

**THE VOLATILITY OF LIQUIDITY AND EXPECTED STOCK  
RETURNS**

A Dissertation

by

FERHAT AKBAS

Submitted to the Office of Graduate Studies of  
Texas A&M University  
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2011

Major Subject: Finance

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## ABSTRACT

The Volatility of Liquidity and Expected Stock Returns. (August 2011)

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Co-Chairs of Advisory Committee: Dr. Sorin M. Sorescu  
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The pricing of total liquidity risk is studied in the cross-section of stock returns. This study suggests that there is a positive relation between total volatility of liquidity and expected returns. Our measure of liquidity is Amihud measure and its volatility is measured using daily data. Furthermore, we document that total volatility of liquidity is priced in the presence of systematic liquidity risk: the covariance of stock returns with aggregate liquidity, the covariance of stock liquidity with aggregate liquidity, and the covariance of stock liquidity with the market return. The separate pricing of total volatility of liquidity indicates that idiosyncratic liquidity risk is important in the cross section of returns.

This result is puzzling in light of Acharya and Pedersen (2005) who developed a model in which only systematic liquidity risk affects returns. The positive correlation between the volatility of liquidity and expected returns suggests that risk averse investors require a risk premium for holding stocks that have high variation in liquidity. Higher variation in liquidity implies that a stock may become illiquid with higher

probability at a time when it is traded. This is important for investors who face an immediate liquidity need and are not able to wait for periods of high liquidity to sell.

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## 1. INTRODUCTION

In this dissertation we document a positive and significant relation between a stock's expected return and its volatility of liquidity. The volatility of liquidity is a stock-specific characteristic that measures the uncertainty associated with the level of liquidity of the stock at the time of trade. The positive correlation between the volatility of liquidity and expected returns suggests that risk averse investors require a risk premium for holding stocks with high variation in liquidity.

Numerous studies have shown that the mean level of liquidity is positively priced in the cross-section of expected returns.<sup>1</sup> The motivation behind examining the second moment of liquidity is that investors who need to trade at random points in time might care about not only the mean but also the volatility of the liquidity distribution. This is the case since liquidity varies over time and higher variation in liquidity implies that a stock may be very illiquid at a time when it is traded. If a stock's liquidity fluctuates within a wider range around its mean compared to otherwise similar stocks, an investor holding the stock may be exposed to a relatively higher probability of low liquidity at the time he needs to sell the stock. The volatility of liquidity captures this risk. Therefore, all else equal, a risk-averse investor may be willing to pay a higher price for a stock that has a lower risk of becoming less liquid at the time of trading, i.e., a stock whose

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This dissertation follows the style of *Journal of Finance*.

<sup>1</sup> See, among others, Amihud and Mendelson (1986, 1989), Brennan and Subrahmanyam (1996), Eleswarapu (1997), Brennan, Chordia and Subrahmanyam (1998), Chalmers and Kadlec (1998), Chordia, Roll and Subrahmanyam (2001), Amihud (2002), Hasbrouck (2009), Chordia, Huh, and Subrahmanyam (2009).

liquidity is less volatile.<sup>2</sup>

We document evidence consistent with this hypothesis. In this study we consider a stock to be illiquid when trading induces negative price impact.<sup>3</sup> Price impact is a major concern to investors because it decreases the potential return from investing in a stock by reducing the price received when the investor attempts to sell the stock. If investors want to sell large amounts in a short period of time, the price impact is of special concern. Therefore, in our empirical analysis we use the price impact of trade based on Amihud (2002) as a measure of liquidity. For each stock, we compute its daily Amihud measures across time. These measures can be interpreted as the daily price response associated with one dollar of trading volume. We use the variation of these measures within a month for each stock as a proxy for the monthly volatility of liquidity for the stock.<sup>4</sup> We find reliable evidence that stocks with high variability in liquidity command higher expected returns. This finding persists across a wide range of robustness checks, which include standard control variables, common risk factors, and

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<sup>2</sup> Amihud and Mendelson (1986) define illiquidity as the cost of immediate execution. They develop a model that links high expected returns with high illiquidity measured by the bid-ask spread. As orders on the buy and sell side arrive randomly, stocks with higher volatility of demand and/or supply face a higher probability of facing a negative liquidity shock (supply greater than demand). Thus, volatility captures the probability that an investor will experience a liquidity shock. This negative liquidity shock imposes a cost on investors in the form of a price impact of trade when they reverse their positions.

<sup>3</sup> Liquidity is a stock characteristic that is difficult to define. Usually, a stock is thought to be liquid if large quantities can be traded in a short period of time without moving the price too much. Studies that use price impact as a measure of liquidity include Brennan and Subrahmanyam (1996), Bertsimas and Lo (1998), He and Mamayasky (2001), Amihud (2002), Pastor and Stambaugh (2003), Acharya and Pedersen (2005), and Sadka (2006). The bid-ask spread has also been used as a measure of liquidity, starting with Amihud and Mendelson (1986). However, it is a less useful measure of liquidity for large investors since large blocks of shares usually trade outside the bid-ask spread (see, e.g., Chan and Lakonishok (1995) and Keim and Madhavan (1996)). In addition, Eleswarapu (1997) finds that the bid-ask spread does not predict returns for NYSE/AMEX stocks, but only for NASDAQ stocks.

<sup>4</sup> More precisely, following Chordia, Subrahmanyam, and Anshuman (2001), the volatility of liquidity is measured as the standard deviation of the daily Amihud measures scaled by their mean. We do this since the mean and standard deviation of liquidity are highly correlated due to the presence of dollar volume in the liquidity measure

different sub-periods. Our estimate of the volatility of liquidity is not sensitive to the measurement horizon and is significant for measurement windows of up to 12 months.

Furthermore, we show that total volatility of liquidity is priced in the presence of systematic liquidity risk: the covariance of stock returns with aggregate liquidity, the covariance of stock liquidity with aggregate liquidity, and the covariance of stock liquidity with the market return. Since total liquidity volatility comes from systematic and idiosyncratic sources, the pricing of total volatility of liquidity in the presence of systematic liquidity betas indicates that idiosyncratic liquidity risk is important in the cross-section of returns. This result has not been shown before. The pricing of idiosyncratic liquidity risk that we document creates a puzzle in light of Acharya and Pedersen (2005) who develop a model in which only systematic liquidity risk affects returns. In particular, total volatility of liquidity affects returns over and above the three liquidity risk effects documented in Acharya and Pedersen (2005): the covariance of stock returns with aggregate liquidity, the covariance of stock liquidity with aggregate liquidity, and the covariance of stock liquidity with the market return.<sup>5</sup>

Using daily data is key to capturing the dimension of liquidity related to short-term variability in trading costs. If an investor faces an immediate liquidity need due to exogenous cash needs, margin calls, dealer inventory rebalancing, forced liquidations, or standard portfolio rebalancing, he needs to unwind his positions in a short period of time. In case of such a liquidity need the investor may not be able to wait for periods of high liquidity to sell the stock, and thus the level of liquidity on the day the investor closes his position is important. This effect will be reinforced if investors are subject to

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<sup>5</sup> Other papers that explicitly study the pricing of systematic liquidity risk include Pastor and Stambaugh (2003) and Sadka (2006), among others

borrowing constraints and cannot borrow easily in case of an urgent consumption need (e.g., see Huang (2003)). The higher a stock's volatility of liquidity, the more likely it is that the investor might end up unwinding his position at a low level of liquidity for the stock, which induces a significant loss of wealth due to a large price impact of trade. Thus, investors will require a compensation for being exposed to this risk.

To the best of our knowledge, this is the first study that documents a positive relation between the volatility of liquidity and average stock returns. Another paper that examines the effect of liquidity variability on stock returns is Chordia, Subrahmanyam, and Anshuman (2001, hereafter CSA). Using turnover and dollar volume as proxies for liquidity and measuring volatility of liquidity using monthly data, they show a strong negative relation between the volatility of liquidity and expected returns. CSA argue that their finding is puzzling since risk averse investors should require a risk premium for holding stocks whose liquidity is volatile.<sup>6</sup>

In contrast to CSA's paper, we document a positive relation between the volatility of liquidity and average returns. This result is new and it is in line with the hypothesis that the inability to wait for periods of high liquidity leads to a risk premium associated with the volatility of liquidity. There are two potential reasons for the difference between our findings and CSA's. First, using daily data rather than monthly observations, we focus on the volatility of liquidity over a shorter time period. The advantage of using daily data is that it allows for the possibility that liquidity may

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<sup>6</sup> Amihud, Mendelson, and Pedersen (2005) also emphasizes the importance of volatility of liquidity and argue that risk averse investors should require compensation for bearing the risk associated with the time variation of liquidity

change within a month. In contrast, calculating volatility using monthly measures of liquidity, as CSA do, implicitly assumes that liquidity is constant within a month.

Therefore, daily data enables us to capture the possibility that a negative liquidity shock and an immediate liquidity need could occur simultaneously over a few days.<sup>7</sup>

Second, our measure of liquidity differs from the one used by CSA. We use the price impact of trade based on Amihud (2002), while CSA use trading volume. Avramov, Chordia, and Goyal (2006) note that trading volume and Amihud's measure of liquidity are only moderately correlated and may capture different aspects of liquidity. While liquidity has many dimensions, we seek to measure the price impact of trade since it is most relevant for our study. If investors seek to uncover their positions in a stock in a short time, we need a measure of liquidity which can capture the possibility of the price moving significantly in the direction of trade. Therefore, we use the price impact of trade as our primary liquidity measure. A key benefit of the Amihud (2002) measure is that it can be estimated over a long sample period with a high frequency. In addition, it gives us the opportunity to measure the volatility of liquidity over a month or a quarter using daily data.

In a robustness analysis we use both our measure of volatility of liquidity and the one proposed by CSA. Namely, the volatility of daily Amihud ratios and the volatility of trading activity over the last 36 months are used in the same regression. Both measures remain significant with a positive sign and a negative sign, respectively.

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<sup>7</sup> We assume that impatient investors take liquidity as given at the time they face a liquidity need. That is, they are unable to wait for periods of high liquidity to reverse their trading positions. If liquidity providers are able to time their trades to align with periods of high liquidity, competition would eliminate their ability to earn a risk premium.

