MODEL-BASED METHODOLOGY FOR BUILDING CONFIDENCE IN A
DYNAMIC MEASURING SYSTEM

A Thesis

by

ISAAC MARK REESE

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

Approved by:

Chair of Committee, Richard J. Malak
Committee Members, Daniel A. McAdams
                              Michael D. Johnson
                              Darren J. Hartl
Head of Department, Andreas Polycarpou

May 2013

Major Subject: Mechanical Engineering

Copyright 2013 Isaac Mark Reese
ABSTRACT

This thesis examines the special case in which a newly developed dynamic measurement system must be characterized when an accepted standard qualification procedure does not yet exist. In order to characterize this type of system, both physical experimentation and computational simulation methods will be used to build trust in this measurement system. This process of establishing credibility will be presented in the form of a proposed methodology.

This proposed methodology will utilize verification and validation methods that apply within the simulation community as the foundation for this multi-faceted approach. The methodology will establish the relationships between four key elements: physical experimentation, conceptual modeling, computational simulations, and data processing. The combination of these activities will provide a comprehensive characterization study of the system.

In order to illustrate the methodology, a case study was performed on a dynamic force measurement system owned by Sandia National Laboratories. This system was designed to measure the force required to pull a specimen to failure in tension at a user-input velocity. The results of the case study found that there was a significant measurement error occurring as the pull event involved large break loads and high velocities. 100 pull events were recorded using an experimental test assembly. The highest load conditions
discovered a force measurement error of over 100%. Using computational simulations, this measurement error was reduced to less than 10%. These simulations were designed to account for the inertial effects that skew the piezoelectric load cells. This thesis displays the raw data and the corrected data for five different pull settings. The simulations designed using the methodology significantly reduced the error in all five pull settings.

In addition to the force analysis, the simulations provide insight into the complete system performance. This includes the analysis of the maximum system velocity as well as the analysis of several proposed design changes. The findings suggest that the dynamic measurement system has a maximum velocity of 28 fps, and that this maximum velocity is unaffected by the track length or the mass of the moving carriage.
DEDICATION

This work is dedicated to my wife Maddie, who supported me throughout the course of this work.
ACKNOWLEDGEMENTS

I would like to thank Dr. Richard Malak for being open to the idea of taking on this project with me. His guidance, insight, and willingness to help made this work possible. I also would like to thank Dr. Michael Johnson, Dr. Darren Hartl, and Dr. Daniel McAdams for participating as my committee members.

Thanks to Mike Hurst and James Love for making this project possible. This work would not have been available without their cooperation and help. All of the testing and funding performed in this thesis was a result of their help and legwork.

I would also like to thank Ben, Edgar, Sean, Isaac (2), and Chuck, whom I worked with on a daily basis, for providing both technical guidance and laughter. Additionally, I would like to thank Kevin, Matt, Scott, and Adam for providing me with a home away from home and a social life outside of graduate school.

Finally, my parents, Maddie’s parents, my brothers, and my sister provided encouragement throughout graduate school through communication and visits from Oklahoma. The eight hour drive to Nardin, OK was not a barrier to my family during my two years at College Station.
NOMENCLATURE

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{LC1}$</td>
<td>Spring constant for force sensor 1</td>
</tr>
<tr>
<td>$K_{LC2}$</td>
<td>Spring constant for force sensor 2</td>
</tr>
<tr>
<td>$K_L$</td>
<td>Spring constant for wire rope lanyard</td>
</tr>
<tr>
<td>$m_1$</td>
<td>Mass attached to force sensor 1</td>
</tr>
<tr>
<td>$m_2$</td>
<td>Mass attached to force sensor 2</td>
</tr>
<tr>
<td>$i_a(t)$</td>
<td>Armature current of motor (state variable)</td>
</tr>
<tr>
<td>$V_{in}(t)$</td>
<td>Input voltage of motor</td>
</tr>
<tr>
<td>$F(t)$</td>
<td>Pull force applied to system</td>
</tr>
<tr>
<td>$\tau_m(t)$</td>
<td>Output torque of motor</td>
</tr>
<tr>
<td>$L_a$</td>
<td>Motor armature inductance</td>
</tr>
<tr>
<td>$R_a$</td>
<td>Motor armature resistance</td>
</tr>
<tr>
<td>$J_m$</td>
<td>Rotational inertia of motor</td>
</tr>
<tr>
<td>$J_z$</td>
<td>Rotational inertia of HPLA</td>
</tr>
<tr>
<td>$\theta_m(t)$</td>
<td>Motor shaft angle</td>
</tr>
<tr>
<td>$\theta_g(t)$</td>
<td>HPLA gear angle</td>
</tr>
<tr>
<td>$N$</td>
<td>Gear reduction ratio</td>
</tr>
<tr>
<td>$r_z$</td>
<td>Effective radius of HPLA</td>
</tr>
<tr>
<td>$K_P$</td>
<td>PID control proportional constant</td>
</tr>
<tr>
<td>$K_I$</td>
<td>PID control integrator constant</td>
</tr>
<tr>
<td>Variable</td>
<td>Description</td>
</tr>
<tr>
<td>----------</td>
<td>-------------</td>
</tr>
<tr>
<td>$K_D$</td>
<td>PID control derivative constant</td>
</tr>
<tr>
<td>$K_T$</td>
<td>Motor torque constant</td>
</tr>
<tr>
<td>$K_e$</td>
<td>Motor voltage constant</td>
</tr>
<tr>
<td>$M$</td>
<td>Maximum velocity input by user</td>
</tr>
<tr>
<td>$b_M$</td>
<td>Viscous damping coefficient for motor</td>
</tr>
<tr>
<td>$b_g$</td>
<td>Viscous damping for HPLA</td>
</tr>
</tbody>
</table>
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>ii</td>
</tr>
<tr>
<td>DEDICATION</td>
<td>iv</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>v</td>
</tr>
<tr>
<td>NOMENCLATURE</td>
<td>vi</td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td>viii</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>x</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>xii</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Problem Background</td>
<td>1</td>
</tr>
<tr>
<td>1.1.1 Motivating Problem Background</td>
<td>2</td>
</tr>
<tr>
<td>1.2 Proposed Solution and Methodology</td>
<td>5</td>
</tr>
<tr>
<td>1.3 Relevant Background Literature</td>
<td>7</td>
</tr>
<tr>
<td>1.4 Contributions</td>
<td>8</td>
</tr>
<tr>
<td>1.5 Thesis Contents</td>
<td>9</td>
</tr>
<tr>
<td>2. TECHNICAL BACKGROUND</td>
<td>11</td>
</tr>
<tr>
<td>2.1 Verification and Validation of Simulations</td>
<td>11</td>
</tr>
<tr>
<td>2.1.1 Verification and Validation Processes and Definitions</td>
<td>12</td>
</tr>
<tr>
<td>2.1.2 Validation Techniques</td>
<td>13</td>
</tr>
<tr>
<td>2.2 Gathering Real System Data through Validation Experimentation</td>
<td>16</td>
</tr>
<tr>
<td>2.2.1 Validation Experimentation</td>
<td>16</td>
</tr>
<tr>
<td>2.2.2 Selection of Instrumentation</td>
<td>17</td>
</tr>
<tr>
<td>3. PROPOSED METHODOLOGY</td>
<td>19</td>
</tr>
<tr>
<td>3.1 Motivation and Application for Methodology</td>
<td>19</td>
</tr>
<tr>
<td>3.2 Description of Process</td>
<td>19</td>
</tr>
<tr>
<td>4. DEMONSTRATION OF METHODS</td>
<td>27</td>
</tr>
<tr>
<td>4.1 Physical Experimentation of Force Tester</td>
<td>28</td>
</tr>
<tr>
<td>4.1.1 Primary System Components</td>
<td>29</td>
</tr>
</tbody>
</table>
4.1.2 Experimental Instrumentation ................................................................. 30
4.1.3 Experimental Testing Procedure ........................................................... 34
4.1.4 Measurement System Operational Data Collection .................................. 35

4.2 Complete System Model and Simulation .................................................. 40
4.2.1 Conceptual Model of Complete System .................................................. 40
4.2.2 Conceptual Model Validation ................................................................. 45
4.2.3 Computational Simulation for Complete System Model ......................... 46
4.2.4 Computational Simulation Verification .................................................... 49
4.2.5 Computational Simulation Operational Validation ............................... 49
4.2.6 Computational Simulation Operational Prediction ................................. 51

4.3 Force Measurement Model and Simulation .............................................. 55
4.3.1 Conceptual Models of Carriage System .................................................. 56
4.3.2 Conceptual Validation of Models ............................................................ 59
4.3.3 Computational Simulation for Carriage System Models ......................... 59
4.3.4 Computational Simulation Verification .................................................... 60
4.3.5 Computational Simulation Operational Validation and Prediction .......... 60
4.3.6 Conceptual Models of Carriage System using Filtering Methods ........... 61
4.3.7 Conceptual Validation of Filtering Model ............................................... 62
4.3.8 Computational Simulation using Filters ............................................... 62
4.3.9 Filter Computational Simulation Verification .......................................... 63
4.3.10 Filter Computational Simulation Operational Validation and Prediction ... 64

