THE PREDICTION OF BUS ARRIVAL TIME

USING AUTOMATIC VEHICLE LOCATION SYSTEMS DATA

A Dissertation

by

RAN HEE JEONG

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

December 2004

Major Subject: Civil Engineering

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ABSTRACT

The Prediction of Bus Arrival Time
Using Automatic Vehicle Location Systems Data. (December 2004)

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Advanced Traveler Information System (ATIS) is one component of Intelligent Transportation Systems (ITS), and a major component of ATIS is travel time information. The provision of timely and accurate transit travel time information is important because it attracts additional ridership and increases the satisfaction of transit users. The cost of electronics and components for ITS has been decreased, and ITS deployment is growing nationwide. Automatic Vehicle Location (AVL) Systems, which is a part of ITS, have been adopted by many transit agencies. These allow them to track their transit vehicles in real-time. The need for the model or technique to predict transit travel time using AVL data is increasing. While some research on this topic has been conducted, it has been shown that more research on this topic is required.

The objectives of this research were 1) to develop and apply a model to predict bus arrival time using AVL data, 2) to identify the prediction interval of bus arrival time and the probabilty of a bus being on time. In this research, the travel time prediction model explicitly included dwell times, schedule adherence by time period, and traffic congestion which were critical to predict accurate bus arrival times. The test bed was a bus route running in the downtown of Houston, Texas. A historical based model, regression models, and artificial neural network (ANN) models were developed to predict bus arrival time. It was found that the artificial neural network models performed considerably better than either historical data based models or multi linear regression models. It was hypothesized that the ANN was able to identify the complex non-linear

relationship between travel time and the independent variables and this led to superior results.

Because variability in travel time (both waiting and on-board) is extremely important for transit choices, it would also be useful to extend the model to provide not only estimates of travel time but also prediction intervals. With the ANN models, the prediction intervals of bus arrival time were calculated. Because the ANN models are non parametric models, conventional techniques for prediction intervals can not be used. Consequently, a newly developed computer-intensive method, the bootstrap technique was used to obtain prediction intervals of bus arrival time.

On-time performance of a bus is very important to transit operators to provide quality service to transit passengers. To measure the on-time performance, the probability of a bus being on time is required. In addition to the prediction interval of bus arrival time, the probability that a given bus is on time was calculated. The probability density function of schedule adherence seemed to be the gamma distribution or the normal distribution. To determine which distribution is the best fit for the schedule adherence, a chi-squared goodness-of-fit test was used. In brief, the normal distribution estimates well the schedule adherence. With the normal distribution, the probability of a bus being on time, being ahead schedule, and being behind schedule can be estimated.

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TABLE OF CONTENTS

Page

CHAPTER Page

CHAPTER Page

CHAPTER Page

LIST OF FIGURES

Page

Page

Page

Page

Page

Page

LIST OF TABLES

Page

Page

Page

Page

Page

Page

Page

Page

CHAPTER I

INTRODUCTION

One component of Intelligent Transportation Systems (ITS) is Advanced Traveler Information System (ATIS), and a major component of ATIS is providing travel time information by different modes to travelers. The provision of timely and accurate transit arrival time information is important because it attracts additional transit ridership and increases the satisfaction of transit users (1-5).

The cost of electronics and components for ITS has been decreased, and ITS deployment is growing nationwide (6, 7). Automatic Vehicle Location (AVL) Systems, which are a part of ITS, have been adopted by many transit agencies, allowing them to track their transit vehicles in real-time (7). While the provision of real-time information, such as bus location, is relatively straightforward, forecasting transit information, such as when a bus will arrive at a particular location, is significantly more complex. Consequently, the need for predicting transit arrival time using AVL data is increasing. While some research on this topic has been conducted, there is still important work to be done (8-12).

1.1 STATEMENT OF PROBLEM

1.1.1 Need to Develop a Bus Arrival Time Prediction Model Using AVL Data

The increase in the transit ridership and the satisfaction of transit users can be achieved by the provision of current traveler information (1-5). In addition, transit operators can identify vehicles that 1) have fallen behind schedule or 2) are in danger of falling behind schedule, and react in a proactive way. For example, bus priority at traffic signals could

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This dissertation follows the style and format of *Transportation Research Record.*

be enabled. Because ITS technologies are deployed nationwide (1, 2, 6, 7), the usage of AVL systems by transit agencies continues to increase (7). Consequently, the need for robust prediction algorithms increases. While there has been some preliminary work in this area, there are a number of questions that need to be considered.

1.1.2 Need to Explicitly Consider Traffic Congestion

In order to predict travel time in an accurate and timely manner, the consideration of traffic conditions is essential, including traffic congestion. The impact of recurrent and non recurrent congestion has been ignored. Lin and Zeng did not consider traffic congestion because their algorithm was formulated primarily for a rural area and because they did not have a system which measured traffic congestion (9). Ojili used one-minute time zones where a bus can run for one minute (10). After finding the current bus location, he predicted the arrival time by counting the estimated number of one-minute time zones between the current location and the given stop. Because the one-minute time zones could be changed by time period or by traffic condition, the model would have to be recalibrated if traffic conditions change. Because Shalaby and Farhan used a Kalman filtering technique, they assumed that the pattern of link travel time is cyclical (13). In summary, a prediction model that explicitly considers traffic congestion is needed.

1.1.3 Need to Explicitly Consider Dwell Times at Bus Stops

The major difference in predicting bus and auto travel times is that the former needs to consider dwell time at bus stops. However, there has been little research that explicitly considers bus dwell times when predicting transit arrival times. Lin and Zeng used regular global positioning systems (GPS), not differential global positioning systems (DGPS), and their GPS unit provided bus location every forty six seconds. They were not able to measure exact dwell time at stops, and consequently they did not explicitly consider it in their model (9). Similar to the previous argument, Ojili (10) and Wall and Dailey (8) did not consider dwell times when they predicted transit arrival time. Shalaby and Farhan assumed that dwell times increase when a bus arrives late because there would be more passengers waiting for the bus (13). In addition, they assumed that dwell time is directly proportional to passenger demand and simply multiplied by 2.5 seconds per passenger. However, they did not consider the fact that the bus driver will stay longer to keep to schedule when they arrive early or that the bus driver can stay at bus stops even though there is no passenger. Because dwell time is a function of the behavior of the bus driver and passengers, a model that can explain the uncertainty of the behavior is required. In summary, a prediction model which explicitly considers dwell time at bus stops is needed.

1.1.4 Need to Consider Schedule Adherence

Transit vehicles have a predefined schedule to follow. Because of this requirement, bus drivers may stay longer at bus stops if they are ahead of schedule and/or to pass some stops if they are behind schedule. In other words, bus schedules control the behavior of bus drivers, dwell times at bus stops, and link travel times. Schedule adherence is the difference between schedule time and actual arrival time. A positive value of schedule adherence means that the bus arrives late and negative value means that the bus arrives early. Consequently, a bus arrival time prediction model should consider schedule adherence as an input variable.

1.1.5 Need to Provide Prediction Intervals

Most existing bus arrival time prediction models only provide the mean value of bus arrival time. For example, the information might be that the bus will arrive in 10 minutes. However, prediction errors tend not to be provided. If a prediction interval is provided with the mean value, it should give more useful information for passengers when making their decisions. For example, the information would be that the bus will arrive in 10 minutes plus or minus 1 minute. Rather than providing a point prediction value, an interval prediction value would be more valuable information for transit users.

1.1.6 Need to Provide the Probability of a Bus Being on Time

On-time performance of a bus is very substantial to transit operators because customers use this to measure quality of service. It would be extremely important to identify, in real-time, whether a given bus is on schedule or not. To measure the on-time performance, the probability of a bus being on time is required. In addition to the prediction interval of bus arrival time, the probability that a given bus is on time is needed to be estimated.

1.2 RESEARCH OBJECTIVES

It is hypothesized that a bus arrival time prediction model considering traffic congestion and dwell time will give a superior result compared to simple conventional prediction models. The objectives of this research are to develop and apply a statistical model to predict bus arrival time using AVL data. Arrival time information can be provided to travelers to help in their decision making and can be used by transit operators to improve their operations. Specific objectives of this research are as follows:

Analyze the characteristics of AVL data, including arrival time, dwell time, and schedule adherence data.
Select reasonable input variables for a bus prediction model.
Cluster input data by time period to implicitly consider traffic congestion.
Develop prediction models including historical data based models, multi linear regression models, and artificial neural network models.
Forecast the bus arrival travel time with three developed models.
Evaluate these three models in terms of prediction accuracy.
Develop a methodology for identifying the prediction interval of the bus arrival time.
Identify a probability function to estimate the probability of a bus being on time.

1.3 RESEARCH FRAMEWORK AND METHODOLOGIES

1.3.1 Perform a Literature Review

Related research reports, journal articles, and Ph. D. dissertations were reviewed. The primary areas of interest are 1) the current state of practice and trends in the provision of traveler information 2) the technology of AVL systems, 3) GPS theory, and 4) the methodology for travel time prediction. The purpose of this task is to ensure that no research relevant to this study is overlooked or inappropriately duplicated.

1.3.2 Collect Data and Define Test Bed

Actual AVL data collected in Houston, Texas, were used as a test bed. The Houston data were collected by Houston Metro buses equipped with DGPS receivers that collect data at 5 seconds intervals. Data were collected over 6 months in 2000 (from June to November). The test bed is route 60, which runs on a congested corridor in Houston. This DGPS provides time, speed, heading, etc., as well as bus location.

There are two test bed sites: a downtown area corridor and a north area corridor. The first corridor has 9 bus stops and is 1.6 kilometer long. Stop 1 and stop 9 are time check points where bus drivers should keep to scheduled time. The second corridor has 25 bus stops and is 4.26 kilometer long. Stop 6 and stop 20 are time check points. The schedule headway during weekday peak period is about 30 minutes and during the weekday non-peak period and weekends is about 1 hour.

1.3.3 Reduce Data and Correct Errors

There are two types of errors associated with GPS data. The first is noise errors added by the U.S. DOD in order to degrade the accuracy of GPS data. This error was corrected by using DGPS. The second type of error is measurement errors. It is anticipated that some of the bus location data were correspond to off-route locations (i.e. parking lot, refueling station, etc.). In addition, even if the bus is located on the road, there would be errors associated with its exact location. An additional source of data error would be missing data. Where the data are missing, existing data were used to calculate input data according to distance. Outliers were also identified when the data are located unreasonably far away from the road.

1.3.4 Cluster Data

The transit schedule and congestion for weekday peak hour, non-peak hour, evening, and weekend, are different. It would be expected that dwell time and link travel time would also be different. To account for these differences, data were clustered by time of the week and time of the day.

1.3.5 Develop Prediction Models

A number of modeling techniques were used including a simple statistical model (historical data), a regression model, and an artificial neural network model. In this research, the input variables were be arrival time, dwell time, and schedule adherence at each stop. To consider traffic congestion, schedule adherence was calculated by subtracting the scheduled data from the actual arrival time. A positive value of schedule adherence means that the bus is delayed at the stop while a negative value means that the bus arrives early. To consider traffic congestion, the link travel times were clustered by time period in task 4. The output variable is arrival time at each stop.

1.3.6 Evaluate Prediction Models

All three model architectures were calibrated. With these calibrated models, the arrival times were predicted. A validation data set was obtained in order to test which model is most appropriate. Predicted arrival times were compared to the observed arrival times from the validation data set. The Mean Absolute Percentage Error (MAPE) was used as the measure of effectiveness (MOE). The MAPE is shown in Equation 1-1. It represents the average percentage difference between the observed value (in this case observed arrival times at a bus stop) and the predicted value (in this case predicted arrival times at a bus stop). Smaller MAPE means that the model predicts more accurately than other models.

$$MAPE = \frac{1}{n}\sum_{i}^{n}\frac{|y_i - y_o|}{y_o} \times 100\% \qquad (1\text{-}1)$$

where,

y_i = Predicted value (i.e. arrival time at given transit stop);

y_o= Observed value (i.e. arrival time at given transit stop);

n = The number of data considered.

1.3.7 Identify the Prediction Interval of the Bus Arrival Time

The model with the smallest MAPE is chosen for the prediction model for the bus arrival time. With the selected model, prediction intervals on these estimates were provided. If ANN models are chosen for the outperformed model, the conventional method for finding prediction interval is not appropriate. In that case, the bootstrap method, which is a statistical method that provides prediction intervals for non-parametric models, was used. In order to statistically test the differences in mean and variance of the three different models, one of several pairwise comparison methods, such as Tukey's procedure was used.

1.3.8 Identify the Probability of a Bus Being on Time

The probability density function of schedule adherence was identified. To determine which distribution was the best fit for schedule adherence, a chi-squared goodness-of-fit test was used. After identifying the best fit probability density function, the probability of a bus being on time, being ahead schedule, and being behind schedule were able to be estimated.

The framework for this research is shown in FIGURE 1-1.

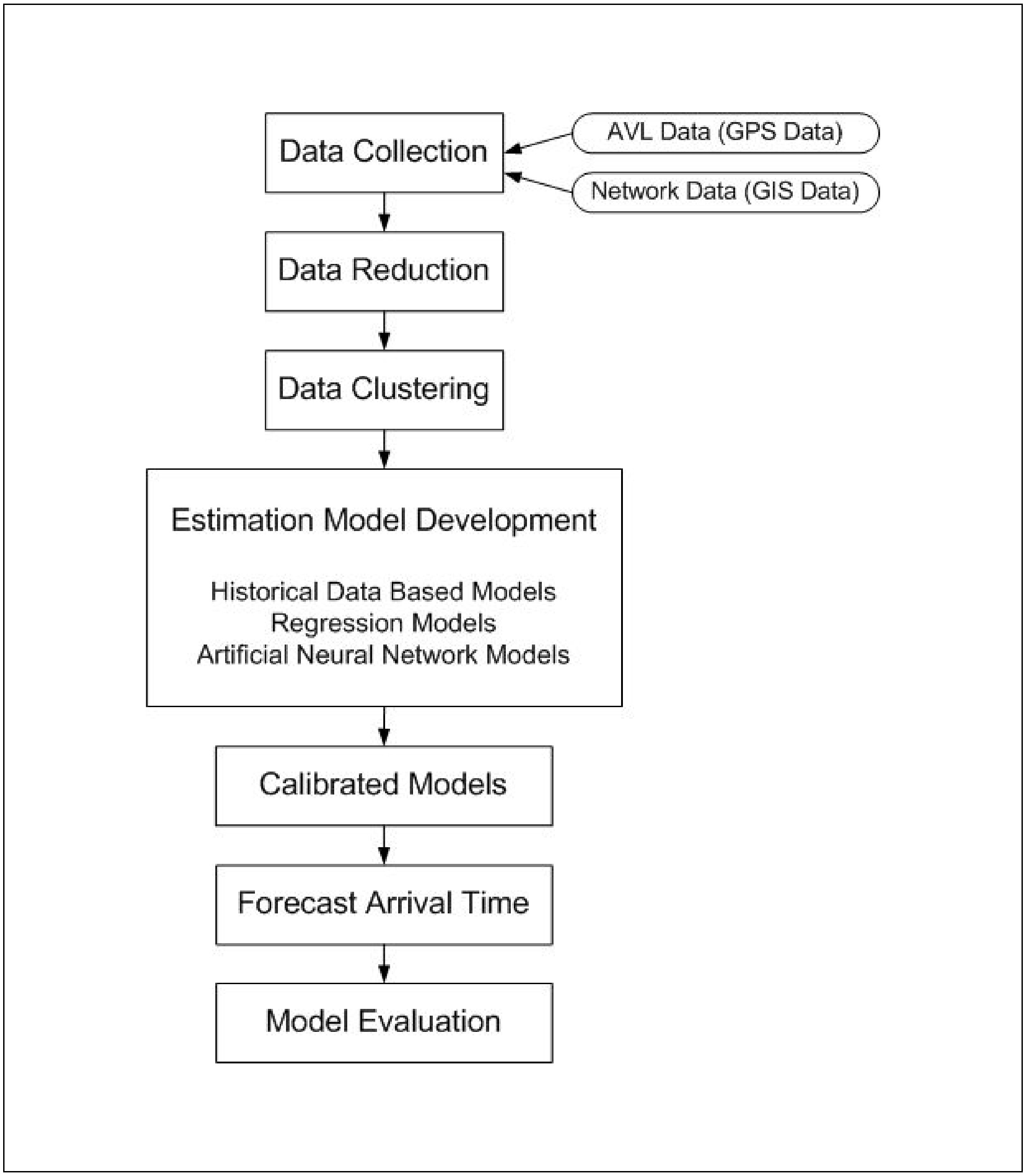

FIGURE 1-1 Framework for Research

1.4 CONTRIBUTION OF THE RESEARCH

Providing travel time information is a major component of ATIS. With the deployment of ATIS, the provision of traveler information can extend the ridership and increase the satisfaction of transit users. Many transit agencies have adopted Automatic Vehicle Location (AVL) Systems and track their transit vehicles in real-time. The need for a model or technique to predict transit arrival time using AVL data is increasing. While some research on this topic has been conducted, there is still important work to be done.

To provide accurate and timely traveler information, consideration of the traffic conditions is essential. However, recent research has not fully considered traffic congestion and dwell times at bus stops, which are critical factors for predicting bus arrival time. This research can provide more accurate prediction of bus arrival time considering traffic congestion and dwell time. In addition to the mean value of arrival time, prediction interval information was provided. This would be more reliable information for both passengers and transit agencies, leading to better operation.

1.5 ORGANIZATION OF THE RESEARCH

This dissertation is organized into eight chapters. Chapter I is an introduction to the research and discusses the background of the problem, statement of the problem, research objectives, research methodology, contribution of the research, and the organization of the dissertation. Chapter II presents a literature reviews on advanced traveler information systems, automatic vehicle location systems, global positioning systems, travel time prediction models, and bus arrival time prediction models. Chapter III describes the details of the test bed and the reduction of the data. Chapter IV describes and graphically depicts the characteristics of the input variables. The development of three bus arrival time prediction models is included in chapter IV, including a historical data based model, a multi linear regression model, and an artificial neural network model. Chapter V discusses the evaluation of the three prediction models. In addition to this, the statistical test for model comparison is conducted in this chapter. Chapter VI discusses the prediction interval of bus arrival time and the probability that a bus is behind schedule. Chapter VII provides contributions and recommendations based on the research. Suggestions for further research are also included in this chapter. The references are followed by a glossary of frequently used terms and acronyms. The appendices also include the results of artificial neural network models with different training and learning functions.

CHAPTER II

LITERATURE REVIEW

2.1 ADVANCED TRAVELER INFORMATION SYSTEMS (ATIS)

2.1.1 Types of Traveler Information Systems

A traveler who wants to move from point A to point B faces a number of decisions including: what transportation mode do I need to use, what time do I need to depart, what transit route or what road do I need to use, etc. Traveler information systems help travelers to make decisions regarding mode, route, and departure time. There are three types of traveler information, and they can be categorized according to the time at which information is provided: pre-trip information, in-terminal/wayside information, and in-vehicle information (1).

Pre-trip information, which is provided before the trip begins, includes transit routes, maps, schedules, fares, park-and-ride lot locations, points of interest, and weather. The information can be distributed by touch-tone telephone, Internet, kiosks, personal pagers, hand-held data receivers, and cable television.

In-terminal/Wayside transit information is provided to transit riders who are already en-route. The information can be distributed by electronic signs, interactive information kiosks, and closed-circuit television monitors.

In-vehicle transit information is provided to transit users while they are in the transit vehicle. Many transit agencies use automated annunciators and in-vehicle displays to provide information about audible and visual next stop, intersection, and transfer point.

2.1.2 Importance of the Provision of Traveler Information

There are a number of reasons why travelers do not use transit. It has been shown that the lack of information and the lack of schedule reliability discourage transit use (1-3, 14). It is hypothesized that if reliable transit information could be provided, travelers would be more likely to use transit. Additionally, it has also been shown that out-of-vehicle waiting time is more critical than in-vehicle travel time when users perceive the quality of transit service (4). Therefore, a reduction in the uncertainty of out-of-vehicle waiting time would enhance the satisfaction of transit users. If accurate arrival time forecasts could be provided to transit users through ATIS, then this uncertainty would be reduced and ridership would increase. Consequently, the provision of accurate and timely traveler information encourages positive attitudes toward transit resulting in increased ridership (1-3, 5). The provision of traveler information is important for transit operators because it not only attracts additional ridership but also increases the satisfaction of current users (1-5). In addition, transit operators can identify vehicles that 1) have fallen behind schedule or 2) are in danger of falling behind schedule, and react in a proactive way. For example, bus priority at traffic signals could be enabled.

2.1.3 Real-time Information

A recent trend, which is directly related to the advances in ITS technologies, is the provision of real-time transit information (2, 15). Real-time transit information includes: transit vehicle arrival time, transit vehicle departure time, current transit vehicle location, speed, and delay. Real-time information is very valuable to transit users because 1) the knowledge of the arrival time can reduce their anxiety related to waiting, and 2) the transit riders can decide whether to wait at transit stops or seek another mode of travel (5). To provide better service for transit patrons, many transit agencies are planning or providing real-time information (6, 16). Real-time data are obtained from Automatic Vehicle Location (AVL) Systems or Automatic Vehicle Identification (AVI) devices, and it can be provided as real-time traveler information directly. However, more commonly the data are processed in order to provide information such as bus arrival time, link travel time, delay, etc., because this is the type of information most valuable to transit users.

2.2 AUTOMATIC VEHICLE LOCATION (AVL) SYSTEMS

Automatic Vehicle Location (AVL) Systems are computer-based vehicle tracking systems. They are also referred to as Automatic Vehicle Monitoring (AVM) Systems or Automatic Vehicle Location and Control (AVLC) Systems (1). They are used in transit, trucking fleets, police cars, ambulances, and for military purposes, and their use in transit continues to grow (1). FIGURE 2-1 shows the schematic display of an AVL system used in transit agencies (2).

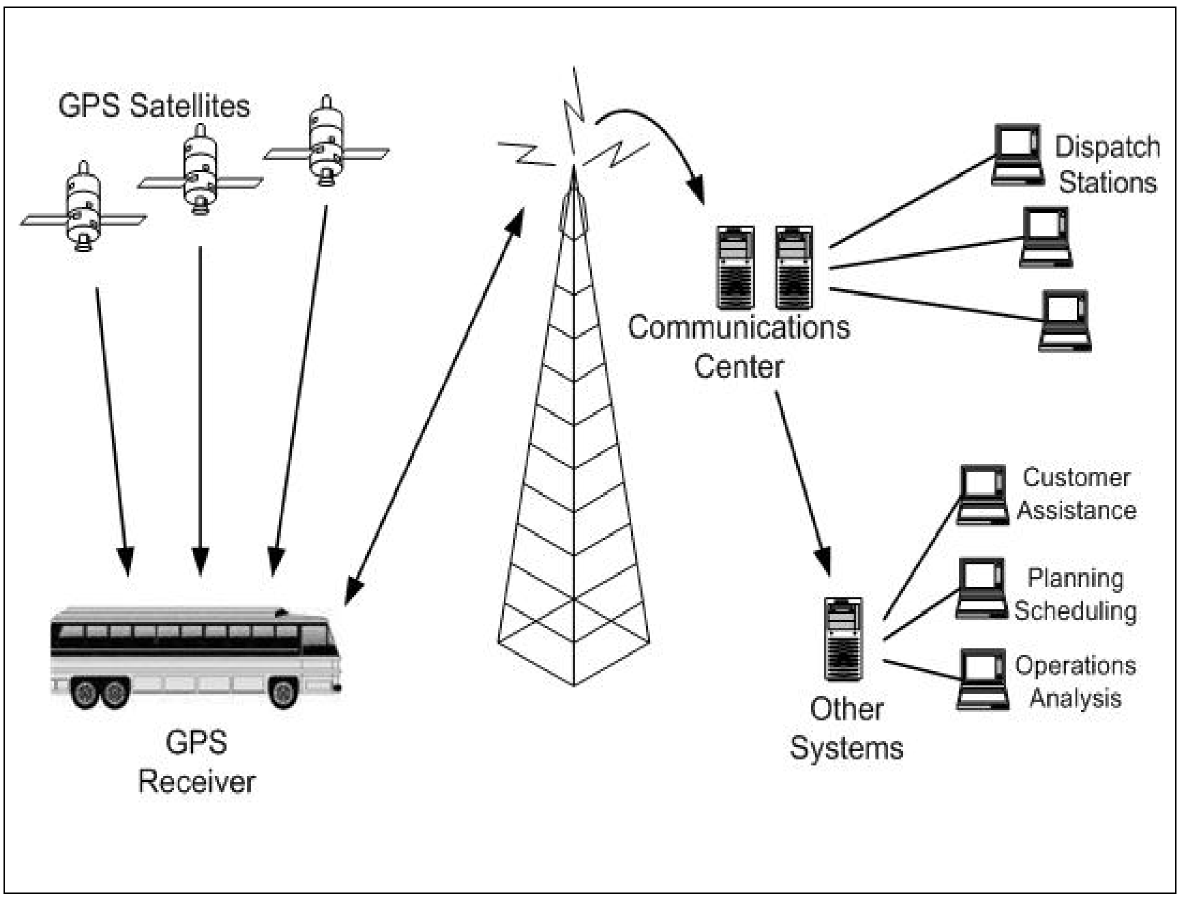

FIGURE 2-1 Schematic Display of an AVL System Used in Transit Agencies

2.2.1 Benefits of AVL Systems

The benefits of AVL systems are as follows:

1) they collect real-time information which can be provided to the public,

2) they improve schedule reliability,

3) they reduce operating and maintenance costs,

4) they improve service efficiency,

5) they enhance safety and security, and

6) the transit agencies can respond more quickly to emergency situations (17-19).

The benefits of AVL systems have been well documented. Schedule adherence improved by 23% in Baltimore, 12.5% in Kansas City, 8.5% in Hamilton, Ontario, and 4.4% in Milwaukee after AVL installation (18, 20). Operating costs were reduced $500,000/year in Kansas City, and $45,000/year in London, Ontario (21). Paratransit ridership increased by 17.5% and paratransit passenger waiting time decreased by 50% in Winston-Salem, North Carolina (21).

2.2.2 Uses of AVL Systems

The first use of AVL technology in transit was in London, England in the late 1950s, and the first use in the United States was in Chicago in the late 1960s (6). A number of transit systems in North America and abroad began to plan for and implement AVL systems during the 1980s (6).

According to the research of Casey in 2002, 322 transit agencies are either operating, implementing, or planning/ testing/ demonstrating AVL systems (16). The number of transit agencies used AVL systems increased by four hundred percent as compared to earlier studies in 1995 (7, 16, 22). An increasing number of transit agencies are planning to install AVL systems because the cost of AVL systems has rapidly dropped (6, 18-22).

2.2.3 Technologies of AVL Systems

AVL systems consist of two technologies, location technology and data transmission technology. “Location technology is used to measure actual real-time position of each vehicle, and data transmission technology is used to relay the information to a central location” (1).

2.2.3.1 LocationTechnology

Location technology includes dead-reckoning, signpost and odometer, global positioning systems (GPS), differential GPS (DGPS), etc. For transit vehicle, a single location technology is usually insufficient for determining position. For instance, tall buildings block the signals and result in multi-path errors. Therefore, the primary location technology is supplemented with another location technology (1). TABLE 2-1 details the advantages and disadvantages of different location technologies.

Dead-reckoning is the most self-determined form of location technology. The transit vehicle determines its own location without the help of external technologies. First, the transit vehicle is told its starting point. The vehicle measures the traveled distance from the starting point by reading the odometer. Then the vehicle determines the traveled direction by compass headings. Dead-reckoning location technology is seldom used by itself because the equipment has to be reset frequently from a known location. Dead-reckoning is usually supplemented by one of other location technologies like signpost or GPS (1). It is relatively inexpensive, but the accuracy degrades with distance traveled (23).

Signpost and odometer uses a series of radio beacons placed along the bus routes. The beacons send a low power signal and the signal is detected by a receiver on the transit vehicle. Then the transit vehicle reports its position to dispatch according to the traveled distance, which is taken from the odometer (1). This technology requires low in-vehicle cost and it is well established and proven. However, additional installation is required with route changes and it can not track the vehicle when a bus is off-route (1, 23).

Global positioning systems (GPS) determine the position using the signals which are transmitted from up to 24 satellites. GPS works anywhere the satellites reach, and it is much more robust than other location technologies. However, satellite signals do not reach underground and they can be interrupted by tall buildings or foliage. Where these problems happen, signpost and odometer can supplement the GPS (1).

The U.S. Department of Defense (DOD) intentionally degraded the accuracy of GPS for safety reason. To correct this interruption, an additional (differential) correction was added and this system is called Differential GPS (DGPS) (1).

TABLE 2-1 The Advantages and Disadvantages by Location Technologies

Type	Advantages	Disadvantages
Dead reckoning	Relatively inexpensive Self-contained on vehicle (no infrastructure costs) Only odometer needed (if on-route is assumed)	Accuracy degrades with distance traveled (errors can accumulate between known locations) - Requires direction indicator and maybe map matching for off-route use Corrupted by uneven road surfaces, steep hills, or magnetic interference
Signpost and odometer	Low in-vehicle cost No blind spots or interference Repeatable accuracy	Requires well-equipped infrastructure No data outside of deployed infrastructure Frequency of updates depends on density of signpost
GPS	Moderately accurate Global coverage Moderate cost per vehicle	Signal attenuation by foliage and tunnels Subject to multi-path errors
DGPS	Very accurate Moderate cost per vehicle	Signal attenuation by foliage and tunnels Subject to multi-path errors Must be within range of differential signal Differential correction must be updated frequently

In the early 1990s, more than 60 percent of transit agencies choose signpost and odometer systems as their AVL location technology (7). However, by 1999, it was found that more than 80 percent of transit agencies choose GPS/DGPS technology (7). FIGURE 2-2 shows the current use of AVL location technologies. The accuracy of AVL systems is critical for transit applications and, ultimately, to increase ridership (6). The use of GPS eliminates the concern about accuracy of information from ground-based AVL systems using odometer readings (6).

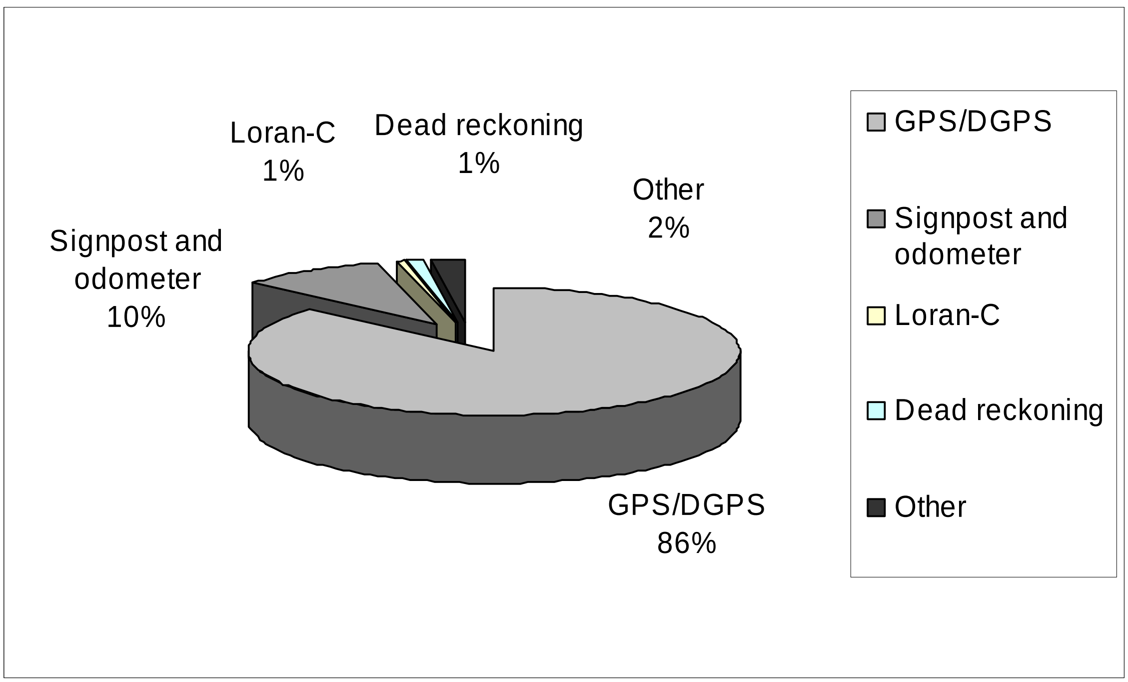

FIGURE 2-2 Current Use of AVL Location Technologies

2.2.3.2 Data Transmission Technology

Position information, regardless of which location technology is adopted, is usually stored on the transit vehicle for some period of time. The information is relayed to the dispatch center in raw form or is processed on-board the vehicle. The two most common data transmission technology are polling and exception reporting (1, 18).

With polling technology, the computer at the dispatch center asks each bus for its location at regular intervals. The accuracy of location is a function of how often the transit vehicle is polled. In addition, the number of radio frequencies which are available in urban areas is limited. Due to this reason, many transit agencies have chosen another technology, exception reporting (1).

Under the exception reporting method, each bus reports its location at a few specified locations or when the bus is found to be off-schedule by some pre-defined tolerance. Exception reporting requires each transit vehicle to know not only its position but also its scheduled position. Many agencies combine the polling and exception reporting methods (1, 18).

2.3 GLOBAL POSITIONING SYSTEMS (GPS)

GPS is a satellite-based navigation system which is funded and controlled by the U.S. Department of Defense (DOD) (23). Even though it was intended for military use, the system has been available for civilian applications world-wide since the 1980s (24). The GPS consists of 24 satellites (see FIGURE 2-3) and transmits the estimated position, velocity, and current time to GPS receivers. To compute position, velocity, and current time, signals from at least four satellites are used (see FIGURE 2-4).

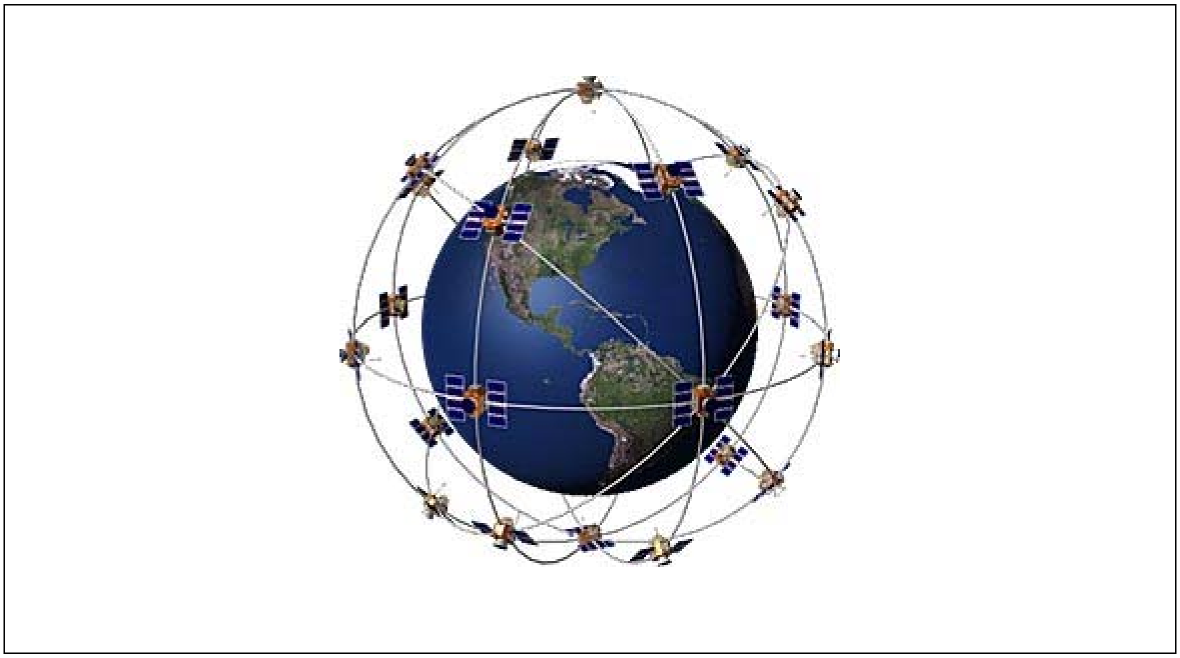

FIGURE 2-3 Twenty Four Satellites of GPS

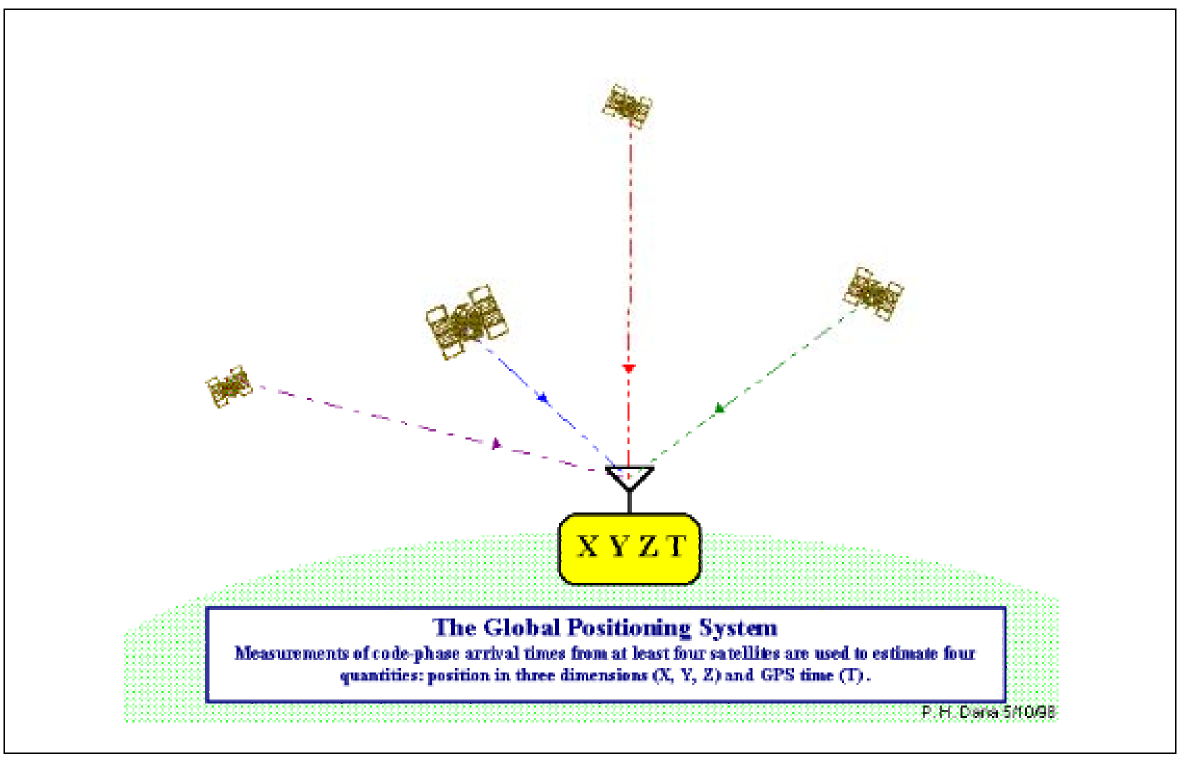

FIGURE 2-4 Calculation of Position, Speed, and Time Using Four Satellites

2.3.1 Accuracy of GPS

GPS has two positioning services, Precise Positioning Service (PPS) and Standard Positioning Service (SPS). PPS is used by authorized users such as U.S. and Allied military while SPS is used by civilian users worldwide (24). For security reasons the DOD intentionally degraded SPS accuracy. The accuracy of PPS was within 22 meters, and the accuracy of SPS was within 100 meters (24). To improve the accuracy of SPS, an additional correction (differential) signal was added, and is called Differential GPS (DGPS) (25). The accuracy of DGPS was better than 10 meters (23).

The SPS accuracy was dramatically improved when the US military removed the intentional degradation to the signal on May 1, 2000 (25). Currently the accuracy of PPS and SPS are the same. The current accuracy of GPS is between 10 and 20 meters, and that of DGPS is between 3 and 5 meters (18, 26).

2.3.2 Use of GPS in Transportation

While traditional methods of data collection techniques in transportation are burdensome, time consuming, and error prone, GPS provides better accuracy, consistency, automation, and easier integration between collected data and the data based on GIS (27-28). Because of the advantages of GPS, a number of studies on data collection using GPS have been conducted (28-32). They used GPS to collect travel time, speed, route choice, and travel surveys. They have shown that the use of GPS for collecting data is easier and more accurate than traditional methods (28-32).

2.4 TRAVEL TIME PREDICTION MODELS

The accurate prediction of link travel time is critical to ITS transit applications. With the development of Advanced Travelers Information Systems (ATIS), the importance of the short-term travel time prediction has increased markedly (33). A number of prediction models, including historical data based models, regression models, time series models and neural network models, have been developed over the years by various transit agencies.

2.4.1 Historical Data Based Models

Historical data based models predict travel time for a given time period using the average travel time for the same time period obtained from a historical data base. These models assume that traffic patterns are cyclical and the ratio of the historical travel time on a specific link to the current travel time reported in real-time will remain constant (34). The procedure requires an extensive set of historical data and it is difficult to install the system in a new setting (34). Real-time models assume that the most recently observed transit travel times will stay consistently into the future. Chen et al developed a prediction algorithm that combined these two approaches. First a historical data base was used to obtain estimated travel time. This time was subsequently adjusted as real-time location data are obtained (35).