Therefore, our finding that the volatility of liquidity is positively related to returns should be viewed as complementary rather than contradictory to the results documented by CSA. CSA offer a possible interpretation of their results using the investor recognition hypothesis of Merton (1987). Namely, the volatility of trading activity for a certain stock might proxy for the heterogeneity of the clientele holding the stock. High volatility could indicate a shift towards a more heterogeneous group of people who want to hold the stock, therefore lowering the required expected return.<sup>8</sup>

Pereira and Zhang (2011) develop a rational model that generates results consistent with CSA's surprising finding. In their model, investors with certain investment horizons time the market by waiting for periods of high liquidity to sell their stocks. The higher a stock's volatility of liquidity, the more likely it is that there will be a point at which liquidity is significantly higher resulting in lower costs of illiquidity for a patient investor. Therefore, Pereira and Zhang (2011) emphasize investors' preference for volatility of liquidity due to upside movements in liquidity. In contrast to Pereira and Zhang (2011), we argue that investors dislike the volatility of liquidity due to the potential of large downside movements in liquidity. Consistent with this hypothesis, we also find that the volatility of liquidity effect on expected returns is stronger in bad economic times when downside movements in liquidity are more likely and borrowing constraints are higher.

In summary, this dissertation contributes to the literature by documenting that the positive effect on returns of the volatility of liquidity is different from previously

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<sup>8</sup> Barinov (2010) argues that controlling for exposure to aggregate market variance explains CSA's results

documented effects such as the mean level of liquidity and systematic liquidity risk. We conjecture that the volatility of liquidity matters most for investors who may face an immediate liquidity need over a relatively short horizon and are unable to adapt their trading to the state of liquidity of their stocks.<sup>9</sup> For example, in August of 1998 Long Term Capital Management had to unwind their positions under highly adverse conditions. A large part of their losses was due to the price impact of trade.<sup>10</sup> Volatility of liquidity is also important for investors that might not be professional traders. For example, a household may have to liquidate its illiquid assets due to consumptions needs. Similarly, a firm may have to liquidate certain assets to undertake a surprise investment opportunity.

The rest of the dissertation is organized as follows. In Section 2 we discuss the construction of our liquidity measure and the data sample. Section 3 documents the main results. Robustness tests are presented in Section 4. Section 5 examines the idiosyncratic component of the volatility of liquidity, and Section 6 concludes.

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<sup>9</sup> Pereira and Zhang (2010) argue that the possibility of an emergency liquidation of a stock with volatile liquidity, combined with an uncertain investment horizon, will command an extra liquidity premium due to the high price impact of trade. This is also in line with the theoretical arguments of Koren and Szeidl (2002) and Huang (2003)

<sup>10</sup> See Lowenstein (2000) for a more detailed story



## 2. EMPIRICAL METHODS

### *2.1 The Main Measure of Liquidity*

If an investor faces an immediate need to sell a stock, he may not be able to adapt his trading to the liquidity state of the stock. Therefore, if he needs to unwind his position in the stock in a short time he might sell at a very unfavorable price due to the high price impact of trade. The price impact of trade is the liquidity measure that we are interested in. The higher the volatility of the price impact, the more the investor should be compensated for holding the stock.

We follow Amihud (2002) and use a measure of liquidity which captures the relation between price impact and order flow. A key benefit of using Amihud's (2002) measure is that it can be estimated over a long sample period at relatively high frequencies. Measures of price impact that use intraday data also provide high frequency observations of liquidity. These measures have high precision, but are not available prior to 1988. Since we require a long sample period for our asset-pricing tests, we use Amihud's measure which is available for a longer time period. Hasbrouck (2009) compares price impact measures estimated from daily data and intraday data, and finds that the Amihud (2002) measure is most highly correlated with trade-based measures. For example, he finds that the correlation between Kyle's lambda and Amihud's measure is 0.82.<sup>11</sup> Similarly, comparing various measures of liquidity,

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<sup>11</sup> Kyle's (1985) lambda is first estimated by Brennan and Subrahmanyam (1996) using intraday trade and quote data. Brennan and Subrahmanyam (1996) estimate lambda by regressing trade-by-trade price change

Goyenko, Holden, and Trzcinka (2009) conclude that Amihud's measure yields significant results in capturing the price impact of trade. They find that it is comparable to intraday estimates of price impact such as Kyle's lambda.<sup>12</sup> Therefore, we use Amihud's ratio as the main liquidity proxy in our study.

## 2.2 Constructing the Volatility of Liquidity

We calculate the daily price impact of order flow as in Amihud (2002):

$$DFIOF_{i,d} = |r_{i,d}| / dvol_{i,d} \quad (1)$$

where  $r_{i,d}$  is the return of stock  $i$  on day  $d$  and  $dvol_{i,d}$  is the dollar trading volume for stock  $i$  on day  $d$ .<sup>13</sup> The higher the daily price impact of order flow is, the less liquid the stock is on that day. Therefore, Amihud's ratio measures illiquidity.

The mean level of illiquidity for month  $t$  is calculated as follows:

$$ILLIQ_{i,t} = \{1/D_{i,t}\} * \sum DFIOF_{i,d} \quad (2)$$

where  $D_{i,t}$  is the number of trading days in month  $t$ .

We use the coefficient of variation as our measure of the volatility of liquidity.<sup>14</sup>

The coefficient of variation is calculated as the standard deviation of the daily price impact of order flow normalized by the mean level of illiquidity:

$$CVILLIQ_{i,t} = SD(DFIOF_{i,d})_t / ILLIQ_{i,t} \quad (3)$$

The reason for using the coefficient of variation is that the mean and the standard

on signed transaction size. Lambda measures the price impact of a unit of trade size and, therefore, it is larger for less liquid stocks. Hasbrouck (2009) uses a similar method to estimate Kyle's lambda

<sup>12</sup> They also compare Pastor and Stambaugh's (2003) gamma and the Amivest liquidity ratio, and conclude that these measures are ineffective in capturing price impact.

<sup>13</sup> We have also tried adjusting DPIOF for inflation as  $DPIOF_{i,d} = |r_{i,d}| / (dvol_{i,d} * infdt)$ , where  $infdt$  is an inflation-adjustment factor. We obtain similar results.

<sup>14</sup> Even though we refer to it as volatility of liquidity, it is actually the volatility of illiquidity since Amihud's ratio measures illiquidity. The higher the volatility of the Amihud ratio within a month, the riskier the stock will be.

deviation of illiquidity are highly correlated. In our empirical analysis we control for the mean level of liquidity and therefore, it is important to have a measure of volatility which is not highly correlated with the mean. The measure derived in equation (3) is our main variable of interest.<sup>15</sup> We examine the relation between this variable and average stocks return and show that they are significantly correlated.

### ***2.3 Data and Descriptive Statistics***

Our main data sample consists of NYSE-AMEX common stocks for the period from January 1964 to December 2009.<sup>16</sup> Following Avramov, Chordia and Goyal (2006), we exclude stocks with a month end price of less than one dollar to ensure that our results are not driven by extremely illiquid stocks. We also require that each stock has at least 10 days with trades each month in order to calculate its volatility of liquidity.<sup>17</sup> Stocks with prices higher than one thousand dollars are excluded. Stocks that are included have at least 12 months of past return data from CRSP and sufficient data from COMPUSTAT to compute accounting ratios as of December of the previous year.

We compute several other stock characteristics in addition to liquidity and the volatility of liquidity. SIZE is the market value of equity calculated as the number of shares outstanding times the month-end share price. BM is the ratio of book value to market value of equity. Book value is calculated as in Fama and French (2002) and measured at the most recent fiscal year-end that precedes the calculation date of market

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<sup>15</sup> Acharya and Pederson (2005) also use daily Amihud measures to construct volatility of liquidity. They use the volatility of liquidity as a sorting variable for portfolios. They do not examine its pricing in the cross-section of stock return

<sup>16</sup> We exclude NASDAQ stocks from the analysis for two reasons. First, Atkins and Dyl (1997) argue that the volume of NASDAQ stocks is inflated as a result of inter-dealer activities. Second, volume data on NASDAQ stocks is not available prior to November 1982.

<sup>17</sup> The results are robust to using at least 15 days with trades.

value by at least three months.<sup>18</sup>

We exclude firms with negative book values. DY is the dividend yield measured by the sum of all dividends over the previous 12 months, divided by the month-end share price. PRC is the month-end share price. In order to control for the momentum effect of Jegadeesh and Titman (1993), we use two different sets of variables. First, following CSA, we include three measures of lagged returns as proxies for momentum. RET 23 is the cumulative return from month t-2 to month t-1, RET46 is the cumulative return from month t-5 to month t-3, and RET712 is the cumulative return from month t-12 to month t-6. The second set of momentum variables includes RET12M, which is the cumulative return from month t-13 to t-2, and RET1M which is the return in the previous month. RET1M controls for monthly return reversal documented by Jegadeesh (1990). IVOL is idiosyncratic volatility calculated as the standard deviation of the residuals from the Fama-French (1993) model, following Ang, Hodrick, Xing, and Zhang (2006). We require at least 10 days of return data to calculate this measure. Spiegel and Wang (2005) argue that liquidity and idiosyncratic volatility are highly correlated and therefore, we check the robustness of our results to the presence of idiosyncratic volatility. Finally, TURN is the turnover ratio measured as the number of shares traded divided by the number of shares outstanding in a given month. We use TURN to ensure that our results are not driven by the volume component of the liquidity measure.<sup>19</sup>

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<sup>18</sup> Book value is defined as total assets minus total liabilities plus balance sheet deferred taxes and investment tax credit minus the book value of preferred stock. Depending on data availability, the book value of preferred stock is based on liquidating value, redemption value, or carrying value, in order of preferences

<sup>19</sup> Our results are robust to including dollar volume among the set of control variables. However, we exclude dollar volume from the reported results since it is highly correlated with both ILLIQ and SIZE.

We match stock returns in month  $t$  to the volatility of liquidity and other stock characteristics in month  $t - 1$ . However, in order to avoid potential microstructure biases and account for return autocorrelations, we measure stock returns as the cumulative return over a 22-day trading period that begins a week after the various stock characteristics are measured. Skipping a week between measuring stock characteristics and future returns also allows us to use the most recent information about the stocks. This is important since we want to capture the dimension of liquidity related to short-term variability in trading costs. In addition, skipping a week assures that there is no overlap between the returns used as dependent variables and the returns used to derive our liquidity measures. Since liquidity varies over time, skipping a longer time interval might result in loss of information relevant for future returns. However, our results are robust to skipping a month and matching stock returns in month  $t$  to stock characteristics in month  $t - 2$ .

Panel A of Table 1 presents time-series averages of monthly cross-sectional statistics for all stocks. There are on average 1,635 firms each month and the total number of observations is 902,308. Our sample of firms exhibits significant variation in market capitalization. The mean firm size is \$2.14 billion, while the largest firm has a market capitalization of \$144.4 billion. Several of the variables exhibit considerable skewness. Therefore, in the empirical analysis from this point on we apply logarithmic transformations to all variables except the ones which may be zero such as the momentum variables, idiosyncratic volatility, and dividend yield.<sup>20</sup> Therefore,

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<sup>20</sup> We have also tried using the original values of CVILLIQ and obtain similar results.