4.4 System Data/Results Processing ............................................................... 64

6. CONCLUSION .............................................................................................. 66

REFERENCES ................................................................................................. 69

APPENDIX A .................................................................................................... 74

APPENDIX B .................................................................................................... 76

APPENDIX C .................................................................................................... 93
LIST OF FIGURES

Figure 1. Dynamic Load Testing Configuration ................................................................. 3
Figure 2. Simple Model Paradigm [4] .......................................................................................... 12
Figure 3. Proposed Methodology for Establishing System Credibility ................................. 20
Figure 4. Implementation of Methodology on Force Test System ....................................... 28
Figure 5. Simplified System Description .................................................................................. 29
Figure 6. Parker HPLA180, Parker MPP Motor, and Aluminum Carriage .......................... 30
Figure 7. Experimental Test Configuration Setup ............................................................... 32
Figure 8. Picture of Experimental Setup .................................................................................. 33
Figure 9. Picture of Each Force Sensor Assembly ................................................................. 33
Figure 10. Set #1 Test #5 (200 lbf 3fps) .............................................................................. 36
Figure 11. Set #2 Test #24 (400 lbf 11fps) ........................................................................... 36
Figure 12. Set #3 Test #44 (600 lbf 16 fps) ......................................................................... 37
Figure 13. Set #4 Test #64 (400 lbf 7.8 fps) ........................................................................ 37
Figure 14. Set #5 Test #84 (600 lbf 7.8 fps) ........................................................................ 38
Figure 15. Accelerometer Data from Test #82 ..................................................................... 39
Figure 16. Encoder Data from Test #82 .............................................................................. 39
Figure 17. Simplified System .................................................................................................. 41
Figure 18. DC Circuit Describing Motor .............................................................................. 41
Figure 19. Simplified Drive System Representation .............................................................. 42
Figure 20. Summary of Forces Associated with Motor and HPLA Drive System ............. 43
Figure 21. Carriage System Model ................................................................. 44
Figure 22. Computational Simulation Logic ..................................................... 47
Figure 23. Simulation Velocity Profile (3ft/s 200lbf) ........................................ 50
Figure 24. Simulation Velocity Profile (7.8 ft/s 400 lbf) ................................... 50
Figure 25. Maximum Velocity Capability .......................................................... 52
Figure 26. System Current During Pull Event .................................................. 52
Figure 27. Required Power to Increase Velocity ................................................. 53
Figure 28. Effect of Carriage Mass on System Performance ............................... 54
Figure 29. Carriage System Model ................................................................. 56
Figure 30. Simulated 1-D Stationary Force Simulation ..................................... 59
Figure 31. Simple Moving Average Filter .......................................................... 63
LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 1</td>
<td>Steps for Establishing Measurement System Credibility</td>
<td>21</td>
</tr>
<tr>
<td>Table 2</td>
<td>Experimental Instrumentation Specifications</td>
<td>31</td>
</tr>
<tr>
<td>Table 3</td>
<td>Description of Instrumentation within System</td>
<td>32</td>
</tr>
<tr>
<td>Table 4</td>
<td>Experimental Data</td>
<td>34</td>
</tr>
<tr>
<td>Table 5</td>
<td>List of Program Inputs and Outputs</td>
<td>46</td>
</tr>
<tr>
<td>Table 6</td>
<td>Constant Values Used in Simulation</td>
<td>48</td>
</tr>
<tr>
<td>Table 7</td>
<td>Contributions of Each Term</td>
<td>58</td>
</tr>
<tr>
<td>Table 8</td>
<td>Simulation Validation Results</td>
<td>61</td>
</tr>
<tr>
<td>Table 9</td>
<td>Moving Average Filtering Results</td>
<td>64</td>
</tr>
</tbody>
</table>
1. INTRODUCTION

1.1 Problem Background

In the realm of engineering, there is no universal qualification procedure for mechanical measurement equipment. To qualify a measurement device is to provide sufficient confidence that the system will consistently perform within a specified accuracy range. This is often performed through a standardized testing procedure designed to either pass or fail the measurement device. In some cases, no standard procedure exists. In these cases, the measurement device must undergo a characterization study in which the device is explored and the standard testing procedures are developed from this characterization study.

An example would be a system that records dynamic load data, such as the impact force exerted by a jackhammer. Most load cells are calibrated statically, but this system would be taking dynamic measurements. The static load tolerance provided in the load cell’s specification sheet could not possibly be regarded as the dynamic tolerance of the measurement system. The measurement system would need to go through a series of tests to understand and characterize the measurement system. Static load cells contained within static load measurement systems are often tested and calibrated using free weights. This is not the same for the dynamic loading event. Load cells are susceptible to vibration and other inertial effects that may also skew the data [1]. As a result, a system...
of this nature would require additional testing to characterize the expected error associated with the measurement.

This thesis focuses on the process of characterizing a complicated dynamic measurement system when there is no standard for qualifying the machine. Before a measurement system can be implemented, it must undergo the appropriate testing to understand and characterize the system. From this study, the qualification and calibration procedures can then be established. A comprehensive characterization study of this type of system will be the focus of this thesis.

1.1.1 Motivating Problem Background

The motivating problem and demonstration to be performed in this thesis consists of a measurement system developed for use at Sandia National Laboratories. The system was designed to perform force measurements at different velocities in order to qualify components under dynamic loading conditions. The system is currently in the testing phase and has yet to be approved for implementation. In order to preserve proprietary information, the exact application of the force tester will not be mentioned in this thesis.
The dynamic force tester, as illustrated in Figure 1, presents the opportunity for an interesting characterization study. The system is designed such that the component is attached to a rigid structure, and is pulled to failure by a steel cable. This steel cable that pulls the component is also attached to a moving sled, or carriage that accelerates to a specified velocity at which the pull event occurs. The piezoelectric load cell that measures the failure force of the component is located on the carriage. This location of the force sensor was selected due to the constraints placed on the design. As a result, it becomes difficult to place confidence in the repeatability and accuracy of this particular measurement.

The force sensor was selected as a piezoelectric load cell due to the high frequency sensing range, which makes it ideal for dynamic loading events. This also results in the detection of high frequency vibration that may affect the measurement. Because of the displacement between the test specimen and the force sensor, significant discrepancies
may occur between the force of interest and the load registered by the transducer due to inertial effects such as vibration.

The machine does have both position and force accuracy requirements. The test system uses a high performance encoder, which will not be assessed and will be assumed to be correct with zero error for the purpose of this study. Historically, dynamic force measurement systems of similar application have been qualified and calibrated using breakable copper coupons. The coupons were designed to break at a desired load and velocity. The failure load tolerances on these coupons were determined experimentally.

Before this dynamic force tester can be implemented, the performance of the system must be studied. The behavior of the force sensor under these dynamic loading conditions must be explained as well as the performance capabilities of the system. The maximum velocity of the system and effects of additional mass must be studied to understand the power constraints on the system. In order to do this, a comprehensive characterization study must be implemented to build confidence in the measurement system. This study presents challenges due to the complexity of characterizing the dynamic force measurement. This problem is the motivation for the work presented in this thesis.
1.2 Proposed Solution and Methodology

The measurement system will be assessed using a combination of simulations and physical experimentation in the form of a proposed methodology. This methodology will be developed as a universal solution to characterizing mechanical measurement systems that do not have standard qualification procedures. This approach will utilize the key definitions and processes developed within the simulation community as the foundation for the methodology.

The ultimate goal of the methodology will be to develop confidence in the system. This confidence in the system’s performance is called as system credibility [2]. In order to properly execute the simulations contained in the proposed methodology, the verification and validation processes developed by Sargent [3, 4], will be modified and extended as the foundation for this proposed multi-faceted approach. The distinction between verification and validation are as follows: Verification is the act of determining that a simulation computer program performs as intended, while validation is the act of determining whether the underlying mathematical model provides a sufficient representation of the system [5]. Both verification and validation of the simulation are implemented in the proposed methodology.

The four main components of the proposed methodology are the physical test system, the conceptual model, computational simulations, and the system data/results. Verification and validation occurs as these components are compared to each other. The
system data/results portion describes the accumulation of simulation data and raw experimental data. The combination of this data and post-processing of the data can allow for inferences about the system to be made and provides a means of understanding physical errors occurring within the system. The conceptual model describes the physics-based models describing the physical test system, which is the physical test of the measurement system. Finally, the computerized model is the computer implementation of the conceptual models. This methodology is described in further detail in Chapter 3. The combination of these activities and the verification and validation steps that occur between them will drive the characterization study of the measurement system and ultimately establish credibility in the measurement system.

The specific system assessed in the case study of this thesis will be simulated using MATLAB software. The experimentation be performed using an experimental test assembly designed specifically for the analysis of the force measurement. The experimental test assembly was designed as a means of measuring the force at the location of the test specimen to provide validation data for this proposed methodology.

This analysis is performed on the dynamic force tester in Chapter 4. The study discovers that there is actually a significant difference between the two force sensor readings well outside of the accuracy requirements. However, using physics-based models that describe the system, this force difference can be accounted for mathematically. The models, simulation, and experiment will provide a detailed assessment of the
measurement system in an effort to increase the credibility of the system and eventually successfully determine that the machine is acceptable for its intended use.