2.4.2 Regression Models

Regression models are conventional approaches for predicting travel time (36). Regression models predict a dependent variable with a mathematical function formed by a set of independent variables (12). To establish a regression model, the dependent variables need to be independent. This requirement limits the applicability of the regression model to the transportation areas because variables in transportation systems are highly inter-correlated (12).

2.4.3 Time Series Models

Time series models assume that the historical traffic patterns will remain the same in the future. The accuracy of Time series models is a function of the similarity between the real-time and historical traffic patterns (12). Variation in historical data or changes in the relationship between historical data and real-time data could significantly cause inaccuracy in the prediction results (37). D'Angelo used a non-linear time series model to predict a corridor travel time on a highway (37). He compared two cases: the first model used only speed data as a variable, while the second model used speed, occupancy, and volume data to predict travel time. It was found that the single variable model using speed was better than the multivariable prediction model.

2.4.4 Kalman Filtering Models

Kalman filtering models have been used extensively in travel time prediction research (12, 38). Chen and Chien (33, 39-40) and Wall and Dailey (8, 41) used Kalman filtering techniques to predict auto travel time. The Kalman filtering model has the potential to adapt to traffic fluctuation with time-dependent parameters (12). These models are effective in predicting travel time one or two time periods ahead, but they deteriorate with multiple time steps (42). Park and Rilett compared neural network models with other prediction models including Kalman filtering techniques to predict freeway link travel time. While the average mean absolute percentage error (MAPE) of neural network models changed from 8.7 for one time period to 16.1 for 5 time periods, that of Kalman filtering changed from 5.7 to 20.1 (42)

2.4.5 Artificial Neural Network Models

Due to their ability to solve complex non-linear relationships, artificial neural network models (ANNs) have been developed for transportation since the early 1990s (43-50). ANN models had better results than those of existing link travel time techniques, including a Kalman filtering model, an exponential smoothing model, a historical profile, and a real-time profile (42, 51-52). In addition, ANN model showed better performance than historical average and autoregressive integrated moving average (ARIMA) models to predict short-term traffic flow (34). While other models are dependent on cyclical traffic data patterns or need independence between dependent and independent variables, ANNs do not require that variables be uncorrelated and/or that they have a cyclic pattern (41).

ANNs emulate the learning process of the human brain (53). They are good at pattern recognition, prediction, classification, etc. ANNs have two stages, training and testing. During the training stage, inductive learning principles are used to learn patterns from a training set data. There are two types of learning processes used: unsupervised and supervised learning. In unsupervised learning, the network attempts to classify the training set data into different groups based on input patterns. In supervised learning, the desired output from output layer neurons is known, and the network adjusts the weight of connections between neurons to produce the desired output (54). During this process, the error in the output is propagated back from one layer to the previous layer by adjusting the weights of the connections (54). This is called the back-propagation method, which is the most frequently used technique in transportation applications (34, 45, 49, and 54). The learning process of ANNs can be continuous so that the models can adapt to changes in environmental characteristics. In other words, ANN models can be considered dynamic prediction models because they can be updated and modified using new online data (41).

2.5 BUS TRAVEL TIME PREDICTION MODELS

In the previous section, travel time prediction models which have been developed were discussed. The travel time prediction models focused on travel time for passenger cars. In this section, travel time prediction models for transit vehicles are reviewed.

In the context of input data source, various types of data source including loop detectors, microwave detectors, radar, etc., have been used. Travel time data can be obtained from these data sources. However, it is not realistic that the entire urban roadway network would be covered by such devices. Thanks to the development of technologies, more reliable data from specific devices that can track the vehicles such as GPS can be collected (33). Recently, a few cities have conducted research on predicting transit travel time using AVL data.

2.5.1 Mathematical Algorithm

Lin and Zeng developed a mathematical algorithm to provide real-time bus arrival information for Blacksburg, Virginia (9). They used bus location data, schedule information, the difference between scheduled and actual arrival time, and waiting time at time-check stops as their main inputs. Their algorithm was primarily developed for rural traveler information systems, and their test bed was a rural area where no congestion exists. Their algorithm could not consider traffic congestion and dwell time at stops. They used regular GPS, not DGPS, and regular GPS gave many GPS errors. Their GPS unit provided bus location data every forty six seconds on average. The minimum sample interval was thirty seconds and about fifteen percent of the data had sample intervals over one minute. In this research, the longest time interval was almost seven minutes. These long intervals of location data could result in inaccurate travel time predictions. According to the authors, this research was the first attempt to estimate arrival time at bus stops based on real-time bus location data. However, their research in 1999 gave inaccurate predictions because they used regular GPS data. As mentioned in previous section, the accuracy of regular GPS dramatically improved since the U.S. DOD remove the intentional degrade by noise in May 2000.

Ojili developed a bus arrival time notification system in College Station, Texas (10). The model breaks the route into one-minute time zones along the bus route. After locating the current bus with respect to these zones, the arrival time is predicted by counting the estimated number of the one-minute time zones between the current location and the given stop. When he decided on the one-minute time zones, he calculated them by averaging the travel time between bus stops and dwell times at stops. For example, when the dwell time at a specific bus stop is about three minutes, the bus stop has three one-minute time zones. The use of static one mile blocks implicitly ignored the effect of congestion and variation in dwell times by the time of the day and the day of the week. For the test bed chosen, this was not critical. However, for larger cities that have wide variations in travel times (due to congestion) and in dwell time (due to passenger demand and/or congestion), the model would have to be calibrated for the specific conditions for each application.

Bae and Kachroo used the dynamics of bus behavior to develop a bus arrival time prediction model (11). The dynamics of bus behavior were simulated based on the ratio between passenger arrival rate and passenger boarding time. They used least square estimation techniques to estimate the parameters for headway and passenger boarding time. Finally, their arrival time prediction model was based on the parameter adaptation algorithm using the parameters for headway and passenger boarding time.

2.5.2 Kalman Filter Model with Historical Data

Wall and Dailey used a combination of both AVL data and historical data to predict bus arrival time in Seattle, Washington (8, 55). Their algorithm consists of two components: tracking and prediction. For the tracking component, they used a Kalman filter model to track a vehicle location. For the prediction component, they used a statistical estimation technique. To produce a distribution of travel times, real-time AVL data were combined with a historical data source. The expected travel time was calculated using the distribution. The predicted travel time is equal to the expected travel time plus the current time. It was found that they could predict bus arrival time with less than 12% error (i.e. when the predicted bus arrival time is 15 minutes, 70 percent of the buses will arrive in between 13 and 17 minutes). However, they did not explicitly deal with dwell time as an independent variable.

Shalaby and Farhan developed a bus travel time prediction model using the Kalman filtering technique (13, 56). They used downtown Toronto data collected with four buses equipped with AVL and automatic passenger counter (APC). They found that Kalman filtering techniques outperformed the historical models, regression models, and time lag recurrent neural network models. They used five-weekday data in May 2001. Four days of data were used for learning and developing models, and one-day data were used for testing. They developed two Kalman filtering algorithms to predict running times and dwell times separately. However, when they developed a historical average model, a regression model, and a time lag recurrent neural network model, they included dwell times in link travel time. They defined a link as the distance between two time check point stops and each link included between 2 and 8 bus stops. Consequently, they predicted dwell time only at time check points, not at every stop. To develop a dynamic, real-time model, they updated the predicted time of bus arrival and departure at time check points. Of the 27 stops on the route, their model was updated at only the six time check points.

2.5.3 Artificial Neural Network Model

Chien et al developed an artificial neural network model to predict dynamic bus arrival time in New Jersey (12). They stated that the back-propagation algorithm, which is the most used algorithm for transportation problems, is unsuitable for on-line application because of its time consuming learning process. Consequently they developed an adjustment factor to modify their travel time prediction using recent observed real-time data (12). They used generated data to predict bus arrival time, and they did not consider dwell time and scheduled data (12). They generated non AVL traffic information, which included traffic volume and passenger demand that AVL can not collect, using Corridor Simulation model (CORSIM). For an actual implementation they assumed they could obtain similar data from Automatic Passenger Counters (APC) and AVL systems. However, typical AVL systems cannot collect these types of data. In addition transit agencies use Automatic Passenger Counters (APC) in only about 40 percent of AVL-use agency (16). Therefore, prediction model which use only AVL data will be more common until APC and other passenger data collection methods are deployed more widely. In summary, prediction model considering traffic congestion and dwell time at bus stops are required in urban congested areas.

In a recent study, it was found that buses spend 20 percent of their service hours stopped at intersections, 23 percent of the time boarding and alighting passengers (dwell time), 5 percent of the time in traffic congestion, and the remaining time, 52 percent, moving (57). According to the report, traffic congestion is clearly not the most significant factor for bus delay. It was hypothesized that in order to improve transit speed, reliability, and on-time performance, the focus should be on the delay caused at bus stops and traffic signals (57). These delays would also be an important factor for prediction of arrival time.

2.6 CONCLUDING REMARKS

Chapter II presented a literature review on advanced traveler information systems, automatic vehicle location systems, global positioning systems, travel time prediction models, and bus arrival time prediction models. According to this literature review, the provision of accurate and timely bus arrival time information can help travelers to make their travel decisions and it encourages positive attitudes toward transit resulting in increased ridership. In addition, transit operators can identify vehicles that 1) have fallen behind schedule or 2) are in danger of falling behind schedule, and react in a proactive way. For example, bus priority at traffic signals could be enabled. Recently, the provision of real-time information can be provided through automatic vehicle location systems. Many transit agencies have used global positioning system especially as the location technology for automatic vehicle location systems. Therefore, there is a need to develop a model for bus arrival time information using automatic vehicle location system data. To predict auto travel time, many models have been developed including historical data based models, regression models, time series models, and artificial neural network models. However, while there is some research on bus arrival time prediction models, there is still important work to be done, as described in chapter I.

CHAPTER III

STUDY DESIGN

In chapter II, a literature review on advanced traveler information systems, automatic vehicle location systems, global positioning systems, travel time prediction models, and bus arrival time prediction models was presented. In this chapter, the process of data collection and data reduction is discussed. In the first section of this chapter, the test bed for this research is detailed. Subsequently, the data reduction procedure and results are presented.

3.1 DATA COLLECTION

3.1.1 Test Bed

3.1.1.1 Description of the Test Bed

AVL data collected in Houston, Texas was used for the test bed. The Houston data were collected by Houston Metro buses equipped with the differential global positioning systems (DGPS) receivers at five-second intervals. This DGPS data provides time, speed, heading, etc. as well as bus location. The data were collected over 6 months in 2000, from June to November. Only the southbound direction was studied in this dissertation.

There are two test bed sites as shown in FIGURE 3-1; the downtown area corridor and the north area corridor. Data from 340 buses were used for this research. A total of 240 buses were used for calibrating the bus arrival time prediction models and 100 buses for evaluating models.

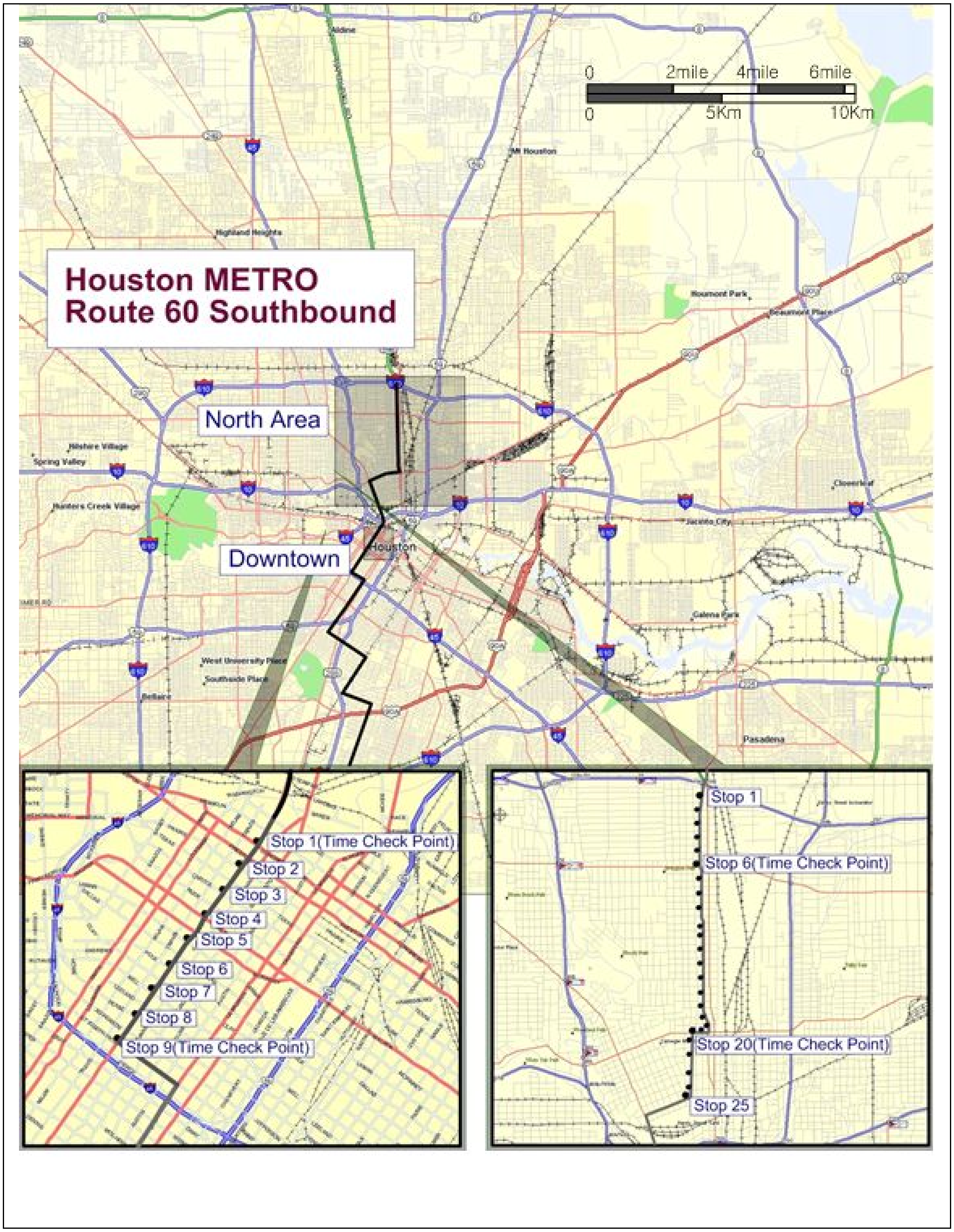

FIGURE 3-1 Map of Route 60 Running in Houston

The test bed 1, the downtown area corridor, has 9 bus stops and is 1.6 kilometers long (1.02 miles). Stop 1 and stop 9 are used as time check points for schedule adherence. The test bed was Route 60 which is highly congested in the morning and afternoon. The test bed 2, the north area corridor, has 25 bus stops and is 4.26 kilometers long (2.66 miles). Stop 6 and stop 20 are used as time check points. These two sites are shown in FIGURE 3-2 and FIGURE 3-3, respectively. The schedule headway during the weekday peak period is 30 minutes and during the weekday non-peak period and weekends is one hour. A detailed schedule by stop for the study period is provided in TABLE A-1 through A-4 of Appendix A.

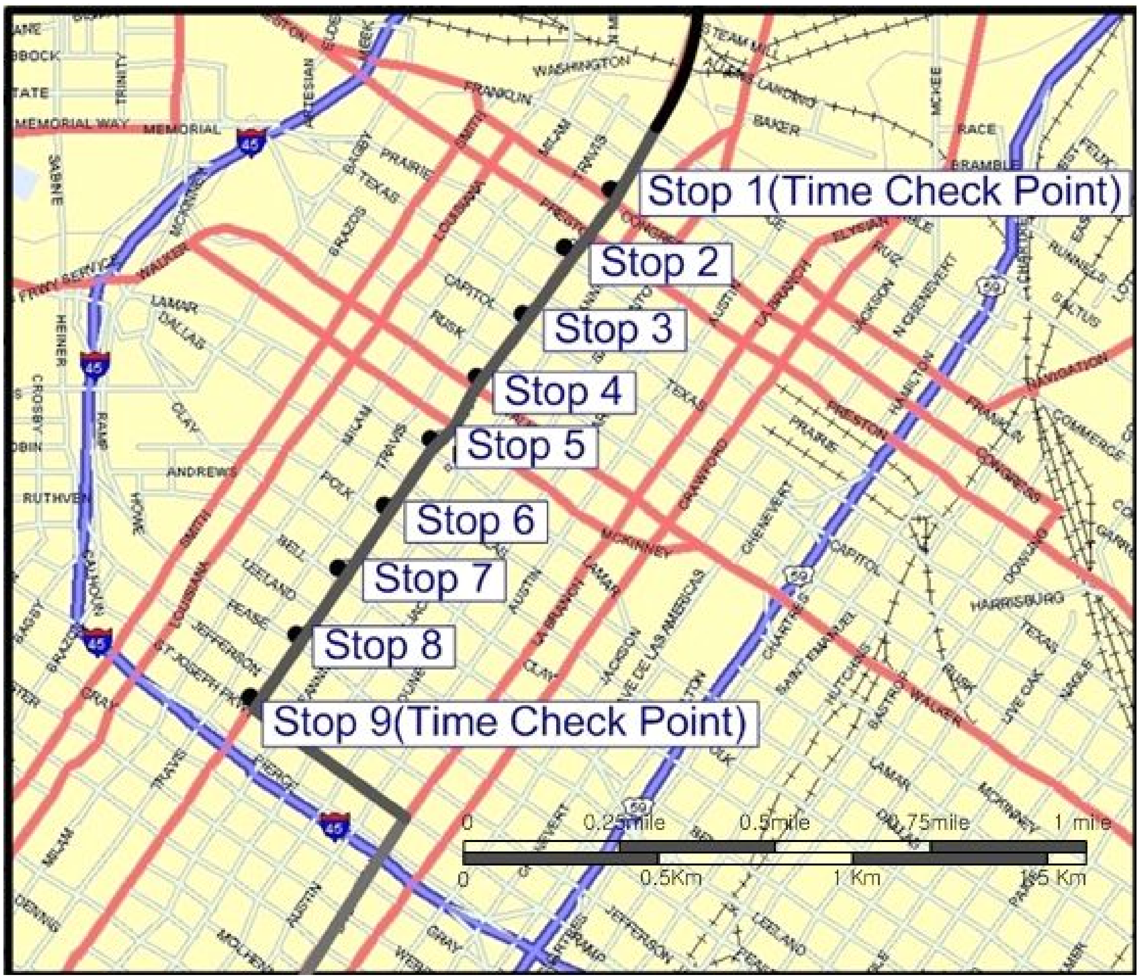

FIGURE 3-2 Map of Route 60 Running in the Test Bed 1 (Downtown)

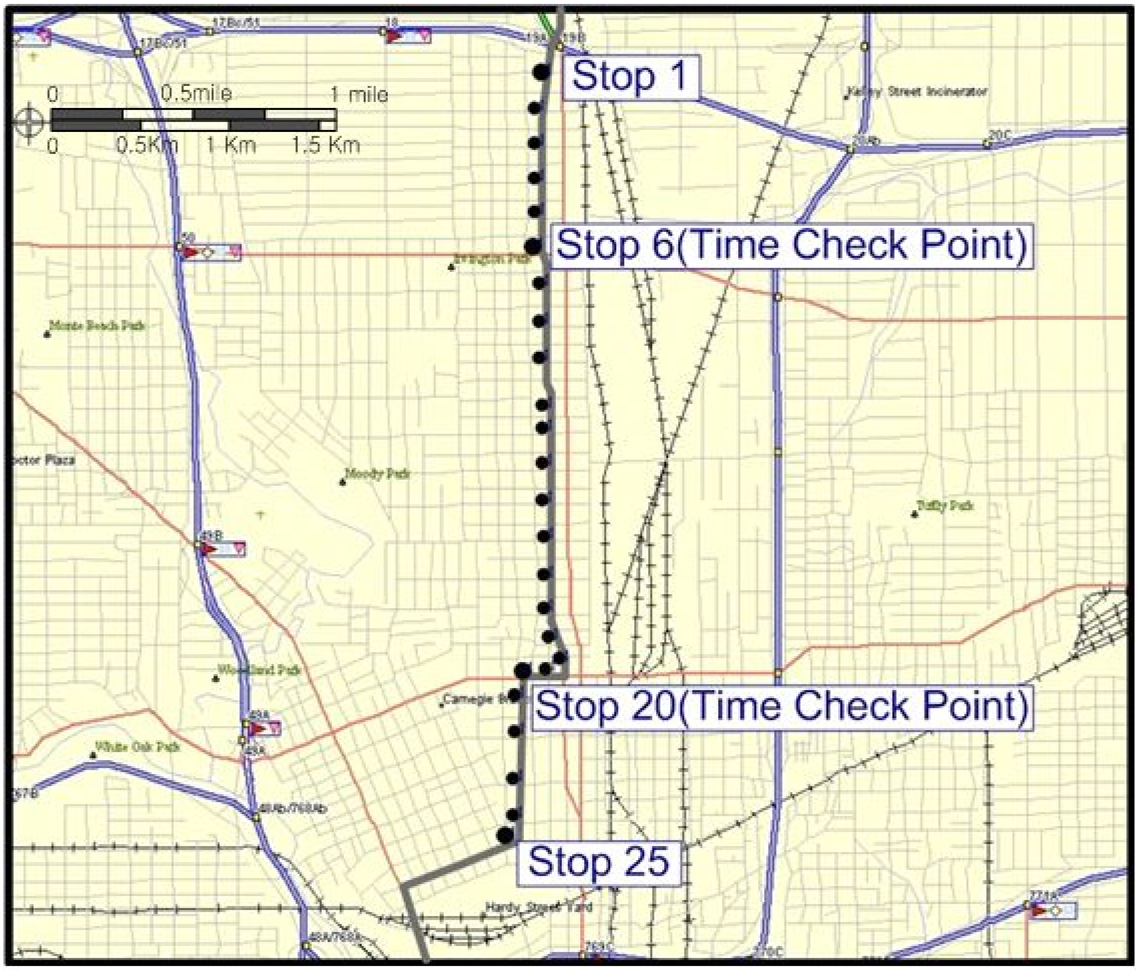

FIGURE 3-3 Map of Route 60 Running in the Test Bed 2 (North Area)

TABLE 3-1 and TABLE 3-2 show the distance between stops. In the downtown area, 9 bus stops are placed along the 1.63 kilometer distance (1.02 miles). The average distance between stops in the downtown area is 0.20 kilometers (0.13 miles). In the north area, 25 bus stops are placed along the 4.26 kilometers distance (2.66 miles). The average distance between stops in the downtown area is 0.18 kilometers (0.11 miles). In the downtown area, the distances between stops are almost the same. However, in the north area, the distance between stops varies from 0.10 kilometer to 0.24 kilometer (from 0.06 miles to 0.15 miles).

TABLE 3-1 Distance between Stops in the Test Bed 1 (Downtown)

Stop number	Distance between stops		Accumulated Distance to stop i	
	miles	Kilometers	miles	Kilometers
1	0.00	0.00	0.00	0.00
2	0.13	0.21	0.13	0.21
3	0.13	0.21	0.26	0.42
4	0.13	0.21	0.39	0.62
5	0.13	0.21	0.52	0.83
6	0.12	0.19	0.64	1.02
7	0.13	0.21	0.77	1.23
8	0.12	0.19	0.89	1.42
9	0.13	0.21	1.02	1.63
	Average 0.13	Average 0.20	Total 1.02	Total 1.63

TABLE 3-2 Distance between Stops in the Test Bed 2 (North Area)

Stop number	Distance between stops		Accumulated Distance to stop i	
	miles	Kilometers	miles	Kilometers
1	0.00	0.00	0.00	0.00
2	0.08	0.13	0.08	0.13
3	0.12	0.19	0.20	0.32
4	0.12	0.19	0.32	0.51
5	0.12	0.19	0.44	0.70
6	0.12	0.19	0.56	0.90
7	0.11	0.18	0.67	1.07
8	0.12	0.19	0.79	1.26
9	0.12	0.19	0.91	1.46
10	0.15	0.24	1.06	1.70
11	0.09	0.14	1.15	1.84
12	0.10	0.16	1.25	2.00
13	0.13	0.21	1.38	2.21
14	0.12	0.19	1.50	2.40
15	0.12	0.19	1.62	2.59
16	0.12	0.19	1.74	2.78
17	0.12	0.19	1.86	2.98
18	0.10	0.16	1.96	3.14
19	0.06	0.10	2.02	3.23
20	0.10	0.16	2.12	3.39
21	0.06	0.10	2.18	3.49
22	0.13	0.21	2.31	3.70
23	0.14	0.22	2.45	3.92
24	0.14	0.22	2.59	4.14
25	0.07	0.11	2.66	4.26
	Average 0.11	Average 0.18	Total 2.66	Total 4.26

3.1.1.2 Number of Observations

TABLE 3-3 and TABLE 3-4 show the number of observations for the test bed 1 and the test bed 2, respectively. All data sets are clustered by time period, because transit vehicles have different schedules by time period, resulting in different travel patterns. The four time periods are weekend, weekday peak, weekday non-peak, and weekday evening. In this research, weekday means Monday through Friday, and weekend means Saturday and Sunday. Weekday peak period data included the bus data when the bus arrived at the first bus stop during 6:15 A.M. ~ 8:15 A.M. and 4:15 P.M. ~ 6:10 P.M. Weekday non-peak period data included the bus data when it arrived before 6:15 A.M., 8:15 A.M. ~ 4:15 P.M., and 6:10 P.M. ~ 7:15 P.M. Weekday evening period data include the bus data when the bus arrived after 7:15 P.M. These time periods were determined by the predetermined bus schedule shown in Appendix A. The bus location data have the current time at specific bus stops, and the time data determined by GPS unit were used to decide in which time period a bus is located.

These tables present the number of observations for each time period. To calibrate and evaluate the prediction models (historical data model, multi linear regression models, and artificial neural network models), 340 data sets were used for the test bed 1, the downtown area, and 326 data for the test bed 2, the north area. The weekend time period has more observations. The number of observations of each time period in the total data was determined by the data collector, the Houston METRO. If the weekday peak period data are more available, the effect of congestion should be analyzed more effectively.

TABLE 3-3 Number of Training Data Sets and Test Data Sets for the Test Bed 1 (Downtown)

Clustering	Training Set	Test Set	Total
Weekend	100	47	147
Weekday peak	53	15	68
Weekday non-peak	65	20	85
Weekday evening	30	10	40
Non-Clustering	240	100	340

TABLE 3-4 Number of Training Data Sets and Test Data Sets for the Test Bed 2 (North Area)

Clustering	Training Set	Test Set	Total
Weekend	96	43	139
Weekday peak	31	13	44
Weekday non-peak	71	32	103
Weekday evening	28	12	40
Non-Clustering	226	100	326

3.1.2 Description of the AVL Data

In this section, the AVL data using GPS is described. The GPS consists of 24 satellites and transmits the estimated position, velocity, and current time to GPS receivers. Typically, these 24 satellites transmit a signal once a second. Because of the wide coverage in the U.S., the location of transit vehicles equipped with GPS can be determined every second. However, to analyze data more conveniently, the AVL systems are configured to transmit the location data less often, such as every five seconds. The interval for the Houston Metro data is about five seconds

3.1.2.1 AVL Raw Data

TABLE 3-5 presents the raw form of AVL data. The AVL data consists of current time, latitude, longitude, altitude, heading, speed, GPS week, ID, etc. The current location of a transit vehicle can be calculated by latitude, longitude, and altitude. These values are straight from the GPS unit, in World Geodetic System 1984 (WGS84) decimal degree. Heading represents the direction that the transit vehicle is moving toward. Time means GPS Time-of-the-Week (TOW), which starts at midnight Sunday in UTC. UTC stands for Coordinated Universal Time, which is also known as GMT, or Greenwich Mean Time. GPS time in UTC format is converted to Central Standard Time. The time combined with GPS week provides the exact date and time stamp of the measurement. Houston Metro used sixteen vehicles for this data collection and ID shows which specific transit vehicle runs for a specific data set. Speed data are calculated within the GPS unit itself.

TABLE 3-5 Structure of AVL Data

	time	latitude	longitude	altitude	heading	speed	GPS week	ID	
GPS	279093	29.73174	-95.31279	0.0000000	0.000	0.000	46	4244	2
GPS	279098	29.73162	-95.31322	0.0000000	0.000	0.000	46	4244	2
GPS	279103	29.73163	-95.31320	0.0000000	0.000	0.000	46	4244	2
GPS	279108	29.73166	-95.31319	0.0000000	0.000	0.000	46	4244	2
GPS	279113	29.73168	-95.31319	0.0000000	0.000	0.000	46	4244	2
GPS	279118	29.73168	-95.31316	0.0000000	0.000	0.000	46	4244	2
GPS	279123	29.73169	-95.31315	0.0000000	0.000	0.000	46	4244	2
GPS	279128	29.73168	-95.31315	0.0000000	0.000	0.000	46	4244	2
GPS	279133	29.73169	-95.31315	0.0000000	0.000	0.000	46	4244	2
GPS	279138	29.73169	-95.31315	0.0000000	0.000	0.000	46	4244	2
GPS	279143	29.73168	-95.31315	0.0000000	0.000	0.000	46	4244	2
GPS	279148	29.73168	-95.31315	0.0000000	0.000	0.000	46	4244	2
GPS	279153	29.73168	-95.31315	0.0000000	0.000	0.000	46	4244	2
GPS	279158	29.73167	-95.31315	0.0000000	0.000	0.000	46	4244	2
GPS	279163	29.73166	-95.31315	0.0000000	0.000	0.000	46	4244	2
GPS	279168	29.73166	-95.31316	0.0000000	0.000	0.000	46	4244	2
GPS	279173	29.73166	-95.31316	0.0000000	0.000	0.000	46	4244	2
GPS	279178	29.73165	-95.31316	0.0000000	0.000	0.000	46	4244	2
GPS	279183	29.73166	-95.31316	0.0000000	0.000	0.000	46	4244	2
GPS	279188	29.73166	-95.31319	0.0000000	0.000	0.000	46	4244	2
GPS	279193	29.73165	-95.31319	0.0000000	0.000	0.000	46	4244	2
GPS	279198	29.73165	-95.31319	0.0000000	0.000	0.000	46	4244	2
GPS	279203	29.73166	-95.31319	0.0000000	0.000	0.000	46	4244	2
GPS	279208	29.73166	-95.31319	0.0000000	0.000	0.000	46	4244	2
GPS	279213	29.73166	-95.31319	0.0000000	0.000	0.000	46	4244	2
GPS	279218	29.73166	-95.31319	0.0000000	0.000	0.000	46	4244	2
GPS	279223	29.73167	-95.31316	0.0000000	0.000	0.000	46	4244	2
GPS	279228	29.73167	-95.31316	0.0000000	0.000	0.000	46	4244	2
GPS	279233	29.73167	-95.31315	0.0000000	0.000	0.000	46	4244	2
GPS	279238	29.73167	-95.31315	0.0000000	0.000	0.000	46	4244	2
GPS	279243	29.73169	-95.31315	0.0000000	0.000	0.000	46	4244	2
GPS	279248	29.73169	-95.31315	0.0000000	0.000	0.000	46	4244	2
GPS	279253	29.73169	-95.31315	0.0000000	0.000	0.000	46	4244	2

3.2 DATA REDUCTION

To predict bus arrival time, the reduction and modification of AVL data are required. AVL data consists of the current location (latitude, longitude, and altitude), current time (time, and GPS week), speed, heading, and vehicle ID. From these raw data, input data for predicting bus arrival time were derived. Input variables for this dissertation are arrival time, dwell time, and schedule adherence at each stop. Before calculating arrival time, dwell time, and schedule adherence, modification of GPS errors is required.

3.2.1 Description of the GPS Error

3.2.1.1 Noise Error

There are two types of error associated with GPS data. The first is noise error added by the U.S. DOD in order to degrade the accuracy of GPS data. This error can be corrected by using DGPS. The Houston Metro data were collected by DGPS and consequently the noise error was already eliminated.

3.2.1.2 Measurement Error

The second type of error is measurement error, which is mainly due to tall buildings in the downtown area. In the north area, this type of error is seldom found.

First, because the tall buildings block communication between satellites and transit vehicles, there are some links with no data. FIGURE 3-4 shows the downtown area of Houston. Dots show the data location. In this figure, more than two blocks do not have location data received from GPS. This type of GPS data error is called “missing data” in this dissertation.

Second, there is off route or off road error. Because of this error, it is anticipated that some of the observed bus locations would be off the roadway. In addition, even if the bus was located on the road, there may be error associated with its exact location. FIGURE 3-4 shows these errors.

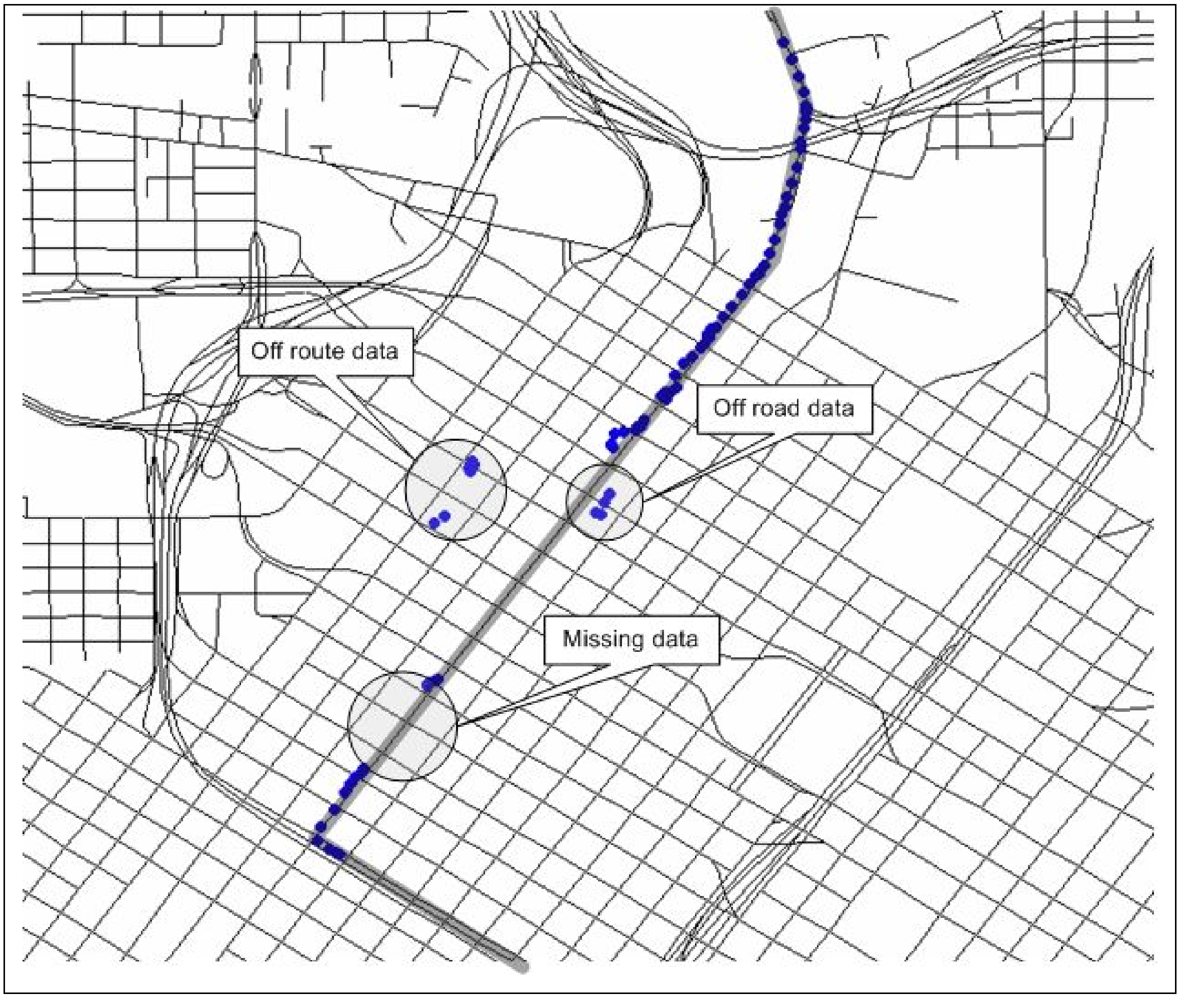

FIGURE 3-4 GPS Measurement Errors

Third, there is another error type called "backward data" in this dissertation. FIGURE 3-5 shows this error. In this section, the data arrive as a sequence of zero speeds. However, because of the aforementioned error, the location of the bus may appear to move. When this movement is backward, it is assumed that the bus stopped. These types of data were corrected as part of the pre processing step.

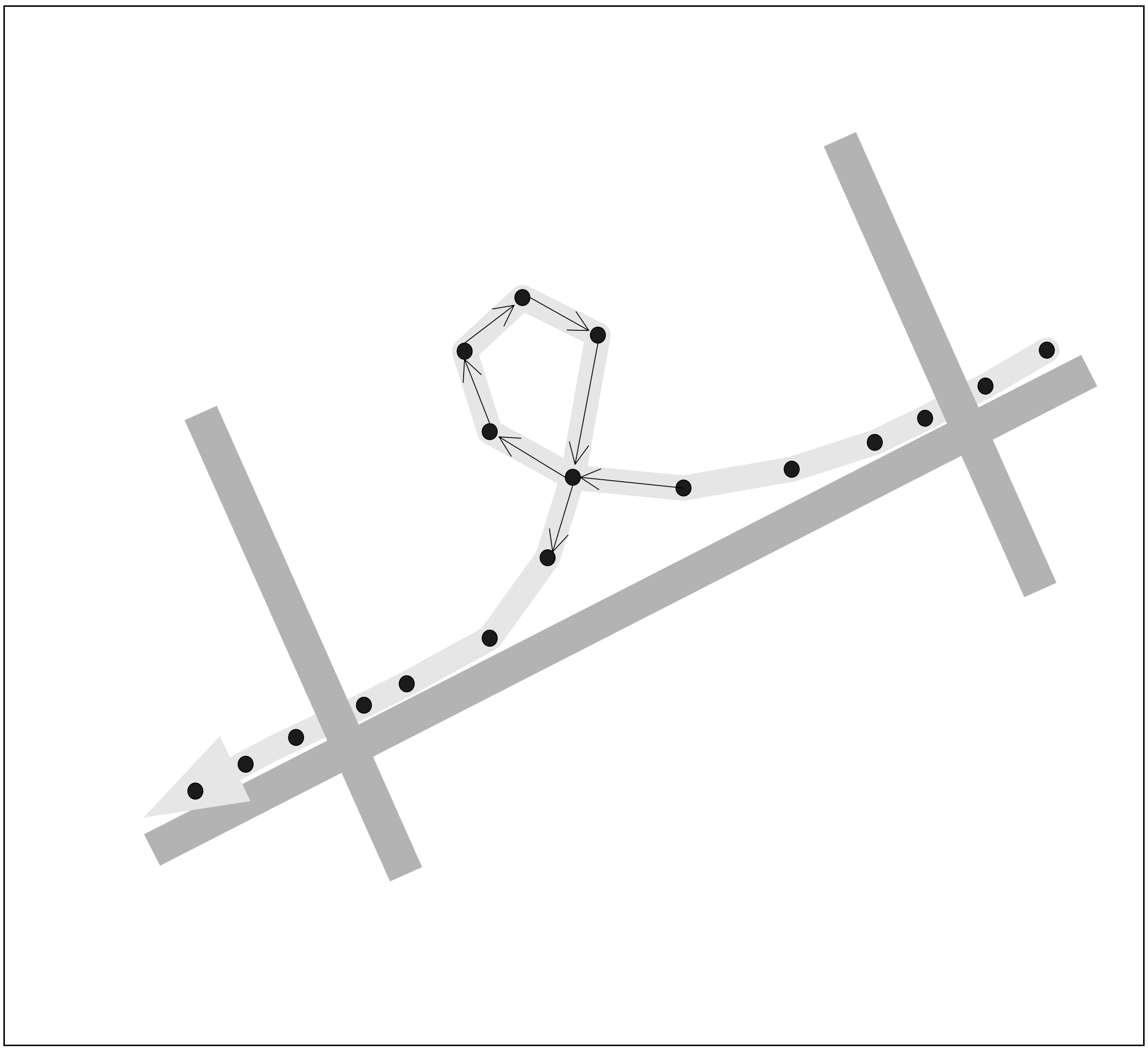

FIGURE 3-5 Backward Data Error

3.2.2 Method for Modifying GPS Error

The GPS measurement errors including missing data, off route/road data, and backward data are modified in this section. With modified GPS location data, input data for bus arrival time will be calculated. The input data are arrival time, dwell time, and schedule adherence at each stop. Even though most of the problematic data were found in the downtown area, north area data were also modified. In north area, missing data and backward data were not found and the major issue was almost off road data. These errors were mainly found in the center of downtown area. The major reason of error was the tall buildings in this area. In the outer part of the downtown area, these errors occurred infrequently. About 23 percent of total data were modified and the exact bus stop location data were used to fix these errors. How large the effect of modifying these errors to the result of the prediction models will not be discussed because the dwell time data can not be estimated without correcting the arrival time data at every stop and because the arrival time data can not be estimated without modifying these errors.

