DISCRETE OPTIMIZATION AND AGENT-BASED SIMULATION FOR REGIONAL EVACUATION NETWORK DESIGN PROBLEM

A Dissertation

by

XINGHUA WANG

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Approved by:

Co-Chairs of Committee,	Halit Üster
	Justin Yates
Committee Members,	Wilbert E. Wilhelm
	Donald Friesen
Head of Department,	Cesar O. Malave

December 2012

Major Subject: Industrial Engineering

ABSTRACT

Natural disasters and extreme events are often characterized by their violence and unpredictability, resulting in consequences that in severe cases result in devastating physical and ecological damage as well as countless fatalities. In August 2005, Hurricane Katrina hit the Southern coast of the United States wielding serious weather and storm surges. The brunt of Katrina's force was felt in Louisiana, where the hurricane has been estimated to total more than $108 billion in damage and over 1,800 casualties. Hurricane Rita followed Katrina in September 2005 and further contributed $12 billion in damage and 7 fatalities to the coastal communities of Louisiana and Texas. Prior to making landfall, residents of New Orleans received a voluntary, and then a mandatory, evacuation order in an attempt to encourage people to move themselves out of Hurricane Katrina's predicted destructive path. Consistent with current practice in nearly all states, this evacuation order did not include or convey any information to individuals regarding route selection, shelter availability and assignment, or evacuation timing. This practice leaves the general population free to determine their own routes, destinations and evacuation times independently. Such freedom often results in inefficient and chaotic utilization of the roadways within an evacuation region, quickly creating bottlenecks along evacuation routes that can slow individual egress and lead to significant and potentially dangerous exposure of the evacuees to the impending storm.

One way to assist the over-burdened and over-exposed population during extreme event evacuation is to provide an evacuation strategy that gives specific information on individual route selection, evacuation timing and shelter destination assignment derived from effective, strategic pre-planning. For this purpose, we present a mixed integer linear program to devise effective and controlled evacuation networks

to be utilized during extreme event egress. To solve our proposed model, we develop a solution methodology based on Benders Decomposition and test its performance through an experimental design using the Central Texas region as our case study area. We show that our solution methods are efficient for large-scale instances of realistic size and that our methods surpass the size and computational limitations currently imposed by more traditional approaches such as branch-and-cut. To further test our model under conditions of uncertain individual choice/behavior, we create an agent-based simulation capable of modeling varying levels of evacuee compliance to the suggested optimal routes and varying degrees of communication between evacuees and between evacuees and the evacuation authority.

By providing evacuees with information on when to evacuate, where to evacuate and how to get to their prescribed destination, we are able to observe significant cost and time increases for our case study evacuation scenarios while reducing the potential exposure of evacuees to the hurricane through more efficient network usage. We provide discussion on scenario performance and show the trade-offs and benefits of alternative batch-time evacuation strategies using global and individual effectiveness measures. Through these experiments and the developed methodology, we are able to further motivate the need for a more coordinated and informative approach to extreme event evacuation.

To *my parents and my fiancé*

ACKNOWLEDGMENTS

I would never have been able to finish my dissertation without the guidance of my committee members, help from friends, and support from my family.

I owe my deepest appreciation to my advisors, Profs. Halit Uster and Justin Yates for their excellent supervision, constant encouragement and persistent help in my research program. It is a great fortune for me to have the opportunity to work with Dr. Uster and Dr. Yates and I really enjoy the wonderful time in their groups.

I am deeply grateful to my committee members, Prof. Wilbert E. Wilhelm, and Prof. Donald Friesen, for their guidance and support throughout the course of this research.

I would like to express my gratitude to my friends and colleagues for they generously share their knowledge and experience, and constantly provide valuable ideas and comments.

Finally, I would like to thank my parents, Chunmei Yang and Tao Wang, my grandmother, Fuqin Wang, my aunt, Chunying Yang, my uncle, Weiming Li, and my fiancé, Chunhua Deng, for their understanding and support throughout the past years.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION

Evacuation from impending extreme events is a complex and integrated operation that necessitates planning and cooperation from government at the local, state and federal levels as well as cooperation between government and non-government organizations to ensure the safety of an exposed population. An effective and well-planed evacuation can reduce damages and fatalities in an extreme event. However, devastating damages and severe fatalities were still incurred in natural disasters in recent years due to the inefficient evacuation process.

On August 29th of 2005, Hurricane Katrina, the deadliest and costliest hurricane in the United States, hit the southern coast of the United States with devastating effects. More than 1,800 people lost their lives and more than $108 billion in damages was incurred (Knabb et al., 2005). In September 2005, Hurricane Rita made landfall between Sabine Pass, Texas, and Johnson Bayou, Louisiana, as a category 3 hurricane, and caused unprecedented damages to numerous Louisiana and Texas communities. More than $12 billion in damages was incurred and seven people died (Knabb et al., 2006). In September 2008, Hurricane Ike, the second-costliest hurricane in the United States and the costliest hurricane in Texas history, made landfall near Galveston, Texas. It caused extensive damage along Louisiana and southeastern Texas coasts, and more than $29 billion in damages was incurred (Berg, 2009). Table 1 presents the 30 costliest mainland United States tropical cyclones from 1900 to 2010, and Table 2 states the 30 deadliest years from 1851-2010 and the costliest years from 1900 to 2010 due to tropical cyclones (Blake et al., 2011) (all dollar values for damage cost mentioned in this chapter are not adjusted for inflation). These well-publicized events brought to light many challenges faced during the expedited evacuation of a largely-populated region. Miscommunications between public and private transportation contractors, uncertainties related to the storm's strength and point of impact and an under-prepared infrastructure all contributed to the storms' effects. Most alarming, however, was the inefficient and chaotic utilization of the region's roadways, which quickly created bottlenecks in traffic that led to significant, serious and dangerous impediments to the population's evacuation. On September 22 of 2005, two days before Hurricane Rita making landfall at Texas coast, 2.5 million people tried to leave Houston and caused 100-mile-long traffic congestions (Blumenthal, 2005).

Though there are many research articles and models that focus on natural disaster evacuation, decidedly few are integrated into evacuation practice and the determination of evacuation policy. It remains the tradition of local and state law enforcement to issue a mandatory evacuation order, which is not accompanied by any additional information and leaves the general population (referred to as self-evacuees) the freedom to determine their actual evacuation time, route of egress and final destination. In the Safety and Preparedness Fact Sheet, the information provided by National Oceanic and Atmospheric Administration, people are encouraged to learn locations of official shelters and determine safe evacuation routes by themselves before the hurricane season (NOAA, 2012). On August 27 of 2005, two days before Hurricane Katrina making landfall, a voluntary evacuation was ordered by Ray Nagin, New Orleans Mayor, at 5:00 pm. Then a mandatory evacuation, the first such order in the city's history, was ordered in the next day at 9:30am (Hauser and Lueck, 2005). People in New Orleans received the evacuation order but without additional information about which specific routes to use and which specific shelters to go. In ascribing such freedom to the self-evacuee, evacuation planners have forfeited their ability to foresee the possible strains and hindrances resulting from an

Table 1 The Costliest 30 Tropical Cyclones in Mainland United States During 1900-2010

Rank	Tropical Cyclone	Year	Category	Damage (U.S.)
1	KATRINA (SE FL, LA, MS)	2005	3	$108,000,000,000
2	IKE (TX, LA)	2008	2	$29,520,000,000
3	ANDREW (SE FL/LA)	1992	5	$26,500,000,000
4	WILMA (S FL)	2005	3	$21,007,000,000
5	IVAN (AL/NW FL)	2004	3	$18,820,000,000
6	CHARLEY (SW FL)	2004	4	$15,113,000,000
7	RITA (SW LA, N TX)	2005	3	$12,037,000,000
8	FRANCES (FL)	2004	2	$9,507,000,000
9	ALLISON (N TX)	2001	TS^a	$9,000,000,000
10	JEANNE (FL)	2004	3	$7,660,000,000
11	HUGO (SC)	1989	4	$7,000,000,000
12	FLOYD (Mid-Atlantic & NE U.S.)	1999	2	$6,900,000,000
13	ISABEL (Mid-Atlantic)	2003	2	$5,370,000,000
14	OPAL (NW FL/AL)	1995	3	$5,142,000,000
15	GUSTAV (LA)	2008	2	$4,618,000,000
16	FRAN (NC)	1996	3	$4,160,000,000
17	GEORGES (FL Keys, MS, AL)	1998	2	$2,765,000,000
18	DENNIS (NW FL)	2005	3	$2,545,000,000
19	FREDERIC (AL/MS)	1979	3	$2,300,000,000
20	AGNES (FL/NE U.S.)	1972	1	$2,100,000,000
21	ALICIA (N TX)	1983	3	$2,000,000,000
22	BOB (NC, NE U.S)	1991	2	$1,500,000,000
22	JUAN (LA)	1985	1	$1,500,000,000
24	CAMILLE (MS/SE LA/VA)	1969	5	$1,420,700,000
25	BETSY (SE FL/SE LA)	1965	3	$1,420,500,000
26	ELENA (MS/AL/NW FL)	1985	3	$1,250,000,000
27	DOLLY (S TX)	2008	1	$1,050,000,000
28	CELIA (S TX)	1970	3	$930,000,000
29	LILI (SC LA)	2002	1	$925,000,000
30	GLORIA (Eastern U.S.)	1985	3	$900,000,000

Note. a : "TS" represents Tropical Storm.

Table 2 The Deadliest 30 Years During 1851-2010 and the Costliest 30 Years During 1900-2010

Ranked On Deaths			Ranked On Unadjusted Damage		
Rank	Year	Deaths	Rank	Year	($ Millions)
1	1900	8,000	1	2005	143,979
2	1893	3,000	2	2004	51,135
3	1928	2,500	3	2008	35,908
4	2005	1,225	4	1992	26,500
5	1881	700	5	2001	9,310
6	1915	550	6	1989	7,670
7	1957	426	7	1999	7,572
8	1935	414	8	1995	5,921
9	1926	408	9	2003	5,600
10	1909	406	10	1996	4,816
11	1906	298	11	1998	4,285
12	1919	287	12	1985	4,000
13	1969	256	13	1979	3,045
14	1938	256	14	1972	2,100
15	1955	218	15	1983	2,000
16	1954	193	16	2002	1,551
17	1972	122	17	1991	1,500
18	1916	107	18	1965	1,445
19	1965	75	19	1969	1,421
20	1960	65	20	1955	985
21	1944	64	21	1994	973
22	1933	63	22	1970	931
23	1999	62	23	1954	756
24	2004	60	24	1964	515
25	1989	56	25	2006	500
26	1966	54	26	1975	490
27	1947	53	27	1961	414
28	1940	51	28	1960	396
29	1964	49	29	1938	306
30	1961	46	30	1980	300

over-capacitated and undirected road network. The result is an immobile population faced with an impending storm and no means of escape to safety.

One way to alleviate the over-burdened and over-exposed population is to provide an evacuation strategy that is more directed while maintaining enough simplicity to be attractive to self-evacuees. Instead of allowing the population to self-direct, the offering of predetermined evacuation routes and sheltering locations would enable policy makers to evaluate the impact of a large scale evacuation on the region and assess its ability to support such movement. An organized evacuation plan with the designated evacuation routes and shelters can also reduce the clearance time to evacuate all population of the affected areas by alleviate traffic congestions, which are caused by undirected self-evacuees. It may also reduce the costs incurred in evacuation process and alleviate individuals' suffering by shorten their traveling times. Thus, preparing a pre-event evacuation plan with the pre-determined evacuation routes and sheltering locations can make evacuation process effective and reduce damages and fatalities. For this purpose, we provide a pre-event strategic evacuation approach with the pre-determined evacuation routes and sheltering locations in this dissertation, while considering the roads capacities to avoid traffic congestions as well as the clearance time to send all evacuees to shelters before disasters happen.

Due to the inherent characteristics of evacuation problem, another challenge, which may make the evacuation process inefficient, is that evacuees may not follow the pre-determined evacuation plan and select their own routes and shelters. In this dissertation, we consider this uncertainty in our multi-agent simulation model, and we test the effectiveness of our pre-determined evacuation plan under this uncertainty.

I.1. Scope of the Dissertation

In this dissertation, we study a regional evacuation network design problem to provide an effective pre-event strategic planning tool. For this purpose, we develop a deterministic optimization model to identify an effective underlying evacuation network, and we design a solution approach for solving large-scale evacuation instances. We also create an agent-based simulation model to test the robustness and applicability of the results of the optimization model under behavioral uncertainty.

I.1.1. Strategic Evacuation Network Design Model

We propose a mixed integer linear program (MIP) called the Strategic Evacuation Network Design (SEND) to devise effective and controlled evacuation networks for sending evacuees from their origins to shelters before extreme events such as hurricanes. The SEND model expands upon the more traditional capacitated multi-commodity network design model as described in section III.1 and provides an evacuation network that directs evacuation zones through the road network to shelter locations while satisfying road network capacity, shelter capacity and evacuation time constraints with an overall objective of cost minimization. Also, we consider the case in which roads and shelters have extra capacities through extra constructions which incur extra costs. It is important to recognize that, in parallel with our objectives in evacuation network design, road capacity is considered at a high level rather than with fine granularity as in a dynamic traffic assignment study. In conjunction with this, we consider a constant (average) traffic speed in modeling and, thus, the traverse time of each road is constant.

Another motivation of our research comes from the methodology itself. Optimization models for regional evacuation problems usually involve large-scale optimization which is difficult to solve. Thus, some of these models were solved by heuristics (Kim et al., 2008; Lu et al., 2005), and some of them were only tested on small networks (Kaufman et al., 1998) (only 4 nodes, 8 physical links, and 4 destinations). However, we develop an effective solution methodology based on Benders decomposition (BD), which can solve large-scale instances (i.e. 47 source nodes, 22 destinations, 512 arcs, 128 nodes, and 1 million variables and 3 million constraints) to optimality in a reasonable time. This solution methodology shows an outstanding performance comparing with the branch-and-cut algorithm. Our approach incorporates several performance enhancements such as surrogate constraints, strengthened Benders cut generation and use of multi-cuts while employing efficient heuristics within the exact BD framework.

We design and implement an experimental design to test our BD technique using a Texas-based evacuation scenario. We show that the SEND model and BD approach can be efficiently and effectively applied to a large-scale evacuation scenario and discuss computational performance as compared to traditional branch-and-cut solution methods.

I.1.2. Multi-agent Simulation Model

As a centralized optimization model, the SEND model is useful under known conditions and perfect information, however, it is not able to account for uncertainty during the evacuation (e.g., individuals who choose their favorable routes but not follow optimal routes or who choose an alternative shelter location). In these cases, evacuees may elongate their travel time or travel distances by selecting routes that look favorable to themselves instead of routes that are a part of the optimal evacuation strategy, causing traffic jams and deteriorating system performance during the evacuation. It is also possible that evacuees may fail to follow directions received as a part of the SEND optimal solution due to difficulties in communication and coordination arising from the chaos and confusion of the emergency situation.

To consider these situations which cannot be handled by a centralized optimization model, we construct a multi-agent simulation (MAS) model and tie our insights and analysis from the MAS to the SEND optimal solution. In the decentralized MAS model, unlike the discrete and centralized SEND problem, every evacuee can make decisions and change decisions during the evacuation. In this way, the MAS model simulates a real-world emergency evacuation situation where evacuees have the freedom to choose their own routes and their own destinations. As part of the MAS model, we examine how varying degrees of compliance and adherence to the optimal SEND strategy impacts system performance through an evacuation. We also prove the effectiveness of the pre-event strategic evacuation plan proposed by the SEND model.

An additional benefit of using the MAS is its added fidelity over the SEND problem. In the latter, each edge is considered to have a finite capacity and the evacuation is managed at a macroscopic level (as opposed to the finer granularity achieved through dynamic traffic assignment studies). The finite capacity assumption is accompanied with the assumption of constant travel speeds and constant travel times for each edge of the network. Unfortunately, none of these assumptions accurately reflect the dynamically changing evacuation environment where speeds and travel times are not constant. The MAS model enables us to better control these variables and account for their impact in assessing evacuation solutions. In the MAS model, we model traffic speed on an edge as a function of the edge's traffic density with edge traffic density being updated dynamically. Moreover, the MAS enables us to model the situation in which evacuees leave in groups at a time sequence. A value is assigned to the range of leaving times for each group. An evacuee may leave at any time in the range of leaving times for his group.

Throughout the evacuation process, it is important to capture and model the interconnectedness of individual evacuees. Various forms of media such as radio and cellular telephones enable evacuees to receive, interpret and act upon near real-time information from other evacuees' observations or from local, state and national governmental agencies. Depending on the method of information sharing and the type of information shared, evacuee behavior may be altered significantly. As an example, individuals may choose an alternative route to avoid a roadway with a major traffic jam based on information they receive from a family member or friend who is currently driving in slow traffic. Evacuees may also choose a different shelter location based on information on shelter status received from the state government through local radio. We use the MAS as a mechanism to study the effects resulting from these shared information modes.

I.2. Contributions

In this dissertation, we provide an effective pre-event strategic planning approach to make evacuation process efficient and successful. Our contributions are summarized as follows:

- By the very nature of the emergency planning, evacuation time is considered as necessitated. However, the approach to incorporating evacuation time to models is a concern in the current studies on evacuation problems. In our study, we track each evacuation route and add a time constraint on it to make sure every evacuee can arrive at a designated shelter within a designated time.
- Though there are many research articles focusing on regional evacuation problems, most of them employed simulation approaches, but not optimization

approaches, as introduced in Chapter II. We develop a mixed integer program to determine the optimal evacuation network based on time and capacity constraints (i.e. capacities of roads and shelters). Our model selects shelters from all potential candidates, chooses evacuation routes, decides flow assignments and minimizes costs.

- Optimization models for regional evacuation problems usually involve large-scale optimization which is difficult to solve. Unlike the articles solving models by heuristics (Kim et al., 2008; Lu et al., 2005) or solving models on small-scale instances (Kaufman et al., 1998), we develop a solution methodology based on the BD approach, which takes advantage of specific characteristics of the SEND problem. This solution methodology can solve large-scale evacuation instances to optimality within a reasonable solution time.

- Unlike many articles testing models on artificial data (e.g. Andreas and Smith, 2009), we design and implement an experiment to test our BD technique using a Texas-based evacuation scenario with real population data and real spatial data. Through this experiment, we show that the SEND model and the BD approach can be efficiently and effectively applied to a large scale evacuation scenario. We also discuss its computational performance as compared to the traditional branch-and-cut solution method which is implemented by CPLEX 12.2.

- For regional evacuation problems, some articles employed simulation approaches at a microscopic level, and some other articles employed optimization approaches at a macroscopic level. However, few are integrated into both of these two levels. In this dissertation, besides developing the SEND model to provide an optimal evacuation plan at a macroscopic level, we construct a MAS model at a microscopic level to simulate the evacuation process under uncertainty, which cannot be handled by a centralized optimization model. We examine how varying degrees of compliance and adherence to the optimal SEND strategy impacts system performance through an evacuation, and prove the effectiveness and robustness of the pre-event strategic evacuation plan proposed by the SEND model. Additionally, time component is added to the MAS model by considering the traffic speed on a road as a non-linear dynamical function of real-time traffic density on this road.

- The MAS model is not only effective to test the pre-event strategic evacuation plan proposed by our SEND model, but also able to test other evacuation strategies on other evacuation networks. It is a tool to evaluate evacuation strategies for decision makers.

I.3. Organization of the Dissertation

The remainder of this dissertation is organized as follows. Chapter II provides a comprehensive review for evacuation problems and discusses relevant literature to better frame the contribution of this study. In Chapter III, we describe the SEND problem and present its assumptions and formulation. We develop a solution approach based on BD by taking advantage of specific characteristics of the SEND problem. We design and implement an experimental design to test our BD technique using a Texas-based evacuation scenario, and we show the efficiency and effectiveness of the SEND model and the BD approach. In Chapter IV, we conduct the MAS model to consider unexpected cases which cannot be handled by the SEND model and consider traffic speed as a variant of real-time traffic density. We design and implement experiments to study the effects of several factors on system performance through an evacuation. Also, we prove the effectiveness of the pre-event strategic evacuation plan proposed by the SEND model. In Chapter V we summarize concluding remarks, potential impacts of this research and possible future research.

CHAPTER II

LITERATURE REVIEW

In recent years, extensive effort has been invested in studying evacuation problems, which are complex integrated problems. In this chapter, we present a comprehensive review of evacuation problems, and then we introduce the works relevant to our study.

II.1. Overview of Emergency Evacuation Problems

Studies on evacuation problems involve various fields such as human behaviors, traffic control strategies, network design, decision making and so on. Based on different study scopes and study objects, models are conducted in two levels. One is the macroscopic evacuation model, which considers the total evacuation time but not the individual behavior (Andreas and Smith, 2009; Chen and Xiao, 2008; Chien and Korikanthimath, 2007; Elmitiny et al., 2007; Kaufman et al., 1998; Kim et al., 2008; Liu et al., 2007; Lu et al., 2005; Mamada et al., 2004; Noh et al., 2009), and the other one is the microscopic evacuation model, which models the individual behavior and the interaction between each evacuee that may influence evacuees' movement (Chen, 2008; Chen et al., 2006; Lamel et al., 2010; Olsson and Regan, 2001). According to different physical evacuation areas, two distinct evacuation problems are studied: building evacuation, e.g., large retailer stores, stadiums, ships, aircraft, etc. (Andreas and Smith, 2009; Hamacher and Tjandra, 2001; Mamada et al., 2004; Olsson and Regan, 2001) and regional evacuation, e.g., nuclear power plants failures, wildfire, floods, hurricanes, etc. (Chen, 2008; Chen et al., 2006; Chen and Xiao, 2008; Chien and Korikanthimath, 2007; Elmitiny et al., 2007; Kaufman et al., 1998; Kim et al., 2008; Lamel et al., 2010; Liu et al., 2007; Lu et al., 2005; Noh et al., 2009; Wei et al., 2008). Building evacuation mainly considers pedestrian evacuation; however, regional evacuation usually focuses on traffic-based evacuation. Moreover, different methodologies are applied to evacuation problems. Some articles simulate evacuation to observe either global (Chien and Korikanthimath, 2007; Elmitiny et al., 2007; Liu et al., 2007; Noh et al., 2009) or individual behavior (Chen, 2008; Chen et al., 2006; Lamel et al., 2010; Olsson and Regan, 2001), and some studies optimize the mathematical model to obtain optimal results (Andreas and Smith, 2009; Chen and Xiao, 2008; Kaufman et al., 1998; Kim et al., 2008; Lu et al., 2005; Mamada et al., 2004). Furthermore, some studies compare the different impacts of simultaneous and staged evacuation (Chen, 2008; Chien and Korikanthimath, 2007). Some articles address the improvement of traffic conditions to reduce evacuation time. For example, Kim et al. (2008) propose the concept of contraflow to increase capacity of routes along the direction of evacuation, and Chen et al. (2007) focus on traffic light timing for traffic flow in urban area. Besides the above studies, some new fields are also explored. Chiu and Mirchandani (2008) propose a behavior-robust feedback information routing (FIR) strategy to further improve system performance.

In Table 3, we illustrate the basic categories of evacuation problems as introduced above. From Table 3, we see that extensive effort has been invested in studies that employ simulation models. Some of the studies that employ optimization models focus on building evacuation. Because regional evacuation optimization problems usually involve large-scale optimization, which is difficult to solve, some studies employ heuristics (Kim et al., 2008; Lu et al., 2005). Kaufman et al. (1998) developed an optimization model for the regional evacuation problem and solved it to optimality; however, their model was only tested on small networks (i.e. only 4 nodes, 8 physical links and 4 destinations). Chen and Xiao (2008) studied on how to control traffic flow to maximize traffic system utilization and tested their approach on several roads

Table 3 Overview for Evacuation Problems

	Scope		Evacuation Region		Methodologies			
					Simulation		Optimization	
	Macro	Micro	Building	Regional	MAS	Non MAS	Exact	Heuristic
Olsson and Regan (2001)		x	x		x			
Andreas and Smith (2009)	x		x				x	
Mamada et al. (2004)	x		x				x	
Chien and Korikanthi-math (2007)	x			x		x		
Elmitiny et al. (2007)	x			x		x		
Liu et al. (2007)	x			x		x		
Noh et al. (2009)	x			x		x		
Chen (2008)		x		x	x			
Chen et al. (2006)		x		x	x			
Lamel et al. (2010)		x		x	x			
Kim et al. (2008)	x			x				x
Lu et al. (2005)	x			x				x
Kaufman et al. (1998)	x			x			x	
Chen and Xiao (2008)	x			x			x	
SEND	x			x			x	
MAS		x		x	x			
SEND-MAS	x	x		x	x		x	

intersections, but not the evacuation network design for the entire regional evacuation. Our research contributes by solving a large-scale regional evacuation network design optimization problem.

II.1.1. Building Evacuation

Hamacher and Tjandra (2001) summarized models and algorithms applied to building evacuation problems. They concluded that the travel time was regarded as the main parameter in all the reviewed papers, with the partial travel time between different nodes being the input and the overall evacuation time being the output. In their research, the authors introduced macroscopic evacuation models that took into account the total evacuation time but no individual behavior. The authors also summarized microscopic evacuation models that simulate the individual behavior and the interaction between each evacuee that may influence evacuees' movement. Macroscopic evacuation models they introduced are mainly based on discrete time dynamic network flow models, which include minimum cost dynamic flow, maximum dynamic flow, universal maximum flow, quickest path and quickest flow. The dynamic network model was described with density dependent travel time, which does not necessarily have to be a constant. Except the continuous time dynamic flow models, multi-criteria optimization problems are also discussed in their paper. Furthermore, Olsson and Regan (2001) compared the calculated theoretical evacuation times and the actual recorded evacuation times, and analyzed human behavior in the evacuation process. Their work emphasized the importance to consider human behavior in evacuation model, and this is also captured by our study. Our MAS model considers the case in which evacuees have the freedom to choose their own favorable routes and shelters.

II.1.2. Regional Evacuation

Besides building evacuation problems, many articles focus on regional evacuation problems, including the evacuation from natural disaster (e.g. wildfire, floods, hurricanes, etc.) (Chen et al., 2006; Lamel et al., 2010; Chen, 2008; LIU et al., 2007; Wolshon and McArdle, 2011; Dow and Cutter, 2002; Simonovic and Ahmad, 2005; Noh et al., 2009) and human-caused disaster (e.g. nuclear power plants failures, terrorism, war etc.) (Wei et al., 2008). Unlike building evacuation that is mainly related with pedestrian evacuation, regional evacuation is usually represented by traffic-based evacuation, and it, as inherent in its name, requires the evacuee to travel long distances, from citywide to statewide. There are other differences between building evacuation problems and regional evacuation problems, due to their inherent characters. In building evacuation problem, people are evacuated to exits, and the number of exits for one building is relatively small. However, in regional evacuation problem, people are evacuated to shelters, and the number of shelters for one region is usually larger than the number of exits for one building. Also, the number of available routes in regional evacuation problems is usually larger than that in building evacuation problems. As a consequence, the size of network in regional evacuation problems is usually larger than that in building evacuation problems, and this can cause the model difficult to solve.

Traffic congestion is a severe problem in evacuation problem, and looking for approaches to alleviate this problem is a challenge in a lot of studies. During a regional evacuation process, many people try to evacuate in a short time frame, so that there are much less freeways available than what's adequate for a smooth evacuation. For example, there are over 2.5 million people as south as Florida and as north as Virginia getting evacuated when Hurricane Floyd approached (Dow and

Cutter, 2002). Also, the evacuation occurred in lieu of Hurricane Katrina in 2005 forced an evacuation involving half an million vehicles and one million people out of New Orleans area. Road capacity of freeway will be too limited compared with what's needed in this case. Apivatanagul et al. (2012) proposed a bi-level optimization model to alleviate traffic congestion. In their model, unlike most of the other models, not all population, who may be affected by disaster, is evacuated to shelters, but their model decided who would stay and who would leave with the purpose to alleviate traffic congestion. The decision of choosing evacuees to leave was made to minimize both of risk and travel time. After the evacuees who would leave were selected, evacuation routes were assigned to these evacuees. Their model alleviated traffic congestion during evacuation, but this alleviation only works without the consideration of human behavior, which may affect the performance of their evacuation plan significant. The evacuees who were not selected to leave may resist to leave, and this may cause traffic congestion and elongate travel time. Human behavior is a very important factor which should be considered in evacuation problem. In our study, we consider human behavior in the MAS model and use it to test the performance of the evacuation plan which is proposed by the SEND model. The results of the tests prove the effectiveness of our evacuation plan. To alleviate traffic congestion, Wolshon and McArdle (2011) proposed that, as an alternative choice, evacuating through secondary and low volume roadways should be integrated with the optimum usage of the main freeways. This study provided an approach to help alleviating traffic congestion in evacuation, but a pre-event evacuation plan with designated routes and designated shelters is still need to guide evacuees and avoid traffic congestion. In our study, SEND model provides an effective evacuation plan which provides guidelines to evacuees.

II.1.3. Simultaneous and Staged Evacuation

Chen (2008) studied the hurricane evacuation of Galveston Island by using agent-based micro-simulation techniques. He compared the time required to evacuate to safe areas of two strategies: simultaneous evacuation and staged evacuation. The most efficient staged evacuation strategy can reduce the evacuation time significantly compared with simultaneous evacuation. In another study, Chien and Korikanthimath (2007) constructed a mathematical model to estimate the evacuation time and delay, and to investigate the relationship between these two quantities. Their article also compared the different influences of simultaneous evacuation and staged evacuation on the evacuation time, and proposed a numerical method to determine the optimal number of evacuation stages. The numerical example shows that the staged evacuation strategy can reduce the evacuation time and delay significantly. The researchers also did the sensitivity analysis of parameters (e.g. demand density, access flow rate and evacuation route length) to the evacuation time and delay. Chien and Korikanthimath (2007) studied a regional evacuation problem by employing a simulation model, and compared the different impacts of simultaneous and staged evacuation. In our study, we consider both of two strategies: simultaneous evacuation and staged evacuation. In SEND model, we consider the case for simultaneous evacuation, and in MAS model we consider the case for staged evacuation. We also test the effect of different number of evacuation stages to total evacuation time, transportation cost, individual evacuation time, and traffic conditions.

