Dynamic Energy Resource Control of Power Electronics in Local Area Power Networks

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***Abstract*—In a local area power system, all components of the system, including sources, loads, and distribution have multiple commitments and responsibilities. These commitments include serving the energy needs of a local load, but also maintaining the efficiency and stability of the overall system. Then, the control law of a power converter should consider these objectives, but also needs to anticipate the reaction of other converters within the power network. A differential game-theoretic approach is proposed to derive sliding surfaces that uses a component's objective and operating characteristics to plan an optimal state trajectory during a transient without the need for communication channels or centralized control. The optimal trajectory includes considerations for maintaining local operation of the converter, as well as the stability of the system as a whole. This paper introduces a geometric control surface based on a change of variables that simply and effectively implements a power buffer function in multiple load converters within a power network and microgrids. The formulation and implementation of the optimal surfaces are presented, in addition to experimental validation of the new power buffer control law.**

***Index Terms*—DC power systems, differential game, distributed control, nonlinear systems, power buffers.**

I. Introduction

In a local area electrical power networks, such as a microgrids [1] or electric ships [2], all components of the system, including sources, loads, and distribution have multiple commitments and responsibilities. Often much of the system comprises power electronic converters for both sources and loads. There has been much work in the stability of microgrids with respect to source control [3] and power-electronic-based energy sources [4]. There has also been much work into the destabilizing effects of constant power electronic loads on a microgrid [5]–[7], but there has not been a proposed method of distributed control of a power electronic converter load that leverages local energy for both the load and system objectives. For example, a point of load power converter (POLC) has a commitment to serve the energy needs of the end load. However, if the power system collapses, the needs of the load cannot be met. Therefore, it is also in the interest of the converter to contribute to the global stability of the microgrid by minimizing nonlinear dynamics and incremental negative impedance [3]–[5]. If the terminal characteristics of a converter are constant power, and the system cannot deliver the demanded power, voltage collapse can result. This situation can result when a transient in the power system limits the energy availability because of ground faults, loss of generation, loads coming on-line, etc.

One method to mitigate the destabilizing effects of constant power loads is the power buffer concept [6]–[8]. A power buffer is a device that mitigates a normally destabilizing event by presenting controlled impedance to the supply during the transient, while local energy is used to maintain constant power to the load until the system can recover. A power buffer may include additional hardware, or may merely be a modification of the controls of an existing active front-end power converter. Prior work has shown that power buffers implemented via control can mitigate the destabilizing effect of constant power by effectively extending the critical clearing time of the system, but have relied on complicated hybrid control approach and extensive parameter tuning [9]. An optimal geometric control approach [10] for a power buffer control was proposed in [6], but was based on a single converter system. In multiconverter local power networks, the actions taken by one element of the system can potentially impact the optimal decisions made by all other elements. A game-theoretic analysis and control method for local power networks was proposed in [11], but only considered the solution of a steady-state operating condition.

This paper proposes a differential game-theoretic [12] approach that uses components objectives and operating characteristics to plan an optimal state trajectory during system disturbances without the need for communication or centralized control. The objectives for individual power converters in local area power network are to serve the energy needs of the load, but it is also in their best interest to maintain the stability of the system. This approach also enables the integration of priority and status into the player's objectives, allowing seamless shifts to alternative operation during an emergency or transient condition. For simplicity, the power systems discussed in this paper are dc with direct application to data center and telecom applications [13], [14]. AC systems will have extra dimensions of frequency and reactive power, but the material presented herein can be extended to these cases without the loss of generality.

II. Game-Theoretic Method

Traditional power-system analysis models load as constant power. Since the power is a square of the bus voltage, the solution is a nonlinear function that requires a numeric solution. However, if the loads are modeled and controlled as variable impedance, the voltages can be found as an analytic closed-form

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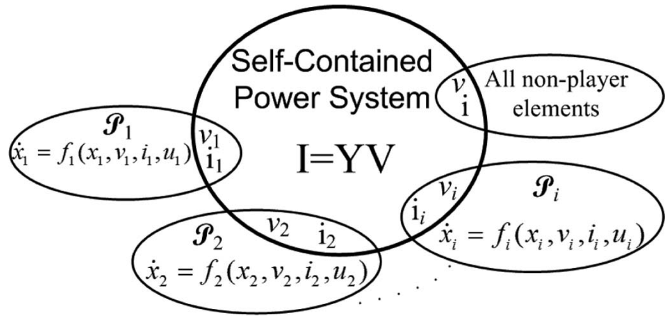


Fig. 1. Generalized game-theoretic method in self-contained power systems.

expression. The input impedance to a POLC can be controlled to supply the prescribed average power to the end load and optimize energy distribution during a transient. If the transients of a power system are modeled as exogenous noise disturbances, then the load control can be formulated as a robust min–max or H^∞ optimal control problem [15], [16]. However, the focus is to develop a paradigm for all energy assets, and then the operation of each will affect the system and alter energy availability for all other loads on the system. Then, an overall system approach is required to develop control laws for each controllable element.

