find the optimal solution. A small interval would help this method to assure a satisfied global optimum, but the computational time will be very huge.

The specific procedure of application of enumeration method in this case study is:

- 1. Setting the minimum and maximum value for  $a_D$ ,  $\beta_0$ ,  $\beta_1$ .
- 2. Setting a reasonable increase interval for  $a_D$ ,  $\beta_0$ ,  $\beta_1$ , then calculate the objective model for all combinations of  $a_D$ ,  $\beta_0$ ,  $\beta_1$  between their minimum and maximum value.
- 3. Taking the combination of  $a_D$ ,  $\beta_0$ ,  $\beta_1$  which leads to the minimum value for the objective model as the solution.
- The Range of  $a_D$ ,  $\beta_0$ ,  $\beta_1$

According the Sun's study (Sun et al., 2013), we set the range for deceleration rate  $(a_D)$  and  $(a_t)$  as:

$$2 * (vo * T - S)/T^2 \le a_D \le 23 \text{ ft/s2} (7m/s2)$$
  
and  $0 \le a_t \le 11.5 \text{ ft/s2} (3.5m/s2)$ 

For  $\beta_0$  and  $\beta_1$ , according to Jinpeng's dissertation (2012), we set the range for them as:

$$0.02 \le \beta_1 \le 0.2$$

then decide the range of  $\beta_0$  as:

$$\beta_1 * \mathbf{v}_0 \le \beta_0 \le 11.5 + \beta_1 * \mathbf{v}_s$$

Another limitation is that the acceleration distance  $(S_A)$  should be shorter than the study region after the intersection (D):

$$S_{A} = \frac{\beta_{0} * t_{A}}{-\beta_{1}} - \frac{\beta_{0}}{\beta_{1}^{2}} (1 - e^{\beta_{1} * t_{A}}) + \frac{v_{s}}{-\beta_{1}} (1 - e^{\beta_{1} * t_{A}}) \le D$$

• The Increase Interval for  $a_D$ ,  $\beta_0$ ,  $\beta_1$ 

The increase interval for  $a_D$  is 1 ft/s2.

The increase interval for  $\beta_0$  is 1.

The increase interval for  $\beta_1$  is 0.01.

This setting of increase interval may be a little large but it's enough to show

something significant, especially considering that a smaller interval only achieves very

limited marginal improvement while costs much more time to run the program.

The unit for each parameter in all tables in this thesis is listed in Table 1.

The unit for each parameter in all tables in this thesis is listed in Table 1. The solution derived from enumeration method and the corresponding performance are summarized in Table 2 and Table 3.

#### **Table 1 Unit of each parameter**

Parameter	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
Unit	mg	S	ft/s <sup>2</sup>	ft/s	ft/s <sup>2</sup>	1/s	ft	S

For Case 1:

Table 2 Solution for Case 1 from EM

W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	876.2964	24.15342	23	39.17396	13	0.04	86.24534	1.76
1	181.3729	26.86668	23	39.17396	2	0.02	956.672	19.31

For Case 2:

#### Table 3 Solution for Case 2 from EM

w	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	1002.661	25.12615	23	18.22195	12	0.03	144.7519	3.73
1	198.3334	31.35997	19	17.80278	4	0.06	1329.602	30.16

One remarkable conclusion is that though the theory and procedure of

enumeration method is simple and straightforward, it takes lots of time to run the whole

combinations of the discrete variables. For each scenario, the average calculation time is 92 seconds.

2) Simplex Optimization (SO)

Simplex is a simple optimization algorithm seeking the vector of parameters corresponding to the global extreme (maximum or minimum) of any n-dimensional function F(x1, x2,..,xn), searching through the parameter space ("search area").

This method is widely used in chemistry researches, the goal may be the search for optimal conditions for obtaining the maximum yield of a compound, e.g. % yield as a function of reflux time and of excess of a particular reagent, or the resolution of two or more chromatographic peaks as a function of the flow rate and the composition of the eluant. Simplex optimization could be easily exercised in MATLAB by using fminsearch function. Function fminsearch uses the Nelder-Mead simplex algorithm as described in Lagarias et al. (1998). This algorithm uses a simplex of n + 1points for n-dimensional vectors x. The algorithm first makes a simplex around the initial guess x0 by adding 5% of each component x0(i) to x0, and using these n vectors as elements of the simplex in addition to x0. Then, according to the rules, the algorithm modifies the simplex repeatedly by either reflect, expand, contract outside, contract inside or shrink as shown in Figure 2.



**Figure 2 Calculation procedure of fminsearch function (From website)** 

The iterations are terminated when no more significant improvement of the response is observed on moving from one simplex to the other and/or the displacements are insignificant.

It should be stressed that when there are local extremes, it is highly probable the algorithm to fail and be trapped there, instead of the global extreme.

The problem requires such inputs as the time period needed when the signal turns green and the queue discharges clearly, the distance of the vehicle from the intersection when the driver decides to decelerate, the study region in distance after the intersection, the normal vehicle operating speed, and the weight of Emissions, and the decision variables include the deceleration rate and two acceleration parameters.

The solution derived from simplex optimization and the corresponding performance are shown in Table 4 -9:

For Case 1:

W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	876.2964	24.15342	23	39.17396	13	0.04	86.24534	1.76
1	876.2964	24.15342	23	39.17396	13	0.04	86.24534	1.76

Table 4 Solution for Case 1 from SO with start point of: ad=23,  $\beta 0=13$ ,  $\beta 1=0.04$ 

#### Table 5 Solution for Case 1 from SO with start point of: ad=23, $\beta$ 0=2, $\beta$ 1=0.02

W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	181.3729	26.86668	23	39.17396	2	0.02	956.672	19.31
1	181.3729	26.86668	23	39.17396	2	0.02	956.672	19.31

Table 6 Solution for Case 1 from SO with start point of: ad=13, β0=9.6, β1=0.15

W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	862.129	24.151	21.030	39.089	12.282	0.020	84.636	1.730
1	298.359	26.758	22.792	39.166	5.162	0.085	1398.965	26.740

For Case 2:

### Table 7 Solution for Case 2 from SO with start point of: ad=23, $\beta 0=12$ , $\beta 1=0.0.3$

W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	1002.661	25.12615	23	18.22195	12	0.03	144.7519	3.73
Ť								
1	1002.661	25.12615	23	18.22195	12	0.03	144.7519	3.73

				vien seen	e point of		, po ., p-	0.00
W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	1054.459	25.148	23.000	18.222	12.349	0.047	149.919	3.840
1	180.305	30.929	23.000	18.222	3.800	0.053	1138.967	26.480

Table 8 Solution for Case 2 from SO with start point of: ad=19,  $\beta$ 0=4,  $\beta$ 1=0.06

Table 9 Solution for Case 2 from SO with start point of: ad=13, β0=9.6, β1=0.15

W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	974.407	25.113	23.000	18.222	11.792	0.020	141.984	3.670
1	1289.667	28.240	13.955	16.871	9.085	0.153	1400.000	28.240

The most remarkable advantage of simplex optimization is that its calculating is very fast. It could provide the solution almost immediately after inputting the command code into MATLAB. The results show that the simplex optimization could achieve some improvement based the start point. For example, in case 2, with start point of: ad=19,  $\beta 0=4$ ,  $\beta 1=0.06$ , the simplex optimization could lead to the better solution and achieve about 9% emissions reduction compared with the start point. However, it's also noticeable that sometimes the simplex optimization fails to work. For example, in case 1, simplex optimization cannot find a better solution with start point of: ad=23,  $\beta 0=2$ ,  $\beta 1=0.02$ . Another remarkable conclusion is that the start point has a significant impact on the solution. For example, in case 1, when minimizing the emissions, with the start point of: ad=23,  $\beta 0=13$ ,  $\beta 1=0.04$ , and ad=23,  $\beta 0=2$ ,  $\beta 1=0.02$ , the difference of emissions

got from these two start point is about 80%. It indicates that this method may fail to find out the global optimal solution without using a good start point.

#### 3) Genetic Algorithm

Genetic algorithm (GA) is a heuristic method for solving optimization problems. Borrowing the concept of biological evolution, GA repeatedly modifies a population of individual solutions, also named chromosomes. The evolution usually starts from a population of randomly generated individuals and is an iterative process, with the population in each iteration called a generation. In each generation, the fitness of every individual in the population is evaluated; the fitness is usually the value of the objective function in the optimization problem being solved. The more fit individuals are stochastically selected from the current population, and each individual's genome is modified (recombined and possibly randomly mutated) to form a new generation. The new generation of candidate solutions is then used in the next iteration of the algorithm. Commonly, the algorithm terminates when either a maximum number of generations has been produced, or a satisfactory fitness level has been reached for the population. During every evolution, based on the fitness function values, the parent chromosomes produce their children through the selection, crossover and mutation rules.

GA is widely used in transportation studies especially in optimization problems. Chakroborty and Mandal (in press) proposed a single GA based algorithm called ROUTER to solve the traveling salesman problem and the single vehicle pick-up and delivery problem. Their study results show that the proposed algorithm was faster than similar algorithms. Chakroborty and Dwivedi (2002) developed a GA based algorithm for transit route design problem. Chan et al. (2002) presented a GA approach to search the optimized path for Traveling Sales Problems. Shandiz et al. (2009) applied GA to solve their proposed method for controlling traffic lights in order to have maximum flow in the route which result in a moving traffic. Lin et al. (2009) used the genetic algorithm to find the shortest time in driving with diverse scenarios of real traffic conditions and varying vehicle speeds. They concluded that the effectiveness of the genetic algorithm is clearly demonstrated when applied on a real map of modern city with very large vertex numbers. Fan and Machemehl (2006) applied a genetic algorithm to systematically examine the underlying characteristics of the optimal bus transit route network design problem with variable transit demand. The results of the case study conducted in their research showed that the genetic algorithms outperform local search methods with multiple starting points and provide no worse solution quality than either simulated annealing or tableau search algorithm. All these studies demonstrate the application and the advantage of GA in transportation studies especially in solving optimization problems.

Figure 3 illustrates the process of GA searching optimal solution in MATLAB. The population size is set to be 1000; other GA parameters such as the selection, crossover, and mutation adopt the default values in MATLAB. The evolution usually starts from a population of randomly generated individuals and is an iterative process, with the population in each iteration called a generation. In each generation, the fitness of every individual in the population is evaluated; the fitness is usually the value of the objective function in the optimization problem being solved. The more fit individuals are stochastically selected from the current population, and each individual's genome is modified (recombined and possibly randomly mutated) to form a new generation. The new generation of candidate solutions is then used in the next iteration of the algorithm.

The problem requires such inputs as the time period needed when the signal turns green and the queue discharges clearly, The distance of the vehicle from the intersection when the driver decides to decelerate, the study region in distance after the intersection, the normal vehicle operating speed, and the weight of Emissions, and the decision variables include the deceleration rate and two acceleration parameters. Figure 4 illustrates the application of GA to solving the optimization problem.



Figure 3 GA toolbox in MATLAB



Figure 4 Application of GA to solving the optimization problem

The solution derived from GA and the corresponding performance are shown in

Table 10 - 17:

For Case 1:

|--|

W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	885.431	24.166	14.509	38.614	12.339	0.022	86.687	1.780
1	161.093	26.749	7.258	36.665	3.061	0.036	837.426	17.160

Table 11 Solution for Case 1 from GA with initial population: ad=23,  $\beta 0=13$ ,  $\beta 1=0.04$ 

w	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	866.881	24.151	22.998	39.174	12.817	0.034	85.217	1.740
1	156.680	26.211	22.968	39.173	3.233	0.040	787.445	15.770

Table 12 Solution for Case 1 from GA with initial population: ad=23, β0=2, β1=0.02

W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	857.466	24.148	22.999	39.174	12.282	0.020	84.195	1.720
1	155.707	28.094	23.000	39.174	1.691	0.020	1398.602	28.070

# Table 13 Solution for Case 1 from GA with initial population: ad=13, $\beta 0=9.6$ , $\beta 1=0.15$

W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	857.466	24.148	22.990	39.174	12.289	0.020	84.192	1.720
1	156.265	26.246	22.708	39.162	2.920	0.034	781.326	15.700

For Case 2:

W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	974.514	25.177	15.397	17.209	11.897	0.024	144.086	3.770
1	172.034	31.507	15.012	17.126	3.308	0.042	1092.762	26.270