II.1.4. Evacuation Decision Making

Evacuation decision making is a complicated process and can therefore be composed of several phases. Those phases, integrally, form an evacuation decision tree. LIU et al. (2007) studied a new aspect in evacuation decision making problem, i.e. grey situation decision. Grey situation decision has been utilized in many other fields, such as site selection of waste sanitary landfill and bidding for equipment purchase. The research also constructed a grey decision model in a framework for multiple periods of a flood disaster. This model evaluated the optimal decision (evacuate or not evacuate) to minimize the total expected cost and the extent of fatalities by considering the potential flood damage, rate of fatalities and evacuation effect index. The model and solution strategy were tested by the data of the river floods in the Netherlands in 1995. Evacuation decision making is also considered in our MAS model, evacuees make their evacuation decisions based on their guidelines, the real time traffic conditions, the information they received, and their personal preference.

II.1.5. Feedback Information Routing Strategy

Current strategies on evacuation traffic management paid most of their attention to increasing network along the evacuation route such as contraflow lanes. However, there are some other routing strategies which are not totally exploited. Chiu and Mirchandani (2008) presented the optimal routing strategies to evacuees who would choose their evacuation routes following a certain rule, and addressed the approach to evaluating the effectiveness of these routing strategies. The article proposed a behavior-robust feedback information routing (FIR) strategy to further improve system performance. The FIR strategy is developed on closed-loop control so that it can respond to the current state of the evacuees and update the guidelines accordingly. It has shown to be very effective and efficient in real-time evacuation traffic management application. In current phase, the guidelines for evacuees are constant but not updated dynamically, and the FIR strategy could be a considered as a future study in our research.

II.1.6. Human Behavior in Evacuation Process

Besides the articles which focus on the development of theoretical and mathematical models, especially the network flow models, there are some other articles investigating another important aspect in evacuation problem, e.g., human behavior. These articles fill the gap between the traditional theoretical evacuation models and the observed behavior. For example, when disaster happens, it is observed that household members seek each other, and then evacuate as a single unit. Obviously, these actions may lead a longer evacuation time than the one that planners have expected. Murray and Mahmassani (2002) addressed this observation in evacuation problems, and modeled this phenomenon in a two-phase model by using two integer programs. The first model is to select a meeting place for all family members. Its objective function is to minimize the maximum distance from the meeting place to each family member's location. The resulting meeting place is used for the second model as a known condition. The second model is to decide which driver is going to pick up which family members and also decide the sequence of pick up. Actually, the second model is a variant of vehicle routing problem, which has already been explored extensively. Simonovic and Ahmad (2005) constructed a simulation model to determine human behavior before and during the flood evacuation. It simulated the acceptance of evacuation orders by evacuees, the number of families to evacuate and the clearance time to evacuate all people to safe areas. This article assessed the effectiveness of different emergency management procedures, with each of which containing the warning method, warning consistency, timing of evacuation order, coherence of community, upstream flooding conditions and different weights for different warning distributions. The experiments were implemented based on the flood evacuation in Red River Basin, Canada. Another aspect of human behavior in evacuation process which has received growing interest over the last several decades is competitive egress behavior. Kirchner et al. (2003) addressed the effect of competitive behavior in emergency evacuation problem. In their model, they introduced a friction parameter μ to distinguish between competitive and cooperative movement. They claimed that competition may increase walking speed of pedestrian in evacuation process. If the door width is larger than the critical door width, competition will decrease the egress time, otherwise it will increase egress time. The authors also used a very interesting experiment to show that the motivation level is very important for the egress time in a narrow aircraft, and then they reproduced this experiment by simulating the evacuation from a room. They also compare the simulation results and the experimental results. These results can provide us some hints for planning evacuation in case of hurricane. It means that moderate competition can increase the speed of evacuees, and the level of competition can actually be controlled by the government. Baker (1991) analyzed the factors affecting the willingness of residents to evacuate, including the risk level of the area, action by public authorities, housing, and so on. Because human behavior is so important in evacuation problems, we consider this factor in our MAS model by giving evacuees freedom to choose their own evacuation routes and shelters. We analyze the effect of evacuees' choices on total evacuation time, individual traveling time, transportation cost, and traffic conditions.

II.2. Traffic Simulation and Dynamic Network

Chen et al. (2006) model and analyze the procedure for hurricane evacuation in the Florida Keys. They built an agent-based micro-simulation model to find the minimum clearance time to evacuate all people in that area. Their paper constructed the decentralized model as agent-based and adopted a real-world instance (i.e. population in Florida Keys in 2000 U.S. Census) to implement the experiments. In our study, we also develop an agent-based simulation model in microscopic level to study effects on total evacuation time, individual traveling time, transportation cost, and traffic conditions. In our MAS model, evacuees are given guidelines but they have freedom to choose their own routes and shelters. Lamel et al. (2010) adapted an existing multi-agent transportation simulation framework to large-scale pedestrian evacuation simulation. A simple queueing model, which considers bottleneck capacities and space constraints, was simulated, and captured the most important aspects of evacuation, such as congestion effects of bottlenecks and clearance time to evacuate to safe areas. This model also has a time-dependent component to reflect changes in the network. The simulation was demonstrated through a case study for Padang, Indonesia. Elmitiny et al. (2007) used the VISSIM traffic simulation model to evaluate a current plan and alternative plans during an emergency situation in a transit facility such as a bus depot. The benefit of traffic rerouting was also investigated. Liu et al. (2007) presented a model reference adaptive control (MRAC) framework for real time traffic management under an emergency evacuation. It controlled traffic flow dynamically to maximize the utilization of the transportation system and minimize fatalities due to traffic accidents and jams. The proposed framework was based on both dynamic network modeling techniques and adaptive control theory. This article also used simulation studies to show that the proposed framework based on MRAC can improve the evacuation performance significantly (measured as the clearance time and the number of victim vehicles). Noh et al. (2009) also used a dynamic transportation simulation model for the evacuation problem, and their model was applied to a case study for flood evacuation in Phoenix, Arizona. Chen and Xiao (2008) proposed an approach for real-time traffic management under emergency evacuation. This approach is different from the predetermined evacuation plans, and it controlled traffic flow dynamically by considering the traffic network as a dynamic system. This approach was used to obtain the minimum evacuation time, and authors showed the effectiveness of the approach in a numerical example. Mamada et al. (2004) developed dynamic network flow models for the building evacuation problem. They introduced the single-sink, two-sink, and k-sink case models, and showed that, if the number of sinks is bounded by some constant, solution time is polynomial. In our MAS model, we also simulate a dynamic transportation system, and we consider traffic speed is a variant as the traffic density on the road. Traffic speed and traverse time of a road is changed dynamically with traffic density.

Some articles addressed the improvement of traffic conditions in order to reduce the evacuation time. Some proposed the concept of contraflow to increase the capacity of routes along the direction of evacuation (e.g. Kim et al., 2008), especially for traffic flow on freeways, and some focus on traffic light timing for traffic flow in urban area. Chen et al. (2007) constructed a simulation model to investigate the influence of traffic light timing on evacuation in urban area, and to study the trade-off between evacuation time and average delay when assessing proposed timing plans.

II.3. Applications of Network Design Problems in Evacuation Problems

Some articles modeled evacuation problems based on networks and solved these problems as network design problems. Andreas and Smith (2009) studied a building evacuation problem based on a staged capacitated tree network, and minimized the expected evacuation penalty over all scenarios. Mamada et al. (2004) also studied a building evacuation network design problem on a tree network. Moreover, Chalmet et al. (1982) constructed three network models for building evacuation problems. The first model is a dynamic model, and the time period is discrete. The other two models are graphical and intermediate models. Besides the studies which used network design problems to study pedestrian evacuation, Kaufman et al. (1998) studied the problem with vehicular traffic. They developed a mixed integer linear program to provide route guidelines to traffic so that travel time can be minimized. However, they only tested their model in a small network (only 4 nodes, 8 physical links, and 4 destinations) and solved it by a basic branch-and-bound algorithm. Due to the high computational cost of traditional time-expanded networks using linear programming approach, Lu et al. (2005) presented a heuristic algorithm, Capacity Constrained Route Planner (CCRP), to produce sub-optimal solution for the evacuation planning problem. In our study, we propose a regional evacuation network design problem and develop a mixed integer linear program to devise effective and controlled evacuation networks for sending evacuees from their origins to shelters before extreme events. We develop an efficient solution methodology to solve large-scale instances to a small optimality gap within a reasonable time.

II.4. Applications of GIS in Evacuation Problems

The evacuation problem always involves the spatial components, so the combination of a geographical information system (GIS) and optimization methodology is desirable. Saadatseresht et al. (2008) proposed a three-step method for evacuation planning. In the first step, safe areas are selected, based on some specific conditions by referencing the maps, satellite images and so on. The second step selects the candidate safe areas, and finds the optimal path between each building block and each candidate safe area using GIS software tools. The third step chooses the optimal safe area for each building block from its candidate safe areas which are selected in the second step. The authors used a two-objective function. Finally, a case study was conducted in a GIS environment, and the results were tested. They used GIS to preprocess before solving the problem (i.e. in the second step), and also obtained results in a GIS environment for visualization to further understand and test their evacuation plan (i.e. in third step). Cova and Church (1997) proposed an approach to identify the communities, which have difficulties in evacuating transportation. They developed an integer programming model and solved this model by a heuristic approach in a GIS context. They conducted a case study on communities in Santa Barbara, California. In our study, we use GIS data to generate our network for computational studies to test the effectiveness and efficiency of the SEND model and our solution methodology. The MAS model is also developed based on this network. Moreover, we use ArcGIS to preprocess our spatial data and visualize the evacuation plan proposed by SEND model.

Based on this literature review, extensive effort has been involved in employing simulation approaches to study the evacuation problem. However, the large-scale regional evacuation network design problems, which are studied by optimization approaches, need to be explored more extensively. Our SEND model makes a contribution in this field. We also consider human behavior and dynamic traffic speed-density model in our MAS model. Also, we use ArcGIS to get a better understanding of our networks and results. Moreover, unlike the other studies, which only employ simulation models or optimization models, we integrate these two parts together to get an overall outcome.

CHAPTER III

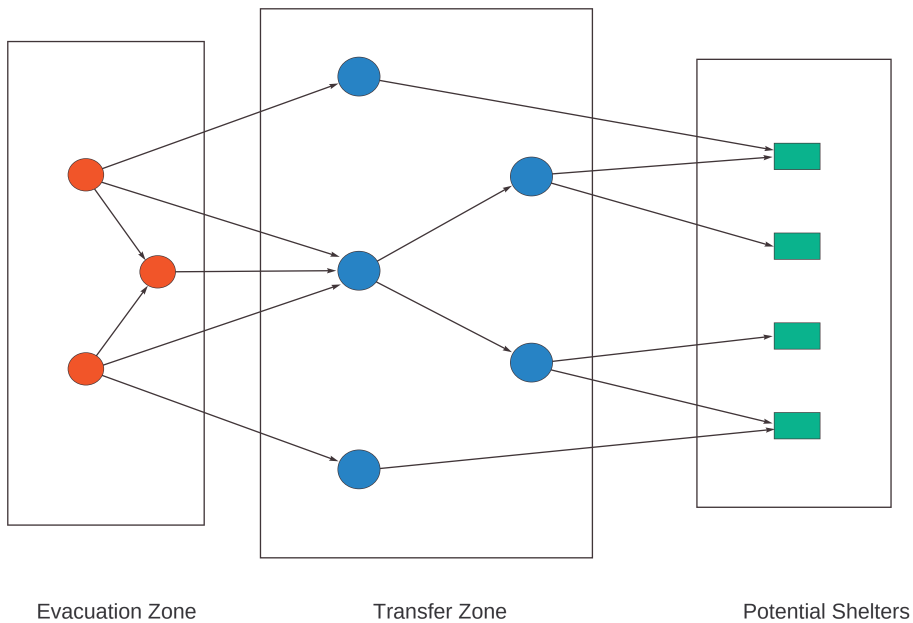
STRATEGIC EVACUATION NETWORK DESIGN PROBLEM

We pose and analyze a regional evacuation network design problem in order to provide a pre-event strategic planning approach. We propose a mixed integer linear program to devise effective and controlled evacuation networks for sending evacuees from their origins to shelters before extreme events such as hurricanes. In this chapter, Section III.1 describes the problem definition and assumptions of SEND problem. Section III.2 provides notations and the formulation of SEND model. Section III.3 describes a solution approach based on BD. Experimental design and computational results are provided in Section III.4. Section III.5 gives a summary for this chapter.

III.1. Problem Definition

In this study, we consider a regional emergency evacuation of a large geographical area, for example, a metropolitan area. On an underlying road network, we define the area to be evacuated (the risk area under threat) in a discretized fashion where the associate set of nodes represents source nodes whose populations are the required outflow. Another set of nodes in this network represents potential shelters which are essentially regions including a set of potential shelters, i.e., a shelter region. A final set of nodes represents potential transfer points (e.g., towns, truck-stops, highway intersections, etc.) which are visited (passed through) by evacuees on their routes from sources to shelters. The problem is illustrated in Figure 1. Origin and destinations nodes are also potential transfer nodes since evacuees could travel through one or several risk and safe areas on their route to destinations. We further assume the following as design characteristics:

Figure 1 Network Illustration for SEND Problem



- The population from one origin can go to several destination nodes, and one destination node can accept flows from several origin nodes. However, between a pair of origin-destination nodes, the population can use at most one path (i.e., the evacuees with the same origin and destination travel along the same route).

- In addition to transportation costs associated with flow, we assume that (undirected) edges, potential shelters and transfer nodes in the underlying network have associated fixed costs. In particular, shelter costs include expenditures to prepare accommodation, safety/security, medical and food supplies; transfer node and edge costs are mainly associated with general maintenance, infrastructure development, readiness to serve evacuees as well as safety/security during an evacuation.

- Furthermore, because each shelter has a space limitation and it should be staffed and provide a certain set of basic supplies such as food, medicine, and other basic necessities of life, and a safe and secure environment for evacuees, we consider an original capacity for each shelter in terms of the number of evacuees it can handle. However, besides the original capacity, each shelter can obtain extra capacity by constructing more facilities and providing more necessaries. As a consequence, the extra capacities of shelters incur extra fixed costs. There is an upper bound for extra capacity that each shelter can obtain.

- Also, we assume that each edge (road segment) in the network has a finite original capacity on the total flow that it can handle in an evacuation event. In each road segment, extra capacity can be obtained by adding new lanes (e.g. employing highway roadside, re-designing a wide four-lane road segment to a five-lane road segment, or paving temporary road segments). Adding new lanes incur extra fixed costs of each edge. It is important to recognize that, in parallel with our objectives in evacuation network design, this capacity is considered at a high level rather than with fine granularity as in a dynamic traffic assignment study. In conjunction with this, we consider a constant (average) traffic speed in the SEND model and, thus, the traverse time of each edge is constant.

- Moreover, to restrict each individual's evacuation time, each road segment (arc) has a specific traversal time, and the sum of the traversal times of the arcs on a path is the traversal time of the path. Traversal time of each evacuation path should be less than or equal to the established evacuation time to guarantee the safety of evacuees and to avoid excessive on-the-road travel times.

- We assume that each edge (undirected) are associated with two arcs (directed), which share the capacity of the corresponding edge. Using one arc on an edge will include the fixed cost of the corresponding edge to the total costs. Also, using both arcs on an edge will only include the fixed cost of the corresponding edge to the total costs once.

Although such a problem is similar to the capacitated multi-commodity network design problem (CMCND) in optimization literature, it has striking differences and associated challenges as we explore in this study. In the CMCND, given a set of commodities defined by unique origin-destination node pairs and flow demands, arcs are selected (from an underlying network) to construct a network on which the commodities are routed without violating arc capacity constraints while minimizing the sum of (variable) flow and (fixed) arc selection costs. CMCND are often applied to telecommunications and transportation networks (Gendron et al., 1998). We use CMCND as a solid foundation to build a more comprehensive model in support of extreme event evacuation, where the commodities in CMCND (loosely) represent a self-evacuee or group of evacuees.

There are mainly four distinct differences between the SEND and the CMCND problems. First, in the CMCND problem, the destination location for each commodity (an origin-destination pair) are known a-priori, but this is not the case in the SEND problem. In the SEND problem, destination (shelter) locations are chosen by the model from a candidate set, and the model opens enough shelters and implicitly determines origin-destination pairs to ensure evacuees sheltering under capacity constraints while minimizing the total costs. Second, in the SEND problem, the selection of transfer nodes, in addition to arcs, is also a part of network design and has an associated fixed cost implication. Third, in the SEND problem, each individual should reach a destination safely within the established evacuation time. The flows considered in the CMCND problem are usually goods, so longer routes for a small part of goods may be allowed. That means for getting a better benefit for the whole group, a small part of this group can make sacrifices. However, in the SEND problem, the flow is an evacuee, and a longer evacuation route means a more dangerous situation. That means no sacrifice is allowed in the SEND problem. Thus, the SEND problem has to monitor the evacuation time for each individual but not only the average or total evacuation time. Fourth, in the SEND problem, shelters' capacities and edges' capacities can be increased within specified ranges, and obtaining the extra capacities can incur extra fixed costs.

III.2. Formulation

We first define the notation employed in our formulation.

Sets

$\mathcal{O}$	Set of origin nodes, $o \in \mathcal{O}$
$\mathcal{D}$	Set of potential destination nodes, $d \in \mathcal{D}$
$\mathcal{I}$	Set of all nodes (equivalently, set of potential transfer nodes), $i, j \in \mathcal{I}$
$\mathcal{A}$	Set of directed arcs $(i, j) \in \mathcal{A}$
$\mathcal{E}$	Set of undirected edges $\{i, j\} \in \mathcal{E}$
$\mathcal{P}_i$	Set of nodes precede node i, $j \in \mathcal{P}_i$ and $(j, i) \in \mathcal{A}$
$\mathcal{S}_i$	Set of nodes succeed node i, $j \in \mathcal{S}_i$ and $(i, j) \in \mathcal{A}$

Parameters

s_o	Population in region $o \in \mathcal{O}$
$q_d^{\mathcal{D}}$	Capacity of destination node $d \in \mathcal{D}$
$q_{ij}^{\mathcal{E}}$	Capacity of undirected edge $\{i, j\} \in \mathcal{E}$
$f_d^{\mathcal{D}}$	Fixed cost for opening a shelter at node $d \in \mathcal{D}$ with original capacity
λ	Upper bound of the increase of each shelter capacity

ξ Magnitude of shelter fixed cost increase per unit extra shelter capacity (e.g. if $\xi = 2$, the capacity of a shelter increases 1 time, and then the fixed cost of this shelter increases 2 times)

$f_i^{\mathcal{I}}$ Fixed cost for using transfer node $i \in \mathcal{I}$

$f_{ij}^{\mathcal{E}}$ Fixed cost for using undirected edge $\{i,j\} \in \mathcal{E}$

$g_{ij}^{\mathcal{E}}$ Fixed cost for adding a new lane at undirected edge $\{i,j\} \in \mathcal{E}$

$b_{ij}^{\mathcal{E}}$ Original number of lanes at undirected edge $\{i,j\} \in \mathcal{E}$

$c_{ij}^{\mathcal{A}}$ Variable cost for one unit flow on arc $(i,j) \in \mathcal{A}$

T Safe evacuation time

t_{ij} Travel time estimate for arc $(i,j) \in \mathcal{A}$

Decision Variables

$r_d^{\mathcal{D}}$ 1 if a shelter is opened at node d with original capacity, 0 not open

$e_d^{\mathcal{D}}$ Magnitude of the increase of shelter d capacity (e.g. the capacity of shelter d increases 2.5 times)

$r_{ij}^{\mathcal{E}}$ 1 if edge $\{i,j\}$ is used with original capacity, 0 not used

$e_{ij}^{\mathcal{E}}$ Number of new lanes added at edge $\{i,j\}$

$r_i^{\mathcal{I}}$ 1 if node i is used as a transfer node, 0 otherwise

z_{odij} 1 if flow from source node o to destination node d traverses arc (i,j), 0 otherwise

x_{odij} Amount of flow from source node o to destination node d on arc (i,j)

m_{od} Fraction of population of source node o going into destination node d

Then, the problem of interest can be formulated as follows:

$$\textbf{(SEND)} \qquad \text{Min} \sum_{o\in\mathcal{O}}\sum_{d\in\mathcal{D}}\sum_{(i,j)\in\mathcal{A}} c_{ij}^{\mathcal{A}}\, x_{odij} + \sum_{\{i,j\}\in\mathcal{E}} f_{ij}^{\mathcal{E}}\, r_{ij}^{\mathcal{E}} + \sum_{\{i,j\}\in\mathcal{E}} g_{ij}^{\mathcal{E}}\, e_{ij}^{\mathcal{E}} + \sum_{i\in\mathcal{I}} f_i^{\mathcal{I}}\, r_i^{\mathcal{I}}$$

$$+\sum_{d\in\mathcal{D}} f_d^{\mathcal{D}}\,(r_d^{\mathcal{D}}+\xi\times e_d^{\mathcal{D}}) \tag{3.1}$$

subject to

$$\sum_{i\in\mathcal{P}_j} z_{odij}\leq 1 \qquad \forall\, o\in\mathcal{O},\forall\, d\in\mathcal{D},\forall\, j\in\mathcal{I} \tag{3.2}$$

$$\sum_{j\in\mathcal{S}_i} z_{odij}\leq 1 \qquad \forall\, o\in\mathcal{O},\forall\, d\in\mathcal{D},\forall\, i\in\mathcal{I} \tag{3.3}$$

$$\sum_{d\in\mathcal{D}} m_{od}=1 \qquad \forall\, o\in\mathcal{O} \tag{3.4}$$

$$z_{odij}\leq x_{odij} \qquad \forall\, o\in\mathcal{O},\forall\, d\in\mathcal{D},\forall\,(i,j)\in\mathcal{A} \tag{3.5}$$

$$x_{odij}\leq s_o z_{odij} \qquad \forall\, o\in\mathcal{O},\forall\, d\in\mathcal{D},\forall\,(i,j)\in\mathcal{A} \tag{3.6}$$

$$z_{odij}\leq \begin{cases} r_i^{\mathcal{I}}, & \text{if } i\notin\mathcal{O} \\ r_j^{\mathcal{I}}, & \text{if } j\notin\mathcal{D} \end{cases} \qquad \forall\, o\in\mathcal{O},\forall\, d\in\mathcal{D},\forall\,(i,j)\in\mathcal{A} \tag{3.7}$$

$$\sum_{o\in\mathcal{O}}\sum_{i\in\mathcal{P}_d} x_{odid}-\sum_{o\in\mathcal{O}}\sum_{i\in\mathcal{S}_d} x_{oddi}\leq q_d^{\mathcal{D}}\,(r_d^{\mathcal{D}}+e_d^{\mathcal{D}}) \qquad \forall\, d\in\mathcal{D} \tag{3.8}$$

$$\sum_{o\in\mathcal{O}}\sum_{d\in\mathcal{D}}(x_{odij}+x_{odji})\leq q_{ij}^{\mathcal{E}}\,(r_{ij}^{\mathcal{E}}+\frac{e_{ij}^{\mathcal{E}}}{b_{ij}^{\mathcal{E}}}) \qquad \forall\,\{i,j\}\in\mathcal{E} \tag{3.9}$$

$$\sum_{j \in \mathcal{S}_i} x_{odij} - \sum_{j \in \mathcal{P}_i} x_{odji} = \begin{cases} m_{od}\, s_o, & \text{if i = o} \\ -m_{od}\, s_o, & \text{if i = d} \\ 0, & \text{otherwise} \end{cases} \qquad \forall\, i \in \mathcal{I}, \forall\, o \in \mathcal{O}, \forall\, d \in \mathcal{D} \tag{3.10}$$

$$\sum_{(i,j) \in \mathcal{A}} z_{odij}\, t_{ij} \leq T \qquad \forall\, o \in \mathcal{O}, \forall\, d \in \mathcal{D} \tag{3.11}$$

$$r_d^{\mathcal{D}}, r_i^{\mathcal{I}}, z_{odij} \in \{0,1\}, x_{odij} \geq 0, 0 \leq e_d^{\mathcal{D}} \leq \lambda \qquad \forall\, o \in \mathcal{O},\, \forall\, d \in \mathcal{D},\, \forall\, i \in \mathcal{I},\ \forall\, (i,j) \in \mathcal{A} \tag{3.12}$$

$$0 \leq m_{od} \leq 1, r_{ij}^{\mathcal{E}} \in \{0,1\}, e_{ij}^{\mathcal{E}} \geq 0 \qquad \forall\, o \in \mathcal{O},\, \forall\, d \in \mathcal{D},\, \forall\, \{i,j\} \in \mathcal{E}. \tag{3.13}$$

The objective function (3.1) minimizes the total evacuation network design cost. Specifically, the first term is the total transportation cost for all flows through arcs. The second term is the fixed cost for using edges with original capacities, and the third term is the fixed cost incurred by adding extra lanes. The fourth term represents the total fixed costs associated with utilized transfer nodes. The fifth term is the fixed cost for open shelters with original capacities, and the last term is the fixed cost incurred by increasing shelter capacities. Constraints (3.2) and (3.3) ensure that there is only one path between an origin node and its (to-be-determined) destination node. Constraints (3.4) represent that the population in each origin is evacuated to some shelter (destination). Note that the variable m_{od} effectively implies the origin-destination node pairs for evacuation. Constraints (3.5) and (3.6) assign the correct values of binary variables based on existence of flows on arcs. Constraints (3.7) require that the flow can pass through a transfer node i only if the node is identified as a transfer node. Constraints (3.8) ensure that, for each destination node, the total inflow is less than or equal to its capacity and this occurs only when the node is decided to be a shelter. If a shelter's original capacity is not enough, this shelter obtains extra capacity; however, the extra capacity that each shelter can obtain is limited. Constraints (3.9) force that the total flow passing through an edge (i, j) does not exceed its capacity and the flow can pass through an edge only if the edge is included in the design. If an edge's original capacity is not enough, new lanes can be added at this edge; however, the number of new lanes are limited. Constraints (3.10) are the flow conservation constraints. Constraints (3.11) require that the evacuation time for each evacuation path (specific for an origin-destination pair) does not exceed the allowed evacuation time. Constraints (3.12) and (3.13) force integrality and feasibility ranges for the decision variables.

III.3. Solution Methodologies

Due to the tremendous number of variables and constraints, SEND is extremely hard to solve, especially for large-scale instances. To tackle this difficulty, we develop our solution methodology based on BD. The reason for choosing BD approach is that BD can solve a complicated mixed integer program by decomposing the entire formulation to two relative simple parts: a master problem and a subproblem, and solving them separately and iteratively (Benders, 1962). From the specific characteristics of SEND, we find that the subproblem of SEND has the integrality property. By taking this advantage, BD can be an effective approach for solving SEND.

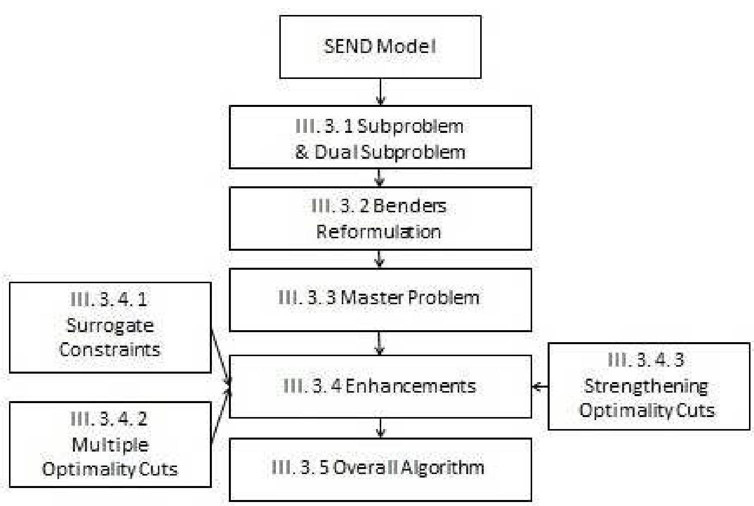
In BD framework, typically, the master problem is a mixed integer program with one continuous variable that is used to integrate the master problem and the subproblem. The subproblem usually only contains continuous variables and uses the solution of the master problem as parameters. As we have introduced, BD works iteratively. At the first iteration, traditionally, master problem is solved without Benders cuts, and the solutions for integer variables are passed to subproblem as coefficients for continuous variables. Then subproblem is solved and generates Benders cuts, which are added and accumulated to master problem as constraints to integrate master problem and subproblem. If the original problem is a minimization problem, since the master problem only contains a part of constraints of original problem, master problem provides a lower bound to the optimal solution of the original problem. On the other hand, the subproblem is solved with the fixed values of integer variables which are passed from the master problem, so the solutions of the fixed integer variables and the corresponding solutions of continuous variables compose an upper bound for the optimal solution of the original problem. Along the iterations, the Benders cuts are accumulated in the master problem. Thus, the optimal solution of the master problem is non-decreased, and the lower bound for the optimal solution of the original problem is improved. With the procedure repeats iteratively, both of the lower bound and the upper bound of the optimal solution of the original problem are updated. Typically, BD approach stops until stop criteria are satisfied (e.g. the gap between the lower bound and the upper bound of the optimal solution of the original problem is within a specific tolerance, or the number of iterations is larger than an established value) (Benders, 1962).

BD is employed to solve the complicated mixed linear problems which can be partitioned to two relative easy problems. This property let BD be a popular approach for solving network design problems (Gzara and Erkut, 2011; Üster and Lin, 2011; Kewcharoenwong and Üster, 2012; Marin and Jaramillo, 2009; Üster and Kewcharoenwong, 2011; Üster and Agrahari, 2011; Easwaran and Üster, 2010, 2009). Recently, BD approach is also used to solve evacuation problems which are modeled in complicated mixed integer linear programs. Andreas and Smith (2009) posed and analyzed a building evacuation problem with a mixed integer linear program, and they developed a solution strategy based on BD to solve their model.