Consider a power system, where $\mathcal{M}$ is the bus set with $\mathbf{m}$ number of buses. Also consider the player set $\mathcal{N}$, such that $\mathcal{N} \subseteq \mathcal{M}$. Then, a player $\mathcal{P}_i$ $(i \in \mathcal{N})$ is a controllable element that effects the movement of energy within the system

$$\mathbf{I} = \mathbf{YV} \tag{1}$$

where $\mathbf{I}$ and $\mathbf{V}$ are the vectors of bus currents and voltages of dimension $\mathbf{m} \times \mathbf{1}$. The symmetric bus admittance matrix $\mathbf{Y}$ is a function of line admittances, and load shunt resistances of dimension $\mathbf{m} \times \mathbf{m}$. A generalized illustration of the game-theoretic method is shown in Fig. 1. It is important to note that each player $\mathcal{P}_i$ $(i \in \mathcal{N})$ is an independently controlled subsystem, which interacts through the $\mathbf{Y}$ bus of the power system. This is then a multidecision maker system, where players can be loads, sources, or network elements.

A source, load, or network player will affect an entry value in (1) and will also have an objective function of the form

$$J_i\left(u_i, u_{-i}\right) = \int f_i(\mathbf{Y}, \mathbf{I}, \mathbf{V}, t, u)dt \quad \forall i \in \mathcal{N} \tag{2}$$

where u_i is the control action taken by player *i*, and u_{-i} are the actions of all other players other than *i*. The objective function *J* can encompass operational parameters such as power, voltage, and current within steady state or transient modes of operation. Then, a Nash equilibrium (NE) [17] exists in the system, if

$$J_i\left(u_i^*, u_{-i}^*\right) \leq J_i\left(u_i, u_{-i}^*\right). \tag{3}$$

The NE described in (3) means that player *i* has optimized its objective, given the control action taken by all other players. It is important to note that all players are independently and actively seeking to optimize their objective and the actions taken by one player can affect the decisions taken by all others. This is the fundamental concept in game theory, in which multiple decision makers in a common system interact to find an equilibrium condition. This approach does not find a system-wide optimal condition, only local Pareto optimality.

Given energy storage in the system and the dynamic nature of the sources and loads, the proposed method is not a static solution, rather a differential game, where each player plans its optimal state trajectory based on the anticipated actions taken by all other players. For example, the constant power characteristic of an end load does not need to translate to the power system if the POLC has some form of local stored energy and is controlled as a power buffer. This energy storage can take the form of a bus capacitor, batteries, spinning inertia, or any other technology. This energy storage satisfies the demand of the end load, while presenting favorable terminal characteristics to the power system, at least for short time periods. Then, the utilization of local stored energy versus the input impedance of the converter is a choice between the stability of the system and the operability of the load.

When two or more POLC's are in a *weak* power system, there can be a shortage of energy, especially during a transient. Then, the strategy implemented by an individual converter not only affects the performance, but also the condition of the entire system. It is straightforward to frame the individual converters as players in a differential game, where energy is the common resource [18]. All power converters have some form of local stored energy that is an asset when the system is no longer able to meet all power requirements.The motivation for a player is to conserve as much stored energy as possible, while also preventing system collapse, and thereby maximizing the critical clearing time of the system. Once local energy has been depleted, and the system fault or event has still not passed, the controller would need to switch to an alternative strategy, namely shutting down or shedding the load.

III. Impedance Model of POLC in Small-Scale Power Systems

In this paper, it is assumed that all players $\mathcal{P}_i$ $(i \in \mathcal{N})$ are loads, such that $\mathcal{L} = \mathcal{N}$. However, other types of players can be included in this approach, such as sources and distribution elements. The bus voltages and current injections [19] are solved from the bus nodal relationship in (1), where $\mathbf{I}$ and $\mathbf{V}$ are the vectors of bus currents and voltages of dimension $\mathbf{m} \times \mathbf{1}$. The symmetric bus admittance matrix $\mathbf{Y}$ is a function of line admittances, and load shunt resistances of dimension $\mathbf{m} \times \mathbf{m}$. The nodal equations in (1) can be partitioned into unknown variables of source current injections (I_s) and bus voltages (V_b), as shown in the following:

$$\begin{bmatrix} I_s \\ 0 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_s \\ V_b \end{bmatrix} \tag{4}$$

where V_s is the source voltage and the bus voltages are $v_i \in V_b$, $i \in \mathcal{M}$. The control variable for a load player is given by $u_i = \dot{r}_i$, where r_i is the shunt resistance to ground at bus $i \in \mathcal{L}$, and the nodal voltages can be found as explicit functions of the control inputs such that

$$v_i = f_i\left(u_i, u_{-i}\right). \tag{5}$$

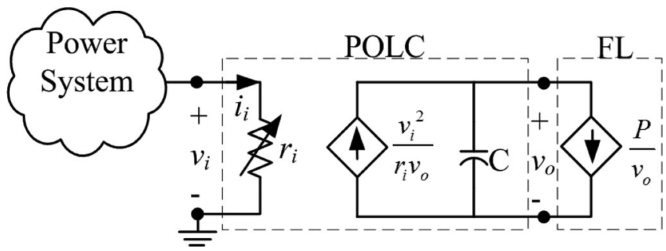


Fig. 2. POLC with local energy storage.

