# On the Performance of an Automatic Frequency Control Loop in Dissimilar Fading Channels in the Presence of Interference

Amin Emad, *Student Member, IEEE,* and Norman C. Beaulieu, *Fellow, IEEE*

***Abstract*—Two effects of a single interference on the performance of an automatic frequency control (AFC) loop are explained, and two measures, average switching rate and mean time to loss of lock, are presented to capture the impact of each effect. These measures are derived in closed-form expressions and single integral formulas in dissimilar fading channels with Rayleigh, Rician, and Nakagami-*m* distributions. The general case of modulated carriers as well as different special cases are considered. Numerical examples are provided to illustrate the effects of different fading scenarios on the performance of an AFC. The effect of maximum Doppler frequency on the performance of the AFC is also investigated.**

***Index Terms*—Automatic frequency control loop, average switching rate, dissimilar fading channels, interference, mean time to loss of lock, Nakagami, outage, Rayleigh, Rician.**

## I. Introduction

In digital receivers, automatic frequency control loops are employed to recover and control the frequency of received signals. They may be used as an independent and necessary part of the receiver (in noncoherent receivers), or they may be used as an acquisition aid to improve the performance of a phase-locked loop (PLL) [1]. The basic components of an AFC are shown in [1, Fig. 8.3-2]. The frequency detector (FD) produces an error voltage which is proportional to the difference between an input signal's carrier frequency and the local frequency of the voltage controlled oscillator (VCO). This error voltage is filtered by the loop filter and then is used to control the local frequency of the VCO. When the local frequency of the VCO equals the carrier frequency of the input signal, the error voltage is equal to zero and the AFC is in steady state.

The performance of an AFC for a single received signal in a noisy environment has been investigated in various references. In [2]-[5], the variance of the frequency error has been used as a measure of the tracking performance of an AFC, while in [6] and [7] the probability of loss of lock of an AFC due to the effect of noise has been employed to evaluate the performance. However, noise is not the only disturbance that can affect the performance of an AFC. Since there are several users in a wireless channel, the presence of more than one signal at the input of a receiver is commonplace. This issue has been investigated in [8] for a single interferer and it has been shown that the AFC will remain locked on the desired signal until the amplitude of the interferer exceeds the amplitude of the desired signal.

Since the amplitudes of the received signals depend on different factors, such as modulation and multipath fading, a jump may occur in an AFC from the desired signal to the interference signal and vice versa. Every jump generates a transient in the receiver filters, which corrupts the performance of the filters and produces an internal receiver outage. The average switching rate (ASR) characterizes how often these jumps occur on average in an AFC. Although the ASR measures the occurrence frequency of outage caused by these jumps, it does not show when the output of the AFC is reliable. In other words, even if the ASR has a small value, the AFC might have locked on the interferer instead of the desired signal, and therefore, the receiver is in outage. A measure that can be used to overcome this shortcoming of the ASR measure is the mean time to loss of lock (MTLL). This parameter measures how long the AFC stays locked on the desired signal, or equivalently, for how long the output of the AFC is reliable.

While outage generated by channel fading has been fully investigated in the literature, there is a dearth of reported results on a receiver's internally generated outage. In reference [9], the ASR and MTLL of an AFC loop with two unmodulated received signals, both subject to Rayleigh fading, have been derived. In [10], these results have been generalized to the case of modulated signals in Rayleigh/Rayleigh, Rician/Rician, and Nakagami/Nakagami channels. However, in many scenarios, the desired signal and the interferer pass through different transmission environments, and therefore, they can have different fading statistics [11]-[14]. For instance, if there is a line-of-sight (LOS) path between the transmitter of the desired signal and the receiver, the channel of the desired signal is best modeled by Rician fading. Meanwhile, there may not be a dominant multipath reflection in the interferer's channel. In this case, the fading affecting the interferer is best modeled by a Rayleigh (pure scattering) or a Nakagami distribution. Other dissimilar channel scenarios may appear as well when there is no LOS path between the transmitter of the desired signal and the receiver. In this letter, we study the performance of an AFC in such mixed fading channels. Rician/Rayleigh, Rayleigh/Rician, Rician/Nakagami, Nakagami/Rician, Nakagami/Rayleigh and Rayleigh/Nakagami scenarios are investigated and closed-form expressions or simple single integral

Paper approved by C. Tepedelenlioglu, the Editor for Synchronization and Equalization of the IEEE Communications Society. Manuscript received June 11, 2010; no revision.

This work was presented in part at the IEEE 69th Vehicular Technology Conference (VTC'09).

A. Emad is with the Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign, IL, USA 61801 (e-mail: emad2@illinois.edu).

N. C. Beaulieu is with the Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB, Canada T6G 2V4 (e-mail: beaulieu@ece.ualberta.ca).

This work was supported in part by an Alberta Informatics Circle of Research Excellence (*i*CORE) scholarship, a Natural Sciences and Engineering Research Council of Canada (NSERC) postgraduate scholarship and an *i*CORE Chair Establishment Grant.

Digital Object Identifier 10.1109/TCOMM.2011.071111.100351

form formulas are derived for the ASR and the MTLL of an AFC. The results are derived for different modulations of the received signals.

The remainder of this letter is organized as follows. In Section II, the system and channel models are described. In Section III and IV, the method of analysis is described. In Section V, some numerical examples are provided, and the performance of an AFC in different scenarios is discussed. Finally, Section VI summarizes the results of this study.

## II. System and Channel Models

In this letter, we consider two modulated received signals subject to multipath fading at the input of an AFC. These signals are modeled as $x_i(t) = s_i(t)A_i(t)\cos(\omega_i t + \theta_i(t) + \Phi_i(t))$ for $i \in \{1,2\}$ where $x_1(t)$ denotes the desired received signal and $x_2(t)$ denotes the interferer; $s_i(t)$ and $\theta_i(t)$ are the amplitude and the phase of the baseband equivalent of the transmitted signal, respectively, and capture the effects of the pulse shape, modulation scheme, and transmitted data. In this model, $A_i(t)$ represents the amplitude of the fading in the channel and $\Phi_i(t)$ is a uniformly distributed random variable in $[0, 2\pi)$ representing the phase of the fading. In this equation, $\omega_i$ denotes the carrier angular frequency. In the rest of the letter, we use $s_i = s_i(t)$ and $A_i = A_i(t)$ for simplicity, and, unless specifically stated otherwise, these are functions of time.