TABLE 3-6 shows the amount of modified error. The total number of GPS location data in the downtown area was 37,968. Among them, 18 percent of data were off road data and 5 percent of data were off route data. Backward data appeared 200 times and it represents 0.53 percent of the total data. Missing data were modified 650 times in the downtown area, while 70 times in the north area. On road data are within plus/minus 25 meters from the center line of the road. The size of one block in the downtown area is about 100 meters. Off road data are between plus/minus 25 and plus/minus 100 meters from the center line. Consequently, off road data are located within half block from the center line of the road. Off route data are outside plus/minus 100 meters from the center line of the road. The largest off route data were located 250 meters from the road.

TABLE 3-6 Numbers of Modified Data

Total number of data	37,968	100.00 %
On road data	29,153	76.78 %
Off road data	6,913	18.21 %
Off route data	1,902	5.01 %
Backward data	200	0.53 %

3.2.2.1 Missing Data

Missing data are modified by extrapolation according to distance between the nearest two observed data. In FIGURE 3-6, there is a missing data area between data i-1 and data i+1. Because only input data at each stop are required, GPS location data are also needed at each bus stop only. In FIGURE 3-6, the location data of data i are required. With the assumption that the traveling speed between data i-1 and data i+1 is constant, the location of data i can be determined using extrapolation according to the distance between two observed data, data i-1 and data i+1. Equation 3-1 is used to estimate the time that the bus was at location i.

$$T_i = T_{i-1} + \left(\frac{X_{i-1,i}}{X_{i-1,i+1}} \times (T_{i+1} - T_{i-1}) \right) \tag{3-1}$$

Where,

$X_{i-1,i+1}$ = Distance between data i-1 and data i+1;

$X_{i-1,i+1} = X_{i-1,i} + X_{i,i+1}$

T_i = GPS Time of data i.

Missing data were modified 650 times in the downtown area, while 70 times in the north area. The raw GPS data were logged every five seconds. Sometimes, the duration of the data were larger than five seconds. TABLE 3-7 shows the number of observation data by the missing duration. For example, the duration of 2,700 data points were ten seconds (i.e. there is one missing logged data.) However, because GPS data at only bus stops are required, the 2700 data points were not treated missing data in this research. The total number of input data for the downtown area was 3,060 (i.e. 340 data set multiply by 9 bus stops). The 650 missing data indicated that about 21 percent of bus stops had missing data. In contrast, only 80 missing data points were modified for the north data and it was about 0.9 percent of bus stops in the area.

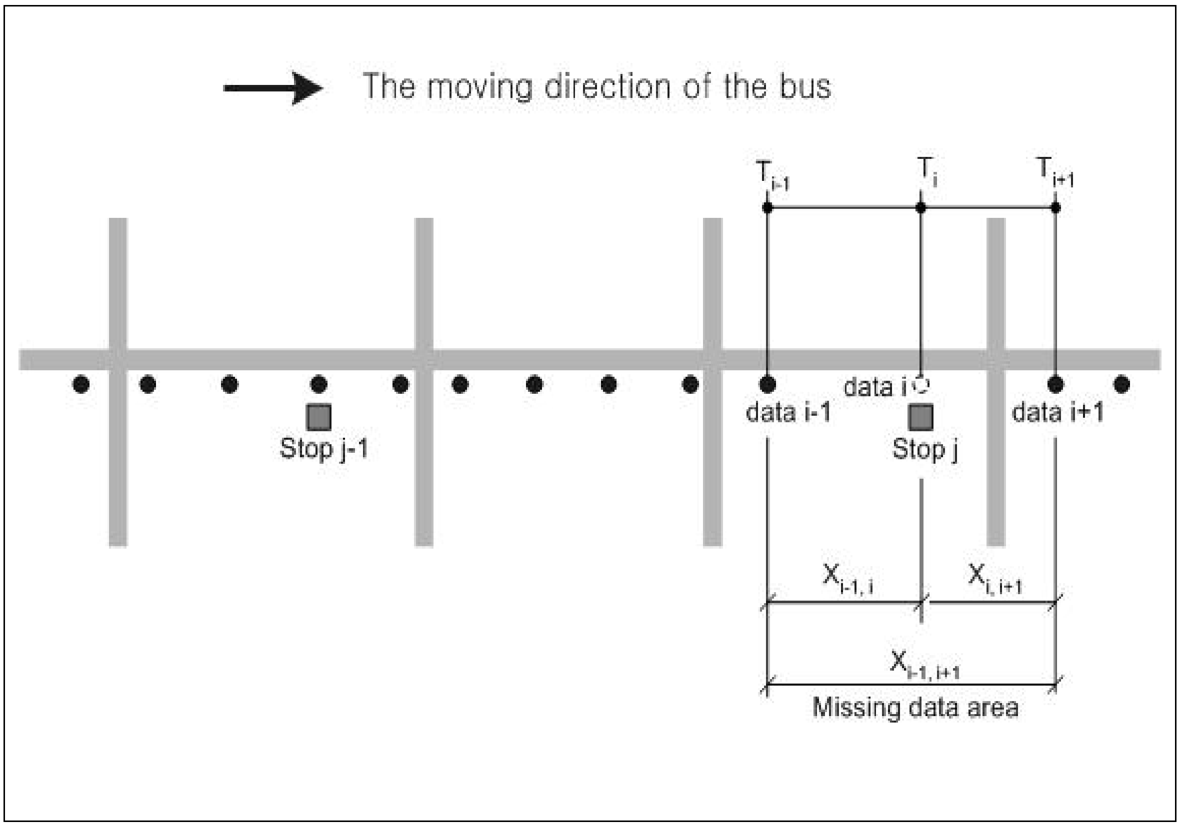

FIGURE 3-6 Modification of Missing Data

TABLE 3-7 Number of Observation Data Points by the Missing Duration

Missing duration	Downtown area	North Area
10 seconds	2,700	1,033
15 seconds	2,060	399
20 seconds	1,650	285
25 seconds	1,473	237
30 seconds	1,285	181
60 seconds	550	70
120 seconds	155	9
180 seconds	59	0
240 seconds	26	0
Total number of data	37,968	72,563

3.2.2.2 Off Route/Road Data

Some GPS location data indicates that the bus is off the road or off the bus route. In this case, the location data are pulled to the road where the bus is running with a 90-degree angle. FIGURE 3-7 illustrates this situation.

On road data are within plus/minus 25 meters from the center line of the road. The size of one block in the downtown area is about 100 meters. Off road data are between plus/minus 25 and plus/minus 100 meters from the center line. Consequently, off road data are located within half block from the center line of the road. Off route data are outside plus/minus 100 meters from the center line of the road. The largest off route data were located 250 meters from the road. The average value of off road data were 37 meters and the standard deviation was 6.4 meters. Total number of GPS location data points in the downtown area was 37,968. Among them, 18 percent of data were off road data and 5 percent of data were off route data.

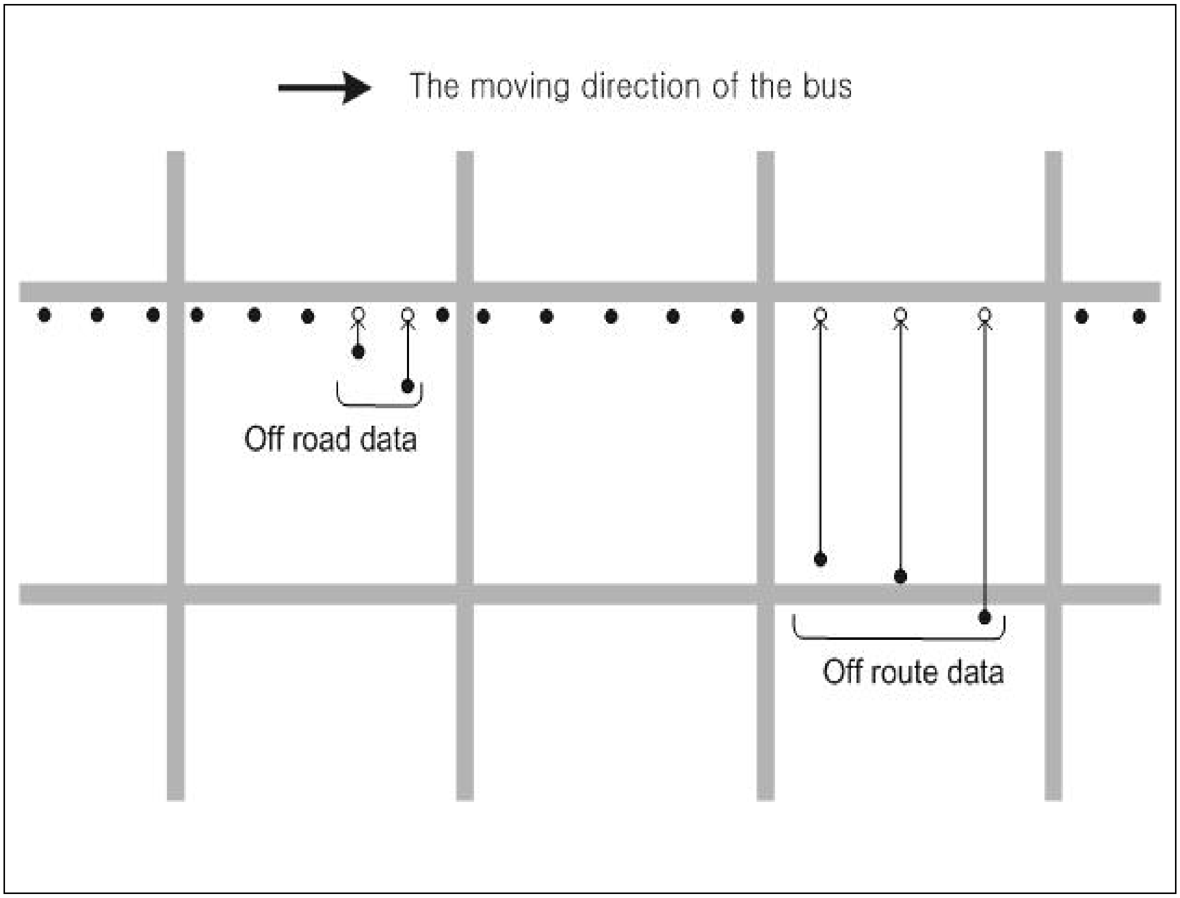

FIGURE 3-7 Modification of Off Route/Road Data

Off route/road data were modified using Equation 3-2 through Equation 3-9. According to FIGURE 3-8,

$$dx = x_{s1} - x_{s9} \qquad (3-2)$$

$$dy = y_{s1} - y_{s9} \qquad (3-3)$$

$$a_1 = \frac{dy}{dx} \qquad (3-4)$$

$$b_1 = y_1 - a_1 x_1 \qquad (3-5)$$

$$a_2 = -\frac{dx}{dy} \quad (\Leftarrow a_1 a_2 = -1) \qquad (3-6)$$

$$b_2 = y - a_2 x \qquad (3-7)$$

$$x_2 = \frac{-(b_1 - b_2)}{a_1 - a_2} \qquad (3-8)$$

$$y_2 = a_1 x_2 + b_1 \qquad (3-9)$$

where,

dx = difference between x coordinate of stop 1 and x coordinate of stop 9;

dy = difference between y coordinate of stop 1 and y coordinate of stop 9;

a_1 = slope of the line connecting stop 1 and stop 9;

b_1 = the point of contact of the line connecting stop 1 and stop 9;

a_2 = slope of the line connecting point P1 and point P2;

b_2 = the point of contact of the line connecting point P1 and point P2;

x_{P2} = x coordinate of point P2;

y_{P2} = y coordinate of point P2.

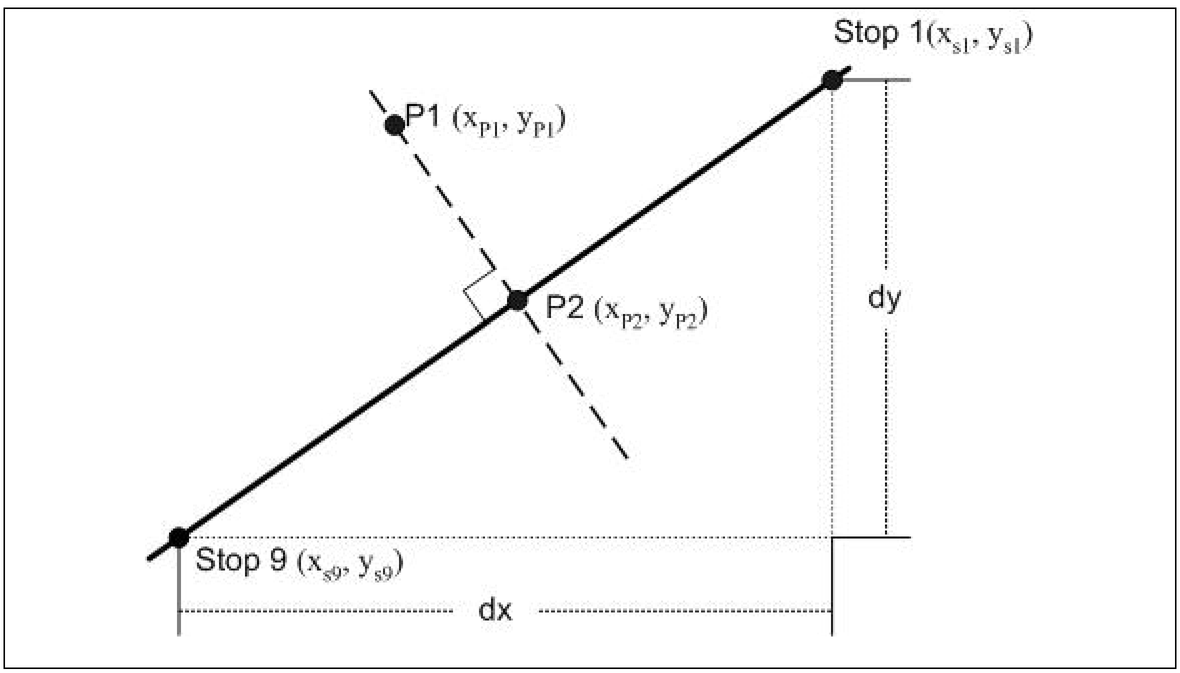

FIGURE 3-8 Illustration of Modifying Off Route/Road Error

3.2.2.3 Backward Data

In FIGURE 3-9, data i+1 is moving backward. Usually the vehicle speed of data point i and backward data point i+1 is zero. In this case, it can be assumed that the data i+1 is stopped at bus stop j. Consequently, the location of data i+1 is replaced with the location of data i for this research. When there are more than two backward data, all backward data have the same location data. Backward data appeared 200 times and it represents 0.53 percent of the total data in the downtown area. In the north area, this type of error occurred infrequently.

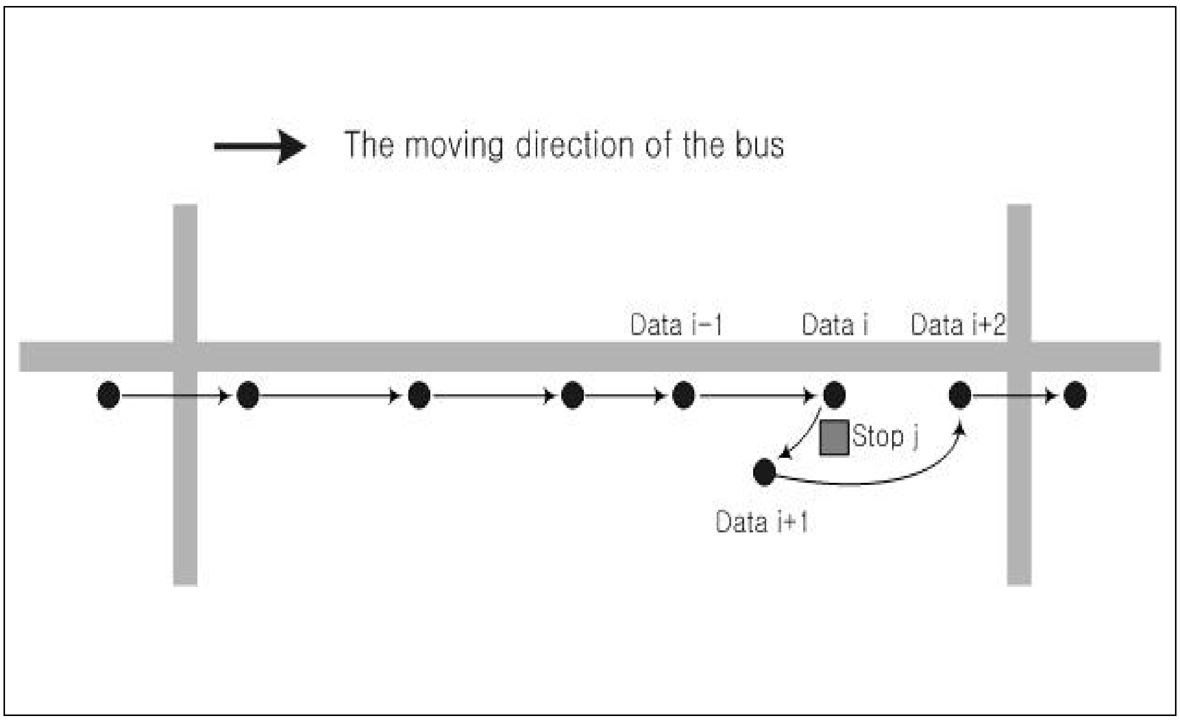

FIGURE 3-9 Modification of Backward Data

3.2.3 Calculating Input Data

In the previous section, GPS measurement error was modified. In this section, with the modified location data, the input data to predict bus arrival time are determined. The three input data are arrival time, dwell time, and schedule adherence at each stop.

3.2.3.1 Arrival Time

The GPS time of the nearest location data from each bus stop (the shortest distance from the bus stop) is considered as the arrival time at the stop. In FIGURE 3-10 the time of data i are the arrival time at bus stop j. Even though the distance between stop j and data i +1 is shorter than the distance between stop j and data i, the time of the nearest data before arriving at the bus stop is considered the arrival time at stop j.

$$A_j = T_i \tag{3-10}$$

where,

$A_j =$ Arrival time at bus stop j;

$T_i =$ GPS Time of the nearest location data i.

However, a bus may stop passing the bus stop sign. Consequently, when the bus is in some cases shown in Equation (3-11) through (3-13), the bus is considered as stopped at the bus stop even if it has positive speed.

Case 1: When the speed of the bus is zero; and

$$V_{i+1} = 0 \tag{3-11}$$

where,

$V_{i+1} =$ Speed of a bus at point i+1.

Case 2: When data i is far more than data i+1; and

$$\left| X_{i,j} - X_{j,i+1} \right| \geq 40 \; meters \tag{3-12}$$

where,

$X_{i,j}$ = Distance between data i and stop j.

Case 3: When the location of bus is less than 5 meters over the bus stop sign.

$$X_{j,i+1} \le 5\ meters \tag{3-13}$$

Even if a bus did not stop due to no passenger demand, the GPS time of the nearest location data from each bus stop is considered as the arrival time at the stop as the same approach. In other words, regardless of whether a bus stops at a bus stop or not, the basic principle of determining the arrival time is that the GPS time of the nearest location data from each bus stop is considered as the arrival time at the stop.

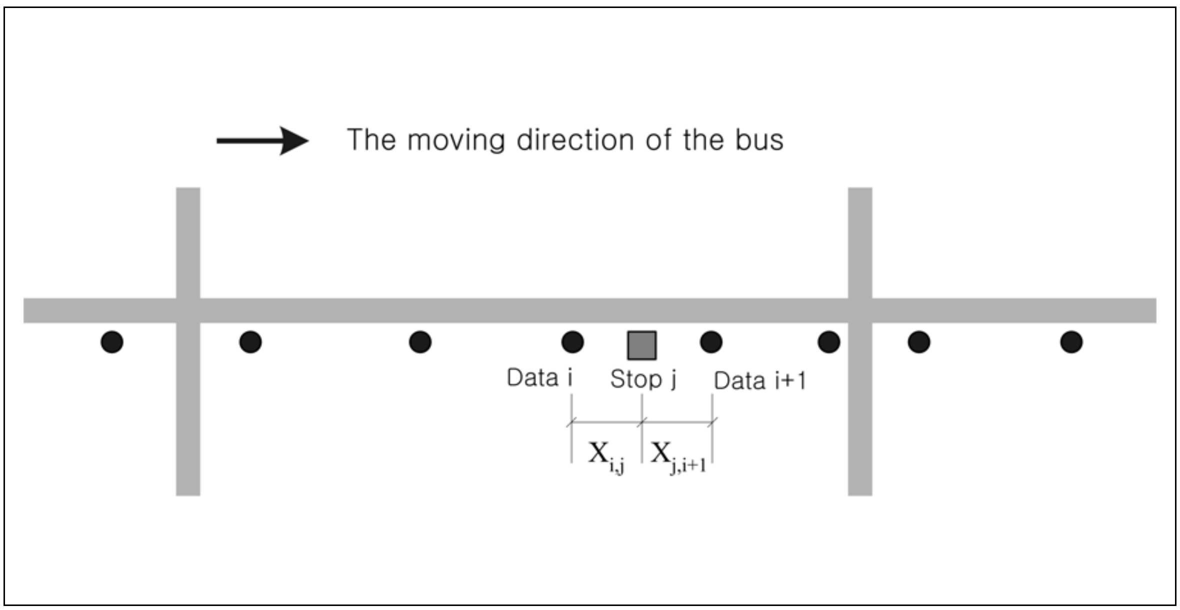

FIGURE 3-10 Calculating Arrival Time at Bus Stop

3.2.3.2 Dwell Time

Dwell time is the time that a bus stays at a given bus stop once it has stopped there. This time depends on the passenger demand, intersection signal, schedule adherence, and the behavior of the bus driver. Dwell time can be measured by the difference between the arrival time and the departure time at the bus stop. Dwell time at bus stop j is equal to the departure time at stop j minus the arrival time at stop j. This is shown in Equation 3-14.

$$W_{jk} = D_{jk} - A_{jk} \qquad \forall_j = 1, N_b \tag{3-14}$$

where,

$W_{jk} =$ Dwell time of bus k at bus stop j;

$D_{jk} =$ Departure time of bus k at bus stop j;

$A_{jk} =$ Arrival time of bus k at bus stop j;

$N_b =$ Last bus stop of test bed b.

For the downtown area this is equal to 9 ($N_1 = 9$) and

for the north area this is equal to 25 ($N_2 = 25$).

b = test bed. Test bed 1 is the downtown area and test bed 2 is the north area.

Departure time is determined when the stopped bus is moving. In other words, when the first location data appears just after the stopped data with speed zero, the time of the first moving data are the departure time of the bus from the bus stop.

$$D_{jk} = T_{i+n} \tag{3-15}$$

where,

$T_{i+n} =$ GPS Time of data i+n.

n = the number of data points that appeared between when the bus stopped and when the bus starts moving.

n^{th} bus is the first moving data after the bus stopped at bus stop j.

When the bus is passing without stopping at bus stop, the dwell time at the bus stop is definitely zero. In FIGURE 3-11, the dwell time at the bus stop j+1 is zero. In the downtown area of Houston, the distance between two stops is relatively short and the bus stops are located every other block. The bus stops are located at the near side of the block and the distance between the bus stop and the intersection is relatively short. Therefore, when the intersection signal is red, the bus tends to stay at the bus stop. Because the intersection signal data for the data collection period is unavailable, the dwell time due to intersection delay could not be determined. In this dissertation, the time when the bus is forced to stay at the bus stop due to the traffic signal, this additional delay is included in the dwell time of the stop.

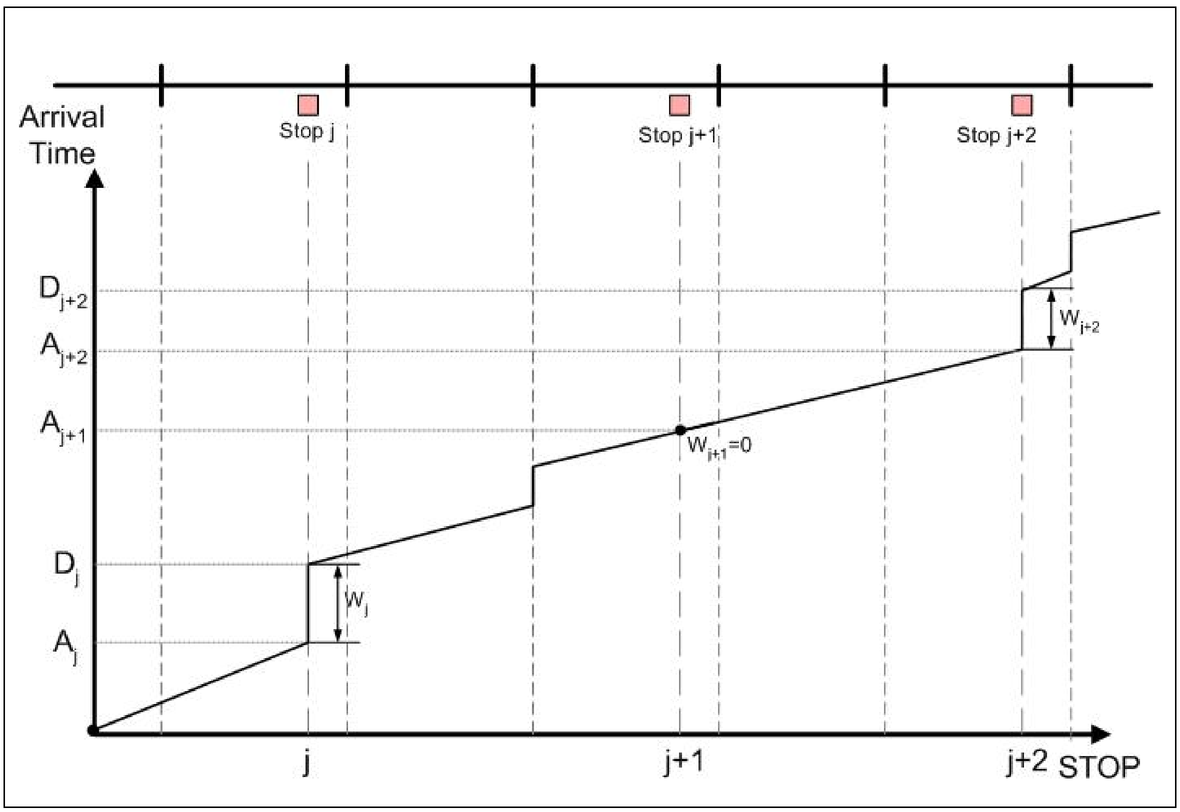

FIGURE 3-11 Calculating Dwell Time at Bus Stop

3.2.3.3 Schedule Adherence

Transit vehicles have a predefined schedule to follow. Because of this requirement, bus drivers may stay longer at bus stops if they are ahead of schedule or by-pass some stops if they are behind schedule. In other words, bus schedule controls the behavior of bus drivers, and consequently can affect the dwell time at bus stops and the link travel time. Schedule adherence is the difference between schedule time and actual arrival time. A positive value of schedule adherence means that the bus arrives late, and a negative value means that the bus arrives early. Schedule adherence at each stop is determined by Equation 3-16.

$$S_{jk} = A_{jk} - P_{jk} \tag{3-16}$$

where,

$S_{jk} =$ Schedule adherence of bus k at bus stop j;

$A_{jk} =$ Arrival time of bus k at bus stop j;

$P_{jk} =$ Predetermined/Scheduled arrival time of bus k at bus stop j.

3.3 DATA CHARACTERISTICS

The transit schedule and the congestion for the weekday peak hour, the non-peak hour, the evening, and the weekend period are different. TABLE A-1 through TABLE A-4 in APPENDIX A list the predetermined schedule data by the time period. The four time periods are weekend, weekday peak, weekday non-peak, and weekday evening. In this research, weekday means Monday through Friday, and weekend means Saturday and Sunday. It was determined based on the predetermined bus schedule shown in Appendix A. Weekday peak period is 6:15 A.M. ~ 8:15 A.M. and 4:15 P.M. ~ 6:10 P.M. Weekday non-peak period is before 6:15 A.M., 8:15 A.M. ~ 4:15 P.M., and 6:10 P.M. ~ 7:15 P.M. Weekday evening period is after 7:15 P.M.

Intuitively it would be expected that dwell time and link travel time would also be different by the time period. To account for these differences, data were clustered by time of the week and time of the day. FIGURE 3-12 through FIGURE 3-17 show the pattern of arrival time, dwell time, and schedule adherence by different time periods. Not surprisingly, these variables are a function of time of day, and there is a wide variation in values.

In general, the variability of the downtown data is larger than that of the north area data. In addition, it was found that the variability of dwell time is larger than that of arrival time. This means that the influence of dwell time on the predicted bus arrival time is relatively large. In the downtown area, stop 1 and stop 9 are time check points. However, drivers stayed longer at stop 1 and stop 5. This phenomenon could have resulted from various reasons, including more demand at stop 5, intersection delay, and the driver staying longer at these locations to stay on schedule. It would be expected that the bus drivers would stay longer at a time check point if they arrive early. However, the pattern of dwell time in FIGURE 3-12 shows that drivers tend to stay when they have passenger demand or intersection delays. In other words, it appears they spent more time at locations where they stopped rather at time check points to keep on schedule. Therefore, for this test bed it can not be assumed that a bus waits at a time check point when it arrives early.

3.3.1 Arrival Time

FIGURE 3-12 shows the plot of arrival time at each stop in the downtown area. It was found that the variability in the weekday non-peak period is larger than that in the weekday peak period and that the weekday peak period gives the least variability in arrival time among four time periods. TABLE 3-8 shows the mean and standard deviation value of the arrival time for the test bed 1, the downtown area. During weekday peak period, the average standard deviation for stop 3 to 9 is about 87 seconds while the average standard deviation for stop 3 to stop 9 is about 151 seconds during weekday non-peak period. This value indicates that the standard deviation of the weekday non-peak period is 173 percent of that of weekday peak period. In other words, the variability in the weekday non-peak period is 73 percent larger than that in the weekday peak period. According to this table, the average standard deviation for stop 3 to stop 9 during weekend is 128 seconds, weekday peak is 87 seconds, weekday non-peak is 151 seconds, and weekday evening is 147 seconds. Consequently, the weekday peak period gives the least variability in arrival time among four time periods. This result is expected given that less variable traffic conditions result in less variability in arrival time in the weekday peak period.

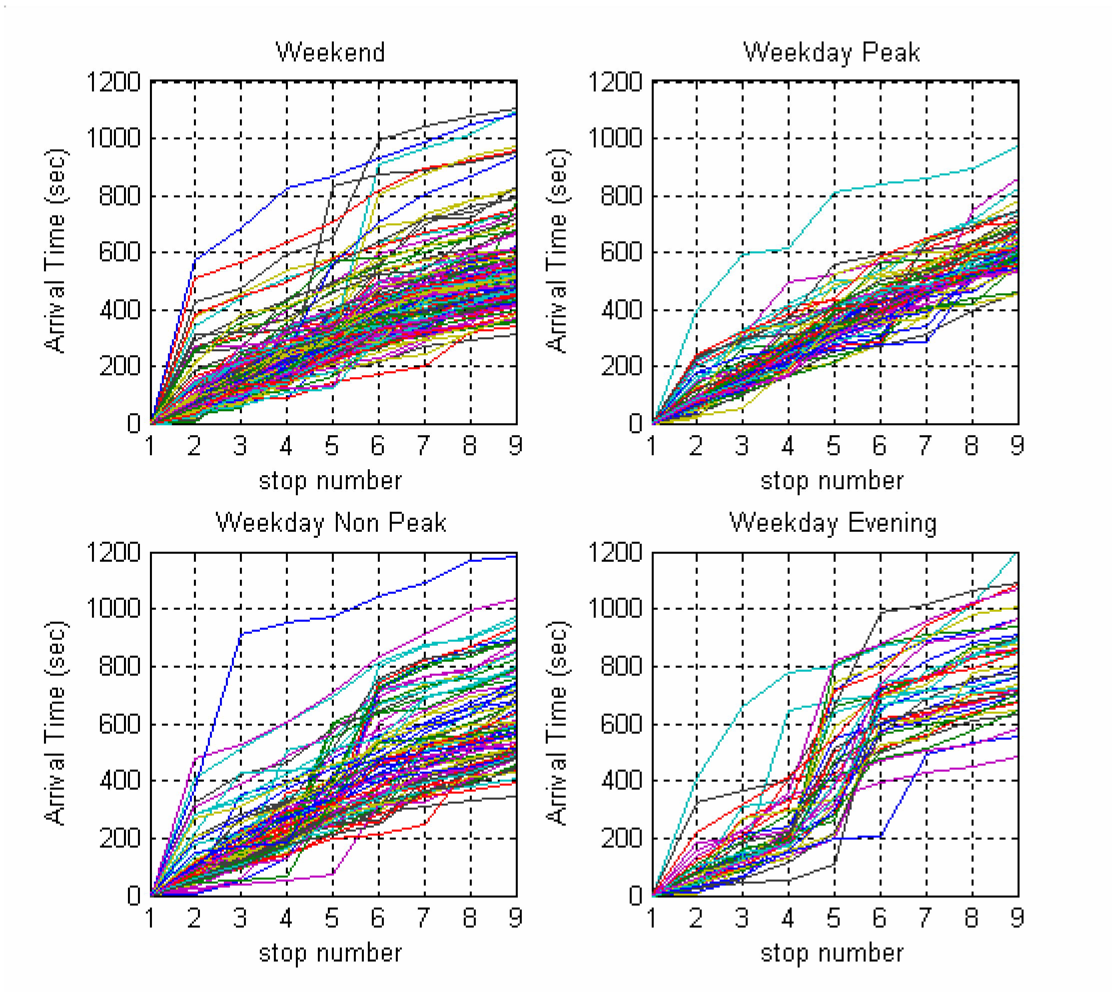

FIGURE 3-12 Arrival Time by Time Period of the Test Bed 1 (Downtown)

TABLE 3-8 Mean and Standard Deviation of Arrival Time for the Test Bed 1 (Downtown)

Time Period	Stop Number	Mean (sec)	Standard Deviation (sec)
Weekend	1	0.5	0.5
	2	99.2	92.3
	3	171.6	96.8
	4	240.5	104.2
	5	319.7	116.0
	6	400.7	141.0
	7	456.4	145.5
	8	502.9	144.5
	9	551.0	147.6
Weekday Peak	1	0.5	0.5
	2	108.9	68.1
	3	191.3	80.9
	4	269.3	81.1
	5	362.1	90.9
	6	424.1	94.9
	7	496.2	88.7
	8	562.8	82.5
	9	621.6	89.9
Weekday Non-Peak	1	0.5	0.5
	2	107.8	87.3
	3	189.6	123.2
	4	265.7	126.8
	5	359.5	131.6
	6	475.8	168.1
	7	544.4	167.6
	8	592.8	166.2
	9	645.2	170.2
Weekday Evening	1	0.5	0.5
	2	83.5	80.1
	3	168.9	108.9
	4	249.4	137.4
	5	454.6	190.8
	6	648.9	145.4
	7	710.8	141.8
	8	769.3	147.8
	9	813.3	157.1

FIGURE 3-13 shows the plot of arrival time at each stop in the north area. TABLE 3-9 through TABLE 3-12 show the mean and standard deviation value of the arrival time for the test bed 2, the north area. During weekday peak period, the standard deviation for stop 3 to 25 is about 57 seconds while the standard deviation for stop 3 to stop 25 is about 80 seconds during weekday non-peak period. This value indicates that the standard deviation of the weekday non-peak period is 141 percent of that of weekday peak period. In other words, the variability in the weekday non-peak period is 41 percent larger than that in the weekday peak period. According to this table, the average standard deviation for stop 3 to stop 25 during weekend is 54 seconds, weekday peak is 57 seconds, weekday non-peak is 80 seconds, and weekday evening is 92 seconds. Like the downtown area, the variability in the weekday peak period is less than that in the weekday non-peak period. However, in case of the north area, the standard deviation in the weekend period is slightly less than that in the weekday peak period. Consequently, the weekend period gives the least variability in arrival time among four time periods. However, in general, the result for the north area is similar to that for the downtown area. In other words, the variability in the weekday peak periods is less than those in the weekday non-peak and weekday evening. This result is expected given that less variable traffic conditions result in less variability in arrival time in the weekday peak period.

In general, the north area had less variability for all time periods. It is expected that the downtown area experiences more variable traffic conditions due to traffic congestion resulting in more variable arrival time.

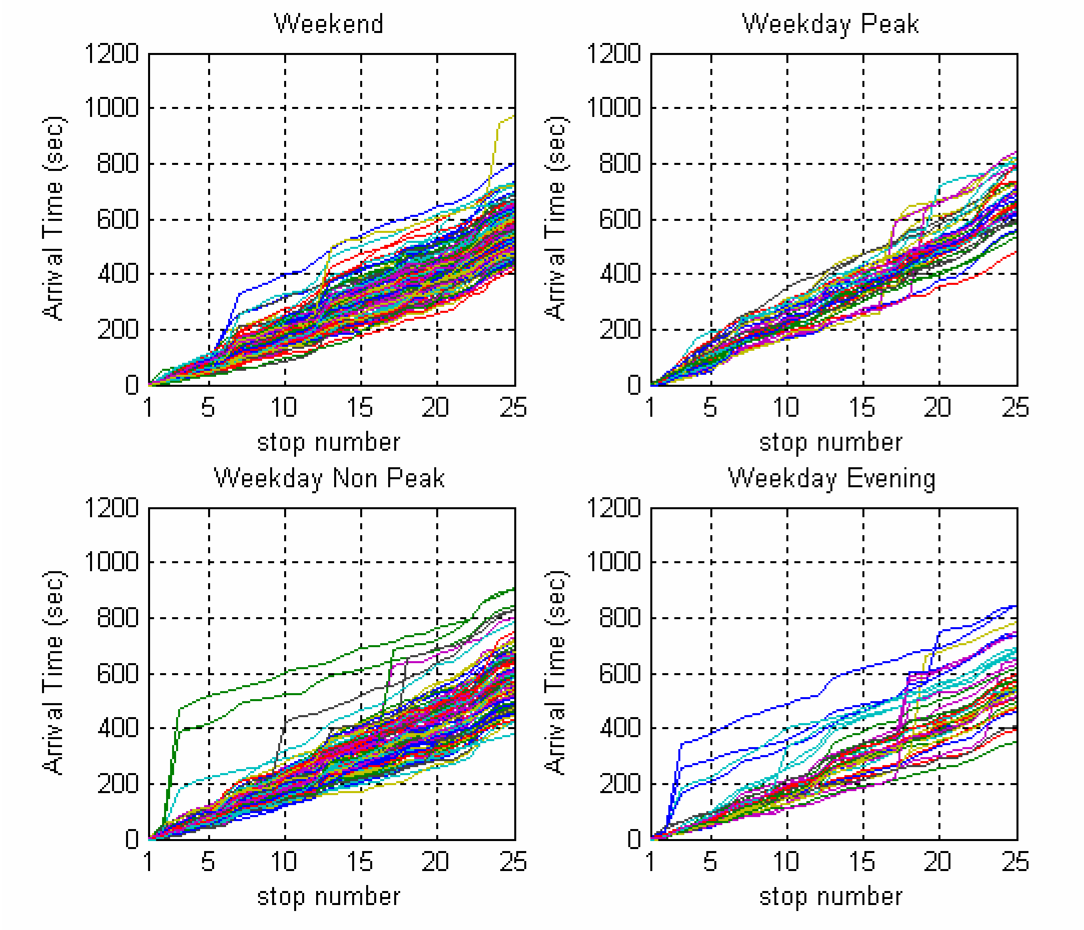

FIGURE 3-13 Arrival Time by Time Period of the Test Bed 2 (North Area)

TABLE 3-9 Mean and Standard Deviation of Arrival Time for the Test Bed 2 (North Area, Weekend)

Stop Number	Mean (sec)	Standard Deviation (sec)
1	0.5	0.5
2	12.9	6.7
3	31.3	10.6
4	47.8	12.6
5	63.3	14.6
6	91.9	25.2
7	127.3	39.4
8	144.7	40.6
9	163.3	41.7
10	185.5	44.2
11	200.7	46.8
12	223.7	50.0
13	272.8	61.2
14	293.8	63.1
15	311.2	64.5
16	331.4	66.1
17	349.1	66.8
18	377.8	69.2
19	391.5	69.8
20	414.8	70.0
21	432.7	71.8
22	466.8	71.7
23	504.6	74.3
24	545.2	82.6
25	569.5	83.4

TABLE 3-10 Mean and Standard Deviation of Arrival Time for the Test Bed 2 (North Area, Weekday Peak)

Stop Number	Mean (sec)	Standard Deviation (sec)
1	0.5	0.5
2	18.9	10.3
3	45.7	19.7
4	72.3	28.5
5	98.9	34.4
6	133.6	36.1
7	174.5	38.6
8	196.1	38.3
9	215.2	39.4
10	242.6	44.1
11	258.6	46.3
12	283.4	50.3
13	320.7	54.6
14	341.6	54.8
15	365.3	56.7
16	387.3	57.3
17	423.8	69.8
18	459.7	73.4
19	487.2	75.0
20	516.2	78.4
21	536.9	80.9
22	569.5	82.2
23	610.5	82.7
24	654.0	84.5
25	684.9	85.6

TABLE 3-11 Mean and Standard Deviation of Arrival Time for the Test Bed 2 (North Area, Weekday Non-Peak)

Stop Number	Mean (sec)	Standard Deviation (sec)
1	0.5	0.5
2	15.7	9.9
3	43.5	57.9
4	62.0	59.2
5	79.4	60.0
6	108.0	60.9
7	140.4	62.3
8	158.5	62.7
9	176.6	63.6
10	202.5	68.7
11	217.3	69.7
12	239.0	71.1
13	278.6	74.6
14	298.0	76.2
15	316.5	77.9
16	335.7	79.7
17	362.3	91.8
18	393.2	95.0
19	407.1	97.6
20	431.5	98.8
21	452.5	100.1
22	484.4	103.2
23	525.6	105.5
24	566.0	106.9
25	591.6	107.3

TABLE 3-12 Mean and Standard Deviation of Arrival Time for the Test Bed 2 (North Area, Weekday Evening)

Stop Number	Mean (sec)	Standard Deviation (sec)
1	0.5	0.5
2	14.4	8.7
3	52.0	66.7
4	68.2	67.7
5	85.3	69.0
6	110.2	71.4
7	139.4	73.0
8	155.1	73.4
9	173.9	76.2
10	200.1	80.4
11	214.5	83.4
12	240.0	83.4
13	285.0	88.8
14	303.7	90.4
15	318.7	91.7
16	336.5	91.7
17	353.9	92.7
18	407.2	105.6
19	428.2	109.4
20	453.6	117.9
21	469.8	119.6
22	495.3	119.4
23	529.2	117.8
24	570.5	114.9
25	589.6	115.8

3.3.2 Dwell Time

FIGURE 3-14 shows the plot of dwell time at each bus stop in the downtown area. Dwell time is a function of passenger demand, intersection signal and delay, schedule adherence, and the behavior of the bus driver.