IV. Dynamic Load Modeling

A POLC is typically a switch-mode power supply that takes energy from the system and converts it to a form usable by the end load. On long time scales, the average input power to the POLC must meet the end-load demand, but modern power electronics are capable of regulating the input power on such small time constants that negative impedance can result [20]. However, one positive factor is that the POLC's always have some minimal energy storage as a part of the power electronics. This energy storage can be leveraged to mitigate the small-signal destabilizing negative impedance of the converter as well as large-signal system transients. This paper will address the large-signal problem, but the proposed method is equally beneficial in the small-signal case. There are many types of POLC's, but a useful example is a motor drive system, where the controllable drive stage converts electrical energy from the system to a form needed to obtain desired torque, speed, and power characteristics of the motor, which is the end load.In this type of system, there is usually an energy storage element in the form of a bus capacitor, but the spinning mass of the motor can also be considered as an energy storage. Then, one method to model a general POLC with local energy storage is to view the input as the controlled impedance and the stored energy as a function of the input and output powers. A model of this type of load can be seen in Fig. 2. Additional constraints and limitations exist depending on the topology of a particular POLC, but the fundamental input impedance and energy balance are common in all energy conversion processes. The state equations for a POLC at bus i are derived from an energy balance between the input and output powers as follows:

$$\dot{e_i} = P_i - P_o = \frac{{v_i}^2}{r_i} - P_o \; \forall v_i \in \mathbf{V}, \quad i \in \mathcal{N} \; \dot{r_i} = u_i \tag{6}$$

where e_i is the local stored energy, P_o is the constant-power end load, r_i is the input resistance value, and *u* is the defined control variable. For the load POLC model in Fig. 2, the energy storage is in the form of a output bus capacitor such that

$$e = \frac{1}{2} C {v_o}^2. \tag{7}$$

However, alternative models of POLC's and energy storage can be used including magnetic or kinetic technologies.

It is assumed that there is a nominal, stable, steady-state operating point. It is also desired to have a minimum energy remaining at the end of a system transient. The boundary conditions on this system are then

$$\begin{aligned} e_i(0) &= e_{io} \\ e_i\left(t_f\right) &\geq e_{if} \\ r_i(0) &= r_{io} \end{aligned} \tag{8}$$

where t_f is terminal time that represents the duration length of system event. The initial (pretransient) conditions e_{io} and r_{io} represent nominal values for energy and input impedance, respectively. The inequality constraint value e_{if} is the enforced minimum value of energy at the terminal time. Once the energy constraint has been violated in a real-time system $(e_i\left(t > t_f\right) < e_{if})$, an alternative control strategy must be enacted to gracefully shut down the load.

V. Player Controls

The objectives of the POLC are to conserve energy for as long as possible, given an energy constrained transient, and to minimize the effects of negative impedance. To benefit of the load and maximize sustaining time, each player wants to reduce the usage of stored energy. From the system perspective, constant impedance minimizes the demands for power under transient conditions. Therefore, a function that captures these objectives is

$$\begin{aligned} J_i = \theta_i\left(e_i\left(t_f\right)\right) + \int_0^{t_f} k_{ei}\left(e_i(t) - e_{io}\right)^2 \\ + k_{ri}\left(r_i(t) - r_{io}\right)^2 + k_{ui}\left(u_i\right)^2 dt \end{aligned} \tag{9}$$

where k_{ei}, k_{ri}, and k_{ui}, are weightings of the energy, resistance, and control components objectives, respectively. The terminal penalty

$$\theta_i\left(e_i\left(t_f\right)\right) = \begin{cases} N\left(e_i\left(t_f\right) - e_{if}\right)^2, & \text{if } e_i\left(t_f\right) \leq e_{if} \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

where *N* is a large positive number, enforces the minimum energy constraint. The cost function in (9) describes the objectives of minimizing the use of stored energy, maintaining the input impedance and minimizing the change in input impedance, while enforcing a terminal penalty for violating the minimum energy constraint. The proposed method in this paper is to find an NE based on individual player objectives (9) such that (3) is satisfied. Alternatively, a generalized NE (GNE) [21], [22] concept could be employed if the player controls (u_i) are restricted based on a feasible power balance of the microgrid system Y_{bus} calculations in (4).

An open-loop control approach is used to solve the optimal response and interaction of the players. The minimum principle is used [23], and then the players' Hamiltonians are as follows:

$$\begin{aligned} H_i = \lambda_i^1\left(\frac{(v_i)^2}{r_i} - P_i\right) + \lambda_i^2\left(u_i\right) + k_{ei}\left(e_i(t) - e_{io}\right)^2 \\ + k_{ri}\left(r_i(t) - r_{io}\right)^2 + k_{ui}\left(u_i\right)^2. \end{aligned} \tag{11}$$

The controls' effort is found from

$$\arg\min_{u_i}\left(H_i\right) = -\frac{\lambda_i^2}{2} \tag{12}$$

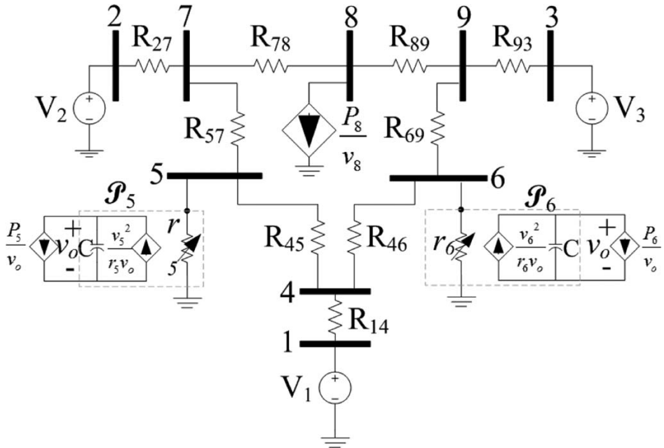


Fig. 3. Nine-bus, two-player example test system.