1.3 Relevant Background Literature

There are multiple key contributors within the modeling and simulation community that will be referenced to establish the framework contained in the modeling and simulation verification and validation portion of this thesis. Although there is no standard methodology or set of definitions for verification and validation, the simulation community has established a set of definitions and methods that will be applied to this thesis [6-11]. The modeling process illustrated in Chapter 2 was the foundation for the methodology presented in this thesis. This work includes recommended strategies for validation as well as multiple processes and paradigms that work towards a universal approach towards modeling and simulation.

Obtaining and applying real world data is another subject heavily utilized in this paper. Both Kleijnen [8] and Robinson [12] provide similar fundamental approaches to gathering and processing real world data. Also, both of these works present cases establishing the difficulty of obtaining real world data and the idea that accurate real data does not actually exist. Additionally, the comparison between simulation data and real world experimental data is hugely important to the application of the methodology presented in this thesis. There are many statistical techniques that can be used for validation as opposed to subjective methods [13-15].
Prediction is also a key element to this thesis. Oberkampf et al. [16] provides a key distinction between validation and prediction. Confidence is obtained in predictions as a result of the physical understanding of the model and system. This work focuses on verification and validation in computation engineering and physics. Provided in this work is the Phenomena Identification and Ranking Table (PIRT), which is essentially a systematic method of prioritizing the adequacy and importance of the physical phenomena, the conceptual model, code verification, experimental adequacy, and validation adequacy.

The selection of sensors and the analysis of transducers under dynamic conditions is also relevant to the work presented in this thesis. Shieh et al [1] provided a methodology and much analysis for selecting the appropriate sensors. This work discusses the different types of force sensors, strain gauges, and accelerometers and how to select the appropriate sensor an application. There are also a few case studies that assess the measurement and experimentation of dynamic systems [17-26]. These studies range from the measurement of forces within robots to the estimation of an impact force induced by a piston slap.

1.4 Contributions

The primary contribution of this thesis is the methodology presented in Chapter 3. The methodology essentially combines methods used within the simulation community to provide a comprehensive analysis of the dynamic force system. The analysis will
include characterizing the measurement system experimentally while simultaneously generating models and simulations to establish a desired level of credibility.

Although the motivation for the work was a unique measurement system, the process could be translated to other measurement study applications. The methodology will be presented as a process that will be utilized to develop measurement system credibility. The process implements the verification and validation of simulation models with the actual physical testing as a proposed methodology.

1.5 Thesis Contents

The remaining content of this thesis will first provide a more detailed technical background in Chapter 2. This will focus on the fundamental components of verification, validation, prediction, and validation experimentation. The definitions developed within the modeling and simulation community can be found in this section as well as historical processes that outline verification and validation of simulations. The various types of validation techniques are listed along with a detailed section detailing data prediction.

Chapter 3 presents the methodology proposed for solving complicated dynamic measurement characterization studies. This chapter details the use of historical verification and validation techniques coupled with the use of experimental procedures. Chapter 4 presents the case study that was the motivation for this research. This chapter details the experiment design that was implemented to provide the database of validation
data as well as the historical validation techniques associated with measurement systems of this style. Next, the chapter discusses two separate simulations. The first iteration simulation provides a model that will be used to predict maximum system performance limits. The second model iteration only assesses the force measurement capabilities of the system. Finally, this chapter summarizes the conclusions that can be made from the experimental data and the simulations.

Chapter 5 presents the final discussion of the work performed in this thesis, including the methodology and case study.
2. TECHNICAL BACKGROUND

This chapter provides the background to the techniques and tools used in the methodology developed in this thesis. Proper modeling and simulation methodology will be presented followed by validation experiment techniques. Additionally, a technical description of the dynamic measurement system will further describe the complexity of this problem.

2.1 Verification and Validation of Simulations

The simulation of the physical measurement system is an essential component of the multi-faceted approach for characterizing the dynamic measurement system. Understanding the basic concepts associated with verification and validation will be essential to the implementation of the methodology presented in this thesis.

A model with even slight flaws could produce invalid results that go unnoticed [8]. Model confidence is often obtained through a series of tests intended to validate the model [6, 27]. The complexity of these tests and the confidence required to establish model validity depends on the model application. The cost of model validation is typically a function of model confidence. Model validation is usually especially costly when a high level of confidence is required [3, 28].
2.1.1 Verification and Validation Processes and Definitions

Many processes exist that provide recommended verification and validation of a simulation. Figure 2 presents a verification and validation process for modeling and simulation. This modeling process will be used as the foundation for modeling and simulation verification and validation in this thesis. In the case of the dynamic force tester, the mathematical models will be simulated in addition to collecting real experimental data for validation. The power, velocity, and expected force measurement will all eventually be assessed in the analysis. Due to the accuracy requirements of the machine, and the interest of this study to develop the credibility of the system, a physical understanding of the force measurement capability will further provide insight into the operational capability of the machine.

![Simple Model Paradigm](image)

**Figure 2. Simple Model Paradigm [4]**
The ultimate goal of this paradigm, as developed by Sargent [3, 4, 9], is to ensure data validity, or provide the necessary confidence that the data associated with the models, simulations, and experiments are sufficient for the intended use of the simulations. As illustrated in Figure 2, the problem entity represents the object or system to be modeled. The conceptual model provides a mathematical representation of this problem entity and the computerized model implements the conceptual model as a computational simulation. Between each of these three blocks on the figure are recommended verification and validation steps designed to build trust in a model and its predictions. The conceptual model is validated through determining that the theories are correct and the model is a correct representation of the problem for the intended purpose and application of the model as it relates to the problem entity being modeled. This is termed conceptual model validation. Once the conceptual model has been validated and the model has been implemented in a simulation, the computerized version of the model must be checked to assure that the conceptual model is correctly implemented. This is performed through computerized model verification. Finally, the computerized model is validated through experimentation with the problem entity through operational validation. The combination of these activities was intended to provide a comprehensive data validation process.

2.1.2 Validation Techniques

Data validation provides a means of assessing the accuracy of the simulation by comparing simulation data to real system data under identical driving input conditions.
There are numerous accepted validation techniques. The selection of a validation technique depends on how observable the system is and the quantity of data that can be collected [29]. The following validation techniques provide both objective and subjective means for determining if a given model or simulation is valid. Both Balci [29] and Sargent [3] provide much of the background and depth in the following techniques.

*Animation and Graphical Comparisons* provide a visual means of subjectively validating the simulation. Graphical representations of the output can be compared with the real data to determine behavioral trends.

*Event validation* is performed by observing events occurring within the system. An example would be a queuing model within a department store, and the events would be the customers that enter and leave the store. The number of customers that enter and leave the store could be validated with the real system.

*Extreme Condition Tests* are operated to test the system at the extreme operational boundaries of the system. For instance, it may be known what the maximum allowable power output of the system is electrically, and the mechanical output should match the maximum available power.

*Face validity* is a subjective means of validation that requires persons or entities knowledgeable in the system to justify the model. The persons involved must have the
capability of providing estimates and utilizing personal intuition to validate the model. This method can be especially useful as an initial means for validation.

*Historical Data Validation* utilizes data taken from the real system in previous tests or during operation in order to test the model. In addition to testing the simulation, this data can be used to construct the model and drive the simulation model.

*Historical Methods* and *Multistage Validation* refer to a set of three methods that are largely philosophical. *Historical Methods* refers to each of the methods individually while *Multistage Validation* combines the methods into a process. The three historic methods are rationalism, empiricism, and positive economics.

*Internal Validation* assesses the internal consistency of the simulation by holding the inputs and parameters constant and performing multiple runs with the simulation. The intent of this technique is to provide enough data to characterize the internal consistency using stochastic methods.

*Predictive Validation* requires that the simulation produce data that has not yet been generated by the system. This data is then compared to the real system to determine if the correlation between the two.
Sensitivity Analysis is the act of varying parameters and inputs in order to analyze the behavior of the output. Sensitivity analysis is an excellent method for determining effective design changes and determining the effect that each parameter or input has on the simulation results.

2.2 Gathering Real System Data through Validation Experimentation

The design of experiments in mechanical systems is essential for providing the necessary validation data. Providing exact data from the real world is not theoretically possible, as this requires the use of a measurement device and there will always be a measurement error. However, depending on the application, and the effort involved with obtaining this data, the validation data can be very accurate.

2.2.1 Validation Experimentation

A validation experiment is the performed to provide real data to be used to validate the computational simulation. Aeschliman et al. [30] provided a list of guidelines for experimental validation that were originally developed for hypersonic fluid flow. The following list provides an expanded generalized version beyond fluid flow intended for use with all simulations of physics-based models:

1. The validation experiment should be designed by all parties involved with both the testing and the computational simulation.

2. All physics of interest, boundary conditions, initial conditions, and geometry information should be factored into the experiment design.
3. The experiment should emphasize the interaction between the simulation and actual system.

4. Although the simulation and experiment should be developed together, the data taken independently.

5. The experimental measurements should be ranked in terms of computational complexity.

6. The experiment should be designed to estimate errors.

These guidelines are appropriate for any physics-based experimental design for which a computational simulation is to be validated. A validation experiment must provide data appropriate for the application. Any unexpected error could invalidate the models and the simulations; therefore it becomes essential to properly design the validation experiment.