In our study, we propose a mixed integer linear program to devise effective and controlled evacuation networks for sending evacuees from their origins to shelters, and we develop a solution methodology to solve SEND model based on BD approach. For SEND, the master problem prescribes facility utilization, and the subproblem contains flow variables and fraction variables to decide the flow assignments.

The formulation for subproblem (SP) is developed in § III.3.1. In § III.3.2, we reformulate SEND to develop the master problem (MP). In § III.3.3, we present the formulation for MP. In § III.3.4, we employ techniques to accelerate BD. At the end of this section, § III.3.5, we present the framework for the overall algorithm. The organization of this section is illustrated in Figure 2.

Figure 2 Organization of Section III.3



III.3.1. Benders Subproblem and Dual Subproblem

Before we introduce the Benders reformulation for the SEND problem, we first present the SP and its dual (DSP). For given values of design variables z_{odij}, $r^{\mathcal{E}}_{ij}$, $r^{\mathcal{D}}_{d}$, $r^{\mathcal{I}}_{i}$, $e^{\mathcal{D}}_{d}$, and $e^{\mathcal{E}}_{ij}$ (obtained as a master problem solution and represented as $\hat{z}_{odij}$, $\hat{r}^{\mathcal{E}}_{ij}$, $\hat{r}^{\mathcal{D}}_{d}$, $\hat{r}^{\mathcal{I}}_{i}$, $\hat{e}^{\mathcal{D}}_{d}$, and $\hat{e}^{\mathcal{E}}_{ij}$ respectively), the SP is extracted from the overall SEND formulation, (3.1)-(3.13), as follows:

$$\textbf{(SP)} \qquad \text{Min} \sum_{o\in\mathcal{O}}\sum_{d\in\mathcal{D}}\sum_{(i,j)\in\mathcal{A}} c^{\mathcal{A}}_{ij}\, x_{odij} \tag{3.14}$$

subject to

$$\sum_{d\in\mathcal{D}} m_{od} = 1 \qquad \forall\, o\in\mathcal{O} \tag{3.15}$$

$$x_{odij} \geq \hat{z}_{odij} \qquad \forall\, o\in\mathcal{O}, \forall\, d\in\mathcal{D}, \forall\,(i,j)\in\mathcal{A} \tag{3.16}$$

$$x_{odij} \leq s_o \hat{z}_{odij} \qquad \forall\, o\in\mathcal{O}, \forall\, d\in\mathcal{D}, \forall\,(i,j)\in\mathcal{A} \tag{3.17}$$

$$\sum_{o\in\mathcal{O}}\sum_{i\in\mathcal{P}_d} x_{odid} - \sum_{o\in\mathcal{O}}\sum_{i\in\mathcal{S}_d} x_{oddi} \leq q^{\mathcal{D}}_{d}\,(\hat{r}^{\mathcal{D}}_{d} + \hat{e}^{\mathcal{D}}_{d}) \qquad \forall\, d\in\mathcal{D} \tag{3.18}$$

$$\sum_{o\in\mathcal{O}}\sum_{d\in\mathcal{D}} (x_{odij} + x_{odji}) \leq q^{\mathcal{E}}_{ij}\,(\hat{r}^{\mathcal{E}}_{ij} + \frac{\hat{e}^{\mathcal{E}}_{ij}}{b^{\mathcal{E}}_{ij}}) \qquad \forall\, \{i,j\}\in\mathcal{E} \tag{3.19}$$

$$\sum_{j\in\mathcal{S}_i} x_{odij} - \sum_{j\in\mathcal{P}_i} x_{odji} = \begin{cases} m_{od}\, s_o, & \text{if } i = o \\ -m_{od}\, s_o, & \text{if } i = d \\ 0, & \text{otherwise} \end{cases} \qquad \forall\, i\in\mathcal{I}, \forall\, o\in\mathcal{O}, \forall\, d\in\mathcal{D} \tag{3.20}$$

$$x_{odij} \geq 0, 1 \geq m_{od} \geq 0 \qquad \forall\, o \in \mathcal{O}, \forall\, d \in \mathcal{D}, \forall\, (i,j) \in \mathcal{A}. \tag{3.21}$$

The solution of SP essentially prescribes origin-destination pairs and flow requirements for each pair (m_{od}) and the routing of flow for the origin-destination pairs (x_{odij}) over the network dictated by the master problem solution.

To obtain the DSP, we define the dual variables ρ_o, μ_{odij}, ω_{odij}, α_d, and θ_{ij} for constraints (3.15), (3.16), (3.17), (3.18), and (3.19), respectively. Additionally, the dual variables for constraint set (3.20) are defined as δ_{od}, σ_{od}, and λ_{iod} for varying right-hand sides in the order given. Then, the DSP is formulated as follows (note that hereafter, if not specified, $o \in \mathcal{O}$, $d \in \mathcal{D}$, and i, $j \in \mathcal{I}$).

$$\textbf{(DSP)} \qquad \text{Max} \sum_{o\in\mathcal{O}} \rho_o + \sum_{o\in\mathcal{O}}\sum_{d\in\mathcal{D}}\sum_{(i,j)\in\mathcal{A}} \hat{z}_{odij}\,\mu_{odij} - \sum_{o\in\mathcal{O}}\sum_{d\in\mathcal{D}}\sum_{(i,j)\in\mathcal{A}} s_o\,\hat{z}_{odij}\,\omega_{odij}$$

$$- \sum_{d\in\mathcal{D}} q_d^{\mathcal{D}}\,(\hat{r}_d^{\mathcal{D}} + \hat{e}_d^{\mathcal{D}})\,\alpha_d - \sum_{\{i,j\}\in\mathcal{E}} q_{ij}^{\mathcal{E}}\,(\hat{r}_{ij}^{\mathcal{E}} + \frac{\hat{e}_{ij}^{\mathcal{E}}}{b_{ij}^{\mathcal{E}}})\,\theta_{ij} \tag{3.22}$$

subject to

$$\mu_{odij} - \omega_{odij} - \theta_{ij} + \lambda_{iod} - \lambda_{jod} \leq c_{ij}^{\mathcal{A}} \qquad i \neq o, d, j \in \mathcal{S}_i, j \neq o, d, j > i \tag{3.23}$$

$$\mu_{odij} - \omega_{odij} + \lambda_{iod} - \lambda_{jod} \leq c_{ij}^{\mathcal{A}} \qquad i \neq o, d, j \in \mathcal{S}_i, j \neq o, d, j = i \tag{3.24}$$

$$\mu_{odij} - \omega_{odij} - \theta_{ji} + \lambda_{iod} - \lambda_{jod} \leq c_{ij}^{\mathcal{A}} \qquad i \neq o, d, j \in \mathcal{S}_i, j \neq o, d, j < i \tag{3.25}$$

$$\mu_{odoj} - \omega_{odoj} - \theta_{oj} + \delta_{od} - \lambda_{jod} \leq c_{oj}^{\mathcal{A}} \qquad i = o, j \in \mathcal{S}_o, j \neq o, d, j > o \tag{3.26}$$

$$\mu_{odoj} - \omega_{odoj} - \theta_{jo} + \delta_{od} - \lambda_{jod} \leq c_{oj}^{\mathcal{A}} \qquad i = o, j \in \mathcal{S}_o, j \neq o, d, j < o \tag{3.27}$$

$$\mu_{odio} - \omega_{odio} - \theta_{io} - \delta_{od} + \lambda_{iod} \leq c_{io}^{\mathcal{A}} \qquad j = o, i \in \mathcal{P}_o, i \neq o, d, o > i \tag{3.28}$$

$$\mu_{odio} - \omega_{odio} - \theta_{oi} - \delta_{od} + \lambda_{iod} \leq c^{\mathcal{A}}_{io} \qquad j = o, i \in \mathcal{P}_o, i \neq o, d, o < i \tag{3.29}$$

$$\mu_{oddj} - \omega_{oddj} + \alpha_d - \theta_{dj} + \sigma_{od} - \lambda_{jod} \leq c^{\mathcal{A}}_{dj} \quad i = d, j \in \mathcal{S}_d, j \neq o, d, j > d \tag{3.30}$$

$$\mu_{oddj} - \omega_{oddj} + \alpha_d - \theta_{jd} + \sigma_{od} - \lambda_{jod} \leq c^{\mathcal{A}}_{dj} \quad i = d, j \in \mathcal{S}_d, j \neq o, d, j < d \tag{3.31}$$

$$\mu_{odid} - \omega_{odid} - \alpha_d - \theta_{id} - \sigma_{od} + \lambda_{iod} \leq c^{\mathcal{A}}_{id} \quad j = d, i \in \mathcal{P}_d, i \neq o, d, d > i \tag{3.32}$$

$$\mu_{odid} - \omega_{odid} - \alpha_d - \theta_{di} - \sigma_{od} + \lambda_{iod} \leq c^{\mathcal{A}}_{id} \quad j = d, i \in \mathcal{P}_d, i \neq o, d, d < i \tag{3.33}$$

$$\rho_o - s_o \, \delta_{od} + s_o \, \sigma_{od} \leq 0 \tag{3.34}$$

$$\rho_o, \mu_{odid}, \omega_{odid}, \alpha_d, \theta_{id} \geq 0. \tag{3.35}$$

Let $\mathcal{L}$ and $\mathcal{V}$ denote the sets of all extreme points and extreme rays in the polyhedron given by all DSP constraints. For each extreme point $l \in \mathcal{L}$, let $\hat{\mu}^l_{odij}$, $\hat{\omega}^l_{odij}$, $\hat{\alpha}^l_d$, $\hat{\theta}^l_{ij}$, $\hat{\rho}^l_o$, $\hat{\delta}^l_{od}$, $\hat{\sigma}^l_{od}$, $\hat{\lambda}^l_{iod}$ and D^l represent the associated values for dual variables and the objective value. If DSP is bounded, let D^* represent the optimal objective value, and then $D^* \geq D^l, \forall\, l \in \mathcal{L}$. Thus, the DSP can be reformulated as $\min_{D \geq 0}\{D : D \geq D^l, \forall\, l \in \mathcal{L}\}$, where

$$\begin{aligned} D^l = & \sum_{o \in \mathcal{O}} \hat{\rho}^l_o + \sum_{o \in \mathcal{O}} \sum_{d \in \mathcal{D}} \sum_{(i,j) \in \mathcal{A}} z_{odij} \hat{\mu}^l_{odij} - \sum_{o \in \mathcal{O}} \sum_{d \in \mathcal{D}} \sum_{(i,j) \in \mathcal{A}} s_o z_{odij} \hat{\omega}^l_{odij} \\ & - \sum_{d \in \mathcal{D}} q^{\mathcal{D}}_d (\hat{r}^{\mathcal{D}}_d + \hat{e}^{\mathcal{D}}_d) \hat{\alpha}^l_d - \sum_{\{i,j\} \in \mathcal{E}} q^{\mathcal{E}}_{ij} (\hat{r}^{\mathcal{E}}_{ij} + \frac{\hat{e}^{\mathcal{E}}_{ij}}{b^{\mathcal{E}}_{ij}}) \hat{\theta}^l_{ij}, \quad \forall\, l \in \mathcal{L}. \end{aligned} \tag{3.36}$$

When DSP is bounded, Benders optimality cuts can be generated as follows:

$$\begin{aligned} D \;\; \geq & \;\; \sum_{o \in \mathcal{O}} \hat{\rho}^l_o + \sum_{o \in \mathcal{O}} \sum_{d \in \mathcal{D}} \sum_{(i,j) \in \mathcal{A}} z_{odij} \hat{\mu}^l_{odij} - \sum_{o \in \mathcal{O}} \sum_{d \in \mathcal{D}} \sum_{(i,j) \in \mathcal{A}} s_o \, z_{odij} \hat{\omega}^l_{odij} \\ - & \;\; \sum_{d \in \mathcal{D}} q^{\mathcal{D}}_d \, (\hat{r}^{\mathcal{D}}_d + \hat{e}^{\mathcal{D}}_d) \, \hat{\alpha}^l_d - \sum_{\{i,j\} \in \mathcal{E}} q^{\mathcal{E}}_{ij} \, (\hat{r}^{\mathcal{E}}_{ij} + \frac{\hat{e}^{\mathcal{E}}_{ij}}{b^{\mathcal{E}}_{ij}}) \, \hat{\theta}^l_{ij}, \quad \forall\, l \in \mathcal{L}. \end{aligned} \tag{3.37}$$

For each extreme ray $v \in \mathcal{V}$, let $\hat{\mu}^v_{odij}$, $\hat{\omega}^v_{odij}$, $\hat{\alpha}^v_d$, $\hat{\theta}^v_{ij}$, $\hat{\rho}^v_o$, $\hat{\delta}^v_{od}$, $\hat{\sigma}^v_{od}$, $\hat{\lambda}^v_{iod}$ represent the corresponding values for dual variables. When DSP is unbounded, Benders feasibility cuts can be generated as follows:

$$\sum_{o\in\mathcal{O}} \hat{\rho}_o^v + \sum_{o\in\mathcal{O}}\sum_{d\in\mathcal{D}}\sum_{(i,j)\in\mathcal{A}} z_{odij}\hat{\mu}_{odij}^v - \sum_{o\in\mathcal{O}}\sum_{d\in\mathcal{D}}\sum_{(i,j)\in\mathcal{A}} s_o z_{odij}\hat{\omega}_{odij}^v$$

$$- \sum_{d\in\mathcal{D}} q_d^{\mathcal{D}}\,(\hat{r}_d^{\mathcal{D}} + \hat{e}_d^{\mathcal{D}})\,\hat{\alpha}_d^v - \sum_{\{i,j\}\in\mathcal{E}} q_{ij}^{\mathcal{E}}\,(\hat{r}_{ij}^{\mathcal{E}} + \frac{\hat{e}_{ij}^{\mathcal{E}}}{b_{ij}^{\mathcal{E}}})\,\hat{\theta}_{ij}^v \;\le\; 0 \qquad \forall\, v \in \mathcal{V}. \tag{3.38}$$

III.3.2. Benders Reformulation

Using the above reformulated representation of DSP, we can reformulate SEND as follows:

(RSEND)

$$\text{Min} \sum_{\{i,j\}\in\mathcal{E}} f_{ij}^{\mathcal{E}}\, r_{ij}^{\mathcal{E}} + \sum_{\{i,j\}\in\mathcal{E}} g_{ij}^{\mathcal{E}}\, e_{ij}^{\mathcal{E}} + \sum_{i\in\mathcal{I}} f_i^{\mathcal{I}}\, r_i^{\mathcal{I}} + \sum_{d\in\mathcal{D}} f_d^{\mathcal{D}}\,(r_d^{\mathcal{D}} + \xi\times e_d^{\mathcal{D}}) + D \tag{3.39}$$

$$\text{subject to} \quad (3.2), (3.3), (3.7), (3.11), (3.12), (3.13), (3.37), (3.38).$$

However, it is not practical to solve this formulation because of the large $|\mathcal{L}|$ and $|\mathcal{V}|$ values. Since not all Benders cuts are binding at optimality, we can relax RSEND by considering only a subset of Benders cuts in each iteration. This relaxed problem is the master problem (MP), which is presented in next subsection. Because SEND is a minimization problem, the optimal objective value of the MP is always a lower bound for SEND.

III.3.3. Benders Master Problem

Based on the discussion in § III.3.2, letting $\mathcal{U}$ and $\mathcal{W}$ be subsets of $\mathcal{L}$ and $\mathcal{V}$, respectively, master problem (MP) can be formulated as follows.

$$\textbf{(MP)} \qquad \text{Min} \sum_{\{i,j\}\in\mathcal{E}} f_{ij}^{\mathcal{E}}\, r_{ij}^{\mathcal{E}} + \sum_{\{i,j\}\in\mathcal{E}} g_{ij}^{\mathcal{E}}\, e_{ij}^{\mathcal{E}} + \sum_{i\in\mathcal{I}} f_i^{\mathcal{I}}\, r_i^{\mathcal{I}} + \sum_{d\in\mathcal{D}} f_d^{\mathcal{D}}\,(r_d^{\mathcal{D}} + \xi \times e_d^{\mathcal{D}}) + D \tag{3.40}$$

$$\text{subject to} \qquad (3.2), (3.3), (3.7), (3.11)$$

$$\begin{aligned} D \geq & \sum_{o\in\mathcal{O}} \hat{\rho}_o^u + \sum_{o\in\mathcal{O}}\sum_{d\in\mathcal{D}}\sum_{(i,j)\in\mathcal{A}} z_{odij}\,\hat{\mu}_{odij}^u - \sum_{o\in\mathcal{O}}\sum_{d\in\mathcal{D}}\sum_{(i,j)\in\mathcal{A}} s_o\, z_{odij}\,\hat{\omega}_{odij}^u \\ & - \sum_{d\in\mathcal{D}} q_d^{\mathcal{D}}\,(r_d^{\mathcal{D}} + e_d^{\mathcal{D}})\,\hat{\alpha}_d^u - \sum_{\{i,j\}\in\mathcal{E}} q_{ij}^{\mathcal{E}}\,(r_{ij}^{\mathcal{E}} + \frac{e_{ij}^{\mathcal{E}}}{b_{ij}^{\mathcal{E}}})\,\hat{\theta}_{ij}^u, \qquad \forall\, u\in\mathcal{U} \end{aligned} \tag{3.41}$$

$$\begin{aligned} & \sum_{o\in\mathcal{O}} \hat{\rho}_o^w + \sum_{o\in\mathcal{O}}\sum_{d\in\mathcal{D}}\sum_{(i,j)\in\mathcal{A}} z_{odij}\,\hat{\mu}_{odij}^w - \sum_{o\in\mathcal{O}}\sum_{d\in\mathcal{D}}\sum_{(i,j)\in\mathcal{A}} s_o z_{odij}\,\hat{\omega}_{odij}^w \\ & - \sum_{d\in\mathcal{D}} q_d^{\mathcal{D}}\,(r_d^{\mathcal{D}} + e_d^{\mathcal{D}})\,\hat{\alpha}_d^w - \sum_{\{i,j\}\in\mathcal{E}} q_{ij}^{\mathcal{E}}\,(r_{ij}^{\mathcal{E}} + \frac{e_{ij}^{\mathcal{E}}}{b_{ij}^{\mathcal{E}}})\,\hat{\theta}_{ij}^w \;\leq\; 0, \qquad \forall\, w\in\mathcal{W} \end{aligned} \tag{3.42}$$

$$r_d^{\mathcal{D}}, r_i^{\mathcal{I}}, z_{odij} \in \{0,1\}, 0 \leq e_d^{\mathcal{D}} \leq \lambda \qquad \forall\, o\in\mathcal{O}, \forall\, d\in\mathcal{D}, \forall\, i\in\mathcal{I}, \forall\,(i,j)\in\mathcal{A} \tag{3.43}$$

$$r_{ij}^{\mathcal{E}} \in \{0,1\}, e_{ij}^{\mathcal{E}} \geq 0 \qquad \forall\,\{i,j\}\in\mathcal{E}. \tag{3.44}$$

In MP, we consider the fixed costs for using edges, the fixed costs for using transfer nodes, and the fixed costs for opening shelters. In MP, the model prescribes the binary variables associated with edges, transhipment nodes, shelters, and route assignments (z_{odij}). MP decides the underlying network for SEND, and then SP prescribes flow assignments based on the underlying network chosen by MP. However, the underlying network chosen by MP may have the connectivity and capacity issues, since these constraints are not in MP and the MP's objective is to minimize network construction costs. Both connectivity and capacity issues may make SP infeasible and cause BD to be inefficient, which is the motivation for us to develop the enhancements for BD in the next subsection.

III.3.4. Algorithmic Enhancements

To this end, we finish developing the basic BD approach. However, if SP is infeasible and DSP is unbounded, DSP generates a feasibility cut that does not improve the lower bound efficiently. Also, the upper bound cannot be updated in the corresponding iteration, because the newly generated objective value for DSP is infinity. On the other hand, if DSP is bounded, it generates an optimality cut based on an extreme point. Optimality cuts improve lower bounds effectively and also may update upper bounds. Obviously, if more optimality cuts are generated, BD can converge quickly; otherwise, BD may converge slowly. Thus, our consideration in accelerating the solution methodology is trying to generate more optimality cuts. Following this idea, if DSP is unbounded, besides adding the feasibility cut to MP, we also generate multiple optimality cuts for MP. Whether SP is feasible or not depends on the values of variables generated in MP, because these values are passed from MP to SP and are used as parameters in SP. Thus, we focus our consideration on how to get MP solutions that form a feasible SP. The most intuitive idea is that, if the MP solution is a part of a feasible solution for SEND, these values can always make SP feasible. If SP is feasible, it can generate an objective value that can form a valid upper bound of SEND. Thus, it is always safe to use the values in any feasible solution of SEND as the corresponding parameters in SP.

Thus, to make the BD approach more efficient, we add the following techniques. First, in the first iteration, we develop a feasible solution (in § III.3.4.2) for SEND and solve DSP using this feasible solution. Then we get an optimality cut and add it to the initial MP (Torres-Soto and Üster, 2011; Easwaran and Üster, 2009). Second, we solve MP with surrogate constraints (in § III.3.4.1). Third, in each iteration, if DSP is infeasible, we generate multiple feasible solutions for SEND (in § III.3.4.2), so we can generate multiple optimality cuts in each iteration (Easwaran and Üster, 2010; Kewcharoenwong and Üster, 2012). If the number of optimality cuts is larger, it can improve the lower bound more effectively. However, it makes MP harder to solve. Thus, it is a trade off in the number of cuts. Also, these feasible solutions are not generated independently and randomly; however, we use information from the solution of MP in the last iteration to generate the feasible solutions which are used in DSP in the current iteration. Fourth, when we generate the optimality cuts, strengthen them and add them to MP (in § III.3.4.3). Because the dual subproblem is highly degenerate and generates multiple optimal solutions, we choose the optimal solution which can generate strengthened cuts that can speed up the convergence rate (Magnanti and Wong, 1981; Roy, 1986; Wentges, 1996). Fifth, we solve MP with early termination criterion in the first several iterations. We give MP a loose gap in the first iteration, and then we decrease this gap gradually in successive iterations. This can save run time for solving MPs and avoid trailing off (Easwaran and Üster, 2010).

Because multiple feasible solutions may be the same in consecutive iterations, this may cause the optimality cuts, which added to MPs to be same in consecutive iterations. This situation may generate the same solutions on successive iterations and may cause the endless loop. Thus, when DSP is unbounded, we add the multiple optimality cuts and a feasibility cut to MP. Moreover, adding feasibility cuts can make the cut pool diverse so that the algorithm is more effective.

III.3.4.1. Surrogate Constraints

Although the surrogate constraints are redundant in the overall SEND model, when added to MP in the BD framework, they help to improve the solution time of MP and/or the quality of lower bounds by providing a higher MP optimal objective value.

Our first set of surrogate constraints to be added to MP concern the total capacity requirements and ensure that the total capacity available at the open shelters is at least equal to the total population evacuated (3.45). Similarly, the total aggregate capacity on the outgoing arcs from origin nodes (3.46) and on the incoming arcs to shelters (3.47) are at least equal to the total evacuee flow.

$$\sum_{d \in \mathcal{D}} q_d^D (r_d^D + e_d^D) \geq \sum_{o \in \mathcal{O}} s_o. \tag{3.45}$$

$$\sum_{o \in \mathcal{O}} \sum_{i \in \mathcal{S}_o} q_{oi}^{\mathcal{E}} (r_{oi}^{\mathcal{E}} + \frac{e_{oi}^{\mathcal{E}}}{b_{oi}^{\mathcal{E}}}) \geq \sum_{o \in \mathcal{O}} s_o \tag{3.46}$$

$$\sum_{i \in \mathcal{P}_d} \sum_{d \in \mathcal{D}} q_{id}^{\mathcal{E}} (r_{id}^{\mathcal{E}} + \frac{e_{id}^{\mathcal{E}}}{b_{id}^{\mathcal{E}}}) \geq \sum_{o \in \mathcal{O}} s_o \tag{3.47}$$

We additionally consider other redundant constraints for addition to MP which, based on our computational tests, contribute to improving lower bounds without a noteworthy computational burden to solving MP. These include

$$\sum_{d \in \mathcal{D}} \sum_{j \in \mathcal{S}_o} z_{odoj} \geq 1, \qquad \forall\, o \in \mathcal{O} \tag{3.48}$$

$$\sum_{o \in \mathcal{O}} \sum_{i \in \mathcal{P}_d} z_{odid} \geq r_d^D, \qquad \forall\, d \in \mathcal{D} \tag{3.49}$$

$$z_{odid} \leq r_d^D, \qquad \forall\, o \in \mathcal{O}, d \in \mathcal{D}, \forall\, i \in \mathcal{P}_d \tag{3.50}$$

$$z_{odij} \leq \begin{cases} r^{\mathcal{E}}_{ij}, & \text{if } j \geq i+1 \\ r^{\mathcal{E}}_{ji}, & \text{if } j < i \end{cases} \qquad \forall\, o \in \mathcal{O}, \forall\, d \in \mathcal{D}, \forall\, (i,j) \in \mathcal{A} \tag{3.51}$$

$$r^{\mathcal{E}}_{ij}\, q^{\mathcal{E}}_{ij} \geq e^{\mathcal{E}}_{ij} \qquad \forall\, \{i,j\} \in \mathcal{E} \tag{3.52}$$

$$r^{\mathcal{D}}_{d}\, q^{\mathcal{D}}_{d} \geq e^{\mathcal{D}}_{d} \qquad \forall\, d \in \mathcal{D} \tag{3.53}$$

Constraints (3.48) require that there is at least one outgoing arc from an origin node and, similarly Constraints (3.49) require that there must be nonzero inflow to a shelter (ensuring at least one incoming arc), if this shelter is opened. Constraints (3.50) ensure that the shelter at node d must be opened if it has nonzero inflow. Constraints (3.51) guarantee that, if a directed arc (i, j) is used, then the corresponding edge $\{i, j\}$ is in solution. Constraints (3.52) and (3.53) ensure that only the used shelters and edges can obtain extra capacities.

III.3.4.2. Generating Feasible Solutions

To improve the efficiency of the Benders decomposition (BD) approach, we heuristically generate and embed feasible solutions of SEND in various stages of the algorithm. First, before the first iteration, we find a feasible solution of SEND and solve a DSP with this feasible solution as its parameters. Then, we generate an optimality cut using the optimal solution of the DSP and add this cut to the initial MP (Wentges, 1996; Easwaran and Üster, 2009). Second, in each iteration, if DSP is infeasible, we determine multiple feasible solutions of SEND so that we can generate multiple Benders optimality cuts in each iteration in addition to the feasibility cut that needs to be added to MP at that iteration.

Also, the multiple feasible solutions of SEND are not generated independently and randomly; however, we use information from the solution of MP in the last iteration to generate these multiple feasible solutions which are used as variable coefficients in DSP in the current iteration. The procedure for generating the feasible solutions of SEND should be very effective and cannot become a burden for the whole solution methodology.

In this section, we introduce the details about how to generate these feasible solutions of SEND. First, we devise a formulation to prescribe a feasible solution (FP) in which we assume the presence of all edges and determine flows by optimizing the transportation cost and fixed costs of shelters under capacity constraints and flow conservation constraints. $mc^{\mathcal{A}}_{ij}$ is the modified variable costs for each arc. To generate multiple feasible solutions of SEND and make them diverse, in each time when we solve FP, we use a new set of $mc^{\mathcal{A}}_{ij}$ in FP by modifying the original arc variable costs $c^{\mathcal{A}}_{ij}$ based on the information from the solution of MP in last iteration.

$$\textbf{(FP)} \qquad \text{Min} \sum_{o\in\mathcal{O}}\sum_{d\in\mathcal{D}}\sum_{(i,j)\in\mathcal{A}} mc^{\mathcal{A}}_{ij}\, x_{odij} + \sum_{d\in\mathcal{D}} f^{\mathcal{D}}_{d}\,(r^{\mathcal{D}}_{d} + \xi \times e^{\mathcal{D}}_{d}) \tag{3.54}$$

subject to

$$\sum_{d\in\mathcal{D}} m_{od} = 1 \qquad \forall\, o \in \mathcal{O} \tag{3.55}$$

$$\sum_{o\in\mathcal{O}}\sum_{i\in\mathcal{P}_d} x_{odid} - \sum_{o\in\mathcal{O}}\sum_{i\in\mathcal{S}_d} x_{oddi} \leq q^{\mathcal{D}}_{d}\,(r^{\mathcal{D}}_{d} + e^{\mathcal{D}}_{d}) \qquad \forall\, d \in \mathcal{D} \tag{3.56}$$

$$\sum_{j\in\mathcal{S}_i} x_{odij} - \sum_{j\in\mathcal{P}_i} x_{odji} = \begin{cases} m_{od}\, s_o, & \text{if } i = o \\ -m_{od}\, s_o, & \text{if } i = d \\ 0, & \text{otherwise} \end{cases} \qquad \forall\, i \in \mathcal{I},\, o \in \mathcal{O},\, d \in \mathcal{D} \tag{3.57}$$

$$\sum_{(i,j)\in\mathcal{A}} x_{odij}\, t_{ij} \leq s_o\, m_{od}\, T \qquad \forall\, o \in \mathcal{O}, \forall\, d \in \mathcal{D}. \tag{3.58}$$

The problem FP is a type of network flow problem and is relatively easy to solve. There is no edge capacity constraint in FP, so flows are not split for a pair of origin-destination. Constraints (3.58) ensure that the time constraints in SEND can be satisfied in FP. Based on the x_{odij} values obtained from FP, we generate the values of the other variables. More specifically, the nonzero values of x_{odij} imply a set of z_{odij}, $r^{\mathcal{E}}_{ij}$, and $r^{\mathcal{I}}_{i}$ variables whose values are all one (corresponding solution vectors are represented as $\hat{z}_{odij}$, $\hat{r}^{\mathcal{E}}_{ij}$, and $\hat{r}^{\mathcal{I}}_{i}$, respectively). If the value of x_{odij} for edge ij is greater than the capacity of this edge, extra capacities should be added, and the values of $\hat{e}^{\mathcal{E}}_{ij}$ can be determined. Thus, a feasible solution of SEND can be obtained from the optimal solution of FP.