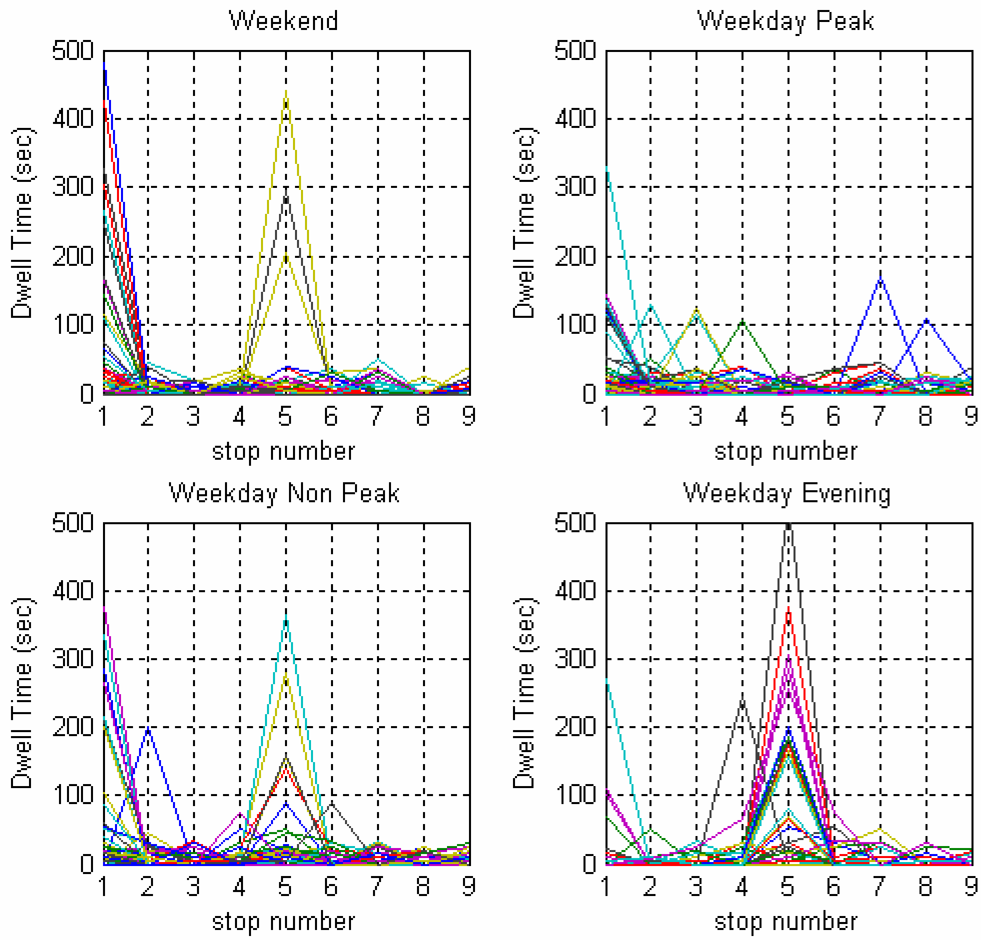

FIGURE 3-14 Dwell Time by Time Period of the Test Bed 1 (Downtown)

TABLE 3-13 presents the mean and standard deviation value of the dwell time for the test bed 1, the downtown area. It was found that the buses stayed for a long time at stop 1 and stop 5. For example, during the weekend period, the mean value of the dwell time at bus stop 1 is 24.7 seconds and at stop 5 is 10.1 seconds, while at other stops is between 0.4 to 3.8 seconds. During the same time period, the standard deviation of the dwell time at stop 1 is 73.2 seconds and at stop 5 is 46.5 seconds, while at other stops, it is between 2.5 to 7.5 seconds.

Stop 1 and stop 9 are the schedule time check points for bus drivers to follow the predefined schedule. Therefore, it is hypothesized that the longer dwell time at stop 1 results from schedule adherence. In other words, when the bus arrived ahead of the schedule, it stayed longer even without passenger demand. However, because stop 5 is not the time check point, it is likely that the high dwell time was due to demand or intersection signal. They might stay at stop 5 to keep on schedule trying not to stop again at stop 9 that has usually less passenger demand.

TABLE 3-13 Mean and Standard Deviation of Dwell Time for the Test Bed 1 (Downtown)

Time Period	Stop Number	Mean (sec)	Standard Deviation (sec)
Weekend	1	24.7	73.2
	2	3.6	7.1
	3	1.5	4.0
	4	3.8	7.2
	5	10.1	46.5
	6	1.9	5.8
	7	2.6	7.5
	8	0.4	2.5
	9	1.8	5.1
Weekday Peak	1	32.1	53.7
	2	9.7	18.4
	3	8.6	21.5
	4	6.8	15.7
	5	3.9	7.3
	6	3.1	7.3
	7	10.4	23.2
	8	4.3	14.8
	9	7.8	10.1
Weekday Non-Peak	1	33.6	76.1
	2	7.3	22.9
	3	2.6	6.4
	4	5.0	11.3
	5	20.9	56.0
	6	3.6	11.8
	7	4.3	8.6
	8	3.0	6.3
	9	3.7	6.9
Weekday Evening	1	15.4	48.4
	2	3.1	8.3
	3	3.2	7.8
	4	12.1	40.0
	5	92.7	125.1
	6	7.3	16.2
	7	6.5	13.0
	8	2.2	6.7
	9	3.3	6.2

TABLE 3-14 shows the total and average dwell time for the test bed 1, the downtown area. According to this table, the average dwell time in the weekend period is 5.6 seconds, in the weekday peak is 9.6 seconds, in the weekday non-peak period is 9.3 seconds, and in the weekday evening is 16.2 seconds. It was found that the dwell time during the weekend period is the shorter than that during other time periods. The dwell time during three weekday periods is larger than that during weekend period. This phenomenon would result from larger passenger demand during weekdays.

Interestingly, relatively long dwell time such as over 200 seconds or 300 seconds was found infrequently during the weekday peak period. TABLE 3-15 shows the number of data that the dwell time is longer than 100, 200, and 300 seconds. It was also found that during the weekday peak period, buses did not stay longer at stops. However, during other time periods, bus drivers stayed longer to follow the schedule. It is likely that the traffic congestion of weekday peak periods resulted in delay and the delayed buses tended not to stay longer at stops. The dwell time of more than 300 seconds would not result from high passenger demand. The extremely long dwell time would include passenger demand, intersection delay, and driver's behavior. It is likely that bus drivers stayed longer when and where they stopped due to demand and intersection delay. This behavior usually did not happen in weekday peak periods.

TABLE 3-14 Total and Average Dwell Time by the Time Period for the Test Bed 1 (Downtown)

Time Period	Total Dwell Time (sec)	Number of Observation	Average Dwell Time (sec)
Weekend	7,382	147	5.6
Weekday Peak	5,895	68	9.6
Weekday Non-Peak	7,140	85	9.3
Weekday Evening	5,828	40	16.2

TABLE 3-15 Number of Long Dwell Time by the Time Period for the Test Bed 1 (Downtown)

Time Period	Over 100 seconds	Over 200 seconds	Over 300 seconds
Weekend	1.1 %	15.9 %	4.0 %
Weekday Peak	2.1 %	1.2 %	0.9 %
Weekday Non-Peak	1.7 %	9.5 %	2.6 %
Weekday Evening	4.4 %	4.8 %	2.1 %

FIGURE 3-15 shows the plot of dwell time at each stop in the north area. In this north area, there are two time check points, stop 6 and stop 20.

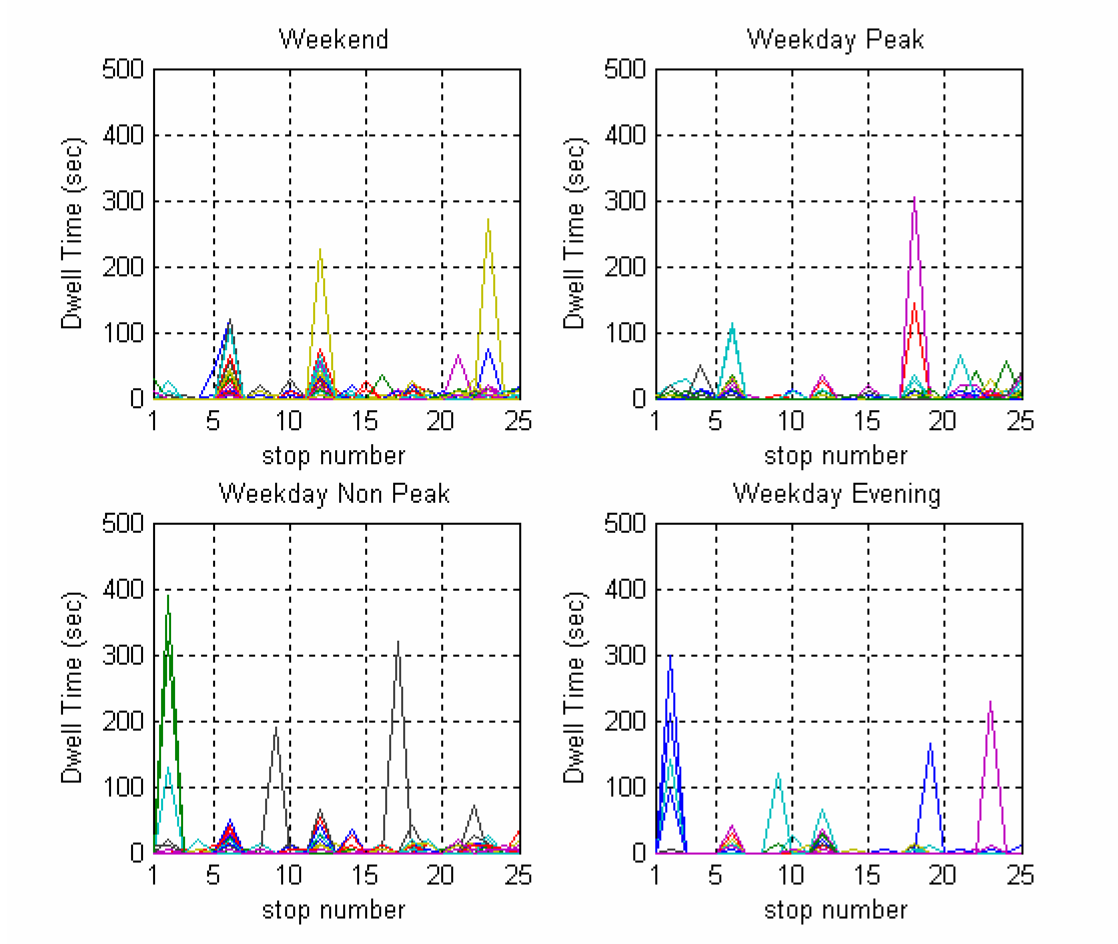

FIGURE 3-15 Dwell Time by Time Period of the Test Bed 2 (North Area)

TABLE 3-16 through TABLE 3-19 show the mean and standard deviation value of the dwell time for the test bed 2, the north area by the time period, weekend, weekday peak, weekday non-peak, and weekday evening, respectively.

It was found that the buses stayed for a long time at stop 6. For example, according to TABLE 3-16, during the weekend period, the mean value of the dwell time at bus stop 6 is 11.4 seconds, while at other stops is between 0.1 to 4.0 seconds. During the same time period, the standard deviation of the dwell time at stop 6 is 19.8 seconds, while at other stops is between 0.6 to 6.0 seconds.

Sometimes, the dwell time at other stops is relatively longer than the dwell time at most stops, it has no regularity. Stop 6 is the schedule time check points for bus drivers to follow the predefined schedule. Therefore, it is hypothesized that the longer dwell time at stop 6 results from schedule adherence. In other words, when the bus arrived ahead of the schedule, it stayed longer even without passenger demand. However, the dwell time at stop 20 has the same pattern.

TABLE 3-16 Mean and Standard Deviation of Dwell Time for the Test Bed 2 (North Area, Weekend)

Stop Number	Mean (sec)	Standard Deviation (sec)
1	0.3	2.7
2	0.4	2.5
3	0.0	0.4
4	0.0	0.0
5	0.4	3.8
6	11.4	19.8
7	0.0	0.0
8	0.3	2.0
9	0.1	0.6
10	0.7	3.2
11	0.0	0.4
12	10.8	23.2
13	0.5	1.9
14	0.3	1.9
15	0.3	2.5
16	0.3	3.0
17	0.3	1.7
18	2.2	5.2
19	0.3	1.8
20	0.2	1.3
21	1.1	6.0
22	1.3	4.4
23	4.0	23.9
24	0.5	1.7
25	0.5	2.4

TABLE 3-17 Mean and Standard Deviation of Dwell Time for the Test Bed 2 (North Area, Weekday Peak)

Stop Number	Mean (sec)	Standard Deviation (sec)
1	0.2	1.1
2	2.5	5.0
3	1.3	5.0
4	3.3	8.4
5	0.1	0.8
6	14.8	23.6
7	0.2	1.1
8	0.0	0.0
9	0.1	0.8
10	0.9	3.5
11	0.0	0.0
12	3.8	7.9
13	0.1	0.8
14	0.6	1.6
15	1.3	3.8
16	0.1	0.8
17	0.0	0.0
18	13.0	50.2
19	0.8	2.6
20	0.0	0.0
21	3.5	10.4
22	2.3	7.3
23	3.2	7.0
24	2.2	8.7
25	6.5	10.4

TABLE 3-18 Mean and Standard Deviation of Dwell Time for the Test Bed 2 (North Area, Weekday Non-Peak)

Stop Number	Mean (sec)	Standard Deviation (sec)
1	0.1	1.1
2	8.7	50.9
3	0.0	0.5
4	0.2	2.0
5	0.2	1.2
6	8.4	10.7
7	0.0	0.5
8	0.3	1.7
9	1.8	18.7
10	0.5	2.2
11	0.0	0.5
12	5.3	11.2
13	0.1	1.5
14	0.8	4.4
15	0.2	1.0
16	0.5	2.1
17	3.1	31.4
18	1.7	5.3
19	1.0	3.4
20	0.0	0.5
21	2.3	5.0
22	2.2	8.0
23	3.0	5.9
24	0.3	1.4
25	1.0	4.3

TABLE 3-19 Mean and Standard Deviation of Dwell Time for the Test Bed 2 (North Area, Weekday Evening)

Stop Number	Mean (sec)	Standard Deviation (sec)
1	0.0	0.0
2	18.8	61.7
3	0.0	0.0
4	0.0	0.0
5	0.0	0.0
6	7.0	11.1
7	0.0	0.0
8	0.0	0.0
9	3.4	19.1
10	1.0	4.1
11	0.3	1.6
12	8.9	13.4
13	0.0	0.0
14	0.1	0.8
15	0.0	0.0
16	0.1	0.8
17	0.0	0.0
18	1.8	4.3
19	4.4	26.1
20	0.0	0.0
21	0.1	0.8
22	0.1	0.8
23	6.8	36.3
24	0.0	0.0
25	0.3	1.6

TABLE 3-20 shows the total and average dwell time for the test bed 1, the downtown area. According to this table, the average dwell time in the weekend period is 1.4 seconds, in the weekday peak is 2.4 seconds, in the weekday non-peak period is 1.7 seconds, and in the weekday evening is 2.1 seconds. It was found that the dwell time during the weekend period is the shorter than that during other time periods. The dwell time during three weekday periods is larger than that during weekend period. This phenomenon would result from larger passenger demand during weekdays. In general, the dwell time in the north area is shorter than that in the downtown area. This result would result from relatively lower demand, less traffic volume, etc.

TABLE 3-20 Total and Average Dwell Time by the Time Period for the Test Bed 2 (North Area)

Time Period	Total Dwell Time (sec)	Number of Observation	Average Dwell Time (sec)
Weekend	5,028	139	1.4
Weekday Peak	2,674	44	2.4
Weekday Non-Peak	4,334	103	1.7
Weekday Evening	2,115	40	2.1

Interestingly, relatively long dwell time such as over 200 seconds was found infrequently during the weekday peak period. TABLE 3-21 shows the number of data that the dwell time is longer than 100, 200, and 300 seconds. It was also found that during the weekday peak period, buses did not stay longer at stops. However, during other time periods, bus drivers stayed longer to follow the schedule. It is likely that the traffic congestion of weekday peak periods resulted in delay and the delayed buses tended not to stay longer at stops. The dwell time of more than 300 seconds would not result from high passenger demand. The extremely long dwell time would include passenger demand, intersection delay, and driver’s behavior. It can be believed that bus drivers stayed longer when and where they stopped due to demand and intersection delay.

This behavior usually did not happen in weekday peak periods. In general, the amount of relatively long dwell time in the north area is less than that in the downtown area.

TABLE 3-21 Number of Long Dwell Time by the Time Period for the Test Bed 2 (North Area)

Time Period	Over 100 seconds	Over 200 seconds	Over 300 seconds
Weekend	0.14 %	5.53 %	0.00 %
Weekday Peak	0.36 %	1.65 %	1.00 %
Weekday Non-Peak	0.19 %	7.13 %	2.40 %
Weekday Evening	0.60 %	5.67 %	0.67 %

3.3.3 Schedule Adherence

FIGURE 3-16 shows the schedule adherence at each stop in the downtown area. Transit vehicles have a predefined schedule to follow. Schedule adherence was calculated by subtracting the scheduled data from the actual arrival time. A positive value of schedule adherence means that bus was delayed at the stop while a negative value means that the bus arrived early.

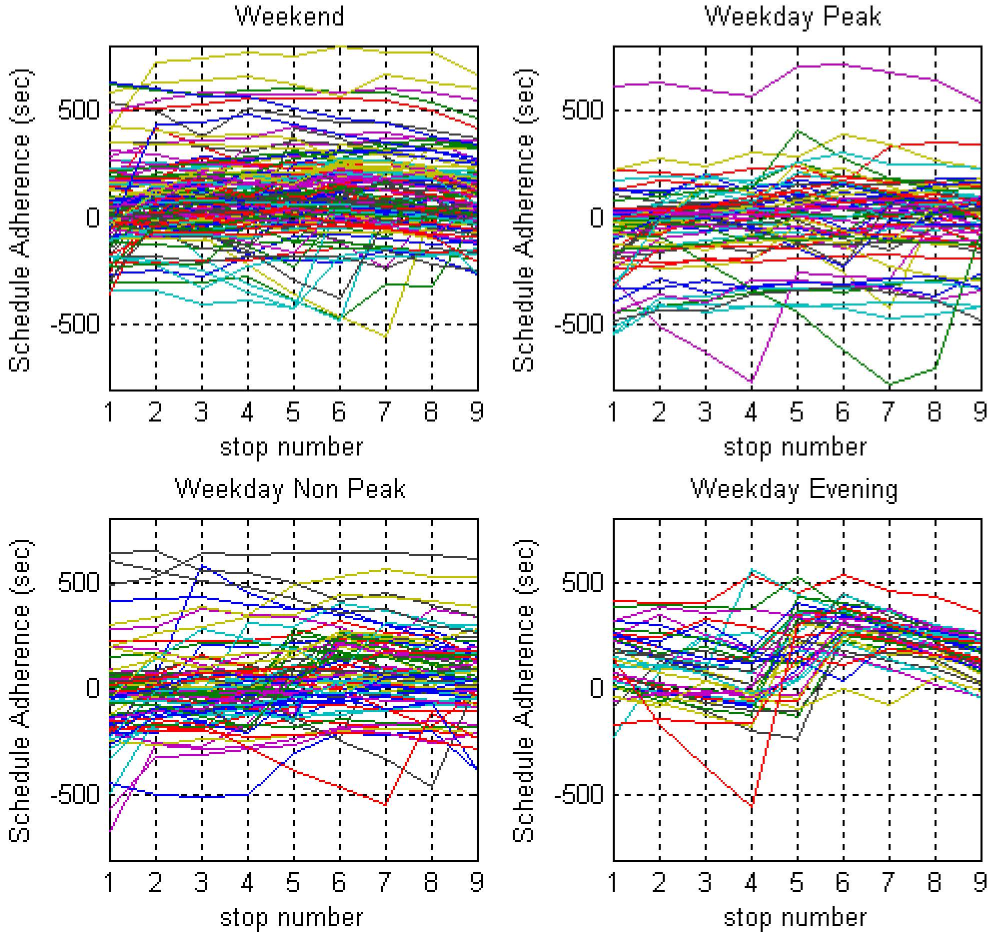

FIGURE 3-16 Schedule Adherence by Time Period of the Test Bed 1 (Downtown)

TABLE 3-2 shows the schedule adherence at each stop in the downtown area. The mean value of the schedule adherence during the weekend period is 91 seconds, weekday peak period is minus 29 seconds, weekday non-peak period is 40 seconds, and weekday evening is 159 seconds.

For this test bed, it was found that there is a pattern for the buses during the weekday peak period to arrive ahead of schedule. This result did not meet the expectation that buses during peak period tend to arrive beyond schedule. This result would be different in other sites and the result can not be generalized.

The standard deviation of the schedule adherence is larger than that of the arrival time and the dwell time. In other words, the variability in the schedule adherence is larger than that in the other two variables.

TABLE 3-22 Mean and Standard Deviation of Schedule Adherence for the Test Bed 1 (Downtown)

Time Period	Stop Number	Mean (sec)	Standard Deviation (sec)
Weekend	1	45.1	179.1
	2	81.3	172.6
	3	85.2	177.2
	4	88.6	182.7
	5	101.0	190.3
	6	118.2	185.2
	7	115.2	177.7
	8	105.0	162.9
	9	80.6	154.8
Weekday Peak	1	-86.6	199.3
	2	-50.4	182.4
	3	-38.2	193.7
	4	-32.9	195.6
	5	3.0	200.7
	6	-14.7	213.0
	7	-16.8	215.3
	8	-8.8	200.4
	9	-15.3	176.6
Weekday Non-Peak	1	-36.3	209.4
	2	3.5	178.2
	3	15.4	196.1
	4	18.6	188.1
	5	48.0	185.0
	6	91.3	187.4
	7	91.3	185.3
	8	71.8	172.0
	9	52.0	168.0
Weekday Evening	1	122.8	145.0
	2	96.9	145.8
	3	81.6	164.4
	4	58.8	203.6
	5	188.1	181.9
	6	282.2	106.7
	7	246.2	95.3
	8	205.6	76.9
	9	147.7	89.7

FIGURE 3-17 shows the schedule adherence at each stop in the north area.

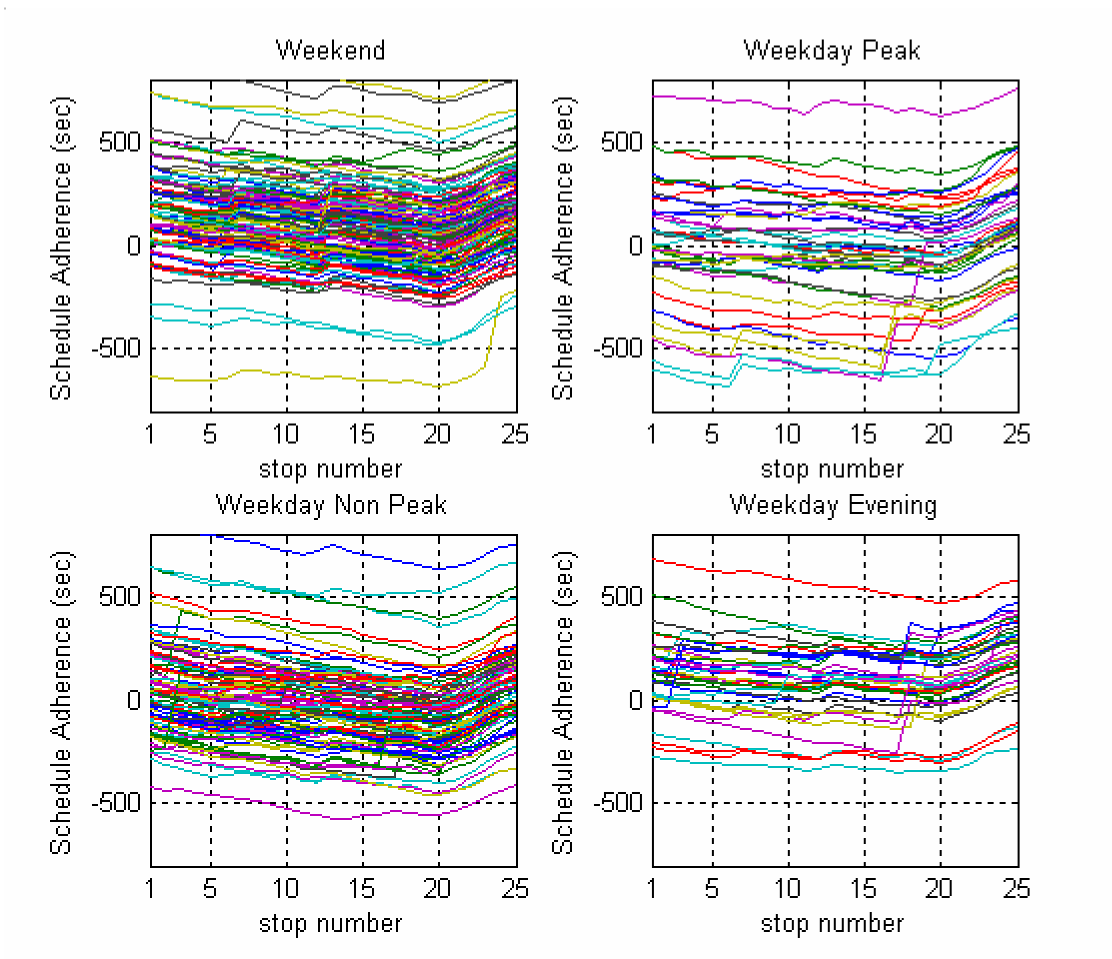

FIGURE 3-17 Schedule Adherence by Time Period of the Test Bed 2 (North Area)

TABLE 3-23 through TABLE 3-26 show the schedule adherence at each stop in the north area by the time period, weekend, weekday peak, weekday non-peak, and weekday evening, respectively.

The standard deviation of the schedule adherence during the weekend period is 202 seconds, weekday peak period is minus 271 seconds, weekday non-peak period is 201 seconds, and weekday evening period is 182 seconds. For this test bed, it was found that the variability in the schedule adherence is larger than that in the arrival time and the dwell time.

Like the downtown area, it was found that there is a pattern for the buses during the weekday peak period to arrive ahead of schedule. In addition to the weekday peak period, the schedule adherence during the weekday non-peak period also has negative values meaning for the buses to arrive ahead of schedule. The standard deviation of the schedule adherence is larger than that of the arrival time and the dwell time. In other words, the variability in the schedule adherence is larger than that in the other two variables.

TABLE 3-23 Mean and Standard Deviation of Schedule Adherence for the Test Bed 2 (North Area, Weekend)

Stop Number	Mean (sec)	Standard Deviation (sec)
1	184.5	207.9
2	168.4	206.8
3	158.9	206.2
4	147.3	205.9
5	134.8	204.2
6	135.4	202.5
7	140.8	202.9
8	128.2	202.7
9	116.8	203.9
10	109.1	202.3
11	94.3	201.6
12	87.2	202.5
13	106.3	205.4
14	97.3	205.3
15	84.7	205.5
16	74.9	205.9
17	62.6	206.2
18	61.3	205.8
19	45.1	205.2
20	38.3	204.1
21	56.2	202.7
22	90.3	199.5
23	128.1	197.3
24	168.8	188.0
25	193.1	185.7

TABLE 3-24 Mean and Standard Deviation of Schedule Adherence for the Test Bed 2 (North Area, Weekday Peak)

Stop Number	Mean (sec)	Standard Deviation (sec)
1	22.5	274.0
2	6.1	276.0
3	-1.9	283.5
4	-10.1	286.3
5	-18.3	289.0
6	-18.4	290.8
7	-7.6	277.3
8	-16.0	277.8
9	-26.9	277.4
10	-29.5	278.1
11	-43.5	280.7
12	-48.7	284.8
13	-41.3	288.4
14	-50.4	289.2
15	-56.8	288.4
16	-64.7	288.0
17	-58.3	261.1
18	-52.3	262.6
19	-54.8	256.0
20	-55.9	248.9
21	-35.2	245.0
22	-2.5	241.7
23	38.5	242.8
24	82.0	242.3
25	112.8	246.3

TABLE 3-25 Mean and Standard Deviation of Schedule Adherence for the Test Bed 2 (North Area, Weekday Non-Peak)

Stop Number	Mean (sec)	Standard Deviation (sec)
1	67.0	216.2
2	49.1	214.9
3	44.2	215.5
4	29.9	214.1
5	14.6	213.2
6	10.5	212.3
7	13.3	211.1
8	1.8	209.9
9	-9.6	208.3
10	-13.3	207.8
11	-28.1	208.0
12	-35.9	207.3
13	-25.9	205.1
14	-36.1	203.8
15	-47.2	201.0
16	-57.6	198.8
17	-60.5	194.2
18	-59.2	191.2
19	-74.9	192.1
20	-79.6	191.9
21	-58.6	189.8
22	-26.7	188.9
23	14.5	187.2
24	54.8	187.6
25	80.5	187.5

TABLE 3-26 Mean and Standard Deviation of Schedule Adherence for the Test Bed 2 (North Area, Weekday Evening)

Stop Number	Mean (sec)	Standard Deviation (sec)
1	127.1	184.1
2	112.9	182.8
3	122.4	188.1
4	110.5	187.6
5	99.5	186.3
6	96.3	187.9
7	99.4	185.7
8	89.0	185.0
9	81.7	186.1
10	81.8	186.8
11	70.1	187.1
12	69.5	184.4
13	88.4	181.4
14	81.0	179.9
15	69.9	180.8
16	61.6	179.5
17	52.9	178.9
18	80.1	179.0
19	75.0	175.4
20	78.2	179.8
21	94.4	179.2
22	119.9	176.4
23	153.8	173.1
24	195.1	177.6
25	214.2	175.2

3.4 CONCLUDING REMARKS

In this chapter, the test bed and the data reduction process were described. To consider traffic congestion for predicting bus arrival time, a bus route running on congested corridor of Houston was selected. To verify the accuracy of the predicted model in a different environment, two test sites were chosen: the down town area and the north area. Even though the Houston Metro used transit vehicles equipped with DGPS, some GPS measurement errors still exist. To predict bus arrival time accurately, accurate input data are essential. The second section of this chapter discussed the characteristics of the errors and the method for modifying these errors. With these modified GPS location data, the three input data were determined. The three input variables were also described in this chapter: arrival time, dwell time, and schedule adherence. Unexpectedly, dwell time shows more variability and uncertainty than arrival time. Consequently, there should be a need for a prediction model that can explain this uncertainty. In chapter IV, three prediction models will be developed.

CHAPTER IV

MODEL DEVELOPMENT

In the previous chapter, the GPS location data were modified and the input data were calculated. With the modified input data, prediction models are developed in this chapter. A number of modeling techniques were used in this study including a simple statistical model (historical data based model), a multi linear regression model, and an artificial neural network model. The input variables are arrival time, dwell time, and schedule adherence at each stop.

In order to consider traffic congestion and different travel patterns, the data set is clustered into four time periods: weekend, weekday peak, weekday non-peak, and weekday evening. In this dissertation, the term "non-clustering data set" indicates the data set before this clustering. The three input variables and link travel time were also clustered by time period. In addition, schedule adherence was calculated by subtracting the scheduled arrival time from the actual arrival time. A positive value of schedule adherence means that bus was delayed at the stop while a negative value means that the bus arrived early. The output variable of the model is the forecast arrival time at each stop.

In this chapter, a number of model specifications M_{mbtn} were developed and tested. The subscript m refers to the model classification and three model classifications were used in this research: a historical data base model, a regression model, and an artificial neural network model. The subscript b refers to the test bed and two test beds were used: the downtown area and the north area. The subscript t refers to the time period and four time periods were used: the weekend period, the weekday peak period, the weekday non-peak period, and the weekday evening period. The subscript n refers to the model number. Model n used the input data of previous n stops. The subscripts are shown in TABLE 4-1.

TABLE 4-1 Model Specifications

Script m	1	Historical data based model
	2	Regression model
	3	Artificial neural network model
Script b	1	Downtown area
	2	North area
Script t	1	Weekend period
	2	Weekday peak period
	3	Weekday non-peak period
	4	Weekday evening period
Script n	1	Model uses input data of stop 1 and predicts arrival time at stop 2 through 9 (for the downtown area) or 25 (for the north area)
	2	Model uses input data of stop 1 to 2 and predicts arrival time at stop 3 through 9 (for the downtown area) or 25 (for the north area)
	3	Model uses input data of stop 1 to 3 and predicts arrival time at stop 4 through 9 (for the downtown area) or 25 (for the north area)
	4	Model uses input data of stop 1 to 4 and predicts arrival time at stop 5 through 9 (for the downtown area) or 25 (for the north area)
	. . . .	. . . .
	23	Model uses input data of stop 1 to 23 and predicts arrival time at stop 24 through 25 (for the north area)
	24	Model uses input data of stop 1 to 24 and predicts arrival time at stop 25 (for the north area)

For example, in the downtown area, the test bed for this research has nine bus stops. Consequently, eight separate models were developed for each of the three techniques analyzed. For example, Model 1 uses the arrival time, dwell time, and schedule adherence data of stop 1 as input variables and predicts the arrival time at stop 2 through stop 9. In contrast, Model 5 uses the input data from the first five stops and predicts arrival times at stops 6 through 9. In the north area, there are 25 bus stops and consequently 24 models were developed using the same approach. The generalization of this model structure is as follows:

Model M_{mbtn} :

Input data = Arrival time, dwell time, and scheduled adherence
at stop 1 through n;

Output = Predicted arrival time at stop n+1 through N_b .

where,

M_{mbtn} = model n with m classification, b test bed, and t time period;

m = model classification;

= 1 (historical data based model),

= 2 (regression model), and

= 3 (artificial neural network model)

b = test bed;

= 1 (test bed 1: downtown area) and

= 2 (test bed 2: north area)

t = time period;

= 1 (weekend period),

= 2 (weekday peak period),

= 3 (weekday non-peak period), and

= 4 (weekday evening period)

n = model number;

N_b = Number of last bus stop of test bed b.

=1 (downtown area) and

=2 (north area)

TABLE 4-2 and TABLE 4-3 show the structure of the models for test beds 1 and 2, respectively..

TABLE 4-2 Model Structure of the Test Bed 1 (Downtown)

Model	Input data	Output
M_{m1t1}	A_{1k} W_{1k} S_{1k}	A_{2k}, A_{3k}, A_{4k}, A_{5k}, A_{6k}, A_{7k}, A_{8k}, A_{9k}
M_{m1t2}	A_{1k}, A_{2k} W_{1k},W_{2k} S_{1k}, S_{2k}	A_{3k}, A_{4k}, A_{5k}, A_{6k}, A_{7k}, A_{8k}, A_{9k}
M_{m1t3}	A_{1k}, A_{2k}, A_{3k} W_{1k},W_{2k},W_{3k} S_{1k}, S_{2k}, S_{3k}	A_{4k}, A_{5k}, A_{6k}, A_{7k}, A_{8k}, A_{9k}
M_{m1t4}	A_{1k}, A_{2k}, A_{3k}, A_{4k} W_{1k},W_{2k},W_{3k},W_{4k} S_{1k}, S_{2k}, S_{3k}, S_{4k}	A_{5k}, A_{6k}, A_{7k}, A_{8k}, A_{9k}
M_{m1t5}	A_{1k}, A_{2k}, A_{3k}, A_{4k}, A_{5k} W_{1k},W_{2k},W_{3k},W_{4k},W_{5k} S_{1k}, S_{2k}, S_{3k}, S_{4k}, S_{5k}	A_{6k}, A_{7k}, A_{8k}, A_{9k}
M_{m1t6}	A_{1k}, A_{2k}, A_{3k}, A_{4k}, A_{5k}, A_{6k} W_{1k},W_{2k},W_{3k},W_{4k},W_{5k},W_{6k} S_{1k}, S_{2k}, S_{3k}, S_{4k}, S_{5k}, S_{6k}	A_{7k}, A_{8k}, A_{9k}
M_{m1t7}	A_{1k}, A_{2k}, A_{3k}, A_{4k}, A_{5k}, A_{6k}, A_{7k} W_{1k},W_{2k},W_{3k},W_{4k},W_{5k},W_{6k},W_{7k} S_{1k}, S_{2k}, S_{3k}, S_{4k}, S_{5k}, S_{6k}, S_{7k}	A_{8k}, A_{9k}
M_{m1t8}	A_{1k}, A_{2k}, A_{3k}, A_{4k}, A_{5k}, A_{6k}, A_{7k}, A_{8k} W_{1k},W_{2k},W_{3k},W_{4k},W_{5k},W_{6k},W_{7k},W_{8k} S_{1k}, S_{2k}, S_{3k}, S_{4k}, S_{5k}, S_{6k}, S_{7k}, S_{8k}	A_{9k}

A_{jk} = Arrival time of bus k at bus stop j;

W_{jk} = Dwell time of bus k at bus stop j;

S_{jk} = Schedule adherence of bus k at bus stop j.

TABLE 4-3 Model Structure of the Test Bed 2 (North Area)

Model	Input data	Output
M_{m2t1}	A_{1k} W_{1k} S_{1k}	A_{2k}, A_{3k}, A_{4k},,,, A_{23k}, A_{24k}, A_{25k}
M_{m2t2}	A_{1k}, A_{2k} W_{1k},W_{2k} S_{1k}, S_{2k}	A_{3k}, A_{4k}, A_{5k},,,, A_{23k}, A_{24k}, A_{25k}
M_{m2t3}	A_{1k}, A_{2k}, A_{3k} W_{1k},W_{2k},W_{3k} S_{1k}, S_{2k}, S_{3k}	A_{4k}, A_{5k}, A_{6k},,,, A_{23k}, A_{24k}, A_{25k}
M_{m2t4}	A_{1k}, A_{2k}, A_{3k}, A_{4k} W_{1k},W_{2k},W_{3k},W_{4k} S_{1k}, S_{2k}, S_{3k}, S_{4k}	A_{5k}, A_{6k}, A_{7k},,,,,,, A_{23k}, A_{24k}, A_{25k}
M_{m2t5}	A_{1k}, A_{2k}, A_{3k}, A_{4k}, A_{5k} W_{1k},W_{2k},W_{3k},W_{4k},W_{5k} S_{1k}, S_{2k}, S_{3k}, S_{4k}, S_{5k}	A_{6k}, A_{7k}, A_{8k},,,, A_{23k}, A_{24k}, A_{25k}
M_{m2t6}	A_{1k}, A_{2k}, A_{3k}, A_{4k}, A_{5k}, A_{6k} W_{1k},W_{2k},W_{3k},W_{4k},W_{5k},W_{6k} S_{1k}, S_{2k}, S_{3k}, S_{4k}, S_{5k}, S_{6k}	A_{7k}, A_{8k},,,, A_{23k}, A_{24k}, A_{25k}
M_{m2t7}	A_{1k}, A_{2k}, A_{3k}, A_{4k}, A_{5k}, A_{6k}, A_{7k} W_{1k},W_{2k},W_{3k},W_{4k},W_{5k},W_{6k},W_{7k} S_{1k}, S_{2k}, S_{3k}, S_{4k}, S_{5k}, S_{6k}, S_{7k}	A_{8k}, A_{9k},,,,, A_{24k}, A_{25k}
. . . .	. . . .	. . . .
M_{m2t22}	A_{1k}, A_{2k}, A_{3k},,,, A_{20k}, A_{21k}, A_{22k} W_{1k},W_{2k},W_{3k},,,,,W_{20k},W_{21k},W_{22k} S_{1k}, S_{2k}, S_{3k},,,,,, S_{20k}, S_{21k}, S_{22k}	A_{23k}, A_{24k}, A_{25k}
M_{m2t23}	A_{1k}, A_{2k}, A_{3k},,,, A_{21k}, A_{22k}, A_{23k} W_{1k},W_{2k},W_{3k},,,,W_{21k},W_{22k},W_{23k} S_{1k}, S_{2k}, S_{3k},,,,,, S_{21k}, S_{22k}, S_{23k}	A_{24k}, A_{25k}
M_{m2t24}	A_{1k}, A_{2k}, A_{3k},,,, A_{22k}, A_{23k}, A_{24k} W_{1k},W_{2k},W_{3k},,,,W_{22k},W_{23k},W_{24k} S_{1k}, S_{2k}, S_{3k},,,,,, S_{22k}, S_{23k}, S_{24k}	A_{25k}

A_{jk} = Arrival time of bus k at bus stop j;

W_{jk} = Dwell time of bus k at bus stop j;

S_{jk} = Schedule adherence of bus k at bus stop j.