2.2.2 Selection of Instrumentation

Typical mechanical systems require sensors, or a device that provides data quantifying the measurement of a system variable, such as force, position, or acceleration. The sensor operates by converting a stimulus, or electrical, pneumatic, hydraulic, or optical input into a measured signal [1]. The selection of sensors is essential to providing both the required accuracy of the system, as well as the appropriate amount of data. Sensor specifications, such as sampling frequency range, measurement tolerance, and sensitivity to factors such as temperature and other environmental variables are utilized to select the appropriate sensor.
Many dynamic systems, similar to the particular system discussed in this thesis, can be characterized with position, force, and accelerometer data. The selection of the appropriate accelerometers, force sensors, and encoders are very important to the both the design of the system and the validation experiment. In order to select the appropriate instrumentation, the specifications for the sensors must be selected by estimating the expected system performance. For example, a human weight scale requires static loading measurement and would be best suited with a strain gauge load cell that does not have a high frequency range, but can have a loading range between 0 and 500 lbf. Additionally, strain gauge load cells tend to have low drift properties, which is ideal for static measurement. Alternatively, a dynamic measurement such as an impact hammer would require a piezoelectric force sensor that has a high frequency range. Appendix D displays multiple figures extracted from the work of Shieh et al. [1] and it is recommended that this work be viewed prior to the selection of any mechanical sensor as it provides a systematic approach to sensor selection.
3. PROPOSED METHODOLOGY

3.1 Motivation and Application for Methodology

This methodology was developed to establish the credibility of a complicated dynamic measurement system through both the use of experimentation and computational simulation. Specifically, the dynamic force tester presented as the case study was the motivation for the activities presented in this methodology. This methodology was designed and intended for any experimental or prototype dynamic mechanical system that demands both accuracy and repeatability of the measurement system, and lacks a formal qualification procedure. The methodology is designed for systems in which the major physical components can be modeled and simulated.

3.2 Description of Process

The proposed methodology provides a multi-faceted approach to establishing measurement credibility. The basic definitions of the terms in the methodology come from the simulation validation and verification literature, some of which was explained in Chapter 2. This multi-staged approach is developed to be an iterative process, in which experiments, models, and simulations should be revised and changed until the desired level of confidence is achieved. This desired credibility should be determined by the stakeholders and for this methodology to be applicable, should require a detailed analysis of the physical phenomena occurring within the system.
The methodology provides a multi-faceted approach to establishing machine credibility through the use of models and experimental measurement validation. Figure 3 illustrates the proposed methodology through which credibility of a dynamic measurement system should be achieved. As can be seen in the figure, the measurement system credibility is the ultimate goal from each activity listed. This suggested approach provides an excellent foundation for which verification and validation activities can be planned and executed for each step of the process.

![Figure 3. Proposed Methodology for Establishing System Credibility](image)

Much of the verification and validation steps for the modeling and simulation represent ideals and relationships developed from previous simulation paradigms [4]. The lower portion of Figure 3 presents the relationships presented earlier relating to verification
and validation. The steps are listed in Table 1 as they apply to a mechanical measurement system:

Table 1. Steps for Establishing Measurement System Credibility

<table>
<thead>
<tr>
<th>#</th>
<th>Step</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Perform Physical Experimentation</td>
</tr>
<tr>
<td>2</td>
<td>Measurement System Operational Data Collection</td>
</tr>
<tr>
<td>3</td>
<td>Analysis and Modeling</td>
</tr>
<tr>
<td>4</td>
<td>Conceptual Model Validation</td>
</tr>
<tr>
<td>5</td>
<td>Computer Programming and Implementation</td>
</tr>
<tr>
<td>6</td>
<td>Computerized Model Verification</td>
</tr>
<tr>
<td>7</td>
<td>Simulation Operational Validation</td>
</tr>
<tr>
<td>8</td>
<td>Simulation Operational Prediction</td>
</tr>
<tr>
<td>9</td>
<td>System Data/Results Processing</td>
</tr>
</tbody>
</table>

This methodology is intended to be an iterative process, and as a result the steps listed in Table 1 may be repeated as the models and simulations are revised. These are the fundamental steps to the methodology and will be explained in further detail as follows:

1. The first step, *perform physical experimentation*, applies to the measurement system being studies. This activity consists of generating validation data for the computational simulations. In the case of the dynamic force tester, a complete test configuration had to be designed and implemented appropriately to collect the data from the system. The measurement system must be carefully designed to provide the desired level of accuracy [31, 32]. This validation data will be utilized to validate the simulation and eventually understand the physical
behavior of the system. If time and resources allow, a proper design of experiments (DOE) should be performed in order to properly obtain the data.

2. *Measurement operational data collection* activity describes the physical collection of data from the current measurement system, including an assessment of the current raw performance, without any corrective or calibration terms. This data will be further assessed in the *data processing* step. The quantity of data required is case dependent.

3. *Analysis and modeling* of the measurement system describes the activity of modeling the mechanical system. Typical mechanical measurement systems can be modeled using electro-mechanical physics-based differential equations. This requires an understanding of dynamics modeling. Dynamics modeling analysis literature for electrical, mechanical, hydraulic, and pneumatic systems provide standard procedures for developing the governing equations of motion [33-35]. These concepts must be understood in order to properly model the system. Additionally, the system must be an observable white box model. The system dynamics must observable enough to be modeled.

4. *Conceptual Model Validation* consists of checking the conceptual models to be a correct representation of the system. This is typically a subjective decision. The constant values, such as initial values, mass values, and coefficients must be
confirmed in this step. Equally important is determining that the equations properly represent the system. This includes assessment of the assumptions made within the system, such as the decision to model a particular system as a 1-dimensional model instead of a 2-dimensional model.

5. *Computer programming and implementation* of the conceptual models is the next step. Although many languages exist, MATLAB will be the preferred programming language of choice for this thesis as it provides many high level functions and graphical capabilities. The selection of the software should be based on both the programmer’s personal preferences and the need for additional tools that are specific to a particular programming software package.

6. *Computerized model verification* is the activity of ensuring that the generated conceptual models are properly implemented in the programming language [36]. In addition to specifically verifying that the equations match the conceptual models, the implementation of the ordinary differential equations requires the selection of the appropriate solver. In MATLAB specifically, there are multiple ordinary differential equation solvers that vary based on the numerical methods that are used to solve the problem and the type of problem that the solver is intended for. To clarify, stiff differential equations, non-stiff differential equations, and moderately stiff differential equations all have solvers contained within MATLAB. Ashino et al [37] provides a more complete analysis of these
methods. Based on the software package, the use of pre-developed functions can improve the simplicity of the computational simulation, but also must be understood prior to implementation.

7. *Simulation operational validation* requires the analysis of the outputs to confirm that the simulation does perform within the applicable domain at the intended level of accuracy. This includes comparison of the simulation with the actual validation data generated from the measurement system. There are many methods for performing this and can be seen in chapter 2. The exact method to be used is case-dependent. It is likely that multiple validation strategies will be used.

8. *Simulation operational prediction* describes the process of generating data from the simulation that can be processed with the actual data from the measurement system. This includes generating test data to be used for understanding the measurement system, and determining the performance limits of the system. This includes predicting errors due to dynamic behavior and determining the operational limits of the equipment. In the case of the dynamic force tester, the velocity is limited due to several factors that must be predicted by the simulation.

9. *System Data/Results Processing* portion of the figure describes the accumulation of simulation data and raw experimental data from the measurement validation step. This step requires the combination of the data collected from the machine
and the data generated from the simulation in an effort to understand the dynamic behavior associated with the machine. The dynamic simulation of the equipment combined with the actual data will allow for a comprehensive understanding of the actual phenomena occurring within the system.

The level of credibility to be achieved is determined by the users of the measurement system. Multiple iterations of the process may be required before sufficient data can illustrate the functionality of the machine. As the machine becomes more complicated, typically more testing and instrumentation is required. For example, an instrument that tests an automobile radiator by observing the temperature within the engine compartment would require testing in multiple climates. The radiator is responsible for maintaining an acceptable temperature threshold of the engine regardless of the external environments. Although one temperature sensor may be sufficient, an entire network of temperature sensors would provide a better understanding of the temperature distribution surrounding the engine block. Depending on the complexity of the heat transfer models, more testing and instrumentation may be required to validate the models. The automobile manufacturer must decide at some point what level of confidence is appropriate in the expected radiator performance.

The proposed methodology takes the ideals developed for modeling and simulation and provides a means of exploring an unknown system without having a standard method for qualifying the system. These steps are intended to be the base structure of the process.
Additionally, these steps can be repeated, as needed to perform the characterization study. The following chapter will utilize these methods to assess the dynamic force tester.
4. DEMONSTRATION OF METHODS

The dynamic force measurement system will be assessed as a demonstration of the methodology detailed in Chapter 3. The deliverables of this study extend beyond exploring the dynamics and accuracy of the force measurement. Because the system is in the design phase, possible system redesigns have been considered to improve the velocity capabilities of the system. Proposed redesigns include increasing the track length and optimizing the mass of the carriage that moves across the surface of the machine. This thesis will examine the performance effects of minimizing the mass of the carriage as well as increasing the track length. As a result, the deliverables of this study can be summarized as follows:

1. Determine if increasing the track length will improve the maximum system velocity.
2. Determine if minimizing the mass of the carriage will improve the maximum system velocity.
3. Define the current dynamic force measurement capabilities.