**Table 1: Summary Statistics**

This table presents time-series averages of cross-sectional summary statistics (Panel A) and monthly cross sectional Pearson's correlations (Panel B) for various stock characteristics. The sample consists of common stocks listed on AMEX and NYSE from January 1964 to December 2009. ILLIQ is the Amihud measure of illiquidity, CVILLIQ is the coefficient of variation of liquidity calculated using equation (3), SIZE is end-of-month price times shares outstanding (in billion dollars), PRC is end-of-month share price, RET23 is the cumulative return from month  $t-3$  to  $t-2$ , RET46 is the cumulative return from month  $t-6$  to  $t-4$ , RET712 is the cumulative return from month  $t-12$  to  $t-7$ , BM is the book-to-market ratio, IVOL is the standard deviation of the residuals from the Fama-French model, DY is dividend yield, TURN is the turnover ratio measured by the number of shares traded divided by the number of shares outstanding, RET12M is the cumulative return over the past twelve months, and RET1M is the return during the previous month. In Panel A, we do not apply log transformations to any of the variables. In Panel B, we apply log transformations to CVILLIQ, ILLIQ, SIZE, BM, 1/PRC, and TURN.

Panel A: Descriptive Statistics									
	MEAN	MEDIAN	STD	MIN	MAX	P1	P25	P75	P99
CVILLIQ	1.09	1.00	0.38	0.44	3.66	0.57	0.84	1.23	2.49
ILLIQ	0.80	0.05	3.46	0.00	68.16	0.00	0.01	0.31	14.50
SIZE	2.14	0.36	7.60	0.00	144.35	0.01	0.09	1.31	33.00
BM	0.94	0.74	0.96	0.01	17.29	0.09	0.45	1.13	4.30
1/PRC	0.10	0.05	0.12	0.00	0.93	0.01	0.03	0.10	0.64
TURN	0.66	0.48	0.77	0.01	12.36	0.03	0.26	0.82	3.45
IVOL	0.02	0.02	0.01	0.00	0.17	0.01	0.01	0.03	0.07
DY	0.03	0.02	0.12	0.00	3.40	0.00	0.00	0.04	0.16
RET23	0.03	0.01	0.16	-0.55	1.71	-0.31	-0.06	0.10	0.54
RET46	0.05	0.03	0.24	-0.64	2.60	-0.40	-0.08	0.15	0.81
RET712	0.07	0.04	0.30	-0.68	3.40	-0.46	-0.09	0.19	1.04
RET1M	0.01	0.01	0.12	-0.46	1.17	-0.24	-0.05	0.06	0.37
RET12M	0.16	0.09	0.48	-0.79	6.32	-0.58	-0.11	0.33	1.81

Panel B: Correlations												
	CVILLIQ	ILLIQ	SIZE	BM	PRC	TURN	IVOL	DY	RET23	RET46	RET712M	RET1M
ILLIQ	0.49											
SIZE	-0.44	-0.94										
BM	0.18	0.32	-0.32									
PRC	0.39	0.78	-0.78	0.33								
TURN	-0.22	-0.43	0.17	-0.17	-0.16							
IVOL	0.15	0.46	-0.48	0.07	0.59	0.22						
DY	-0.02	-0.08	0.09	0.14	-0.06	-0.07	-0.13					
RET23	0	-0.06	0.04	-0.15	-0.12	0.13	0.09	-0.03				
RET46	-0.05	-0.09	0.06	-0.21	-0.16	0.08	-0.07	-0.03	0			
RET712	-0.08	-0.11	0.08	-0.25	-0.18	0.09	-0.07	-0.01	0.02	0.03		
RET1M	0.02	-0.02	0.03	-0.11	-0.08	0.12	0.16	-0.02	0.69	0.01	0.02	
RET12M	-0.1	-0.15	0.1	-0.34	-0.25	0.17	-0.04	-0.04	0.39	0.56	0.69	0.27

when we write CVILLIQ from now on we are referring to the natural logarithm of the variable. The same applies for all other variables except IVOL, DY, and return-related variables.

Panel B of Table 1, we present time-series averages of monthly cross-sectional Pearson's correlations. The correlation between SIZE and ILLIQ is -0.94 which is in line with the evidence that smaller firms are less liquid. We utilize multiple regression specifications in our empirical analysis to ensure that the results are not contaminated by this high correlation. The correlation between CVILLIQ and ILLIQ is positive (0.49) and at a moderate level compared to the correlation (0.93) between  $SD(DPIOF_{i,j})_t$  and ILLIQ. The correlation between the level and volatility of liquidity is similar to the one reported in CSA when they use the coefficient of variation of dollar volume and turnover over the past 36 months. In addition, since we use both ILLIQ and CVILLIQ in our multivariate regressions, the concern that part of the effect of CVILLIQ on future returns might be due to the correlation of ILLIQ with other variables should be alleviated.

Finally, the correlation between IVOL and CVILLIQ is 0.15, indicating that these two variables do not capture the same effect even though they both include the stock return.

### 3. EMPIRICAL RESULTS

#### *3.1 Portfolio Approach*

We begin the analysis using a portfolio approach where we assign stocks to portfolios based on the variation of liquidity, CVILLIQ, and other firm characteristics such as size, illiquidity, momentum, and book-to-market. This is a standard approach, pioneered by Jegadeesh and Titman (1993), which reduces the variability in returns. Each month, we assign stocks into 3 categories based on various firm characteristics. Then we further sort stocks into quintiles based on CVILLIQ. All stocks are held for a month after skipping a week after portfolio formation. Monthly portfolio returns are calculated as equally-weighted or value-weighted averages of the returns of all stocks in the portfolio.

Table 2 present the average returns of portfolios sorted by CVILLIQ alone and by characteristics and CVILLIQ. The first panel contains the results for the univariate sort on CVILLIQ using both equal- and value-weighted returns. According to the results, as CVILLIQ increases the average returns also increase which is in line with the prediction that stocks with higher volatility of liquidity have higher average returns. The difference between the highest and lowest CVILLIQ quintiles (CV5-CV1) is 32 basis points per month for equally-weighted returns. The difference is significant with a t-statistic of 2.73.

We also calculate the abnormal returns of the high-minus-low volatility of liquidity strategy (CV5- CV1) using the Fama-French (1993) model. The alpha is 30



basis points and significant at the 1% level. Similar results hold for value-weighted returns. When we use the Fama-French model augmented with momentum and aggregate liquidity, the results are qualitatively identical. In that model, the alpha is 25 basis points and significant at the 1% level.<sup>21</sup>

In the second panel of Table 2, we first sort stocks into three groups, S1, S2, and S3 based on SIZE, where S1 represents small stocks and S3 represents large stocks. We then independently sort stocks into quintiles based on CVILLIQ. The intersection of the two sorts creates 15 portfolios which are held for a month after skipping a week after portfolio formation. The results show that the difference between the extreme CVILLIQ quintiles, CV5 and CV1, decreases as firm size increases. While the difference between CV5 and CV1 for small stocks is 41 basis points per month and significant, it decreases to an insignificant 10 basis points per month for large stocks. However, the positive relation between the volatility of liquidity and returns is not confined to the smallest size group; it is also present among medium cap stocks.

The Fama-French alpha of the CV5-CV1 strategy is 62 basis points per month for small stocks. The Fama-French model augmented with momentum and liquidity yields an alpha of 39 basis points. Overall, the results suggest that the volatility of liquidity effect is strongest among small stocks.

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<sup>21</sup> The aggregate liquidity factor is constructed using 9 equally-weighted portfolios sorted on size and illiquidity. Every month, we sort stocks into 3 groups (Small, Medium, and Big) according to their end of-previous-month market capitalization. Then we further sort stocks into three groups (High, Medium, and Low) according to their average monthly Amihud illiquidity. Each portfolio is rebalanced monthly. The liquidity factor is the average return on three high illiquidity portfolios minus the average return on three low illiquidity portfolios:  $ILL = 1/3( HighSmall + HighMedium + HighBig ) - 1/3( LowSmall + LowMedium + LowBig )$ .

**Table 2: Average Portfolio Returns**

This table presents average returns (in % form) for various portfolios. The first set of portfolios involves a single sort on the volatility of liquidity, CVILLIQ. The other sets of portfolios involve a double sort on a stock characteristic (size, illiquidity, momentum, book-to-market and contemporaneous return) and the volatility of liquidity. The volatility of liquidity is computed as the coefficient of variation of daily Amihud ratios within a month, as in equation (3). The following variables are measured in logs: CVILLIQ, size, illiquidity, and book-to-market. The sample consists of common stocks listed on AMEX and NYSE from January 1964 to December 2009. The portfolios are rebalanced every month and we skip a week between portfolio formation and the holding period. The table also presents the average returns of the high-minus-low volatility of liquidity strategy, CV5-CV1, within each sort, together with the corresponding Fama-French alphas (FF3), and the alphas from the Fama-French model augmented with momentum and aggregate liquidity (FF5). Newey-West  $t$ -statistics are shown below the average returns.

Mean Portfolio Returns									
	All Stocks		Size			Illiquidity			
	EW	VW	Small	Medium	Large	IL1	IL2	IL3	
CVILLIQ1	1.01	0.8	1.03	1.06	0.93	0.94	1.05	1.05	
CVILLIQ2	1.11	0.9	1.21	1.23	0.99	1.02	1.18	1.15	
CVILLIQ3	1.15	0.9	1.22	1.24	1.01	1.06	1.21	1.19	
CVILLIQ4	1.26	1	1.35	1.31	1.07	1.09	1.28	1.35	
CVILLIQ5	1.33	1.1	1.44	1.3	1.03	1.12	1.25	1.43	
CV5 – CV1	0.32	0.3	0.41	0.24	0.1	0.17	0.2	0.38	
t-statistic	2.73	2.7	2.57	2.44	1.09	1.78	2.21	2.85	
FF3 alphas	0.3	0.2	0.62	0.34	0.1	0.2	0.28	0.56	
t-statistic	3.04	2.3	3.78	3.69	1.09	1.99	3.14	3.8	
FF5 alphas	0.25	0.3	0.39	0.29	0.09	0.21	0.23	0.39	
t-statistic	3.55	3.2	2.47	3.39	1	2.07	2.63	2.75	

  

	Momentum			Book to Market			Contem. Return			
	M1	M2	M3	BM1	BM2	BM3	R1	R2	R3	
CVILLIQ1	0.67	1	1.29	0.85	1.03	1.26	1.06	1.11	0.89	
CVILLIQ2	0.8	1.1	1.4	0.97	1.04	1.39	1.27	1.16	0.93	
CVILLIQ3	0.86	1.1	1.44	0.97	1.12	1.39	1.32	1.19	0.96	
CVILLIQ4	1.02	1.2	1.59	1.02	1.21	1.48	1.46	1.3	1.01	
CVILLIQ5	1.05	1.4	1.77	1.03	1.25	1.55	1.43	1.39	1.14	
CV5 – CV1	0.39	0.4	0.48	0.18	0.23	0.28	0.37	0.28	0.25	
t-statistic	2.62	3.2	4.2	1.46	1.86	2.07	2.58	2.21	1.8	
FF3 alphas	0.43	0.4	0.47	0.16	0.28	0.43	0.39	0.25	0.19	
t-statistic	3.03	3.6	4.47	1.54	2.73	3.44	2.94	2.18	1.58	

In the remainder of Table 2, we perform additional double-sorts using control variables that have been shown to affect returns: illiquidity (ILLIQ), momentum (RET12M), book- to-market (BM), and contemporaneous return (RET1M). The result suggest that the average return of the high-minus-low volatility of liquidity strategy (CV5-CV1) is higher for less liquid stocks (ILL3), value stocks (BM3), and contemporaneous losers (R1). While past performance over the previous 12 months does not seem to be related to the volatility of liquidity when we use raw returns or the Fama-French model, the effect appears to be more pronounced among winners when we use the Fama-French model augmented with momentum and liquidity.