The statistics of $A_i$ and its time derivative, $\dot{A}_i$, depend on the moments of the received signals [15]. These moments are defined as

$$b_{in} = (2\pi)^n \int_{f_i - f_{m_i}}^{f_i + f_{m_i}} (f - f_i)^n W_i(f) df \qquad i = 1, 2 \quad (1)$$

where $f_i$ is the carrier frequency, $f_{m_i}$ is the maximum Doppler frequency, and $W_i(f)$ is the power spectral density (PSD) of the unmodulated received signal. If the PSD of the received signals are symmetric around the carrier frequencies, $A_i$ and $\dot{A}_i$ are independent [15]-[17]. When $A_i$ has a Rayleigh distribution, the joint probability density function (JPDF) of $A_i$ and $\dot{A}_i$, $f_{A_i,\dot{A}_i}(\alpha_i, \dot{\alpha}_i)$, is given by

$$f_{A_i,\dot{A}_i}(\alpha_i, \dot{\alpha}_i) = \frac{\alpha_i}{\sigma_i^2} \exp\left(-\frac{\alpha_i^2}{2\sigma_i^2}\right) \frac{1}{\sqrt{2\pi\dot{\sigma}_i^2}} \exp\left(-\frac{\dot{\alpha}_i^2}{2\dot{\sigma}_i^2}\right) \quad (2)$$

where $\sigma_i^2 = b_{i0}$ and $\dot{\sigma}_i^2 = b_{i2}$. When $A_i$ has a Rician distribution,

$$f_{A_i,\dot{A}_i}(\alpha_i, \dot{\alpha}_i) = \frac{\alpha_i}{\sigma_i^2} \exp\left(-\frac{\alpha_i^2}{2\sigma_i^2} - K_i\right) I_0\left(\sqrt{2K_i}\frac{\alpha_i}{\sigma_i}\right) \times \frac{1}{\sqrt{2\pi\dot{\sigma}_i^2}} \exp\left(-\frac{\dot{\alpha}_i^2}{2\dot{\sigma}_i^2}\right) \quad (3)$$

where $K_i$ is called the Rice factor and is equal to the ratio of the power in the LOS component to the power in the scatter component, $I_0(\cdot)$ is the zero-order modified Bessel function of the first kind [18], $\sigma_i^2 = b_{i0}$, and $\dot{\sigma}_i^2 = b_{i2}$. When $A_i$ has a Nakagami distribution with Nakagami fading parameter, $m_i$, where $m_i = n_i/2$ and $n_i$ is an integer, the JPDF of $A_i$ and $\dot{A}_i$ is equal to [19]

$$f_{A_i,\dot{A}_i}(\alpha_i, \dot{\alpha}_i) = \frac{2m_i^{m_i}\alpha_i^{2m_i-1}}{\Gamma(m_i)(2\sigma_i^2)^{m_i}} \exp\left(-\frac{m_i\alpha_i^2}{2\sigma_i^2}\right) \times \frac{1}{\sqrt{2\pi}\dot{\sigma}_i} \exp\left(-\frac{\dot{\alpha}_i^2}{2\dot{\sigma}_i^2}\right) \quad (4)$$

where $\dot{\sigma}_i^2 = \frac{b_{i2}}{m_i}$ and $\sigma_i^2 = b_{i0}$.

Note that in this study, we consider independent channels such that $A_1$ is independent of $A_2$ and $\dot{A}_1$ is independent of $\dot{A}_2$.

## III. Average Switching Rate

In [8, Ch. 19], it has been shown that if the difference between the carrier frequencies of the desired signal and the interferer, $|f_1 - f_2|$, is much smaller than the carrier frequencies, and also the modulations of the received signals are slow compared to $|f_1 - f_2|$, the AFC will lock on the signal with the larger amplitude. These conditions are fulfilled in the case of adjacent-channel interference (ACI), most of the time in the case of abutting-channel interference[1], and sometimes in the case of cochannel interference [8, Ch. 19]. Considering these conditions, one can find the ASR of an AFC using the relative statistics of the received signals.

In order to find the average switching rate, one can define $Z = s_1A_1 - s_2A_2$, the difference between the amplitude of the desired signal, $x_1(t)$, and the interference signal, $x_2(t)$. If the AFC is locked on $x_1(t)$, a jump occurs if the amplitude of $x_2(t)$, $s_2A_2$, becomes greater than the amplitude of $x_1(t)$, $s_1A_1$. This is equivalent to $Z = s_1A_1 - s_2A_2$ becoming less than zero. If the AFC is locked on $x_2(t)$, a jump occurs if $s_1A_1$ becomes greater than $s_2A_2$, which is equivalent to $Z$ becoming greater than zero. As a result, the average switching rate of an AFC, $N$, is equal to the average zero-crossing rate (negative-going plus positive-going) of $Z$, $N_Z(0)$. One has [20]

$$N = N_Z(0) = \int_{-\infty}^{\infty} |\dot{z}| f_{Z,\dot{Z}}(0, \dot{z}) d\dot{z} \quad (5)$$

where $f_{Z,\dot{Z}}(z, \dot{z})$ is the JPDF of $Z$ and its time derivative, $\dot{Z}$. One can use the definition of the conditional probability density function [21], $f_{Z,\dot{Z}}(z, \dot{z}) = f_{\dot{Z}|Z}(\dot{z}|z) f_Z(z)$, to find

$$N = f_Z(0) \int_{-\infty}^{\infty} |\dot{z}| f_{\dot{Z}|Z}(\dot{z}|0) d\dot{z} \quad (6a)$$

where

$$f_Z(0) = \int_{y=0}^{\infty} \frac{1}{s_1 s_2} f_{A_1}\left(\frac{y}{s_1}\right) f_{A_2}\left(\frac{y}{s_2}\right) dy. \quad (6b)$$