4.1 HISTORICAL DATA BASED MODELS

In this section, a simple statistical model, (i.e. a historical data based model), is developed. The historical data based models for link travel time and bus arrival time are shown in Equations 4-1 and 4-2, respectively. FIGURE 4-1 illustrates link travel time, dwell time, and arrival time. First, the link travel time between transit stops is calculated. It can be seen in Equation 4-1 that this is a function of the difference in the average time of arrival of the downstream stops and the average departure time (i.e. arrival time + dwell time) at the upstream stop. Subsequently a recursive formula is used to predict the arrival time at the remaining stops as shown in Equation 4-2. Link travel time does include stopped delay at intersections but does not include dwell times. The arrival time calculations are done only at transit stops and only when the bus first arrives at a given stop. These constraints could be generalized, but they were useful for limiting the number of models that were calibrated in this study.

$$\tau_{it} = \alpha_{j+1,t} - (\alpha_{jt} + \varpi_{jt}) \qquad \forall_i = M, N_b, \quad \forall_t = 1, T \qquad (4\text{-}1)$$

where,

τ_{jt} = Estimated link travel time from stop i to stop j+1 departing during time period t;

α_{jt} = Average arrival time at stop j departing during time period t;

ϖ_{jt} = Average dwell time at stop j departing during time period t ;

T = Number of time periods. For the test bed, this is equal to 4 ;
weekend, weekday peak, weekday off-peak, and weekday evening ;

M = Current bus stop. i.e. from 1 to $N_b - 1$.

$$A_{kjt} = A_{kMt} + \sum_{i=M}^{N-1} \tau_{jt} + \sum_{i=M}^{N-1} \varpi_{jt} \qquad \forall_j = M+1, N, \quad \forall_t = 1, T \qquad (4\text{-}2)$$

where,

A_{kjt} = Forecast arrival time for bus k at bus stop j departing during time period t;

A_{kMt} = Observed arrival time for bus k at current stop M departing during time period t.

The results of the historical data based model can be seen in next chapter with the results of the other models.

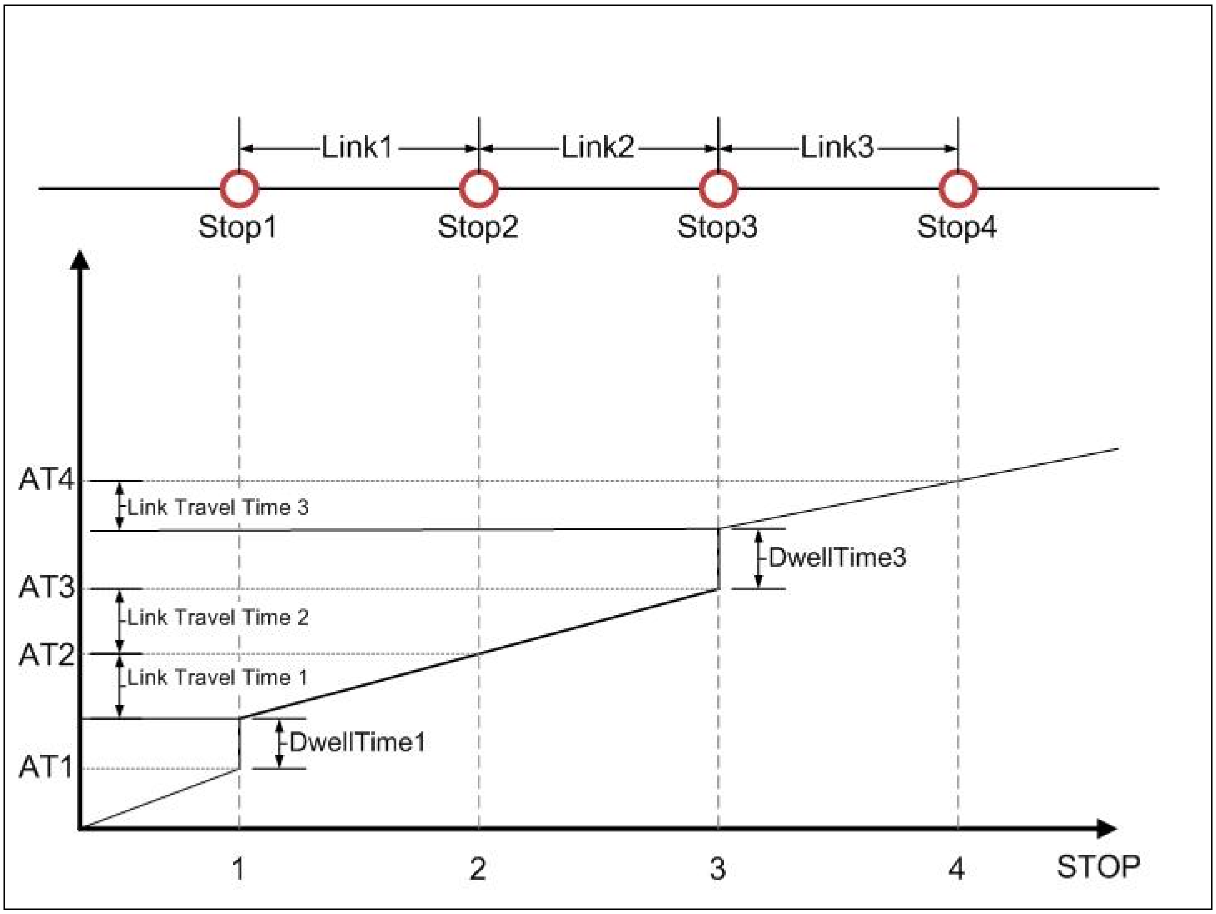

FIGURE 4-1 Arrival Time, Link Travel Time and Dwell Time

4.2 MULTI LINEAR REGRESSION MODELS

In this section, multi linear regression models are developed. To develop regression models, the input variables should not be inter-correlated. In this research, arrival time, and dwell time and schedule adherence at each stop were used as the input variables. The correlation coefficients are calculated by Equation 4-3. A value close to one means that the two independent variables are highly correlated and they should not both be used in the regression model.

$$Correlation\ Coefficients = \frac{Covariance(X_1 X_2)}{\sqrt{Covariance(X_1 X_1) Covariance(X_2 X_2)}} \quad (4\text{-}3)$$

From the analysis of the correlation coefficients, it was found that the correlation coefficient of the arrival time at previous stop and the arrival time at next stops is too high (near 0.9) and can not be used as an independent variable. Therefore, the distance from current stop to each stop is used as an independent variable. Consequently, distance from current stop to each stop, dwell time, and schedule adherence at each stop are chosen as the independent variables for multi linear regression models. TABLE 4-4 and TABLE 4-5 give the result of correlation coefficients of the downtown area and the north area, respectively. The values of the coefficients between independent variables were less than 0.15. Consequently, the three variables were chosen for use as the independent variables for the regression models.

TABLE 4-4 Correlation Coefficients of the Test Bed 1 (Downtown)

	Distance	Dwell Time	Schedule Adherence	Dependent variable
Distance	1.000	-0.127	0.127	0.836
Dwell Time	-0.127	1.000	-0.150	-0.111
Schedule Adherence	0.127	-0.150	1.000	0.247
Dependent variable	0.836	-0.111	0.247	1.000

TABLE 4-5 Correlation Coefficients of the Test Bed 2 (North Area)

	Distance	Dwell Time	Schedule Adherence	Dependent variable
Distance	1.000	-0.003	-0.059	0.925
Dwell Time	-0.003	1.000	-0.032	0.005
Schedule Adherence	-0.059	-0.032	1.000	-0.057
Dependent variable	0.925	0.005	-0.057	1.000

With three independent variables, many regression models can be developed. To select the best multi linear regression models, a forward stepwise regression technique was used. TABLE 4-6 and TABLE 4-7 show the results of the stepwise regression for the downtown area and the north area, respectively. Forty one regression models are analyzed with this stepwise regression technique. Among the forty one models, only eleven models showed a reasonable R^2 value and had statistically significant values of beta (i.e. the coefficients of the independent variables). According to the results of the stepwise regression technique, dwell time was not found to be statistically significant and therefore it was not be used to develop regression models. Consequently, the other two variables, distance and schedule adherence, are used as independent variables and to predict the dependent variable, arrival time at each stop. Among the eleven regression models, five models were chosen to predict bus arrival time because some models had unreasonable results (i.e. extremely large MAPE, which indicates the prediction errors). These regression models shown in Equation 4-4 through Equation 4-8 were tested in this research. Arrival time at the bus stop is predicted by Equation 4-9.

TABLE 4-6 Stepwise Regression of the Test Bed 1 (Downtown)

No.	Independent variables						R^2	Significance of betas		
	L	L^2	S	S^2	W	W^2		b1	b2	b3
1	O						0.6996	O		
2		O					0.6221	O		
3			O				0.0612	O		
4				O			0.0098	O		
5					O		0.0124	O		
6						O	0.0061	O		
7	O	O					0.7030	O	O	
8	O		O				0.7199	O	O	
9	O			O			0.7083	O	O	
10	O				O		0.6997	O	X	
11	O					O	0.6999	O	X	
12		O	O				0.6506	O	O	
13		O		O			0.6333	O	O	
14		O			O		0.6227	O	X	
15		O				O	0.6227	O	X	
16			O	O			0.0615	O	X	
17			O		O		0.0669	O	O	
18			O			O	0.0634	O	X	
19				O	O		0.0236	O	O	
20				O		O	0.0168	O	O	
21					O	O	0.0158	O	O	
22	O	O	O				0.7221	O	O	O
23	O	O		O			0.7111	O	O	O
24	O	O			O		0.7030	O	O	X
25	O	O				O	0.7032	O	O	X
26	O		O	O			0.7220	O	O	O
27	O		O		O		0.7201	O	O	X
28	O		O			O	0.7199	O	O	X
29	O			O	O		0.7084	O	O	X
30	O			O		O	0.7087	O	O	X
31	O				O	O	0.7003	O	X	X
32		O	O	O			0.6532	O	O	O
33		O	O		O		0.6506	O	O	X
34		O	O			O	0.6506	O	O	X
35		O		O	O		0.6342	O	O	X
36		O		O		O	0.6341	O	O	X
37		O			O	O	0.6227	O	X	X
38			O	O	O		0.0675	O	X	O
39			O	O		O	0.0639	O	X	X
40			O		O	O	0.0697	O	O	X
41				O	O	O	0.0270	O	O	O

L: Distance from current stop to stop i

S: Schedule adherence

W: Dwell time

TABLE 4-7 Stepwise Regression of the Test Bed 2 (North Area)

No.	Independent variables						R^2	Significance of betas		
	L	L^2	S	S^2	W	W^2		b1	b2	b3
1	O						0.8557	O		
2		O					0.8309	O		
3			O				0.0033	O		
4				O			0.0027	O		
5					O		0.0000	X		
6						O	0.0000	X		
7	O	O					0.8605	O	O	
8	O		O				0.8555	O	X	
9	O			O			0.8555	O	X	
10	O				O		0.8555	O	X	
11	O					O	0.8554	O	X	
12		O	O				0.8326	O	O	
13		O		O			0.8312	O	O	
14		O			O		0.8309	O	X	
15		O				O	0.8309	O	X	
16			O	O			0.0049	O	O	
17			O		O		0.0033	O	X	
18			O			O	0.0033	O	X	
19				O	O		0.0028	O	X	
20				O		O	0.0027	O	X	
21					O	O	0.0001	X	X	
22	O	O	O				0.8607	O	O	O
23	O	O		O			0.8607	O	O	X
24	O	O			O		0.8606	O	O	X
25	O	O				O	0.8605	O	O	X
26	O		O	O			0.8555	O	X	X
27	O		O		O		0.8555	O	X	X
28	O		O			O	0.8555	O	X	X
29	O			O	O		0.8556	O	X	X
30	O			O		O	0.8555	O	X	X
31	O				O	O	0.8558	O	O	O
32		O	O	O			0.8327	O	O	X
33		O	O		O		0.8326	O	O	X
34		O	O			O	0.8326	O	O	X
35		O		O	O		0.8313	O	O	X
36		O		O		O	0.8313	O	O	X
37		O			O	O	0.8314	O	O	O
38			O	O	O		0.0050	O	O	X
39			O	O		O	0.0049	O	O	X
40			O		O	O	0.0034	O	X	X
41				O	O	O	0.0029	O	X	X

L: Distance from current stop to stop i

S: Schedule adherence

W: Dwell time

Regression 1: $\tau_{Mjkt} = b_0 + b_1 L_{Mjk}$ (4-4)

Regression 2: $\tau_{Mjkt} = b_0 + b_1 L_{Mjk}{}^2$ (4-5)

Regression 3: $\tau_{Mjkt} = b_0 + b_1 L_{Mjk}{}^2 + b_2 S_{Mkt}$ (4-6)

Regression 4: $\tau_{Mjkt} = b_0 + b_1 L_{Mjk}{}^2 + b_2 S_{Mkt}{}^2$ (4-7)

Regression 5: $\tau_{Mjkt} = b_0 + b_1 L_{Mjk} + b_2 L_{Mjk}{}^2 + b_3 S_{Mkt}$ (4-8)

where,

$\tau_{Mjkt} =$ Travel time from current stop M to stop j for bus k

departing during time period t, j = M, N_b;

$L_{Mjk} =$ Distance from stop M to stop j for bus k;

$S_{Mkt} =$ Schedule adherence of bus k at bus stop M departing during time period t;

Equal to observed arrival time at current stop M (A_{Mkt}) minus scheduled arrival time;

M = Current bus stop. i.e. from 1 to N-1;

$N_b =$ Number of last bus stop of test bed b.

For the downtown area this is equal to 9 ($N_1 = 9$) and

for the north area this is equal to 25 ($N_2 = 25$).

$$A_{kjt} = A_{kMt} + \tau_{Mjk} \quad (4\text{-}9)$$

where,

$A_{kjt} =$ Forecast arrival time for bus k at bus stop j departing during time period t;

$A_{kMt} =$ Observed arrival time for bus k at current stop M departing during time period t.

All three model architectures were calibrated using the calibration and testing data sets. The Mean Absolute Percentage Error (MAPE) is used as the measure of effectiveness

(MOE) in this dissertation. The MAPE is shown in Equation 4-10. This represents the average percentage difference between the observed value (in this case arrival time at a transit stop) and the predicted value (in this case predicted arrival time at a transit stop). In general, the smaller the MAPE value the better the prediction model.

$$MAPE = \frac{1}{n}\sum_{i}^{n}\frac{|y_i - y_o|}{y_o} \times 100\% \qquad (4\text{-}10)$$

where,

y_i Predicted value (i.e. arrival time at given transit stop);

y_o Observed value (i.e. arrival time at given transit stop);

n Number of observations.

TABLE 4-8 through TABLE 4-13 shows the MAPE result for the five different regression models. In this section, MAPE was used to select the best multi linear regression model among the five regression models. According to TABLE 4-8, in the downtown area, regression 1 gives the best results for the non-clustering, the weekend, the weekday non-peak, and the weekday evening period. Regression 5 gives the best results for the weekday peak period. However, according to TABLE 4-9 through TABLE 4-13, the results are slightly different for the north area. Regression 5 gives the best results for the non-clustering, weekend, and weekday non-peak periods. Regression 1 gives the best results for the weekday peak and weekday evening periods. In general, regression 1 and regression 5 give the best results for both the downtown area and the north area. TABLE 4-14 and TABLE 4-15 present the best results for the downtown area and the north area, respectively. Consequently, the best regression model shown in these tables will be used to evaluate the three different prediction models.

TABLE 4-8 MAPE for Different Linear Regression Models of the Test Bed 1 (Downtown)

Time Period	Model	Reg. 1	Reg. 2	Reg. 3	Reg. 4	Reg. 5
Non Clustering	1	33.94	39.86	42.48	40.96	36.65
	2	27.56	26.63	27.94	28.18	29.38
	3	25.07	24.55	24.86	25.88	25.87
	4	23.11	23.70	24.64	24.83	23.15
	5	22.68	23.56	23.44	24.31	22.21
	6	22.03	23.38	23.45	24.04	21.45
	7	21.97	24.51	24.65	24.91	21.02
	8	22.40	27.58	27.43	27.71	20.70
	Average	24.85	26.72	27.36	27.60	25.05
Weekend	1	35.09	42.61	43.50	42.52	36.56
	2	24.66	24.30	24.64	24.78	26.74
	3	22.29	22.48	21.99	22.87	23.54
	4	20.57	21.55	21.30	22.01	21.20
	5	19.87	20.99	20.97	21.14	19.68
	6	19.80	21.32	21.27	21.24	19.03
	7	19.87	22.77	22.90	22.72	18.58
	8	20.30	26.28	26.36	26.21	18.15
	Average	22.81	25.29	25.37	25.44	22.94
Weekday Peak	1	24.19	29.96	30.02	30.49	24.31
	2	19.11	19.26	19.11	19.78	20.03
	3	15.22	16.36	15.91	16.87	16.08
	4	13.85	15.74	15.10	16.03	13.88
	5	13.28	14.62	14.13	15.04	12.99
	6	12.00	14.05	13.78	14.66	11.23
	7	11.98	15.05	15.08	15.62	10.63
	8	12.20	18.43	18.48	18.76	9.90
	Average	15.23	17.93	17.70	18.41	14.88
Weekday Non-Peak	1	31.41	35.73	37.66	38.69	33.56
	2	28.72	28.67	28.68	31.47	29.40
	3	25.58	26.12	25.05	28.47	25.87
	4	21.74	23.42	22.83	25.36	22.33
	5	21.39	23.10	22.27	24.47	21.46
	6	19.85	21.59	21.25	22.50	20.41
	7	19.32	21.66	22.63	21.72	20.11
	8	19.84	24.60	25.13	24.71	20.35
	Average	23.48	25.61	25.69	27.17	24.19
Weekday Evening	1	28.91	36.93	46.11	32.57	39.86
	2	25.18	24.95	31.36	23.24	28.52
	3	22.50	22.95	24.93	20.93	22.60
	4	17.65	21.35	19.68	19.83	16.64
	5	16.19	20.53	18.06	18.16	15.79
	6	16.03	18.38	17.42	17.12	15.48
	7	16.03	18.99	17.54	17.88	15.08
	8	17.25	23.16	19.08	20.67	14.62
	Average	19.97	23.41	24.27	21.30	21.07

TABLE 4-9 MAPE for Different Linear Regression Models of the Test Bed 2 (North Area, Non-Clustering)

Time Period	Model	Reg. 1	Reg. 2	Reg. 3	Reg. 4	Reg. 5
Non Clustering	1	22.61	52.65	50.85	52.32	21.25
	2	18.51	28.02	27.56	28.02	18.69
	3	17.84	22.14	21.99	22.20	17.85
	4	17.50	19.58	19.56	19.65	17.18
	5	16.98	18.29	18.32	18.34	16.56
	6	16.69	18.02	18.0	18.08	16.29
	7	16.45	17.76	17.80	17.83	16.01
	8	16.19	17.48	17.53	17.53	15.77
	9	16.00	17.22	17.27	17.27	15.61
	10	15.75	16.94	17.00	16.98	15.45
	11	15.38	16.61	16.71	16.68	15.20
	12	15.02	16.33	16.40	16.36	14.97
	13	14.83	15.73	15.78	15.74	14.71
	14	14.66	15.22	15.27	15.22	14.50
	15	14.48	14.83	14.90	14.83	14.32
	16	14.31	14.60	14.69	14.60	14.10
	17	14.01	14.48	14.55	14.47	13.96
	18	13.84	14.33	14.41	14.32	13.80
	19	13.62	14.19	14.28	14.18	13.58
	20	13.53	14.23	14.31	14.22	13.47
	21	13.48	14.36	14.40	14.35	13.37
	22	13.46	14.53	14.51	14.52	13.26
	23	13.43	14.70	14.63	14.67	13.14
	24	13.48	14.66	14.55	14.62	13.03
	Average	15.50	18.20	18.14	18.21	15.25

TABLE 4-10 MAPE for Different Linear Regression Models of the Test Bed 2 (North Area, Weekend)

Time Period	Model	Reg. 1	Reg. 2	Reg. 3	Reg. 4	Reg. 5
Weekend	1	23.55	52.86	51.46	52.57	19.10
	2	17.81	28.27	27.91	28.23	17.52
	3	16.87	22.30	22.28	22.31	17.05
	4	16.74	19.84	19.95	19.87	16.73
	5	16.36	18.65	18.83	18.68	16.25
	6	16.24	18.62	18.85	18.65	16.13
	7	16.00	18.31	18.51	18.33	15.80
	8	15.78	17.95	18.13	17.96	15.54
	9	15.68	17.63	17.78	17.62	15.40
	10	15.50	17.33	17.46	17.32	15.29
	11	15.17	17.02	17.15	16.99	15.08
	12	14.96	16.69	16.80	16.65	14.96
	13	14.60	15.83	15.91	15.78	14.48
	14	14.24	15.02	15.06	14.95	14.00
	15	13.92	14.31	14.34	14.24	13.59
	16	13.69	13.76	13.77	13.68	13.29
	17	13.50	13.40	13.34	13.26	13.03
	18	13.26	12.98	12.97	12.90	12.76
	19	13.10	12.68	12.69	12.60	12.52
	20	13.10	12.52	12.55	12.44	12.40
	21	13.12	12.38	12.40	12.31	12.24
	22	13.21	12.36	12.35	12.29	12.11
	23	13.30	12.28	12.26	12.21	11.92
	24	13.42	12.22	12.19	12.16	11.75
	Average	15.13	17.72	17.71	17.67	14.54

TABLE 4-11 MAPE for Different Linear Regression Models of the Test Bed 2 (North Area, Weekday Peak)

Time Period	Model	Reg. 1	Reg. 2	Reg. 3	Reg. 4	Reg. 5
Weekday Peak	1	16.77	50.61	50.57	49.27	20.04
	2	14.88	22.55	22.55	22.40	14.57
	3	13.77	16.19	16.19	16.41	13.51
	4	12.96	13.82	13.83	14.18	12.81
	5	12.41	12.87	12.87	13.31	12.27
	6	12.22	12.68	12.69	13.25	12.11
	7	12.09	12.40	12.40	12.92	11.97
	8	12.12	12.06	12.06	12.55	11.96
	9	12.16	11.76	11.76	12.21	11.96
	10	12.06	11.65	11.64	12.07	11.94
	11	11.80	11.64	11.63	12.03	11.81
	12	11.59	11.42	11.41	11.78	11.74
	13	11.52	11.24	11.24	11.56	11.82
	14	11.45	11.29	11.28	11.54	11.95
	15	11.31	11.53	11.52	11.76	12.04
	16	11.02	11.94	11.93	12.12	12.02
	17	10.38	12.38	12.37	12.45	11.68
	18	10.14	12.92	12.91	12.88	11.69
	19	9.99	13.65	13.64	13.47	11.74
	20	9.97	14.65	14.64	14.33	11.93
	21	9.89	15.85	15.83	15.37	12.10
	22	9.56	16.75	16.73	16.20	11.94
	23	9.07	17.28	17.26	16.67	11.49
	24	8.60	17.07	17.06	16.38	11.00
	Average	11.57	15.26	15.25	15.30	12.42

TABLE 4-12 MAPE for Different Linear Regression Models of the Test Bed 2 (North Area, Weekday Non-Peak)

Time Period	Model	Reg. 1	Reg. 2	Reg. 3	Reg. 4	Reg. 5
Weekday Non Peak	1	22.80	52.72	50.37	52.49	23.18
	2	19.73	28.46	27.58	28.68	20.07
	3	19.40	22.58	21.97	22.67	19.18
	4	19.08	19.95	19.50	19.96	18.52
	5	18.68	18.71	18.32	18.66	18.02
	6	18.42	18.41	18.10	18.38	17.77
	7	18.36	18.31	18.04	18.31	17.67
	8	18.28	18.19	18.01	18.25	17.60
	9	18.21	18.12	17.99	18.22	17.58
	10	18.05	17.96	17.87	18.09	17.50
	11	17.73	17.75	17.68	17.90	17.31
	12	17.39	17.56	17.51	17.72	17.12
	13	17.28	17.32	17.27	17.49	17.04
	14	17.22	17.23	17.23	17.40	17.03
	15	17.12	17.21	17.25	17.38	17.02
	16	16.95	17.24	17.31	17.38	16.95
	17	16.59	17.15	17.19	17.28	16.69
	18	16.45	17.17	17.21	17.32	16.62
	19	16.21	17.16	17.16	17.32	16.45
	20	16.10	17.32	17.29	17.49	16.37
	21	16.15	17.70	17.63	17.89	16.48
	22	16.11	18.02	17.87	18.21	16.45
	23	16.12	18.38	18.14	18.53	16.40
	24	16.15	18.30	17.97	18.37	16.30
	Average	17.69	19.96	19.69	20.06	17.56

TABLE 4-13 MAPE for Different Linear Regression Models of the Test Bed 2 (North Area, Weekday Evening)

Time Period	Model	Reg. 1	Reg. 2	Reg. 3	Reg. 4	Reg. 5
Weekday Evening	1	22.39	45.87	44.36	45.49	32.40
	2	17.57	26.23	27.83	26.08	25.46
	3	16.22	22.16	24.99	22.02	22.98
	4	15.78	20.24	23.87	20.10	21.40
	5	15.37	19.49	23.36	19.35	20.33
	6	14.94	19.28	22.85	19.13	19.33
	7	14.47	18.91	22.15	18.75	18.37
	8	14.14	18.55	21.44	18.38	17.61
	9	13.86	18.16	20.76	17.99	17.04
	10	13.51	17.64	20.10	17.49	16.56
	11	13.19	17.19	19.38	17.04	16.20
	12	12.91	16.49	18.45	16.33	15.76
	13	12.73	15.42	17.23	15.25	15.18
	14	12.61	14.46	16.11	14.28	14.74
	15	12.58	13.76	15.20	13.56	14.44
	16	12.73	13.37	14.54	13.14	14.27
	17	12.80	13.37	14.29	13.16	14.21
	18	12.52	12.87	13.61	12.70	13.78
	19	12.36	12.45	12.89	12.29	13.29
	20	12.25	12.22	12.30	12.11	12.83
	21	12.24	12.17	11.81	12.12	12.35
	22	12.26	12.32	11.53	12.28	11.96
	23	12.23	12.50	11.32	12.45	11.47
	24	12.51	12.55	11.40	12.49	11.44
	Average	13.92	17.40	18.82	17.25	16.81

TABLE 4-14 Best Regression of the Test Bed 1 (Downtown)

Model	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
	Reg. 1	Reg. 1	Reg. 5	Reg. 1	Reg. 1
1	33.94	35.09	24.31	31.41	28.91
2	27.56	24.66	20.03	28.72	25.18
3	25.07	22.29	16.08	25.58	22.50
4	23.11	20.57	13.88	21.74	17.65
5	22.68	19.87	12.99	21.39	16.19
6	22.03	19.80	11.23	19.85	16.03
7	21.97	19.87	10.63	19.32	16.03
8	22.40	20.30	9.90	19.84	17.25
Average	24.85	22.81	14.88	23.48	19.97

TABLE 4-15 Best Regression of the Test Bed 2 (North Area)

Model	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
	Reg. 5	Reg. 5	Reg. 1	Reg. 5	Reg. 1
1	21.25	19.10	16.77	23.18	22.39
2	18.69	17.52	14.88	20.07	17.57
3	17.85	17.05	13.77	19.18	16.22
4	17.18	16.73	12.96	18.52	15.78
5	16.56	16.25	12.41	18.02	15.37
6	16.29	16.13	12.22	17.77	14.94
7	16.01	15.80	12.09	17.67	14.47
8	15.77	15.54	12.12	17.60	14.14
9	15.61	15.40	12.16	17.58	13.86
10	15.45	15.29	12.06	17.50	13.51
11	15.20	15.08	11.80	17.31	13.19
12	14.97	14.96	11.59	17.12	12.91
13	14.71	14.48	11.52	17.04	12.73
14	14.50	14.00	11.45	17.03	12.61
15	14.32	13.59	11.31	17.02	12.58
16	14.10	13.29	11.02	16.95	12.73
17	13.96	13.03	10.38	16.69	12.80
18	13.80	12.76	10.14	16.62	12.52
19	13.58	12.52	9.99	16.45	12.36
20	13.47	12.40	9.97	16.37	12.25
21	13.37	12.24	9.89	16.48	12.24
22	13.26	12.11	9.56	16.45	12.26
23	13.14	11.92	9.07	16.40	12.23
24	13.03	11.75	8.60	16.30	12.51
Average	15.25	14.54	11.57	17.56	13.92

4.3 ARTIFICIAL NEURAL NETWORK (ANN) MODELS

4.3.1 Structure of Artificial Neural Network (ANN) Models

In this dissertation, a fully connected multilayer neural network model was chosen. The backpropagation neural network, arguably the most popular algorithm for transportation use, was adopted in this research. The ANN architecture used in the research has three layers: an input layer, a hidden layer, and an output layer. The structure of this three-layer ANN model is shown in FIGURE 4-2. A complete description of the ANN training and testing process may be found elsewhere (53).

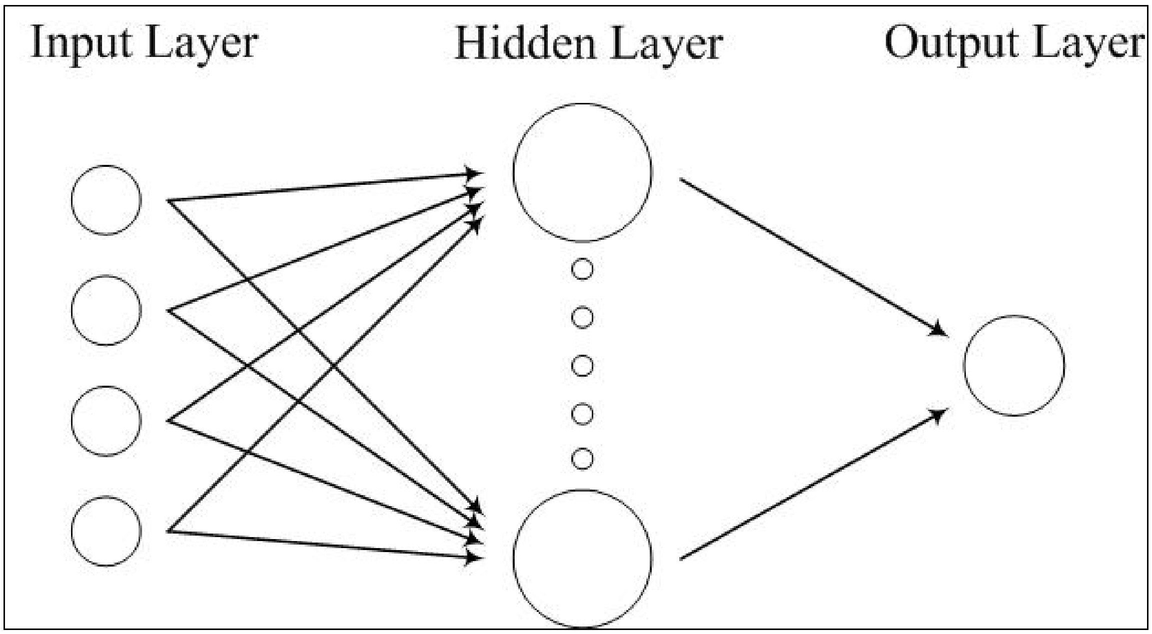

FIGURE 4-2 Input-Output Structure of the ANN Models

The hidden layer generates weight and bias parameter during the training process. The optimal values were based on minimizing prediction error. Initial parameters were randomly generated and these parameters influence on the prediction results. There were three types of parameters: the number of hidden neurons, weight, and bias parameters. In order to study the impact that the number of neurons had on the final result, a total of one to thirty neurons were tested. It was found that the number of neurons did not substantially impact on the result of ANN models. Therefore, in this dissertation, fifteen different number of neurons, from one to fifteen inclusive, were used to save running time.

4.3.2 Selecting Input Variables

Similar to the previous techniques, arrival time, dwell time, and schedule adherence were used as input, as shown in Equation 4-11.

$$\tau_{Mjkt} = f(A_{jkt}, W_{jkt}, S_{jkt}) \tag{4-11}$$

where,

τ_{Mjkt} = Travel time from current stop M to stop j for bus k departing during time period t, j = M, N_b;

A_{jkt} = Arrival time for bus k at bus stop j departing during time period t;

W_{jkt} = Dwell time of bus k at bus stop j departing during time period t; and

S_{jkt} = Schedule adherence of bus k at bus stop j departing during time period t Equal to observed arrival time at current stop M (A_{Mk}) minus scheduled arrival Time.

In this section, different combinations of the three input variables are tested. To select input variables for Artificial Neural Network Models, seven different scenarios were conducted. There were three input variables; arrival time, dwell time, and schedule adherence. Model I to III used each of the three variables, Model IV to VI used a pair of variables, and Model VII used all three variables. TABLE 4-16 and TABLE 4-17 show the MAPE results of these seven models.

For the downtown area, Model VII shows the best results. In the north area, however, Model V shows better results than Model VII. It should be noted that in the downtown test bed Model V had, on average, 1.38 percent higher MAPE value. This small difference in results means that dwell time is not very useful for predicting bus arrival time at this site. Choosing variables is highly dependent on the characteristics of each site and driver behavior. TABLE 4-18 shows that running time for Model V and Model

VII are, on average, within 4.99 percent of each other. This means that using three variables as opposed to two does not appreciably effect computation time. Consequently in this dissertation, all three variables are used to develop the artificial neural network models.

TABLE 4-16 MAPE of Artificial Neural Network Models of the Test Bed 1 (Downtown, Non-Clustering)

Model	I	II	III	IV	V	VI	VII
	AT	DT	SA	AT, DT	AT, SA	DT, SA	AT,DT, SA
1	34.37	25.78	14.11	26.00	14.09	12.91	13.84
2	19.59	22.44	11.04	19.58	5.35	10.05	6.10
3	18.01	21.38	10.50	17.91	4.61	9.58	4.92
4	17.43	20.61	9.86	17.42	4.55	9.86	3.62
5	12.05	16.12	9.04	9.27	4.09	9.97	3.50
6	7.46	15.84	8.59	7.98	3.04	10.49	3.46
7	5.20	15.63	9.09	4.79	3.10	9.95	2.63
8	4.06	14.52	9.05	3.90	2.30	7.73	2.48
Average	14.77	19.04	10.16	13.36	5.14	10.06	5.07

AT: Arrival time;

DT: Dwell time;

SA: Schedule Adherence.

TABLE 4-17 MAPE of Artificial Neural Network Models of the Test Bed 2 (North Area, Non-Clustering)

Model	I AT	II DT	III SA	IV AT, DT	V AT, SA	VI DT, SA	VII AT,DT, SA
1	20.03	19.94	10.65	19.92	11.78	10.72	12.35
2	15.29	15.68	8.58	14.68	6.81	8.70	8.31
3	13.36	15.06	7.76	13.91	5.14	9.12	6.40
4	13.33	15.08	7.70	13.42	5.26	8.48	6.05
5	12.40	14.59	6.51	13.32	5.37	7.66	5.02
6	10.47	13.64	6.19	10.42	4.15	7.06	4.75
7	9.32	12.92	5.88	9.99	1.58	6.63	1.73
8	9.06	13.29	5.32	9.45	1.50	6.23	1.64
9	8.80	14.08	5.56	10.23	1.32	6.16	1.82
10	7.79	13.86	5.29	8.34	0.95	5.99	1.84
11	7.71	13.78	4.91	8.30	0.97	5.69	1.72
12	7.48	13.87	4.60	8.23	1.04	5.97	1.70
13	6.02	13.43	4.65	6.62	0.92	6.16	1.84
14	6.27	13.19	4.96	6.61	0.83	5.92	1.45
15	5.81	12.94	4.45	6.73	0.78	5.66	1.45
16	5.93	15.18	4.54	6.87	0.70	5.41	1.61
17	5.21	12.95	4.35	5.91	0.62	5.20	1.18
18	4.74	12.40	4.38	4.30	0.79	4.82	1.28
19	3.70	11.97	4.22	3.83	0.78	4.65	1.52
20	3.42	12.90	3.90	3.57	0.70	4.40	1.16
21	3.21	12.09	3.83	3.42	0.27	4.16	1.34
22	3.09	12.27	3.69	3.60	0.52	3.97	1.12
23	2.49	15.77	3.68	2.52	0.26	3.49	0.70
24	1.56	14.32	3.48	2.16	0.19	3.71	1.08
Average	7.77	13.97	5.38	8.18	2.22	6.08	2.88

TABLE 4-18 Running Time of Artificial Neural Network Models

Model	I	II	III	IV	V	VI	VII
Area	AT	DT	SA	AT, DT	AT, SA	DT, SA	AT,DT, SA
Downtown	7 min	13 min	**27 min**	13 min	23 min	**28 min**	21 min
North Area	4 hours 31 min	7 hours 33 min	**19 hours 57 min**	7 hours 38 min	11 hours 1 min	**19 hours 44 min**	11 hours 34 min

4.3.3 Selecting the Training Functions

Twelve different training functions were tested with the downtown and the north area data and these are listed in TABLE 4-19. TABLE 4-20 and TABLE 4-22 show the running time by training function for the downtown area and the north area, respectively. TABLE 4-21 and TABLE 4-23 show the average MAPE by training function for the downtown area and the north area, respectively. In Appendix B, TABLE B-1 through TABLE B-12 (for the downtown area) and TABLE B-13 through TABLE B-24 (for the north area) show the MAPE of eight models and average MAPE by training function.

According to TABLE 4-21 and TABLE 4-23, the Bayesian Regularization training function and the Levenberg-Marquardt Backpropagation training function outperformed the other eleven training functions. The average MAPE of the Bayesian Regularization training function was approximately twelve percent lower than that of the Levenberg-Marquardt Backpropagation training function for the downtown area, and 55 percent for the north area. It may be seen in TABLE 4-20 and TABLE 4-22, the running time of the Bayesian Regularization training function was 102 minutes for the downtown area and 63 hours 7 minutes for the north area while the running time of the Levenberg-Marquardt Backpropagation training function was 40 minutes for downtown and 19 hours 7 minutes for the north area. The running time of the Bayesian Regularization training function was approximately 255 percent higher than that of the Levenberg-Marquardt Backpropagation training function for the downtown area and 330 percent for the north area. Therefore, the Levenberg-Marquardt Backpropagation training function was chosen as the best training function for this research in terms of efficiency and accuracy.