Overall, the portfolio approach suggests that the positive relation between the volatility of liquidity and average returns is a separate effect which is different than the well documented size, momentum and book-to-market effects. In addition, the volatility of liquidity effect does not seem to be concentrated only among a small portion of the sample of stocks.

### ***3.2 Regression Approach***

In this section we extend the portfolio analysis from before by performing cross-sectional regressions. These regressions allow us to control for various other stock characteristics that may potentially affect the relation between the volatility of liquidity and returns. More precisely, we use Fama-MacBeth (1973) regressions in which the dependent variables are excess returns. The main independent variable is the coefficient of variation of illiquidity, CVILLIQ. We adjust the Fama-MacBeth t-statistics for heteroskedasticity and autocorrelation of up to 8 lags.

The results are presented in Table 3. Panel A presents results using the real values of the independent variables. There are three columns in Panel A, each one corresponding to a different regression specification. In column 1, we use ILLIQ, SIZE, BM, DY, 1/PRC, RET23, RET46, and RET712. These are the same control variables as the ones used in CSA. In column 2, we use an alternative set of return variables, RET12M and RET1M, to control for both past returns and returns contemporaneous to CVILLIQ. The variable RET1M is included to take into account the monthly reversal effect documented by Jegadeesh (1990). Since the calculation of CVILLIQ involves return and volume data, in column 3 of Table 3 we include idiosyncratic return volatility, IVOL, and turnover, TURN, to the set of control variables. Price is excluded from the analysis in column 3 since it is highly correlated with market size, illiquidity, and idiosyncratic volatility. However, the results are not affected if price is included in the regression.

The results show that CVILLIQ is positively and significantly related to expected returns in all specifications. The illiquidity level, ILLIQ, is not significant in columns 1 and 2, which may be a result of the presence of both price and size in the same regression. However, the level of illiquidity is significantly positive in column 3, which is in line with Amihud and Mendelson (1986) and Amihud (2002). Since ILLIQ and SIZE are strongly correlated, there might be a potential multicollinearity problem in a regression that includes both of these variables. In untabulated results, we exclude SIZE from the model and the coefficient on ILLIQ becomes significantly positive in all specifications. When we exclude ILLIQ instead, the coefficient on SIZE is significantly

**Table 3: Fama-MacBeth Regression Estimates Using Individual Security Data**

This table presents the results from Fama-MacBeth regressions in which the dependent variables are stock returns and the independent variables are various stock characteristics. The sample consists of common stocks listed on AMEX and NYSE from January 1964 to December 2009. The stock characteristics are defined in Table 1. All variables are measured in logs except for DY, IVOL, RET712, RET46, RET23, RET1M, and RET12M. Panel A uses the actual values of the independent variables, while Panel B uses their decile ranks standardized between zero and one. The coefficients are multiplied by 100. Newey-West  $t$ -statistics are reported below the coefficients. The cross-sectional adjusted  $R^2$  is reported in the last row.

	Panel A: Real Values			Panel B: Decile Ranks		
	1	2	3	1	2	3
CVILLIQ	0.36	0.35	0.2	0.3	0.28	0.22
	5.39	5.48	3.05	5.15	4.68	4.22
ILLIQ	-0.05	-0.04	0.18	-0.38	-0.34	0.4
	-0.97	-0.81	2.97	-1.46	-1.28	1.76
SIZE	-0.11	-0.12	0.04	-0.61	-0.59	-0.41
	-1.8	-1.83	0.55	-2.32	-2.11	-1.65
DY	0.7	0.64	-0.01	0.25	0.19	0.08
	0.78	0.68	-0.01	1.46	1.06	0.49
RET712	0.96			0.95		
	5.17			5.56		
RET46	1.02			0.66		
	3.51			3.2		
RET23	-0.22			-0.29		
	-0.68			-1.62		
1/PRC	0.03	0		0.17	0.14	
	0.26	-0.03		0.81	0.68	
BM	0.25	0.22	0.21	0.6	0.59	0.57
	3.9	3.39	3.38	4.43	4.28	4.03
TURN			0.23			0.34
			2.82			1.88
IVOL			-26.82			-0.5
			-6.5			-3.43
RET1M		-0.01	-0.9		-0.57	-0.56
		-3.3	-2.2		-3.99	-3.79
RET12M		0.6	0.59		1.07	1.07
		2.69	2.7		3.98	4.07
Adj.R <sup>2</sup>	0.07	0.06	0.07	0.07	0.06	0.07

negative in all specifications. The relation between return and the volatility of liquidity is not affected by these modifications.

Note that the coefficient on turnover has a positive sign in the third specification. This result differs from the findings of CSA who show that TURN has a negative effect on expected returns. In untabulated tests we find that the coefficient on TURN becomes negative once CVILLIQ, ILLIQ, and IVOL are excluded from the regression. Turnover is used in the literature as a proxy for liquidity or divergence of opinion among investors. Since ILLIQ and IVOL are such proxies as well, it is possible that the positive coefficient on TURN in model 3 is a result of the interaction between all these variables. To ensure that our results are not driven by this interaction, we repeat the analysis within different turnover groups and find similar results.

Instead of using the real values of the independent variables, in Panel B of Table 3 we first transform the independent variables into decile ranks and then standardize the ranks with values between zero and one. This rank transformation has two advantages: it makes the coefficient interpretation more intuitive and comparable across variables, and it minimizes the effect of outlier observations. Panel B shows that the results are similar and somewhat stronger compared to the results in Panel A. The results in column 3 suggest that, after controlling for various firm characteristics, stocks in the highest CVILLIQ decile earn on average 22 basis points per month more than stocks in the lowest CVILLIQ decile.

Overall, the results in Table 3 suggest that the volatility of liquidity is significantly positively related to average returns. This relation persists over and above

the positive correlation between the level of illiquidity and returns. This is in line with the hypothesis that investors want to be compensated for holding stocks whose liquidity is more volatile.

### ***3.3 Regression Approach within Size and Illiquidity Groups***

As mentioned earlier, the high correlation between size and illiquidity may cause potential multicollinearity problems and bias our results. In this section we perform additional tests to ensure that the main results are not driven by this correlation. Every month we sort stocks based on size or illiquidity and run Fama-MacBeth regression within each size or illiquidity group. This way we control for one of the correlated variables and allow the other one to vary within each group. For the sake of brevity we report the results using the 3rd model from our previous analysis, but the results are similar for models 1 and 2.

In Panel A of Table 4, we report Fama-MacBeth regressions within each size category. The results suggest that the positive relation between CVILLIQ and returns is stronger among smaller stocks. Furthermore, the level of illiquidity is significant and positive in all size groups. When we move from larger to smaller stocks, the volatility of liquidity and the illiquidity effects get stronger. Overall, the results suggest that, after controlling for the size effect, both the mean and the second moment of illiquidity are positively related to expected stock returns.

**Table 4: Fama-MacBeth Regression Estimates by Size and Illiquidity Groups**

This table presents the results from Fama-MacBeth regressions in which the dependent variables are stock returns and the independent variables are various stock characteristics. The sample consists of common stocks listed on AMEX and NYSE from January 1964 to December 2009. The stock characteristics are defined in Table 1. All variables are measured in logs except for DY, IVOL, RET1M, and RET12M. Panel A shows results within three separate size groups, while Panel B displays results within three separate illiquidity groups. In both panels the actual values or the decile ranks of the independent variables are used. The coefficients are multiplied by 100. Newey-West t-statistics are reported below the coefficients. The cross-sectional adjusted  $R^2$  is reported in the last row

	Panel A: Regressions by Size Group						Panel B: Regressions by Illiquidity Group					
	Real Values			Decile Ranks			Real Values			Decile Ranks		
	Small	Med.	Large	Small	Med.	Large	ILLIQ1	ILLIQ2	ILLIQ3	ILLIQ1	ILLIQ2	ILLIQ3
CVILLIQ	0.34	0.15	0.08	0.55	0.11	0.06	0.13	0.13	0.52	0.1	0.11	0.65
	3.06	1.76	0.98	4.79	1.59	0.95	1.39	1.66	5.15	1.32	1.69	5.92
ILLIQ	0.33	0.14	0.08	1.52	0.67	0.67						
	5.95	2.81	2.39	2.86	2.53	2.49						
SIZE							-0.07	-0.09	-0.3	-0.57	-0.42	-1.02
							-1.65	-1.77	-5.05	-1.61	-1.82	-2.98
DY	-1.88	-0.22	2.02	-0.05	0.06	0.36	1.3	-0.08	-1.55	0.32	0.07	0.04
	-1.2	-0.17	1.42	-0.27	0.33	1.83	0.96	-0.08	-0.83	1.66	0.4	0.21
BM	0.3	0.18	0.06	0.9	0.42	0.22	0.05	0.17	0.31	0.19	0.43	0.87
	3.78	2.55	0.84	4.61	2.7	1.4	0.62	2.3	4.12	1.13	2.66	4.77
TURN	0.35	0.2	0.15	0.56	0.5	0.47	0.04	0.12	0.09	0.25	0.28	0.34
	4.36	2.65	2.27	1.71	2.77	2.72	0.55	2.01	1.32	1.51	1.96	1.09
IVOL	-32.98	-29.67	-20.8	-0.89	-0.5	-0.3	-21.49	-29.03	-22.93	-0.31	-0.49	-0.63
	-7.75	-5.87	-3.17	-4.52	-3.49	-1.79	-3.31	-5.93	-5.17	-1.9	-3.08	-3.08
RET1M	-1.02	-0.42	-1.44	-0.61	-0.43	-0.62	-0.72	0.11	-1.53	-0.43	-0.4	-0.72
	-2.06	-0.88	-2.69	-3.14	-2.78	-4.16	-1.22	0.24	-3.07	-2.72	-2.54	-3.73
RET12M	0.81	0.49	0.6	1.54	0.94	0.73	0.49	0.44	0.94	0.79	0.93	1.49
	3.99	1.97	2.36	5.42	3.41	2.7	1.94	1.68	4.61	2.86	3.3	5.41
Adj.R <sup>2</sup>	0.04	0.06	0.1	0.04	0.06	0.1	0.11	0.07	0.04	0.11	0.06	0.04



In Panel B of Table 4, the Fama-MacBeth regressions are performed within each illiquidity category. The coefficient on CVILLIQ is positive and significant among the less liquid stocks. The significance level of the CVILLIQ coefficient decreases as liquidity increases, but it still remains positive among the most liquid stocks. A similar pattern is observed for the SIZE coefficient as the sign is negative for all illiquidity groups but significant only among the least liquid stocks (ILLIQ3). Once again the results are similar if the independent variables are measured in decile ranks.

