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Calibration and temperature profile of a tungsten filament lamp

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Abstract
The goal of this work proposed for undergraduate students and teachers is the calibration of a tungsten filament lamp from electric measurements that are both simple and precise, allowing to determine the temperature of tungsten filament as a function of the current intensity. This calibration procedure was first applied to a conventional filament lamp (lamp used in automotive lighting) and then tested on a standard tungsten ribbon lamp. The calibration procedure developed was checked by determining the calibration point of the tungsten ribbon lamp with an accuracy of 2%. In addition, for low current intensity, it was observed that the temperature of the filament was not uniform; an explanation is proposed by considering a simple heat transfer model.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Radiation of heated bodies is commonly taught during undergraduate courses, generally to introduce modern physics with a presentation of Planck's law and Wien's displacement law. In addition, during a sensor course about photometric sensors such as CCD detectors or photodiodes, it is common to explain a calibration procedure obtained by recording the signal they issue for a given wavelength in analysing a heated body for which both the temperature and emissivity are known [1]. The body of reference chosen is often a laboratory black body whose main drawback is to be limited to relatively low temperatures (below 2000 K most commonly). Another possibility is the use of a standard lamp (primary or secondary tungsten ribbon lamp) whose price is relatively high, and for which the temperature at a given value of current intensity is known.

In this paper, we present a calibration procedure for a tungsten filament lamp with electrical measurements combining simplicity and precision [2], actually proposed to undergraduate

students, and allowing to investigate the fundamental laws governing the radiation of heated bodies (Stefan–Boltzmann law, etc).

When the current intensity is low, the observation of the filament indicates that its temperature is not uniform; only its central part reaches high temperature: a simple heat transfer model explains this behaviour.

2. The radiative emission of a heated opaque body

The surface of an opaque body at the temperature T is the source of a continuous emission of radiation whose energy distribution against wavelength is given by Planck's law, based on the black body model, which by definition absorbs all the wavelengths λ that it receives [3, 4]. Quantitatively, it is common to consider the spectral luminance L_λ^0 given by

$$L_\lambda^0 = \frac{2hc^2}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{\lambda kT}\right) - 1} \tag{1}$$

with

- h: Planck's constant ($h = 6.6 \times 10^{-34}$ J s)
- k: Boltzmann's constant ($k = 1.38 \times 10^{-23}$ J K^{-1})
- c: speed of light in vacuum ($c = 3 \times 10^8$ m s^{-1}).

L_λ^0 represents the flux emitted per unit of apparent surface, per unit solid angle and per unit wavelength (unit: W m^{-2} sr^{-1} m^{-1}).

When the body is an emissive surface S of a black body, the power emitted in the half-space is given by the Stefan–Boltzmann law:

$$P^0 = S\sigma T^4 \tag{2}$$

where σ is Stefan's constant ($\sigma = 5.67 \times 10^{-8}$ W K^{-4} m^{-2}). When the considered body is not a black body, previous equations must be multiplied by the emissivity $\varepsilon(\lambda, T)$ lower than 1.

3. The thermophysical data of tungsten

Tungsten is the metal having the highest melting temperature ($T_F = 3695$ K). Therefore, it was widely studied since the beginning of 20th century to permit the production of filaments of incandescent lamps [5].

Since the filament of the lamp is essentially a resistance heated by the Joule effect [6–8], the necessary data to calculate the resistance as a function of temperature are the resistivity ρ and the coefficient of thermal expansion β of tungsten. These quantities are tabulated as a function of temperature, and β is given as a percentage of the filament's length ℓ_0 at 300 K [9].

For a temperature T, the resistance R of a tungsten thread is given by $R(T) = \rho(T)\ell(T)/S_f(T)$, where S_f is the section of the wire assumed to be constant over its length. The section, not necessarily circular, can be expressed in terms of a characteristic length d. We have $S = Cd^2$, where C is a proportionality constant depending on the geometry of the section. To fix ideas, $C = \pi/4$ in the case of a circular section by selecting the wire diameter as a characteristic length d.