This characterization will implement multiple simulations along with the use of physical experimentation to provide the sufficient analysis to accomplish these tasks. The methodology will be followed as the system validation data is collected and the simulations are generated. Figure 4 illustrates how the methodology will be utilized to characterize this system.
As illustrated in Figure 4, two simulations will be developed in this study of the dynamic force measurement system. The first simulation will model the entire system from the motor to the pull specimen. This simulation will focus on the power limitations of the system. The second simulation will focus specifically on the dynamic force measurement and explore how the effect of the inertial loading on the moving force sensor. These two simulations will both be performed with MATLAB.

4.1 Physical Experimentation of Force Tester

This section will develop both step 1 and step 2 of the methodology developed in Chapter 3. The physical experiment will be performed by designing a test configuration that will be used to provide the validation data for the simulations. Although a proper design of experiments was not performed, the results from this series of tests will provide a sufficient demonstration of the methodology.
Previous dynamic force measurement systems have been qualified using test specimens designed specifically to break at a desired load and velocity. Rather than using these coupons, this study will utilize a force sensor to measure the force exerted at the point of the test specimen. These coupons will not be used to qualify this machine. Instead, these coupons will be implemented as load selectors only. That is, the coupons will be used in the system to determine the load range at which the failure will occur. This physical test setup, coupled with computational analysis, will provide the comprehensive analysis of the system.

4.1.1 Primary System Components
The complete drive system can be separated into five major components: The rotary servo motor, the gear box, the high performance linear actuator, the carriage, and the lanyard. The system will be modeled as an electro-mechanical system. The interface between each component of the complete system can be simplified as illustrated in Figure 5:

![Figure 5. Simplified System Description](image)
The gear box currently has a 3:1 speed reduction gear ratio and this interfaces with the high performance linear actuator (HPLA). Figure 6 displays the HPLA that converts the motor torque into a linear force that is used to drive the carriage for the pull event. The experimental test configurations will interface with the lanyard as illustrated previously in Figure 5.

4.1.2 Experimental Instrumentation

As noted earlier, selecting the proper sensors is essential to meeting measurement requirements. Due to the high velocity impact loading event that is taking place, a large quantity of data must be taken to record the dynamic event. Also, because the event
occurs rather quickly, the signal drift is not a concern [1]. The following table lists the selected instrumentation and the individual specifications.

<table>
<thead>
<tr>
<th>Table 2. Experimental Instrumentation Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Name</strong></td>
</tr>
<tr>
<td>-------------------------</td>
</tr>
<tr>
<td>Force sensors</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Tri-axial Accelerometer</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Signal Conditioner</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>DAQ</td>
</tr>
<tr>
<td>Linear Encoder</td>
</tr>
</tbody>
</table>

From Table 2, it can be seen that the sampling frequency range capabilities were selected to be very high. The instrumentation will be adjusted to sample 50,000 data points per second. This will ensure that any effect due to vibration is detected by the sensor. In addition to the stationary force sensor, a triaxial accelerometer will be placed at the moving force sensor location to quantify the vibration that may occur at this location within the system.
Figure 7 displays the physical locations of the instrumentation on the measurement system. The rigid frame and force sensor mount assembly on the left hand side of the figure illustrate the test configuration that was designed for this study. The force sensor on the left hand side will represent the “true” load of interest as experienced by the test specimen as it is pulled to failure. The summary of the instrumentation for gathering physical data can be seen in Table 3.

Table 3. Description of Instrumentation within System

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stationary Force sensor</td>
<td>&quot;True&quot; force at location of test specimen</td>
</tr>
<tr>
<td>Moving Force sensor</td>
<td>Force taken from permanent force sensor location</td>
</tr>
<tr>
<td>Breakable Copper Coupon</td>
<td>Specimen designed to break at certain load and velocity</td>
</tr>
<tr>
<td>Tri-axial Accelerometer</td>
<td>Vibration occurring at location of moving force sensor</td>
</tr>
<tr>
<td>Linear Encoder</td>
<td>Position of Carriage</td>
</tr>
</tbody>
</table>
Figure 8 and 9 display the actual test setup during a pull test. The transducers as well as the copper pull specimen can be seen in the images.

**Figure 8. Picture of Experimental Setup**

**Figure 9. Picture of Each Force Sensor Assembly**
4.1.3 Experimental Testing Procedure

The experimental testing procedure as specified in this section represents step 1 of the proposed methodology. The testing procedure consisted of five categories of tests. Each category consists of a given velocity and force (selected by the copper coupon) at which the test will take place. The velocity is achieved by programming the system to attain this velocity at the location of the pull event, or when the lanyard goes taught. The force is achieved by utilizing the historic breakable copper coupons. These copper coupons were specifically designed to break at a particular force and velocity. The five tests that will be performed with the system can be seen Table 4:

Table 4. Experimental Data

<table>
<thead>
<tr>
<th>Set #</th>
<th>Load (lbf)</th>
<th>Sample Size</th>
<th>Velocity (fps)</th>
<th>Acceleration (g’s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
<td>20</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>400</td>
<td>20</td>
<td>11</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>600</td>
<td>20</td>
<td>16</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>400</td>
<td>20</td>
<td>7.8</td>
<td>-1.24</td>
</tr>
<tr>
<td>5</td>
<td>600</td>
<td>20</td>
<td>7.8</td>
<td>-1.24</td>
</tr>
</tbody>
</table>

These tests were selected because they are historic tests performed on dynamic force measurement systems. The first three sets will be performed at constant velocity, with negligible acceleration. The final two sets will expose the system to significant acceleration at the point of impact. The total number of tests performed was 100. These tests will be used to generate the validation data for the simulations. Each test will record data from the moving piezoelectric force sensor, the stationary piezoelectric force sensor, the triaxial accelerometer, and the position encoder. As a result, each test will
generate five columns each containing 100,000 data points taken over two seconds. The force difference will be quantified as the percent difference associated with the moving force sensor. The stationary force sensor will be regarded as true force data. Again, the force difference will be defined as follows for these tests:

\[
\% F_{diff} = \left| \frac{F_{stationary} - F_{move}}{F_{stationary}} \right| \times 100
\]  

(1)

4.1.4 Measurement System Operational Data Collection

The generation of raw data from the measurement system represents step 2 of the proposed methodology. 100 sets of data were taken in an effort to characterize the system. The results of the lower velocity tests displayed a very low force difference between the stationary force sensor and the moving force sensor. This was not true for the higher velocity pulls. The higher velocity sets of data returned significant differences in the force sensor data. The velocity data, however, was very consistent. The maximum velocity experienced during each run contained very little variance. Figures 8-12 display the relationship between the moving force sensor data and the stationary force sensor data. Each of the five figures only displays data from one test out of each set of 20.
Figure 10. Set #1 Test #5 (200 lbf 3fps)

Figure 11. Set #2 Test #24 (400 lbf 11fps)
Figure 12. Set #3 Test #44 (600 lbf 16 fps)

Figure 13. Set #4 Test #64 (400 lbf 7.8 fps)
As can be seen in Figures 10-14, there is a significant difference between the moving force sensor measurement and the stationary force sensor measurement, especially during the more violent events (higher velocity and load). This implies that there is some inertial effect occurring between the stationary force sensor and the moving force sensor.

In addition to the data from the two force sensors, the position data from the encoder and the acceleration data located the moving force sensor were recorded for each test. Figures 15-16 display the data taken from the encoder and the data taken from the accelerometer for one of the tests taken in the fifth set of data.
The results of the five sets of data have established that significant amplification of the moving force sensor is occurring in the system. This is either a result of electrical amplification, or a result of some inertial effect. The following sections will model this system and attempt to explore and define the force difference term. The results from these tests will be used as validation data for the computational simulations.
4.2 Complete System Model and Simulation

The first of the two simulations will present a complete system analysis. This simulation will be performed to explore the power limits of the system, specifically the velocity limits of the system. This portion of the analysis represents steps 3-8 of the methodology presented in Chapter 3.

4.2.1 Conceptual Model of Complete System

This section details step 3 of the methodology. The first conceptual model will consist of physics-based mathematical differential equations to describe the system. The system will be modeled as an electro-mechanical system that operates off of standard 460 VAC. The dynamic force tester can described as a dynamic system with seven primary states. The system can essentially be modeled with an electrical circuit describing the brushless servo motor. In order to model the system, there are several key assumptions. The motor will be analyzed as a DC motor because the motor specification sheet provides the necessary DC analysis constants. In addition, the high performance linear actuator (HPLA) provides equations in the specification for the moment of inertia. These equations can be seen in the appendix. Finally, the gear box will be assumed to have negligible losses and the lanyard and both force sensors will be modeled as springs with a constant that will allow the pull force to be modeled as a function of distance. Figure 17 illustrates the simplification of the system.
The motor circuit will be directly modeled as a DC motor circuit given the values listed in the product manual. The simple DC circuit is illustrated in Figure 18. The equations can be derived as follows:

\[
\frac{d[i_a(t)]}{d(t)} = -\frac{R_a}{L_a}i_a(t) - \frac{K_e}{L_a}\dot{\theta}_m(t) + \frac{V_{in}(t)}{L_a}
\]  

(2)
The HPLA is slightly more complicated. The product specification guide combines both the load being pulled by the HPLA with the belt inertia into one inertia value. Using the provided equations, it is possible to obtain an approximation of the inertia of the belt including the mass of the carriage. This exact calculation can be seen in the Appendix.