The framework of the heuristic algorithm is presented in Algorithm 1. Note that for the first iteration in BD, FP uses the original variable costs ($c^{\mathcal{A}}_{ij}$) rather than the modified variable costs ($mc^{\mathcal{A}}_{ij}$), i.e., $\omega = 0$.

Algorithm 1 Generate Feasible Solutions

1: **initialize** multiplier vector W and **MP** solution $\hat{z}_{odij}$, $\hat{r}^{\mathcal{E}}_{ij}$, $\hat{r}^{\mathcal{D}}_{d}$, and $\hat{r}^{\mathcal{I}}_{i}$
2: **for** each ω in the multiplier vector W **do**
3:     **for** each edge $\{i, j\} \in \mathcal{E}$ **do**
4:         $mc^{\mathcal{A}}_{ij} = c^{\mathcal{A}}_{ij} + (1 - \hat{r}^{\mathcal{E}}_{ij})\,(f^{\mathcal{E}}_{ij}/q^{\mathcal{E}}_{ij})\,\omega$
5:         $mc^{\mathcal{A}}_{ji} = c^{\mathcal{A}}_{ji} + (1 - \hat{r}^{\mathcal{E}}_{ij})\,(f^{\mathcal{E}}_{ij}/q^{\mathcal{E}}_{ij})\,\omega$
6:     **end for**
7:     **for** each shelter $d \in \mathcal{D}$ **do**
8:         $mc^{\mathcal{A}}_{id} = c^{\mathcal{A}}_{id} + (1 - \hat{r}^{\mathcal{D}}_{d})\,(f^{\mathcal{E}}_{id}/q^{\mathcal{E}}_{id})\,\omega$
9:     **end for**
10:     **for** each transfer node $i \in \mathcal{I}$ **do**
11:         $mc^{\mathcal{A}}_{ij} = c^{\mathcal{A}}_{ij} + (1 - \hat{r}^{\mathcal{I}}_{i})\,(f^{\mathcal{I}}_{i}/q^{\mathcal{E}}_{ij})\,\omega$
12:         $mc^{\mathcal{A}}_{ji} = c^{\mathcal{A}}_{ji} + (1 - \hat{r}^{\mathcal{I}}_{i})\,(f^{\mathcal{I}}_{j}/q^{\mathcal{E}}_{ij})\,\omega$
13:     **end for**
14:     **if** current iteration is the first iteration in BD **then**
15:         Solve **FP** using the original variable costs $c^{\mathcal{A}}_{ij}$
16:     **else**
17:         Solve **FP** using the modified variable costs $mc^{\mathcal{A}}_{ij}$
18:     **end if**
19:     Update $\hat{z}_{odij}$, $\hat{r}^{\mathcal{E}}_{ij}$, $\hat{r}^{\mathcal{D}}_{d}$, $\hat{r}^{\mathcal{I}}_{i}$, $\hat{e}^{\mathcal{E}}_{ij}$, and $\hat{e}^{\mathcal{D}}_{d}$
20:     Pass $\hat{z}_{odij}$, $\hat{r}^{\mathcal{E}}_{ij}$, $\hat{r}^{\mathcal{D}}_{d}$, $\hat{r}^{\mathcal{I}}_{i}$, $\hat{e}^{\mathcal{E}}_{ij}$, and $\hat{e}^{\mathcal{D}}_{d}$ to **DsP** as coefficients variables and solve **DsP**
21: **end for**

The detailed approach to generate the modified variable costs $mc^{\mathcal{A}}_{ij}$ is outlined in lines 3-13 of Algorithm 1. The idea for modifying the variable costs is to use the information from the solution of MP as follows. If an edge is not selected in MP solution, this edge can be considered as the one with a less priority by increasing the variable costs of corresponding arcs, since SEND is a cost minimization problem. We use similar approaches to use the information related to shelters and transhipment nodes from the solution of MP. The parameter ω is a coefficient chosen to represent variations in instance parameters, especially the relative magnitudes of arc capacity and cost parameters. We test the value of ω on a series of numbers from 0.5 to

100, and select a few numbers to compose the multiplier vector W based on their performance of improving the upper bound of the objective value of SEND in BD framework.

III.3.4.3. Strengthening Benders Cuts

We observe that, in our numerical studies, the Benders optimality cuts obtained as outlined above are rarely effective in facilitating generation of good lower bounds. Main reason for this can be attributed to the fact that the Benders subproblem is essentially a network flow problem with multiple optimal solutions. In such a situation, it is possible that one can generate multiple alternative Benders optimality cuts, each of which corresponding to a different optimal dual subproblem solution. Then, it is clear that we are interested in choosing, among these optimal solutions, the one that provides a strong Benders optimality cut. For this purpose, Magnanti and Wong (1981) define the strongness of a cut is as follows: in an optimization problem $\mathrm{Min}_{y\in\mathcal{Y},\, z\in\mathcal{R}}\{z : f(u)+y\,g(u) \leq z, \forall\, u \in U\}$, if $f(u_1)+y\,g(u_1) \geq f(u_2)+y\,g(u_2) \forall\, y \in \mathcal{Y}$ with a strict inequality for at least one $y \in \mathcal{Y}$, then the cut $f(u_1) + y\,g(u_1) \leq z$ is stronger than the cut $f(u_2) + y\,g(u_2) \leq z$.

Thus, we develop an approach for our formulation to generate the strengthened Benders cuts by solving the DSP in a two phase approach (Roy, 1986; Wentges, 1996; Easwaran and Üster, 2009; Üster and Agrahari, 2011). In this approach, given the values of variables $\hat{z}_{odij}$, $\hat{r}^{\mathcal{E}}_{ij}$, $\hat{r}^{\mathcal{D}}_{d}$, $\hat{r}^{\mathcal{I}}_{i}$, $\hat{e}^{\mathcal{E}}_{ij}$, and $\hat{e}^{\mathcal{D}}_{d}$ from MP, we first solve the DSP and record the values of dual variables associated with the non-zero coefficients in the DSP objective function. Those dual variable values dictate the value of the DSP optimal objective value and must be kept as they are. However, the dual variables that are associated with the zero coefficients can take any value (with some exceptions as given below) without affecting optimality since they are nullified regardless. Thus, to obtain strengthened bounds, we solve the following optimization problem 2PDsP as a second phase problem:

$$\textbf{(2PDSP)} \qquad \text{Max} \sum_{o\in\mathcal{O}}\sum_{d\in\mathcal{D}}\sum_{(i,j)\in\mathcal{A}} \hat{z}_{odij}\,\mu_{odij} - \sum_{o\in\mathcal{O}}\sum_{d\in\mathcal{D}}\sum_{(i,j)\in\mathcal{A}} s_o\,\omega_{odij}$$

$$- \sum_{d\in\mathcal{D}} q_d^{\mathcal{D}}\,(1+\lambda)\,\alpha_d - \sum_{\{i,j\}\in\mathcal{E}} q_{ij}^{\mathcal{E}}\,(1+\frac{1}{b_{ij}^{\mathcal{E}}})\,\theta_{ij} \qquad (3.59)$$

$$\text{subject to} \quad (3.23) - (3.34).$$

The 2PDsP model is obtained from DSP as follows. We exclude the first term ($\sum_o \rho_o$) since it is not factored by any dual solution and thus constant after the first phase is solved. We leave the second term as it is since the μ_{odij} variables corresponding to $\hat{z}_{odij}$ with zero value cannot be changed as this leads to unboundedness in 2PDsP (or, equivalently, infeasibility of SP due to constraint (3.16) which forces all x_{odij} to be at least one, if all z_{odij} is set to non-zero values). In the last three terms, we fix the values of ω_{odij}, α_d, and θ_{ij} associated with non-zero coefficients as obtained in the first phase and treat the others as decision variables. The constraint set is modified accordingly via fixing the above mentioned variable values from the first phase. A combined set of solutions obtained in the first and the second phases is used to generate a Benders optimality cut of the form (3.41).

III.3.5. Overall Algorithm

To this end, we already introduced each piece of the approach in details, and then we present the integrated framework in Algorithm 2 as follows, where UB is the upper bound for the objective value of SEND in BD framework and LB is the lower bound for the objective value of SEND in BD framework.

Algorithm 2 Benders Decomposition Algorithm

initialize LB = $-\infty$, UB = ∞. Initialize ϵ and `CoefficientArrary`
Develop a feasible solution for **SEND** to obtain $\hat{z}_{odij}$, $\hat{r}^{\mathcal{E}}_{ij}$, $\hat{r}^{\mathcal{D}}_{d}$, $\hat{e}^{\mathcal{E}}_{ij}$, $\hat{e}^{\mathcal{D}}_{d}$, and $\hat{r}^{\mathcal{I}}_{i}$
while `gap` $> \epsilon$ **do**
    Substituting $\hat{z}_{odij}$, $\hat{r}^{\mathcal{E}}_{ij}$, $\hat{r}^{\mathcal{D}}_{d}$, $\hat{e}^{\mathcal{E}}_{ij}$, $\hat{e}^{\mathcal{D}}_{d}$, and $\hat{r}^{\mathcal{I}}_{i}$ to **DSP** and solve **DSP**
    if DSP is unbounded **then**
        Generate a feasibility cut
        for each ω in the multiplier vector W **do**
            Generate a feasible solution (refer to **Algorithm 1**)
            Substituting $\hat{z}_{odij}$, $\hat{r}^{\mathcal{E}}_{ij}$, $\hat{r}^{\mathcal{D}}_{d}$, $\hat{e}^{\mathcal{E}}_{ij}$, $\hat{e}^{\mathcal{D}}_{d}$, and $\hat{r}^{\mathcal{I}}_{i}$ to **DSP** and solve **DSP**
            Solve **2PDSP** to generate a strengthened optimality cut
            Update UB
        end for
        Add the feasibility cut and the multiple strengthened optimality cuts to **MP**
    end if
    Solve **MP** with early termination criteria to obtain $\hat{z}_{odij}$, $\hat{r}^{\mathcal{E}}_{ij}$, $\hat{r}^{\mathcal{D}}_{d}$, $\hat{e}^{\mathcal{E}}_{ij}$, $\hat{e}^{\mathcal{D}}_{d}$, and $\hat{r}^{\mathcal{I}}_{i}$, and the LB
    `gap` = (UB-LB)/UB
end while
return UB and the corresponding solution

III.3.6. Other BD Enhancements Tested on the SEND Problem

Benders Decomposition is well known to study mixed integer programming, and there are a lot of variations proposed in recent years. Before we develop our own solution methodology, we did a comprehensive review in this field. In our problem, for most of iterations, the dual subproblems are unbounded, so Benders cuts are generated from extreme rays, called feasibility cuts. However, these feasibility cuts cannot improve lower bound efficiently. To tackle this difficulty, we focus on two variations: accelerating Benders decomposition by local branching, and improving Benders decomposition using maximum feasible subsystem. We review these studies and employ them in our problem. However, they do not show good performances in computational studies. Thus, they are not included in our solution approach.

III.3.6.1. Accelerate Benders Decomposition by Local Branching

In 2009, Rei et al. (2009) proposed a new variation to accelerate Benders decomposition by local branching. The main idea of local branching is to divide the feasible region of the original problem to several small pieces and find the optimal solution in each piece. There are two purposes for using local branching in Benders decomposition: first, find a better upper bounds by using local search; second, generate optimality cuts to obtain the better lower bounds by adding multiple cuts. The scheme for applying local branching in Bender decomposition frame is presented in Algorithm 3. The mechanism of local branching is similar to the one for branch-and-bound algorithm. (x^t, y^t) is considered as the current feasible solution, and x^t is the solution for integer variables, y^t is the solution for continuous variables. The distance between x^t and x is measured by Hamming distance function and represented as $\Delta(x, x^t)$. If x is the solution for binary variables, the distance function is very simple; however, if x is the solution for general variables, the case becomes much more complicated (Fischetti and Lodi, 2003). Based on this distance function, set S can be divided to two subsets S_1 and S_2. For all x in subset S_1, $\Delta(x, x^t) \leq k$; for all x in subset S_2, $\Delta(x, x^t) \geq k + 1$. Thus, the original problem is divided to two subproblems P_t and $\overline{P}_t$. Problem P_t is the original problem plus the additional constraint $\Delta(x, x^t) \leq k$, and Problem $\overline{P}_t$ is the original problem plus the additional constraint $\Delta(x, x^t) \geq k+1$. Based on this setting, the detail steps for local branching algorithm is presented in Algorithm 4.

Algorithm 3 Apply Local Branching in BD

1: Initialize $i = 0$
2: Start with a solution (x^i, y^i)
3: **while** Gap between upper bound and lower bound $> \varepsilon$ **do**
4: $\quad i{+}{+}$
5: $\quad$ Local branching and generating multiple feasible solutions (find the minimum objective value as upper bound)
6: $\quad$ Add the multiple optimality cuts to MP to improve the lower bound
7: $\quad$ Solve MP to get a new solution (x^i)
8: **end while**

Algorithm 4 Local Branching Algorithm

1: Initialize k, and (x^t, y^t) is the current solution
2: Generate the two subproblems P_t and $\overline{P}_t$
3: **while** Finish exploring the feasible region **do**
4: $\quad$ Solve subproblem P_t
5: $\quad$ **if** P_t is feasible **then**
6: $\quad\quad$ Check the value of the objective function Obj
7: $\quad\quad$ **if** the current $Obj <$ the last Obj **then**
8: $\quad\quad\quad$ Divide the feasible region of $\overline{P}_t$ as before using the distance function $\Delta(X, X^{t+1})$, creating the new subproblems
9: $\quad\quad\quad$ Change the subproblem P_t to $\overline{P}_t$
10: $\quad\quad$ **else**
11: $\quad\quad\quad$ Go to step 4
12: $\quad\quad$ **end if**
13: $\quad$ **else**
14: $\quad\quad$ Increase the size of k, $k = k + 1$
15: $\quad\quad$ Go to step 4
16: $\quad$ **end if**
17: **end while**

This variation of Benders decomposition can be applied to our problem. In our case, we have four groups of binary variables r_d, r_i, r_{ij}, and z_{odij}. Since the binary variables z_{odij} are four dimensional variables, the local branching for this group can be very time consuming. Also, the solutions for variables r_d have little impact for the other variables, since there may be a lot of choice for routes even the shelters' locations are fixed. Furthermore, the solutions for r_i are very related to the solutions for r_{ij}. Thus, we decide to do local branching on binary variables r_{ij}. When the original problem is complicated, it is hard to solve problem P_t. To save the solution time, we solve the subproblem P_t using Benders decomposition. However, with the size of the neighborhood defined by the distance function increase, it is still very difficult to solve the subproblem P_t. For circumvent this difficulty, we employ the mechanism that master problem is not solved to optimality in first several iterations. We test this algorithm in our computational study. However, the test results show that this algorithm does not work for our problem, since our problem is involved with high dimensional variables. The test results are showed in Table 4.

We use the data in Class 3 to test the Benders decomposition with the local branch (BDLB), and use the Case I parameters. The data and the parameters are introduced in Chapter IV. The results are listed in the Table 4. The column "Solution Time" is the solution time under the designated stop criterion. The column "Gap" is the gap at which tests stop. The stop criterion is set as: optimality gap $< 3\%$ or number of iterations ≥ 5. For all networks, BDLB can not solve the problem to less than 3% gap within 5 iterations. We also test the same instances for BD without local branch (i.e. the traditional BD). There is no feasible solution in 5 iterations, so there is no upper bound and no gap between upper bound and lower bound. Although, BDLB performs better than the traditional BD, BDLB still cannot solve our problem to a small gap within a reasonable time.

Table 4 Tests for BD with Local Branch in Class 3

	BDLB		
	Solution Time	Gap (%)	Solved
Network 1	8742	10.5	N
Network 2	8673	9.8	N
Network 3	8847	10.9	N

III.3.6.2. Improving Benders Decomposition Using Maximum Feasible Subsystem (MFS)

In 2010, Saharidis and Ierapetritou (2010) presented an approach to improve Benders decomposition using MFS cut generation strategy. As introduced at the beginning of this chapter, if a dual subproblem is bounded, an optimal solution (an extreme point for the solution space) is found, and an optimality cut is generated. Otherwise, an extreme ray is found, and a feasibility cut is generated. Unlike the optimality cuts, feasibility cuts have few contribution in improving the lower bound in Benders decomposition. Thus, if the number of feasibility cuts is large, the convergent rate for Benders decomposition is slow (Saharidis and Ierapetritou, 2010). To tackle this obstacle, every time when a feasibility cut is generated, an additional optimality cut is produced. This additional optimality cut is produced by the modified subproblem. For obtaining a feasible solution, a minimum number of constraints are relaxed from the original subproblem, and then the modified subproblem is produced. This additional optimality cut is an extreme point in the solution space of the modified subproblem. However, an arbitrary choice for the extreme point may not have the most contribution to improve the lower bound. Thus, Saharidis and Ierapetritou (2010) suggest to find the MFS cuts. The strategy for generating the MFS cuts can be achieved in two steps: first, find the maximum feasible subsystem of the original subproblem; second, relax all infeasible constraints to find a feasible solution. In general, a mixed integer model can be represented as follows.

(Initial Problem)

$$\begin{aligned}
&\text{Min } \ c^T x + d^T y \\
&\text{subject to} \\
&Ax + By \leq b \\
&Fy \leq p \\
&x \in R^n_+, y \in Z^q_+
\end{aligned}$$

By fixing the values for integer variables y, the subproblem has the following form:

(Sub Problem)

$$\begin{aligned}
&\text{Min } \ c^T x + d^T \overline{y} \\
&\text{subject to} \\
&Ax \leq b - B\overline{y} \\
&x \in R^n_+
\end{aligned}$$

When the subproblem is infeasible, to determine its maximum feasible set, the following problem is solve, where M is a big positive number.

(Extended Sub Problem)

$$\begin{aligned}
&\text{Min } \ w_1 + w_2 + \cdots + w_m \\
&\text{subject to} \\
&Ax - MIw \leq b - B\overline{y}
\end{aligned}$$

$$x \in R_+^n, w = \{0, 1\}$$

If $w = 0$, that means the corresponding constraint should be included in the maximum feasible subsystem; otherwise, the corresponding constraint should be removed from the subproblem to make it feasible. Based on this idea, the primal Max FS problem (PMFSP) is generated. Assuming $w_1 = 1, w_2 = w_3 = \cdots = w_n = 0$, the PMFSP is formulated as follows.

(PMFSP)

$$\text{Min } c^T x + d^T \overline{y}$$

subject to

$$Ax_1 \leq b - B\overline{y}_1 + M$$

$$Ax_2 \leq b - B\overline{y}_2$$

$$\cdots$$

$$Ax_m \leq b - B\overline{y}_m$$

$$x \in R_+^n$$

In the generated MFS cut, due to the complimentary slackness theorem, the dual variables that corresponding to the relaxed constraints are zero. The MFS cuts are added to the master problem to improve the lower bound.

We applied this variation of Bender decomposition to our problem. However, ESP is a mixed integer problem, and it is hard to solve especially for the large scale problem. After we formulate the ESP, we see that it is almost the same size as our original problem, and it is really difficult to solve for the large scale instances. We test this algorithm in our computational study using the data in Class 3. However, the ESP problem cannot be solved within 10 minutes, since this problem is solved repeatedly in each iteration. The long solution time for ESP can be a huge burden for the whole solution methodology. Thus, this algorithm is removed from the candidates pool for our solution methodologies.

III.4. Computational Study

In this section, we conduct two experiments. In the first experiment, we test the performance of our emergency evacuation model and the proposed solution algorithm, and we conduct this experiment based on an evacuation scenario in coastal Texas. We benchmark the performance of our solution methodology against a traditional branch and cut (B&C) solution strategy.

Second, we conduct an experiment to evaluate the effect of three parameters: T (the established safe evacuation time), λ (how many times the shelter capacity can increase at most), and ξ (fixed cost for generating extra capacity of shelter).

We use C++ to implement the proposed solution algorithm, and we use CPLEX 12.2 (64 bits) with default settings to solve the MP and the DSP in the BD framework. Also, we use the same version of CPLEX with identical settings to solve the original problem with B&C approach. All machines used have 2.4 GHZ Intel Core 4 CPU processors with 8 GB RAM. All spatial analysis is conducted using ArcGIS 10 on identical machines. The remainder of this section is organized as follows. In § III.4.1, the generation of the underlying networks are presented. In § III.4.2.1, we conduct an experiment to prove the effectiveness and the efficiency of the SEND model and the BD approach, and we benchmark the performance of the accelerated BD approach against the traditional B&C solution strategy. In § III.4.2.2, we conduct an experiment to study the effects of parameters T, λ and ξ on the optimal solution.

III.4.1. Network Generation

For having a basis of our evacuation scenario, we obtain the spatial data (i.e. traffic networks and county-divisions) and the population data for Texas from the U.S. Census Bureau. The spatial data comes in the format of TIGER files from the 2009 U.S. Census Bureau and all population data is from the 2000 Economic Census. To develop a scaled evacuation scenario capable of testing the accelerated BD approach against the B&C (CPLEX 12.2), we define our underlying network by choosing a part of the primary and the secondary roads from the real traffic network of Texas. We choose our potential sheltering areas from the 2009 Texas State shelter hubs which is released by the Texas Department of Public Safety (DPS). We define a potential sheltering area as an area which may include one potential shelter or a few potential shelters. Figure 3 illustrates all 17 Texas Shelter Hubs, and we choose the central portion of this map (the portion below the bold-black line) as the study area to develop our scenario. In this study area, 9 counties are considered as potential sheltering areas: Brazos, Walker, Dallas, Tarrant, McLennan, Travis, Bexar, Nacogdoches and Smith. We consider 5 coastal Texas counties as the affected areas where the residents need to be evacuated to shelters, and these 5 counties are the evacuation zones designated by Texas DPS for 2009 hurricane evacuation. They are Matagorda, Brazoria, Galveston, Chambers, and Harris. Each affected area may include one origin or several origins. Population for each affected area is considered as the number of evacuees in this area and is provided in Table 5. Population for each potential sheltering area is used to evaluate the capacity of this area to accommodating evacuees and is presented in Table 6.

Figure 4 illustrates the condensed network used in our scenario. We select a part of the primary and the secondary roads in our study area, which is the portion below

Figure 3 Texas State Hurricane Shelter Hubs in 2009

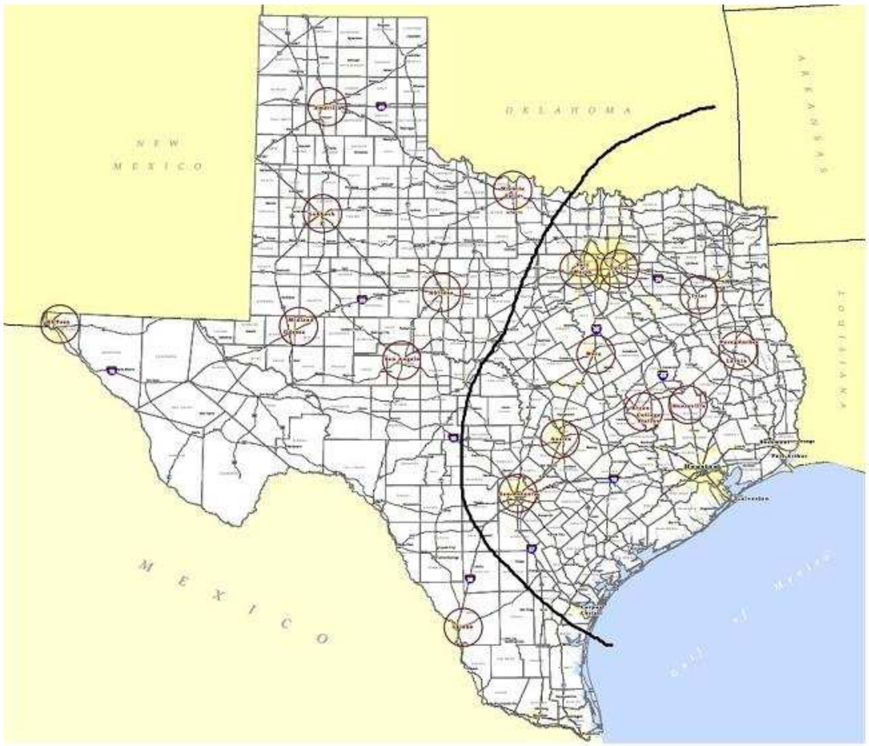


Table 5 Population of the Affected Areas in 2000 U.S. Census

Index	County	Population
1	Matagorda	37,265
2	Brazoria	301,044
3	Galveston	288,239
4	Chambers	26,031
5	Harris	3,984,349

Table 6 Population of the Potential Sheltering Areas in 2000 U.S. Census

Index	County	Population
1	Brazos	152,415
2	Walker	61,758
3	Dallas	2,218,899
4	Tarrant	1,446,219
5	McLennan	213,517
6	Travis	812,280
7	Bexar	1,392,931
8	Nacogdoches	59,203
9	Smith	174,706

the bold-black line in Figure 3, to construct our underlying network. The roads in our underlying network are highlighted in red in Figure 4. All junctions in this network are considered as transhipment nodes in the SEND problem. Figure 4 also illustrates the potential sheltering areas (9 counties) and the affected areas (5 counties). The numbers in the parenthesis after each potential sheltering area presented in the legend bar are the indices of potential shelters in this potential sheltering area for one class of our experiment as introduced next.

The 5 affected areas and the 9 potential sheltering areas are used as a basis to generate an extended experiment to test the SEND model and the accelerated BD approach. We split the 5 affected areas and the 9 potential sheltering areas by zip-code to create a maximum of 47 origins and 22 shelters, respectively. As shown in Table 7, we derive 4 classes networks, which have different numbers of origins, shelters, nodes and arcs, to test the SEND model and the accelerated BD approach. In Figure 4, the numbers in the parenthesis after each potential sheltering area in the legend bar is the indices of potential shelters in this potential sheltering area in Class 1, which has 14 potential shelters. For each class, we modify the network presented in Figure 4 to generate two new networks, so we have three networks for each class. In our experiment, all data is real data or generated based on real data, except the parameters of road capacities and the fixed costs of edges and transfer nodes. To test the robustness of the SEND model and the accelerated BD approach, we set road capacities at two levels (a low level and a high level) and set fixed costs of edges and transfer nodes at two levels (a low level and a high level). Thus, we have 4 cases for parameters of road capacities and fixed costs as presented in Table 8, and these 4 cases are labeled as I, II, III, and IV.

In our computational study, we make several assumptions to ensure consistency. For each origin, we assume that the entire population within that area leave from

Figure 4 Study Network in Central Texas Area

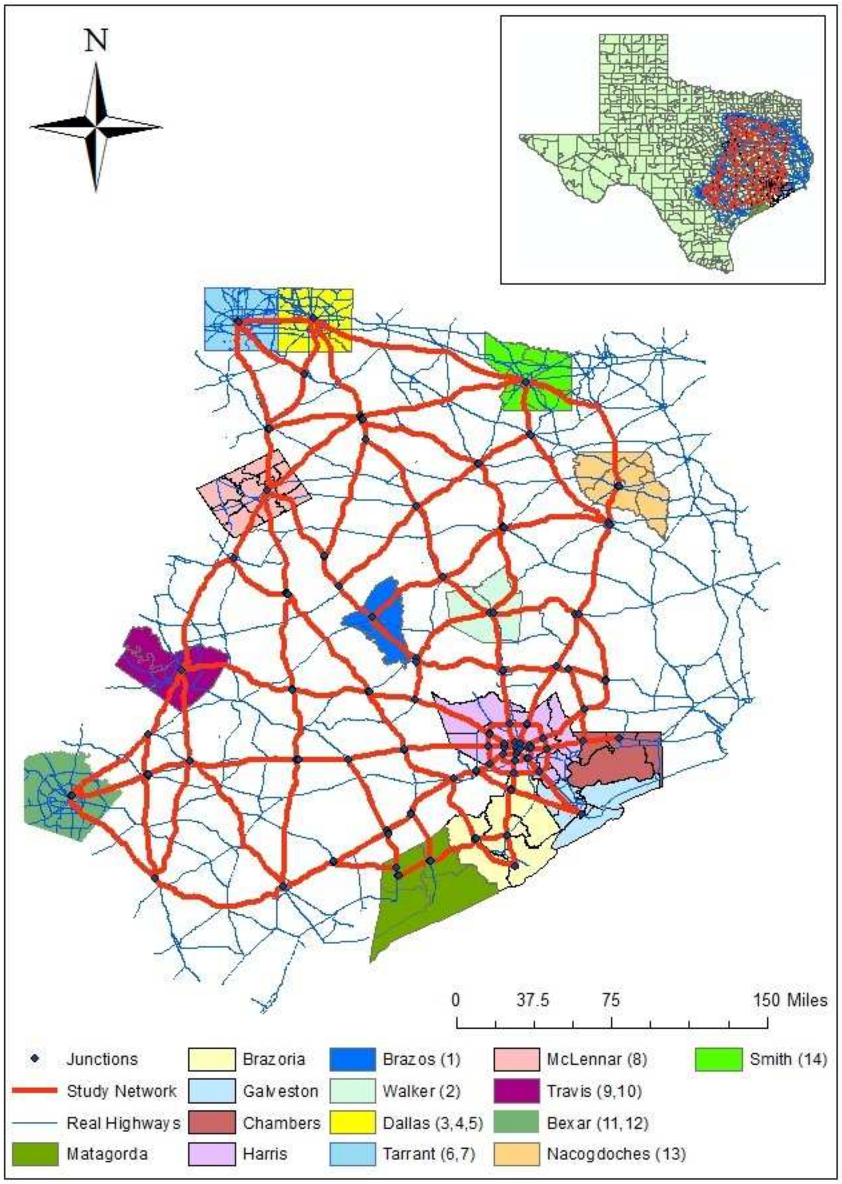


Table 7 Four Classes Networks

	Origins	Shelters	Nodes	Arcs	Variables			Constraints
					Intg.	Cont.	Total	
Class 1	12	14	94	346	58,409	58,296	116,705	338,832
Class 2	18	18	99	400	129,917	129,924	259,841	745,429
Class 3	24	22	108	462	244,297	244,464	488,761	1,392,446
Class 4	47	22	128	512	529,814	530,442	1,060,256	3,046,895

Table 8 Four Cases for Parameters of Road Capacity and Fixed Costs

		Roads Capacity	
		High	Low
Fixed Costs	Low	I	II
	High	IV	III

the centroid of that area, and the travel distance within that area can be ignored. We make these assumptions reasonable by letting each origin represent a small area. Additionally, each origin is composed of a few zip-code areas such that the variation of populations among origins is small. The population of each origin is the sum of the populations of zip-code areas which compose the origin. Using the same approach, we identify each potential shelter by combining a few zip-code areas. Similarly, the population of each potential shelter is the sum of the populations of zip-code areas which compose the potential shelter. We assume that the original capacity of a shelter has a linear relationship with its population. Moreover, we realize that the total capacity of potential shelters should be larger than the total population of origins; otherwise, the problem is infeasible. Thus, we set the total original capacity of potential shelters as 1.5 times of the total population of the origins. Each road segment (i.e. an edge) has a capacity, and we define this capacity as $2000 * \upsilon$ cars/hour, where υ is a coefficient.