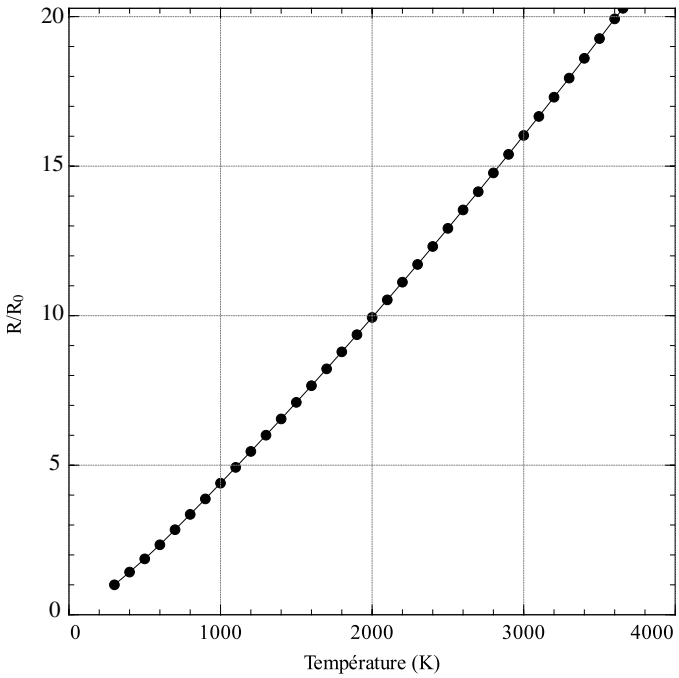

Figure 1. Curve representative of $R(T)/R_0$ as a function of the temperature T for tungsten.

Let us calculate the variation of resistance $R(T)$ with respect to the resistance R_0 at a temperature of 300 K chosen for reference. We have $R_0 = \rho_0\ell_0/S_{f0}$ or, introducing the length d_0 at 300 K, $R_0 = \rho_0\ell_0/Cd_0^2$. For a temperature T, the resistance $R(T)$ is

$$R(T) = \frac{\rho(T)\ell(T)}{Cd^2(T)} \Rightarrow R(T) = \frac{\rho(T)\left[\ell_0 + \frac{\beta\ell_0}{100}\right]}{C\left[d_0 + \frac{\beta d_0}{100}\right]^2}.$$

The ratio $R(T)/R_0$ is then

$$R(T)/R_0 = \frac{100\rho(T)}{[100+\beta]\rho_0}. \tag{3}$$

Relation (3) provides, on the basis of the data available in the literature, the ratio $R(T)/R_0$ as a function of the temperature T. The graph of $R(T)/R_0$ against T is given in figure 1: we note that when there is an increase of temperature of 3000 K, the resistance is multiplied by about 15.

Using a least-squares method, we chose to determine the equation of a parabola passing through the points $(R(T)/R_0, T)$ with a relative error less than 0.1% for high temperatures (see equation (4)).

$$R(T)/R_0 = -0.524 + 0.004\,66T + 2.84 \times 10^{-7}T^2 \tag{4}$$

4. Calibration procedure of a tungsten filament lamp

The measurements were carried out using a Philips lamp Model E4-2DT W21W designed to operate under a nominal voltage of 12 V, for a power of 21 W. The experimental set-up

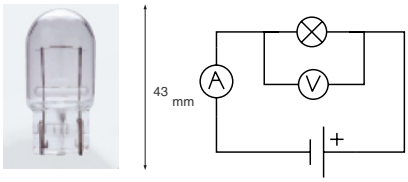

Figure 2. Tungsten filament lamp used (left image) and electrical mounting (right image).

(figure 2) includes a dc power supply Convergie/Fontaine (type ASF1000/20.50) allowing a direct reading of the current intensity I and a digital voltmeter to measure the voltage drop U across the lamp.

By varying the current intensity I from low values (near zero) to about 2.5 A, the measurement of voltage U at the terminals of the lamp allows the resistance R of the lamp for each value of I ($R = U/I$) to be determined. During the experiment, it is necessary to wait until the lamp temperature stabilizes before increasing the pair of values (U, I).

Particular care is needed to find the resistance of the lamp at room temperature R_0 that influences the quality and precision of the procedure calibration. The measurement points (R, I) at very low current have been extrapolated to zero current using a parabolic model: $R(I) = R_0 + AI^2$. Using a least-squares method, the parameter A and the value of the resistance R_0 were found.