 Table 14 Solution for Case 2 from GA with default initial population

Table 15 Solution for Case 2 from GA with initial population of : ad=23,  $\beta$ 0=12,  $\beta$ 1=0.03

W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	993.242	25.121	22.999	18.222	12.046	0.030	143.885	3.710
1	171.217	31.410	20.888	18.023	3.166	0.039	1087.291	26.080

Table 16 Solution for Case 2 from GA with initial population of : ad=19,  $\beta$ 0=4,  $\beta$ 1=0.06

W	Emissions	Travel Time	ad	vs	β0	β1	Sa	ta
0	960.277	25.106	22.997	18.222	11.868	0.020	140.672	3.640
1	171.590	31.607	19.712	17.891	3.041	0.036	1095.680	26.420
						•		

Table 17 So	olution for (	Case 2 from C	<b>FA with initi</b>	al population (	of : ad=13, β0=9.6,
β1=0.15					

W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	964.987	25.108	22.998	18.222	11.889	0.022	141.125	3.650
1	171.340	31.484	17.559	17.599	3.203	0.039	1088.827	26.180

The results show that the difference of the solutions derived from GA and their corresponding performance are very small even with various initial population, which indicates that the start point hardly impact the final solution. For example, in case 1, the largest difference of emissions got from various initial population is only about 3%. Another conclusion is that the calculation time of GA is acceptable, in this case, the average calculation time is 17 seconds.

#### 4) Compare of three methods

The results derived from EM, SP and GA show that:

Compared with EM, GA has the advantage of not only could find the optimal solution but also save lots of calculation time.

Compared with SP, GA takes more time to calculation, but GA has the advantage of could find the optimal solution without a good start point. This advantage is significant as it's difficult even impractical to predict a good start point when solve the proposed optimization problem. In addition, an average calculation time of 17 seconds is acceptable considering its powerful ability to find the optimal solution.

So overall, GA is determined to be the most practicable and efficient method to solve the proposed optimization problem and will be applied in the following sensitivity study.

#### Sensitivity study

In the sensitivity study, the optimal deceleration rate and acceleration parameters will be solved in various scenarios with different S, T and  $w_i$ , and their performance

including ravel time and emissions will be evaluated. GA is used to find the optimal solution. When applying the GA, the population size is 1000, the initial population is  $[ad=13, \beta_0=9.6, \beta_1=0.15]$ . For each scenario, it requires that running GA at least twice and accept the optimal result as the final solution.

26 groups of case study were conducted as summarized in Table 18:

**Table 18 Scenarios of Case Study** 

S (ft)	T (s)
400	8, 10, 12, 14, 16, 18, 20, 22
600	12, 14, 16, 18, 20, 22
800	14, 16, 18, 20,22, 24
1000	18, 20, 22, 24 , 26, 28

For each group, for w from 0 to 1 with increase interval of 0.5 (totally 3 scenarios), the optimal  $a_D$ ,  $\beta_0$ ,  $\beta_1$  were solved respectively, and the corresponding emissions and travel time were estimated. The meaning of different values of w is shown in Table 19:

## Table 19 Meaning of Different Values of w

Value of <i>w</i>	Meaning
0	Minimizing the travel time.
0.5	Treating travel time and emissions equally.
1	Minimizing the emissions.

The results are shown in the Figure 5 - 18 and Table 20 - 33.

## Table 20 Emissions when S=400ft

Emissions		T (s)										
(mg)												
W	8	10	12	14	16	18	20	22				
0	415.0	857.4	938.0	953.1	972.7	983.0	964.9	1003.3				
	0	7	6	0	9	4	9	8				
0.5	67.27	158.3	163.7	166.4	173.7	170.9	171.8	173.39				
		7	9	7	0	0	1					
1	67.05	156.2	163.1	166.1	169.8	170.4	171.3	172.42				
		7	6	8	0	0	4					



Figure 5 Emissions when S=400ft

Travel Time (s)		T (s)								
W	8	10	12	14	16	18	20	22		
0	23.92	24.15	24.40	24.62	24.81	24.98	25.11	25.22		
0.5	25.42	26.32	27.77	28.95	29.76	30.64	31.23	31.69		
1	25.42	26.25	27.99	29.14	30.67	30.70	31.48	32.30		

Table 21 Travel Time when S=400ft



Figure 6 Travel Time when S=400ft

Emissions (mg)		T (s)								
w	12	14	16	18	20	22				
0	421.67	724.27	970.76	958.51	968.73	955.39				
0.5	74.11	83.39	162.77	166.29	167.16	171.34				
1	71.20	83.31	162.51	166.13	166.97	168.53				

Table 22 Emissions when S=600ft



Figure 7 Emissions when S=600ft

Table 23	Travel	Time	when	S=600ft
Table 25	Iravei	1 me	when	5=00011

Travel Time (s)	T (s)					
W	12	14	16	18	20	22
			10	10		
0	23.92	24.06	24.22	24.38	24.53	24.66
0.5	25.47	26.90	26.72	26.74	28.48	29.14
1	25.83	26.91	26.76	27.72	28.53	29.29



Figure 8 Travel Time when S=600ft

Emissions (mg)		T (s)					
W	14	16	18	20	22	24	
0	124.19	423.63	657.00	851.67	950.00	946.05	
0.5	40.01	73.31	84.87	162.08	165.19	167.01	
1	39.23	73.17	84.01	160.91	164.51	166.29	

Table 24 Emissions when S=800ft



Figure 9 Emissions when S=800ft

Travel Time (s)	T (s)					
W	14	16	18	20	22	24
0	23.87	23.92	24.02	24.14	24.25	24.37
0.5	24.36	25.83	26.13	26.43	26.96	27.73
1	24.39	25.84	26.52	26.88	26.97	27.73

Table 25 Travel Time when S=800ft



Figure 10 Travel Time when S=800ft

Emissions (mg)	T (s)					
W	18	20	22	24	26	28
0	196.65	430.30	613.26	779.68	917.88	932.88
0.5	59.63	79.41	85.66	93.77	165.70	168.00
1	58.32	75.24	85.44	93.68	165.49	166.59

Table 26 Emissions when S=1000ft



Figure 11 Emissions when S=1000ft

Travel Time (s)	T (s)					
W	18	20	22	24	26	28
0	23.87	23.92	24.00	24.09	24.18	24.28
0.5	24.30	25.83	26.14	26.73	27.09	27.23
1	24.33	26.00	26.15	26.76	27.17	27.23