One notable observation is that CVILLIQ is more significant among small and illiquid stocks. A possible explanation might be that illiquid stocks have low average levels of liquidity and therefore, a high volatility of the liquidity distribution implies that investors in illiquid stocks may face even lower levels of liquidity at a point when they need to trade. Liquid stocks, on the other hand, may expose investors to this risk to a lower extent since their liquidity distributions have higher means.

Overall, the results in Table 4 suggest that our previous findings are not driven by multicollinearity biases due to the high correlation between size and illiquidity. Controlling for this correlation, the coefficient on CVILLIQ is still positive and significant.

## 4. ROBUSTNESS

### *4.1 Risk Adjusted Returns*

The Fama-MacBeth regressions that we run previously use non-risk-adjusted excess returns as the dependent variables and relate them to the volatility of liquidity and other firm characteristics. Since we do not control for systematic risk, it might be possible that our previous results are driven by exposure to some well-known risk factors. In order to control for this possibility, we follow Brennan, Chordia, and Subrahmanyam (1998) and examine the relation between risk-adjusted returns and the volatility of liquidity. The risk-adjustment is done relative to the Fama-French model augmented with momentum and aggregate liquidity. It is important to control for exposure to aggregate liquidity since previous studies have shown that return sensitivity to market liquidity is priced (e.g., Pastor and Stambaugh (2003)). Factor loadings are estimated using a 60-month rolling window and the Dimson (1979) procedure with one lag is used to adjust the estimated factor loadings for possible thin trading.<sup>22</sup>

In addition, previous studies have shown that asset liquidity changes over time and this time variation is governed by a significant common component in the liquidity across assets (see, e.g., Chordia, Roll, and Subrahmanyam (2000), Hasbrouck and Seppi (2001), Amihud (2002), and Korajczyk and Sadka (2008)). Therefore, the volatility of liquidity for a given stock might be driven by exposure to aggregate market liquidity or the aggregate market return. Therefore, it is possible that our measure of the

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<sup>22</sup> The results are not sensitive to using the Dimson (1979) adjustment. We also use the Fama-French three-factor model to adjust for risk and obtain similar results.

volatility of liquidity captures the risk associated with the covariance of a stock's liquidity with aggregate market liquidity or the market return (see Acharya and Pederson (2005)). If a stock becomes illiquid when the market as a whole is illiquid, investors would like to be compensated for holding this security. Furthermore, if a stock becomes illiquid if the market overall is doing badly, then this security will require a higher expected return as well. To address these two covariance effects, we include two additional variables in our regressions. These variables are a stock's illiquidity sensitivity to aggregate market illiquidity,  $\beta_{L,L}$ , and its sensitivity to the market return,  $\beta_{L,M}$ . The betas for each stock are derived from a regression of the stocks illiquidity on the market illiquidity and the market return using daily data within a month.<sup>23</sup> Aggregate market illiquidity is calculated as the equally-weighted average of stocks' daily illiquidity measures. If our volatility of liquidity measure, CVILLIQ, captures the covariance between the assets's illiquidity and aggregate market illiquidity, or the asset's illiquidity and the market return, then the coefficient on CVILLIQ should be insignificant in the presence of  $\beta_{L,L}$  and  $\beta_{L,M}$ .

Table 5 presents the results from risk-adjusted Fama-MacBeth regressions. Panel A contains the coefficients from standard Fama-MacBeth regressions with real values of the independent variables, Panel B shows the coefficients from standard Fama-MacBeth regressions with decile ranks of the independent variables, and Panel C presents purged estimators.

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<sup>23</sup> We obtain similar results if the sensitivities are derived from univariate regressions.

**Table 5: Fama-MacBeth Regression Estimates: Using Risk-Adjusted Returns as the Dependent Variables**

This table presents the results from Fama-MacBeth regressions in which the dependent variables are risk-adjusted returns and the independent variables are various stock characteristics. The risk-adjustment is based on the Fama-French model augmented with momentum and aggregate liquidity and the Dimson procedure with one lag.  $\beta_{L,L}$  and  $\beta_{L,M}$  are the respective coefficient estimates from monthly multivariate regressions of daily firm-level Amihud measures on daily aggregate Amihud measures and daily market returns within each month. The label "Raw" in Panel A refers to the standard Fama-MacBeth coefficients, the label "Decile" in Panel B refers to the decile ranks of the Fama-MacBeth coefficients, while the label "Purged" in Panel C refers to the intercept terms from time-series regressions of the Fama-MacBeth coefficients on the factors. The sample consists of common stocks listed on AMEX and NYSE from January 1964 to December 2009. The stock characteristics are defined in Table 1. All variables are measured in logs except for DY, IVOL, RET1M, RET12M,  $\beta_{L,L}$  and  $\beta_{L,M}$ . The coefficients are multiplied by 100. Newey-West t-statistics are reported below the coefficients. The cross-sectional adjusted  $R^2$  is reported in the last row.

	Panel A: Raw		Panel B: Decile		Panel C: Purged	
	1	2	1	2	1	2
CVILLIQ	0.21	0.19	0.24	0.24	0.19	0.18
	2.99	2.65	3.81	3.88	2.28	2.1
$\beta_{L,L}$		0		0.07		0
		-0.27		1.3		-0.18
$\beta_{L,M}$		-275.21		-0.17		-302.16
		-0.77		-2.72		-0.79
ILLIQ	0.2	0.19	0.22	0.17	0.19	0.18
	3.49	3.21	0.97	0.76	2.61	2.32
SIZE	0.09	0.08	-0.34	-0.36	0.07	0.06
	1.42	1.21	-1.5	-1.58	0.92	0.74
DY	0.16	0.18	0.13	0.13	0.29	0.29
	0.28	0.3	1.17	1.21	0.42	0.43
BM	0.12	0.12	0.34	0.33	0.03	0.03
	2.87	2.92	3.28	3.22	0.76	0.8
TURN	0.21	0.2	0.16	0.15	0.22	0.22
	2.69	2.6	1.02	0.97	2.31	2.18
IVOL	-26.54	-27.53	-0.48	-0.48	-28.51	-29.21
	-6.49	-6.8	-3.98	-4.01	-6.77	-6.99
RET1M	-2.37	-2.38	-1.05	-1.03	-3.19	-3.2
	-4.42	-4.44	-4.74	-4.71	-5.26	-5.26
RET12M	0.37	0.37	0.61	0.61	0.24	0.24
	2.15	2.14	2.33	2.33	1.27	1.26
Adj.R <sup>2</sup>	0.03	0.04	0.03	0.03		

The purged estimators are computed as the constant terms from OLS regressions of monthly Fama-MacBeth coefficient estimates on factor returns. Each Panel uses two specifications: the third model from Table 3 and the same model augmented with  $\beta_{L,L}$  and  $\beta_{L,M}$ .<sup>24</sup> According to the results, the coefficient on CVILLIQ remains positive and significant in all panels of Table 5.

Acharya and Pederson (2005) find that the covariance between an asset's illiquidity and the market return is significantly negatively related to stock returns. This is the case since investors are willing to accept a lower expected return on a security that is less illiquid in a down market. The results in Table 5 on  $\beta_{L,M}$  are consistent with Acharya and Pederson (2005). However,  $\beta_{L,M}$  is significant only when the decile ranks of the independent variables are used. A possible explanation behind the insignificant coefficient on  $\beta_{L,M}$  in Panels A and C could be the presence of considerable skewness in the cross-sectional distribution of  $\beta_{L,M}$ .

Overall, the results in table 5 suggest that the volatility of liquidity does not simply capture the covariance between individual stock liquidity and aggregate market liquidity or return. To the extent that  $\beta_{L,L}$  and  $\beta_{L,M}$  measure systematic liquidity risk, the separate pricing of CVILLIQ indicates that idiosyncratic volatility risk is important in the cross-section of returns. The coefficient on CVILLIQ remains significantly positive under various risk adjustments and control variables, indicating that it captures an effect different from a stock's return (or liquidity) exposure to aggregate liquidity or other standard return factors.

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<sup>24</sup> The results are similar for other sets of control variables.

## 4.2 Sub-Sample Analysis

In this section we examine whether the results are robust across different sample periods. We divide the sample in two periods, before and after 1987. The motivation for choosing 1987 comes from Amihud, Mendelson and Wood (1990) who argue that investors' perception of illiquidity have changed drastically after the crash of October 1987 and investors have realized that markets are not as liquid as before the crash. Panel A of Table 6 presents results using the real values of the independent variables, while Panel B presents results using decile ranks. The two sample periods are from 1964:01 to 1987:12 and from 1988:01 to 2009:12. Panel A, shows that the coefficient on CVILLIQ is significant and positive in both sub-periods when using the real values of the independent variables. Panel B shows that the same result holds when we use decile ranks for the variables.

An interesting observation from Panel A of Table 6 is that illiquidity is positively related to returns in both sample periods, but it is significant only during the 1964:01 to 1987:12 period. When we exclude size from our analysis to control for multicollinearity, illiquidity becomes significant in the later period but the effect is relatively weaker compared to the earlier period.<sup>25</sup> Overall, the results suggest that the volatility of liquidity is significantly related to average returns in both time periods that we examine.

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<sup>25</sup> Ben-Raphael, Kadan, and Wohl (2009) show that both the sensitivity of stock returns to illiquidity and the illiquidity premia have declined over the past four decades. They claim that the proliferation of index funds and exchange-traded funds, and enhancements in markets that facilitate arbitrage activity might explain their results.

**Table 6: Fama-MacBeth Regression Estimates: Sub-Period Analysis**

This table presents the results from Fama-MacBeth regressions in which the dependent variables are stock returns and the independent variables are various stock characteristics. The sample consists of common stocks listed on AMEX and NYSE for two sample periods, 1964:01 to 1987:12 and 1988:01 to 2009:12. The stock characteristics are defined in Table 1. All variables are measured in logs except for DY, IVOL, RET1M, and RET12M. Panel A uses the actual values of the independent variables, while Panel B uses their decile ranks standardized between zero and one. The coefficients are multiplied by 100. Newey-West  $t$ -statistics are reported below the coefficients. The cross-sectional adjusted  $R^2$  is reported in the last row.