In order to find $f_{\dot{Z}|Z}(\dot{z}|0)$, one can differentiate $Z$ with respect to time and use the definition of $Z$ to find $A_1$ as a function of $Z$ and $A_2$. Consequently, $\dot{Z} = s_1\dot{A}_1 - s_2\dot{A}_2 + \left(\frac{\dot{s}_1 s_2}{s_1} - \dot{s}_2\right) A_2 + \frac{\dot{s}_1}{s_1} Z$. Note that $\dot{A}_1$ and $\dot{A}_2$ are both zero-mean normally distributed random processes, and $\dot{A}_1$ and $\dot{A}_2$ are independent. As a result, if $Z$ and $A_2$ are given, $\dot{Z}$ is

[1] In [8] the signals sent in the immediate neighbour channel of the desired signal are called the abutting-channel interference while the signals sent in other adjacent channels of the desired signal are called the adjacent-channel interference.

equal to the summation of two scaled zero-mean independent random processes and a constant. Consequently,

$$f_{\dot{Z}|A_2,Z}(\dot{z}|\alpha_2,0)=\frac{1}{\sqrt{2\pi\Omega}}\exp\left[-\frac{\left(\dot{z}-\alpha_2\left(\frac{\dot{s}_1 s_2}{s_1}-\dot{s}_2\right)\right)^2}{2\Omega}\right] \quad (7)$$

where $\Omega = s_1^2\dot{\sigma}_1^2 + s_2^2\dot{\sigma}_2^2$. Moreover, using the fundamental theorem for transformations of a random variable [21], it can be easily shown that

$$f_{A_2|Z}(\alpha_2|0) = \frac{s_2}{s_1} f_{A_1}\left(\frac{s_2}{s_1}\alpha_2\right). \quad (8)$$

Consequently, using the theorem of total probability [21],

$$f_{\dot{Z}|Z}(\dot{z}|z=0) = \int_0^\infty f_{\dot{Z}|A_2,Z}(\dot{z}|\alpha_2,0) f_{A_2|Z}(\alpha_2|0)d\alpha_2 \quad (9)$$

and the ASR can be found using eqs. (6)-(9).

One should note that in the special case that the desired transmitted signal is a replica of the interference signal, i.e., $s_1(t) = ks_2(t)$ where $k$ is a constant, $\dot{s}_1(t)s_2(t) = \dot{s}_2(t)s_1(t)$ and therefore $f_{\dot{Z}|Z}(\dot{z}|0) = \frac{1}{\sqrt{2\pi\Omega}}\exp\left(-\frac{\dot{z}^2}{2\Omega}\right)$. Consequently, the ASR in this case is equal to

$$N=f_Z(0)\int_{-\infty}^{\infty}|\dot{z}|\frac{1}{\sqrt{2\pi\Omega}}\exp\left(-\frac{\dot{z}^2}{2\Omega}\right)d\dot{z}=\sqrt{\frac{2\Omega}{\pi}}f_Z(0). \quad (10)$$

It is worth mentioning that the case where the signals are unmodulated is a special case of (10).

The general results for the ASR of Rayleigh/Rician, Rayleigh/Nakagami, and Nakagami/Rician scenarios are presented in Appendix A. In addition, Table I reviews some important special cases for each of these scenarios. In this table "Ray", "Ric", and "Nak" stand for Rayleigh, Rician, and Nakagami fading, respectively. Note that the ASRs for Rician/Rayleigh, Nakagami/Rayleigh, and Rician/Nakagami channels can be easily found using these results by interchanging the roles of the desired signal and the interference signal (i.e., switching index 1 to 2 and index 2 to 1). For the sake of brevity, the flip results are not presented.

## IV. Mean Time to Loss of Lock

The MTLL is a measure that indicates the average time an AFC locks on the desired signal before jumping to the interference signal. The MTLL for unmodulated signals in fading channels can be found using [22]

$$T = \frac{2F}{N} \quad (11)$$

where $N$ is the ASR of the AFC and $F$ is the probability that the amplitude of the desired signal is larger than the amplitude of the interference signal, i.e., $F = Pr[s_1A_1 > s_2A_2]$. In order to find $F$, we introduce the transformation $Y = \frac{s_1A_1}{s_2A_2}$ and $W = s_2A_2$. The Jacobian of this transformation is equal to $\frac{s_1s_2}{w}$ and therefore $f_{Y,W}(y,w) = \frac{w}{s_1s_2}f_{A_1,A_2}\left(\frac{yw}{s_1},\frac{w}{s_2}\right)$. Since $A_1$ and $A_2$ are independent, integrating over $w$ yields $f_Y(y) = \int_0^\infty \frac{w}{s_1s_2}f_{A_1}\left(\frac{yw}{s_1}\right)f_{A_2}\left(\frac{w}{s_2}\right)dw$, and therefore, $F = Pr[s_1A_1 > s_2A_2] = Pr[Y > 1]$ can be obtained using

$$F = \int_{y=1}^\infty\int_{w=0}^\infty \frac{w}{s_1s_2}f_{A_1}\left(\frac{wy}{s_1}\right)f_{A_2}\left(\frac{w}{s_2}\right)dwdy. \quad (12)$$