Knowing the experimental value of the report $R(T)/R_0$ for each value of the current, we can determine the temperature T of the filament using relation (4) (solution of a quadratic equation easily performed using a spreadsheet). Finally, we obtain a calibration curve in the form of the temperature T of the filament as a function of the current intensity I (figure 3).

From a pure energy viewpoint [10], the heated filament is the place of the conversion of electrical energy into heat and into electromagnetic radiation. Considering that a steady state is reached, there is an equality between the power $P = IU$ dissipated by the Joule effect and the sum of powers transferred by heat between the filament at the temperature T and the 'environment' (temperature T_a), and by radiation (Stefan–Boltzmann law). Denoting by α the coefficient of heat exchange, including the phenomena of heat conduction and heat convection, ε the emissivity of tungsten and S the surface emissivity of the filament, we have

$$UI = \alpha\,(T - T_a) + S\varepsilon\sigma T^4. \tag{5}$$

From the data of U and I, it is straightforward to compute the power UI and then consider the quantity UI/T^4. According to equation (5), we have

$$\frac{UI}{T^4} = \alpha\frac{(T - T_a)}{T^4} + S\varepsilon\sigma. \tag{6}$$

Equation (6) contains two terms: the first term is preponderant at low temperature, and the second term is constant, provided that ε is constant as a function of temperature.

Figure 4 presents the graph of the quantity UI/T^4 as a function of T, which is fully consistent with the forecasts announced earlier and demonstrates the validity of the measurements. For high temperatures, the curve is horizontal and permits the determination of the emitting surface S of the tungsten filament.

The value of the emissivity of tungsten at the wavelength of 0.6 μm averaged on the temperature is equal to 0.44 [6]. From the value of $S\varepsilon\sigma$ shown in figure 4, we can extract the value of the product ℓd. Considering the resistance $R_0 = 0.501\ \Omega$ and the resistivity of

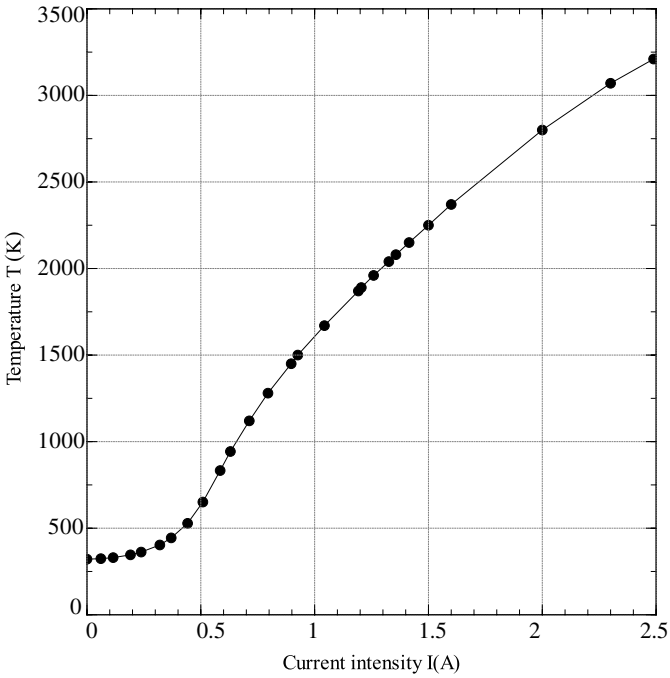

Figure 3. Calibration curve giving the temperature of the filament lamp as a function of the current intensity.