TABLE 4-19 Lists of Training Functions

No	Function
1	Batch Training with Weight and Bias Learning Rule
2	BFGS Quasi-Newton Backpropagation
3	Bayesian Regularization
4	Powell-Beale Conjugate Gradient Backpropagation
5	Fletcher-Powell Conjugate Gradient Backpropagation
6	Gradient Descent Backpropagation
7	Gradient Descent with Adaptive Learning Rate Backpropagation
8	Levenberg-Marquardt Backpropagation
9	One Step Secant Backpropagations
10	Resilient Backpropagation
11	Sequential Order Incremental Update
12	Scaled Conjugate Gradient Backpropagation

TABLE 4-20 Running Time by Training Function of the Test Bed 1 (Downtown)

No	Function	Running time
1	Batch Training with Weight and Bias Learning Rule	6 min
2	BFGS Quasi-Newton Backpropagation	29 min
3	Bayesian Regularization	102 min
4	Powell-Beale Conjugate Gradient Backpropagation	13 min
5	Fletcher-Powell Conjugate Gradient Backpropagation	12 min
6	Gradient Descent Backpropagation	5 min
7	Gradient Descent with Adaptive Learning Rate Backpropagation	6 min
8	Levenberg-Marquardt Backpropagation	40 min
9	One Step Secant Backpropagations	11 min
10	Resilient Backpropagation	5 min
11	Sequential Order Incremental Update	2 min
12	Scaled Conjugate Gradient Backpropagation	10 min

* This running time includes training, testing, and calculation of prediction errors

TABLE 4-21 Average MAPE of Different Training Functions of the Test Bed 1 (Downtown)

No	Functions	Average MAPE				
		Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
1	Batch Training with Weight and Bias Learning Rule	109.46	134.35	146.59	136.10	118.91
2	BFGS Quasi-Newton Backpropagation	12.28	11.11	9.80	12.96	11.19
3	Bayesian Regularization	5.09	4.12	4.82	7.88	5.08
4	Fletcher-Powell Conjugate Gradient Backpropagation	11.64	11.55	9.99	13.84	10.38
5	Powell-Beale Conjugate Gradient Backpropagation	12.94	11.49	9.29	13.92	10.60
6	Gradient Descent Backpropagation	146.68	156.29	163.00	145.96	144.74
7	Gradient Descent with Adaptive Learning Rate Backpropagation	27.95	29.02	25.79	26.53	26.65
8	Levenberg-Marquardt Backpropagation	5.18	4.28	5.33	8.65	7.18
9	One Step Secant Backpropagations	14.83	13.17	12.16	15.73	13.34
10	Resilient Backpropagation	12.66	11.92	10.26	15.24	12.74
11	Sequential Order Incremental Update	294.74	309.56	301.65	265.92	185.52
12	Scaled Conjugate Gradient Backpropagation	13.17	11.96	10.67	13.96	11.77

TABLE 4-22 Running Time by Training Function of the Test Bed 2 (North Area)

No	Function	Running time
1	Batch Training with Weight and Bias Learning Rule	20 min
2	BFGS Quasi-Newton Backpropagation	10 hr 23 min
3	Bayesian Regularization	63 hr 7 min
4	Powell-Beale Conjugate Gradient Backpropagation	45 min
5	Fletcher-Powell Conjugate Gradient Backpropagation	40 min
6	Gradient Descent Backpropagation	18 min
7	Gradient Descent with Adaptive Learning Rate Backpropagation	19 min
8	Levenberg-Marquardt Backpropagation	19 hr 7 min
9	One Step Secant Backpropagations	44 min
10	Resilient Backpropagation	19 min
11	Sequential Order Incremental Update	8 min
12	Scaled Conjugate Gradient Backpropagation	26 min

* This running time includes training, testing, and calculation of prediction errors

TABLE 4-23 Average MAPE of Different Training Functions

No	Functions	Average MAPE				
		Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
1	Batch Training with Weight and Bias Learning Rule	213.88	229.69	216.61	238.56	225.36
2	BFGS Quasi-Newton Backpropagation	8.62	7.59	7.38	8.80	8.53
3	Bayesian Regularization	2.25	0.90	1.34	2.43	1.60
4	Fletcher-Powell Conjugate Gradient Backpropagation	8.07	6.81	6.55	8.01	7.87
5	Powell-Beale Conjugate Gradient Backpropagation	9.21	8.40	7.21	8.92	9.17
6	Gradient Descent Backpropagation	219.58	234.88	219.69	243.25	228.73
7	Gradient Descent with Adaptive Learning Rate Backpropagation	31.77	33.32	33.36	30.03	35.45
8	Levenberg-Marquardt Backpropagation	2.77	1.93	4.68	4.39	5.09
9	One Step Secant Backpropagations	11.18	9.80	8.29	11.29	11.21
10	Resilient Backpropagation	8.05	8.02	6.95	8.76	8.81
11	Sequential Order Incremental Update	316.69	314.69	280.97	298.85	297.30
12	Scaled Conjugate Gradient Backpropagation	10.06	8.83	7.68	9.35	9.56

4.3.4 Selecting the Learning Functions

In this section, fourteen different learning functions were tested. TABLE 4-24 lists the name of the fourteen learning functions. TABLE 4-25 and TABLE 4-27 show the running time by learning functions for the downtown area and the north area, respectively. TABLE 4-26 and TABLE 4-28 show the average MAPE by learning function for the downtown area and the north area, respectively. In Appendix B, TABLE B-25 to TABLE B-38 and TABLE B-39 to TABLE B-52 show the average MAPE by learning function.

TABLE 4-26 and TABLE 4-28 show the average MAPE for each of the fourteen learning functions tested for the downtown area and the north area, respectively. It can be seen that the results from these functions were not significantly different. For example, the average difference was only 0.04 percent. However, the Perceptron Weight and Bias Learning Function outperformed the other thirteen learning functions by approximately 5.30 percent for the test bed 1, the downtown area. According to TABLE 4-25 and TABLE 4-27, the running times of these fourteen learning functions are almost the same. Consequently, the Perceptron Weight and Bias Learning Function was chosen for this research.

TABLE 4-24 Lists of Training Functions

No	Function
1	Conscience Bias Learning Function
2	Gradient Descent Weight/bias Learning Function
3	Gradient Descent with Momentum Weight/bias Learning Function
4	Hebb Weight Learning Function
5	Hebb with Decay Weight Learning Function
6	Instar Weight Learning Function
7	Kohonen Weight Learning Function
8	LVQ1 Weight Learning Function
9	LVQ2 Weight Learning Function
10	Outstar Weight Learning Function
11	Perceptron Weight and Bias Learning Function
12	Normalized Perceptron Weight and Bias Learning Function
13	Self-organizing Map Weight Learning Function
14	Widrow-Hoff Weight and Bias Learning Rule

TABLE 4-25 Running Time by Learning Functions of the Test Bed 1 (Downtown)

No	Function	Running time
1	Conscience Bias Learning Function	40 min
2	Gradient Descent Weight/bias Learning Function	39 min
3	Gradient Descent with Momentum Weight/bias Learning Function	41 min
4	Hebb Weight Learning Function	40 min
5	Hebb with Decay Weight Learning Function	41 min
6	Instar Weight Learning Function	41 min
7	Kohonen Weight Learning Function	41 min
8	LVQ1 Weight Learning Function	42 min
9	LVQ2 Weight Learning Function	43 min
10	Outstar Weight Learning Function	41 min
11	Perceptron Weight and Bias Learning Function	42 min
12	Normalized Perceptron Weight and Bias Learning Function	40 min
13	Self-organizing Map Weight Learning Function	39 min
14	Widrow-Hoff Weight and Bias Learning Rule	41 min

TABLE 4-26 Average MAPE of Different Learning Functions for the Test Bed 1 (Downtown)

No	Functions	Average MAPE by the Time period					
		Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening	Average
1	Conscience Bias Learning Function	5.28	4.14	5.91	8.05	7.57	6.19
2	Gradient Descent Weight/bias Learning Function	5.19	4.03	5.26	8.61	7.33	6.08
3	Gradient Descent with Momentum Weight/bias Learning Function	5.16	5.20	5.20	8.37	8.24	6.43
4	Hebb Weight Learning Function	5.17	4.55	5.48	8.51	7.50	6.24
5	Hebb with Decay Weight Learning Function	4.72	4.25	5.15	8.66	8.00	6.16
6	Instar Weight Learning Function	5.33	4.32	5.63	8.97	7.66	6.38
7	Kohonen Weight Learning Function	5.27	3.85	5.16	8.58	7.82	6.14
8	LVQ1 Weight Learning Function	5.18	4.28	5.33	8.65	7.18	6.12
9	LVQ2 Weight Learning Function	5.31	4.33	5.30	8.79	7.77	6.30
10	Outstar Weight Learning Function	5.19	4.21	5.18	8.62	7.60	6.16
11	Perceptron Weight and Bias Learning Function	5.13	4.31	5.04	7.87	7.17	5.90
12	Normalized Perceptron Weight and Bias Learning Function	5.13	4.70	5.22	8.16	8.61	6.36
13	Self-organizing Map Weight Learning Function	4.98	4.25	5.30	8.26	7.42	6.04
14	Widrow-Hoff Weight and Bias Learning Rule	5.04	4.55	5.48	8.57	8.65	6.46

TABLE 4-27 Running Time by Learning Function of the Test Bed 2 (North Area)

No	Function	Running time
1	Conscience Bias Learning Function	19hr 24min
2	Gradient Descent Weight/bias Learning Function	19hr 16min
3	Gradient Descent with Momentum Weight/bias Learning Function	19hr 21min
4	Hebb Weight Learning Function	19hr 31min
5	Hebb with Decay Weight Learning Function	19hr 59min
6	Instar Weight Learning Function	19hr 23min
7	Kohonen Weight Learning Function	19hr 32min
8	LVQ1 Weight Learning Function	19hr 20min
9	LVQ2 Weight Learning Function	19hr 42min
10	Outstar Weight Learning Function	19hr 45min
11	Perceptron Weight and Bias Learning Function	19hr 18min
12	Normalized Perceptron Weight and Bias Learning Function	19hr 25min
13	Self-organizing Map Weight Learning Function	19hr 31min
14	Widrow-Hoff Weight and Bias Learning Rule	19hr 19min

TABLE 4-28 Average MAPE of Different Learning Functions for the Test Bed 2 (North area)

No	Functions	Average MAPE by the Time period					
		Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening	Average
1	Conscience Bias Learning Function	2.84	1.97	4.42	4.10	5.06	3.68
2	Gradient Descent Weight/bias Learning Function	2.83	1.99	4.25	4.42	5.21	3.74
3	Gradient Descent with Momentum Weight/bias Learning Function	2.75	1.97	4.54	4.58	5.06	3.78
4	Hebb Weight Learning Function	2.84	1.95	4.39	4.20	5.05	3.69
5	Hebb with Decay Weight Learning Function	2.87	1.95	4.43	4.42	5.07	3.75
6	Instar Weight Learning Function	2.75	1.91	4.35	4.53	5.11	3.73
7	Kohonen Weight Learning Function	2.82	1.98	4.42	4.51	5.22	3.79
8	LVQ1 Weight Learning Function	2.84	1.92	4.43	4.51	5.17	3.77
9	LVQ2 Weight Learning Function	2.90	1.94	4.26	4.30	5.26	3.73
10	Outstar Weight Learning Function	2.77	2.05	4.16	4.64	5.05	3.73
11	Perceptron Weight and Bias Learning Function	2.82	1.92	4.39	4.36	5.12	3.72
12	Normalized Perceptron Weight and Bias Learning Function	2.83	1.96	4.48	4.24	5.28	3.76
13	Self-organizing Map Weight Learning Function	2.89	1.93	4.50	4.40	5.14	3.77
14	Widrow-Hoff Weight and Bias Learning Rule	2.77	1.96	4.27	4.08	5.11	3.64

4.3.5 Number of Neurons

Fifteen different numbers of hidden neurons, from one to fifteen inclusive, were tested. TABLE 4-29 and FIGURE 4-3 show the MAPE by the different number of hidden neurons. After the ANN models were tested with the fifteen different neurons, the best number of neuron was selected for each ANN model. Consequently, the downtown area, as an example, could have a different number of neurons than that of the north area and the weekend time period could have a different number of neurons than that of weekday peak time period. It should be noted that the results from the fifteen different neurons were very similar so for this test bed the results would not be appreciably different if the number of neurons were the same for all models.

TABLE 4-29 Average MAPE by Different Number of Neurons

No. of Neurons	model 1	model 2	model 3	model 4	model 5	model 6	model 7	model 8
1	24.17	13.55	13.22	7.79	7.55	5.79	4.40	3.14
2	17.52	9.76	8.41	7.22	4.12	4.16	3.67	3.02
3	16.83	7.92	8.34	7.37	6.07	3.61	2.79	5.15
4	16.46	10.25	4.70	5.95	4.73	5.45	3.97	4.68
5	17.24	10.37	5.76	5.71	6.41	4.51	5.63	3.32
6	15.03	6.13	7.76	5.34	2.77	6.75	5.40	3.78
7	14.76	5.58	5.37	4.38	5.90	4.11	3.89	3.90
8	19.53	8.13	6.96	7.11	5.64	3.85	2.60	4.30
9	15.30	5.94	4.72	4.36	6.55	6.87	3.38	4.07
10	15.77	7.32	4.68	6.99	7.72	5.04	3.26	3.77
11	15.92	7.14	7.15	6.38	5.67	4.37	3.95	3.05
12	17.89	6.30	6.74	5.99	9.68	3.96	3.79	4.18
13	17.92	6.14	4.33	10.84	5.65	5.37	3.50	6.10
14	21.78	5.62	6.46	8.03	6.69	4.57	2.71	4.76
15	22.16	6.66	4.72	4.83	5.80	4.47	3.96	7.58

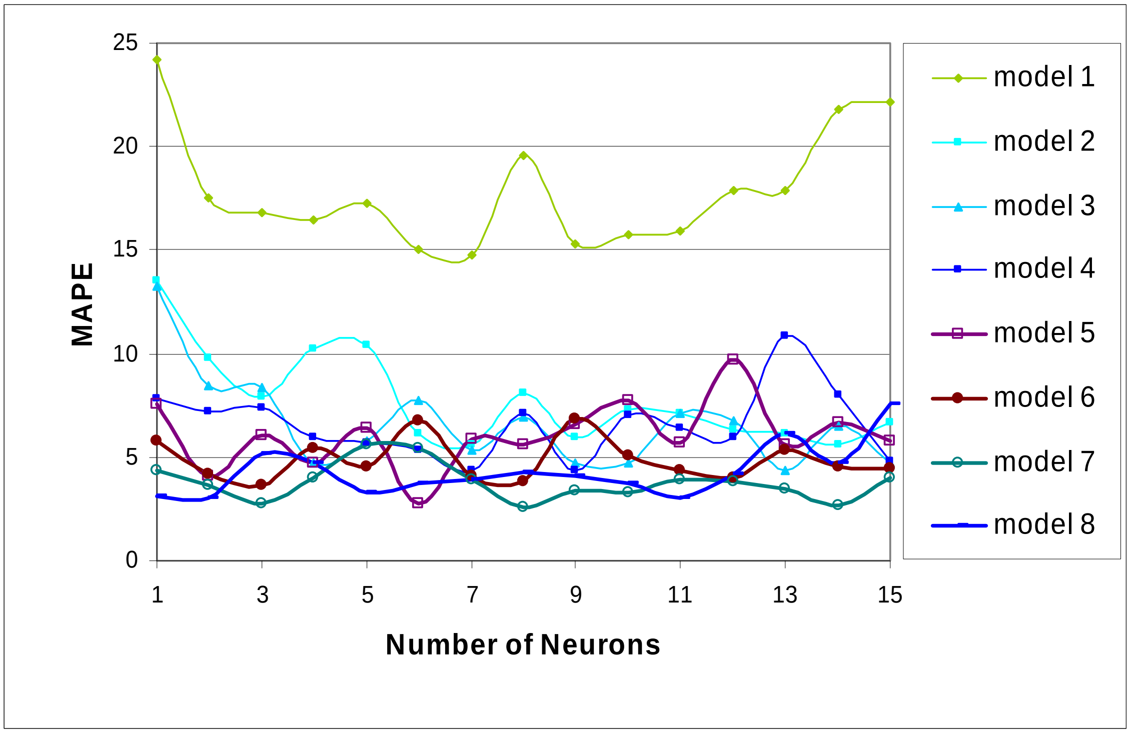

FIGURE 4-3 Average MAPE by Different Number of Neurons

4.4 CONCLUDING REMARKS

In this chapter, three prediction models were developed. The models includes historical data based models, multi linear regression models, and artificial neural network models. For all three, the input data are arrival time, dwell time, and schedule adherence at each stop. However, link travel time was used instead of arrival time for the historical data based model. For regression models, it was found that dwell time was not statistically significant and the arrival time was highly inter-correlated with other variables. Consequently, distance from current stop to each stop and schedule adherence at each stop were used as the independent variables. In case of ANN models, three layer feedforward ANN models were used. Backpropagation with the Levenberg-Marquardt training function and the Perceptron Weight and Bias learning function were used for the training process. Fifteen different hidden neurons were also used and the best neuron resulting in the smallest prediction error was used to predict bus arrival time.

In chapter V, these three prediction models will be evaluated and statistically tested in order to identify the best modeling approach for the test bed.

CHAPTER V

MODEL EVALUATION

Three different models for bus arrival time prediction were developed in the previous chapter: historical data based models, multi linear regression models, and artificial neural network models. In this chapter, these models are compared in terms of the accuracy of predicted bus arrival time discussed in the previous section. In addition to this, these three different models are tested statistically.

5.1 MAPE OF THE HISTORICAL DATA BASED MODELS

TABLE 5-1 and TABLE 5-2 show the historical model MAPE for each of the eight (for the downtown area) and 24 models (for the north area) of the five clustering options. It can be seen that the MAPE decreases as the prediction time decreases. For example, the MAPE for model 1 of the downtown area, which predicts the arrival time at the next 8 stops, is considerably higher than for model 8, which predicts the arrival time at the last stop, all other things being equal. It can also be seen that the clustering in the data leads to a smaller MAPE. For example, the average MAPE of the downtown area over all eight models for any of the clustering groups is smaller than for the non-clustering MAPE. The average MAPE of the north area over all eight models for three of the clustering groups is smaller than the non-clustering MAPE. The only exception is the weekday non-peak period which had a higher MAPE. The fact that clustering data gave better results would be expected because the clustering explicitly accounts for different levels of congestion and demand levels associated with different times of the day. Interestingly, the lowest MAPE of the downtown area was for the weekday peak. It is hypothesized that congestion reduces variability in travel times and that this makes the historic model more accurate for this time period. It is important to note, however, that the overall error rate is still relatively high. For example, during the weekday peak in the downtown area, the average prediction error at transit stop 1 would still be on the order of twenty-five percent.

TABLE 5-1 MAPE for Historical Data Based Models of the Test Bed 1 (Downtown)

Model	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
1	33.94	35.93	25.22	31.15	24.63
2	19.73	15.89	12.03	21.24	16.30
3	17.19	13.87	10.87	15.96	16.99
4	17.09	13.05	9.15	17.07	15.16
5	11.60	9.54	7.91	11.74	15.51
6	8.31	6.71	9.88	9.15	6.84
7	5.32	5.10	6.63	4.74	4.50
8	3.84	4.12	4.51	3.65	2.63
Average	14.63	13.03	10.78	14.34	12.82

TABLE 5-2 MAPE for Historical Data Based Models of the Test Bed 2 (North Area)

Model	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
1	20.02	18.22	16.63	21.60	19.15
2	16.51	15.59	11.98	17.96	16.42
3	13.29	13.91	11.44	14.98	8.57
4	12.69	13.26	11.41	14.54	8.85
5	12.23	12.65	10.77	14.52	8.68
6	10.68	11.52	10.37	11.47	8.38
7	9.20	9.18	6.96	10.88	7.45
8	8.92	8.77	7.05	10.77	7.70
9	8.85	8.45	7.35	10.64	7.38
10	7.71	7.92	6.80	9.07	5.04
11	7.57	7.48	6.54	9.21	5.10
12	7.12	7.24	7.40	8.15	5.19
13	5.45	4.33	6.40	6.75	5.47
14	5.46	4.43	7.10	6.43	5.53
15	5.42	4.22	7.39	6.44	6.46
16	5.41	4.24	7.89	6.17	7.01
17	4.79	4.08	4.66	4.62	7.96
18	4.32	4.02	4.81	5.02	3.56
19	3.50	3.16	3.46	4.37	2.10
20	3.35	3.13	3.48	4.30	1.91
21	3.18	2.98	2.94	4.15	1.46
22	2.65	2.39	2.48	3.47	1.45
23	2.21	2.02	1.96	2.58	2.25
24	1.39	1.24	1.22	1.69	1.30
Average	7.58	7.27	7.02	8.74	6.43

FIGURE 5-1 and FIGURE 5-2 show the tendency for the MAPE results to decrease as prediction time decreases. Both the downtown area and the north area had similar tendencies. However, the MAPE of the downtown area shown in FIGURE 5-1 is higher than that of the north area shown in FIGURE 5-2. In chapter 4, it was found that the values of input variables for the downtown area had more variability than those of the north area. Therefore, the larger MAPE of the downtown area is a reasonable result. In other words, more variable traffic conditions due to traffic congestion leads to more prediction error.

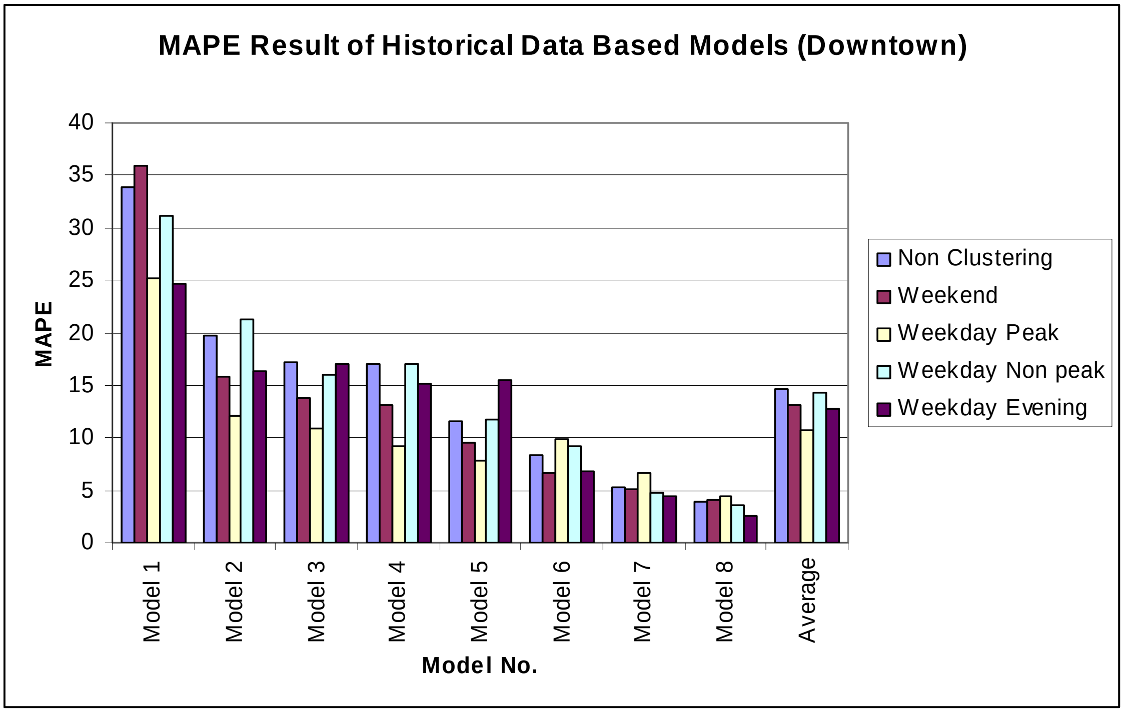

FIGURE 5-1 MAPE Result of Historical Data Based Models of the Test Bed 1 by Time Period (Downtown)

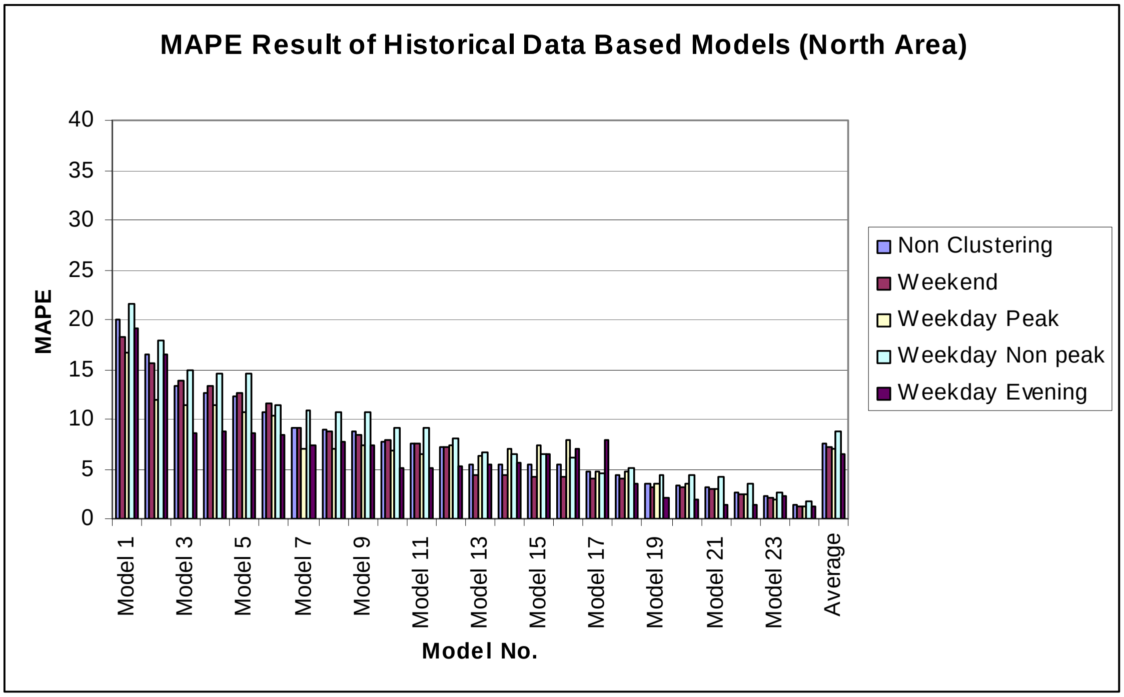

FIGURE 5-2 MAPE Result of Historical Data Based Models of the Test Bed 2 by Time Period (North Area)

FIGURE 5-3 and FIGURE 5-4 show that clustering data leads to more precise prediction results. Both the downtown area and the north area had similar patterns. However, the MAPE of the downtown area shown in FIGURE 5-2 is higher than that of the north area shown in FIGURE 5-4. It is expected that clustering the data results in less variability in the input variables, which in turn, results in more precise prediction results.

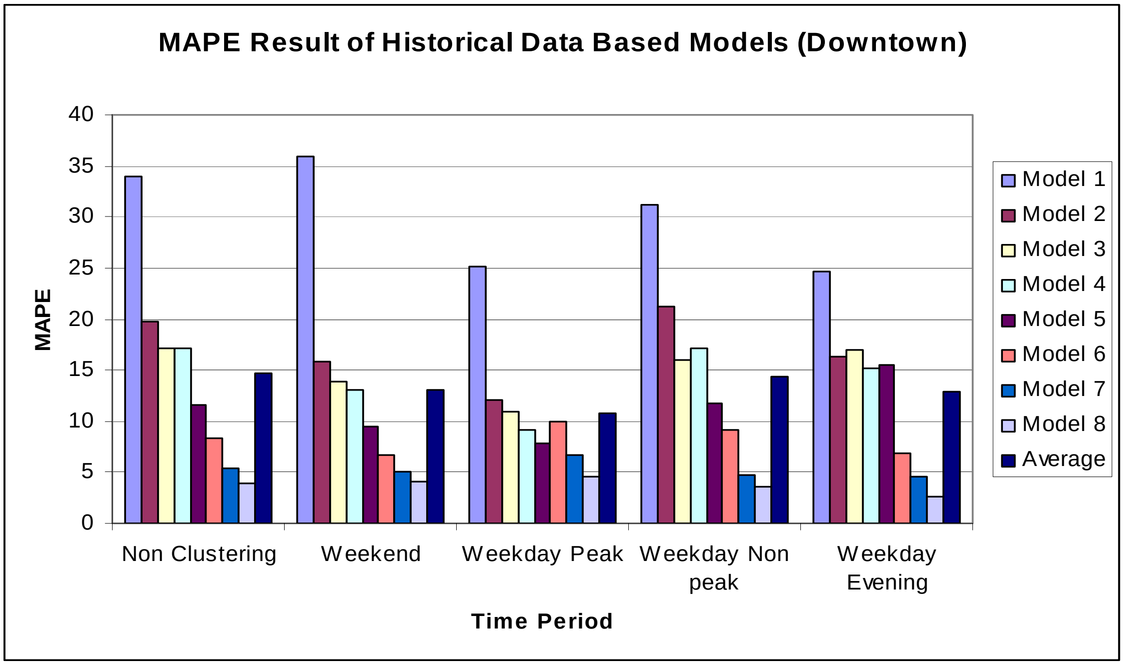

FIGURE 5-3 MAPE Result of Historical Data Based Models of the Test Bed 1 by Model (Downtown)

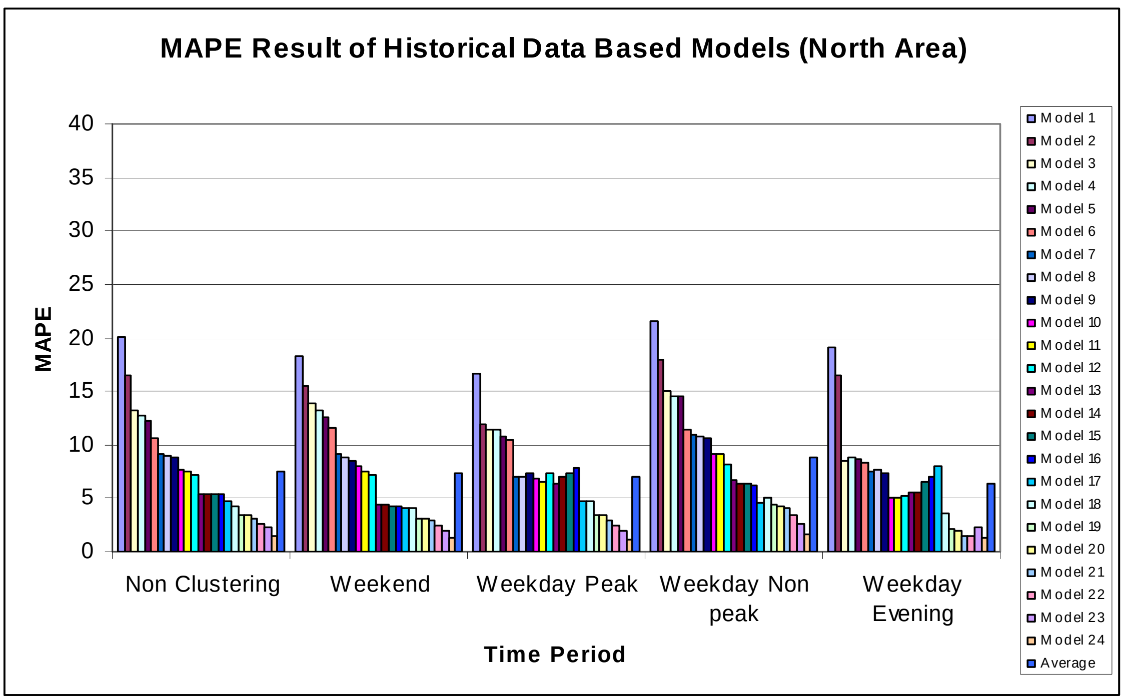

FIGURE 5-4 MAPE Result of Historical Data Based Models of the Test Bed 2 by Model (North Area)

5.2 THE MAPE OF REGRESSION MODELS

The MAPE results for the Multiple Linear Regression (MLR) models are shown in TABLE 5-3 and TABLE 5-4. The results for the five model specifications for each of the eight model types (for the downtown area) and the 24 model types (for the north area) are shown for each of the five clustering options. Similar to the historic model analysis, the MLR regression results tend to improve as prediction time decreases. In addition, the clustering results provided better results than the non-clustering approach. Interestingly, the lowest MAPE of the regression models of both the downtown area and the north area was for the weekday peak. It is hypothesized that the congestion reduces the variability in travel times and this makes the models more accurate for this time period. The use of real-time schedule adherence data did not significantly improve the results. It is hypothesized that there is a non-linear relationship between arrival time and schedule adherence and this caused the relatively poor results. In addition, the various non-linear model specifications were unable to capture this phenomenon. For this test bed the historic model gave superior results, in terms of MAPE, in comparison to the MLR results.

TABLE 5-3 MAPE of Multi Linear Regression Models of the Test Bed 1 (Downtown)

Model	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
	Reg. 1	Reg. 1	Reg. 5	Reg. 1	Reg. 1
1	33.94	35.09	24.31	31.41	28.91
2	27.56	24.66	20.03	28.72	25.18
3	25.07	22.29	16.08	25.58	22.50
4	23.11	20.57	13.88	21.74	17.65
5	22.68	19.87	12.99	21.39	16.19
6	22.03	19.80	11.23	19.85	16.03
7	21.97	19.87	10.63	19.32	16.03
8	22.40	20.30	9.90	19.84	17.25
Average	24.85	22.81	14.88	23.48	19.97

TABLE 5-4 MAPE of Multi Linear Regression Models of the Test Bed 2 (North Area)

Model	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
	Reg. 5	Reg. 5	Reg. 1	Reg. 5	Reg. 1
1	21.25	19.10	16.77	23.18	22.39
2	18.69	17.52	14.88	20.07	17.57
3	17.85	17.05	13.77	19.18	16.22
4	17.18	16.73	12.96	18.52	15.78
5	16.56	16.25	12.41	18.02	15.37
6	16.29	16.13	12.22	17.77	14.94
7	16.01	15.80	12.09	17.67	14.47
8	15.77	15.54	12.12	17.60	14.14
9	15.61	15.40	12.16	17.58	13.86
10	15.45	15.29	12.06	17.50	13.51
11	15.20	15.08	11.80	17.31	13.19
12	14.97	14.96	11.59	17.12	12.91
13	14.71	14.48	11.52	17.04	12.73
14	14.50	14.00	11.45	17.03	12.61
15	14.32	13.59	11.31	17.02	12.58
16	14.10	13.29	11.02	16.95	12.73
17	13.96	13.03	10.38	16.69	12.80
18	13.80	12.76	10.14	16.62	12.52
19	13.58	12.52	9.99	16.45	12.36
20	13.47	12.40	9.97	16.37	12.25
21	13.37	12.24	9.89	16.48	12.24
22	13.26	12.11	9.56	16.45	12.26
23	13.14	11.92	9.07	16.40	12.23
24	13.03	11.75	8.60	16.30	12.51
Average	15.25	14.54	11.57	17.56	13.92

FIGURE 5-5 and FIGURE 5-6 show the MAPE results of the regression models by model for the downtown area and the north area, respectively. Similar to the historical data based model analysis, the MAPE tends to increase as prediction time increase. However, this tendency is not strong in comparison to the historical data based models. Similar to the previous analysis, the MAPE of the downtown area is higher than that of the north area.

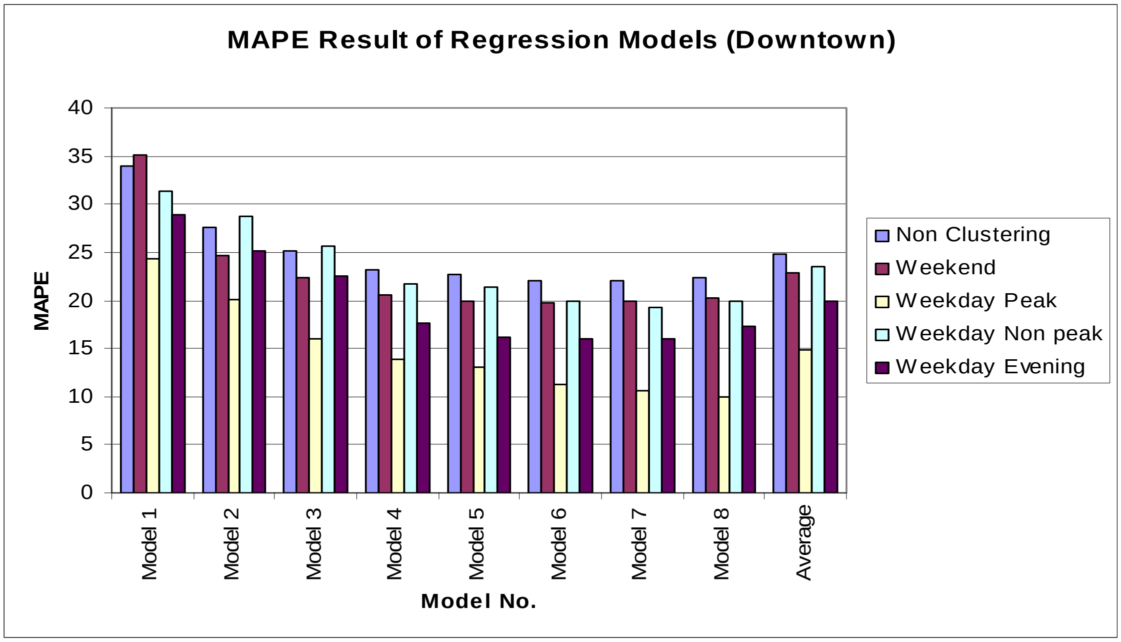

FIGURE 5-5 MAPE Result of Regression Models of the Test Bed 1 by Time Period (Downtown)

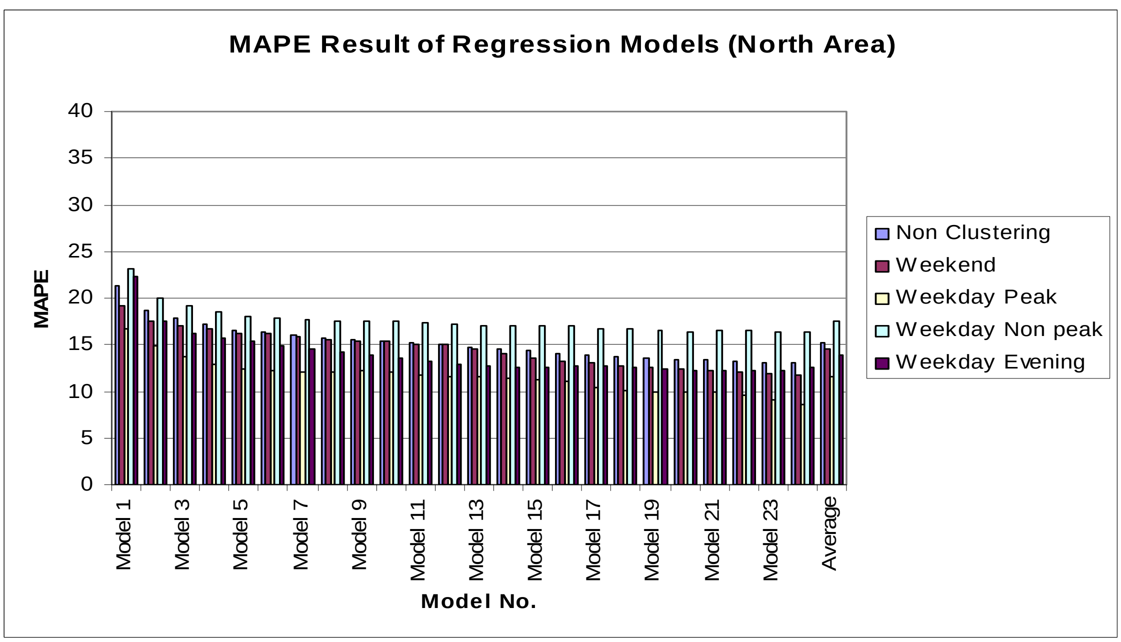

FIGURE 5-6 MAPE Result of Regression Models of the Test Bed 2 by Time Period (North Area)

FIGURE 5-7 and FIGURE 5-8 show that clustering data leads to more precise prediction results. It is expected that clustering data have less variability in input variables resulting in more precise prediction results. However, the MAPE of the weekday non-peak period is the largest one in the north area, while the MAPE of the non-clustering data set is larger than that of the other four clustering data set in the downtown area.

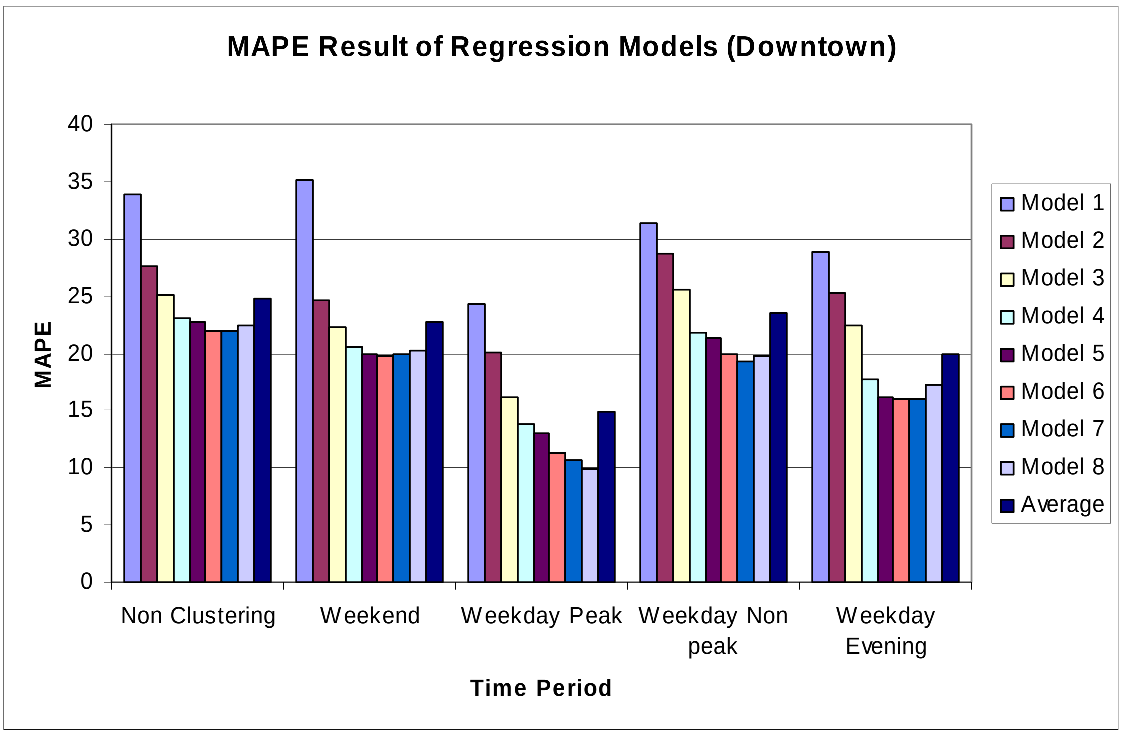

FIGURE 5-7 MAPE Result of Regression Models of the Test Bed 1 by Model (Downtown)

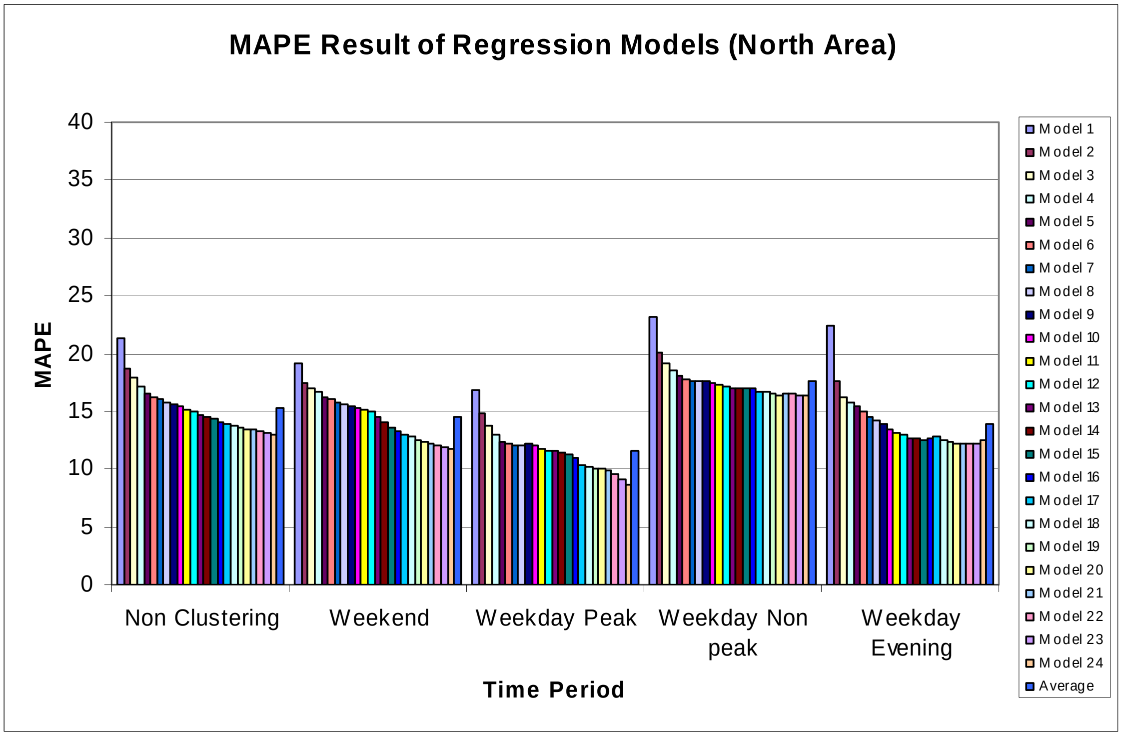

FIGURE 5-8 MAPE Result of Regression Models of the Test Bed 2 by Model (North Area)

5.3 MAPE OF ARTIFICIAL NEURAL NETWORK MODELS

TABLE 5-5 and TABLE 5-6 show the MAPE results for the optimal ANN model for each of the eight model types (for the downtown area) and the 24 model types (for the north area) of five clustering options. Similar to the previous two models the MAPE decreased as prediction time decreased. In contrast to the previous two techniques, the clustering resulted in poorer results than the non-clustering option. It is hypothesized that the ANN, as a universal function approximator, was able to identify the non-linear relationships associated with the different clusters. While in general the clustering should not do worse than the non-clustering option, it is hypothesized that there may not have been enough observations to adequately fit the functions. If more observations were available the results of the two approaches might have been more similar.