TABLE I
THE AVERAGE SWITCHING RATE OF AN AFC IN FADING

| Fading | System Model | Average Switching Rate |
|---|---|---|
| Ray/Ric | $s_1=s_2=1$<br>$\dot{s}_1=\dot{s}_2=0$ | $\sqrt{\frac{\dot{\sigma}_1^2+\dot{\sigma}_2^2}{(\sigma_1^2+\sigma_2^2)^3}}\sigma_1\sigma_2\exp(-K_2)\,{}_1F_1\left(\frac{3}{2};1;\frac{K_2\sigma_1^2}{\sigma_1^2+\sigma_2^2}\right)$ |
| Ray/Ric | $s_1(t)=ks_2(t)$ | $\frac{{d_3}^{-3/2}\sqrt{\Omega}}{s_1^2s_2^2\sigma_1^2\sigma_2^2}\exp(-K_2)\,{}_1F_1\left(\frac{3}{2};1;\frac{K_2}{s_2^2\sigma_2^2d_3}\right)$ |
| Ray/Ric | $s_1(t)\neq s_2(t)$<br>$\dot{s}_1(t)\neq 0$<br>$\dot{s}_2(t)\neq 0$ | $\sqrt{\frac{\pi}{2}}\frac{{d_3}^{-3/2}}{s_1^2s_2^2\sigma_1^2\sigma_2^2}\exp(-K_2)\,{}_1F_1\left(\frac{3}{2};1;\frac{K_2}{s_2^2\sigma_2^2d_3}\right)$<br>$\times\int_{-\infty}^{\infty}\left[1+d_2\sqrt{\frac{\pi}{d_1}}\dot{z}\exp\left(\frac{\dot{z}^2d_2^2}{d_1}\right)\text{erfc}\left(-\frac{\dot{z}d_2}{\sqrt{d_1}}\right)\right]$<br>$\times M_1|\dot{z}|\exp\left(-\frac{\dot{z}^2}{2\Omega}\right)d\dot{z}$ |
| Ray/Nak | $s_1=s_2=1$<br>$\dot{s}_1=\dot{s}_2=0$ | $\sqrt{\frac{\dot{\sigma}_1^2+\dot{\sigma}_2^2}{\pi}}\frac{\Gamma(m_2+\frac{1}{2})}{\Gamma(m_2)}\frac{2m_2^{m_2}}{{\sigma_2}^{2m_2}\sigma_1^2}\left(\frac{m_2}{\sigma_2^2}+\frac{1}{\sigma_1^2}\right)^{-m_2-\frac{1}{2}}$ |
| Ray/Nak | $s_1(t)=ks_2(t)$ | $\frac{\Gamma(m_2+\frac{1}{2})}{\Gamma(m_2)}\frac{2m_2^{m_2}\sqrt{\Omega/\pi}}{(s_2^2\sigma_2^2)^{m_2}s_1^2\sigma_1^2}\left(\frac{m_2}{s_2^2\sigma_2^2}+\frac{1}{s_1^2\sigma_1^2}\right)^{-m_2-\frac{1}{2}}$ |
| Ray/Nak | $\dot{s}_1(t)\neq 0$<br>$\dot{s}_2(t)\neq 0$ | $\int_{-\infty}^{\infty}\left[1+d_2\sqrt{\frac{\pi}{d_1}}\dot{z}\exp\left(\frac{\dot{z}^2d_2^2}{d_1}\right)\text{erfc}\left(-\frac{\dot{z}d_2}{\sqrt{d_1}}\right)\right]$<br>$\times\frac{\sqrt{2}m_2^{m_2}\Gamma(m_2+\frac{1}{2})M_1}{\Gamma(m_2)(s_2^2\sigma_2^2)^{m_2}s_1^2\sigma_1^2{d_4}^{m_2+\frac{1}{2}}}\|\dot{z}\|\exp\left(-\frac{\dot{z}^2}{2\Omega}\right)d\dot{z}$ |
| Nak/Ric | $s_1=s_2=1$<br>$\dot{s}_1=\dot{s}_2=0$ | ${}_1F_1\left(m_1+\frac{1}{2};1;\frac{K_2\sigma_1^2}{\sigma_1^2+\sigma_2^2m_1}\right)$<br>$\times\sqrt{\frac{\dot{\sigma}_1^2+\dot{\sigma}_2^2}{\pi}}\frac{2m_1^{m_1}\Gamma(m_1+\frac{1}{2})\exp(-K_2)}{\Gamma(m_1){\sigma_1}^{2m_1}\sigma_2^2d_6^{m_1+\frac{1}{2}}}$ |
| Nak/Ric | $s_1(t)=ks_2(t)$ | $\frac{2m_1^{m_1}\Gamma(m_1+\frac{1}{2})\exp(-K_2)\,{}_1F_1\left(m_1+\frac{1}{2};1;\frac{K_2}{s_2^2\sigma_2^2d_6}\right)}{\Gamma(m_1)(s_1^2\sigma_1^2)^{m_1}s_2^2\sigma_2^2d_6^{m_1+\frac{1}{2}}\sqrt{\pi/\Omega}}$ |
| Nak/Ric | $s_1(t)\neq s_2(t)$<br>$\dot{s}_1(t)\neq 0$<br>$\dot{s}_2(t)\neq 0$ | $\frac{2\Gamma(m_1+\frac{1}{2})\Gamma(2m_1){m_1}^{2m_1}{s_2}^{2m_1-2}}{\Gamma^1(m_1)(s_1^2\sigma_1^2)^{2m_1}\sigma_2^2(2d_5)^{m_1}d_6^{m_1+\frac{1}{2}}\sqrt{\pi\Omega}}$<br>$\times{}_1F_1\left(m_1+\frac{1}{2};1;\frac{K_2}{s_2^2\sigma_2^2d_6}\right)$<br>$\times\int_{-\infty}^{\infty}\|\dot{z}\|\exp\left[\left(\frac{d_2^2}{d_5}-\frac{1}{2\Omega}\right)\dot{z}^2-K_2\right]D_{-2m_1}\left(-\frac{2d_2\dot{z}}{\sqrt{d_5}}\right)d\dot{z}$ |

Appendix A presents closed-form expressions for $F$ in Rayleigh/Rician, Rayleigh/Nakagami, and Nakagami/Rician fading scenarios. Note that from these expressions, $F$ in Rician/Rayleigh, Nakagami/Rayleigh, and Rician/Nakagami scenarios can be easily found using the following method. Let $F_{1/2}$ denote the expression for $F$ in the distribution1/distribution2 scenario and $F'_{1/2}$ be the expression obtained from $F_{1/2}$ by interchanging the roles of the desired signal and the interference signal (i.e., switching index 1 to 2 and index 2 to 1). Then, $F$ in the distribution2/distribution1 scenario ($F_{2/1}$) is equal to $F_{2/1} = 1 - F'_{1/2}$.