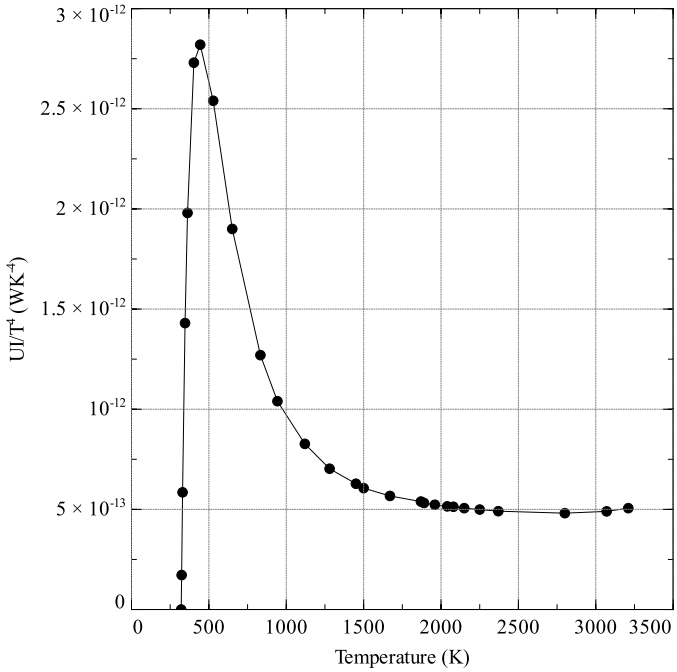

Figure 4. Curve giving UI/T^4 against T.

Figure 5. Photograph of the standard tungsten ribbon lamp.

tungsten at room temperature $\rho_0 = 5.6 \times 10^{-8}\ \Omega\,\mathrm{m}$, we obtain the ratio ℓ/d^2. These data provide access to the dimensions of the filament: $\ell = 2.6$ cm and $d = 6.1 \times 10^{-5}$ m.

5. Procedure applied to a standard lamp

The temperature calibration procedure presented above was applied to a standard tungsten ribbon lamp (lamp OSRAM WI 17G, figure 5) calibrated on a unique value of the current I. The calibration by the manufacturer indicates a temperature of 2689 K for a current intensity equal to 14.1 A.

The curves giving the temperature of the lamp as a function of the current intensity and UI/T^4 against T are shown in figure 6. The value of the temperature determined for a current intensity of 14.1 A (calibration point) is equal to 2760 K, which corresponds to a relative error of 2.6% with respect to the certified calibration temperature. The quantification of the uncertainties (electrical measurements and application of equation (4)) shows that the experimental error is lower than 2%. Given the aging of this lamp, with evaporation of tungsten and its deposition on the internal bulb wall as shown in figure 5, it is clear that its calibration is not up-to-date and helps explain the difference between the temperature measured at 14.1 A and the temperature calibration.

6. Temperature pattern of the filament at a low current intensity

For the tungsten lamp studied in section 4, when the current intensity is less than 1 A, the filament of the bulb does not have a uniform temperature. Red light emission only appears on the central area of the filament, where the temperature is maximum (see figure 7).

To explain this behaviour, it is necessary to study the heat transfer phenomena in the filament.

The filament is put along an axis $z'Oz$ with the origin O at the centre of the filament (figure 8). The thermal conductivity κ of tungsten is taken independent of temperature

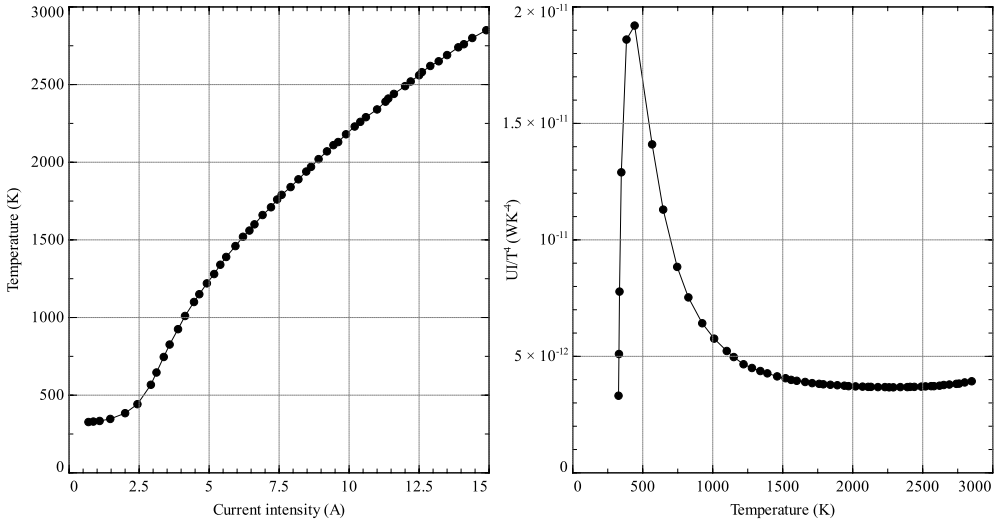

Figure 6. Curves giving the temperature T of the standard tungsten ribbon lamp against the current intensity (left) and UI/T^4 against T (right).