Table 27 Travel Time when S=1000ft



Figure 12 Travel Time when S=1000ft

## Table 28 Emissions when T=18s

Emissions (mg)	T (s)					
W	400	600	800	1000		
0	983.04	958.51	657.00	196.65		
0.5	170.90	166.29	84.87	59.63		
1	170.40	166.13	84.01	58.32		



Figure 13 Emissions when T=18s

Table 29 Travel Time when T=18s	Table 2	29 ]	Fravel	Time	when	T=18s
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Travel Time (s)	T (s)					
W	400	600	800	1000		
0	24.98	24.38	24.02	23.87		
0.5	30.64	26.74	26.13	24.30		
1	30.70	27.72	26.52	24.33		



Figure 14 Travel Time when T=18s

## Table 30 Emissions when T=20s

Emissions (mg)	T (s)					
W	400	600	800	1000		
0	964.99	968.73	851.67	430.30		
0.5	171.81	167.16	162.08	79.41		
1	171.34	166.97	160.91	75.24		



Figure 15 Emissions when T=20s

Table 31 Travel	Time whe	en T=20s
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Travel Time (s)	T (s)				
W	400	600	800	1000	
0	25.11	24.53	24.14	23.92	
0.5	31.23	28.48	26.43	25.83	
1	31.48	28.53	26.88	26.00	



Figure 16 Travel Time when T=20s

## Table 32 Emissions when T=22s

Emissions (mg)	T (s)							
W	400	600	800	1000				
0	1003.38	955.39	950.00	613.26				
0.5	173.39	171.34	165.19	85.66				
1	172.42	168.53	164.51	85.44				



Figure 17 Emissions when T=22s

## Table 33 Travel Time when T=22s

Travel Time (s)	T (s)							
W	400	600	800	1000				
0	25.22	24.66	24.25	24.00				
0.5	31.69	29.14	26.96	26.14				
1	32.30	29.29	26.97	26.15				



Figure 18 Travel Time when T=22s

Based on the results of the sensitivity analysis, it's can be concluded that:

- From minimizing travel time (w=0) to minimizing emissions (w=1), the emissions reduces significantly by about 84%, while the travel time only increase about 22%. It because when only concerns travel time, the acceleration rate is very large so that lead to a large value of VSP, and result in large emissions rate.
- When concerning travel time and emissions with equal weight (w=0.5), the optimal solutions of  $\beta_0$ ,  $\beta_1$  are almost same as when only minimizing emissions (w=1). So their corresponding travel time and emissions are almost overlap in the above figures especially for the emissions. From w=0 to w=0.5, the emissions also reduces significantly for about 84%, while the travel time only increase about 19%. It because the difference of emissions

between when w=0 and w=1 respectively is much larger than that of travel time. For emissions, the difference is usually 1000 or more, while for travel time, it's only about 5 or less.

- When S is fixed, with T increases, the general trends of both of Emissions and Travel Time are increasing. It because larger T leads to smaller vs, thus lead to more fuel consumption and acceleration time to accelerate to vo from vs, finally result in larger emissions and travel time.
- When S is fixed, the increase of emissions is relatively dramatic with certain increase of T than others. For example, when S=400, the increase of emissions is larger when T changes from 8 to 10. And after that, the increases of emissions are not as noticeable as this certain increase of T. For S=600, such certain increase of T is from 12 to 14, for S=800 is form 16 to 18, for S=1000 is from 20 to 22.
- When T is fixed, with S increases, the general trends of both of Emissions and Travel Time are decreasing. It because larger S leads to larger vs, thus lead to less fuel consumption and acceleration time to accelerate to vo from vs, finally result in decreasing of emissions and travel time.
- When T is fixed, the decrease of emissions is relatively dramatic with certain increase of S than others. For example, when T=18, the decrease of emissions is larger when S changes from 800 to 1000. And before that, the decrease of emissions are not as noticeable as this certain increase of S. For T=20, such certain increase of S is still from 800 to 1000.

• Difference of both emissions and travel time between minimizing emissions and minimizing travel time increase if T increase, while decrease while S increase. When T is very small, the difference is small, and when T increase, the emissions obtained from minimizing travel time increase dramatically.

#### **Baseline study**

It's noticeable that when minimizing the emissions, the corresponding acceleration rate is small, thus result in a long acceleration time, and when minimizing the travel time, the corresponding acceleration rate is large, thus result in a short acceleration time. While in practice, some drivers may not like to change their deceleration and acceleration rate to apply the recommended speed trajectory.

This part aims to investigate the benefits of eco-driving under such situation: the drivers would always keep their driving habit during decelerating and accelerating process, which means that the drivers would apply fixed deceleration rate ( $a_D$ ) and acceleration parameters ( $\beta_0$  and  $\beta_0$ ). For eco-driving, the drivers would decelerate in advance to avoid fully stop at the stop line, while for normal-driving, the drivers would firstly remain their normal operation speed (vo), and then decelerate to fully stop at the stop line.

Assuming that both the eco-driving and normal driving strategy will apply the following deceleration rate and acceleration parameters:

a<sub>D</sub>: 13 ft/s<sup>2</sup> (4 m/s<sup>2</sup>)  
$$\beta_0$$
: 9.6

## $\beta_1: 0.15$

In which, the deceleration rate is 4 m/s2, and the maximum acceleration rate is 3 m/s2.

Two cases are evaluated:

Case 1: S=400 ft, D=1400 ft, vo=58.667 ft/s (40 mile/h)

Case 2: S=800 ft, D=1400 ft, vo=58.667 ft/s (40 mile/h)

The results are shown in the Figure 19 - 22 and Table 34 - 37, and more detail

information are shown in Table 39 - 64.