	Panel A: Real Values		Panel B: Decile Ranks	
	1964-1987	1988-2009	1964-1987	1988-2009
CVILLIQ	0.17	0.23	0.23	0.21
	2.27	2.09	3.39	2.62
ILLIQ	0.29	0.06	0.39	0.42
	4.25	0.62	1.48	1.08
SIZE	0.1	-0.04	-0.57	-0.23
	1.31	-0.33	-1.7	-0.62
DY	-0.55	0.6	-0.02	0.18
	-0.37	1.4	-0.07	1.06
BM	0.25	0.18	0.6	0.53
	2.63	2.11	3.2	2.5
TURN	0.2	0.27	0.01	0.7
	2.05	1.97	0.06	2.29
IVOL	-32.87	-20.18	-0.52	-0.48
	-5.33	-3.91	-2.96	-2
RET1M	-1.82	0.12	-0.81	-0.27
	-3.22	0.23	-4.88	-1.16
RET12M	1.02	0.12	1.36	0.74
	5.37	0.3	6.74	1.49
Adj. $R^2$	0.08	0.05	0.08	0.05

We further split our sample into good and bad states of the business cycle. The motivation for doing this comes from recent theoretical research that relates crisis periods to declines in asset liquidity. Several models predict that sudden liquidity dry-

ups may occur due to demand effects such as market participants engaging in panic selling, supply effects such as financial intermediaries not being able to provide liquidity, or both.<sup>26</sup> These models predict that the demand for liquidity increases in bad times as investors liquidate their positions across many assets.

At the same time, the supply of liquidity decreases in bad times as liquidity providers hit their funding constraints. In addition, borrowing constraints are tighter in bad times. Investors, who cannot borrow easily in case of an emergency consumption need, would have to liquidate their positions. As a result, the uncertainty associated with an asset's liquidity is likely to increase around crisis periods and become a stronger concern for investors. Therefore, we conjecture that the volatility of liquidity effect will be stronger during bad economic times.

We use the growth rate of industrial production as an indicator of good or bad economic times. The advantage of this variable is that it is a contemporaneous indicator of the business cycle. Data on the level of industrial production comes from the website of the Federal Reserve Bank of St. Louis. Industrial production growth (IND) is defined as the first difference in the log of industrial production. To capture crisis periods, we split the sample in two parts: one corresponding to the 10% lowest observations of IND (bad times), the other corresponding to the rest of the observations. We compute the average return of the equally-weighted high-minus-low volatility of liquidity strategy (CV5-CV1) within each sub-sample. Untabulated results show that the average CV5-CV1 return is 1.01% per month in bad times and 0.23% per month the rest of the time.

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<sup>26</sup> See Gromb and Vayanos (2002), Morris and Shin (2004), Vayanos (2004), Garleanu and Pedersen (2007), and Brunnermeier and Pedersen (2009), among others.



The difference between the two is statistically significant. If we define bad times as the 25% (50%) lowest observations of IND, the average CV5-CV1 return is 0.61% (0.44%) in bad times and 0.22% (0.21%) the rest of the time. Therefore, the results suggest that the expected return premium for stocks with high volatility of liquidity is higher in bad times and increases with the severity of the crisis period.

#### ***4.3 Comparing Daily and Monthly Measures of the Volatility of Liquidity, Based on Amihud, Turnover, and Dollar Volume***

Chordia, Subrahmanyam, and Anshuman (2001) compute the volatility of turnover and dollar volume over the past 36 months and show that it is negatively related to average returns. This result seems to be in contrast to what we document so far, since we find that the volatility of liquidity is positively related to expected returns. One possible explanation for this apparent discrepancy could be that we use a different measure of liquidity than CSA and we use daily data to estimate its volatility. To address this issue in greater detail, in this section we examine separately two sets of regressions. In the first case, we focus on volatility of liquidity estimated from daily data using three different variables: the Amihud ratio, turnover, and dollar volume. In the second case, we use volatility of liquidity estimated from monthly data using the same three separate variables. We define the new variables as follows: the coefficient of variation of daily turnover, estimated using a method similar to equation (3) is CVTURN. Similarly, the coefficient of variation of daily dollar volume is CVDVOL. Also, for each stock and every month we calculate the coefficient of variation of liquidity by using the past 36 monthly observations of the Amihud ratio. We call the resulting coefficient of variation

CVILLIQ36 to distinguish it from our previous measure CVILLIQ which is computed using daily data within a month. The coefficient of variation of monthly turnover over the last 36 months is CVTURN36, and the coefficient of variation of monthly dollar volume over the last 36 months is CVDVOL36. The last two variables are the ones used by CSA to document a negative relation between the volatility of liquidity and average returns.

The results are presented in Table 7. As before, we apply log transformations to the newly defined variables since they exhibit skewness. We use the same control variables as before. In Panel A the volatility of liquidity is estimated with daily data, in Panel B it is estimated with monthly data, while in Panel C it is estimated with both daily and monthly data. For the sake of brevity, we use only one specification in terms of control variables; however, the results are similar under different specifications. According to the results in Panel A, the coefficient on CVILLIQ is positive and significant in the presence of CVTURN or CVDVOL. The coefficient of variation of daily turnover and dollar volume are negatively related to average returns, but the effect is not significant in the presence of CVILLIQ.

The results in Panel A suggest that investors fear volatility in the daily price impact of trade. Therefore, the difference between CSA's and our results could stem from using the price impact as a measure of liquidity and also estimating the volatility of liquidity using daily data within a month. The volatility of the daily price impact is associated with a positive return premium. We conjecture that the reason for this is that traders who have immediate liquidity needs cannot time their trades so that they occur

only during periods of low price impact. The more volatile the price impact of trade, the higher is the probability that an immediate liquidity need might be executed at low levels of liquidity.

In Panel C, we implement a direct comparison between our measure of the volatility of liquidity, CVILLIQ, which uses daily data, and CSA's measures CVTURN36 and CVDVOL36 which use monthly data. This test represents a significant challenge for the CVILLIQ measure since it is based on daily data and therefore, it is likely to be noisier than CVTURN36 and CVDVOL36. The results show that both measures remain significant and have opposite signs when included in the same regression. Therefore, the volatility of liquidity based on daily Amihud ratios within a month, CVILLIQ, contains separate information about expected returns relative to the volatility of liquidity estimated from monthly turnover and dollar volume.

As argued earlier, this could reflect two complementary rather than opposing effects. The negative coefficient on CVTURN36 may reflect the possibility that high variability of trading activity means a higher chance to sell when liquidity is favorable for investors who can time the market (see Pereira and Zhang (2011)).

On the other hand, we argue that some traders may not be able to time the market and, therefore, will require a risk premium for holding stocks with higher variation in liquidity. This is reflected in the positive coefficient on CVILLIQ. An alternative explanation for the negative coefficient associated with CVTURN36 (CVDVOL36) could be that the variability of turnover (dollar volume) could measure firm- specific

**Table 7: Fama-MacBeth Regression Estimates: Comparing Daily and Monthly Measures of the Volatility of Liquidity Based on Amihud, Turnover, and Dollar Volume**

This table presents the results from Fama-MacBeth regressions in which the dependent variables are stock returns and the independent variables are various stock characteristics. The sample consists of common stocks listed on NYSE and AMEX for the period from January 1964 to December 2009. The stock characteristics are defined in Table 1. The volatility of liquidity is measured in six separate ways: the coefficient of variation of daily Amihud ratios within a month (CVILLIQ), the coefficient of variation of daily turnover within a month (CVTURN), the coefficient of variation of daily dollar volume within a month (CVDVOL), the coefficient of variation of monthly Amihud ratios over the last 36 months (CVILLIQ36), the coefficient of variation of monthly turnover over the last 36 months (CVTURN36), and the coefficient of variation of monthly dollar volume over the last 36 months (CVDVOL36). Panel A contains only daily measures of the volatility of liquidity, while Panel B contains only monthly measures of the volatility of liquidity. Panel C contains both. All variables are measured in logs except for DY, IVOL, RET1M, and RET12M. The coefficients are multiplied by 100. Newey-West  $t$ -statistics are reported below the coefficients. The cross-sectional adjusted  $R^2$  is reported in the last row.

Panel A: Daily			Panel B: Monthly			Panel C: Daily and Monthly		
	1	2		1	2		1	2
CVILLIQ	0.17	0.18	CVILLIQ36	0.04	0.04	CVILLIQ	0.17	0.17
	2.85	2.98		0.37	0.36		2.86	2.84
CVTURN	-0.08		CVTURN 36	-0.26		CVTURN 36	-0.27	
	-1.08			-4.15			-4.4	
CVDVOL		-0.13	CVDVOL36		-0.15	CVDVOL36		-0.15
		-1.72			-2.06			-1.96
ILLIQ	0.22	0.24	ILLIQ	0.21	0.21	ILLIQ	0.2	0.2
	2.94	3.13		3.56	3.52		3.1	3.09
SIZE	0.08	0.08	SIZE	0.03	0.04	SIZE	0.03	0.04
	1.04	1.16		0.43	0.57		0.39	0.5
DY	0.1	0.1	DY	-0.05	-0.06	DY	0.1	0.09
	0.13	0.14		-0.06	-0.08		0.14	0.13
BM	0.19	0.19	BM	0.18	0.18	BM	0.19	0.19
	3.13	3.13		3.08	3.09		3.09	3.11
TURN	0.26	0.27	TURN	0.25	0.25	TURN	0.25	0.25
	2.65	2.77		3.35	3.39		2.86	2.9
IVOL	-25.4	-25.5	IVOL	-25.2	-25.21	IVOL	-24.4	-24.37
	-6.12	-6.11		-5.88	-5.93		-5.56	-5.63
RET1M	-0.88	-0.86	RET1M	-0.9	-0.9	RET1M	-0.87	-0.88
	-2.09	-2.07		-2.18	-2.18		-2.08	-2.09
RET12M	0.58	0.58	RET12M	0.61	0.61	RET12M	0.6	0.59
	2.7	2.7		3.1	3.06		2.71	2.66
Adj.R <sup>2</sup>	0.07	0.07	Adj.R <sup>2</sup>	0.07	0.07	Adj.R <sup>2</sup>	0.07	0.07

negative relation between the variability of trading activity and returns. He shows that at the firm level, high variability of turnover implies high uncertainty and low aggregate volatility risk. Therefore, firms with high variability of turnover beat the CAPM in periods of increasing market variance and this explains their low expected returns. Other studies that use turnover as a proxy for firm-specific uncertainty or investor disagreement include Harris and Raviv (1993) and Blume, Easley, and O'Hara (1994), among others. Turnover is found to be high if prices fluctuate a lot, if traders disagree about firm value, or if they receive a lot of information about the firm (e.g., Karpoff (1987)). On the other hand, the effect of volume on returns is ambiguous. While CSA find a negative relations, Gervais, Kaniel and Milgelgrin (2001) documents that unusual volume is positively related to future returns.