## V. Numerical Examples and Discussion

In the numerical examples, the important special case of two-dimensional isotropic scattering and an omnidirectional receiving antenna is considered. In this case, $b_{i2} = 2\pi^2 f_{m_i}^2 b_{i0}$ for $i = 1, 2$ where $f_{m_i}$ is the maximum Doppler frequency [16]. Moreover, we assume that $f_m = f_{m_1} = f_{m_2}$ and $s_1 = s_2 = 1$. In each scenario, the signal-to-interference ratio (SIR) is defined as $\text{SIR} = 10\log_{10}\left(\frac{E[\alpha_1^2 s_1^2]}{E[\alpha_2^2 s_2^2]}\right)$ where $E[\cdot]$ denotes the expectation. Note that when $\alpha_i$ is Rayleigh, Rician, and Nakagami distributed ($i \in \{1,2\}$), $E[\alpha_i^2]$ is equal to $2b_{i0}$, $2b_{i0}(1 + K_i)$, and $2b_{i0}$, respectively [23].

In Fig. 1, the ASR (normalized to $f_m$) of an AFC versus the SIR is shown in a Rician/Nakagami scenario where SIR =

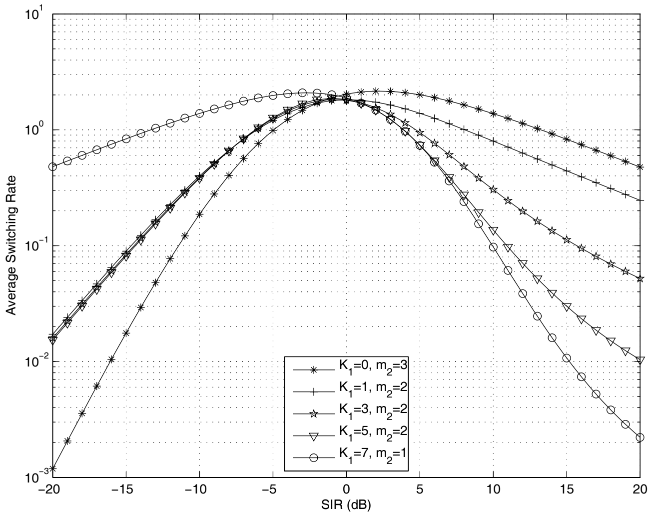


Fig. 1. The ASR (normalized to $f_m$) of an AFC in Rician/Nakagami fading channels.

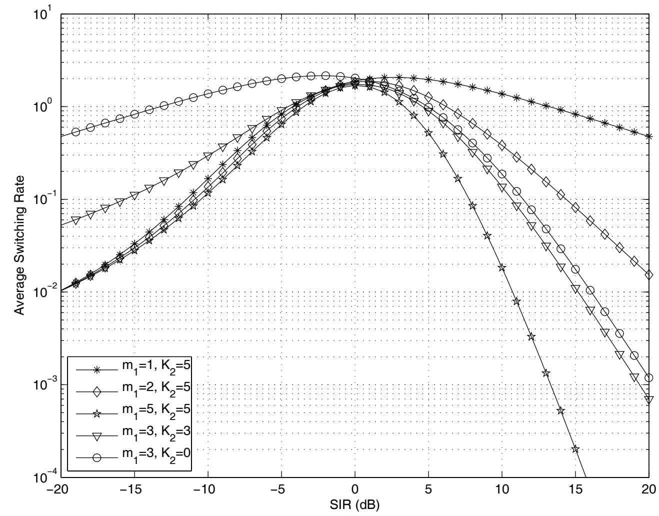


Fig. 2. The ASR (normalized to $f_m$) of an AFC in a Nakagami/Rician fading scenario.

$10 \log_{10}\left(\frac{b_{10}(1+K_1)}{b_{20}}\right)$. Note that $K_i = 0$ and $m_i = 1$ are equivalent to Rayleigh fading. One can observe that on the left side of the figure, the shapes of the curves are determined by $m_2$, while on the right side of this figure, the shapes of the curves are mainly determined by $K_1$. In other words, at small values of SIR, the parameters of the interference signal affect the values of ASR and at large values of SIR, it is primarily the parameters of the desired signal that determine the ASR. This can be easily observed by tracking the behaviour of the ASR curves in which $m_2 = 2$ and $K_1 = 1, 3, 5$. In these cases, the ASR curve is almost the same when the SIR is much smaller than zero; however, as the SIR increases, the ASR decreases more significantly for larger values of $K_1$. The reason for this behaviour is that at small values of SIR, the interference signal has greater power than the desired signal and therefore $Z = s_1A_1 - s_2A_2$ has almost the same behaviour as $-s_2A_2$, and therefore, it is the characteristics of the interference signal that determines the ASR. On the other hand, at large values of SIR, the difference between the amplitude of the received signals is almost equal to the amplitude of the desired signal, and therefore, the desired signal has the greatest influence on the value of the ASR. As expected, a similar behaviour can be seen in Fig. 2 where the ASR of an AFC is shown for a Nakagami/Rician scenario. In this case, the SIR is defined as $\text{SIR} = 10 \log_{10}\left(\frac{b_{10}}{b_{20}(1+K_2)}\right)$.