Table 34 Emissions when S=4	DOft
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Emissions (mg)	T (s)								
	8	9	10	11	12	13			
Decelerate in	729 41	1603.90	2289.43	2473.87	2474.89	2475.29			
advance	727.71	8	6	8	5	9			
Normal Driving	2484.8	2484.87	2484.89	2484.91	2484.93	2484.95			
Normal-Driving	5	7	7	6	5	5			



Figure 19 Emissions when S=400ft

Table 35 Travel Time when S=400ft

Travel time (s)	T (s)							
	8	9	10	11	12	13		
Decelerate in advance	24.301	24.776	25.215	25.595	25.917	26.196		
Normal-Driving	29.031	29.031	29.031	29.031	29.031	29.031		



Figure 20 Travel Time when S=400ft

Emissions (mg)	T (s)							
Linissions (mg)	16	17	18	19	20	21	22	
Decelerate in	720.7	1084.	1542.3	1894.7	2203.3	2439.8	2498.4	
advance	6	2	7	6	6	2	1	
Normal Driving	2496.	2496.	2496.2	2496.3	2496.3	2496.3	2496.3	
Normai-Driving	2	2	8	0	2	4	6	

## Table 36 Emissions when S=800ft



Figure 21 Emissions when S=800ft

Travel time (s)	T (s)							
	16	17	18	19	20	21	22	
Decelerate in advance	24.280	24.508	24.738	24.946	25.143	25.329	25.490	
Normal-Driving	29.031	29.031	29.031	29.031	29.031	29.031	29.031	



Figure 22 Travel Time when S=800ft
Based on the results of the baseline study, it's can be concluded that:

- Eco-driving strategy which requires drivers to decelerate in advance finds its benefits on reduction in both emissions and travel time compared with normal-driving strategy. For example, in scenario of S=400ft and T=8s, Decelerate in advance could lead to about 70% reduction in emissions and 16% reduction in travel time. The reason for such reduction is that, if the drivers decelerate in advance so that make sure that they can pass intersection without fully stop, they just need to accelerate from a speed larger than zero (vs) but not from zero, thus would decrease time and fuel consumption needed for acceleration, and result in less emissions and travel time.
- For normal-driving strategy, the travel time remains unchanged being 29s regardless different T and S. It because in all scenarios, the drivers need to accelerate from speed of zero to the normal operation speed (vo), and then remain vo until reaching to the end of the study region. With the fixed acceleration parameters, such acceleration time and cruise time will be same. When S is fixed, the emissions almost remain same regardless different T. For example, in Case 1, the emissions remains approximate 2485mg regardless T increase from 8s to 13s. It because different T only impact the idling time, while the emissions rate of idling is too small compared with that of acceleration (idling emissions rate: 0.019mg/s, acceleration emissions

rate:>80mg/s). And as the acceleration process is same for all scenarios, the total emissions will almost remain the same as well.

- For eco-driving strategy (decelerate in advance), both of emissions and travel time will be different with various T and S. When S is fixed, both of emissions and travel time will increase as the T increases, especially for the emissions. For example, in Case 2, the emissions and travel time increase by about 246% and 5% respectively as T increase from 16s to 22s. The reason for the dramatic change of emissions lies in the acceleration process. As different T will lead to different vs, the larger T is, the smaller vs is. And it takes more time and fuel consumption to accelerate from a smaller vs to the normal operation speed, thus result in longer travel time and larger emissions.
- For eco-driving strategy (decelerate in advance), the emissions will firstly increase as the T increases until it become approximately same as that of normal-driving strategy, then almost remain constant regardless the T continue increasing. For example, in Case 1, the emissions of eco-driving increases from about 730mg to 2470mg (approximately same as the emissions of normal-driving: 2480mg) as T increases from 8s to 11s, and after that, the emissions of eco-driving remains about 2470mg regardless the T continue increasing . The reason for such stable condition lies in the mechanism of calculation of emissions rate. The emissions rate is determined by VSP and speed, and when speed is small (<25 mile/hr), the emission rate will also be very small thus the acceleration process before

speed get to 25 mile/hr only contributes little to the total emissions. So when T increases to a value which make vs smaller than 25 mile/hr, the total emissions will almost remain the same regardless T continue increasing. In the above example, when T=11s, the vs=23.2 mile/hr <25 mile/hr, so the emissions will almost remain the same regardless T continue increasing to 13s.

- Based on above analysis, it can be concluded that there existing a critical T (TC) for each S, and when T is smeller than TC, the eco-driving strategy could achieve remarkable emissions reduction, while when T is larger than TC, the eco-driving strategy only could lead to little emissions reduction. TC can be inverse calculated by setting vs=25 mile/hr. In Case 1 and Case 2, TC is approximate 11s and 21s respectively, and it can be demonstrated by Figure 19 and Figure 21.
- When T is larger than TC, eco-driving strategy still could achieve benefits
  on saving travel time although it would not reduce emissions remarkably any
  more. For example, in Case 1, when T=13s>TC=11s, the eco-driving
  strategy only achieve 0.4% in emissions reduction, but could save
  approximate 10% travel time.

For scenario of S=400ft and T=10s, we denote the results derived from completely eco-driving with w=1 as Case 1, the results derived from completely ecodriving with w=0 as Case 2, the results derived from only decelerating in advance as Case 3, and the results derived from normal-driving as Case 4. Table 38 summaries the performance of these four cases, and Figure 23 shows the speed trajectory of these four cases.

	•	W	Emissions	Travel Time	ad	VS	β0	β1
Case 1	Eac driving	1	156	26.25	22.71	39.16	2.92	0.03
Case 2	Eco-arrying	0	857	24.15	22.99	39.17	12.29	0.02
Case 3	Decelerate in adv	ance	2289	25.22	13.00	38.42	9.60	0.15
Case 4	Normal-drivin	g	2485	29.03	13.00	0.00	9.60	0.15

**Table 38 Summary of four cases** 



Figure 23 Speed Trajectory of four cases

Table 38 demonstrates that the eco-driving strategy when minimizing the emissions could significantly reduce the emissions with a satisfied travel time compared with other three strategies. The normal-driving strategy leads to the worst performance both on emissions and travel time. Though the strategy of decelerating in advance works better than the normal-driving strategy, it would generate much more emissions compared with the eco-driving strategy when minimizing the emissions (about 93% more emissions), but only save little travel time (about 4% less travel time), which demonstrates the remarkable potential of completely eco-driving strategy on reducing emissions without significantly increasing the travel time. Figure 23 shows that the speed trajectory derived from eco-driving strategy is more smooth than that from normal-driving.