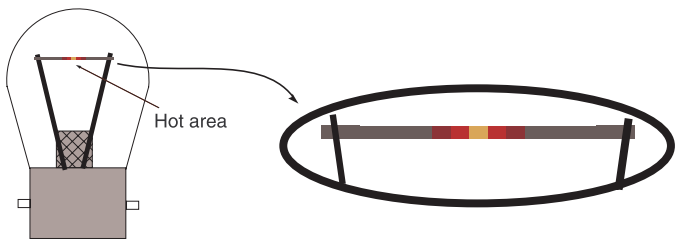

Figure 7. Behaviour of the filament for low values of current intensity.

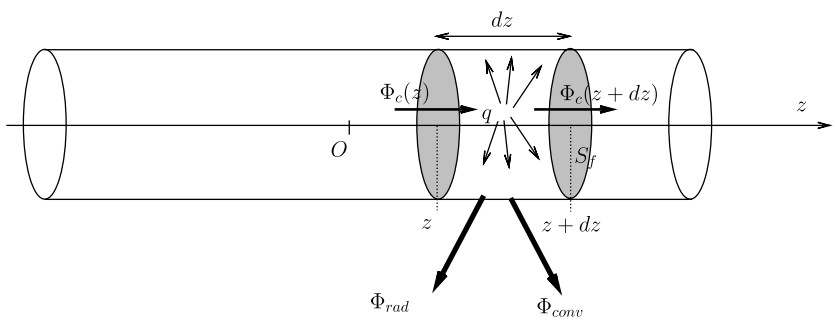

Figure 8. Modelling of heat transfer processes in the filament.

($\kappa = 174$ W m^{-1} K^{-1}). We assume that the temperature distribution in the filament has reached a steady state, and that the temperature is uniform across a diameter. Heat dissipation by the Joule effect in the wire is represented by a heat source term q (in W m^{-3}) given by

$$q = \frac{RI^2}{\ell S_f} \rightarrow q = \frac{16\rho I^2}{\pi^2 d^4}.$$

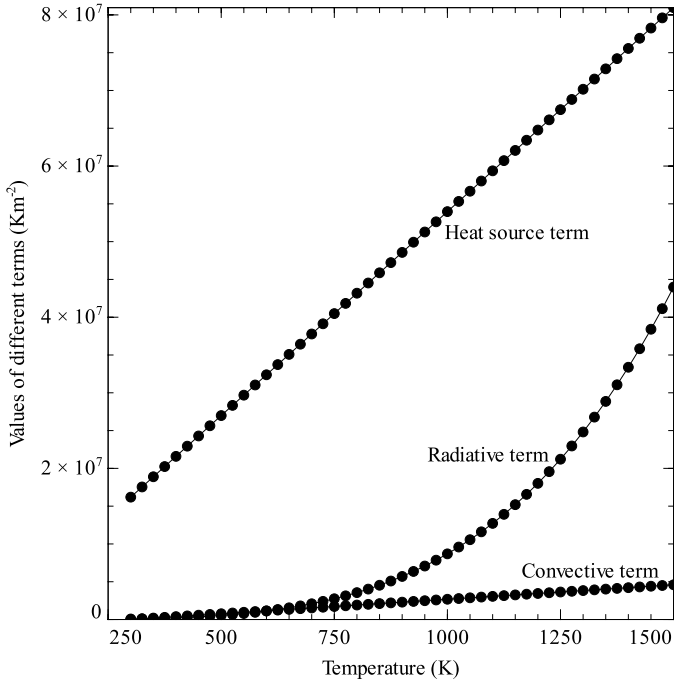

Figure 9. Plot of the different terms of equation (7).