The next step is to define the free body diagrams to obtain the equations of motion for the mechanical system. Two free body diagrams will represent both the motor shaft interfacing with the HPLA and can be reduced to the following models illustrated in Figure 19:

![Figure 19. Simplified Drive System Representation](image)

Therefore, using these equations, it is possible to obtain an accurate approximation of the inertia of the belt including the mass of the carriage. The dynamics model can be reduced to two simple free body diagrams displayed in Figure 20.
Figure 20. Summary of Forces Associated with Motor and HPLA Drive System

\[ F_{bm} = r_g^2 b_m \dot{x}_3 \]  \hspace{1cm} (3)

\[ F_{bg} = r_g^2 b_g \dot{x}_3 \]  \hspace{1cm} (4)

\[ \sum M = \tau_m(t) - F_N r_m - F_{bm} r_m = J_m \ddot{\theta}_m \]  \hspace{1cm} (5)

\[ \sum M = F_N r_g - F_{bg} r_g - F_t(t) r_g = J_Z \ddot{\theta}_g \]  \hspace{1cm} (6)

\[ \left( \frac{\tau_m}{r_m} - \frac{J_m \ddot{\theta}_m}{r_m} \right) r_g - F_{bg} r_g - F_{in} r_g = J_Z \ddot{\theta}_g \]  \hspace{1cm} (7)

\[ N = \frac{\theta_m}{\theta_g} = \frac{r_g}{r_m} \]  \hspace{1cm} (8)
The carriage system will initially be modeled as simply one resistive force in the form of a spring that represents the lanyard. As a result, there is a direct relationship that can be made between the angle of the motor and the position of the carriage as displayed in Figure 21.

![Figure 21. Carriage System Model](image)

The relationship between the position of the carriage and the angle of the HPLA can be defined in the following equation:

$$\theta_m = N\theta_g$$  \hspace{1cm} (9)

$$x = \theta_g r_g$$  \hspace{1cm} (10)

$$\ddot{x} = \frac{N r_g \tau}{J_z + J_m} - \frac{F_{in} r_g^2}{J_z + J_m} - \frac{(r_g^2 b_m + r_g^2 b_g) \dot{x}}{J_z + J_m}$$  \hspace{1cm} (11)

The final set of governing differential equations can be seen as follows:
As provided by the spec sheet for the motor, the torque can be directly related to the current as follows:

$$\tau = K_t i_a(t) \quad (12)$$

Finally the governing differential equations for this problem become:

$$d\dot{x}(t) = \ddot{x} \quad (13)$$

$$\ddot{x} = \frac{N r_g K_t i_a}{J_z + J_m} - \frac{F_{in} r_g^2}{J_z + J_m} - \frac{(r_g^2 b_m + r_g^2 b_g) \dot{x}}{J_z + J_m} \quad (14)$$

$$\frac{d[i_a(t)]}{d(t)} = -\frac{R_a}{L_a} i_a(t) - \frac{K_e}{L_a} \dot{\theta}_m(t) + \frac{V_{in}(t)}{L_a} \quad (15)$$

4.2.2 Conceptual Model Validation

The activity of validating the conceptual model is listed as step 4 in the methodology. As described in the methodology, the conceptual models must be validated after each model is generated. This includes proving that the assumptions are valid and that each model is an appropriate representation of the system for the model’s intended application. The intention of the first iteration of the conceptual models is to capture the system’s velocity capabilities. This includes understanding the system velocity and power constraints. This validation occurs by comparing the governing equations with the actual system. The conceptual model was validated by confirming the value of each constant used in the
differential equations and confirming that the usage of a one dimensional model would suffice to assess the system performance.

4.2.3 Computational Simulation for Complete System Model

The development of the simulations is step 5 of the methodology described in Chapter 3. These models will be implemented using MATLAB. Specifically, the differential equations will be implemented using one of MATLAB’s ordinary differential equation (ODE) solvers. The simulation utilizes a PID controller to track a user-input velocity profile similar to the actual system. Three discrete operational conditions dictate the pull force load as follows:

\[
F_{in}(t) = \begin{cases} 
0 & x < x_{lanyard} \\
 k(x - x_{lanyard}) & x \geq x_{lanyard} \\
0 & F_{in} > F_{failure}
\end{cases}
\]  

(16)

This simulation allowed the position, velocity, current, and force to be observed. The force was simply a linear function of distance for this simulation. The list of inputs and outputs can be seen in Table 5.

<table>
<thead>
<tr>
<th>Table 5. List of Program Inputs and Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inputs</td>
</tr>
<tr>
<td>Desired Velocity Profile</td>
</tr>
<tr>
<td>Maximum Acceleration</td>
</tr>
<tr>
<td>Break Force Load</td>
</tr>
<tr>
<td>Lanyard Length</td>
</tr>
</tbody>
</table>

46
The exact logic implemented in the program implemented if-then statements to provide the three discrete conditions to simulate the system. Figure 22 displays the logic implemented into the MATLAB script.

Figure 22. Computational Simulation Logic
The simulation was performed over a combination of velocity and load scenarios to examine the system. The constants for the equations developed for the simulation can be seen in Table 6:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rₐ</td>
<td>Motor Armature Resistance</td>
<td>0.28</td>
<td>Ohms</td>
</tr>
<tr>
<td>Jₘ</td>
<td>Rotational inertia of motor</td>
<td>2.60E-03</td>
<td>kgx m/s²</td>
</tr>
<tr>
<td>M_c</td>
<td>Mass of Carriage</td>
<td>47</td>
<td>Kg</td>
</tr>
<tr>
<td>J_z</td>
<td>Rotational inertia of HPLA</td>
<td>0.2335</td>
<td>kgx m/s²</td>
</tr>
<tr>
<td>N</td>
<td>Gear Ratio</td>
<td>3</td>
<td>-</td>
</tr>
<tr>
<td>r_z</td>
<td>Effective radius of HPLA</td>
<td>6.68E-02</td>
<td>m</td>
</tr>
<tr>
<td>K_T</td>
<td>Motor Torque Constant</td>
<td>1.591</td>
<td>Nx m/A_{rms}</td>
</tr>
<tr>
<td>K_e</td>
<td>Motor Voltage Constant</td>
<td>0.9185</td>
<td>V_{rms}/(rad/s)</td>
</tr>
<tr>
<td>b_M</td>
<td>Viscous damping coefficient for motor</td>
<td>10*</td>
<td>N/s</td>
</tr>
<tr>
<td>b_g</td>
<td>Viscous damping for HPLA</td>
<td>10*</td>
<td>N/s</td>
</tr>
<tr>
<td>i_a(max)</td>
<td>Maximum current circuit can experience</td>
<td>66.46</td>
<td>Amp_{rms}</td>
</tr>
<tr>
<td>V_{in}(max)</td>
<td>Maximum voltage applied to circuit</td>
<td>325.2</td>
<td>V_{rms}</td>
</tr>
</tbody>
</table>

*denotes that the value was obtained through calibration
4.2.4 Computational Simulation Verification

In order to perform step 6 of the methodology, the computational simulation must be verified as a proper implementation of the developed physics-based mathematical models. This step included the selection of the correct MATLAB solver to assess the ordinary differential equations. Additionally, the equations were confirmed to be properly implemented in the simulation.

4.2.5 Computational Simulation Operational Validation

The validation of the computational simulation performs step 7 of the methodology. The velocity model and simulation do not have set accuracy requirements. This portion of the analysis was performed to answer a few key questions about the system. As a result, face validation, or allowing persons knowledgeable of the system to subjectively decide if the simulation is valid for its intended use. The designer of the dynamic force system provided this validation based on the operational output of the simulation. Figure 23-22 display operational capabilities of the simulation. The simulation follows the input velocity profile very similar to the real system and the simulation also reacts to the force impact very similar to the real system.
Figure 23. Simulation Velocity Profile (3ft/s 200lbf)

Figure 24. Simulation Velocity Profile (7.8 ft/s 400 lbf)
Predicting the system performance limits is step 8 in the methodology. Because the simulation was validated to be appropriate for simulating the velocity of the machine, there are questions about the machine that can be answered using the computational simulation as follows:

1. What is the maximum system achievable velocity?
2. Will a reduced carriage mass improve system velocity capabilities?
3. Does increasing the track length improve system velocity?

These questions will be answered by generating performance data with this complete system simulation. The simulation provided very clear answers to the questions about the system.