# CHAPTER V

#### **CONCLUSIONS AND FUTURE WORK**

This research developed an optimization model for eco-driving at signalized intersection to determine the optimal eco-driving trajectory (the speed profile) at a signalized intersection, which aims to achieve the minimization of a linear combination of emissions and travel time. Then enumeration method, simplex optimization and genetic algorithm were investigated to determine a practicable and efficient method to solve the proposed optimization problem. As various scenarios of distance from the vehicle to the intersection, queue discharging time and weights of emission/travel time will lead to different optimal trajectories and different emissions and travel times. A sensitivity study was conducted to analyze and compare the performance of the optimal solution in various scenarios of different such parameters. In addition, a baseline study was conducted to investigate the benefits of eco-driving when drivers only decelerate in advance but not apply the recommended speed trajectory. The research resulted in the following conclusions:

1. Genetic algorithm is a practicable and efficient method to solve the proposed optimization problem because of its advantage of could find the optimal solution regardless whether has a good start point and a acceptable calculation time.

2. Eco-driving strategy could achieve satisfied reduction in emissions without significantly increasing travel time. When concerning travel time and emissions with equal weight, the optimal solutions of  $\beta_0$ ,  $\beta_1$  are almost same as when only minimizing

emissions. When S is fixed, with T increases, the general trends of both of Emissions and Travel Time are increasing. When T is fixed, with S increases, the general trends of both of Emissions and Travel Time are decreasing. Difference of both emissions and travel time between minimizing emissions and minimizing travel time increase if T increase, while decrease while S increase. When T is very small, the difference is small, and when T increase, the emissions obtained from minimizing travel time increase dramatically. Overall, Emissions is more sensitive to various scenarios than travel time.

3. Eco-driving still could achieve remarkable reduction in emissions as long as the drivers decelerate earlier even though the they would not apply the recommended speed trajectory. There exists a critical T (TC) for each S. When T is smeller than TC, the eco-driving strategy could achieve remarkable emissions reduction, and when T is larger than TC, the eco-driving strategy could only lead to little emissions reduction, but still could achieve benefits on saving travel time.

#### **Future work**

1. This research only focused on the emissions type of CO, while other emissions like CO2 or NOx has different emissions rate, so the optimal speed trajectory and the corresponding performance of eco-driving when considering other emissions will be different as well. It's necessary to explore the application the proposed model on more other types of emissions and evaluate the performance and benefits of eco-driving on different types of emissions.

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2. The traffic emissions model is critical in this research as it is directly involved in the calculation of emissions thus has a significant impact on the optimal solution. This study only applied the MOVES model. While there are also many other widely-used traffic emissions model like NCSU model and CMEM model. It's necessary to explore the application of the proposed model and evaluate the corresponding performance and benefits of eco-driving when using other traffic emissions model.

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#### APPENDIX A

# Solutions of sensitivity study:

## Table 39 Solution when S=400ft and T=8s

W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	414.996	23.922	22.995	49.786	12.631	0.023	41.723	0.77
0.5	67.266	25.417	17.709	49.717	2.968	0.047	1394.314	25.32
1	67.049	25.421	10.678	49.509	3.223	0.052	1396.448	25.36

#### Table 40 Solution when S=400ft and T=10s

W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	857.466	24.148	22.99	39.174	12.289	0.02	84.192	1.72
0.5	158.366	26.316	22.043	39.135	2.705	0.030	794.818	16
1	156.265	26.246	22.708	39.162	2.920	0.034	781.326	15.7

### Table 41 Solution when S=400ft and T=12s

W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	938.061	24.403	23.000	32.050	12.560	0.033	109.169	2.4
0.5	163.786	27.775	22.995	32.050	3.122	0.038	922.175	19.63
1	163.158	27.991	17.006	31.529	2.911	0.033	935.269	20.07

W	Emissions	Travel Time	ad	vs	β0	β1	Sa	ta
0	953.097	24.621	22.991	27.015	12.073	0.021	121.587	2.83
0.5	166.470	28.948	21.933	26.931	3.311	0.042	1001.156	22.15
1	166.183	29.137	21.160	26.864	3.005	0.035	1002.433	22.36

## Table 42 Solution when S=400ft and T=14s

#### Table 43 Solution when S=400ft and T=16s

W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	972.790	24.810	22.994	23.300	11.969	0.020	130.429	3.17
0.5	173.703	29.755	22.979	23.299	3.640	0.049	1077.022	24.25
1	169.798	30.672	13.666	21.911	2.840	0.032	1070.758	25.06

#### Table 44 Solution when S=400ft and T=18s

W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	983.044	24.983	22.993	20.459	12.099	0.029	139.097	3.49
0.5	170.901	30.641	22.995	20.459	3.265	0.041	1066.123	24.95
1	170.402	30.700	19.158	20.061	3.331	0.042	1072.032	25.11

W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
					,	,		
0	964.987	25.108	22.998	18.222	11.889	0.022	141.125	3.65
0.5	171.811	31.233	22.983	18.220	3.288	0.041	1085.968	25.88
1	171.340	31.484	17.559	17.599	3.203	0.039	1088.827	26.18

## Table 45 Solution when S=400ft and T=20s

# Table 46 Solution when S=400ft and T=22s

W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	1003.381	25.225	22.995	16.418	11.871	0.023	144.840	3.83
0.5	173.388	31.686	22.944	16.413	3.332	0.042	1099.878	26.57
1	172.417	32.297	19.275	16.039	2.931	0.034	1111.550	27.38

#### Table 47 Solution when S=600ft and T=12s

W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	421.666	23.921	22.995	49.859	12.550	0.021	41.783	0.77
0.5	74.114	25.465	12.667	49.738	2.642	0.042	1399.703	25.46
1	71.200	25.827	1.703	46.194	1.844	0.023	1058.128	20

Table 48 Solution when S=600ft and T=14s
--

W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	724.271	24.061	22.991	42.449	12.676	0.028	72.305	1.43
0.5	83.387	26.900	22.997	42.449	1.991	0.026	1262.109	24.55
1	83.315	26.912	22.824	42.445	1.919	0.024	1256.725	24.47

## Table 49 Solution when S=600ft and T=16s

W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	970.763	24.224	23.000	36.853	12.709	0.033	93.267	1.95
0.5	162.774	26.718	18.149	36.667	3.239	0.040	839.876	17.17
1	162.506	26.757	18.150	36.667	2.988	0.035	834.649	17.12