Alternatively, CSA's findings might be related to short sale restrictions. We explain this potential relation below. High volatility of trading activity might be related to an increase in investor disagreement. On the other hand, high volatility of trading activity might be due to high volume shocks which attract investors' attention. If a certain stock experiences an increase in investor disagreement or if it attracts the attention of investors, under short sale restrictions the price of the stock will reflect the views of optimistic investors and this will lead to overvaluation as argued by Miller (1977). Since the overvaluation would be temporary, the price will converge to its fundamental value as the uncertainty about the stock is resolved. Therefore, the documented negative relation between the volatility of trading and returns might be a result of overvaluation. In that case, we would expect that the result documented by CSA

would be stronger among stocks with high short sale restrictions. We test this hypothesis below.

We measure short sale restrictions using residual institutional ownership (RESIO), following Nagel (2005). Data on institutional ownership are obtained from 13-F filings, available from Thomson Financial for the period 1980-2009. We define Institutional Ownership (IO), as the sum of the holdings of all institutions for each stock in each quarter, divided by the number of shares outstanding obtained from CRSP. Stocks that have available return data but no reported institutional holdings are assumed to have zero institutional ownership. IO values below 0.0001 and above 0.9999 are replaced with 0.0001 and 0.9999, respectively. Following Nagel (2005), every quarter we regress the logit transformation of institutional ownership of each stock on  $\log(\text{SIZE})$  and  $\log(\text{SIZE})^2$ . Next we average the slope coefficients of the independent variables over time. The residuals of the regressions stand for Residual Institutional Ownership (RESIO). We average the residual IO for each stock over the course of the past ten quarters to obtain average residual IO. Following Nagel (2005), RESIO is lagged two quarters so that the results are not driven by the short-term outperformance of institutional investors' trades (see Chen, Jegadeesh, and Wermers (2000) and Yan and Zhang (2009)). We divide stocks in two groups according to high and low RESIO, and we run Fama-McBeth regressions within each group. Our measure of the volatility of liquidity, CVILLIQ, and CSA's measure are both included in each regression. The results are presented in table 8.

**Table 8: Fama-MacBeth Regression Estimates: The Role of Short Sale Restrictions**

This table presents the results from Fama-MacBeth regressions in which the dependent variables are stock returns and the independent variables are various stock characteristics. The sample consists of common stocks listed on NYSE and AMEX for the period from January 1983 to December 2009. The stock characteristics are defined in Table 1. The volatility of liquidity is measured in three separate ways: the coefficient of variation of daily Amihud ratios within a month (CVILLIQ), the coefficient of variation of monthly turnover over the last 36 months (CVTURN36), and the coefficient of variation of monthly dollar volume over the last 36 months (CVDVOL36). In each Panel we perform regressions within two separate residual institutional Ownership (RESIO) groups (i.e. above and below median (RESIO)). Panel A contains both CVTURN36 and CVILLIQ, while Panel B contains CVDVOL36 and CVILLIQ. The coefficients are multiplied by 100. Newey-West t-statistics are reported below the coefficients. The cross-sectional adjusted  $R^2$  is reported in the last row.

	Panel A: Volatility of Turnover		Panel B: Volatility of Dollar Volume	
	Low RESIO	High RESIO	Low RESIO	High RESIO
CVILLIQ	0.23 2.49	0.21 2.11	0.22 2.38	0.2 2.02
CVTURN36	-0.32 -3.6	-0.2 -1.94		
CVDVOL36			-0.28 -2.81	-0.19 -1.87
ILLIQ	0.22 1.84	0.13 1.25	0.22 1.87	0.13 1.28
SIZE	-0.06 -0.46	0.05 0.43	-0.04 -0.35	0.06 0.51
DY	-0.36 -0.44	0.75 1.95	-0.37 -0.45	0.67 1.74
BM	0.36 4.61	0.2 2.55	0.35 4.62	0.2 2.52
TURN	0.37 2.48	0.47 4.35	0.39 2.58	0.47 4.53
IVOL	-13.01 -2.13	-37.26 -6.5	-12.79 -2.13	-37.09 -6.55
RET1M	-0.23 -0.39	-0.81 -1.66	-0.23 -0.39	-0.8 -1.63
RET12M	0.19 0.64	0.4 0.95	0.19 0.67	0.43 1.01
Adj.R <sup>2</sup>	0.08	0.07	0.08	0.07

Table 8 shows that the negative relationship between the volatility of trading activity over the past 36 months and returns is more pronounced among stocks with high short sale restrictions. While the coefficient on CVTURN36 is -0.32 in the low RESIO group and it increases to -0.20 in the high RESIO group. The difference between the two coefficients is significant at the 1% level. On the other hand, the coefficient on CVILLIQ is significant in both groups and similar in magnitude across groups. Similar results hold when the volatility of trading activity is measured by CVDOL36. Overall these results suggest that CSA's findings might be related to overvaluation under short sale restrictions. This is consistent with the argument proposed by Miller (1977).

#### ***4.4 Alternative Measurement Periods for the Volatility of Liquidity***

So far our results are based on the volatility of liquidity measured from daily data within a month. However, since the Amihud ratio includes returns, it might be the case that our measure is capturing some short-term return autocorrelations that cannot be adjusted for with our control variables. In addition, it might be possible to get more precise estimates of the volatility of liquidity by using a larger sample of daily Amihud ratios. Therefore, in this section we investigate whether our results are robust to alternative measurement periods for our key variable, CVILLIQ.

We use 3, 6, 9, and 12 months of daily Amihud ratios to compute four alternative measures of the volatility of liquidity. We stop at 12 months since we want a balance between a more precise measure of CVILLIQ and more recent information about the liquidity of the stock. Since liquidity varies over time, going beyond 12 months to



**Table 9: Fama-MacBeth Regression Estimates: Using Different Measurement Periods for the Volatility of Liquidity**

This table presents the results from Fama-MacBeth regressions in which the dependent variables are stock returns and the independent variables are various stock characteristics. The sample consists of common stocks listed on AMEX and NYSE for period from January 1964 to December 2009. The stock characteristics are defined in Table 1. All variables are measured in logs except for DY, IVOL, RET1M, and RET12M. The volatility of liquidity, CVILLIQ, is measured as the coefficient of variation of daily Amihud ratios within 3, 6, 9, or 12 months. Panel A uses the actual values of the independent variables, while Panel B uses their decile ranks standardized between zero and one. The coefficients are multiplied by 100. Newey-West t-statistics are reported below the coefficients. The cross-sectional adjusted  $R^2$  is reported in the last row.

	Panel A: Real Values				Panel B: Decile Ranks			
	3 mo.	6 mo.	9 mo.	12 mo.	3 mo.	6 mo.	9 mo.	12 mo.
CVILLIQ	0.34	0.35	0.3	0.21	0.25	0.33	0.27	0.21
	3.64	2.78	2.26	1.61	3.7	3.56	2.65	2.1
ILLIQ	0.17	0.17	0.17	0.18	0.41	0.36	0.4	0.43
	2.78	2.74	2.84	3.16	1.75	1.52	1.74	1.93
SIZE	0.04	0.04	0.04	0.05	-0.36	-0.36	-0.36	-0.36
	0.57	0.58	0.59	0.72	-1.45	-1.44	-1.45	-1.47
DY	0	0.02	0.04	0.07	0.07	0.09	0.09	0.08
	0	0.03	0.05	0.09	0.47	0.53	0.53	0.53
BM	0.21	0.21	0.21	0.21	0.57	0.56	0.56	0.57
	3.37	3.33	3.35	3.38	4	3.93	3.97	4
TURN	0.23	0.23	0.23	0.24	0.34	0.35	0.35	0.35
	2.83	2.88	2.94	3.08	1.9	1.92	1.94	1.96
IVOL	-27.15	-27.61	-27.85	-27.93	-0.52	-0.54	-0.56	-0.56
	-6.64	-6.74	-6.83	-6.85	-3.61	-3.71	-3.83	-3.85
RET1M	-0.93	-0.92	-0.91	-0.9	-0.57	-0.57	-0.57	-0.56
	-2.29	-2.25	-2.23	-2.2	-3.89	-3.89	-3.86	-3.82
RET12M	0.58	0.56	0.56	0.57	1.05	1.04	1.04	1.05
	2.65	2.59	2.6	2.64	4.01	4.04	4.07	4.13
Adj. $R^2$	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07

measure the volatility of liquidity might not give sufficient weight to the most recent

variation in price impact. Table 9 presents the results using our main specification.

The coefficient on CVILLIQ is significantly positive in all cases except when we use 12 months of daily data to estimate the volatility of liquidity and the real values of the control variables. Therefore, the positive relation between the volatility of liquidity and average returns appears to be robust to the number of daily observations used in calculating the volatility of liquidity.

#### ***4.5. Expected Volatility of Liquidity***

We are interested in the relation between expected returns and ex-ante volatility of liquidity. However, it is not straightforward to test this relation empirically. Our analysis so far uses lagged volatility of liquidity as a proxy for the ex-ante volatility of liquidity. If the volatility of liquidity is time-varying, lagged volatility of liquidity alone may not adequately forecast expected volatility of liquidity. Therefore, we estimate a cross-sectional model of expected volatility of liquidity that uses additional predictive variables. Specifically, we run a cross-sectional regression of CVILLIQ, measured over the same holding period as returns, on firm characteristics measured at the end of the previous month. In the cross-sectional regressions we use two lags of CVILLIQ, SIZE, BM, IVOL, RET1M, RET12M, ILLIQ, and TURN. Then we use the fitted values of CVILLIQ from the cross-sectional regressions as independent variables in the subsequent Fama-MacBeth regressions. The results are in Table 10.

The predicted value of CVILLIQ, FCVILLIQ, is significantly positively related to average returns in all specifications. Therefore, our main results are robust to this alternative estimate of the volatility of liquidity.

**Table 10: Fama-MacBeth Regression Estimates: Using the Predicted Value of the Volatility of Liquidity**

This table presents the results from Fama-MacBeth regressions in which the dependent variables are stock returns and the independent variables are various stock characteristics. The sample consists of common stocks listed on AMEX and NYSE for period from January 1964 to December 2009. The stock characteristics are defined in Table 1. The volatility of liquidity is estimated from a cross-sectional model. Specifically, we run a cross-sectional regression of CVILLIQ, measured over the same holding period as returns, on firm characteristics measured at the end of the previous month. In the cross-sectional regressions we use two lags of CVILLIQ, SIZE, BM, IVOL, RET1M, RET12M, ILLIQ, and TURN. Then we use the fitted values of CVILLIQ from the cross-sectional regressions as independent variables in the subsequent Fama-MacBeth regressions. The fitted value is denoted by FCVILLIQ. Panel A uses the actual values of the independent variables, while Panel B uses their decile ranks standardized between zero and one. The coefficients are multiplied by 100. Newey-West t-statistics are reported below the coefficients. The cross-sectional adjusted  $R^2$  is reported in the last row.