TABLE 5-5 MAPE of Artificial Neural Network Models of the Test Bed 1 (Downtown)

Model	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
1	14.76	10.79	7.11	17.96	11.04
2	5.58	4.11	5.75	7.65	9.92
3	4.33	4.15	5.79	7.86	8.70
4	4.36	3.73	5.01	7.07	7.64
5	2.77	3.05	5.14	6.82	6.30
6	3.61	2.52	3.70	5.52	4.70
7	2.60	3.21	3.51	5.58	4.29
8	3.02	2.91	4.30	4.52	4.74
Average	5.13	4.31	5.04	7.87	7.17

TABLE 5-6 MAPE of Artificial Neural Network Models of the Test Bed 2 (North Area)

Model	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
1	11.77	4.93	8.52	14.33	8.09
2	8.05	3.53	5.89	10.37	8.58
3	6.69	3.08	6.05	10.15	6.48
4	5.90	3.11	5.48	8.57	5.72
5	5.09	2.50	5.05	6.18	5.84
6	4.80	2.14	5.02	6.08	5.25
7	1.77	2.22	4.82	4.16	5.21
8	1.66	1.91	4.28	4.83	6.65
9	1.74	1.95	4.78	3.49	5.05
10	1.77	1.98	5.11	3.35	6.50
11	1.79	1.78	3.33	3.63	4.87
12	1.38	1.57	3.83	3.09	5.23
13	1.29	1.78	4.22	2.78	5.12
14	1.60	1.64	3.85	2.77	4.58
15	1.51	1.56	6.12	3.17	4.90
16	1.40	1.58	3.30	2.27	4.83
17	1.73	1.22	4.22	2.05	5.15
18	1.17	1.24	2.89	1.74	3.63
19	1.53	1.35	4.36	2.64	3.72
20	0.92	1.30	3.18	2.34	4.43
21	0.93	0.93	3.39	1.89	3.72
22	1.08	0.70	2.92	1.39	3.07
23	0.70	0.66	2.44	0.89	4.04
24	1.36	1.37	2.27	2.53	2.15
Average	2.82	1.92	4.39	4.36	5.12

FIGURE 5-9 and FIGURE 5-10 show the MAPE results of ANN models by model for the downtown area and the north area, respectively. These figures show the tendency for the MAPE results to decrease as prediction time decreases. Both the downtown area and the north area show the same tendencies. However, the MAPE of the downtown area shown in FIGURE 5-9 is higher than that of the north area shown in FIGURE 5-10. In the previous chapter, it was found that the values of input variables of the downtown area had more variability than those of the north area. Therefore, the larger MAPE of the downtown area is a reasonable result. In other words, more variable traffic conditions due to traffic congestion leads to higher prediction errors.

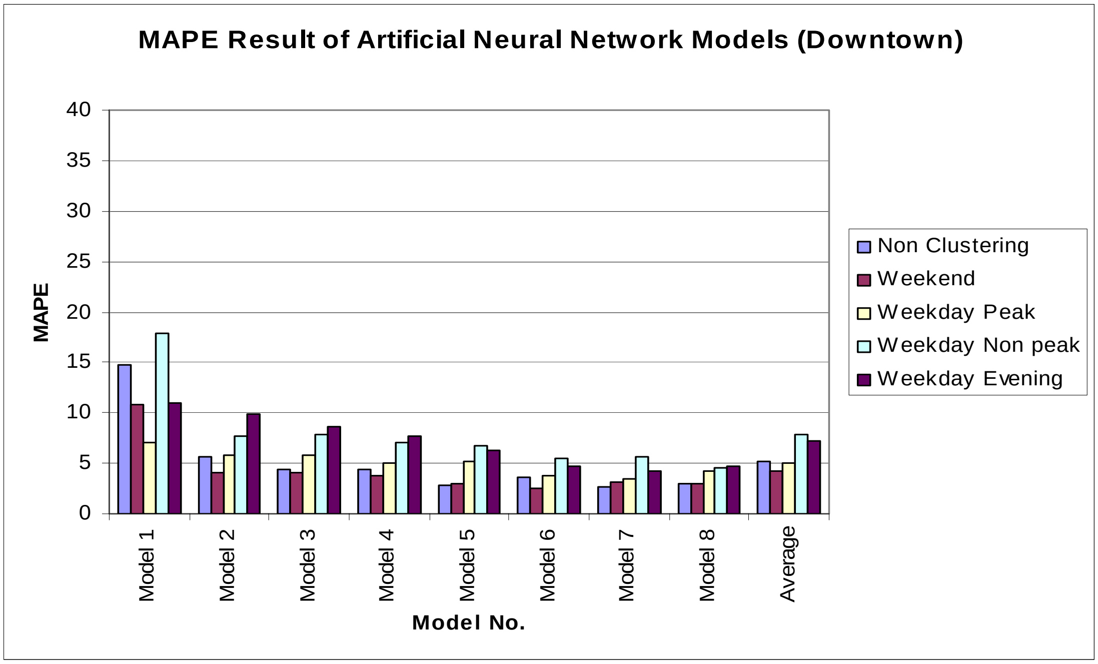

FIGURE 5-9 MAPE Result of Artificial Neural Network Models of the Test Bed 1 by Time Period (Downtown)

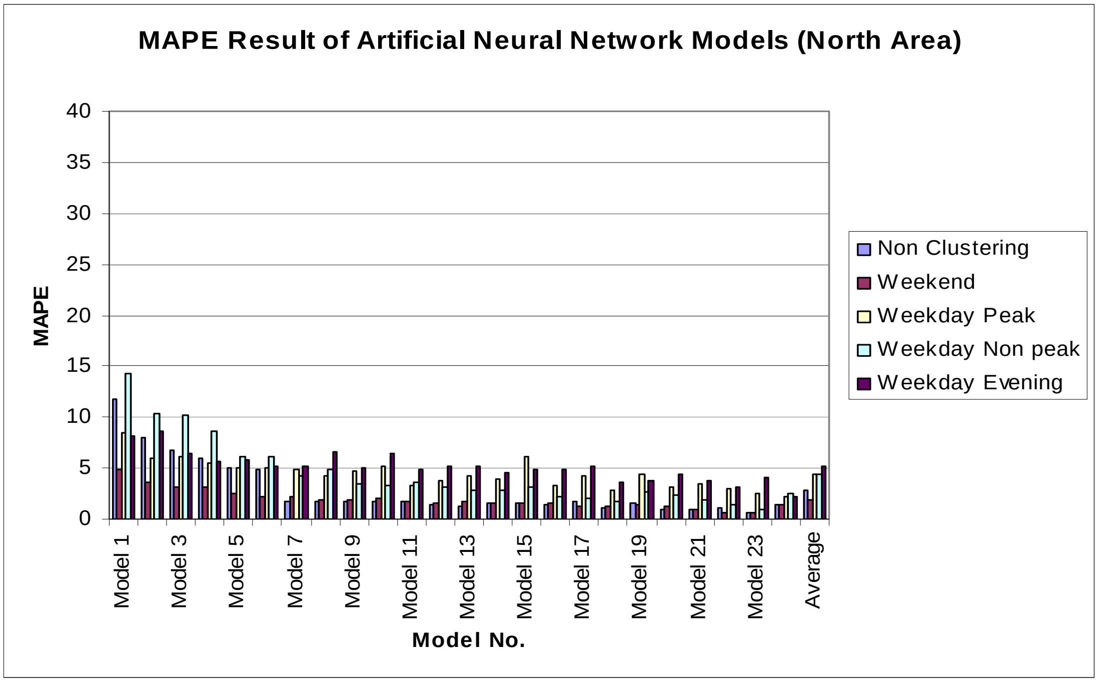

FIGURE 5-10 MAPE Result of Artificial Neural Network Models of the Test Bed 2 by Time Period (North Area)

FIGURE 5-11 and FIGURE 5-12 show the MAPE results by time period. Unlike the historical data based models and multi linear regression models, the MAPE of clustering data is not smaller than that of non-clustering data.

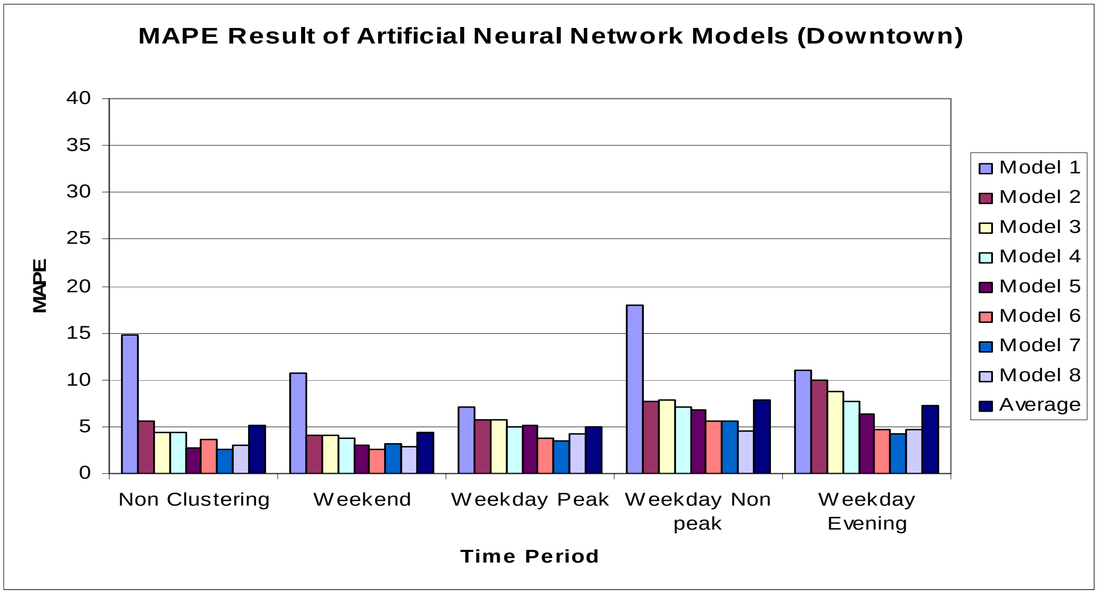

FIGURE 5-11 MAPE Result of Artificial Neural Network Models of the Test Bed 1 by Model (Downtown)

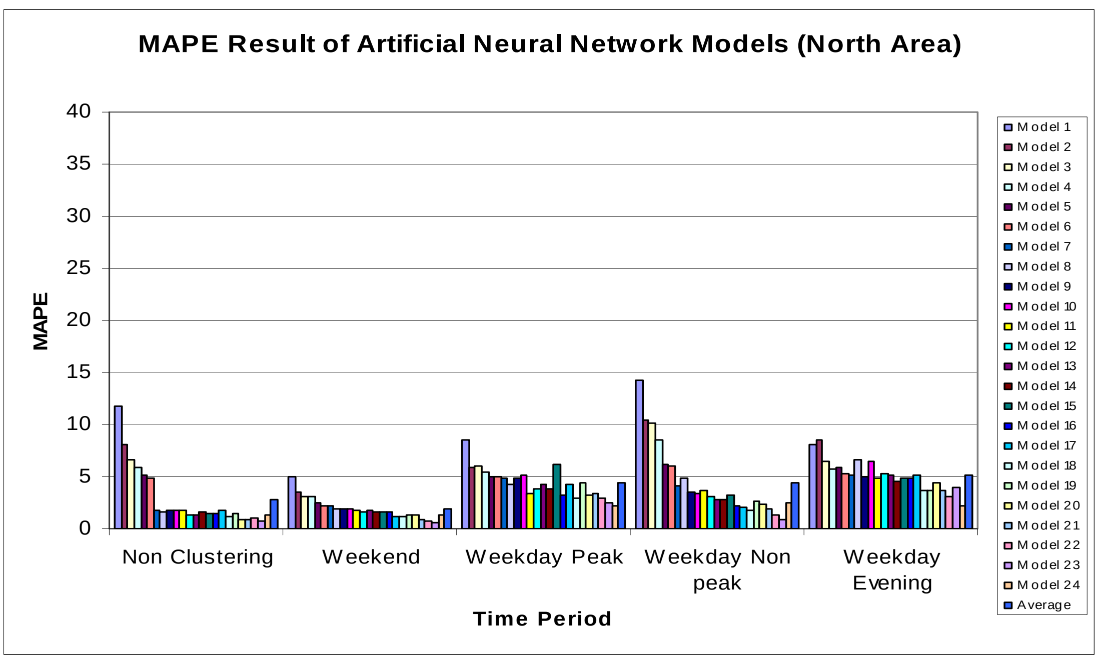

FIGURE 5-12 MAPE Result of Artificial Neural Network Models of the Test Bed 2 by Model (North Area)

5.4 EVALUATION OF PREDICTION MODELS

The Average Mean Absolute Percent Error (MAPE) was used to measure model performance in this dissertation. TABLE 5-7, and FIGURE 5-13 through FIGURE 5-14 show the results of the model evaluation. For both the downtown area and the north area, the artificial neural networks give better results in terms of prediction accuracy. In the previous chapter, the need for a prediction model that can explain the uncertainty in input variables became clear. It is hypothesized that the ANN models that were able to identify the non-linear relationship among the input variables and therefore they were able to give the best results.

TABLE 5-7 Average MAPE of Prediction Models

Site	Time Period	Historical Data Based Models	Regression Models	Artificial Neural Network Models
Downtown	Non-Clustering	14.63	24.85	5.13
	Weekend	13.03	22.81	4.31
	Weekday Peak	10.78	14.88	5.04
	Weekday Non-Peak	14.34	23.48	7.87
	Weekday Evening	12.82	19.97	7.17
North area	Non-Clustering	7.58	15.25	2.82
	Weekend	7.27	14.54	1.92
	Weekday Peak	7.02	11.57	4.39
	Weekday Non-Peak	8.74	17.56	4.36
	Weekday Evening	6.43	13.92	5.12

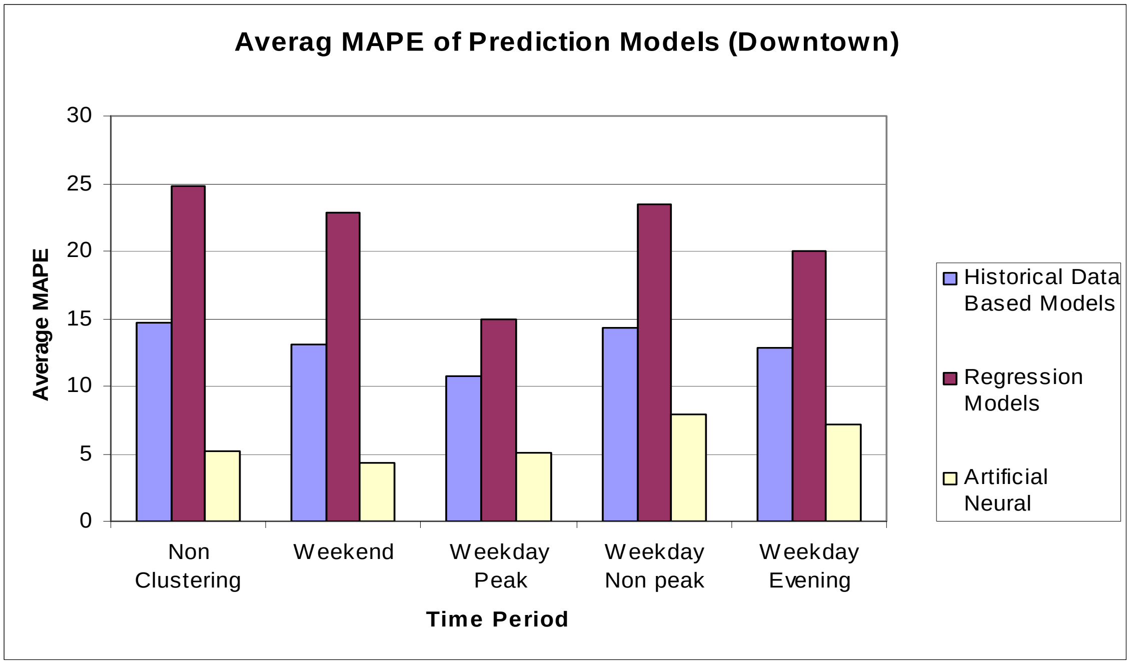

FIGURE 5-13 MAPE of Prediction Models of the Test Bed 1 (Downtown)

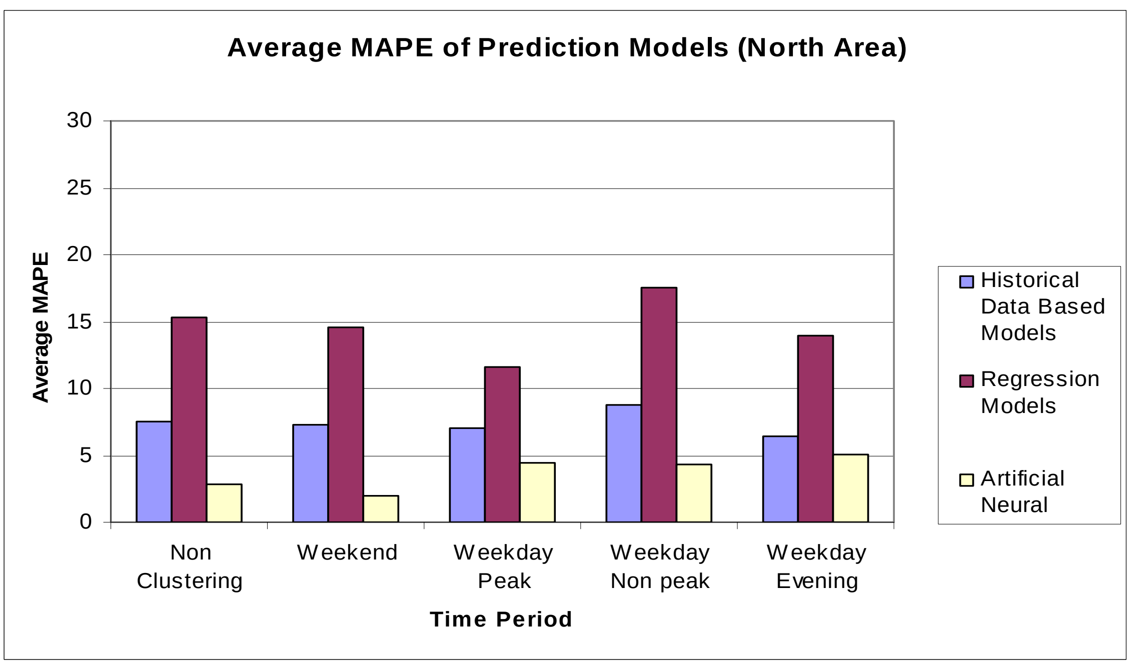

FIGURE 5-14 Average MAPE of Prediction Models of the Test Bed 2 (North Area)

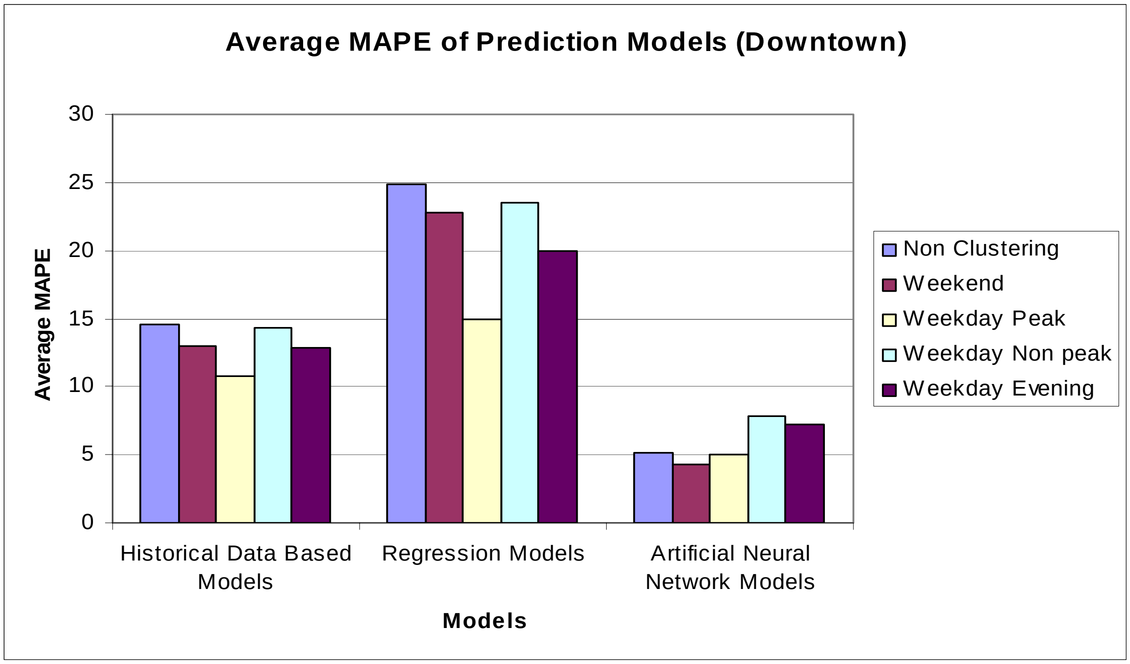

FIGURE 5-15 Average MAPE of Prediction Models of the Test Bed 1 (Downtown)

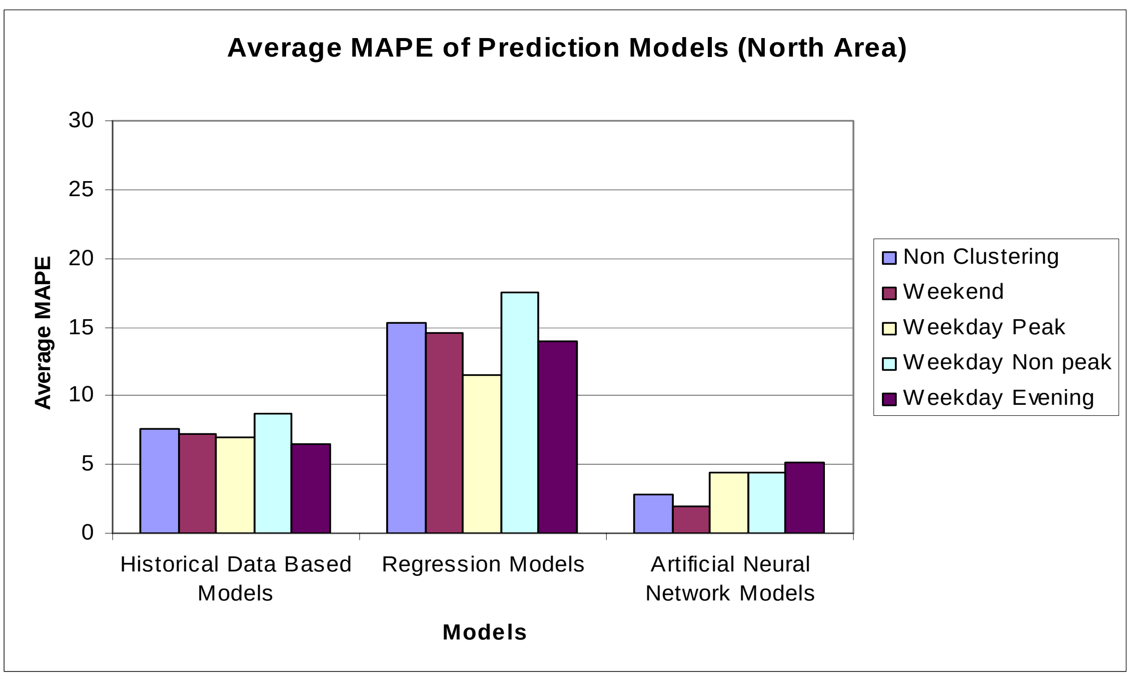

FIGURE 5-16 Average MAPE of Prediction Models of the Test Bed 2 (North Area)

The most important point to note is that the ANN had the lowest MAPE as compared to the historic model and the MLR model. On average, the ANN models had a 55 percent improvement with respect to the best historic model and a 71 percent improvement compared to the best MLR models in the downtown area. For the north area, the ANN models were 49 percent and 74 percent better as the historical data based model and the regression models, respectively. TABLE 5-8 presents these figures. It is hypothesized that the use of historic data (representing congestion) coupled with real-time schedule adherence data (representing real-time congestion and demand inputs) resulted in the better performance of the ANN model.

TABLE 5-8 Improvement of ANN Models with Respect to Historical Data Based Models and Regression Models

Site	Time Period	Historical Data Based Models	Regression Models
Downtown	Non-Clustering	65 %	79 %
	Weekend	67 %	81 %
	Weekday Peak	53 %	66 %
	Weekday Non-Peak	45 %	66 %
	Weekday Evening	44 %	64 %
	Average	55 %	71 %
North area	Non-Clustering	63 %	82 %
	Weekend	74 %	87 %
	Weekday Peak	37 %	62 %
	Weekday Non-Peak	50 %	75 %
	Weekday Evening	20 %	63 %
	Average	49 %	74 %

5.5 MODEL COMPARISON

Three different models for bus arrival time prediction were developed in the previous chapter: a historical data based model, a regression model, and a artificial neural network model. These models were compared in terms of the accuracy, as measured by MAPE, of predicted bus arrival time in the previous section. In this section, these three different models are tested statistically using Tukey's procedure (the T method).

To check statistically the differences among the predicted models, Tukey's procedure was performed. Let

I = the number of models being compared, 3 (a historical data based model, a multi linear regression model, and an artificial neural network model);
J = the number of elements of a sample data set;
$MAPE_H$ = MAPE of the historical data based models;
$MAPE_R$ = MAPE of the regression models;
$MAPE_N$ = MAPE of the artificial neural network models;
The null hypotheses and alternative hypotheses were set as
H_0 : $MAPE_H = MAPE_R = MAPE_N$ and
Ha : $MAPE_N \leq MAPE_H \leq MAPE_R$
$\alpha = 0.05$ (The level of significance per test is 0.05).

Tukey's procedure involves the use of another probability distribution called the Studentized range distribution. The distribution depends on two parameters: a numerator degree of freedom m and a denominator degree of freedom v. Let $Q_{\alpha,m,v}$ denote the upper-tail α critical value of the Studentized range distribution with m numerator degree of freedom and v denominator degree of freedom (analogous to F_{α,v_1,v_2}). Values of $Q_{\alpha,m,v}$ are given and can be used to obtain simultaneous confidence intervals for all pairwise differences $\mu_i - \mu_j$. (58)
Tukey's procedure is described as follows:

Step 1: Select α and find $Q_{\alpha,I,I(J-1)}$ from appropriate tables (58)

Step 2: Determine $w = Q_{\alpha,I,I(J-1)}\sqrt{MSE / J}$

MSE = mean square for error

Step 3: List the sample means in increasing order and underline those pairs that differ by less than w. Any pair of sample means not underscored by the same line corresponds to a pair of true treatments means that are judged significantly different.

In this dissertation, α is equal to 0.05, and $Q_{\alpha,I,I(J-1)} = Q_{0.05,3,27} = 3.49$. The MSE value for the non-clustering, weekend, weekday peak, weekday non-peak, and weekday evening are shown in TABLE 5-9 through TABLE 5-13, respectively. w values are show in TABLE 5-14. Step 3 of Tukey's procedure is shown in TABLE 5-15. The mean MAPE of the three prediction models is arranged in increasing order. For the non-clustering data set, the w value is 2.03, and the difference between the pairs $MA\overline{P}E_H$ - $MA\overline{P}E_N$ and $MA\overline{P}E_R$ - $MA\overline{P}E_H$ is 9.37 and 10.10, respectively. These two values are significantly higher than 2.03. This means the MAPE of the historical data based models and the MAPE of the regression models are significantly higher than the MAPE of the artificial neural network models. For all five time periods, the w values are significantly smaller than the difference between every pair. Consequently, the MAPE of the artificial neural network models is statistically smaller than the MAPE of the historical data based models and the MAPE of the regression models with a significance level of 0.05. Consequently, the ANN method was adopted as the preferred prediction method.

TABLE 5-9 ANOVA Table for Non-Clustering Data Set

Source of variation	df	Sum of squares	Mean square	F
Models	2	977.22	488.61	145.11
Error	27	90.91	3.37	F crit
Total	29	1068.13		3.35

F crit: F critical value with significance level of 0.05

TABLE 5-10 ANOVA Table for Weekend Data Set

Source of variation	df	Sum of squares	Mean square	F
Models	2	3992.05	1996.02	1428.87
Error	27	37.72	1.40	F crit
Total	29	4029.77		3.35

TABLE 5-11 ANOVA Table for Weekday Peak Data Set

Source of variation	df	Sum of squares	Mean square	F
Models	2	2339.92	1169.96	2405.34
Error	27	13.13	0.49	F crit
Total	29	2353.06		3.35

TABLE 5-12 ANOVA Table for Weekday Non-Peak Data Set

Source of variation	df	Sum of squares	Mean square	F
Models	2	977.22	488.61	145.11
Error	27	90.91	3.37	F crit
Total	29	1068.13		3.35

TABLE 5-13 ANOVA Table for Weekday Evening Data Set

Source of variation	df	Sum of squares	Mean square	F
Models	2	450.58	225.29	348.51
Error	27	17.45	0.65	F crit
Total	29	468.03		3.35

TABLE 5-14 w **Values**

Clustering	$Q_{0.05,3,27}$	MSE	w
Non-Clustering	3.49	3.37	2.03
Weekend	3.49	1.40	1.31
Weekday peak	3.49	0.49	0.77
Weekday Non-Peak	3.49	3.37	2.03
Weekday evening	3.49	0.65	0.89

TABLE 5-15 Result of Tukey's Procedure

<table>
<tr><th>Clustering</th><th colspan="2">$MA\overline{P}E_N$</th><th colspan="2">$MA\overline{P}E_H$</th><th colspan="2">$MA\overline{P}E_R$</th></tr>
<tr><td rowspan="2">Non-Clustering</td><td colspan="2">5.30</td><td colspan="2">14.67</td><td colspan="2">24.77</td></tr>
<tr><td></td><td colspan="2">9.37</td><td colspan="2">10.10</td><td></td></tr>
<tr><td rowspan="2">Weekend</td><td colspan="2">5.03</td><td colspan="2">13.19</td><td colspan="2">32.54</td></tr>
<tr><td></td><td colspan="2">8.16</td><td colspan="2">19.35</td><td></td></tr>
<tr><td rowspan="2">Weekday peak</td><td colspan="2">6.70</td><td colspan="2">10.87</td><td colspan="2">27.17</td></tr>
<tr><td></td><td colspan="2">4.17</td><td colspan="2">20.47</td><td></td></tr>
<tr><td rowspan="2">Weekday Non-Peak</td><td colspan="2">10.07</td><td colspan="2">14.56</td><td colspan="2">23.78</td></tr>
<tr><td></td><td colspan="2">4.49</td><td colspan="2">9.22</td><td></td></tr>
<tr><td rowspan="2">Weekday evening</td><td colspan="2">10.52</td><td colspan="2">13.30</td><td colspan="2">19.77</td></tr>
<tr><td></td><td colspan="2">2.78</td><td colspan="2">6.47</td><td></td></tr>
</table>

5.6 CONCLUDING REMARKS

In this chapter, the three prediction models developed in chapter IV were evaluated and statistically tested. The ANN models give superior results than historical data based models and regression models in term of prediction accuracy. It was hypothesized that the ANN, as a universal function approximator, was able to identify the non-linear relationships associated with the different clusters. It is found that MAPE decreases as prediction time decrease. It was also seen that clustering data leads to smaller MAPE in historical data based models and regression models. However, for the ANN models, clustering the data did not give a smaller MAPE. While in general the clustering should not do worse than the non-clustering option, it is hypothesized that there may not have been enough observations to adequately fit the functions. If more observations were available, the results between the two approaches might have been more similar. It was also found that the input data at five previous bus stops could give about five percent of MAPE.

The three prediction models were statistically tested with Tukey's procedure. It was found that the MAPE of artificial neural networks is statistically smaller than the MAPE of the historical data based models and the MAPE of the regression models with a significance level of 0.05. Consequently, the ANN method was adopted as the preferred prediction method. In chapter VI, three issues related to bus arrival time will be studied: the prediction interval of bus arrival time, the probability of a bus being on time, and the feasibility for real-time application.

CHAPTER VI

PREDICTION INTERVAL, THE PROBABILITY OF A BUS BEING ON TIME, AND REAL-TIME APPLICATION

Having access to accurate and timely travel time information would be very useful to transit patrons as well as transit authorities. For transit users, the prediction interval of bus arrival time is very important information for making travel decisions. For example, when the predicted bus arrival time is 30 minutes, no one knows what time exactly the bus will arrive. This was shown in chapter III where the variability (standard deviation) of the arrival times in the test bed 1 during the weekday non-peak period was 150 seconds. Because variability in travel time (both waiting and on-board) is extremely important for transit choice, it would also be useful to extend the model to provide not only estimates of the travel time but also prediction intervals. It is hypothesized if transit users were to receive prediction interval information, they can make better decisions.

In addition, on-time performance of a bus is very significant to transit operators because customers use this information to measure quality of service and because operators use this information to decide when to activate strategies such as bus priority. It would be extremely important to identify, in real-time, whether a given bus is on schedule or not. To measure the on-time performance, the probability of a bus being on time is required. The prediction interval can be used to identify, in real-time, the probability that a given bus is on time.

Thanks to the wide-spread deployment of AVL technology, obtaining real-time bus location data in most major urban areas is possible. Real-time information on the current location of a given bus is would be useful for passengers. However, real-time bus location data should be processed into real-time bus arrival information because this information is more useful for travelers. While getting real-time location data is relatively easy, predicting real-time bus arrival times is more complicated. Intuitively, when using complex models, predicting real-time arrival information every second or every five seconds would be very difficult. A methodology for developing a real-time prediction model will be discussed later in this chapter.

In chapter V, three different prediction models for bus arrival time were evaluated and statistically tested. It was found that the artificial neural network models gave the best results in terms of prediction accuracy. Subsequently, in this chapter, based on the ANN models, a methodology for identifying the prediction interval of the bus arrival time and the probability of a given bys being on time is developed. Because ANNs are non parametric models, conventional techniques for calculating prediction intervals can not be used. Consequently, a new computer-intensive method, known as the bootstrap technique, is used to obtain the prediction interval of bus arrival time. In addition, the probability that a given bus is on time was calculated as part of this work. The methodology for real-time application is also discussed in the last section.

6.1 PREDICTION INTERVAL

Because the ANN model is a non-parametric method, a bootstrap technique was chosen to obtain the prediction interval for bus arrival time. The bootstrap technique is a recently developed computer-based method for conducting statistical inference. In general, a narrower prediction interval indicates a more precise arrival time estimate.

6.1.1 Bootstrap Technique

The bootstrap technique is a computer-intensive method used to make statistical inferences about an estimate. FIGURE 6-1 shows the schematic diagram of the bootstrap technique. The original data set consists of n elements of data, $x = (x_1, x_2, x_3, ..., x_n)$. From the data set, a statistical estimate $\hat{\theta}$ is calculated. Subsequently, B bootstrap samples are generated from the original data set using a computer and a specific random sampling strategy.. Each bootstrap sample has n elements generated by sampling with replacement n times from the original data set. The first bootstrap sample has $x_1^* = (x_1^*, x_2^*, x_3^*, ..., x_n^*)$ and the Bth bootstrap sample has $x_B^* = (x_1^*, x_2^*, x_3^*, ..., x_n^*)$. The value of the statistics $\hat{\theta}^*$, the bootstrap replications, for each bootstrap sample are calculated. Typically, the value of B is set in the range of 100 to 200 (58-59). A major advantage of the bootstrap technique is that, because it is computer based, as many bootstrap replications can be calculated as desired (58-59).

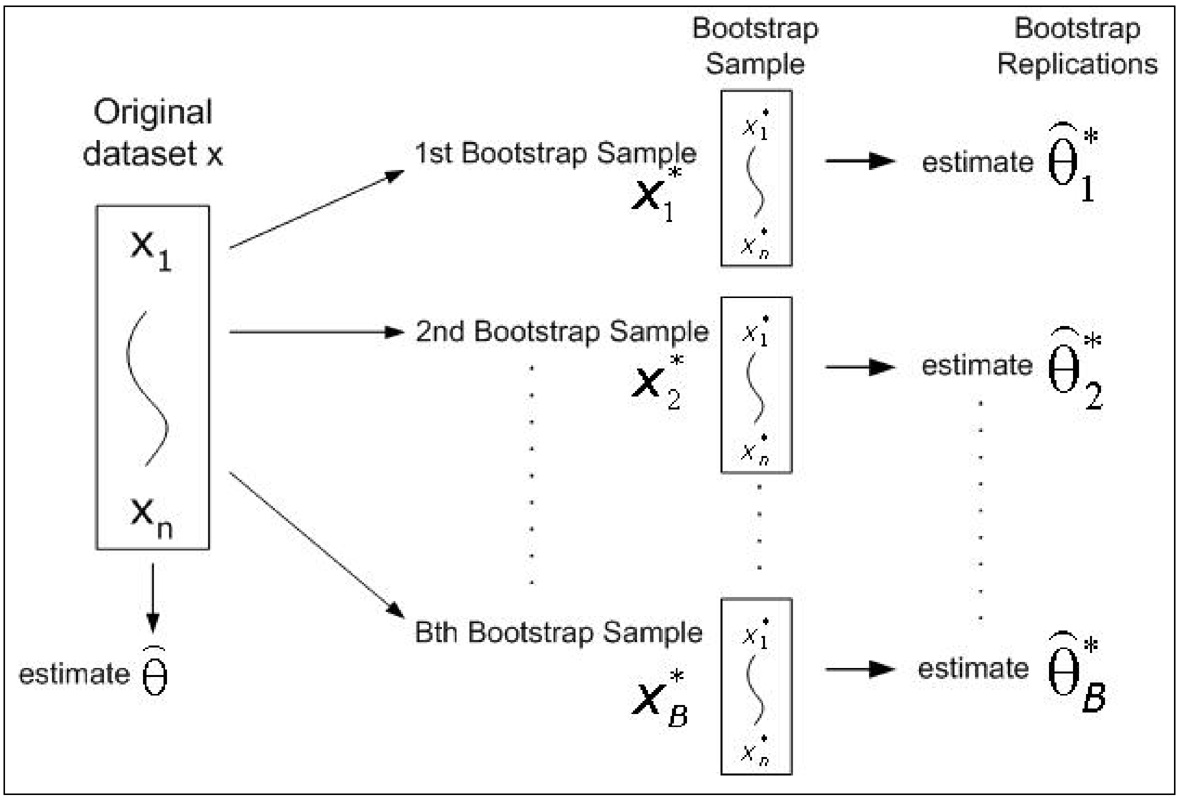

FIGURE 6-1 A Schematic Diagram of a Bootstrap Technique

The bootstrap procedure for the prediction interval of the arrival time for the ANN models is described as follows:

Step1: Generate a bootstrap sample from the original data set

Sampling is random and with replacement

Step 2: The ANN model runs with the bootstrap sample data generated in Step 1

Step 3: Repeat Step 1 and Step 2 B times

Step 4: Calculate mean ($\bar{\hat{\gamma}}$) and variance (S^2) of B bootstrap replications $\hat{\theta}^*$

Step 5: Calculate the prediction interval of arrival time with Equation 6-1

$$\text{Prediction Interval} = \bar{\hat{\gamma}} \pm t'_{\alpha/2,n-2}\sqrt{S^2 + \frac{S^2}{n}} \qquad (6\text{-}1)$$

where,

$\bar{\hat{\gamma}}$ = mean of predicted bus arrival time;

S^2 = variance of predicted bus arrival time;

$t'_{\alpha/2,n-2} =$ critical values $t_{\alpha,v}$ for the t distribution.