and the costates are found from

$$\dot{\lambda}_i^1 = -\frac{\partial H_i}{\partial e_i} \tag{13}$$

$$\dot{\lambda}_i^2 = -\frac{\partial H_i}{\partial r_i} \tag{14}$$

with boundary conditions of

$$\lambda_i^1 (t_f) = \frac{\partial}{\partial e_i (t_f)} \theta (e_i (t_f)) \tag{15}$$

$$\lambda_i^2 (t_f) = 0. \tag{16}$$

The solution of (12) will yield the control of the converters modeled in (6) and will give an optimal trajectory in the energy-resistance (e-r) plane of each load during the system transient. This calculated trajectory will extend the critical clearing time of the system and maintain the end load for the maximum duration until local energy is depleted, as defined in (8). To implement this method, the calculated optimal e-r trajectory can be used as a sliding-mode, switching surface in a real-time system.

VI. Illustrative Example

The operation of each element in the power system affects all other elements, whether they are sources (generation), intelligent controllable loads, passive resistive loads, or active constant-power loads. The system shown in Fig. 3 will be used for the duration of this paper to illustrate the principles of the proposed method. For simplicity, this example system is the dc, but the proposed control method can be directly extended to a higher dimension ac system. This system has nine buses ($\mathcal{M} = \{1, 2, 3 \ldots 9\}$), three sources, three-loads, and two-players ($\mathcal{N} = \{5, 6\}$). The load at bus 8 is modeled as a constant power. If the players $\mathcal{P}_5$ and $\mathcal{P}_6$ maintained constant-power characteristic ($\dot{e} = 0$), the entire power system could collapse given a system transient such as a loss in generation or bus fault. Therefore, the loads must react to the transient and each other to minimize their respective reaction functions. This is a nonzero sum differential game because we are considering dynamic state trajectories and the sum of the players' cost functions does not

TABLE I
Experimental Steady-State Parameters

	e_{io}	e_{if}	r_{iss}	$V_{onominal}$	C	P
$\mathcal{P}_5$	16 J	6.4 J	8.25 Ω	100 V	3.2 mF	250 W
$\mathcal{P}_6$	10 J	4 J	8.25 Ω	100 V	2 mF	250 W
Bus_8	-	-	8.25 Ω	-	-	250 W

necessarily be equal to zero

$$\sum_{\mathcal{L}} J_i = J_5 + J_6 \neq 0. \tag{17}$$

This means that the gain of one player is not necessarily the losses of the others. Ultimately, in the power system game, there is a finite amount of energy, and the decisions of the players influence the distribution. If the players act wisely, then the optimal amount of energy is given to the load. However, for bad decisions, the energy could be dissipated in loss through the distribution lines.

To illustrate the operation and method proposed in this paper, the system in Fig. 3 will be used for both numeric theoretical solutions and experimental results. For simplicity, all line parameters are modeled as 1-Ω resistors, and the voltage sources at buses 1, 2, and 3 are 48 Vdc. The load-player parameters are given in Table I. The minimum energy (e_f) for the loads was arbitrarily chosen to be 40% of the starting energy. Since $\mathcal{P}_5$ has a larger capacitor, the stored energy is also larger for the given nominal bus voltage.

A. Solution Methods

To solve this problem,a turn-based boundary value problem (BVP) method [24] was used, in which the control of the players was taken as a vector of discrete time points such that

$$u_i = [u_i^1, u_i^2, \ldots, u_i^k] \tag{18}$$

where *k* is the number of points on the interval 0 to t_f [25]. If the solver requires a control in between one of these discrete points in time, then a first-order hold is used to interpolate the value. The turn-based solution in this differential game means that one player solves for the optimal control *u*, while all other players hold their control vectors constant. This is one current drawback to this method as the solution is found through an iterative process of "turns" that results in an open-loop control that is optimized for a single system event. However, it can be shown that an optimized strategy for one system event is still beneficial for other events, albeit suboptimal.

Fig. 4 shows the convergence of the control vectors in time and states for select rounds in the solution process. In this example, the two players $\mathcal{P}_5$ and $\mathcal{P}_6$ differ in stored energy. The fault time (t_f) for this example was chosen to be 100 ms. It is seen how the players react to the decision made by the opponent. In this example, in the first round, $\mathcal{P}_5$ maintains constant impedance and loses energy very rapidly, and $\mathcal{P}_6$ optimizes the trajectory to conserve energy and minimize the cost function. Likewise, in subsequent rounds, the active player must react to the choice the other player made in the previous round. Convergence occurs in round 12 [see Fig. 4(e) and (f)], when the players no longer change any point along their control vectors (within a small

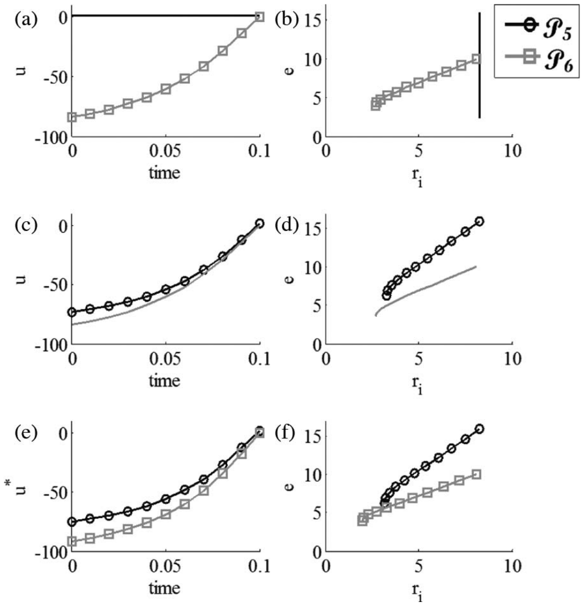


Fig. 4. Differential game solution. Iteration 1: (a) controls (b) states. Iteration 2: (c) controls (d) states. Final solution, Iteration 12: (e) controls (f) states.