III.4.2. Computational Experiments

III.4.2.1. Experiment for Testing Efficiency of BD Approach

As introduced above, for each class in Table 7, we generate 3 networks. Also, for each network, we have 4 cases for parameters of road capacities and fixed costs of edges and transfer nodes. For each case, we generate 5 random instances by a uniform distribution. Thus, there are totally 240 instances (i.e. there are 4 class types, 3 generated networks, 4 cases for parameters and 5 random instances) tested in our experiment. We use a uniform distribution to randomly generate fixed costs for edges and transhipment nodes, as shown in Table 9. We assume that the capacity for each lane in the whole evacuation process is 2000 times a coefficient υ, where υ is 36 for low level capacity and 48 for high level capacity. The capacity of an edge is the product of the capacity of each lane and the number of lanes on this edge. Also, we assume that the original capacity of a shelter is proportional to its population (i.e. a larger town can accommodate more evacuees) and assume that the total capacity of shelters is greater than the total evacuation population. Moreover, we assume that the fixed cost for opening a shelter with original capacity is proportional to its original capacity. Thus, we create parameters of shelter capacities and shelter fixed costs as shown in Table 9. The parameter λ means that how many times the shelter capacity can increase at most (i.e. if λ is 5, a shelter can increase its capacity by 5 times). The parameter ξ is the fixed cost for generating extra capacity of shelters (e.g. if ξ is 2, the fixed cost of a shelter increases 2 times while its capacity increases 1 time).

Table 9 Parameters in Experimental Design

E^a Capacity	L^b	2000*number of lanes*36
	H^c	2000*number of lanes*48
Fixed Cost for Edges	L^b	Primary: Uniform[150, 250]; Secondary: Uniform[100, 200]
	H^c	Primary: Uniform[200, 300]; Secondary: Uniform[150, 250]
Fixed Cost for Nodes	L^b	Primary: Uniform[200, 300]; Secondary: Uniform[150, 250]
	H^c	Primary: Uniform[250, 350]; Secondary: Uniform[200, 300]
S^d Capacity		Its Population*1.5*Total Origin Population/Total Shelter Population
S^d Fixed Cost		Its Capacity*450000/Total Shelter Capacity
Evacuation Time		16 Hours
λ		5
ξ		2

Note. a: ”E” represents Edge; b: ”L” represents Low; c: ”H” represents High; d: ”S” represents Shelter.

Table 10 present comparisons of the time required to obtain the solution by

BD and B&C approaches for instances in Class 1, 2, 3, and 4, respectively. The optimality gap for cases I and II is set as 3% in Class 1,2,3, and 3.6% in Class 4. The optimality gap for cases III and IV is 3.6% in Class 1,2,3, and 4.5% in Class 4. Table 11 reports the average number of iterations required by BD.

Table 10 Average Solution Times for BD and B&C approaches

	Class 1		Class 2		Class 3		Class 4	
	BD	B&C	BD	B&C	BD	B&C	BD	B&C
Case I	12.5	364.0	40.4	1693.3	276.4	4791.0*	608.7	> 7200
Case II	4.1	383.1	25.5	1216.7	143.7	2955.9	540.6	> 7200
Case III	8.4	420.9	39.5	1780.3	216.4	3911.0*	459.1	> 7200
Case IV	4.0	423.3	27.3	1792.2	149.9	3264.4*	377.1	> 7200

*Note. *: not all instances can be solved within 2 hours, and the average solution time is calculated from the solvable instances.*

Table 11 Average Number of Iterations

	Class 1	Class 2	Class 3	Class 4
Case I	2.4	2.8	3.6	2.8
Case II	1.0	1.8	2.3	2.5
Case III	1.7	2.7	3.1	2.2
Case IV	1.0	1.9	2.3	1.9

The results reported in Table 10 indicate that the accelerated BD approach performs much better than the traditional B&C strategy in solving the SEND model. By using the accelerated BD approach, the average solution time decreases dramatically for instances in Class 1 and Class 2. For instances in Class 3, by using the traditional B&C strategy, there are 4 out of 15 instances cannot be solved in 2 hours for case I; and there are 6 and 2 instances cannot be solved in 2 hours for case III and IV, respectively. For those instances which cannot be solved by B&C in Class 3, there is even no feasible solution founded in 2 hours. Furthermore, for all instances in Class 4, B&C is unable to find a feasible solution within 2 hours while BD can obtain optimal solutions around 10 minutes. Thus, we can conclude that our proposed solution methodology can solve the SEND model in large-scale instances efficiently, and the computational performance of our proposed solution methodology is much better than the traditional B&C strategy in solving the SEND problem.

III.4.2.2. Experiment for Parameters Sensitivity Analysis

In experiment II, we analyze the effect of three parameters: T, λ, and ξ. Through this experiment, we look for the difference of locations of open shelters and the difference of the usages of extra shelter capacities. We use p to represent the usages of extra shelter capacities (i.e. if $p = 0.2$, the shelter capacity increases by 20 percent), and p is less than or equal to λ. For this purpose, we test T in 6 levels (in hours): 24, 22, 20, 18, 16 and 14. λ is tested in 2 levels: 0.5 and 5, and ξ is tested in 2 levels: 1.2 and 2. We test these 3 parameters on one instance in Class 1 with parameters of case I, and we run 24 tests (i.e. there are 6 values for T, 2 values for λ, and 2 values for ξ).

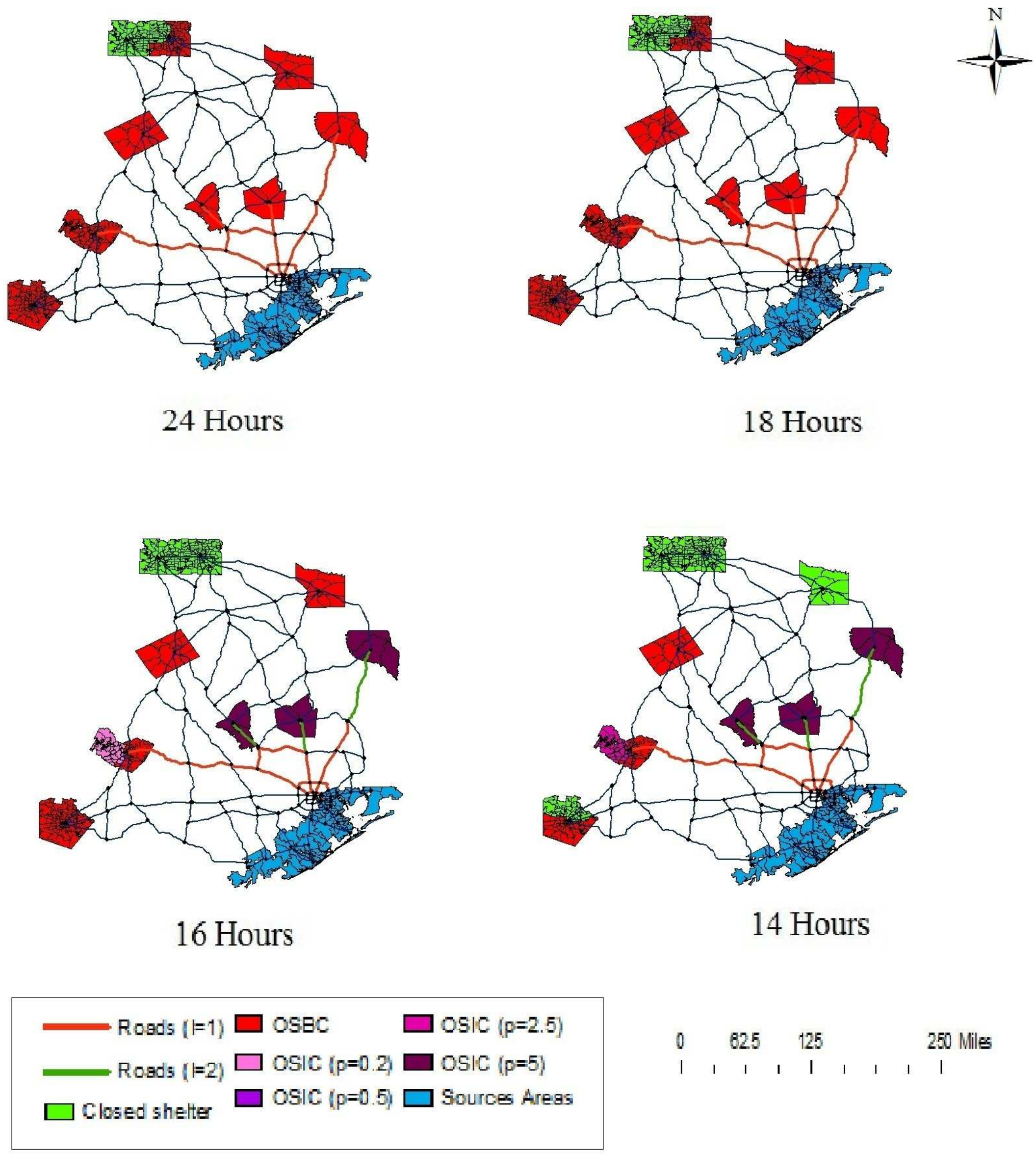
To see the effect of the established safe evacuation time T on the optimal solution, we fix the value of λ and ξ and look at the optimal solutions under different T. From these solutions, we find that when the established safe evacuation time decreases, the nearby shelters are used rather than the far shelters. This can be understood intuitively, since arriving far shelters may need longer traveling time which may be larger than the established safe evacuation time. As shown in Table 12 and Figure 5, $\lambda = 5$ and $\xi = 2$, when the established safe evacuation time is 24 hours, almost every potential shelter is open, except the three farthest shelters, and no open shelter requires extra capacity. The shelters in red are the open shelters, and the shelters in purple-colors are the shelters with extra capacity. The depth of purple indicates the different usages of extra shelter capacities. Because the fixed cost for using one unit of extra shelter capacity is larger than the fixed cost for using one unit of original shelter capacity, the SEND model always tries to open new shelters without giving open shelters extra capacities to satisfy the flow requirements. When the established safe evacuation time is 24 hours, the time constraints are loose, so evacuees have enough time to travel to far shelters instead of congesting at the closed shelters to incur more fixed cost for requiring extra shelter capacities. When the established safe evacuation time decreases to 18 hours, there is no difference from the case of 24 hours. It means that the time constraints are not bounded when the safe evacuation time is 24 hours. When the established safe evacuation time is 16 hours, less far shelters are open, and the nearby shelters require extra capacities to satisfy the flow demand. In this case, because the established safe evacuation time is not long enough to travel to far shelters and few shelters are open, some of open shelters have to have extra capacities to satisfy the total flow demand. When the established safe evacuation time is 14 hours, comparing to the case of 16 hours, less number of far shelters are open, and nearby shelters requires more extra capacities to satisfy the total flow demand. Thus, if the established safe evacuation time T decreases, the SEND model will open more nearby shelters rather than far shelters, and it will force the nearby shelters to use the extra capacities to satisfy the total flow demand. Moreover, we find that when the established safe evacuation time decreases, the roads connecting to the nearby shelters need more extra capacities. As shown in Figure 5, each highlighted red line is a road which has a new lane added, and each highlighted green line is a road which has two new lanes added. When the established safe evacuation time is 24 hours and 18 hours, there are roads with one added lane but no road having two added lanes. When the established safe evacuation time decreases to 16 hours and 14 hours, three roads connecting to the nearby shelters require more capacities, and each of them have two new lanes added. The flows on roads are related to the inflows of the shelters which are connected to the roads. For the three nearby shelters, the capacity of each one increases by 5 times, so the roads connecting to these shelters need more extra capacities to satisfy flow demand. This is the reason why the roads connecting to these shelters have two new lanes added.

Table 12 Open Shelter Locations with $\lambda = 5$ and $\xi = 2$

Shelter Index		1	2	3	4	5	6	7	8	9	10	11	12	13	14
$T = 24$	Open Shelter	1	1	0	1	1	0	0	1	1	1	1	1	1	1
	Extra Capacity	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$T = 18$	Open Shelter	1	1	0	1	1	0	0	1	1	1	1	1	1	1
	Extra Capacity	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$T = 16$	Open Shelter	1	1	0	0	0	0	0	1	1	1	1	1	1	1
	Extra Capacity	5	5	0	0	0	0	0	0	0.2	0	0	0	5	0
$T = 14$	Open Shelter	1	1	0	0	0	0	0	1	1	1	0	1	1	0
	Extra Capacity	5	5	0	0	0	0	0	0	2.5	0	0	0	5	0

To test the effect of λ, we fix the value of T and ξ and analyze the results under different λ. When λ is smaller, more open shelters may have extra capacities. This

Figure 5 Open Shelter Locations with $\lambda = 5$ and $\xi = 2$



can be understand intuitively. If λ is smaller, since the maximum total capacity (original capacity plus the extra capacity) that a shelter can obtain is smaller, more shelters needs to have extra capacities to satisfy the total demands. As shown in Table 13 and Figure 6 ($T = 16$ and $\xi = 2$), when λ is 5, less open shelters have extra capacities than when λ is 0.5. Moreover, from Figure 6 ($T = 16$ and $\xi = 2$), we see that when λ is smaller, the roads use less extra capacities. When λ is 0.5, there are roads with one added lane but no road having two added lanes. When λ is 5, there are three roads with two added lanes. As introduced above, the flows on roads are related to the inflows of the shelters which are connected to the roads. Since when λ is 5, nearby shelters use more extra capacities, the roads connecting to these shelters also use more extra capacities.

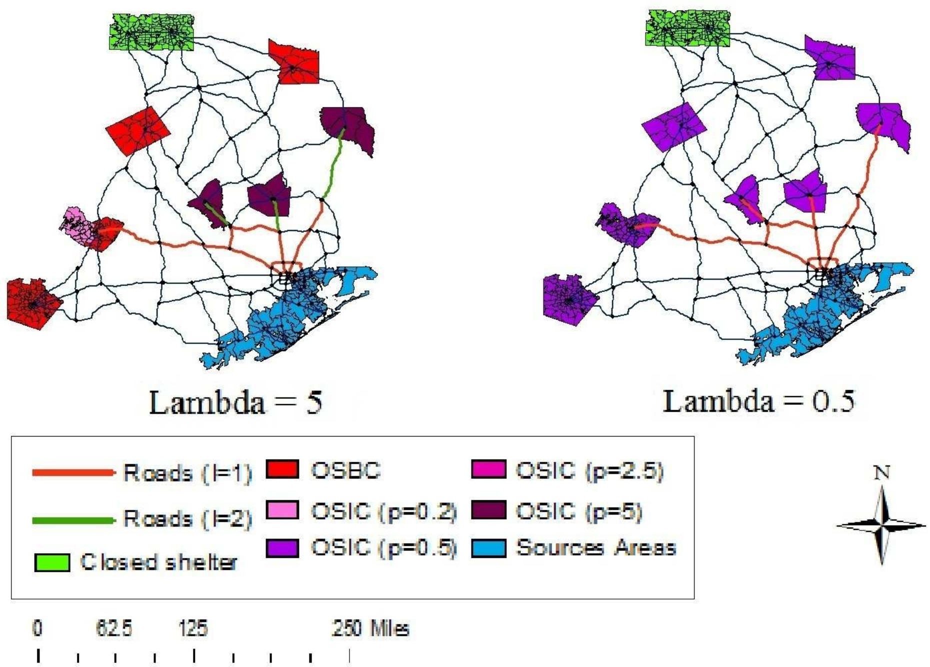
Table 13 Open Shelter Locations with $T = 16$ and $\xi = 2$

Shelter Index		1	2	3	4	5	6	7	8	9	10	11	12	13	14
$\lambda = 0.5$	OS^a	1	1	0	0	0	0	0	1	1	1	1	1	1	1
	EC^b	0.5	0.5	0	0	0	0	0	0.5	0.5	0.5	0.5	0.5	0.5	0.5
$\lambda = 5$	OS^a	1	1	0	0	0	0	0	1	1	1	1	1	1	1
	EC^b	5	5	0	0	0	0	0	0	0.2	0	0	0	5	0

Note. a: "OS" represents Open Shelter; b: "EC" represents Extra Capacity

To analyze the effect of ξ, we fix the value of T and λ and check the optimal solutions under different ξ. When ξ is smaller, the closer shelters may be used rather than the farther shelters, since using the farther shelters cause more transportation costs and more fixed costs for edges and transfer nodes. However, only using closer shelters may cause more fixed costs due to using extra shelter capacities, since the original capacities of closer shelters may not satisfy the total demand. Thus, there is a trade off between using farther shelters and closer shelters. It depends on the

Figure 6 Open Shelter Locations with $T = 16$ and $\xi = 2$



value of ξ. If the fixed costs for using extra shelter capacities is not larger than the extra costs due to traveling to the farther shelters, the model may use less shelters, which are closer to origins, and make these closer shelters have extra capacities; otherwise, the farther shelters are opened. Table 14 and Figure 7 show the open shelter locations with $\lambda = 5$ and $\xi = 1.2$. Comparing the Figure 5 and Figure 7, less shelters are open in Figure 7, and these open shelters are closer to origins. Also, more open shelters have extra capacities. Moreover, we find that the usages of road capacities are same for four cases. Because for the nearby shelters which have extra capacities, the usage of shelter extra capacities are same, the roads connecting to these shelters use same extra capacities for four maps.

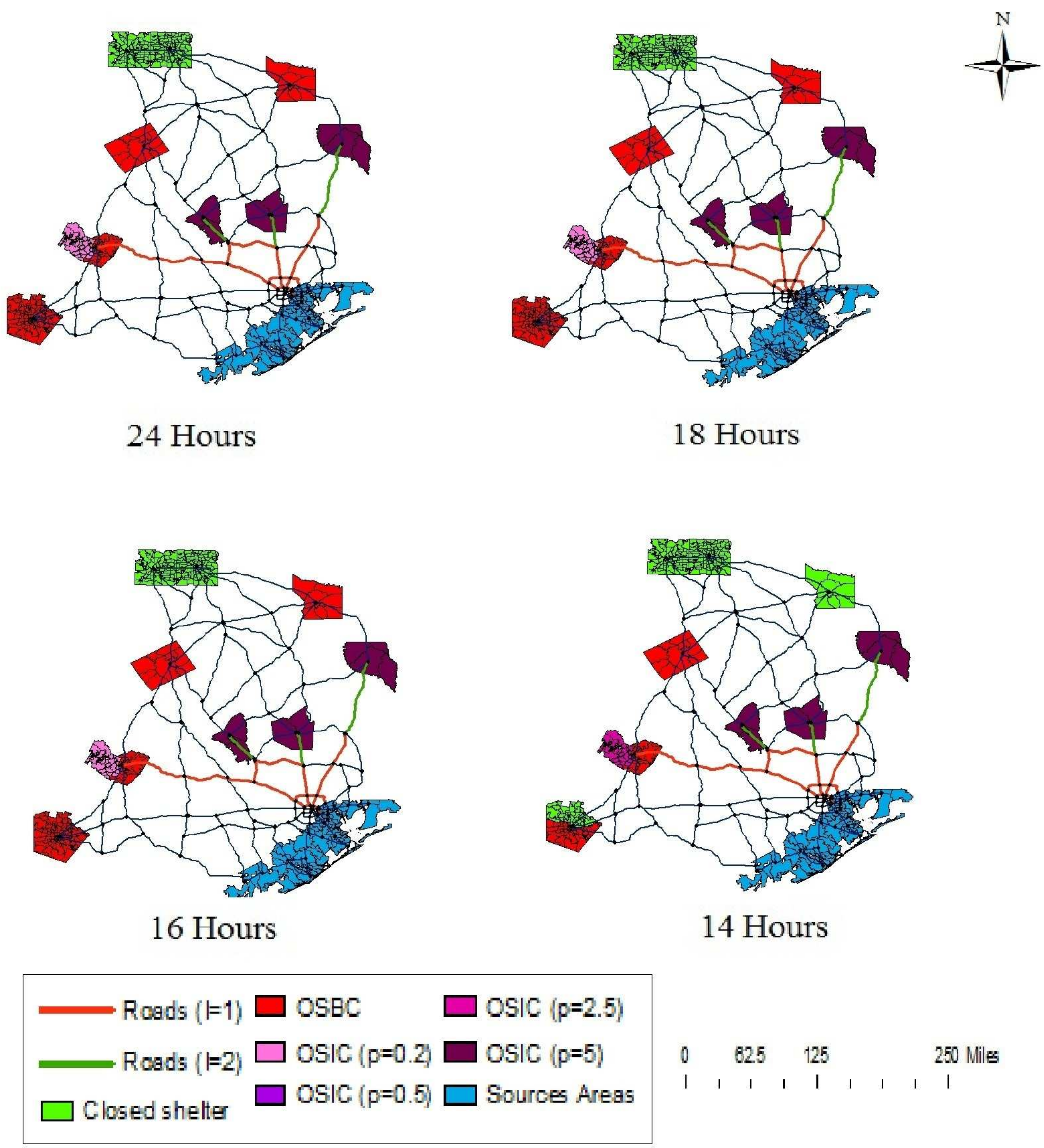
Table 14 Open Shelter Locations with $\lambda = 5$ and $\xi = 1.2$

Shelter Index		1	2	3	4	5	6	7	8	9	10	11	12	13	14
$T = 24$	Open Shelter	1	1	0	0	0	0	0	1	1	1	1	1	1	1
	Extra Capacity	5	5	0	0	0	0	0	0	0.2	0	0	0	5	0
$T = 18$	Open Shelter	1	1	0	0	0	0	0	1	1	1	1	1	1	1
	Extra Capacity	5	5	0	0	0	0	0	0	0.2	0	0	0	5	0
$T = 16$	Open Shelter	1	1	0	0	0	0	0	1	1	1	1	1	1	1
	Extra Capacity	5	5	0	0	0	0	0	0	0.2	0	0	0	5	0
$T = 14$	Open Shelter	1	1	0	0	0	0	0	1	1	1	0	1	1	0
	Extra Capacity	5	5	0	0	0	0	0	0	2.5	0	0	0	5	0

III.5. Summary

In this chapter, we pose and analyze a regional evacuation network design problem in order to provide a pre-event strategic planning tool for this purpose. We propose a mixed integer linear program to devise effective and controlled evacuation

Figure 7 Open Shelter Locations with $\lambda = 5$ and $\xi = 1.2$



networks for sending evacuees from their origins to shelters before extreme events such as hurricanes happen. The SEND model determines the optimal evacuation routes based on time and capacity constraints. Also, it selects shelters from a set of potential shelter candidates and decides flow assignments on the optimal routes while minimizing the total evacuation cost.

To solve this model for large scale instances, we develop an efficient solution methodology based on BD approach, which takes advantage of specific characteristics of the SEND problem. We utilize a few techniques to accelerate BD approach: adding surrogate constraints to MsP to improve the lower bound of the objective value of SEND in BD framework, solving MsP with a loose optimality gap in the first several iterations, adding multiple optimality cuts in each iteration by generating multiple feasible solutions of SEND heuristically, and strengthening Benders optimality cuts.

We design and implement an experimental design to test our BD technique using a Texas-based evacuation scenario. The SEND model and BD approach can be efficiently and effectively applied to a large scale evacuation scenario, and we benchmark the computational performance of our BD technique against the traditional branch-and-cut solution methods, which are implemented by CPLEX 12.2. We also design and implement an experiment to study the effects of parameters T, λ, and ξ on the optimal solution of SEND.

CHAPTER IV

MULTI-AGENT SIMULATION PROBLEM

In the SEND problem, we construct an optimization MIP model to analyze a regional evacuation network design problem in order to provide a pre-event strategic planning tool. The optimization MIP model determines the optimal evacuation network based on time and capacity constraints. It selects shelters from all potential candidates, chooses evacuation routes and decides flow assignments while minimizing the total costs. However, a centralized optimization model cannot handle unexpected situations, such as people not following the designated evacuation routes and/or not going to the designated shelters. In this case, evacuees may choose the routes or destinations that look favorable to themselves but not the routes or destinations recommended by the optimal evacuation plan. This may cause traffic jams in some road segments and make evacuees suffer a longer evacuation time. Furthermore, due to the difficulties in communication and coordination, especially for a large population in a chaotic emergency situation, evacuees may fail to follow the evacuation instructions because of misunderstandings and confusion. These situations may cause the optimal evacuation plan to not be achieved smoothly and successfully.

To handle these unexpected situations and to check the robustness of our optimization model, we conduct a multi-agent simulation (MAS) model. In the decentralized MAS model, every evacuee can make decisions and change those decisions during evacuation. The MAS model simulates the situation in which evacuees have the freedom to choose their own routes and their own destinations after they have been told the designated routes as guidelines. In the MAS problem, we study the effect of probabilities for people following the designated routes and the designated shelters on the total evacuation time, the traffic jam situation and the traveling time for individuals.

In our optimization model, there is no time component considered. We consider each edge as having a finite capacity on the total flow that it can handle in an evacuation event. This capacity is considered at a macroscopic level rather than with fine granularity as in a dynamic traffic assignment study. As a consequence, we consider a constant traffic speed and a constant traverse time for each edge. However, it is more complicated in real-world situations. Traffic speed and traverse time are normally not constant, but are related to traffic density on the road. To consider traffic speed as a variant with traffic density, we include traffic speed as a function of traffic density to the MAS model, so traffic speed and traverse time are changed dynamically with traffic density. Moreover, the MAS enables us to model the situation in which evacuees leave in groups at a time sequence. A value is assigned to the range of leaving times for each group. An evacuee may leave at any time in the range of leaving times for his group.

Furthermore, information sharing is an important difference between a centralized system and a decentralized system. In a centralized system, information sharing is assumed as perfect for the whole system; however, in a decentralized system, this is not the case. In evacuation problems, the information, which can influence the performance of the system, may or may not be shared perfectly. For example, evacuees may not know real-time traffic conditions and the status of shelters. In the MAS problem, the interactions between evacuees are considered as a type of approach to sharing information. We consider two types of information shared in the system.

- Information shared among evacuees-If there is slow traffic on a road segment, people who are driving on this road may call their connections (e.g. their friends, their relatives and their colleagues) to inform them the slow traffic.

Then people who receive this message may make a detour.

- Information sent from a radio station to all evacuees-we consider a radio station broadcast as another approach to sharing information. The radio station broadcasts the real-time traffic conditions and the status of shelters to all drivers (i.e. shelters are full or not). Evacuees may change their routes based on this received information.

We study the effect of the shared information on the evacuation performance.

IV.1. Literature Review

In recent decades, studies on the agent-based system have aroused more attentions. Agent-based system considers each agent as a subject, and each agent only considers itself and its environment to make its own decisions. The traditional and also the most common system, centralized system, has a few critical drawbacks: first, it is hard to make changes in the centralized system; second, in real-world cases, the quality of information may be not as good as we expected, also it may be very expensive to get good quality information, so each agent in the system may only access limited information; third, once errors happen in the centralized system, it may cause a fatal harm to the whole system. However, agent-based system lets each agent to make its own decisions and lets agents to communicate and negotiate with each other. Thus, the agent-based system has more flexibility, less complexity, and better error tolerance, comparing with the centralized system (Krothapalli and Deshmukh, 1999). Now, agent-based system is considered as a good alternative to the centralized system in many fields.

Recently, multi-agent systems are applied to evacuation problems to consider human behavior in a microscopic level. Chen et al. (2006) studied the evacuation problem of the Florida Keys by developing an agent-based simulation model. The objective of their model was to figure out the minimum clearance time to evacuate all population in that area. Chen (2008) employed a multi-agent system to simulate a regional evacuation problem and compared the performance of simultaneous and staged evacuation strategies. Their study claimed that the most efficient staged evacuation strategy can shorten the total evacuation time. In our MAS problem, we develop a multi-agent system to study a regional evacuation problem. We also investigate the effectiveness of staged evacuation strategies, and we study the effects of the number of stages on the evacuation performance.

In the recent years, several practical agent-based modeling toolkits has been developed to let individuals to develop agent-based applications. Nikolai and Madey (2009) made a comprehensive survey for all agent-based toolkits based on 5 characteristics. They listed the programming languages which are required to develop models in these toolkits, and they introduced the operating system which are needed to run these toolkit. Also, they introduced the type of license to manage these platforms. Moreover, the primary domain and technical support level of these toolkit are presented in their survey. Some studies also made surveys on the agent-based modeling toolkits (Railsback et al., 2006; Tobias and Hofmann, 2004; Castle and Crooks, 2006; Serenko and Detlor, 2002).