#### Table 50 Solution when S=600ft and T=18s

W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	958.514	24.381	22.992	32.507	12.346	0.026	106.909	2.34
0.5	166.289	26.738	14.824	32.001	2.816	0.032	935.179	20.01
1	166.126	27.715	16.348	32.137	3.337	0.042	932.110	19.74

W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	968.727	24.529	22.998	29.046	12.244	0.026	117.031	2.66
0.5	167.159	28.482	21.529	28.976	3.237	0.040	973.970	21.22
1	166.968	28.527	17.816	28.744	3.248	0.040	974.835	21.28

## Table 51 Solution when S=600ft and T=20s

#### Table 52 Solution when S=600ft and T=22s

W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	955.390	24.663	23.000	26.233	12.189	0.026	124.976	2.93
0.5	171.338	29.136	18.205	25.935	3.506	0.046	1028.303	22.8
1	168.528	29.290	21.481	26.154	3.053	0.036	1009.862	22.64

#### Table 53 Solution when S=800ft and T=14s

W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	124 190	23 865	22 992	57 139	13 368	0.033	7 525	0.13
U	124.170	25.005	<i>LL.))L</i>	57.157	15.500	0.055	1.525	0.15
0.5	40.007	24.364	0.221	55.935	1.756	0.029	1393.340	24.25
1	39.226	24.393	0.219	55.826	1.527	0.025	1392.197	24.26

W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	423.629	23.921	22.998	49.895	12.638	0.023	41.223	0.76
0.5	73.311	25.833	1.275	46.175	1.817	0.023	1057.776	20
1	73.169	25.837	1.278	46.200	1.826	0.023	1063.399	20.1

## Table 54 Solution when S=800ft and T=16s

#### Table 55 Solution when S=800ft and T=18s

W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	656.996	24.021	22.993	44.191	12.705	0.027	65.872	1.28
0.5	84.874	26.130	22.998	44.191	2.202	0.030	1231.784	23.56
1	84.008	26.516	11.077	43.897	1.788	0.022	1193.744	23

#### Table 56 Solution when S=800ft and T=20s

W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	851.673	24.137	22.999	39.605	12.553	0.027	83.110	1.69
0.5	162.079	26.427	22.490	39.596	3.048	0.036	767.547	15.35
1	160.910	26.881	22.734	39.600	4.521	0.073	1397.023	26.83

W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	950.003	24.255	22.994	35.849	12.221	0.020	95.561	2.02
0.5	165.192	26.961	21.530	35.812	2.945	0.034	857.408	17.72
1	164.514	26.969	15.948	35.606	3.127	0.038	857.830	17.72

#### Table 57 Solution when S=800ft and T=22s

#### Table 58 Solution when S=800ft and T=24s

W	Emissions	Travel Time	ad	vs	β0	β1	Sa	ta
0	946.048	24.372	22.998	32.724	12.286	0.024	105.713	2.31
0.5	167.013	27.728	20.598	32.649	2.896	0.033	918.479	19.52
1	166.285	27.731	14.430	32.332	3.054	0.036	917.698	19.51

#### Table 59 Solution when S=1000ft and T=18s

W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	196.647	23.871	22.997	55.544	12.680	0.021	15.415	0.27
0.5	59.631	24.303	0.473	54.572	8.070	0.137	1339.985	23.28
1	58.321	24.333	0.442	54.426	7.814	0.133	1382.199	24.03

Table 60 Solution v	when S=	:1000ft and	T=20s
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W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	430.296	23.921	22.995	49.917	12.833	0.027	41.234	0.76
0.5	79.411	25.830	1.022	46.194	2.554	0.038	1257.389	23.57
1	75.244	26.001	1.021	46.182	1.896	0.024	1063.259	20.09

#### Table 61 Solution when S=1000ft and T=22s

W	Emissions	Travel Time	ad	vs	β0	β1	Sa	ta
0	613.262	23.998	22.997	45.277	12.605	0.024	61.357	1.18
0.5	85.663	26.139	22.755	45.275	1.738	0.021	1137.314	21.67
1	85.445	26.148	21.071	45.261	1.716	0.021	1128.423	21.51

#### Table 62 Solution when S=1000ft and T=24s

W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	770 670	24.099	22.005	41 206	17 (00	0.020	76 600	1.52
0	//9.0/9	24.000	22.993	41.390	12.000	0.029	/0.000	1.33
0.5	93.767	26.725	22.999	41.396	2.067	0.027	1323.121	25.92
1	93.677	26.763	22.825	41.394	2.038	0.027	1318.329	25.84

W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	917.883	24.181	22.989	38.108	12.425	0.024	88.124	1.82
0.5	165.698	27.087	5.183	36.665	3.210	0.039	840.597	17.19
1	165.494	27.175	5.184	36.666	2.977	0.035	836.049	17.15

## Table 63 Solution when S=1000ft and T=26s

#### Table 64 Solution when S=1000ft and T=28s

W	Emissions	Travel Time	ad	VS	β0	β1	Sa	ta
0	932.883	24.275	22.999	35.290	12.348	0.024	97.861	2.08
0.5	168.003	27.230	18.018	35.167	2.780	0.031	875.816	18.24
1	166.589	27.232	15.412	35.069	3.124	0.038	869.852	18.05