In order to determine the maximum velocity, the simulation was given a velocity profile with a maximum velocity of 30 fps. Additionally, the break force was set to zero and the track length parameter was extended to explore the benefits of lengthening the track. This will answer the first two questions because the system will attempt to achieve 30 fps, which was beyond the initial design requirement of the machine. The results can be seen in the following figures.
Figure 25. Maximum Velocity Capability

Figure 26. System Current During Pull Event
Figure 25 illustrates the behavior of the simulation as it attempts to achieve 30 fps. It can clearly be seen that the maximum velocity capability on the machine is the 28 fps. Additionally, even with an extended track length, the maximum velocity overshoots to 28 fps and dampens out at 25 fps. It achieves this velocity before 2.5 feet, which is about half the track length. Therefore, it clearly has a sufficient track length to achieve maximum velocity. Figure 26 displays the current during the pull. It illustrates that the system hits the current limit of 66 amps as specified as a constant.

Figure 25 illustrates the required power to increase the velocity. The current system requires around 21 kW to achieve the maximum velocity. Increasing the power to 25 kW would allow the system to achieve 30 fps.
The next analysis will calculate the effect that the mass has on the system. This simulation was performed under the same conditions. No load and extended track length. However, the mass of the carriage will be varied from 1kg to 61 kg over 5 different test runs.

![Velocity Profile](image)

**Figure 28. Effect of Carriage Mass on System Performance**

From Figure 28, it can be concluded that the mass has little effect on the system velocity. From the plot above, it can be seen that the mass does slightly change the maximum velocity initially, but then settles down to a constant 25 fps after the overshoot. This is due to the friction and viscous damping occurring within motor and HPLA. The conclusions are as follows:
1. The maximum achievable system velocity is 25 fps. The system can hit as high as 28 fps due to the momentum carried by the system from the acceleration. However, the system levels off at 25 fps.

2. Increasing the length of the track will not improve the performance of the system. The current track length is sufficient for achieving maximum velocity.

3. The mass of the carriage does not affect the velocity capabilities of the system. As can be seen in the figures, the mass only affects the no-load overshoot velocity.

This complete system provided a comprehensive system model. The ability to probe the simulation and easily adjust inputs and parameters made the simulation valuable for this analysis. The conclusions answered multiple questions about the possible redesign of the system.

4.3 Force Measurement Model and Simulation

This second simulation will repeat steps 3-8 to address the force measurement capability. From the experimental results, it is clear that the measurement system produces raw data outside the force accuracy requirements of the components being tested. Historical data validation will be used to both construct the model and drive the simulation. Two conceptual models will be developed and will therefore illustrate the suggested iterative approach of the methodology. The conceptual model was revised in order to reduce the force difference.
4.3.1 Conceptual Models of Carriage System

The development of another conceptual model will repeat step 3 for this second simulation. From the experimental results, the force data registered by the moving force sensor is clearly being amplified by some dynamic event. In order to identify this event, a separate model and simulation will take place. The model will be revised to take position data from the carriage and output the force at the point of the load specimen. Additionally, a 1-D model will be generated describing two force sensors and the lanyard.

Figure 29. Carriage System Model

The differential equations associated with positions $x_1$ and $x_2$ can be seen as follows:

$$
\ddot{x}_1(t) = -\frac{(K_L + K_{LC1})}{m_1}(x_1(t) - x_1(0)) + \frac{K_L}{m_1}(x_2(t) - x_2(0))
$$

(17)
\[ \ddot{x}_2(t) = \frac{K_L}{m_2}(x_1(t) - x_1(0)) - \frac{(K_L + K_{LC2})}{m_2}(x_2(t) - x_2(0)) + \frac{K_{LC2}}{m_2}(x_3(t) - x_3(0)) \]  
(18)

\[ F_{stationary} = K_{LC1}(x_1(t) - x_1(0)) \]  
(19)

\[ F_{move} = K_{LC2}\left(x_3(t) - x_3(0) - (x_2(t) - x_2(0))\right) \]  
(20)

\[ F_{stationary}(t) = F_{move}(t) - m_1\ddot{x}_1(t) - m_2\ddot{x}_2(t) \]  
(21)

Because accelerometer data for \( \ddot{x}_2 \) can be obtained from the experimental data, it is possible to use this as an input. However, \( \ddot{x}_1 \) is not observable with the current experimental configuration directly from an accelerometer.

The acceleration of \( M_1 \) can be accounted for using the force data at that location. In order to do this, the relationship between force and position can be utilized to generate an equation for the acceleration at \( M_1 \).

\[ x_1(t) = \frac{F_{stationary}(t)}{K_{LC1}} - x_1(0) \]  
(22)

\[ \ddot{x}_1(t) = \frac{\ddot{F}_{stationary}(t)}{K_{LC1}} \]  
(23)
Using Equation 17, it can easily be checked what the actual impact is of this term from the experimental data. By doing so, it can be concluded that this term is negligible due to the low mass and low acceleration. The force becomes very small as a result and can be taken out of the stationary force relationship. The maximum observed acceleration from the experimental data using this force relationship was 3.53 g’s. This translates to a maximum force of less than 10 lbf due to the acceleration of $M_1$. This result suggests that the $M_1\ddot{x}_1$ term could contribute 1.25% of the moving force cell difference. As a result, this term can be ignored due to this small contribution. The results are displayed in Table 7.

<table>
<thead>
<tr>
<th>Set #</th>
<th>Velocity (fps)</th>
<th>Avg Stationary Load (lbf)</th>
<th>$a_{1 \text{max}}$ (g’s)</th>
<th>$a_{2 \text{max}}$ (g’s)</th>
<th>$m_1a_1$ (lbf)</th>
<th>$m_2a_2$ (lbf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>212.1</td>
<td>0.617</td>
<td>27.9</td>
<td>1.56</td>
<td>70.6</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
<td>584.08</td>
<td>1.98</td>
<td>114</td>
<td>5.02</td>
<td>290</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>688.82</td>
<td>2.7</td>
<td>330</td>
<td>6.85</td>
<td>839</td>
</tr>
<tr>
<td>4</td>
<td>7.8</td>
<td>588.92</td>
<td>2.29</td>
<td>199</td>
<td>5.81</td>
<td>504</td>
</tr>
<tr>
<td>5</td>
<td>7.8</td>
<td>792.87</td>
<td>3.53</td>
<td>225</td>
<td>8.95</td>
<td>572</td>
</tr>
</tbody>
</table>

*Note: Carriage Experienced Acceleration of -1.24g’s when pull occurred.*

From this model, the input will be the position data ($x_3$), and the output will be the force associated with $K_{LC1}$, representing the force sensor at the location of the pull specimen. This will be validated using the experimental data.

$$F_{\text{stationary}}(t) = F_{\text{move}}(t) - m_2\ddot{x}_2(t)$$  \hspace{1cm} (24)
Using this one-dimensional physics-based model of the system, this revised model will determine the dynamic effects occurring during the impact event that may affect the measurement.

4.3.2 Conceptual Validation of Models

Conceptual model validation will be performed again, revisiting step 4 of the methodology. This model is very simple therefore validation that this 1-D model appropriately models the system is a simple task.

4.3.3 Computational Simulation for Carriage System Models

The simulation of the conceptual model, or step 5 in the methodology, will be performed using the historic data to drive the simulation. Once again, the simulation will be performed in MATLAB. The general structure of this simulation can simply be reduced to the following:

![Simulated 1-D Stationary Force Simulation](image)

**Figure 30. Simulated 1-D Stationary Force Simulation**
This simple 1-dimensional simulation provides a means of accounting for the impact vibration that the carriage is exposed to during the pull. The moving force data and accelerometer data illustrated in Figure 30 represent actual data collected from the experimental testing. The simulated stationary force data output represents the corrected moving force data. This simulation will be implemented in MATLAB and will simply take the accelerometer data and moving force data to determine if the inertial effects can be simplified into a 1-dimensional system.

4.3.4 Computational Simulation Verification

In order to perform the methodology, step 6 requires the verification of the computational simulation. This simulation processes the historical moving force data by subtracting the inertial effects as described in Equation 23. Therefore, the simulation can be validated by ensuring that the simulation performs the correct mathematical operations with the data.

4.3.5 Computational Simulation Operational Validation and Prediction

Validation of the simulation as specified in step 7 of the methodology will be performed again. The validation will be performed by comparing the historic stationary force sensor data against the simulated force sensor data.
Table 8. Simulation Validation Results

<table>
<thead>
<tr>
<th>Sample Size</th>
<th>Load (lbf)</th>
<th>Velocity (fps)</th>
<th>Avg Simulated Stationary Load (lbf)</th>
<th>Avg Stationary Load (lbf)</th>
<th>Avg Force difference (%)</th>
<th>Std Dev (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>200</td>
<td>3</td>
<td>215.28</td>
<td>212.1</td>
<td>1.50%</td>
<td>1.30%</td>
</tr>
<tr>
<td>20</td>
<td>400</td>
<td>11</td>
<td>658.85</td>
<td>584.08</td>
<td>12.80%</td>
<td>6.32%</td>
</tr>
<tr>
<td>20</td>
<td>600</td>
<td>15</td>
<td>726.02</td>
<td>688.82</td>
<td>5.40%</td>
<td>4.06%</td>
</tr>
<tr>
<td>20</td>
<td>400*</td>
<td>7.8</td>
<td>654.29</td>
<td>588.92</td>
<td>11.10%</td>
<td>7.31%</td>
</tr>
<tr>
<td>20</td>
<td>600*</td>
<td>7.8</td>
<td>1047.78</td>
<td>792.87</td>
<td>32%</td>
<td>11.23%</td>
</tr>
</tbody>
</table>

*Note: Carriage Experienced Acceleration of -1.24g’s when pull occurred.