By reviewing these surveys, we select 4 open-source agent-based modeling toolkits as candidates for consideration. Table 15 lists the comparison for these 4 agent-based modeling toolkits: SeSAM, NetLogo, MASON, and Repast. Among these 4 toolkits, SeSAM is the easiest one to learn and use; however, the size of a model developed in SeSAM is limited. Thus, SeSAM is removed from our candidate pool. MASON is good to build large-scale agent-based models, but it requires the significant JAVA knowledge. Compared to MASON, NetLogo and Repast are easier to get

Table 15 Comparison for Open Source MAS Softwares

	Primary Domain	Programming Language	Model Size
SeSAM	General purpose	Visual Programming	Ten thousands of agents
NetLogo	Social and natural sciences	NetLogo	No limit
MASON	General purpose	Java	Millions of agents
Repast	Social sciences	Java, Python, C++	No limit

started. Since Repast has a rich set of developed tools, we select Repast to build our MAS model. Although there is no limitation on the size of a model in NetLogo and Repast theoretically, they may hit some limits that are inherent in the underlying JAVA Virtual Machine and/or operating system.

IV.2. Problem Definition

In MAS, we study a regional evacuation problem in an agent-based system. In this system, each evacuee has the ability to make decisions and change decisions based on personal preference or through the exchange of information between agents. In other words, MAS enables individual evacuees to select their evacuation route and their shelter destination. We now discuss some of the parameters and characteristics of our MAS in detail.

- Evacuation Performance - four major evacuation performance indicators are observed: total evacuation time, individual travel time, system-wide traffic conditions and total transportation cost.
  - Total evacuation time is defined as the difference in time between the first evacuee leaving an origin to the last evacuee arriving at a shelter.

- Individual travel time is defined for each evacuee as the difference in time between that evacuee leaving an origin and arriving at a shelter.
- Traffic conditions is the defined as the number of roads with traffic jam.
- Transportation cost associated with flow is defined as same as in the SEND model in Chapter III, and it is $\sum_{o\in\mathcal{O}}\sum_{d\in\mathcal{D}}\sum_{(i,j)\in\mathcal{A}} c_{ij}^{\mathcal{A}} x_{odij}$. Transportation cost is the only cost considered in MAS model, because in MAS problem all facilities are assumed to be available or opened already.

- Evacuee Decision-Making - evacuees use the designated routes and the designated shelters recommended by the SEND optimization as a guideline. At any network intersection, the agent may change their route or shelter destination based on personal compliance rates as well as on real-time traffic conditions and the status of shelters (i.e., full or not full). Personal compliance rates are assigned at prior to running MAS and remain constant for each agent through each simulation.

- Evacuee Travel Time - Each agent evacuates on a path from his origin to a shelter with available spaces. The traverse time of this evacuation path is the sum of the traverse time of the arcs on this path. As a curved line segment, each arc is divided to a finite number of straight line segments, which are the GIS data in the shape file of the traffic network. The traverse time of each arc is the sum of the traverse time of straight line segments on this arc. An evacuee's traverse time of a straight line segment is the ratio of the length of this straight line segment to the travel speed on this straight line segment.

- Evacuee Travel Speed - As introduced above, each arc is divided to a finite number of straight line segments. We assume that an evacuee's travel speed does not change while driving on one straight line segment. When an evacuee arrives at an end node of a straight line segment, his travel speed is changed based on the current traffic density on this arc. The relationship between the traffic speed and the traffic density on the road is introduced in subsection IV.3.4 in details. Because the length of straight line segments are small (e.g. most of them are less than 100 meters), evacuees' travel speed can be considered as near real-time changed travel speed.

- Evacuee Departure - Evacuees are organized into groups which are assigned separate departure times for leaving an origin. The size of these groups and the proximity of their departure times have significant impact on overall evacuation performance. Establishing smaller group sizes will lead to staggered evacuation times, resulting in smaller sets of edge users and helping to decrease edge traffic density. Longer lead times between consecutive groups helps to decrease the amount of network users while also helping to decrease edge traffic density. Smaller groups and longer lead times, however, result in individuals evacuating closer to time-zero (i.e., landfall) of an extreme event and thereby increasing the populations evacuation risk.

- Evacuee Communication - evacuees can send real-time traffic condition information to one another as one method of agent communication. This is akin to an evacuee driving on a slow road calling his/her friends and family and encouraging them to choose an alternate route. Additionally, all agents are capable of receiving real-time system information in similar fashion to a radio announcing traffic jam or changes to shelter status via an FM or AM broadcast. All shared information has the potential to influence evacuee decision-making during the evacuation.

We develop the MAS model as a way to study the effects of individual decision-making and information sharing on traffic conditions, total evacuation time, individual travel time and overall transportation cost. The effects of individual compliance assumptions, information sharing, and a-priori decisions on the number of evacuation groups and the timing of their departure all influence evacuation success and performance. MAS enables us to strategically study the interactions and interdependencies of these assumptions and decisions within a realistic evacuation environment. In this way, we are also able to evaluate the effectiveness and robustness of the original SEND optimal route and shelter allocations of our optimal evacuation plan.

IV.3. Model

For constructing MAS model, there are three types of input data for the model. The first data set is the optimal solution from our optimization model. Evacuees use the optimal routes and shelter locations as guidelines. The second type of data is obtained from GIS. It is geographic information: counties from which people should evacuate, the populations in these counties, shelters locations, transfer nodes, and available roads. MAS problem uses the same network as SEND problem. However, in MAS problem, all potential shelter locations are opened, and all transfer nodes and edges are available. The third data set is the capacity of each road, the capacity of each shelter, and the transportation cost for routing one unit flow through each arc.

IV.3.1. Parameters and Sets

First, we introduce the notations employed in MAS model as follows.

Sets

$\mathcal{SPS}_i$ Set of shortest paths from transfer node i to all shelters

$\mathcal{TPS}_{i,e}$ Set of total paths, composed by $\mathcal{SPS}_i$ and OP_e

$\mathcal{APS}_{e,i,t}$ Set of available paths for evacuee e at transfer node i and time t

$\mathcal{FSS}_t$ Set of full occupied shelters at time t

$\mathcal{RPS}_{e,i,t}$ Set of available paths without slow traffic or traffic jam for evacuee e at transfer node i and time t

$\mathcal{STR}_t$ Set of road segments with slow traffic at time t

$\mathcal{TJR}_t$ Set of road segments with traffic jam at time t

$\mathcal{STP}_{e,i,t}$ Set of available paths with slow traffic for evacuee e at transfer node i and time t

$\mathcal{TJP}_{e,i,t}$ Set of available paths with traffic jam for evacuee e at transfer node i and time t

Parameters

OP_e Optimal path of evacuee e

v_o Traffic speed when traffic density is equal to road capacity

k_j Jam density when traffic speed is equal to zero

P Probability to follow the optimal paths and the optimal shelter locations

G Number of groups in which people start to evacuate

RT_g Range of leaving time for group g, $g = 1, \cdots, G$

IS_1 Binary value for the 1st type of information sharing: 1 means that information is shared among evacuees; otherwise, it is 0

IS_2 Binary value for the 2nd type of information sharing: 1 means that information is sent from the radio station to all evacuees; otherwise, it is 0

Variables

v_t Traffic speed at time t

k_t Traffic density at time t

IV.3.2. Structures

We construct MAS model by employing Repast JAVA. The inherent structure of models built by Repast includes three components: contexts, projections, and agents. Repast manual states that context, as a main function in Repast, performs as a data structure to organize agents from both a modeling perspective and a software perspective, and also context may include a few sub-contexts. Repast manual also claims that projections are interaction networks or relationships between agents, and projections are associated with contexts. Agents are the "intelligent" units which can make decisions under certain conditions by only considering its own situation and its environment. For example, if the main context is a country, and each sub-context is for each city in this country. The agents can be the residences in each city, and the projection for each sub-context can be the road network connecting each agent's house in this city.

In MAS model, there are three contexts: main context, person context (main's sub-context), and junction Context (main context's sub-context, person context's sib-context). For main context, there is no projection. In person context, there is one projection, which is a geography projection. The geography projection is for GIS environment, and it includes coordinates, shapes, lengths, etc.. In MAS, the geography projection includes the length of roads, the coordinates of nodes which compose and sketch the roads, the coordinates of origins, the coordinates of shelters, and the coordinates of transfer nodes. Person context contains four types of agents: vehicles, roads, a radio station, and destinations. In junction context, there are two projections, which are a geography projection and a network projection. The network projection is the network relationship between two objects. In MAS model, it maps edges to their vertices. The MAS model structure inspired by the structure of agent-based crime simulation model by Malleson (2008).

IV.3.3. Agents and Interactions

In MAS model, each vehicle is considered as one agent. Drivers can decide to follow the optimal evacuation routes or choose their own routes, and also they can adjust their paths, according to the real time traffic conditions, the status of shelters, and their personal preferences. When a driver arrives at a transfer node, he has a chance to make a decision: which route will be chosen to follow. At transfer node i, each driver receives a route list, which contains k shortest paths from the current location to each shelter. Since the number of shelters is $|\mathcal{D}|$, so the number of shortest paths in the route list is $k \times |\mathcal{D}|$. This set of shortest paths in the route list is defined as the set $\mathcal{SPS}_i$. The set $\mathcal{SPS}_i$ and the optimal path OP_e for evacuee e, which is used as a guideline, compose the total paths set $\mathcal{TPS}_{e,i}$ for evacuee e at transfer node i. The procedure for a driver making his decisions is presented in Algorithm 5. First, the evacuee e check whether he is sill driving on OP_e. If he is, he still has the chance to follow OP_e, and the available paths set for evacuee e at transfer node i at time t is $\mathcal{APS}_{e,i,t} = \mathcal{TPS}_{e,i}$. Otherwise, it is not a choice for him to follow OP_e, and $\mathcal{APS}_{e,i,t} = \mathcal{TPS}_{e,i} \setminus OP_e$. Second, the driver checks whether he receives messages about the status of shelters (i.e. a shelter is full or not). The set of full occupied shelters at time t is $\mathcal{FSS}_t$. If he does, he checks whether the paths in $\mathcal{APS}_{e,i,t}$ use the shelters in $\mathcal{FSS}_t$ as destinations. If a path in $\mathcal{APS}_{e,i,t}$ reaches a shelter in $\mathcal{FSS}_t$, this path is deleted from $\mathcal{APS}_{e,i,t}$. That means this path is no longer a candidate of his route. A value is assigned to the probability of the selection of each path in $\mathcal{APS}_{e,i,t}$. The set of paths which has no slow traffic or traffic jam is defined as $\mathcal{RPS}_{e,i,t}$, and it is initialized as $\mathcal{RPS}_{e,i,t} = \mathcal{APS}_{e,i,t}$. Third, the driver checks whether he receives messages about real time traffic conditions about slow traffic or traffic jam. The set of road segments with slow traffic at time t is $\mathcal{STR}_t$, and the set of road segments with traffic jam at time t is $\mathcal{TJR}_t$. If a path in $\mathcal{APS}_{e,i,t}$ contains a road segment in $\mathcal{STR}_t$ or $\mathcal{TJR}_t$, this path is added to the set of paths with slow traffic $\mathcal{STP}_{e,i,t}$ for evacuee e at transfer node i at time t or the set of paths with traffic jam $\mathcal{TJP}_{e,i,t}$ for evacuee e at transfer node i at time t respectively. This path has a less priority to be selected, and it is deleted from $\mathcal{RPS}_{e,i,t}$. The probability for choosing this path decreases. Fourth, based on the updated probability associated with each path in $\mathcal{APS}_{e,i,t}$, the driver chooses his route from $\mathcal{APS}_{e,i,t}$. Now set $\mathcal{APS}_{e,i,t}$ is composed by $\mathcal{RPS}_{e,i,t}$, $\mathcal{STP}_{e,i,t}$, and $\mathcal{TJP}_{e,i,t}$. After a driver makes his decision, he drives along with the chosen path until he arrives at a next transfer node. Then, he has a chance to make another decision based on the real traffic conditions, the status of shelters, and his preference. This procedure repeats until the driver arrives at a shelter which has available spaces. The flow chart for evacuees are presented in Figure 8.

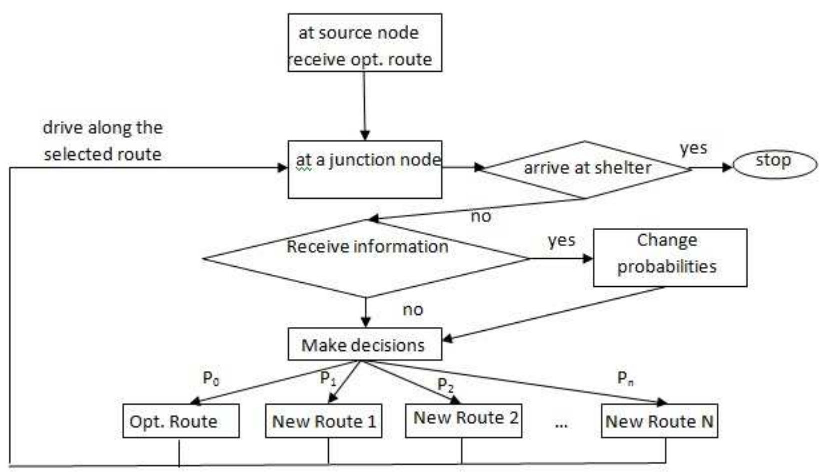
Algorithm 5 Procedure for Evacuees Choosing Their Routes

Initialize $\mathcal{TPS}_{i,e}$= $\mathcal{SPS}_i$+OP_e, $\mathcal{APS}_{e,i,t}$, $\mathcal{TJP}_{e,i,t}$, $\mathcal{STP}_{e,i,t}$, $\mathcal{RPS}_{e,i,t} = \emptyset$
while Arrive at a transfer node & not arrive at a non-full shelter **do**
  if Drive on OP **then**
    $\mathcal{APS}_{e,i,t} = \mathcal{TPS}_{i,e}$
  else
    $\mathcal{APS}_{e,i,t} = \mathcal{TPS}_{i,e} \backslash OP_e$
  end if
  if Receive messages about the status of shelters **then**
    for (Shelter s : $\mathcal{FSS}_t$) **do**
      for (Path p : $\mathcal{APS}_{e,i,t}$) **do**
        if the destination of p is s **then**
          $\mathcal{APS}_{e,i,t} = \mathcal{APS}_{e,i,t} \backslash p$
        end if
      end for
    end for
  end if
  A value is assigned to the probability of the selection of each path in $\mathcal{APS}_{e,i,t}$, and $\mathcal{RPS}_{e,i,t} = \mathcal{APS}_{e,i,t}$
  if Receive messages about traffic jam **then**
    for (Road Segment r : $\mathcal{TJR}_t$) **do**
      for (Path p : $\mathcal{APS}_{e,i,t}$) **do**
        if p contains r **then**
          Decrease the probability for choosing p
          Add p to $\mathcal{TJP}_{e,i,t}$, and delete p from $\mathcal{RPS}_{e,i,t}$
        end if
      end for
    end for
  end if
  GO to Algorithm 6
end while

Algorithm 6 Continue for Procedure for Evacuees Choosing Their Routes

if Receive messages about slow traffic **then**
  for (Road Segment r : $\mathcal{STR}_t$) **do**
    for (Path p : $\mathcal{APS}_{e,i,t}$) **do**
      if p contains r **then**
        Decrease the probability for choosing p
        Add p to $\mathcal{STP}_{e,i,t}$, delete p from $\mathcal{RPS}_{e,i,t}$
      end if
    end for
  end for
end if
Choose one path from $\mathcal{APS}_{e,i,t}$ according probabilities

Figure 8 Flow Chart for Evacuees



Other agents in MAS model are roads, a radio station, and destinations. Each road is considered as an agent, which has capacities and real time traffic flow. The radio station is considered as an agent, and it send message to all drivers about real time traffic conditions and the status of shelters. Each destination is also considered as an agent, and it has capacity and status which indicate whether the shelter is full or not.

Interactions among agents are an essential part of MAS model. It is a significant difference from MAS model to an optimization model. The interactions may influence the performance of whole MAS system, so they are an interesting part we study on MAS problem. There are four types of interactions among agents. The first set is interactions among evacuees. Since there are social relationships between evacuees. An evacuee is connected with M other evacuees, and they may be friends, relatives, colleagues and so on. If an evacuee is driving in a slow traffic, he is willing to send message to his connections. The persons who receive the message will decrease the probability for choosing this road. The second interaction set is the interactions between roads and the radio station. If there is traffic jam on a road segment, this road segment sends a message to the radio station to let the radio station know its traffic condition. However, the road segment only sends its traffic condition to the radio station when there is traffic jam on this road. The third interaction set is the interactions between shelters and the radio station. If a shelter is full occupied, it sends its full occupied status to the radio station. A shelter only send message to the radio station when its status changes to full occupied. The fourth interaction set is the interactions between the radio station and all evacuees. After the radio station receiving messages from roads or shelters, it forwards these messages to evacuees (i.e. it broadcasts the congestion on roads and the status of shelters). Thus, evacuees can avoid the congested roads and change their routes to the shelters which still have available spaces.

IV.3.4. Traffic Speed-Density Model

To consider traffic speed on each road segment as a variant with traffic density, we include traffic speed as a function of traffic density to MAS model. We use the classic Speed-Density Model: Greenberg Model. Greenberg (1959) proposed a model for real time traffic flow. By assuming the traffic flow as a continuous fluid, he used fluid dynamic principles to conduct the relationships among traffic speed, traffic density, and the traffic flow. The model is as follows:

$$\textbf{(Greenberg)} \qquad k_t = k_j \, e^{(-v_t/v_o)} \tag{4.1}$$

where v_o is the traffic speed at $q_{ij}^{\mathcal{E}}$. $q_{ij}^{\mathcal{E}}$ is the road capacity. v_t is the traffic speed at time t. k_j is the jam density when traffic speed is zero. k_t is the traffic density at time t. Then, traffic speed can be a function of traffic density as follows:

$$\textbf{(Greenberg)} \qquad v_t = -v_o \ln(k_t/k_j) \tag{4.2}$$

From the above equation, we can easily get the derivations as follows. If the traffic density k_t decreases, the traffic speed v_t can increase. If the traffic density k_t is less than the jam density k_j, the traffic speed is greater than zero; otherwise, the traffic speed is zero. However, in our simulation, we give the jam speed a real small value but not zero to make sure the vehicles in system can move. In the case of $k_t < k_j$, if $k_t \leq e^{-1}k_j$, v_t is greater or equal to v_o; otherwise, v_t is less than v_o. If $k_t = q_{ij}^{\mathcal{E}}$, $v_t = v_o$ based on the definition of v_o, and if $k_t/k_j = e^{-1}$, $v_t = v_o$. Thus, $k_j = \frac{q_{ij}^{\mathcal{E}}}{e^{-1}}$.

IV.4. Computational Study

In this section, we conduct five experiments to study the effects of five factors on the performance of evacuation process. We evaluate the performance of evacuation process in four perspectives: total evacuation time, individual traveling time, traffic conditions, and transportation cost.

In the first experiment, we test the effect of the probability, at which evacuees follow the optimal evacuation route, on the performance of evacuation process. We benchmark the performance of the evacuation in which evacuees may not follow the optimal evacuation route exactly (i.e. evacuees may have 70% probabilities to follow the optimal evacuation routes, or 30% probabilities, or even 0% probabilities) against the case in which evacuees follow the optimal evacuation route exactly. Through this experiment, we analyze the effect of probability of evacuees following the optimal evacuation routes, and we prove the robustness and the effectiveness of the strategic evacuation plan proposed by SEND model.

Second, we conduct an experiment to test the influence of evacuees leaving in groups with different leaving time on the performance of evacuation process. Evacuees leave in groups in time sequence and each group has its own leaving time. We assume that these leaving times are not overlapped. Since the total population is constant, more leaving groups means less population in each group. Also, a wide range of leaving time causes a rare population density evacuating at one time unit. However, a big number of groups or/and a wide range of leaving time may cause the groups, which are scheduled at the rear part of the sequence, leave at a late time, and result the evacuees, which leave in the late groups, in a risky situation. In this experiment, we analyze and evaluate the influence of this interesting part on the performance of evacuation process.

Third, we design an experiment to study the effect of information sharing in the decentralized MAS problem. We study two types of information shared in MAS model. The first type of the shared information is the messages, which are sent from evacuees to their connections (e.g. their friends, relatives, colleagues etc.), about real time traffic conditions (i.e. which road segments have slow traffic). The second type of shared information is the broadcast, which is sent from a radio station to all evacuees, about real time traffic conditions (i.e. which road segments have traffic jam) and the status of shelters (i.e. shelters are full occupied or not). Slow traffic is defined as the traffic flow with a speed less than v_o, which value is defined as 40 mph; traffic jam is defined as the traffic flow with a density equal or greater than k_j. From the derivations in subsection IV.3.4, $k_j = \frac{q_{ij}^{\mathcal{E}}}{e^{-1}}$, where $q_{ij}^{\mathcal{E}}$ is defined as 2000*1.5=3000 cars/per lane/per hour to keep consistent with the type I parameters in SEND model. We define the traffic speed in traffic jam as v_j (i.e. the value for v when $k = k_j$), which value is 5 mph but not 0 mph, to insure that the evacuees do not stop before they arrive at shelters, which have available spaces. The traffic speed and traffic density parameters, which are used in all five experiments in this chapter, are reported in Table 16. Through this experiment, we analyze and evaluate the effect of information sharing on the performance of evacuation process.

Table 16 Parameters for Traffic Speed and Traffic Density

v_o	v_j	$q_{ij}^{\mathcal{E}}$	k_j
40 mph	5 mph	3000 cars/per lane/per hour	$q_{ij}^{\mathcal{E}}/e^{-1}$

Fourth, we test the performance of the evacuation network which has extra edges' capacities added to some specific road segments. The locations where extra edges' capacities are added are a part of the optimal solution of SEND model. We benchmark the performance of the evacuation in which the network has extra edge-capacities against the case in which the network has no extra edge-capacities. By comparing these two cases, we analyze the effect of road capacities on the performance of evacuation process, and we prove the effectiveness of the construction of extra edge-capacities which is proposed by SEND model.

Last, we conduct an experiment to test the performance of the evacuation routes proposed by SEND model. We benchmark the performance of evacuation in which evacuees follow their own favorable routes (i.e. the shortest paths from their origins to the shelters which are recommended by SEND model) against the case in which evacuees follow the designated routes proposed by SEND model. In this experiment, we prove the effectiveness of the evacuation routes, which are proposed by SEND model, by analyzing its effect on the performance of evacuation process.

We develop all experiments based on the same traffic network, which is used in the SEND problem in Chapter III. The network size for all experiments in this section is Class 3, which is introduced in Table 7 in § III.4.1. We choose one instance of SEND problem in Class 3 with type I parameters as a benchmark instance(BISP) for MAS problem. The optimal routes and the optimal shelters (OROS), which are a part of the optimal solution of BISP, are used as the designated routes and the designated shelters to guide evacuees. To consider computer memory issue, we down-scale the population in evacuating areas by 500 to run all instances in simulation. That means we consider 500 vehicles as one agent in MAS model, comparing 1 vehicle considered as a unit in SEND model (i.e. assuming each vehicle has 4 passengers). However, to compare and contrast the solutions of SEND model and MAS model, we simulate the case, in which evacuees follow OROS exactly, on the same scale level (i.e down-scale population by 500). We use JAVA to code our MAS model in Repast Simphony environment. All machines used have 2.4 GHZ Intel Core 4 CPU processors with 8 GB RAM. All spatial analysis is conducted using ArcGIS 10 on the same machines. The remainder of this article is organized as follows. From subsection IV.4.1 to subsection IV.4.5, experiment I to experiment V are presented respectively, and their solutions are also analyzed respectively.

IV.4.1. Experiment for Effects of Varying Degrees of Compliance to the Optimal SEND Strategy on System Performance

As one of the significant difference from the decentralized MAS model to the centralized optimization model, agents' ability of having intelligence and freedom cause that the system can explore at different perspectives. In SEND problem, evacuees have no freedom to choose evacuation routes and shelters. Every decision in the system is decided by SEND model whose objective is to minimize the total costs while satisfying capacity constraints and time constraints, and each evacuee is assumed to follow the decision of SEND model exactly. However, in MAS system, evacuees may not follow the designated routes and the designated shelters proposed by SEND model, and they can make their own decisions to choose their favorable routes based on the real time traffic conditions, the status of shelters, and their personal preferences. However, the freedom of evacuees may cause traffic jam in some road segments, induce a longer total evacuation time, make individuals suffer a longer traveling time, and even cause a higher transportation cost in the whole system. Thus, it is important to check how the probabilities (P), at which evacuees decide to follow the optimal evacuation routes and shelters, influence the performance of evacuation process. The objective of experiment I is to study the influence of P in different levels on the performance of evacuation process. Moreover, experiment I shows the robustness and the effectiveness of the evacuation plan which is proposed by SEND model.

To study the influence of P on total evacuation time, individual traveling time, traffic conditions, and transportation cost. We test P in four levels: 0%, 30%, 70%, and 100%. 0% presents the case in which evacuees do not have any guidelines with designated routes and designated shelters. 100% presents the case in which evacuees follow the designated routes and designated shelters exactly. 30% means that evacuees have 30% probability to follow the designated routes and designated shelters, if the optimal route OP_e is in the set $\mathcal{RPS}_{e,i,t}$. Also, for each level of probability, we design two cases: even choices and uneven choices. The even choices are defined as a case in which each route in $\mathcal{RPS}_{e,i,t}$, besides OP_e if $OP_e \in \mathcal{RPS}$, has a same probability RP to be chosen. The probability for selecting a path in $\mathcal{STP}_{e,i,t}$ and $\mathcal{TJP}_{e,i,t}$ is a half of RP. Uneven choices are defined as a case in which only the shortest path in $\mathcal{RPS}_{e,i,t}$, besides OP_e if $OP_e \in \mathcal{RPS}_{e,i,t}$, has a major probability to be chosen, but the other paths in $\mathcal{RPS}_{e,i,t}$ have minor probabilities to be chosen. The minor probability for choosing one path in $\mathcal{RPS}_{e,i,t}$ is 2%, and the probability for choosing a path in $\mathcal{STP}_{e,i,t}$ and $\mathcal{TJP}_{e,i,t}$ is 1%. Since there are four levels for P and two cases for each level, there are $8 - 1 = 7$ cases (i.e. when $P = 100\%$, even choices and uneven choices are a same case). For each case, we test 10 random instances, so there are totally 70 instances.

For evaluating the effect of P on the performance of evacuation process, we fix other factors which may also influence the performance of evacuation process. Figure 9 shows 4 evacuation zip-zones, from coast to inland, which is recommended by Texas DPS in 2009 for hurricane evacuation. These zip-zones are shown in hatched yellow, yellow, green and orange respectively. According to the locations of origins and the population of origins, we divide 24 origins in our problem to 4 groups, illustrated in Figure 10, to simulate the evacuation zip-zones recommended by Texas DPS. The 4 groups of origins, from coast to inland, are colored in red, yellow, green, and orange respectively. Evacuees leave in groups as the division of their origins. We use 2 hours as the range of leaving time for each group (i.e. $RT_1 = RT_2 = RT_3 = RT_4 = 2$). Tier 1 is assumed to start leaving at time 0, so tier 2, 3, and 4 start to leave at 2 hours later, 4 hours later, and 6 hours later respectively. We assume that all evacuees in the previous tier leave before the start leaving time of the next tier. Moveover, there are messages, which is sent from evacuees to their connections, about which road segments have slow traffic (i.e. $IS_1 = 1$), and there is broadcast, which is sent from the radio station to all evacuees, about which road segments have traffic jam and which shelters are full occupied (i.e. $IS_2 = 1$). Moreover, people evacuate in the traffic network, where extra edge-capacities are added as recommended by the optimal solution of BISP.