	Panel A: Real Values			Panel B: Decile Ranks		
	1	2	3	1	2	3
FCVILLIQ	1.49	1.31	0.89	0.48	0.37	0.33
	4.25	3.69	2.67	2.5	1.81	2.19
ILLIQ	-0.1	-0.1	0.13	-0.66	-0.5	0.1
	-2.5	-1.97	1.86	-2.92	-2.18	0.41
SIZE	-0.1	-0.13	0.01	-0.68	-0.62	-0.52
	-2.2	-2.1	0.17	-2.68	-2.35	-2.13
DY	0.73	0.67	0	0.24	0.18	0.06
	0.83	0.71	0	1.4	1.03	0.4
RET712	0.97			0.96		
	5.22			5.82		
RET46	1.04			0.68		
	3.58			3.36		
RET23	-0.1			-0.24		
	-0.4			-1.37		
1/PRC	0.03	0			0.14	
	0.28	-0.03			0.68	
BM	0.23	0.21	0.2		0.58	0.56
	3.59	3.14	3.15		4.12	3.92
TURN			0.22			0.28
			2.66			1.66
IVOL			-26.69			-0.49
			-6.13			-3.42
RET1M		-1.2	-0.84	0.16	-0.54	-0.53
		-3.21	-2.07	0.8	-3.9	-3.68
RET12M		0.62	0.59	0.58	1.11	1.07
		2.85	2.71	4.21	4.26	4.15
Adj.R <sup>2</sup>	0.07	0.06	0.07	0.07	0.06	0.07

#### ***4.6. Additional Robustness Checks***

In this section we address some remaining concerns about the main results. We report our findings which are not tabulated in the dissertation but are available upon request. First, since our findings are stronger among small stocks, it might be the case that the findings are driven by the January effect documented by Keim (1983) (see also Tinic and West (1986), Eleswarapu and Reinganum (1993), and Amihud (2002)). In separate regressions and sorting analysis we control for the January effect and find similar results.

Second, the volatility of liquidity measure CVILLIQ might capture an interaction effect between past returns and trading volume. For example, Cooper (1999) and Lee and Swaminathan (2001) document that return continuations accentuate with volume, while Avramov et al. (2006) show that the short term return reversals accentuate with volume. Accordingly, we include an interaction term between trading volume and past returns and trading volume and contemporaneous returns in the Fama-MacBeth regressions. We find that the coefficient on CVILLIQ remains positive and significant.

Third, Acharya and Pederson (2005) argue that a 30% cap should be imposed on the Amihud ratio measure since anything greater than that due to low volume days might be unreasonable. We show that our results are not affected by imposing a 30% cap on the daily components of the liquidity measure.

In addition, Asparouhova et al. (2010) show that microstructure-induced noise in prices can lead to biases in empirical asset pricing tests. We employ the correction for this bias suggested by Asparouhova et al. (2010) and we find that the results remain

robust. Therefore, it is unlikely that our main results are driven by microstructure-induced noise.

Finally, to ensure that our results are not driven by a non-linear relation between illiquidity and future returns, we include  $ILLIQ$ -squared in the regressions and find similar results. In addition, return skewness does not seem to influence our main findings.

## 5. THE IDIOSYNCRATIC COMPONENT OF VOLATILITY OF LIQUIDITY

Our results so far indicate that total volatility of liquidity is positively related to expected returns. In this section we present a formal approach of extracting the idiosyncratic component of liquidity risk. Our goal is to examine whether this is the component that drives the cross-sectional pricing abilities of total volatility of liquidity.

We extract the idiosyncratic volatility of liquidity for each stock  $i$  in every month  $t$  using daily data within the month. In particular, we regress daily firm-level illiquidity on daily excess market returns and daily changes in market illiquidity:

$$DPIOF = \alpha_{it} + \beta_{illiq,i,t}^R * ExcessMKTRET_{d,t} + \beta_{illiq,i,t}^I * \Delta MKTILLIQ_{d,t} + e_{i,d,t} \quad (4)$$

and the measure of idiosyncratic liquidity volatility is:

$$CVILLIQ^{idios} = SD(e_{i,d,t}) / ILLIQ_{i,t} \quad (5)$$

The coefficient  $\beta_{illiq}^R$  measures the covariance of stock illiquidity while the coefficient  $\beta_{illiq}^I$  measures the covariance of stock illiquidity with aggregate market illiquidity. Both of these reflect systematic variations in firm-level illiquidity.

We further estimate systematic variation in stock returns by regressing daily firm-level excess returns on daily excess market returns and daily changes in market illiquidity:

$$R_{i,d,t} = \alpha_{it} + \beta_{r,i,t}^R * ExcessMKTRET_{d,t} + \beta_{r,i,t}^I * \Delta MKTILLIQ_{d,t} + u_{i,d,t} \quad (6)$$

where  $\beta_{r,i,t}^R$  is market betas and  $\beta_{r,i,t}^I$  is the covariance of stock return and aggregate market liquidity.

The four beta coefficients from equations (4) and (6) are very similar to the systematic liquidity and return risks examined by Acharya and Pedersen (2005). Our objective is to test whether idiosyncratic liquidity risk measured by  $CVILLIQ^{idios}$  is priced in the presence of these four betas. Motivated by Acharya and Pedersen (2005), we additionally control for the covariance between daily stock returns and daily stock illiquidity over month  $t$ ,  $COV(r, illiq)$ . We also control for the skewness of daily returns,  $SKEW$ .

The results are presented in Table 11. The results show that idiosyncratic liquidity risk  $CVILLIQ^{idios}$  is positively and significantly related to expected returns in all specifications. After controlling for various firm characteristics and risk exposures, stocks in the highest  $CVILLIQ^{idios}$  percentile earn on average 27 basis points per month more than stocks in the lowest  $CVILLIQ^{idios}$  percentile when we use excess returns as dependent variable. The magnitude is similar when we use risk-adjusted returns.

Acharya and Pederson (2005) find that the covariance between an asset's illiquidity and the market return is significantly negatively related to stock returns. This is the case since investors are willing to accept a lower expected return for a security that is less illiquid in a down market. The results in Table 11 are consistent with Acharya and Pederson (2005). However  $\beta_{illiq}^R$  is significant only when the decile ranks of the independent variables are used.

**Table 11: Fama-MacBeth Regression Estimates: Using Idiosyncratic Volatility of Liquidity**

This table presents the results from Fama-MacBeth regressions in which the dependent variables are stock returns and the independent variables are various stock characteristics. The sample consists of common stocks listed on AMEX and NYSE from January 1964 to December 2010. The stock characteristics are defined in Table 1 and Section V. In Panel A the dependent variables are excess stock returns, while in Panel B the dependent variables are risk-adjusted stock returns. Risk-adjustment is based on the Fama-French 3-factor model augmented with a momentum factor. In both panels the independent variables are various stock characteristics in both percentile ranks (standardized between zero and one) and real values. When real values of independent variables are used, we apply log transformations to SIZE, BM, and TURN. To minimize microstructure issues, one week is skipped between measurement of the independent and dependent variables. Coefficient estimates are multiplied by 100. Newey-West t-statistics are reported below the coefficients. The cross-sectional adjusted  $R^2$  is reported in the last row.

	Panel A: Real Values			Panel B: Decile Ranks		
	1	2	3	1	2	3
FCVILLIQ	1.49	1.31	0.89	0.48	0.37	0.33
	4.25	3.69	2.67	2.5	1.81	2.19
ILLIQ	-0.12	-0.1	0.13	-0.66	-0.5	0.1
	-2.54	-1.97	1.86	-2.92	-2.18	0.41
SIZE	-0.13	-0.13	0.01	-0.68	-0.62	-0.52
	-2.16	-2.1	0.17	-2.68	-2.35	-2.13
DY	0.73	0.67	0	0.24	0.18	0.06
	0.83	0.71	0	1.4	1.03	0.4
RET712	0.97			0.96		
	5.22			5.82		
RET46	1.04			0.68		
	3.58			3.36		
RET23	-0.13			-0.24		
	-0.42			-1.37		
1/PRC	0.03	0			0.14	
	0.28	-0.03			0.68	
BM	0.23	0.21	0.2		0.58	0.56
	3.59	3.14	3.15		4.12	3.92
TURN			0.22			0.28
			2.66			1.66
IVOL			-26.69			-0.49
			-6.13			-3.42
RET1M		-1.2	-0.84	0.16	-0.54	-0.53
		-3.21	-2.07	0.8	-3.9	-3.68
RET12M		0.62	0.59	0.58	1.11	1.07
		2.85	2.71	4.21	4.26	4.15
Adj.R2	0.07	0.06	0.07	0.07	0.06	0.07



A possible explanation behind the insignificant coefficient on  $\beta^R_{\text{illiq}}$  when we use real values of the independent variables could be the presence of considerable skewness in the cross sectional distribution of  $\beta^R_{\text{illiq}}$ . The covariance between an asset's return and the market illiquidity,  $\beta^R_{\text{illiq}}$ , has a positive and significant coefficient. The significance fades away when we use real values of the independent variables which again could be due to extreme outliers. Finally the covariance between stock illiquidity and market illiquidity is insignificant in any specification.

Overall, the results in Table 11 suggest that idiosyncratic volatility of liquidity is significantly positively related to average returns. This relation persists over and above the correlation between systematic liquidity risk and returns. Therefore, the previously documented relation between total volatility of liquidity and expected returns is driven by the idiosyncratic component of liquidity volatility. This result is puzzling in light of Acharya and Pedersen (2005) who document the pricing of systematic liquidity betas only.

## 6. CONCLUSION

In this study we find that the volatility of liquidity is positively related to future returns. The positive correlation between the volatility of liquidity and expected returns suggests that risk averse investors require a risk premium for holding stocks that have high variation in liquidity. Our results are robust to various control variables, systematic risk factors, and different sub-periods. Higher variation in liquidity implies that a stock may become illiquid with higher probability at a time when it is traded. This is important for investors who may face an immediate liquidity need due to exogenous cash needs, margin calls, dealer inventory rebalancing, or forced liquidations. In case of such liquidations, the investor may not be able to time the market by waiting for periods of high liquidity and thus, the level of liquidity on the day of the liquidity need is important. Overall all our results suggest that besides the mean level of liquidity, the second moment of liquidity also matters and is significantly related to future returns.

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