## **APPENDIX B**

Operating Mode	Emission Rate (mg/s)	Operating Mode Description	Vehicle- Specific Power (VSP <i>t</i> , kW/tonne)	Vehicle Speed (v <sub>t</sub> ,mi/hr)	Vehicle Acceleration ( <i>a</i> , mi/hr- sec)
0	0.277777778	Deceleration/Braking			at ≤ -2.0 OR (at < -1.0 AND at-1 <-1.0 AND at-2 <-1.0)
1	0.019444444	Idle		$-1.0 \le v_t < 1.0$	
11	0.083333333	Coast	$VSP_t < 0$	$0 \le v_t < 25$	
12	0.277777778	Cruise/Acceleration	$0 \leq VSP_t < 3$	$0 \le v_t < 25$	
13	0.555555556	Cruise/Acceleration	$3 \leq VSP_t < 6$	$0 \le v_t < 25$	
14	1.666666667	Cruise/Acceleration	$6 \leq VSP_t < 9$	$0 \le v_t < 25$	
15	1.111111111	Cruise/Acceleration	$9 \leq \text{VSP}_t < 12$	$0 \le v_t < 25$	
16	1.666666667	Cruise/Acceleration	$12 \leq VSP_t$	$0 \le v_t < 25$	
21	0.166666667	Coast	$VSP_t < 0$	$25 \le v_t < 50$	
22	0.833333333	Cruise/Acceleration	$0 \leq VSP_t < 3$	$25 \le v_t < 50$	
23	1.666666667	Cruise/Acceleration	$3 \leq VSP_t < 6$	$25 \le v_t < 50$	
24	1.388888889	Cruise/Acceleration	$6 \leq VSP_t < 9$	$25 \le v_t < 50$	
25	2.777777778	Cruise/Acceleration	$9 \leq \text{VSP}_t < 12$	$25 \le v_t < 50$	
27	8.333333333	Cruise/Acceleration	$12 \leq VSP < 18$	$25 \le v_t < 50$	
28	83.33333333	Cruise/Acceleration	$18 \leq VSP \leq 24$	$25 \le v_t < 50$	
29	222.2222222	Cruise/Acceleration	$24 \leq VSP < 30$	$25 \le v_t < 50$	
30	472.2222222	Cruise/Acceleration	$30 \leq VSP$	$25 \le v_t < 50$	
33	0.111111111	Cruise/Acceleration	$VSP_t < 6$	$50 \le v_t$	
35	0.277777778	Cruise/Acceleration	$6 \leq \text{VSP}_t < 12$	$50 \le v_t$	
37	0.833333333	Cruise/Acceleration	$12 \leq VSP < 18$	$50 \le v_t$	
38	5.55555556	Cruise/Acceleration	$18 \leq VSP < 24$	$50 \le v_t$	
39	8.333333333	Cruise/Acceleration	$24 \leq VSP < 30$	$50 \le v_t$	
40	33.33333333	Cruise/Acceleration	$30 \le VSP$	$50 \le v_t$	

# Table 65 Emissions rate of CO from MOVES (EPA, 2009)

#### **APPENDIX C**

#### MATLAB code for GA:

```
function O=GA(v)
vo=58.667;
T=10;
D=1400;
S=400;
EEo=168.002997174117;
TTo=24.2754258368859;
ad=v(1):
b0=v(2);
b1 = v(3);
w=0;
sq=ad*T^2-2*vo*T+2*S;
if sq>=0
  vs=vo-ad*T+sqrt(ad*(ad*T^2-2*vo*T+2*S));
  td=(vo-vs)/ad;
  ta1 = log((b0/b1-vo)/(b0/b1-vs))/-b1;
  ta = floor(ta1*100)/100;
  Sa=b0*ta/b1-b0*(1-exp(-b1*ta))/b1^{2}+vs*(1-exp(-b1*ta))/b1;
  if Sa<=D && Sa>0 && ad<23 && ad>0 && b0<11.5+b1*vs && b0>b1*vo &&
b1>=0.02 && b1<=0.2
    ta1=floor(ta);
    tacr=ta-ta1;
    tac=(D-Sa)/vo;
    vt=vs;
    Ea=0;
    ERd=0.2778;
    Ed=ERd*td;
    vs1=vs*3600/5280;
    if vs1<25
      ERbc=0.2778;
    elseif 25<= vs1 && vs1<31.5
      ERbc=0.8333;
    else
      ERbc=1.6667;
    end
    Ebc=ERbc*(T-td);
    for ta2=1:ta1*10
      at=b0-b1*vt;
```

```
vt1=vt*3600/5280;
  VSP=0.3227*at*vt1+0.0954*vt1+0.0000272*vt1^3;
  vt=vt+at/10;
  if VSP<3 && vt1<25
    ERa=0.2778;
  elseif 3<=VSP && VSP<6 && vt1<25
    ERa=0.5556;
  elseif 6<=VSP && VSP<9 && vt1<25
    ERa=1.6667;
  elseif 9<=VSP && VSP<12 && vt1<25
    ERa=1.1111;
  elseif VSP>=12 && vt1<25
    ERa=1.6667;
  elseif VSP<3 && vt1>=25
    ERa=0.8333;
  elseif 3<=VSP && VSP<6 && vt1>=25
    ERa=1.6667;
  elseif 6<=VSP && VSP<9 && vt1>=25
    ERa=1.3889;
  elseif 9<=VSP && VSP<12 && vt1>=25
    ERa=2.7778;
  elseif 12<=VSP && VSP<18 && vt1>=25
    ERa=8.3333:
  elseif 18<=VSP && VSP<24 && vt1>=25
    ERa=83.3333;
  elseif 24<=VSP && VSP<30 && vt1>=25
    ERa=222.2222;
  elseif VSP>=30 && vt1>=25
    ERa=472.2222;
  end
  Ea=Ea+ERa/10;
end
at=b0-b1*vt;
vt=vo;
vt1=vt*3600/5280;
VSP=0.3227*at*vt1+0.0954*vt1+0.0000272*vt1^3;
if VSP<3 && vt1<25
  ERa=0.2778;
elseif 3<=VSP && VSP<6 && vt1<25
  ERa=0.5556;
elseif 6<=VSP && VSP<9 && vt1<25
  ERa=1.6667;
elseif 9<=VSP && VSP<12 && vt1<25
  ERa=1.1111;
```

```
elseif VSP>=12 && vt1<25
      ERa=1.6667;
    elseif VSP<3 && vt1>=25
      ERa=0.8333;
    elseif 3<=VSP && VSP<6 && vt1>=25
      ERa=1.6667;
    elseif 6<=VSP && VSP<9 && vt1>=25
      ERa=1.3889;
    elseif 9<=VSP && VSP<12 && vt1>=25
      ERa=2.7778;
    elseif 12<=VSP && VSP<18 && vt1>=25
      ERa=8.3333;
    elseif 18<=VSP && VSP<24 && vt1>=25
      ERa=83.3333;
    elseif 24<=VSP && VSP<30 && vt1>=25
      ERa=222.2222;
    elseif VSP>=30 && vt1>=25
      ERa=472.2222;
    end
    Ea=Ea+ERa*tacr;
    ERac=1.6667;
    Eac=ERac*tac;
    E1=Ed+Ebc+Ea+Eac;
    TT1=ta+tac;
    O=w*E1/EEo+(1-w)*TT1/TTo;
  else
    O=20;
  end
else
  O=25;
end
```