In Table 17, the data in the first column is the probabilities at which evacuees follow OROS; the number in the second column is the average total evacuation time in hours; the third column shows time increase comparing the current probability level and the 100% probability level; the fourth column presents the average transportation costs in the whole evacuation process; the last column states the cost increase comparing the current probability level and the 100% probability level. The average total evacuation time increases as P decreases, and the average transportation costs increases as P decreases. It means that when evacuees have more willingness to follow OROS, the total evacuation time can be less and the transportation cost can be saved. For the case in which evacuees have no OROS as guidelines (i.e. $P = 0$), the total evacuation time and the transportation cost are the largest ones among all cases. That means OROS is an effective guideline, for the cases with even choices, to save the total evacuation time and save the transportation cost. Table 18 presents the similar results for the cases with uneven choices. Among these cases, when evacuees do not have OROS as guidelines, it causes the longest total evacuation time and

Figure 9 Hurricane Evacuation Zip-Zones in 2009 from Texas DPS

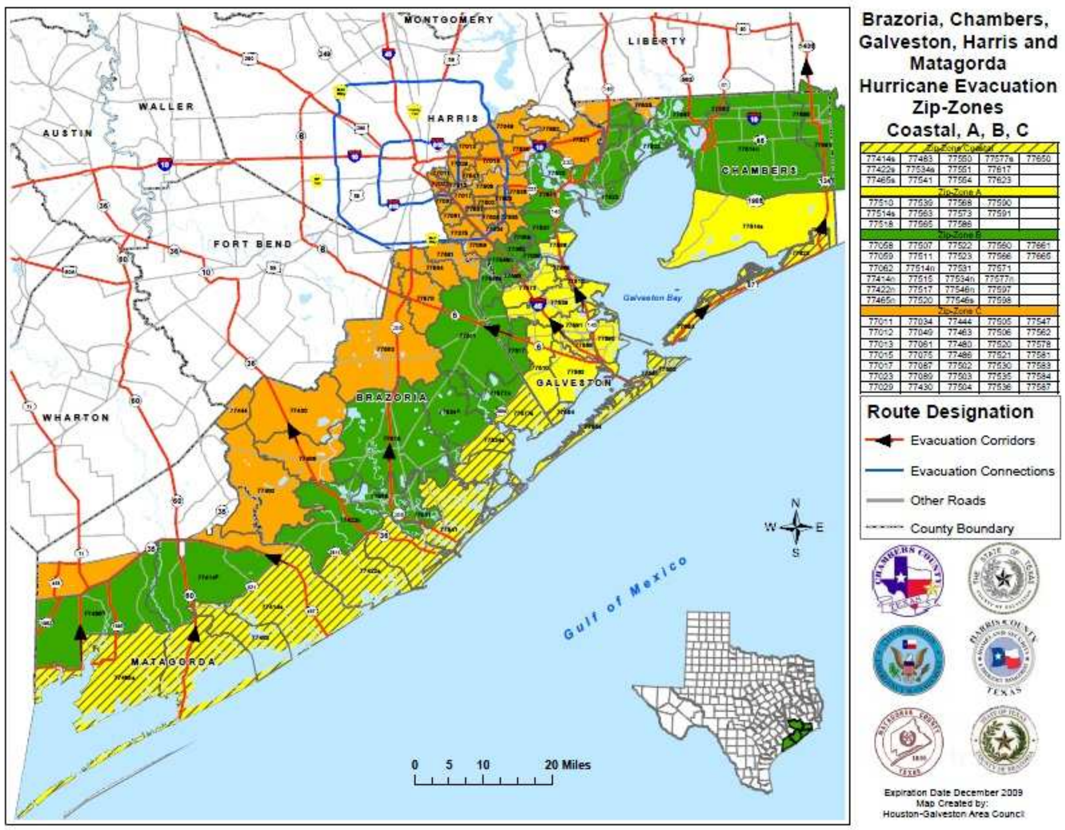
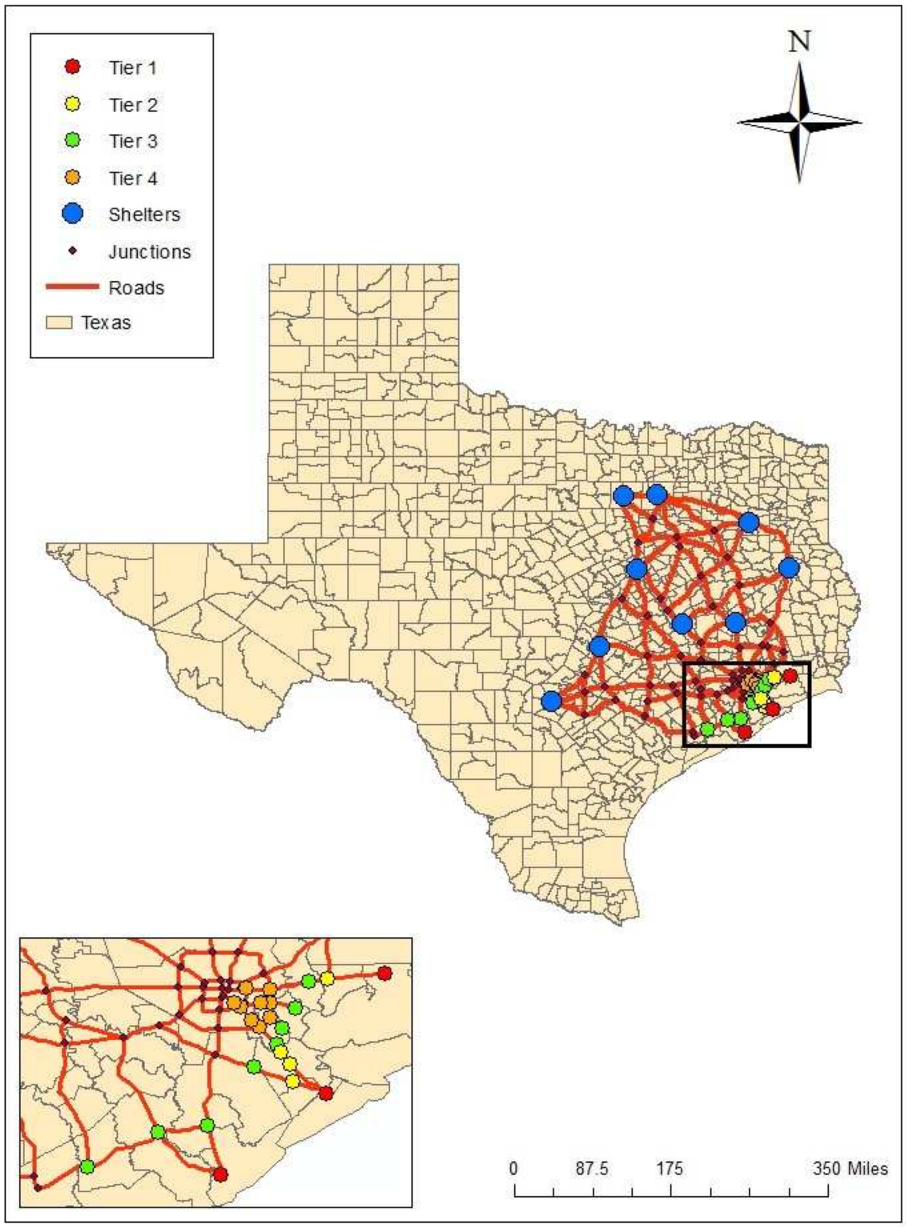


Figure 10 4-Group Division of Origins



the most transportation cost. Also, when evacuees have more willingness to follow OROS, both of total evacuation time and transportation cost can be saved. Thus, OROS is effective, for both of the cases with even choices and uneven choices, to save total evacuation time and transportation cost. Moreover, OROS is generated by considering the road-capacity constraints, it avoids the situation of traffic jam. If evacuees do not follow OROS, it has more opportunities to cause traffic jam. This can be proved by the results in Table 19. It shows that, in both of the cases with even choices and uneven choices, the number of road segments with traffic jam increases with the decrease of P. Also, without using OROS as guidelines, evacuees may have to change their target shelters on their way because their target shelters are already full, and this may cause the whole trip to be longer. Thus, SEND model provides an effective pre-event evacuation plan which can save total evacuation time, save transportation cost, and improve traffic situation, not only when evacuees follow this plan exactly but also when evacuees use this plan as guidelines to help their decisions on routes and shelters.

Table 17 Comparison of Evacuation Time and Cost for Even Choices at Different P Level

	P	Average Total Evacuation Time	Time Increase	Transportation Cost	Cost Increase
	0%	20.41	93%	468534	73%
	30%	20.02	89%	446114	64%
	70%	18.71	43%	385590	42%
Benchmark	100%	10.58	0%	271577	0%

Table 20 compares the total evacuation time and the transportation cost between the cases with even choices and the cases with uneven choices. At each P level, the cases with uneven choices can save total evacuation time the the transportation

Table 18 Comparison of Evacuation Time and Cost for Uneven Choices at Different P Level

	P	Average Total Evacuation Time	Time Increase	Transportation Cost	Cost Increase
	0%	13.99	32%	337708	24%
	30%	13.82	31%	329328	21%
	70%	13.36	21%	318862	15%
Benchmark	100%	10.58	0%	271577	0%

Table 19 Comparison of the Number of Roads with Traffic Jam at Different P Level

P	Even	Uneven
0%	0.4	3.0
30%	0.2	1.9
70%	0.1	0.8
100%	0	0

cost, comparing to the cases with even choices. Because in the cases with uneven choices, the shortest path has a major probability to be chosen, evacuees have more opportunities to follow a shorter path comparing to the cases with even choices.

In summary, Figure 11 and Figure 12 illustrate the relationship between total evacuation time and P, the relationship between transportation cost and P, for both the cases with even choices and uneven choices. Also, the relationship between the cases with even choices and uneven choices are presented in these two figures.

Besides studying the influence of P on the performance of evacuation process at a macro level (i.e. the influence on the total evacuation time), we also study the influence at a micro level (i.e. the influence on the individual traveling time). Table 21 presents the statistic for individuals' traveling time. The data in the first column is the probability at which evacuees follow OROS; the second column presents the per-

Table 20 Comparison of Evacuation Time and Cost between the Even Choices and the Uneven Choices

	Average Total Evacuation Time			Transportation Cost		
P	Even	Uneven	Decrease	Even	Uneven	Decrease
0%	20.41	13.99	31%	468534	337708	28%
30%	20.02	13.82	31%	446114	329328	26%
70%	18.71	13.36	29%	385590	318862	17%
100%	10.58	10.58	0%	271577	271577	0%

Figure 11 Comparison of Total Evacuation Time at Different P Level

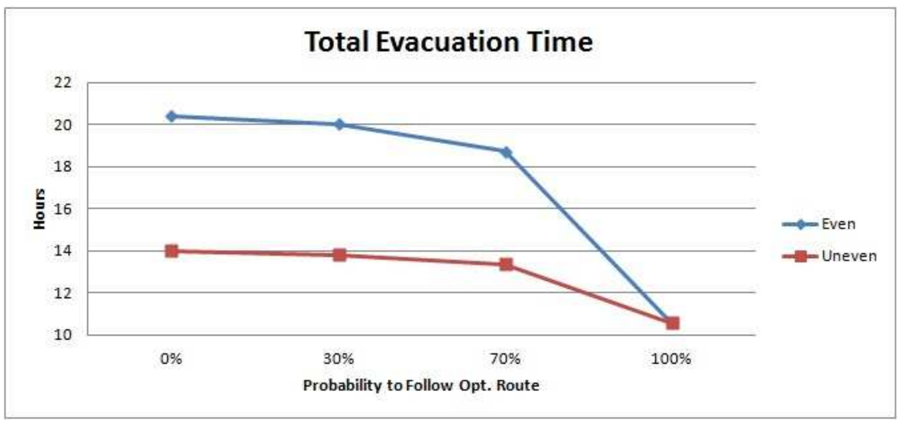
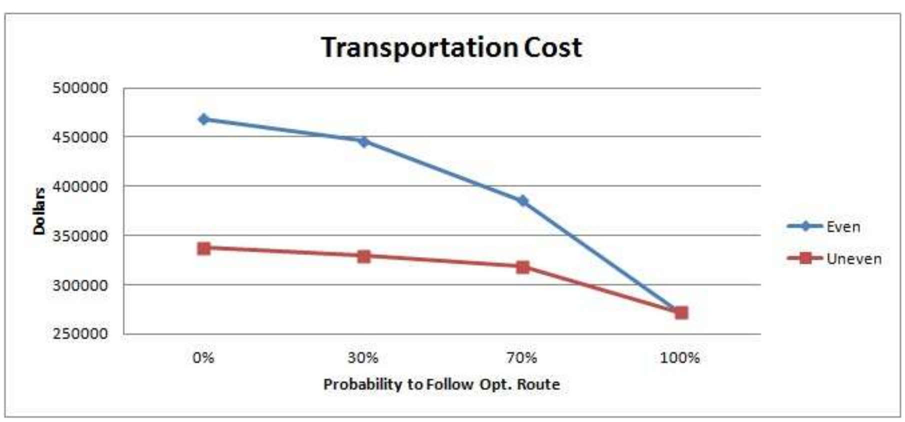


Figure 12 Comparison of Transportation Cost at Different P Level



centage of evacuees whose traveling time is less than 300 minutes (5 hrs); the third column states the 90% percentile for all individuals' traveling time. The percentage of evacuees, whose traveling time is less than 5 hrs, increases with the increase of P, and the 90% percentile for all individuals' traveling time decreases with the increase of P. These trends, which are illustrated in Figure 13 and Figure 14, means that if evacuees have more willingness to follow OROS, more evacuees can arrive at shelters within 5 hrs, and most of individuals suffer a shorter travel. If evacuees do not use OROS as guidelines, their individual traveling times are longer than the cases in which they use OROS as guidelines. Thus, OROS is an effective guideline to save individuals' traveling times. In conclusion, SEND model can provide a pre-event evacuation plan which can not only improve the performance of evacuation process in macro level by saving total evacuation time, saving transportation cost, and improving traffic situations, but also contribute in micro level by alleviating individuals' suffering.

Table 21 Individuals' Traveling Time in Experiment I

P	Percentage of Evacuees (T. T. < 300 mins)	90% Percentile for T. T. (mins)
0%	97%	267
30%	97%	259
70%	98%	247
100%	100%	175

IV.4.2. Experiment for Effects of Varying Groups and Varying Leaving Times on System Performance

Since time component is included in MAS model, we consider evacuees leave in groups in a time sequence, and a range of leaving time is associated with each group. We assume that these leaving times are not overlapped (i.e. all evacuees in the previous

Figure 13 Percentage of Evacuees with Traveling Time < 300 mins in Experiment I

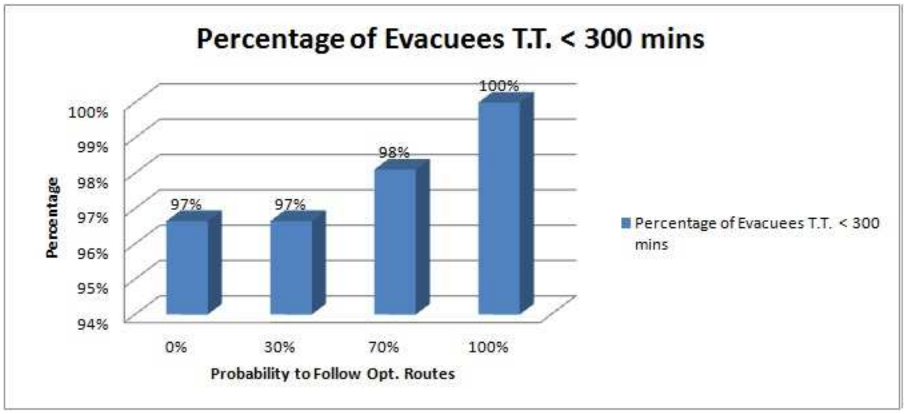
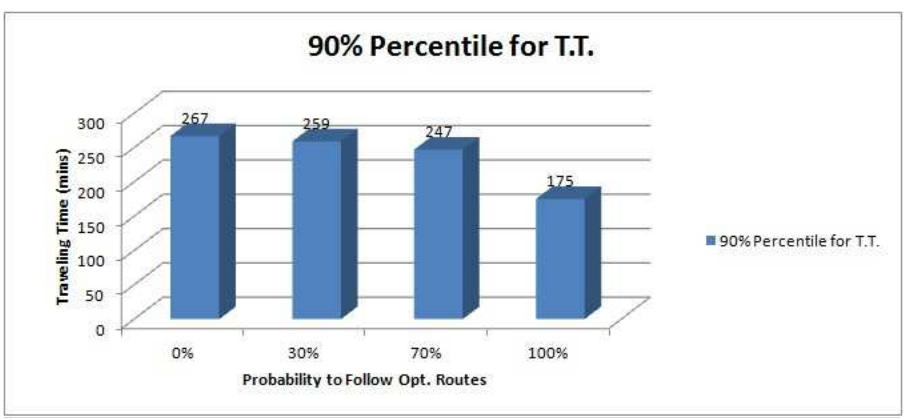


Figure 14 90% Percentile for Individuals' Traveling Time in Experiment I



tier leave before the start leaving time of the next tier). Because total population to evacuate is constant, more groups means less population in each group, and a wide range of leaving time for each group means a rare population density evacuating at one time unit. These may cause light traffic density and fast traffic flow on roads. However, more groups or wider ranges of leaving times can cause the groups, which are scheduled at the rear part of the sequence, leaving at a late time. This may cause a longer total evacuation time and may make the evacuees who leaves at a late time in a dangerous situation. Thus, there is a trade-off between traffic density and the gap of leaving time between two consecutive groups.

To study the influence of the number of groups (G) and the range of leaving time for each group (RT_g) on the performance of evacuation process, we conduct experiments on 3 levels for G: 2, 3, and 4. For $G = 2$, we test $RT_1 = RT_2$ on 3 levels: 2 hrs, 3 hrs, and 4 hrs. For $G = 3$, we test RT_g on 1 level, $RT_1 = 2hrs$, $RT_2 = 4hrs$, $RT_3 = 2hrs$ (e.g. evacuees in tier 1 leaves from 8am to 10am; evacuees in tier 2 leaves from 10am to 2pm; evacuees in tier 3 leaves from 2pm to 4pm). For $G = 4$, we test RT_g on 1 level, $RT_1 = RT_2 = RT_3 = RT_4 = 2hrs$, which are the values used in Experiment I in subsection IV.4.1. Based on the 4-group division which are generated to simulate the evacuation zip-zones recommended by Texas DPS, we generate the divisions for 2 groups and 3 groups. We combine the tier 1 and 2 in the 4-group division to compose the tier 1 in the 2-group case, and the tier 2 in the 2-group division is composed by the tier 3 and 4 in the 4-group case. For the 3-group division, its tier 1 and 3 are the tier 1 and 4, respectively, in the 4-group case, but its tier 2 is composed by the tier 2 and 3 in the 4-group case. The groups are generated by considering the evacuation zip-zones recommended by Texas DPS, considering the locations of origins, and considering the population of origins. In summary, origins are divided to 2 groups (illustrated in Figure 15), 3 groups (illustrated in Figure 16), and 4 groups (illustrated in Figure 10), so that their population can evacuate in groups. Because there are 3 levels for G, 1 level for RT_g when $G = 3 or 4$, and 3 levels for RT_g when $G = 2$, there are 5 cases with different G and RT_g. For each case, we test 10 random instances, so there are 50 instances tested in this experiment.

For evaluating the effect of G and RT_g on the performance of evacuation process, we fix other factors which may also influence the performance of evacuation process. We set $P = 30\%$ (i.e. evacuees have 30% probability to follow OROS), and set $IS_1 = IS_2 = 1$ (i.e. there is information shared between evacuees and their connections, and there is information sent from the radio station to all evacuees). Moreover, people evacuate in the traffic network, where extra edge-capacities are added as recommended by the optimal solution of BISP.

Table 22 presents the average total evacuation time, the average transportation cost, and the average number of roads with traffic jam, when evacuees leave in different groups and with different ranges of leaving time. The data in the first column and the second column is the values of G and RT_g. The third column presents the average total evacuation time, and the fourth column states the average transportation cost. The last column claims the average number of roads with traffic jam. Comparing the first three rows, evacuees leaves in 2 groups, but the range of leaving time for each group is different. When the range of leaving time increases, the total evacuation time increases, because a wide range of leaving time means that the evacuees in tier 2 leave at a late time. Recalling the definition of the total evacuation time, which is the time from the first evacuee starting to leave from an origin to the last evacuee arriving at a shelter which has available spaces, if a part of evacuees leaves at a late time, the total evacuation time may increase. Also, comparing row 1, row 4, and row 5, more groups and/or wider ranges of leaving time cause the total evacuation time to increase. This shows our expectation before experiments,

Figure 15 2-Group Division of Origins

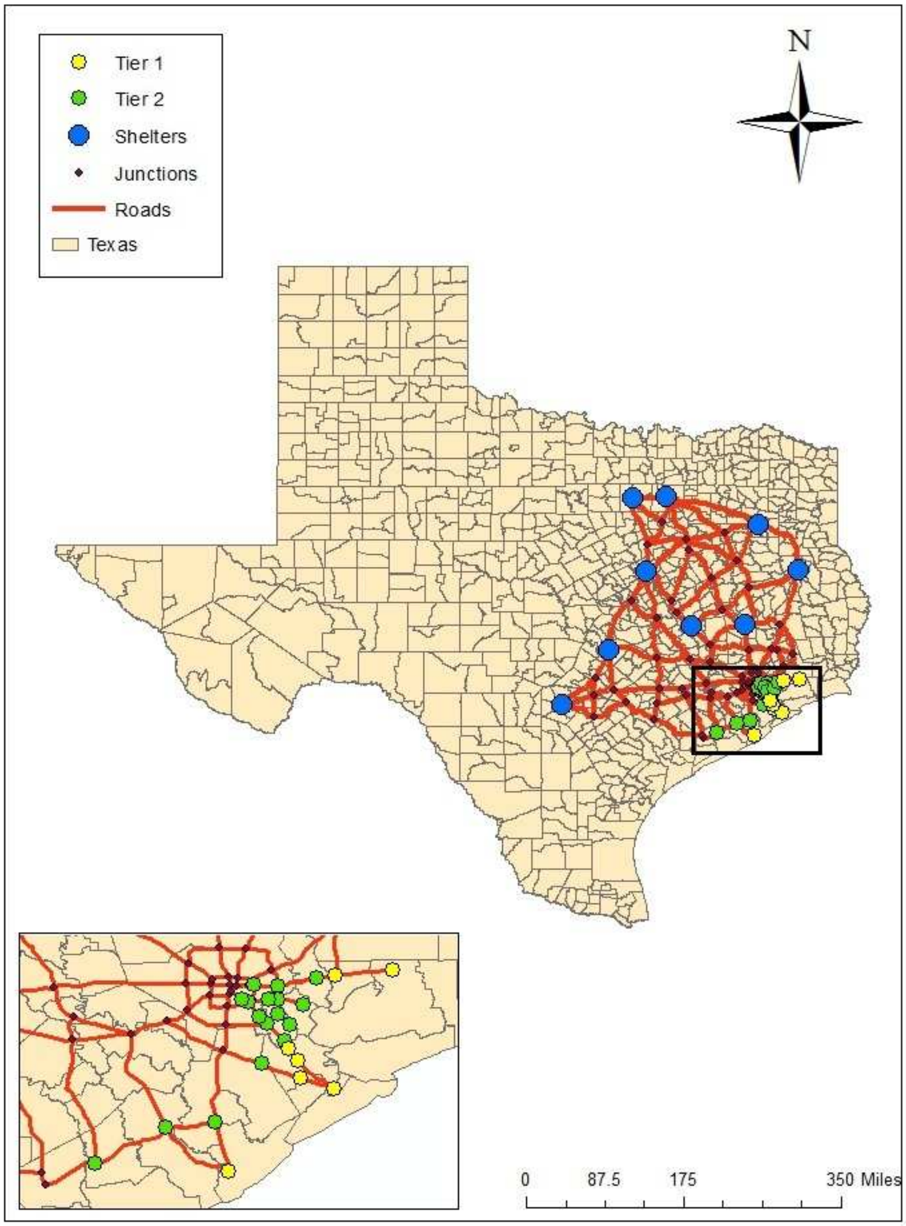
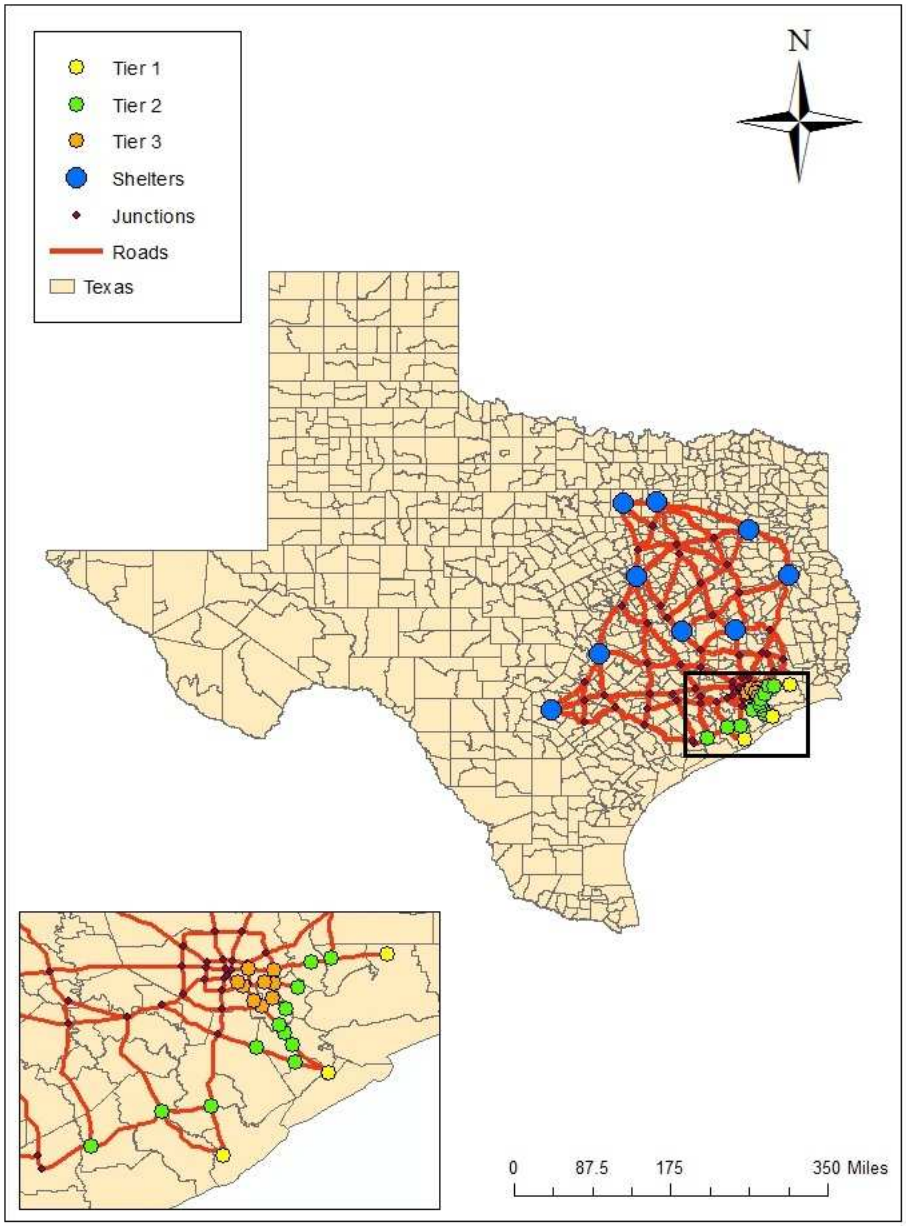


Figure 16 3-Group Division of Origins



and it illustrates a disadvantageous effect of more groups and/or wider ranges of leaving time on the total evacuation time. However, more groups and/or wider ranges of leaving time can reduce the number of roads with traffic jam efficiently, according to the results presented in the last column of Table 22. This also proves our expectation of this experiment: more groups and/or wider ranges of leaving time can cause lighter traffic density and improve traffic situation. Furthermore, when the number of groups or/and the range of leaving time increase, the transportation cost decrease in generally but not strictly. Because the transportation cost is related to the lengths of evacuation routes but not time component, less roads with traffic jam may cause evacuees have less probabilities to detour and to avoid longer trips. However, the number of roads with traffic jam is not the only factor to influence the transportation cost, which are also affected by evacuees' choices on their routes. Especially when the number of roads with traffic jam is small, fewer evacuees have to detour, and the influence on the transportation cost is not significant. Thus, the transportation cost does not decrease strictly with the increase of G and/or RT_g.

Table 22 Comparison of Evacuation Time and Cost with Different Groups and Different Leaving Time

G	RT_g	Time (Hrs)	Cost	No. of Roads with Traffic Jam
2	[2, 2]	11.40	342577	11.7
2	[3, 3]	12.33	340507	9.9
2	[4, 4]	13.43	332848	6.1
3	[2, 4, 2]	13.17	327416	5.6
4	[2, 2, 2, 2]	13.82	329328	1.9

More groups or/and wider ranges of leaving time do not improve the total evacuation time, but they have an advantageous effect on individuals' traveling time. Table 23 presents the statistic of individuals' traveling time for evacuees leaving in different groups with different leaving time. The data in the first and the second column is the values of G and RT_g; the second column presents the percentage of evacuees whose traveling time is less than 300 minutes (5 hrs); the third column states the 90% percentile for all individuals' traveling time. The percentage of evacuees, whose traveling time is less than 5 hrs, increases with the increase of G or/and RT_g, and the 90% percentile for all individuals' traveling time decreases with the increase of G or/and RT_g. These trends, which are illustrated in Figure 17 and Figure 18, means that if evacuees leaves in more groups or/and leaves with a wider time range, more evacuees can arrive at shelters within 5 hrs, and most of individuals suffer a shorter traveling time. In Figure 17 and Figure 18, case I is the case where $G = 2, RT_1 = RT_2 = 2$; case II is the case where $G = 2, RT_1 = RT_2 = 3$; case III is the case where $G = 2, RT_1 = RT_2 = 4$; case IV is the case where $G = 3, RT_1 = 2, RT_2 = 4, RT_3 = 2$; case V is the case where $G = 4, RT_1 = RT_2 = RT_3 = RT_4 = 2$. Thus, more groups or/and wider ranges of leaving time can improve the performance of evacuation process in a micro level by alleviating individuals' suffering.