numerical error value). This represents the NE of the game because neither player has incentive to change their decisions at any point in time. It is also seen in Fig. 4 that the exact state-plane solutions can be approximated by a piecewise linear relationship of the form

$$r_i^* = \begin{cases} \dfrac{e_i - intercept}{slope} & \text{if } r_i > R_{\min} \\ R_{\min} & \text{otherwise.} \end{cases} \quad (19)$$

This simplified piecewise linear solution for the optimal trajectory is then used as a control surface for the POLC. For example, if the POLC is a switch-mode power converter, then (19) can be used in a hysteretic switching process. The implementation of such a process is outside the scope of this paper.

B. Numeric Solution of Trajectory

The solution of the controller is found numerically from the MATLAB boundary value problem ODE solver command "bvp4c.m". The solution code gives exact optimal trajectories for both load players and determines the approximate piecewise manifold to be implemented in hardware. The optimal state-plane trajectories for $\mathcal{P}_5$ and $\mathcal{P}_6$ are shown in Fig. 5 for a ground fault at bus 2 for $t_f = 100$ ms. Note in Fig. 5(a) that the trajectory for $\mathcal{P}_5$ remains constant resistance. This is because the fault is closest to $\mathcal{P}_6$ and has little affect on $\mathcal{P}_5$. The theoretical optimal switching surface for $\mathcal{P}_5$ is a constant resistance at 8.25 Ω . However, for practical implementation, the surface intersection with the desired steady-state operating point must have a nonvertical slope to settle to that equilibrium point. The piecewise linear surface parameters are then shown in Table II.

As observed in Fig. 6, the trajectory for $\mathcal{P}_5$ has to reduce impedance to conserve energy at a much higher rate than $\mathcal{P}_6$ for a fault at bus 2, which is much closer to $\mathcal{P}_5$ than $\mathcal{P}_6$. The available

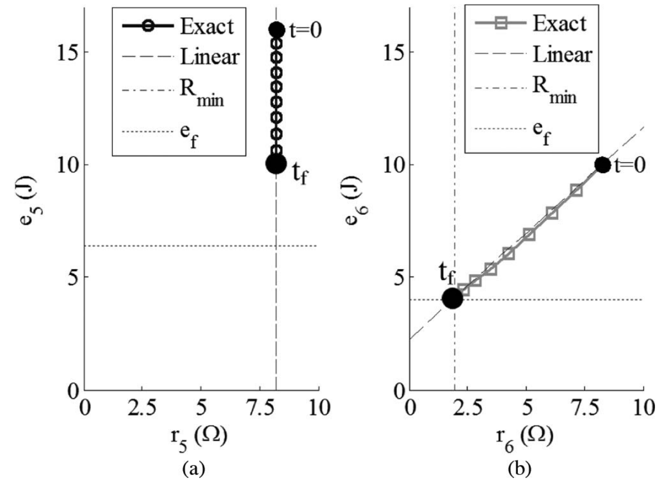


Fig. 5. Theoretic optimal trajectory solution for (a) $\mathcal{P}_5$ and (b) $\mathcal{P}_6$ for example system with fault at bus 3.

TABLE II
CONTROLLER PARAMETERS FOR FAULT AT BUS 3

	Slope (J/Ω)	Intercept (J)	R_{min} (Ω)
$\mathcal{P}_5$	10	-61	7
$\mathcal{P}_6$	0.9	3.65	2