Table 8. Simulation Validation Results displays the results of the validation study. The average simulated stationary load describes the simulation output average over all 20 pulls per category. The error describes the error associated with this simulation output. As can be seen from the results, the maximum average error that occurred during constant velocity was 12.8%.

Although this model significantly reduces the error, additional modeling must be performed in order to further understand what is producing the error and how to correct for it. This simple 1-dimensional model is not sufficient for the application.

4.3.6 Conceptual Models of Carriage System using Filtering Methods

The proposed methodology is intended to be an iterative process. Step 3 will be performed again in an attempt to model the force during the pull event because the 1-D physics-based model of the force sensor was not sufficient. Because the force sensor is a
piezoelectric force sensor between to metal plates, it is highly susceptible to the effects of vibration [38]. A low pass filter will be implemented to reduce the effects of vibration on the moving force sensor. The low pass filter will essentially be implemented by taking the moving average of the moving force sensor data. The equation for the moving average can be seen as follows:

$\bar{X}_i = \frac{X_{i-N} + X_{i-N+1} + X_{i-N+2} + \cdots + X_i}{N}$  \hspace{1cm} (25)

This is a common means of implementing a low pass filter, where $N$ is the moving average population size.

**4.3.7 Conceptual Validation of Filtering Model**

This ultimate goal of this model is to provide a low pass filter to remove the high frequency vibration data from the moving force sensor results. The conceptual validation, or *step 4* in the methodology, can easily be performed in this case because the model is simply a moving average of the moving force sensor data.

**4.3.8 Computational Simulation using Filters**

This final computational simulation or *step 5*, of the data took a moving average of the moving force data using the built in MATLAB function. The population size that was taken was 140 points, or the equivalent of 2.9 ms.
The input moving force data represents the raw force sensor data from the dynamic force tester. The simulated stationary force data on the output represents the moving force data after the moving average has been taken to reduce the high frequency vibrations.

4.3.9 Filter Computational Simulation Verification

Verification of the simulation as specified in step 6 of the methodology requires justification that the moving average has been correctly implemented. The use of the MATLAB function makes this verification simple. The function does correctly implement the moving average equation as listed in Equation 19.
4.3.10 Filter Computational Simulation Operational Validation and Prediction

This computational implementation of the model simply requires taking the moving average of the moving force sensor data. Table 9 displays that the force difference was significantly reduced. Again, this model will be validated using historical validation data from which to drive and assess the simulation.

<table>
<thead>
<tr>
<th>Sample Size</th>
<th>Load (lbf)</th>
<th>Velocity (fps)</th>
<th>Avg Simulated Stationary Load (lbf)</th>
<th>Avg Stationary Load Reading (lbf)</th>
<th>Avg Force Diff (%)</th>
<th>Std Dev %</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>200</td>
<td>3</td>
<td>217.54</td>
<td>212.0954</td>
<td>2.50%</td>
<td>1.30%</td>
</tr>
<tr>
<td>20</td>
<td>400</td>
<td>11</td>
<td>451.2848</td>
<td>584.0846</td>
<td>5.29%</td>
<td>6.32%</td>
</tr>
<tr>
<td>20</td>
<td>600</td>
<td>15</td>
<td>711.2655</td>
<td>688.8237</td>
<td>3.81%</td>
<td>4.06%</td>
</tr>
<tr>
<td>20</td>
<td>400*</td>
<td>7.8</td>
<td>719.6</td>
<td>588.9165</td>
<td>22.20%</td>
<td>7.31%</td>
</tr>
<tr>
<td>20</td>
<td>600*</td>
<td>7.8</td>
<td>616.4</td>
<td>792.8722</td>
<td>16.31%</td>
<td>11.23%</td>
</tr>
</tbody>
</table>

*Note: Carriage Experienced Acceleration of -40 ft/s² when pull occurred.

From the results listed in Table 9, it can be seen that the error has been significantly reduced. This implies that the moving average does cancel out the high frequency signals due to vibration during the impact event. Therefore, this filtering technique can be used to provide an accurate representation of the force that is actually being experienced at the location of the specimen.

4.4 System Data/Results Processing

The final step in the methodology, step 9, consists of the collection of all results and data taken during the simulation and physical experimentation to draw conclusions on the
credibility of the system, which is the ultimate goal of this methodology. From this characterization study, the following conclusions can be made about the system:

The maximum velocity is not affected by the track length or the mass of the carriage. The acceleration capabilities of the carriage allow the system to achieve top speed within 2.5 feet, which is less than half of the useable track length. This top speed is limited by the viscous damping and friction within the system. Because it can reach the top speed in 2.5 feet, increasing the length of the track would provide no significant advantage.

The moving force sensor is significantly affected by the inertial effects of the system and registers force data much higher than the force value of interest, especially when the impact event is more violent. The two models that addressed these inertial effects significantly reduced this force error. Utilizing a moving average with the moving load data could be implemented into this force measurement system to automatically remove these high frequency events. A mechanical implementation of vibration control including the selection of a more vibration-resistive force sensor would greatly reduce the error as well [39]. These inertial effects must be addressed before the machine can guarantee any sort of accuracy with the dynamic force measurement.
6. CONCLUSION

Characterizing a dynamic mechanical measurement system presents many challenges, mostly due to inertial effects such as vibration. The transducers implemented in these systems are often calibrated under static conditions, therefore the advertised accuracy of the transducers are not representative of the system as a whole. This type of system must be characterized to determine the performance of the system. A system of this nature can typically be tested experimentally, but this does not always explain the physical phenomena that may be causing the discrepancy between the measurements. The use of models and simulations provides an understanding of the physics of the measurement system. However; a simulation can only be trusted once it has been validated. In order to validate a simulation, real system data must be provided. The methodology presented in this thesis combines modeling and simulation with experimentation for the specific purpose of analyzing a dynamic force system.

The foundation for the methodology is derived from the simulation community to develop the proposed methodology for addressing measurement systems. The verification and validation process developed by Sargent [4] was utilized to generate a comprehensive process with the sole purpose of developing measurement system credibility. The case study displayed the use of this methodology as a means of characterizing a dynamic force measurement system. After the methodology had been implemented on the system, the performance limits were identified. Because
piezoelectric force sensors are very sensitive to vibration, the study found that more
violent tests resulted in a larger error. This measurement error was then assessed using
both physics-based models and a moving average to provide a physics-based explanation
of what was causing the error.

The primary complications with the methodology are both cost and the ease of system
modeling. Cost can be defined in terms of both time and money. This methodology
requires the development of simulations as well as experimental data. This results in the
need for a facility, personnel, and the appropriate instrumentation. The case study
assessed in this thesis was performed over the course of six months. This includes the
design and construction of the experimental configurations. A high interest in the
characterization of a system must be expressed for this methodology to be a
consideration.

Although this methodology was developed for use with a very unique problem, the
methodology could be expanded to characterize any dynamic system requiring a difficult
measurement. Additionally, this methodology could be implemented to confirm that
historical measurement qualification procedures are appropriate with current
measurement systems. The knowledge of the governing physics behind these dynamic
systems would allow engineers to discover flaws in these qualification systems and
provide a new set of standards if needed.
The key benefit to this methodology is the comprehensive approach of using both experimentation and simulation techniques to explore a system. Further investigation is warranted to study how to identify the circumstances for which this methodology would be appropriate for use. A tradeoff study analyzing the benefits of using this methodology versus the cost would accomplish this.
REFERENCES


APPENDIX A

POSITION, VELOCITY, AND ACCELERATION DATA FROM ENCODER

The position, velocity, and acceleration data had very little variance within each of the five categories of pulls. The following figures display the acceleration, velocity, and position graphs representing each set of data.

3 ft/s 200 lbf coupon data

11 ft/s 400 lbf coupon data
15 ft/s 600 lbf coupon data

7.8 ft/s and -40 ft/s² 400 lbf coupon data

7.8 ft/s and -40 ft/s² 600 lbf coupon data
APPENDIX B

FORCE DATA FROM ALL EXPERIMENTAL TESTS

(excluding test 21, 40, 46, 65)
APPENDIX C

DEVELOPMENT OF HPLA INERTIAL VALUES

The product specification guide for the Parker HPLA180 combines the load being pulled by the HPLA with the belt inertia into one inertia value. The variables needed for this inertia calculation are provided in the specifications as follows:

- Carriage Length (Lcarriage)
- Total length of track (LProfile)
- Toothed belt length (LROH) = \( 2(LProfile) - Lcarriage + 1190 \)
- Effective radius of toothed pulley (RA)
- Stroke of Pull Motion (stroke)
- Length of toothed belt (LR) = \( 2(stroke) + LROH \)
- Mass of toothed belt per meter (mR1M)
- Mass of toothed belt (mR) = \( LR \times mR1M \)
- Mass of Carriage (mNL)
- Additional mass moment of inertia caused by belt mass (JR) = \( mR \times RA^2 \)
- Additional mass moment of inertia caused by payload mass (JNL) = \( mNL \times RA^2 \)
- Additional mass moment of inertia (JZ) = \( JNL + JR \)