The effect of the Nakagami parameter $m$ and the Rice factor $K$ can also be examined in these two figures. Comparing the curves in Fig. 1 for $m_2 = 2$ shows that for a specific value of SIR, increasing the value of $K_1$ decreases the ASR. In other words, the performance of AFC improves as the Rice factor of the desired signal increases. Since for a specific value of SIR, increasing the Rice factor is equivalent to increasing the power in the deterministic part of the fading (LOS component), as compared to the random part of the fading (scattering component), the ASR decreases when the Rice factor increases and the fading decreases. In the extreme case where the Rice factor tends to infinity, the desired signal becomes deterministic and the ASR is almost equal to zero for large values of SIR[2]. Similarly, Fig. 2 shows that for a constant value of $K_2$, increasing the value of $m_1$ decreases the ASR at a specific value of SIR.

In addition, one can see that in all these curves, the worst case scenario happens when the received signals have comparable powers. This is expected since having comparable powers means that the slightest change in the amplitude of the received signals causes one of them to become larger than the other and therefore results in a switch in the AFC. One should note that in general, the ASR is small both at small values of SIR and large values of SIR. Although this shows that there is less switching in both of these regions, and therefore the AFC has superior performance with respect to ASR, it does not show whether the output of the AFC is reliable. Since at small values of SIR the AFC is locked on the interference signal for most of the time, the output of the AFC is not reliable. In order to capture the impact of this phenomenon we can use the MTLL.

Figs. 3 and 4 show the MTLL (multiplied by $f_m$) of an AFC in Rician/Nakagami and Nakagami/Rician scenarios, respectively, for different fading parameters. As expected, the MTLL increases in all of the curves as the SIR increases, which shows that the reliability of the AFC's output increases. Moreover, one can see that decreasing the fading of the desired signal by increasing the Nakagami parameter or the Rice factor increases the MTLL and therefore improves the performance of the AFC.

One should note that Figs. 1 and 2 show the ASR normalized to the maximum Doppler frequency, $f_m$, versus the SIR where $f_m = f_{m_1} = f_{m_2}$. As a result, increasing $f_m$ (while keeping other parameters unchanged) increases the ASR and therefore deteriorates the performance of the AFC. However if $f_{m_1} \neq f_{m_2}$, the ASR (normalized to $f_{m_1}$) increases as the ratio $f_{m_2}/f_{m_1}$ increases. This behaviour can be observed in Fig. 5 in which the ASR of an AFC versus $f_{m_2}/f_{m_1}$ is shown

[2]Note that in this case, there still might be switching. This is because the interference signal is not completely deterministic and therefore there is a nonzero probability that its amplitude becomes larger than the desired signal.

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.



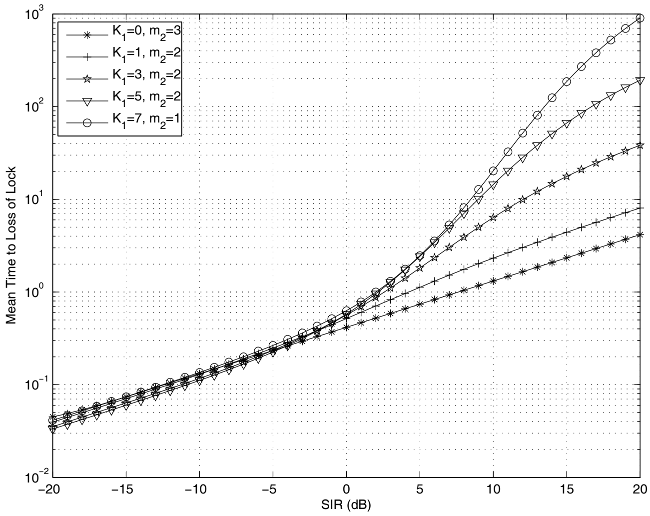


Fig. 3. The MTLL (multiplied by $f_m$) of an AFC in a Rician/Nakagami fading scenario.

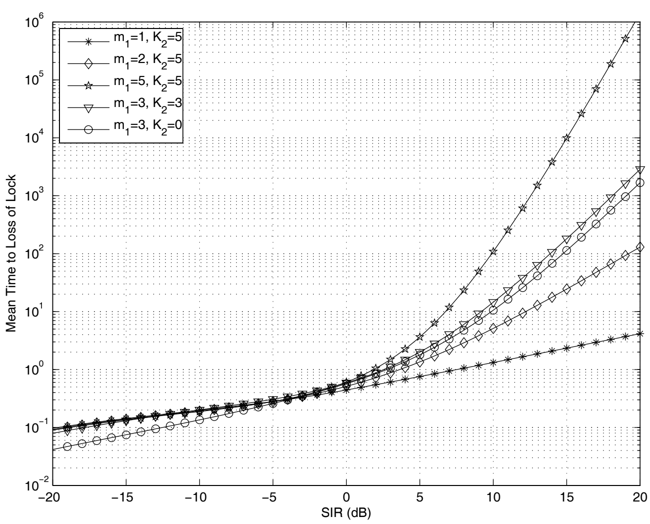


Fig. 4. The MTLL (multiplied by $f_m$) of an AFC in a Nakagami/Rician fading scenario.

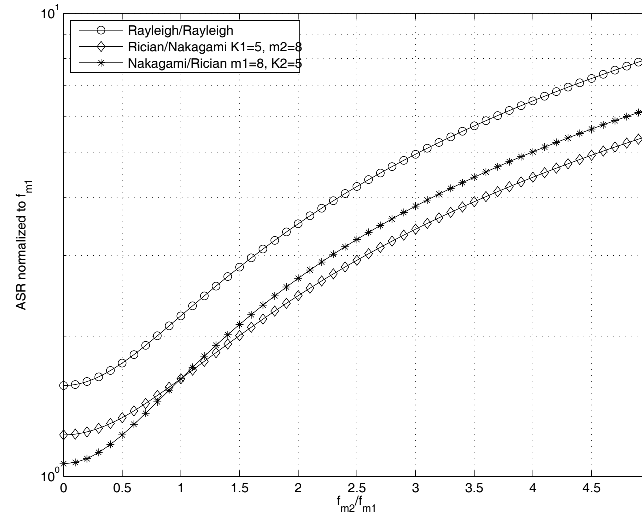


Fig. 5. The ASR (normalized to $f_{m_1}$) of an AFC for SIR $=0$ dB in different fading scenarios.