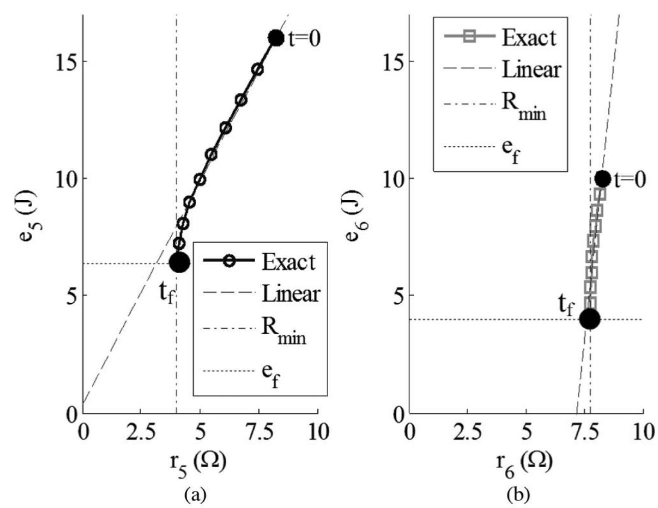


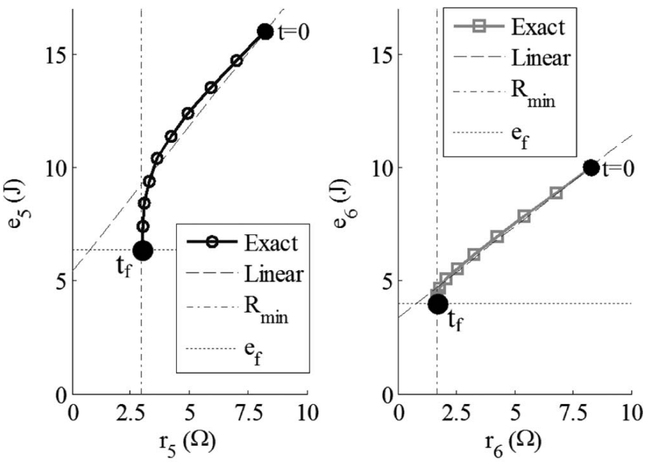
Fig. 6. Theoretic optimal trajectory solution for (a) $\mathcal{P}_5$ and (b) $\mathcal{P}_6$, for example, system with fault at bus 2.

TABLE III
CONTROLLER PARAMETERS FOR FAULT AT BUS 2

	Slope (J/Ω)	Intercept (J)	R_{min} (Ω)
$\mathcal{P}_5$	1.9	1.5	4
$\mathcal{P}_6$	10	-55	7.7

energy to $\mathcal{P}_5$ is restricted more than to $\mathcal{P}_6$. The trajectory for $\mathcal{P}_6$ is almost constant resistance. If $\mathcal{P}_6$ would have chosen constant power (horizontal), that would have compounded the energy restriction seen by $\mathcal{P}_5$ and may have resulted in collapse. The piecewise linear surface parameters are then given in Table III.

Fig. 7 shows the solution to the boundary value problem for the two players (a) $\mathcal{P}_5$ and (b) $\mathcal{P}_6$ for a ground fault at bus 1 for a fault time of $t_f = 100$ ms. Since the fault is at bus 1, the players



(a) (b)

Fig. 7. Theoretic optimal trajectory for (a) $\mathcal{P}_5$ and (b) $\mathcal{P}_6$, for a ground fault at bus 1.

TABLE IV
CONTROLLER PARAMETERS FOR FAULT AT BUS 1

	Slope (J/Ω)	Intercept (J)	R_{min} (Ω)
$\mathcal{P}_5$	1.35	5.8	3.16
$\mathcal{P}_6$	0.95	3.29	2

share the voltage sag effects equally. Notice how both players have to reduce input impedance to conserve stored energy for the duration of the fault. The piecewise approximations are also shown in Fig. 7 with piecewise linear surface parameters given in Table IV.

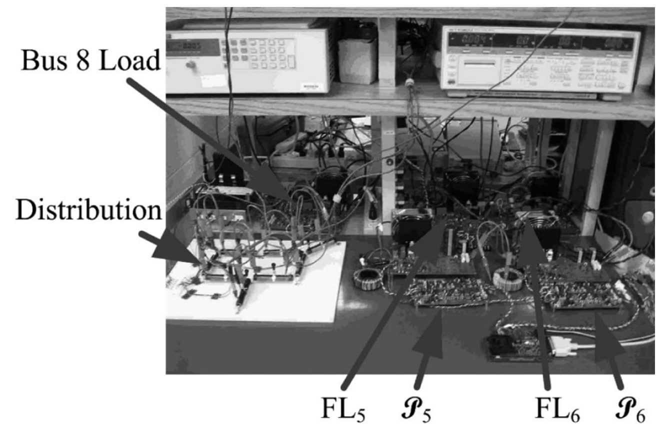


Fig. 8. Experimental hardware apparatus of a nine-bus power-electronics-based power network.

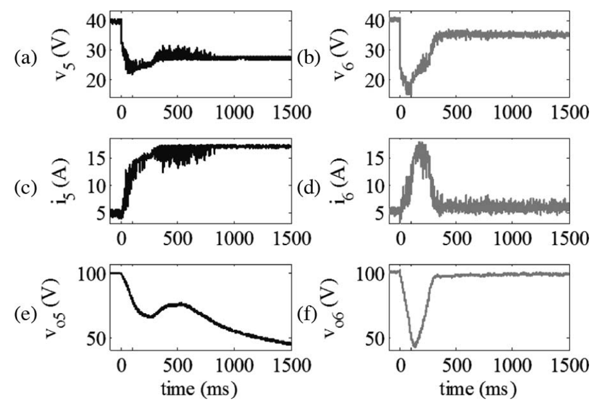


Fig. 9. Experimental converter data for $\mathcal{P}_5$ and $\mathcal{P}_6$, constant power load, with ground fault at 2. (a) Bus 5 voltage. (b) Bus 6 voltage. (c) $\mathcal{P}_5$ load current. (d) $\mathcal{P}_6$ load current. (e) $\mathcal{P}_5$ output voltage. (f) $\mathcal{P}_6$ output voltage.