The resistivity ρ as a function of the temperature T is simulated using a linear law $\rho = aT$, with $a = 3 \times 10^{-10}\ \Omega\,\mathrm{m\ K^{-1}}$. The energy balance for the volume of the cylindrical wire located between z and $z + \mathrm{d}z$ is

Conductive flux at z + local heat production

= conductive flux at $z + \mathrm{d}z$ + radiative flux + convective flux

$$\Phi_c(z) + qS_f\mathrm{d}z = \Phi_c(z + \mathrm{d}z) + \Phi_{\mathrm{rad}} + \Phi_{\mathrm{conv.}}$$

The conductive flux is written using Fourier's law [4] as

$$\Phi_c(z) = -\kappa\left(\frac{\mathrm{d}T}{\mathrm{d}z}\right)_z S_f$$

$$\Phi_c(z + \mathrm{d}z) = -\kappa\left(\frac{\mathrm{d}T}{\mathrm{d}z}\right)_{z+\mathrm{d}z} S_f = -\kappa\left[\left(\frac{\mathrm{d}T}{\mathrm{d}z}\right)_z + \left(\frac{\mathrm{d}^2T}{\mathrm{d}z^2}\right)_z \mathrm{d}z\right] S_f.$$

The convective and radiative fluxes are given by the following expressions, where h is a coefficient of natural convection with a value of about 10 W m^{-2} K:

$$\Phi_{\mathrm{rad}} = \epsilon\sigma\pi d\,\mathrm{d}zT^4$$

$$\Phi_{\mathrm{conv}} = h\pi d\,\mathrm{d}z(T - T_a).$$

The equation of the conservation of energy becomes, after grouping and simplifying of different terms,

$$\frac{\mathrm{d}^2T}{\mathrm{d}z^2} + \frac{16I^2a}{\kappa\pi^2d^4}T = \frac{4\epsilon\sigma}{\kappa d}T^4 + \frac{4h}{\kappa d}(T - T_a). \tag{7}$$

Table 1. Emissivity of tungsten [5].

Temperature	Wavelength (μm)									
(K)	0.25	0.3	0.35	0.4	0.5	0.6	0.7	0.8	0.9	1
1600	0.448	0.482	0.478	0.481	0.469	0.455	0.444	0.431	0.413	0.390
1800	0.442	0.748	0.476	0.744	0.465	0.452	0.440	0.425	0.407	0.385
2000	0.436	0.474	0.473	0.747	0.462	0.448	0.436	0.419	0.401	0.381
2200	0.429	0.470	0.470	0.471	0.458	0.445	0.431	0.415	0.896	0.378
2400	0.422	0.465	0.466	0.468	0.455	0.441	0.427	0.408	0.391	0.372
2600	0.418	0.461	0.464	0.464	0.451	0.437	0.423	0.404	0.386	0.369
2800	0.411	0.456	0.461	0.461	0.448	0.434	0.419	0.400	0.383	0.367

For temperatures below 1500 K and for a current intensity of 0.5 A, we plotted in figure 9 the different contributions of equation (7).

It is clear that the contribution of the heat source term at low temperature is largely dominant, and equation (7) can be reduced to

$$\frac{\mathrm{d}^2 T}{\mathrm{d}z^2} + \frac{16 I^2 a}{\kappa \pi^2 d^4} T = 0 \tag{8}$$

whose solution is

$$T(z) = A \cos \sqrt{\frac{16 I^2 a}{\kappa \pi^2 d^4}} z + B \sin \sqrt{\frac{16 I^2 a}{\kappa \pi^2 d^4}} z,$$

where A and B are the integration constants. Given the symmetry of the problem ($T(z)$ is an even function of z), we retain only the cosine solution, with $A = T_{\max}$, maximum temperature obtained in $z = 0$. The inverse of the quantity $\sqrt{\frac{16 I^2 a}{\kappa \pi^2 d^4}}$ represents the typical length over which the temperature of the filament varies significantly. Numerically, the typical length is 4.3 mm, in good agreement with the experimental observations.

7. Conclusion

The procedure for the calibration of a tungsten filament lamp presented in this paper is relatively simple to set up. The validity of the procedure has been shown through the evaluation of UI/T^4 as a function of the temperature, whose behaviour is entirely consistent with theory.

Finally, to be able to calibrate a photometric sensor using the radiation emitted by a tungsten filament of known surface temperature, it is necessary to consider relation (1) giving the spectral radiance of the black body multiplied by the emissivity of tungsten according to the temperature T and the wavelength λ (see table 1).

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