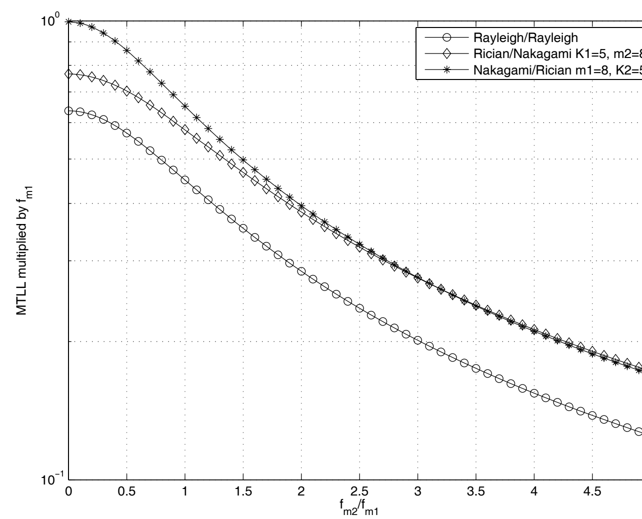


Fig. 6. The MTLL (multiplied by $f_{m_1}$) of an AFC for SIR $=0$ dB in different fading scenarios.

for SIR $=0$ dB for the three cases of Rayleigh/Rayleigh, Rician/Nakagami, and Nakagami/Rician fading.

Similarly, the MTLL in Figs. 3 and 4 is normalized by $f_m$ (through multiplication by $f_m$) and therefore an increase in the value of $f_m$ corresponds to a decrease in the value of the MTLL. In the case of unequal maximum Doppler frequencies, increasing the ratio of $f_{m_2}/f_{m_1}$ decreases the MTLL, i.e., deteriorates the performance of the AFC. This can be observed in Fig. 6, which shows the MTLL (multiplied by $f_{m_1}$) versus $f_{m_2}/f_{m_1}$ for SIR $=0$ dB in Rayleigh/Rayleigh, Rician/Nakagami, and Nakagami/Rician fading scenarios.

## VI. Conclusion

In this letter, closed-form expressions and single integral form formulas were derived for the average switching rate and the mean time to loss of lock of an AFC operating in dissimilar fading channels. Different combinations of Rayleigh, Rician, and Nakagami-*m* fading channels were considered. Numerical examples were provided to illustrate the effect of a single interference and multipath fading on the performance of an AFC. The effect of maximum Doppler frequency on the ASR and MTLL of an AFC in these channels was discussed. The case of channels with unequal maximum Doppler frequencies was considered and numerical examples were provided to illustrate the behaviour of an AFC in these channels.

## Appendix A
## Expressions for the ASR and $F$ in Dissimilar Fading Channels

### A. Rayleigh/Rician

In this scenario,

$$N = \int_{-\infty}^{\infty} \left[ 1 + d_2 \sqrt{\frac{\pi}{d_1}} \dot{z} \exp\left(\frac{\dot{z}^2 d_2^2}{d_1}\right) \operatorname{erfc}\left(-\frac{\dot{z} d_2}{\sqrt{d_1}}\right)\right] \times M_1 f_Z(0) |\dot{z}| \exp\left(-\frac{\dot{z}^2}{2\Omega}\right) d\dot{z} \tag{13a}$$

where

$$f_Z(0)=\sqrt{\frac{\pi}{2}}\frac{{d_3}^{-3/2}}{s_1^2 s_2^2\sigma_1^2\sigma_2^2}\exp(-K_2)\,{}_1F_1\left(\frac{3}{2};1;\frac{K_2}{s_2^2\sigma_2^2 d_3}\right) \quad (13b)$$

and $M_1 = \frac{1}{2d_1\sigma_1^2\sqrt{2\pi\Omega}}\left(\frac{s_2}{s_1}\right)^2$, $d_1 = \frac{s_2^2}{2s_1^2\sigma_1^2} + 2\Omega d_2^2$, $d_2 = \frac{(\dot{s}_2 s_1 - \dot{s}_1 s_2)}{2\Omega s_1}$, and $d_3 = \frac{s_1^2\sigma_1^2 + s_2^2\sigma_2^2}{s_1^2 s_2^2\sigma_1^2\sigma_2^2}$.

In this scenario, $F$ is equal to

$$F = \frac{1}{s_2^2\sigma_2^2 d_3}\exp\left(-\frac{K_2}{s_1^2\sigma_1^2 d_3}\right). \quad (14)$$

### B. Rayleigh/Nakagami

In this scenario, one has

$$N=\int_{-\infty}^{\infty}\left[1+d_2\sqrt{\frac{\pi}{d_1}}\dot{z}\exp\left(\frac{\dot{z}^2 d_2^2}{d_1}\right)\operatorname{erfc}\left(-\frac{\dot{z}d_2}{\sqrt{d_1}}\right)\right] \quad (15)$$
$$\times\frac{\sqrt{2}m_2^{m_2}\Gamma(m_2+\frac{1}{2})M_1}{\Gamma(m_2)(s_2^2\sigma_2^2)^{m_2}s_1^2\sigma_1^2{d_4}^{m_2+\frac{1}{2}}}|\dot{z}|\exp\left(-\frac{\dot{z}^2}{2\Omega}\right)d\dot{z}$$

where $d_4 = \frac{1}{s_1^2\sigma_1^2} + \frac{m_2}{s_2^2\sigma_2^2}$ and $M_1$, $d_1$, and $d_2$ are defined in (13) and it can be shown using (12) that

$$F = \frac{m_2^{m_2}}{\left(\frac{s_2^2\sigma_2^2}{s_1^2\sigma_1^2}+m_2\right)^{m_2}}. \quad (16)$$