C. Experimental Results

To verify the dynamic energy game theory, the example system was built in the laboratory, as shown in Fig. 8. This local area power network has 5 switch-mode power converters and 16 channels of data collected, including the voltage at each bus, and the load input currents as well as the POLC output voltage. It is impractical to collect the data from this experiment with an oscilloscope. Therefore, a National Instruments data acquisition (NIDAQ) card and PC were used to collect data at a sample rate of 5 kHz. This acquisition rate does not capture each switching cycle, but is fast enough that the switch boundary is apparent. All data and plots in this section are experimental laboratory data collected from hardware. The data was processed, formatted, and plotted with MATLAB. A nonlinear hysteretic controller was constructed from Texas Instrument DSP processor and analog circuitry to implement the optimal game-theoretic trajectories of $\mathcal{P}_5$ and $\mathcal{P}_6$.The control parameters *slope*, *intercept*, and $R_{\min}$ for both players were found from the theoretical solution to the boundary value problem described in Section IV-A. The POLCs at $\mathcal{P}_5$ and $\mathcal{P}_6$ are boost converter topologies that then feed a constant-power buck converter for the final load. Then, the switch action for the boost converters are governed by the control law

$$q = \begin{cases} 0 & \left(\dfrac{v_i}{i_i} - r_i^*\right) - \dfrac{h}{2} > 0 \\ 1 & \left(\dfrac{v_i}{i_i} - r_i^*\right) + \dfrac{h}{2} < 0 \end{cases} \tag{20}$$

where r_i^* is the calculated resistance reference from (19), v_i and i_i are the measured input voltage and current of the converter, respectively, and *h* is the hysteresis band parameter. The converter at bus 8 is a constant-power buck converter [26].

1) Constant Power POLC Without Optimized Trajectories: For this example, a ground fault at bus 3 was imposed at $t = 0$ s for a duration of 100 ms such that $t_{\text{fault}} = \{t : 0 \leq t < 100 \text{ ms}\}$. The POLC's at buses 5 and 6 were controlled with a traditional pulse width modulation (PWM)voltage control process that in effect produces constant power. This example illustrates the response of the system *without* the game-theoretic trajectory control. The experimental data for $\mathcal{P}_5$ and $\mathcal{P}_6$ are shown in Fig. 9. The time-based data in Fig. 9 are then transformed into impedance and energy, and plotted in state-plane view, as shown in Fig. 10. In Fig. 10, it is seen that $\mathcal{P}_6$ recovers its energy and returns to nominal impedance, but $\mathcal{P}_5$ is caught at the minimum impedance ($R_{\min}$) and is incapable of recovering

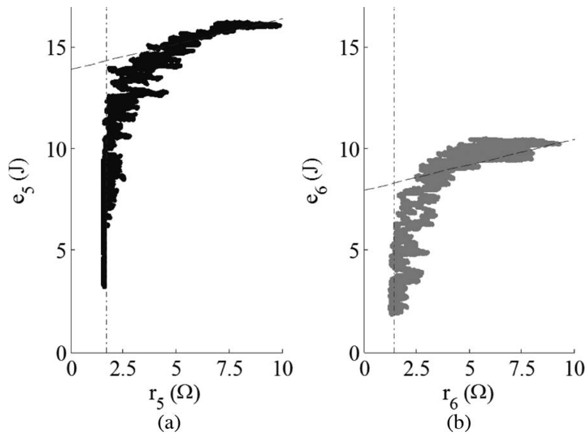


Fig. 10. Experimental state-plane trajectories for players at (a) $\mathcal{P}_5$ and (b) $\mathcal{P}_6$, constant power load, with a ground fault at bus 2.

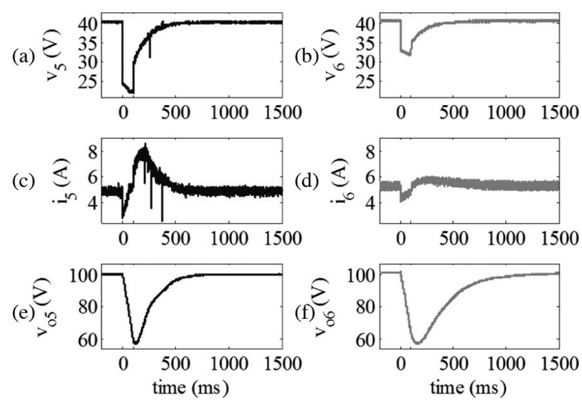


Fig. 11. Experimental converter data for $\mathcal{P}_5$ and $\mathcal{P}_6$, optimized for fault at bus 2, with ground fault at 2. (a) Bus 5 voltage. (b) Bus 6 voltage. (c) $\mathcal{P}_5$ load current. (d) $\mathcal{P}_6$ load current. (e) $\mathcal{P}_5$ output voltage. (f) $\mathcal{P}_6$ output voltage.

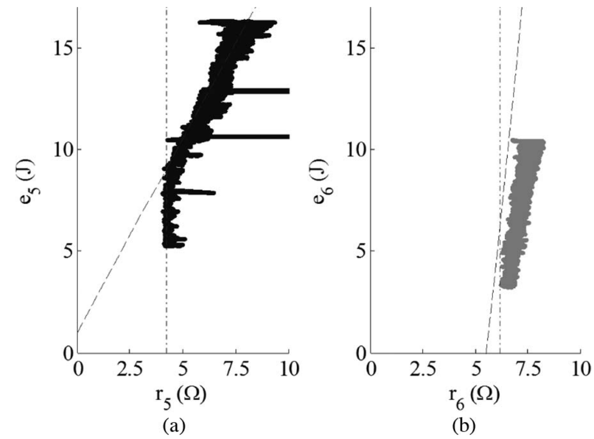


Fig. 12. Experimental state-plane trajectories for players at (a) $\mathcal{P}_5$ and (b) $\mathcal{P}_6$, optimized for fault at bus 2, with a ground fault at bus 2.