Table 23 Individuals' Traveling Time in Experiment II

G	RT_g	Percentage of Evacuees (T. T. < 300 mins)	90% Percentile for T. T. (mins)
2	[2, 2]	91%	295
2	[3, 3]	96%	274
2	[4, 4]	96%	272
3	[2, 4, 2]	95%	274
4	[2, 2, 2, 2]	97%	259

Figure 17 Percentage of Evacuees with Traveling Time < 300 mins in Experiment II

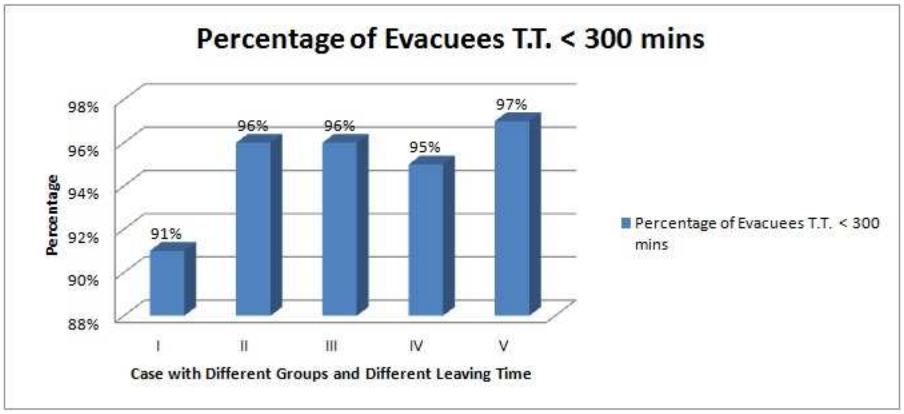
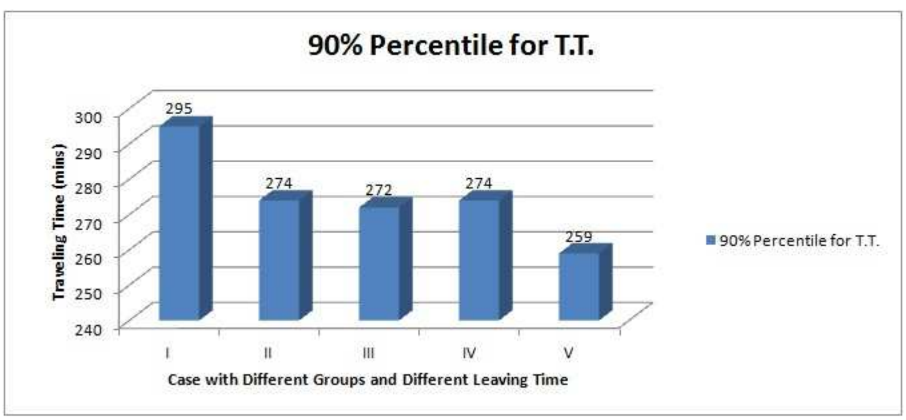


Figure 18 90% Percentile for Traveling Time in Experiment II



IV.4.3. Experiment for Effects of Varying Shared Information on System Performance

Information sharing is a significant difference between the centralized system and the decentralized system. In the centralized system, it is assumed that information is shared perfectly in the whole system; but in decentralized system, not every agent can receive the real time information. In MAS model, as the factors which may influence the performance of evacuation process, the real time traffic conditions and the status of shelters may not be known by each evacuee in time. To study the importance for sharing these information to evacuees in time, we test two types of information sharing (IS_1 and IS_2): evacuees send messages about slow traffic to their connections; a radio station broadcasts on traffic jam and status of shelters to all evacuees. Slow traffic is defined as the traffic flow with the speed less than v_o; traffic jam is defined as the traffic flow with the density bigger than or equal to k_j.

To test the influence of IS_1, we set IS_1 in 2 levels: 0 (i.e. evacuees do not send messages to their connections) and 1 (i.e. evacuees send message to their connections). We also set IS_2 in 2 levels: 0 (i.e. the radio station do not broadcasts), and 1 (i.e. the radio station broadcasts to all evacuees). Thus, there are 4 cases by combining the different levels of IS_1 and IS_2. For each case, we test 10 random instances, so there are 40 random instances tested in this experiment. For evaluating the effect of IS_1 and IS_2 on the performance of evacuation process, we fix other factors which may also influence the performance of evacuation process. We set $P = 30\%$ (i.e. evacuees have 30% probabilities to follow OROS), and set $G = 4$, and $RT_1 = RT_2 = RT_3 = RT_4 = 2$ (i.e. evacuees leave in 4 groups, and the range of leaving time for each group is 2 hours).

We first test instances in the traffic network with extra capacities of edges, which are recommended by the optimal solution of BISP. However, as presented in Table 24, when the type of information sharing changes, the total evacuation time, the transportation cost, and the number of roads with traffic jam do not change obviously. When the network has extra capacities of edges, the traffic situation is improved. Because the information of real time traffic condition is only sent when the road segments has slow traffic or traffic jam, the frequency for sending these messages is low, and the influence of the different types of information sharing is not obvious.

Table 24 Comparison for Different Type of Information Sharing In Network with Extra Edge Cap.

$[IS_1, IS_2]$	Time (Hrs)	Cost	No. of Roads with Traffic Jam
[0, 0]	14.01	329575	2.1
[1, 0]	13.98	330094	2.0
[0, 1]	13.91	330322	2.1
[1, 1]	13.82	329328	1.9

To observe the influence of information sharing on the performance of evacuation process, we test instances in the network without extra edge-capacities added. Table 25 presents the average total evacuation time, the average transportation cost and the average number of roads with traffic jam for the cases with different type of information sharing. The data in the first column is the types of information sharing; the second column presents the average total evacuation time; the third column states the average transportation cost; and the last column claims the average number of roads with traffic jam. Comparing row 1 with row 2 and row 1 with row 3, both of the average total evacuation time and the average transportation cost decrease. This means that by sharing both of these two types of information, the total evacuation time and the transportation cost can be saved. Also, comparing the difference from row 1 to row 2 and the difference from row 1 to row 3, there are bigger saves on the average total evacuation time and the average transportation cost by sharing the second type of information. Thus, the broadcast sent from the radio station to all evacuees has more significant influence on the total evacuation time and the transportation cost, because it is sent to all evacuees and it contains two types of information: traffic jam and status of shelters. Moreover, we find an interesting effect of sharing the second type of information on the number of roads with traffic jam. When the radio station send messages to evacuees, the number of roads with traffic jam increases, because evacuees try to avoid the roads, which are labeled as "roads with traffic jam" by the radio station, but congest on other roads. Thus, the number of roads with traffic jam increases when people hear the broadcast from the radio station.

Table 25 Comparison for Different Type of Information Sharing In Network Without Extra Edge Cap.

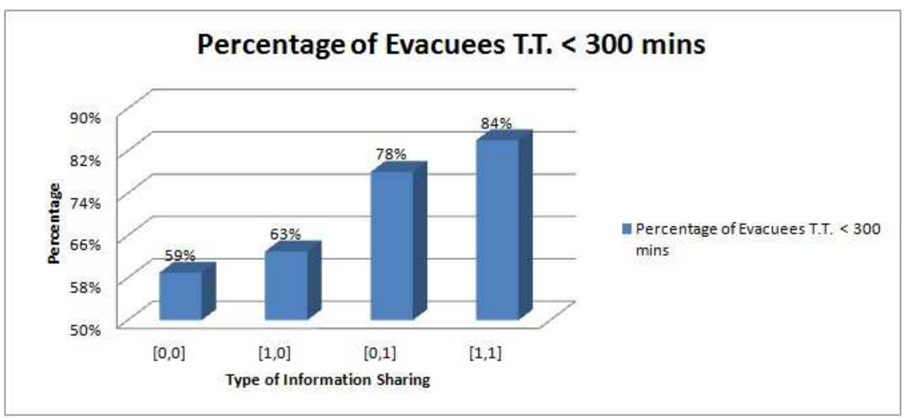
$[IS_1, IS_2]$	Time (Hrs)	Cost	No. of Roads with Traffic Jam
[0, 0]	18.08	332260	26.8
[1, 0]	17.73	331963	24.8
[0, 1]	15.93	340470	45.7
[1, 1]	15.69	337744	45.2

Table 26 presents the statistic of individuals' traveling time for the cases with different types of information sharing. The data in the first and the second column is the values of IS_1 and IS_2; the third column presents the percentage of evacuees whose traveling time is less than 300 minutes (5 hrs); the fourth column states the 90% percentile for all individuals' traveling time. By sharing both of these two types of information, individuals' traveling time decreases. The broadcast sent from the radio station to all evacuees has more significant effect on individuals' traveling time. These trends are illustrated in Figure 19 and Figure 20. Thus, sharing information improve the performance of evacuation process in a micro level by alleviating individuals' suffering, and sharing the second type of information have more significant effect.

Table 26 Individuals' Traveling Time in Experiment III

$[IS_1, IS_2]$	Percentage of Evacuees (T. T. $<$ 300 mins)	90% Percentile for T. T. (mins)
[0, 0]	59%	487
[1, 0]	63%	438
[0, 1]	78%	373
[1, 1]	84%	373

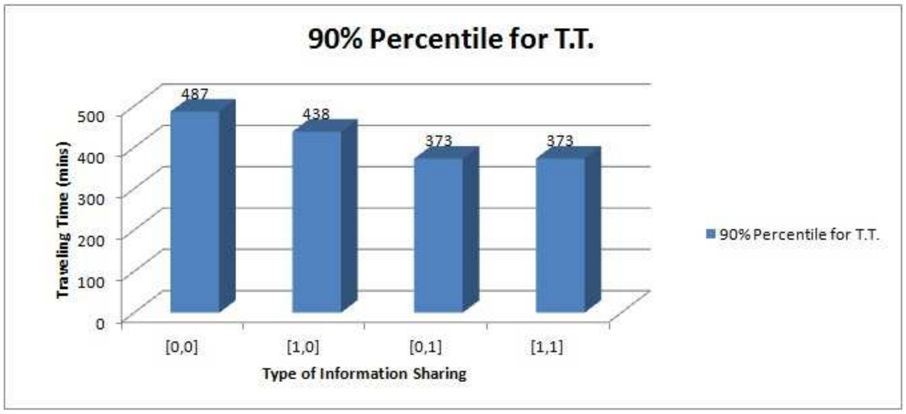
Figure 19 Percentage of Evacuees with Traveling Time $<$ 300 mins in Experiment III



IV.4.4. Experiment for Effects of Road Capacities on System Performance

In SEND model, extra edge-capacities are allowed to add to increase roads' capacities. To prove the effectiveness of this decision, we test two cases in this experiment. Case 1 is the case in which evacuation is conducted in the traffic network without extra

Figure 20 90% Percentile for Traveling Time in Experiment III



edge-capacities; case 2 is the one in which evacuation is conducted in the traffic network with extra edge-capacities which are proposed by the optimal solution of BISP. We test 10 instances for each case, so there are 20 instances are tested in this experiment. We fix other factors as follows: $P = 30\%$ (i.e. evacuees have 30% probability to follow OROS), $G = 4$, $RT_1 = RT_2 = RT_3 = RT_4 = 2$ (i.e. evacuees leave in 4 groups, and the range of leaving time for each group is 2 hours), and $IS_1 = IS_2 = 1$ (i.e. there is information shared between evacuees and their connections, and there is information sent from the radio station to all evacuees).

Table 27 presents the performance of evacuation network with extra edge-capacities and the performance of evacuation network without extra edge-capacities. The first column indicates whether there are extra edge-capacities in the evacuation network. The second column states the average total evacuation time; the third column presents the average transportation cost; and the fourth column claims the average number of roads with traffic jam. When evacuation network has extra edge-capacities, the total evacuation time and the transportation cost can be saved. Also, by adding extra capacities to 27 road segments (i.e. edges), the average number of roads with traffic jam decreases from 45.2 to 1.9. Thus, the construction of extra edge-capacities, which is proposed by SEND model, is an effective strategy to improve traffic condition, save total evacuation time, and save transportation cost.

Table 27 Comparison of Networks with and Without Extra Edge-capacities

Extra Edge Cap.	Time (Hrs)	Cost	No. of Roads with Traffic Jam
No	15.69	337744	45.2
Yes	13.82	329328	1.9

IV.4.5. Experiment for Effects of Routes Selection on System Performance

In this experiment, we benchmark the performance of the case in which evacuees follow their own favorable routes (i.e. the shortest paths from their origins to the shelters recommended by BISP) against the case in which evacuees follow $OROS$ in BISP. We first compare the difference of lengths from shortest paths to ORs. Table 28 presents the statistic for increase of lengths from shortest paths to ORs. Table 29 presents the statistic of individuals' traveling time for evacuees leaving in shortest paths or leaving in ORs. The data in the first column is the types of paths (i.e. the shortest paths or the optimal paths); the second column presents the percentage of evacuees whose traveling time is less than 300 minutes (5 hrs); the third column shows the 90% percentile for all individuals' traveling time. For the case in which evacuees use shortest paths, the percentage of evacuees, whose traveling time is less than 5 hrs, is smaller than the case in which evacuees use OR_es. Also, for the case in which evacuees use shortest paths, the 90% percentile for all individuals' traveling time is bigger than the case in which evacuees use OR_es. Thus, the evacuation routes, which is proposed by SEND model, is effective to alleviate individuals' suffering.

Table 28 Increase of Lengths From Shortest Paths to Opt. Paths)

Max	Min	Average	STDEV	Median
21%	0%	4%	0.05	2%

Table 29 Individuals' Traveling Time in Experiment V

Type of Paths	Percentage of Evacuees (T. T. < 300 mins)	90% Percentile for T. T. (mins)
Opt	89%	242
Shortest	87%	247

IV.5. Summary

Due to the difficulties in communication and coordination, especially for a large population, in a chaotic emergency situation, evacuees may fail to follow the evacuation instructions because of misunderstandings and confusion, or evacuees may just want to make their own choices on evacuation routes and shelters. To consider these situations which cannot be handled by a centralized optimization model, we construct MAS model to study the case in which evacuees can make their own decisions and change decisions along their evacuation, even they have been told the designated routes and shelters as guidelines. Also, we include time component to MAS model. Rather than a constant value, traffic speed is considered as a nonlinear dynamical function of traffic density in MAS model. Thus, traffic speed and traverse time are changed dynamically with real time traffic density. Moreover, in MAS model, evacuees leave in groups at time sequence, and a range of leaving time is assigned to each group. Furthermore, unlike the perfectly information sharing in centralized system, two types of information sharing are considered in MAS model: evacuees send messages about slow traffic to their connections, and a radio station broadcasts on traffic jam and status of shelters to all evacuees.

After constructing MAS model, we conduct five experiments to study the effects of five factors on the performance of evacuation process by evaluating the total evacuation time, the transportation cost, the traffic conditions, and the individuals' traveling time. These five factors are the probabilities at which evacuees follow the designated routes and shelters, the number of groups and the range of leaving time for each group, the type of information sharing, the edge-capacities in traffic network, and the evacuation routes. Through these experiments, we prove that the evacuation plan proposed by SEND model is effective to shorten the total evacuation time, save the transportation cost, improve the traffic conditions, and alleviate individuals' suffering.

CHAPTER V

CONCLUSIONS AND FUTURE DIRECTIONS

This dissertation concentrates on analyzing a regional evacuation network design problem in order to provide a pre-event strategic planning tool. For this purpose, We propose two models: a strategic evacuation network design model and a multi-agent simulation model.

In this chapter, conclusions of this dissertation are summarized in Section V.1, and future research directions are presented in Section V.2.

V.1. Conclusions

We propose a MIP model called SEND to devise effective and controlled evacuation networks for sending evacuees from their origins to shelters before extreme events such as hurricanes. The SEND model determines an optimal set of evacuation routes based on time and capacity constraints. Additionally, the model selects shelters from a set of potential shelter candidates and decides flow assignments on the optimal routes while minimizing the total evacuation cost.

To solve this model for large scale instances, we develop an efficient solution methodology based on the BD approach, which takes advantage of specific characteristics of the SEND problem. We utilize a few technics to accelerate the BD approach. First, we add surrogate constraints to MsP to improve the lower bound of the objective value of SEND in the BD framework. Second, we solve MsP with a loose optimality gap in the first iteration, and then we decrease this loose gap gradually in the consecutive iterations. Third, we include multiple optimality cuts to MsP, instead of one, in each iteration by generating multiple feasible solutions of SEND heuristically. Last, we strengthen Benders optimality cuts to improve the lower bound of the objective value of SEND in the BD framework.

We design and implement an experiment to test our BD technique using a Texas-based evacuation scenario. The SEND model and the BD approach can be efficiently and effectively applied to a large-scale evacuation scenario, and we benchmark the computational performance of our BD technique against the traditional branch-and-cut solution method, which is implemented by CPLEX 12.2. We also design and implement an experiment to study the effects of parameters T, λ, and ξ on the optimal solution of the SEND model.

Although the SEND model is useful under known conditions and perfect information, it is not able to account for uncertainties during evacuation processes. Considering the uncertainty that evacuees do not follow the optimal SEND strategy, we develop the MAS model in which every evacuee can make decisions and change decisions during the evacuation. In this way, the MAS model simulates a real-world emergency evacuation situation where evacuees have the freedom to choose their own routes and their own destinations. Additionally, by adding the time component at a fine granularity to the MAS model, we model traffic speed on an edge as a function of the traffic density of the edge while traffic density is being updated dynamically. Moreover, in the MAS model, we test staged evacuation strategies, in which evacuees leave in groups at a time sequentially. A value is assigned to the range of leaving times for each group. Furthermore, we consider two types of information shared in the system: one is shared between agents and their connections, and the other is sent from a radio station to all agents.

While developing the MAS model, we design and implement five experiments to investigate the effects of five factors on evacuation performance. We evaluate evacuation performance in four perspectives: total evacuation time, individual travel time, system-wide traffic conditions and total transportation cost. First, we examine how varying degrees of compliance to the optimal SEND strategy impacts evacuation performance. In this experiment, we also prove the effectiveness of the optimal evacuation routes and shelters, which are recommended by the SEND model. Second, we investigate the effectiveness of staged evacuation strategies and we study the effects of the number of stages and the leaving times on the evacuation performance. Third, we investigate how varying types of shared information impacts the evacuation performance. Fourth, we benchmark the performance of the evacuation conducted on the evacuation network with extra edge capacities, recommended by the SEND model, against the evacuation conducted on the evacuation network without extra edge capacities. Through this experiment, we prove the effectiveness of the evacuation network design proposed by the SEND model. Last, we benchmark the performance of the evacuation in which evacuees follow their own favorable routes (i.e. the shortest paths from their origins to the specific shelters, and their destination shelters are recommended by the SEND model) against the evacuation in which evacuees follow the optimal evacuation routes (i.e. both of the routes and the shelters are recommended by the SEND model). Through this experiment, we prove that the evacuation routes, which are proposed by the SEND model, are effective to shorten individuals' traveling time.

V.2. Future Directions

A few extensions of this study may be possible.

- Time component in SEND model: In Chapter III, the SEND model is not developed at a fine granularity level for the time component, and this can be explored as a future study. However, by introducing the time component at a fine granularity level, the size of the MIP model will increase dramatically and make the MIP model extremely hard to solve to optimality. Thus, an effective solution methodology should be developed for the new model.

- Uncertainties in evacuation: In Chapter IV, we consider an uncertainty in the evacuation process: evacuees may not follow the designated routes and shelters and they may choose their own routes and shelters. Besides this, other uncertainties in the evacuation process can be considered in future studies, e.g. traffic accidents on roads, damages of some roads and hurricanes that make landfall while the evacuation is in progress. Considering these uncertainties, a model can simulate a more comprehensive situation and handle a more complicated case.

- Consider individuals' characters: In Chapter IV, all evacuees are assigned an equivalent degree of compliance to the optimal SEND strategy. However, based on their personal characters, evacuees may have varying degrees of compliance to the optimal SEND strategy. Baker (1991) stated that evacuees with different ages may have different preferences for reactions in evacuation (e.g. whether evacuees decide to leave or stay). Thus, instead of considering a uniform degree of compliance to the optimal SEND strategy, it is more proper to consider it based on individuals' characters.

REFERENCES

Andreas, A. K., J. C. Smith. 2009. Decomposition algorithms for the design of a nonsimultaneous capacitated evacuation tree network. *Networks* **53(2)** 91–103.

Apivatanagul, P., R.A. Davidson, L.K. Nozick. 2012. Bi-level optimization for risk-based regional hurricane evacuation planning. *Natural Hazards* **60**(2) 567–588.

Baker, E. J. 1991. Hurricane evacuation behavior. *International Journal of Mass Emergencies and Disasters* **9** 287–310.

Benders, J. F. 1962. Partitioning procedures for solving mixed-variables programming problems. *Numerische Mathematik* **4** 238–252.

Berg, R. 2009. Tropical cyclone report Hurricane Ike. *National Hurricane Center* URL `http://www.nhc.noaa.gov/pdf/TCR-AL092008_Ike_3May10.pdf`. [Online; accessed 10-September-2012].

Blake, E.S., C.W. Landsea, E.J. Gibney. 2011. The deadliest, costliest and most intense United States tropical cyclones from 1851 to 2010 (and other frequently requested hurricane facts). *National Oceanic and Atmospheric Administration Technical (NOAA) Memorandum NWS NHC-6* URL `http://www.nhc.noaa.gov/pdf/nws-nhc-6.pdf`. [Online; accessed 10-September-2012].

Blumenthal, R. 2005. Miles of traffic as texans heed order to leave. URL `http://www.nytimes.com/2005/09/23/national/nationalspecial/23storm.html?pagewanted=all`. [Online; accessed 10-September-2012].

Castle, C., A. Crooks. 2006. Principles and concepts of agent-based modelling for developing geospatial simulations. Working paper 110, Centre for Advanced Spatial Analysis, University College London.

Chalmet, L. G., R. L. Francis, P. B. Saunders. 1982. Network models for building evacuation. *Management Science* **28** 86–105.

Chen, M., L. Chen, E. MillerHooks. 2007. Traffic signal timing for urban evacuation. *Journal of Urban Planning and Development* **133** 30–42.

Chen, X. 2008. Microsimulation of hurricane evacuation strategies of galveston island. *The Professional Geographer* **60** 160–173.

Chen, X., J. W. Meaker, F. B. Zhan. 2006. Agent-based modeling and analysis of hurricane evacuation procedures for the Florida Keys. *Natural Hazards* **38** 321–338.

Chen, Y., D. Xiao. 2008. Real-time traffic management under emergency evacuation based on dynamic traffic assignment. *International Conference on Automation and Logistics, Qingdao, China, September 1-3, 2008* 1376–1380.

Chien, S. I., V. V. Korikanthimath. 2007. Analysis and modeling of simultaneous and staged emergency evacuations. *Journal of Transportation Engineering* **133** 190–197.

Chiu, Y., P.B. Mirchandani. 2008. Online behavior-robust feedback information routing strategy for mass evacuation. *IEEE Transactions on Intelligent Transportation Systems* **9**(2) 264–274. URL `http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=4538005`.

Cova, T. J., R. L. Church. 1997. Modelling community evacuation vulnerability using gis. *Int. J. Geographical Information Science* **11**(8) 763–784.

Dow, K., S.L. Cutter. 2002. Emerging hurricane evacuation issues: Hurricane Floyd and South Carolina. *Natural Hazards Review* **3**(1) 12–18.

Easwaran, G., H. Üster. 2009. Tabu search and benders decomposition approaches for a capacitated closed-loop supply chain network design problem. *Transportation Science* **43** 301–320.

Easwaran, G., H. Üster. 2010. A closed-loop supply chain network design problem with integrated forward and reverse channel decisions. *IIE Transactions* **42** 779–792.

Elmitiny, N., S. Ramasamy, E. Radwan. 2007. Emergency evacuation planning and preparedness of transit facilities. *Transportation Research Record* **1992** 121–126.

Fischetti, M., A. Lodi. 2003. Local branching. *Mathematical Programming* **98** 23–47.

Gendron, B., T.G. Crainic, A. Frangioni. 1998. Multicommodity capacitated network design. B. Sanso, P. Soriano, eds., *Telecommunications Network Planning*. Kluwer Academic Publishers, Dordrecht, The Netherlands 1–19.

Greenberg, H. 1959. An analysis of traffic flow. *Operations Research* **7**(1) 79–85.

Gzara, F., E. Erkut. 2011. Telecommunications network design with multiple technologies. *Telecommunication Systems* **46** 149–161.

Hamacher, H. W., S. A. Tjandra. 2001. Mathematical modelling of evacuation problems : A state of art. *Pedestrian and Evacuation Dynamics* **24**(24) 227266. URL `http://www.forschungsplattform.de/zentral/download/berichte/bericht24.pdf`.

Hauser, C., T.J. Lueck. 2005. Mandatory evacuation ordered for New Orleans as storm nears. URL http://www.nytimes.com/2005/08/28/national/29katrinacnd.html?_r=0. [Online; accessed 10-September-2012].

Kaufman, D. E., J. Nonis, R. L. Smith. 1998. A mixed integer linear programming model for dynamic route guidance. *Transportation Research Part B-Methodological* **32** 431–440.

Kewcharoenwong, P., H. Üster. 2012. Benders decomposition algorithms for the fixed-charge relay network design in telecommunications. Forthcoming in *Telecommunication Systems*.

Kim, S., S. Shekhar, M. Min. 2008. Contraflow transportation network reconfiguration for evacuation route planning. *IEEE Transactions on Knowledge and Data Engineering* **20**(8) 1115–1129. URL http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=4384487.

Kirchner, A., H. Klupfel, K. Nishinari, A. Schadschneider, M. Schreckenberg. 2003. Simulation of competitive egress behavior: Comparison with aircraft evacuation data. *Physica A-Statistical Mechanics and its Applications* **324** 689–697.

Knabb, R.D., D.P. Brown, J.R. Rhome. 2006. Tropical cyclone report Hurricane Rita. *National Hurricane Center* URL http://www.nhc.noaa.gov/pdf/TCR-AL182005_Rita.pdf. [Online; accessed 10-September-2012].

Knabb, R.D., J.R. Rhome, D.P. Brown. 2005. Tropical cyclone report Hurricane Katrina. *National Hurricane Center* URL http://www.nhc.noaa.gov/pdf/TCR-AL122005_Katrina.pdf. [Online; accessed 10-September-2012].

Krothapalli, N.K.C., A.V. Deshmukh. 1999. Design of negotiation protocols for multi-agent manufacturing systems. *International Journal of Production Research* **37** 1601–1624.

Lamel, G., D. Grether, K. Nagel. 2010. The representation and implementation of time-dependent inundation in large-scale microscopic evacuation simulations. *Transportation Research Part C* **18** 84–98.

Liu, H. X., J. X. Ban, W. Ma, P. B. Mirchandani. 2007. Model reference adaptive control framework for real-time traffic management under emergency evacuation. *Journal of Urban Planning and Development* **133** 43–50.

LIU, Y., X. ZHANG, W. ZHANG. 2007. *Multiple Periods Flood Disaster Evacuation Grey Decision Model*. Ieee 18–20. URL `http://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=04443389`.

Lu, Q., B. George, S. Shekhar. 2005. Capacity constrained routing algorithms for evacuation planning: A summary of results. *LNCS* **3633** 291–307.

Magnanti, T. L., R. T. Wong. 1981. Accelerating benders decomposition: Algorithmic enhancement and model selection criteria. *Operations Research* **29** 464–484.

Malleson, N. 2008. Agent-based crime simulation. URL `http://crimesim.blogspot.com/2008/05/using-repast-to-move-agents-along-road.html`. [Online; accessed 12-August-2011].

Mamada, S., K. Makino, S. Fujishige. 2004. Evacuation problems and dynamic network flows. *SICE Annual Conference, Sapporo, Japan, August 4-6, 2004* 530–535.

Marin, A., P. Jaramillo. 2009. Urban rapid transit network design: accelerated benders decomposition. *Annals of Operations Research* **169** 35–53.

Murray, P. M., H. S. Mahmassani. 2002. Model of houshold trip chain sequencing in an emergency evacuation. *TRB 2003 Annual Meeting* .

Nikolai, C., G. Madey. 2009. Tools of the trade: A survey of various agent based modeling platforms. *Journal of Artificial Societies and Social Simulation* **12**(2) 2. URL `http://jasss.soc.surrey.ac.uk/12/2/2.html`.

NOAA. 2012. Safety and preparedness fact sheet. URL `http://www.nws.noaa.gov/os/hurricane/pdfs/hurricane-safety_flyer.pdf`. [Online; accessed 10-September-2012].

Noh, H., Y. Chiu, H. Zheng, M. Hickman, P. Mirchandani. 2009. Approach to modeling demand and supply for a short-notice evacuation. *Transportation Research Record* **2091** 91–99.

Olsson, P. A., M. A. Regan. 2001. A comparison between actual and predicted evacuation times. *Safety Science* **38** 139–145.

Railsback, S., S. Lytinen, S. Jackson. 2006. Agent-based simulation platforms: Review and development recommendations. *Simulation* 609–623.

Rei, W., J. Cordeau, M. Gendreau, P. Soriano. 2009. Accelerating benders decomposition by local branching. *INFORMS Journal on Computing* **21** 333–345.

Roy, T. J. Van. 1986. A cross decomposition algorithm for capacitated facility location. *Operations Research* **34** 145–162.

Saadatseresht, M., A. Mansourian, M. Taleai. 2008. Evacuation planning using multiobjective evolutionary optimization approach. *European Journal of Operational Research* **198** 305–314.

Saharidis, G. K.D., M. G. Ierapetritou. 2010. Improving benders decomposition using maximum feasible subsystem (mfs) cut generation strategy. *Computers and Chemical Engineering* **34** 1237–1245.

Serenko, A., B. Detlor. 2002. Agent toolkits: A general overview of the market and an assessment of instructor satisfaction with utilizing toolkits in the classroom. Working Paper.

Simonovic, S. P., S. Ahmad. 2005. Computer-based model for flood evacuation emergency planning. *Natural Hazards* **34** 25–51.

Tobias, R., C. Hofmann. 2004. Evaluation of free java-libraries for social-scientific agent based simulation. *Journal of Artificial Societies and Social Simulation* **7**(1) 1. URL `http://jasss.soc.surrey.ac.uk/7/1/6.html`.

Torres-Soto, J.E., H. Üster. 2011. Dynamic-demand capacitated facility location problems with and without relocation. *International Journal of Production Research* **49** 3979–4005.

Üster, H., H. Agrahari. 2011. A benders decomposition approach for a distribution network design problem with consolidation and capacity considerations. *Operations Research Letters* **39** 138–143.

Üster, H., P. Kewcharoenwong. 2011. Strategic design and analysis of a relay network in truckload transportation. *Transportation Science* **45** 505–523.

Üster, H., H. Lin. 2011. Integrated topology control and routing in wireless sensor networks for prolonged network lifetime. *AD HOC Networks* **9** 835–851.

Wei, H., Q. Zeng, H. Hu, X. Wang, A. R. Kukreti. 2008. Integrated urban evacuation planning framework for responding to human-caused disasters over a surface transportation network. *Transportation Research Record* **2041** 29–37.

Wentges, P. 1996. Accelerating benders' decomposition for the capacitated facility location problem. *Mathematical Methods for Operations Research* **44** 267–290.

Wolshon, B., B. McArdle. 2011. Traffic impacts and dispersal patterns on secondary roadways during regional evacuations. *Natural Hazards Review* 19–27.