### C. Nakagami/Rician

In this scenario,

$$N = \frac{2\Gamma\left(m_1+\frac{1}{2}\right)\Gamma(2m_1)m_1^{2m_1}s_2^{2m_1-2}\,{}_1F_1\left(m_1+\frac{1}{2};1;\frac{K_2}{s_2^2\sigma_2^2 d_6}\right)}{\Gamma^2(m_1)(s_1^2\sigma_1^2)^{2m_1}\sigma_2^2(2d_5)^{m_1}d_6^{m_1+\frac{1}{2}}\sqrt{\pi\Omega}}$$
$$\times\int_{-\infty}^{\infty}|\dot{z}|\exp\left[\left(\frac{d_2^2}{d_5}-\frac{1}{2\Omega}\right)\dot{z}^2-K_2\right]D_{-2m_1}\left(-\frac{2d_2\dot{z}}{\sqrt{d_5}}\right)d\dot{z} \quad (17)$$

where $d_5 = 4\Omega d_2^2 + \frac{m_1}{\sigma_1^2}\left(\frac{s_2}{s_1}\right)^2$ and $d_6 = \frac{1}{s_2^2\sigma_2^2} + \frac{m_1}{s_1^2\sigma_1^2}$. Then,

$$F=\int_0^{\infty}\frac{w\Gamma\left(m_1,\frac{m_1 w^2}{2s_1^2\sigma_1^2}\right)}{s_2^2\sigma_2^2\,\Gamma(m_1)}\exp\left(\frac{-w^2}{2s_2^2\sigma_2^2}-K_2\right)I_0\left(\frac{\sqrt{2K_2}w}{s_2\sigma_2}\right)dw. \quad (18)$$

## REFERENCES


[1] H. Meyr and G. Ascheid, *Synchronization in Digital Communications*, vol. I. Wiley, 1990.

[2] F. D. Natali, "AFC tracking algorithms," *IEEE Trans. Commun.*, vol. 32, no. 8, pp. 935-947, Aug. 1984.

[3] ——, "Noise performance of a cross-product AFC with decision feedback for DPSK signals," *IEEE Trans. Commun.*, vol. 34, no. 3, pp. 303-307, Mar. 1986.

[4] A. N. D'Andrea and U. Mengali, "Noise performance of two frequency-error detectors derived from maximum likelihood estimation methods," *IEEE Trans. Commun.*, vol. 42, pp. 793-802, Feb./Mar./Apr. 1994.

[5] A. N. D'Andrea, A. Ginesi, and U. Mengali, "Frequency detectors for CPM signals," *IEEE Trans. Commun.*, vol. 43, no. 2/3/4, pp. 1828-1837, Feb./Mar./Apr. 1995.

[6] M. J. Ferguson and P. E. Mantey, "Automatic frequency control via digital filtering," *IEEE Trans. Audio Electroacoust.*, vol. AU-16, no. 3, pp. 392-397, Sep. 1968.

[7] S. Aguirre and S. Hinedi, "Two novel automatic frequency tracking loops," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 25, no. 5, pp. 749-760, Sep. 1989.

[8] E. J. Baghdady, *Lectures on Communication System Theory*. McGraw-Hill, 1961.

[9] G. W. Lank and L. S. Reed, "Average time to loss of lock for an automatic frequency control loop with two fading signals, and a related probability distribution," *IEEE Trans. Inf. Theory*, vol. IT-12, no. 1, pp. 73-75, Jan. 1966.

[10] A. Emad and N. C. Beaulieu, "Performance of an AFC loop in the presence of a single interferer in a fading channel," *IEEE Trans. Commun.*, vol. 58, no. 12, pp. 3386-3391, Oct. 2010.

[11] Y. D. Yao and A. U. H. Sheikh, "Outage probability analysis for micro-cell mobile radio systems with co-channel interferers in Rician/Rayleigh fading environment," *IEE Electron. Lett.*, vol. 26, pp. 864-866, June 1990.

[12] ——, "Investigation into cochannel interference in microcellular mobile radio systems," *IEEE Trans. Veh. Technol.*, vol. 41, pp. 114-123, May 1992.

[13] J. R. Haug and D. R. Ucci, "Outage probability of microcellular radio systems in a Rayleigh/Rician fading environment," in *Proc. IEEE Int. Conf. Commun.*, June 1992, pp. 312.4.1–312.4.5.

[14] J. C. Lin, W. C. Kao, Y. T. Su, and T. H. Lee, "Outage and coverage considerations for microcellular mobile radio systems in a shadowed-Rician/shadowed-Nakagami environment," *IEEE Trans. Veh. Technol.*, vol.48, pp. 66-75, Jan. 1999.

[15] W. C. Jakes, Jr., editor, *Microwave Mobile Communications*. Wiley, 1974.

[16] G. L. Stüber, *Principles of Mobile Communications*, 2nd edition. Kluwer, 2000.

[17] S. O. Rice, "Statistical properties of a sine wave plus noise," *Bell Sys. Tech. J.*, vol. 27, pp. 109-157, Jan. 1948.

[18] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, 9th edition. Dover, 1970.

[19] M. D. Yacoub, J. E. V. Bautista, and L. G. de Rezende Guedes, "On higher order statistics of the Nakagami-*m* distribution," *IEEE Trans. Veh. Technol.*, vol.48, pp. 790-794, May 1999.

[20] S. O. Rice, "Mathematical analysis of random noise," *Bell Sys. Tech. J.*, vol. 24, pp. 46-156, Jan. 1945.

[21] A. Papoulis, *Probability, Random Variables, and Stochastic Processes*, 2nd edition. McGraw-Hill, 1984.

[22] S. O. Rice, "Distribution of the duration of fades in radio transmission," *Bell Sys. Tech. J.*, vol. 37, pp. 581-635, May 1958.

[23] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*, 2nd edition. Wiley, 2005.