TABLE V
POSTFAULT EXPERIMENTAL RESULTS SUMMARY

	v_5	i_5	v_{o5}	v_6	i_6	v_{o6}
Non-optimize (Fig 9)	Low	High	C	R	R	R
Optimized (Fig 11)	R	R	R	R	R	R

C:Collapses R:Recovers

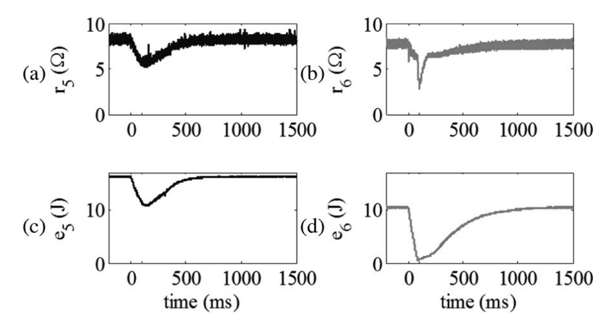


Fig. 13. Transformed experimental data for $\mathcal{P}_5$ and $\mathcal{P}_6$, optimized for fault at bus 2, with ground fault at 3. (a) $\mathcal{P}_5$ input resistance. (b) $\mathcal{P}_6$ input resistance. (c) $\mathcal{P}_5$ stored energy. (d) $\mathcal{P}_6$ stored energy.

its energy. This example shows that constant-power and low-impedance characteristics are not beneficial to the system. It is then important to analyze the system and design the player's control surface to optimize energy usage and balance priority and performance objectives. The following experimental results will show the system *with* the proposed game-theoretic trajectory control method.

2) Optimized State Trajectories: For this experiment, the state trajectory controllers for $\mathcal{P}_5$ and $\mathcal{P}_6$ were used for a 100-ms ground fault at bus 2. The theoretic optimal trajectories are given in Fig. 6 with the piecewise controller parameters given in Table III. The experimental results are shown in Fig. 12. The data in Fig. 12 are then transformed into impedance and energy, and plotted in the state plane shown in Fig. 13. It is seen that the experimental results closely resemble the theoretic results and demonstrates the effectiveness of the proposed method.

3) Comparison of Experimental Results: The results of the experiments for the optimized (see Figs. 11 and 12) and nonoptimized (see Figs. 9 and 10) are summarized in Table V. The results in Fig. 11 show that the proposed control method enables the system to completely recover and to return to nominal operation after the fault at bus 3 clears. However, Fig. 9 shows the same fault in the nonoptimized case causes the $\mathcal{P}_5$ not to recover. While the experiment in Fig. 9 did not collapse the entire system, the system did not recover to the preferred state. These results show that the proposed game-theoretic-distributed controls can mitigate a system fault without the need for centralized control or communications.

4) Nonoptimized Trajectories: Real-time solutions for all possible system events are impractical. Therefore, solutions using the game-theoretic approach in this paper are required to be computed offline and imbedded into the local controller. It is anticipated that a small subset of all optimal solutions would still yield beneficial results, albeit suboptimal. To illustrate this point, another experiment was conducted with the players $\mathcal{P}_5$ and $\mathcal{P}_6$ optimized for a fault at bus 2, but with the ground fault imposed at bus 3 at $t = 0$ s for a duration of 100 ms. The state trajectories for both players are shown in Fig. 13 and projected on the state plane in Fig. 14. In this example, the fault is farther away from $\mathcal{P}_5$, therefore the voltage sag at bus 5 is less than expected. Little energy is lost from $\mathcal{P}_5$. However, because of the constant-impedance nature of $\mathcal{P}_6$, it loses most of its stored

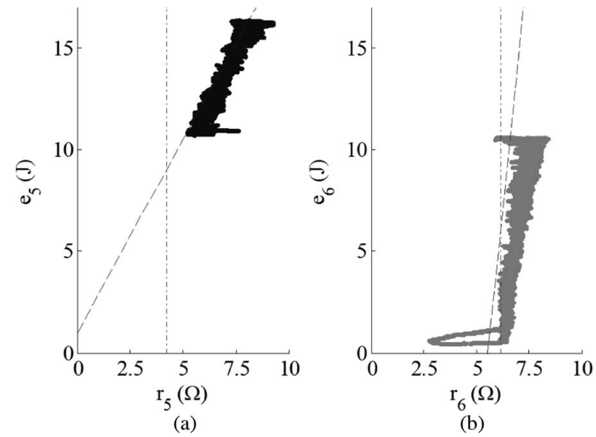


Fig. 14. Experimental state-plane trajectories for players at (a) $\mathcal{P}_5$ and (b) $\mathcal{P}_5$, optimized for fault at bus 2, with a ground fault at bus 3.

energy before the fault is cleared. It is important to note that while the controllers were optimized for a fault at bus 2 and the actual fault was at bus 3, the system still remained stable and was able to recover to its original operating point.

VII. Conclusion

Local area power networks are a collection of many types of energy entities including sources, loads, and networks that may have conflicting interests in the operational parameters of the overall system. These systems can have small stability margins due to the small difference in magnitude of the load demand and the source-generation capacity. This paper presents the application of game theory to the dynamic trajectory control to the operation analysis of system components based on stored energy as a system resource to be utilized. The simple and brief examples given illustrate how local energy can be utilized to alter the system trajectories for the benefit of the load and the system. For an active load, the "players" control strategy needs to factor in the actions of all other components in the power system. This includes other active loads with local energy storage, as in the given examples, but also the reactions of any independently controllable network element.

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