To calculate a prediction interval of bus arrival time, the ANN model for the non-clustering downtown data was used. According to TABLE 5-5, the MAPE of models 1, 2, 3, and 4 were 14.76, 5.58, 4.33, and 4.36, respectively and the MAPE of model 5 was 2.77. In other words, when the input data consists of the five previous bus stops, the model could predict bus arrival time with less than three-percent error. Consequently, ANN model 5 with non-clustering downtown data was used to calculate the prediction interval of bus arrival time. The number of data for model training, n, is 240. A sensitivity analysis on the number of bootstrap samples (B) was conducted to identify the relationship between B and the interval. In this research, three values were used: 100, 200, and 1000.

6.1.2 Result of the Prediction Interval of Bus Arrival Time

The results of the prediction interval of bus arrival time are shown in TABLE 6-1 through TABLE 6-3 and FIGURE 6-2. TABLE 6-1 shows the result of the prediction interval when B is equal to 100. It can be seen that the arrival time at stop 6 is 447.5 seconds and the prediction interval is plus/minus 55.3 seconds. TABLE 6-2 shows the result of the prediction interval when B is equal to 200. It can be seen that the arrival time and the prediction interval are 447.5 seconds plus/minus 57.8 seconds. TABLE 6-3 shows the result of the prediction interval when B is equal to 1000. It can be seen that they are 447.4 seconds plus/minus 60.6 seconds. While the prediction interval tends to increase with B, the values are all relatively close together.

In contrast, the prediction interval at stop 9 shows a large difference for different values of B. When B is equal to 100, the prediction interval of bus arrival time at stop 9 is plus/minus 111.3 seconds as shown in TABLE 6-1. When B is equal to 200 and 1000, the prediction intervals of arrival time at stop 9 are plus/minus 83.1 seconds and 83.4 seconds, respectively as shown in TABLE 6-2 and TABLE 6-3. It appears that a larger B such as 200 or 1000 gives a better prediction interval. However, the results between 200 and 1000 bootstrap samples were only 0.5 percent different. Consequently, for this test bed, it was decided that 200 bootstrap samples were enough to calculate the prediction interval of bus arrival time.

TABLE 6-1 Results of Prediction Interval (B=100)

Arrival time at stop	Mean (sec)	Variance (sec)	Prediction interval (sec)			
			Lower	Upper	Lower	Upper
Stop 6	447.5	27.9	392.2*	502.8	-55.3	55.3
Stop 7	511.2	25.3	461.2	561.2	-50.0	50.0
Stop 8	564.4	34.1	496.9	632.0	-67.5	67.5
Stop 9	611.2	56.2	499.9	722.5	-111.3	111.3

* : 95% significance level was used to calculate the prediction interval

TABLE 6-2 Results of Prediction Interval (B=200)

Arrival time at stop	Mean (sec)	Variance (sec)	Prediction interval (sec)			
			Lower	Upper	Lower	Upper
Stop 6	447.5	29.5	389.7	505.3	-57.8	57.8
Stop 7	511.0	26.0	460.1	561.9	-50.9	50.9
Stop 8	564.3	27.7	510.0	618.7	-54.4	54.4
Stop 9	611.8	42.4	528.6	694.9	-83.1	83.1

TABLE 6-3 Results of Prediction Interval (B=1000)

Arrival time at stop	Mean (sec)	Variance (sec)	Prediction interval (sec)			
			Lower	Upper	Lower	Upper
Stop 6	447.4	30.9	386.7	508.0	-60.6	60.6
Stop 7	511.1	30.0	452.3	570.0	-58.8	58.8
Stop 8	564.7	27.2	511.3	618.0	-53.3	53.3
Stop 9	612.5	42.6	529.1	695.9	-83.4	83.4

FIGURE 6-2 shows the difference of the prediction interval with three different B values. In this figure, the prediction interval at stop 9 has the largest prediction interval. In addition, the bootstrap sample of B = 100 had the largest interval at this stop.

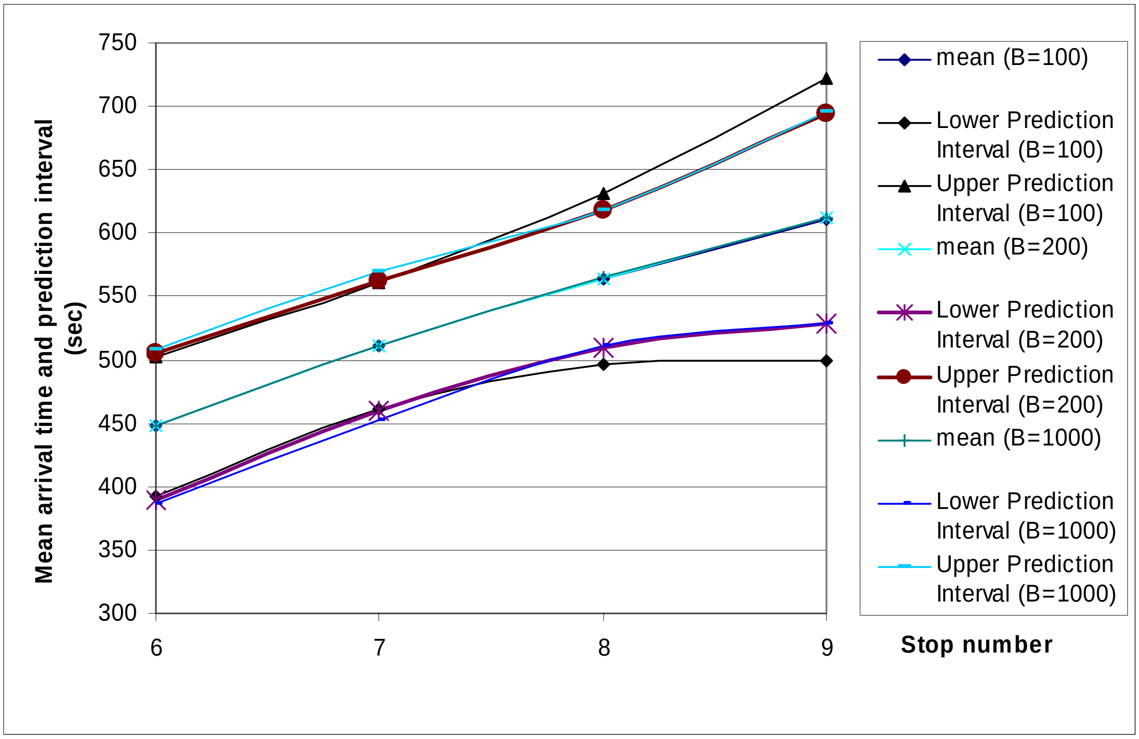

FIGURE 6-2 Prediction Interval of Bus Arrival Time

The population prediction interval for the MLR can be readily calculated using Equation 6-2 (58).

$$\hat{\beta}_0 + \hat{\beta}_1 x^* \pm t_{\alpha/2,n-2} \cdot s\sqrt{1 + \frac{1}{n} + \frac{n(x^* - \bar{x})^2}{n\sum x_i^2 - (\sum x_i)^2}} \qquad (6\text{-}2)$$

TABLE 6-4 shows the prediction interval of the bus arrival time estimated by the ANN models and the regression models. Both prediction intervals are for the population, not for the mean (58-59). The prediction interval of regression model for the mean can be seen in TABLE 6-4 and it was calculated using Equation 6-3 (58). It can be seen that the ANN models had smaller prediction intervals than the regression models. For example, at stop 6, the ANN prediction interval is between 389.7 seconds and 505.3 seconds while the regression model prediction interval is between 152.5 seconds and 678.4 seconds.

Intuitively, the former would be preferred by transit users and operators because it has a smaller interval.

TABLE 6-4 Prediction Interval by ANN Models and Regression Models

Bus stop	ANN (for population)		Regression (for population)		Regression (for mean)	
	Lower Prediction Interval	Upper Prediction Interval	Lower Prediction Interval	Upper Prediction Interval	Lower Prediction Interval	Upper Prediction Interval
Stop 6	389.7	505.3	152.5	678.4	409.4	421.5
Stop 7	460.1	561.9	233.5	759.4	489.3	503.6
Stop 8	510.0	618.7	308.3	834.2	562.6	579.8
Stop 9	528.6	694.9	389.3	915.2	641.8	662.6

$$\hat{\beta}_0 + \hat{\beta}_1 x^* \pm t_{\alpha/2,n-2} \cdot s\sqrt{\frac{1}{n} + \frac{n(x^* - \bar{x})^2}{n\sum x_i^2 - (\sum x_i)^2}} \qquad (6\text{-}3)$$

FIGURE 6-3 shows that the prediction interval by the regression models gave wider prediction intervals at all stops than that by the ANN models.

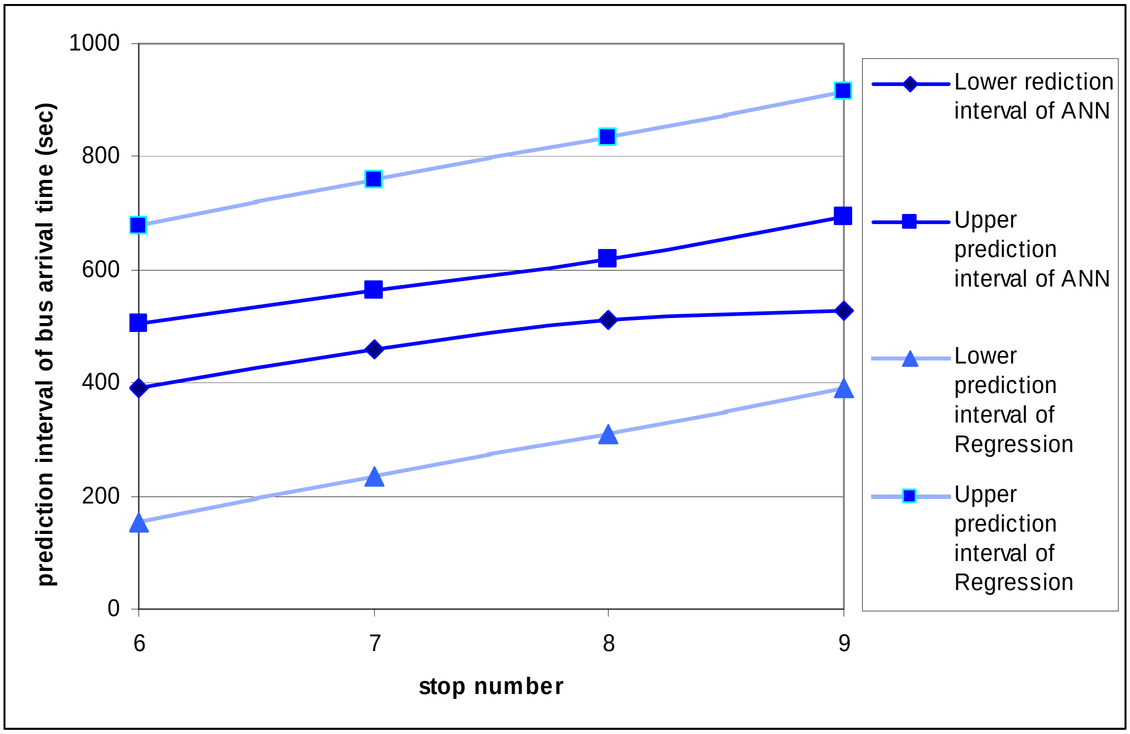

FIGURE 6-3 Prediction Interval by ANN Models and Regression Models

6.2 PROBABILITY OF A BUS BEING ON TIME

On-time performance of a bus is very important metric used by transit operators because customers use this to measure quality of service. In addition, certain transit operation strategies, such as bus priority, require this information as an input. Consequently, it would be extremely important to identify, in real-time, whether a given bus is on schedule or not. To estimate the on-time performance in real-time, the probability of a bus being on time is required. In this section, the probability of a bus being on time, being ahead of the schedule, or being behind schedule is studied.

6.2.1 Characteristics of Schedule Adherence

FIGURE 6-4 through FIGURE 6-8 shows the observed histogram of schedule adherence by the time period. The histograms of schedule adherence by time period and bus stop can be seen in FIGURE C-1 through FIGURE C-45 in APPENDIX C. A positive value of schedule adherence means that the bus is behind schedule and a negative value of schedule adherence means that the bus is ahead of schedule. While each stop has a slightly different shape, these are some general trends.

FIGURE 6-4 shows the observed histogram of schedule adherence for the non-clustering data set. According to this figure, the distribution seems to follow a gamma distribution or a normal distribution. FIGURE 6-6 shows the observed histogram of schedule adherence for weekday peak period and FIGURE 6-7 shows the observed histogram of schedule adherence for weekday non-peak period. The mean of schedule adherence is approximately zero and these graphs are symmetric and bell-shaped. FIGURE 6-5 shows the observed histogram of schedule adherence for the weekend period and FIGURE 6-8 shows the observed histogram of schedule adherence for weekday evening period. In contrast to the weekday peak and non-peak period, it should be noted that the histograms have a long right tail and a short left tail for the weekend and weekday evening periods. Therefore, both the gamma distribution and the normal distribution are tested using a chi-squared goodness-of-fit test.

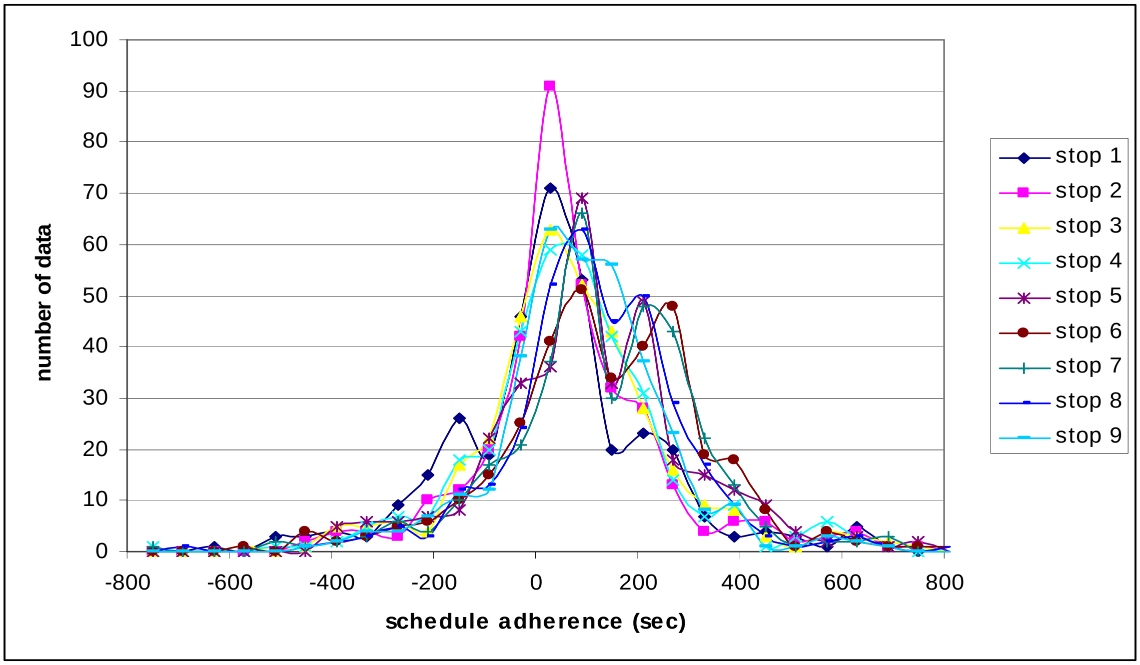

FIGURE 6-4 Schedule Adherence of Non-Clustering

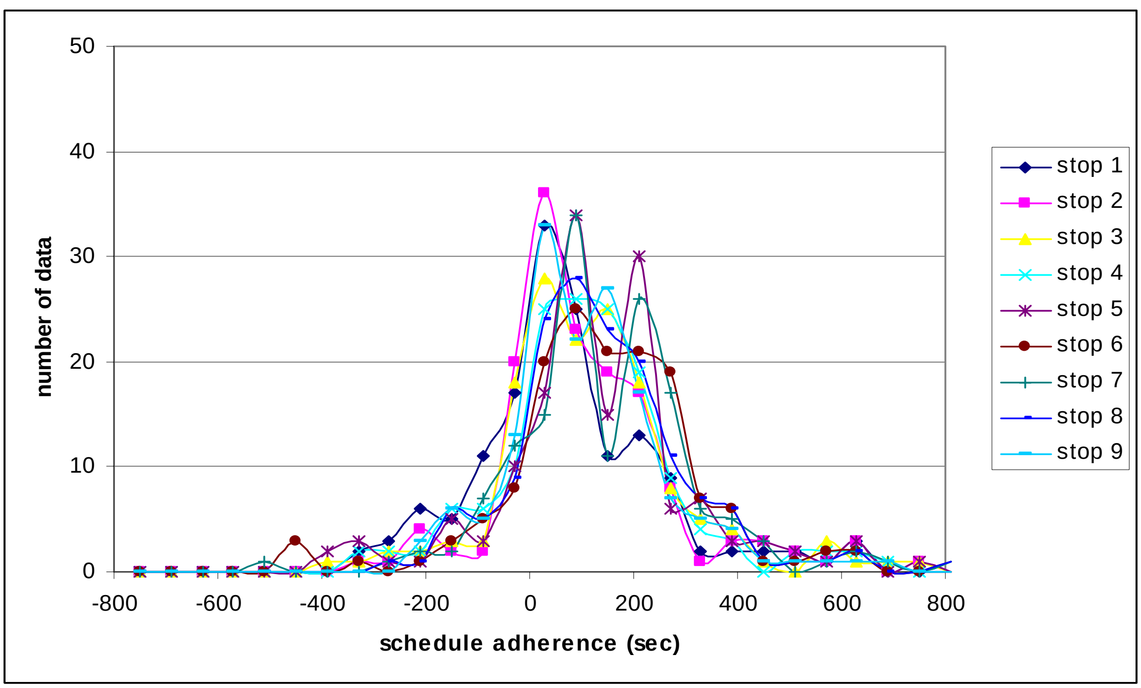

FIGURE 6-5 Schedule Adherence of Weekend

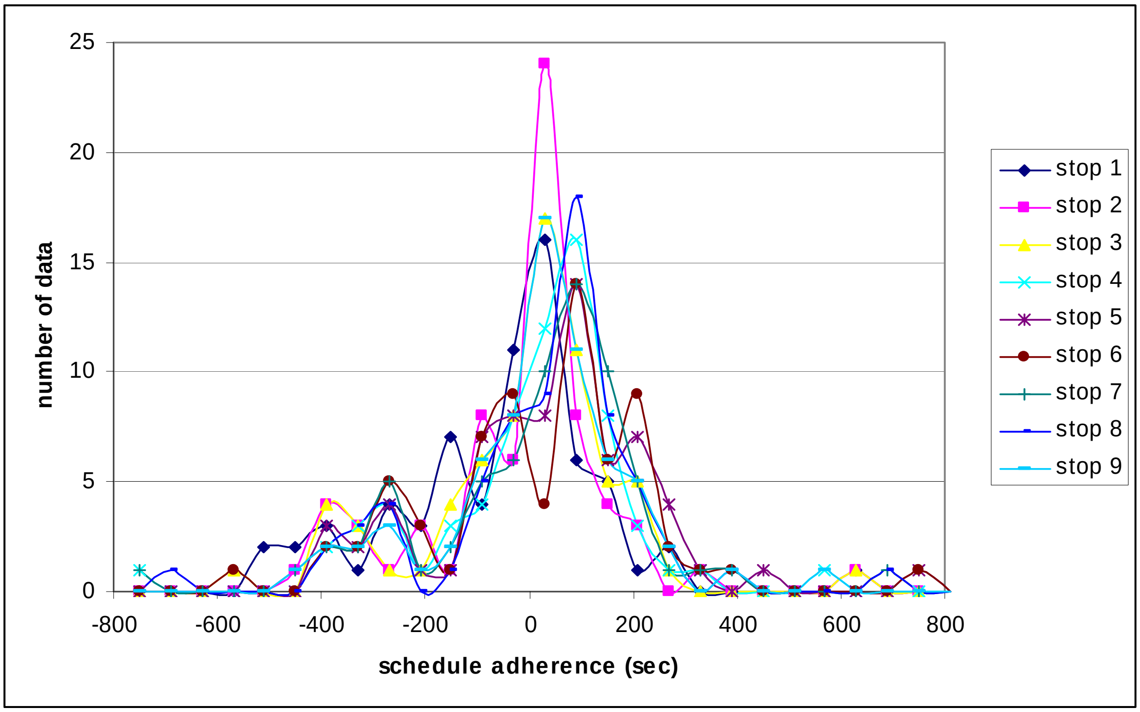

FIGURE 6-6 Schedule Adherence of Weekday Peak

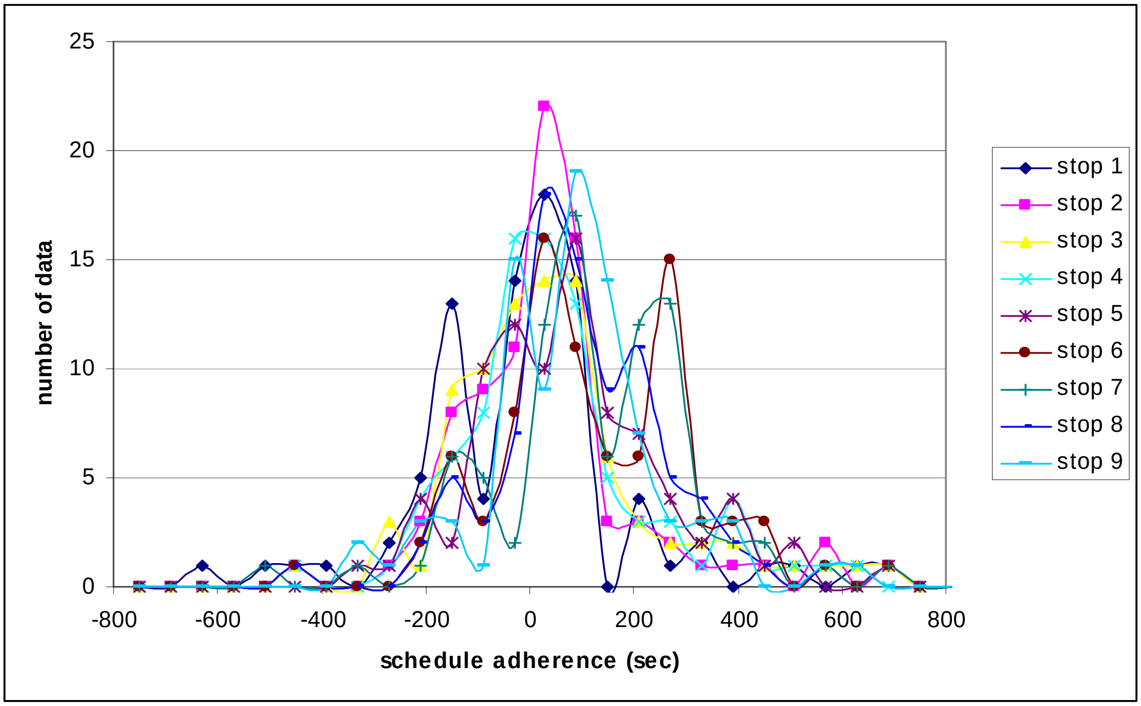

FIGURE 6-7 Schedule Adherence of Weekday Non-Peak

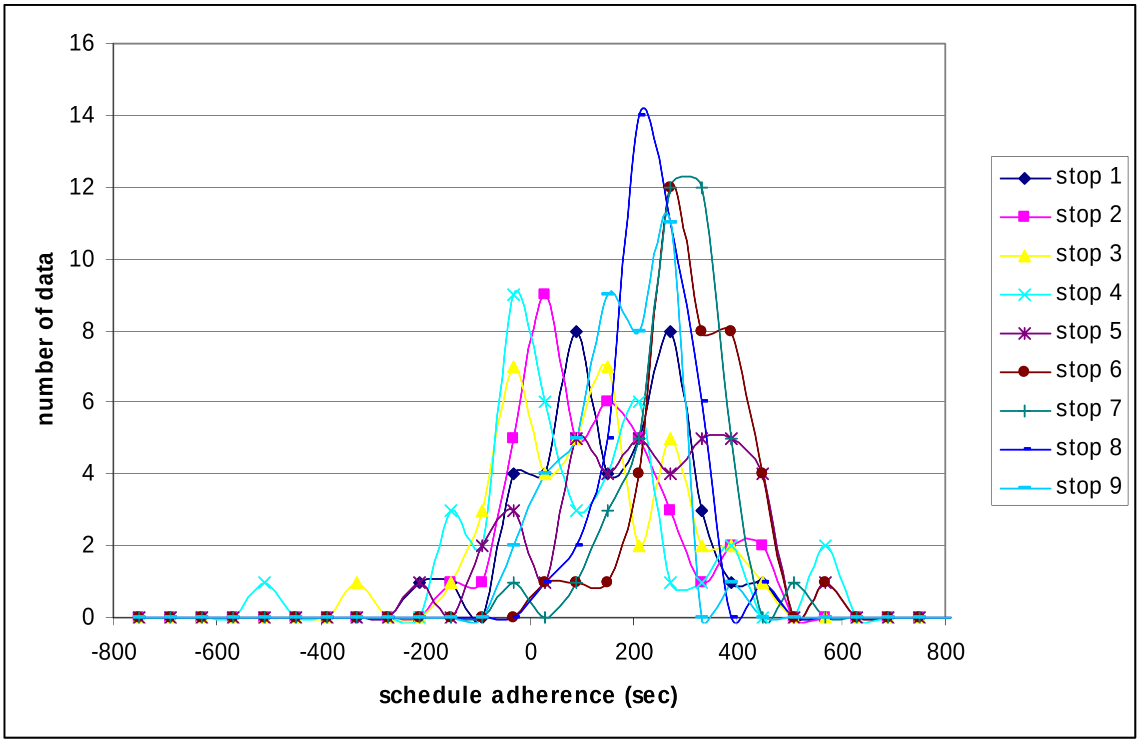

FIGURE 6-8 Schedule Adherence of Weekday Evening

6.2.2 Gamma Distribution

The chi-squared goodness-of-fit test for gamma distribution follows the steps below (60).

Step 1: Calculate mean and standard deviation

Step 2: Select an appropriate value of K that affects the shape of the distribution

The K values can vary from 0 to ∞; if K is selected to be 1, the resulting distribution takes the form of a negative exponential distribution. The K value can be determined using Equation 6-4.

$$K = \frac{\mu - \alpha}{s} \tag{6-4}$$

where,

μ = mean;

α = a value that effects the shift of the distribution,

the α value is greater than or equal to zero,

in this dissertation α value is zero;

s = standard deviation.

Step 3: Calculate λ value which is a function of mean and K value

λ value can be determined using Equation 6-5

$$\lambda = \frac{K}{\mu - \alpha} \tag{6-5}$$

Step 4: Calculate the Gamma function [Γ(K)]. The [Γ(K)] is equal to (K-1)!. If K is a integer such as 1,2,3, etc. the Gamma function is simply 0!, 1!, 2!, etc. However if K is a non integer value such as 4.602, the Gamma function is 3.602!, which is not so easily calculated. A Gamma function table is given in statistical books (60).

Step5: Solve probability density function f(t) for various desired values of schedule adherence t using Equation 6-6.

$$f(t) = \frac{\lambda}{\Gamma(K)}[\lambda(t-\alpha)]^{K-1} e^{-\pi(t-\alpha)} \tag{6-6}$$

where,

f(t) = probability density function;

λ = parameter that is a function of mean and user-specified parameters, K and α;

K = user-selected parameter between 0 and ∞ that affects the shape of distribution;

α = user-selected parameter greater than or equal to zero
that affects the shift of distribution (seconds);
t = schedule adherence (seconds);
Γ(K) = the Gamma function, equivalent to (K-1)!.

Step 6: Calculate probability of grouped schedule adherence using Equation 6-7

$$P(t \leq x < t + \Delta t) = [\frac{f(t) + f(t + \Delta t)}{2}]\Delta t \tag{6-7}$$

where, Δt is equal to sixty seconds

In this dissertation, the schedule adherence is grouped in sixty-second intervals.

Step 7: Calculate frequencies of grouped schedule adherence using Equation 6-8

$$F(t \leq x < t + \Delta t) = N[P(t \leq x < t + \Delta t)] \tag{6-8}$$

where,

N = total number of observations;

$F(t \leq x < t + \Delta t)$ = predicted number of schedule adherence in the schedule adherence group $t \leq x < t + \Delta t$.

Step 8: Calculate chi-square value using Equation 6-9

$$\chi^2_{cal} = \sum_{i=1}^{I} \frac{(f_o - f_t)^2}{f_t} \tag{6-9}$$

where,

χ^2_{cal} = calculated chi-square value;

f_O = observed number of frequency of observations in schedule adherence interval I;

ft = theoretical number of frequency of expected observations in schedule adherence interval I;

i = any time interval of schedule adherence;

I = total number of intervals.

Step 9: Compare the values of χ^2_{cal} and χ^2, where χ^2 is the table value from a statistics book with two user-selected parameters α and degrees of freedom. The null hypothesis H_O is that the Gamma distribution gives a good fit of schedule adherence. If χ^2_{cal} is greater than χ^2, H_O is rejected, meaning a poor fit. If χ^2_{cal} is less than χ^2, H_O is accepted meaning a good fit.

In Step 1, the mean and standard deviations of schedule adherence is calculated. TABLE 6-5 shows the values by time period. In Step 2 through Step 4, the k values, the λ values, and the Gamma function [Γ(K)] are determined by Equation 6-4 and Equation 6-5, respectively. These values are shown in TABLE 6-6. In Step 5 through 7, the probability density function, probability, and frequencies for grouped schedule adherence are calculated using Equation 6-6 through Equation 6-8, respectively. These results are shown in TABLE 6-7 through TABLE 6-11 by time period. As can be seen in these tables, all five time periods give the same result, namely that χ^2_{cal} is greater than χ^2. Therefore the null hypothesis (H_O) is rejected. This means that the gamma distribution gives a poor fit of schedule adherence. FIGURE 6-9 through 6-13 shows the difference between the observed schedule adherence and the predicted schedule adherence by the Gamma distribution. It can be seen that there are large differences between the two distributions. Consequently, the normal distribution will be applied for the goodness-of-fit test in the next section.

TABLE 6-5 Mean and Standard Deviation of Schedule Adherence

Time Period	Number of Observations	Mean (sec)	Standard Deviation (sec)
Non-Clustering	340	62	189
Weekend	147	99	174
Weekday peak	68	-28	179
Weekday non-peak	85	43	200
Weekday evening	40	151	146

TABLE 6-6 Parameters for Gamma Distribution

Time Period	K value	λ value	Gamma Function, [Γ(K)]
Non-Clustering	4.602	0.0053	13.4158
Weekend	5.212	0.0057	33.2084
Weekday peak	4.353	0.0056	9.5127
Weekday Non-Peak	4.270	0.0050	8.5079
Weekday evening	6.589	0.0069	337.95

TABLE 6-7 Probability and Predicted Frequencies of Non-Clustering

Group	Schedule Adherence, x (sec)	Observed frequency, f_O	Probability, P(x)	Predicted Frequency, F(x)	$\frac{(f_o - f_t)^2}{f_t}$
1	-809 ~-750	0	0.00014	0	0.0
2	-749 ~ -690	0	0.00135	0	0.0
3	-689 ~ -630	0	0.00502	2	2.0
4	-629 ~ -570	0	0.01163	4	4.0
5	-569 ~ -510	1	0.02054	7	5.1
6	-509 ~ -450	2	0.03059	11	7.4
7	-449 ~ -390	3	0.04054	15	9.6
8	-329 ~ -270	4	0.04939	18	10.9
9	-269 ~ -210	6	0.05645	20	9.8
10	-209 ~ -150	7	0.06140	22	10.2
11	-149 ~ -90	14	0.06416	23	3.5
12	-89 ~ -30	18	0.06489	23	1.1
13	-29 ~ 30	35	0.06386	23	6.3
14	31 ~ 90	57	0.06141	22	55.7
15	91 ~ 150	58	0.05789	21	65.2
16	151 ~ 210	37	0.05364	19	17.1
17	211 ~ 270	37	0.04896	18	20.1
18	271 ~ 330	25	0.04410	16	5.1
19	331 ~ 390	12	0.03926	14	0.3
20	391 ~ 450	10	0.03458	12	0.3
21	451 ~ 510	4	0.03017	11	4.5
22	511 ~ 570	2	0.02610	9	5.4
23	571 ~ 630	3	0.02240	8	3.1
24	631 ~ 690	3	0.01909	7	2.3
25	691 ~ 750	1	0.01616	6	4.2
26	751 ~ 810	1	0.01360	5	3.2
27	811 ~ 870	0	0.01138	4	4.0
Calculated Chi-square value, χ^2_{cal}					260.2
Table value of χ^2, α=0.05, df = 27-1-2=24					36.4

χ^2_{cal} is greater than χ^2 : H_O is rejected

TABLE 6-8 Probability and Predicted Frequencies of Weekend

Group	Schedule Adherence, x (sec)	Observed frequency, f_O	Probability, P(x)	Predicted Frequency, F(x)	$\frac{(f_o - f_t)^2}{f_t}$
1	-809 ~-750	0	0.00004	0	0.0
2	-749 ~ -690	0	0.00058	0	0.0
3	-689 ~ -630	0	0.00265	1	1.0
4	-629 ~ -570	0	0.00715	1	1.0
5	-569 ~ -510	0	0.01416	2	2.0
6	-509 ~ -450	0	0.02308	4	4.0
7	-449 ~ -390	0	0.03287	5	5.0
8	-329 ~ -270	1	0.04246	7	5.1
9	-269 ~ -210	1	0.05093	8	6.1
10	-209 ~ -150	2	0.05765	9	5.4
11	-149 ~ -90	4	0.06230	9	2.8
12	-89 ~ -30	5	0.06480	10	2.5
13	-29 ~ 30	13	0.06529	10	0.9
14	31 ~ 90	26	0.06403	10	25.6
15	91 ~ 150	27	0.06135	9	36.0
16	151 ~ 210	20	0.05762	9	13.4
17	211 ~ 270	20	0.05317	8	18.0
18	271 ~ 330	11	0.04831	7	2.3
19	331 ~ 390	5	0.04329	7	0.6
20	391 ~ 450	4	0.03832	6	0.7
21	451 ~ 510	2	0.03354	5	1.8
22	511 ~ 570	1	0.02907	5	3.2
23	571 ~ 630	1	0.02496	4	2.3
24	631 ~ 690	2	0.02126	3	0.3
25	691 ~ 750	1	0.01796	3	1.3
26	751 ~ 810	0	0.01507	3	3.0
27	811 ~ 870	1	0.01257	2	0.5
Calculated Chi-square value, χ^2_{cal}					144.9
Table value of χ^2, α=0.05, df = 27-1-2=24					36.4

χ^2_{cal} is greater than χ^2 : H_O is rejected

TABLE 6-9 Probability and Predicted Frequencies of Weekday Peak

Group	Schedule Adherence, x (sec)	Observed frequency, f_O	Probability, P(x)	Predicted Frequency, F(x)	$\frac{(f_o - f_t)^2}{f_t}$
1	-809 ~-750	0	0.00032	0	0.0
2	-749 ~ -690	0	0.00266	0	0.0
3	-689 ~ -630	0	0.00886	1	1.0
4	-629 ~ -570	0	0.01876	1	1.0
5	-569 ~ -510	0	0.03075	2	2.0
6	-509 ~ -450	1	0.04292	3	1.3
7	-449 ~ -390	3	0.05371	4	0.3
8	-329 ~ -270	2	0.06211	4	1.0
9	-269 ~ -210	3	0.06767	5	0.8
10	-209 ~ -150	2	0.07037	5	1.8
11	-149 ~ -90	2	0.07051	5	1.8
12	-89 ~ -30	6	0.06852	5	0.2
13	-29 ~ 30	9	0.06491	4	6.3
14	31 ~ 90	13	0.06018	4	20.3
15	91 ~ 150	12	0.05478	4	16.0
16	151 ~ 210	6	0.04906	3	3.0
17	211 ~ 270	5	0.04333	3	1.3
18	271 ~ 330	2	0.03780	3	0.3
19	331 ~ 390	1	0.03261	2	0.5
20	391 ~ 450	1	0.02786	2	0.5
21	451 ~ 510	0	0.02359	2	2.0
22	511 ~ 570	0	0.01982	1	1.0
23	571 ~ 630	0	0.01653	1	1.0
24	631 ~ 690	0	0.01369	1	1.0
25	691 ~ 750	0	0.01127	1	1.0
26	751 ~ 810	0	0.00923	1	1.0
27	811 ~ 870	0	0.00752	1	1.0
Calculated Chi-square value, χ^2_{cal}					67.4
Table value of χ^2, α=0.05, df = 27-1-2=24					36.4

χ^2_{cal} is greater than χ^2 : H_O is rejected

TABLE 6-10 Probability and Predicted Frequencies of Weekday Non-Peak

Group	Schedule Adherence, x (sec)	Observed frequency, f_O	Probability, P(x)	Predicted Frequency, F(x)	$\frac{(f_o - f_t)^2}{f_t}$
1	-809 ~-750	0	0.00026	0	0.0
2	-749 ~ -690	0	0.00209	0	0.0
3	-689 ~ -630	0	0.00695	1	1.0
4	-629 ~ -570	0	0.01481	1	1.0
5	-569 ~ -510	0	0.02459	2	2.0
6	-509 ~ -450	1	0.03492	3	1.3
7	-449 ~ -390	0	0.04456	4	4.0
8	-329 ~ -270	1	0.05265	5	3.2
9	-269 ~ -210	1	0.05871	5	3.2
10	-209 ~ -150	3	0.06257	6	1.5
11	-149 ~ -90	6	0.06430	6	0.0
12	-89 ~ -30	6	0.06414	6	0.0
13	-29 ~ 30	11	0.06241	5	7.2
14	31 ~ 90	15	0.05947	5	20.0
15	91 ~ 150	15	0.05565	5	20.0
16	151 ~ 210	6	0.05127	5	0.2
17	211 ~ 270	6	0.04660	4	1.0
18	271 ~ 330	5	0.04184	4	0.3
19	331 ~ 390	2	0.03717	3	0.3
20	391 ~ 450	2	0.03270	3	0.3
21	451 ~ 510	1	0.02852	3	1.3
22	511 ~ 570	1	0.02469	2	0.5
23	571 ~ 630	1	0.02121	2	0.5
24	631 ~ 690	1	0.01811	2	0.5
25	691 ~ 750	1	0.01537	1	0.0
26	751 ~ 810	0	0.01298	1	1.0
27	811 ~ 870	0	0.01090	1	1.0
Calculated Chi-square value, χ^2_{cal}					71.4
Table value of χ^2, α=0.05, df = 27-1-2=24					36.4

χ^2_{cal} is greater than χ^2 : H_O is rejected

TABLE 6-11 Probability and Predicted Frequencies of Weekday Evening

Group	Schedule Adherence, x (sec)	Observed frequency, f_O	Probability, P(x)	Predicted Frequency, F(x)	$\frac{(f_o - f_t)^2}{f_t}$
1	-809 ~-750	0	0.00000	0	0.0
2	-749 ~ -690	0	0.00009	0	0.0
3	-689 ~ -630	0	0.00067	0	0.0
4	-629 ~ -570	0	0.00249	0	0.0
5	-569 ~ -510	0	0.00631	0	0.0
6	-509 ~ -450	0	0.01247	1	1.0
7	-449 ~ -390	0	0.02073	1	1.0
8	-329 ~ -270	0	0.03034	1	1.0
9	-269 ~ -210	0	0.04031	2	2.0
10	-209 ~ -150	0	0.04964	2	2.0
11	-149 ~ -90	1	0.05749	2	0.5
12	-89 ~ -30	1	0.06332	3	1.3
13	-29 ~ 30	4	0.06686	3	0.3
14	31 ~ 90	3	0.06813	3	0.0
15	91 ~ 150	4	0.06733	3	0.3
16	151 ~ 210	5	0.06479	3	1.3
17	211 ~ 270	6	0.06091	2	8.0
18	271 ~ 330	8	0.05610	2	18.0
19	331 ~ 390	4	0.05074	2	2.0
20	391 ~ 450	3	0.04514	2	0.5
21	451 ~ 510	1	0.03957	2	0.5
22	511 ~ 570	0	0.03423	1	1.0
23	571 ~ 630	0	0.02925	1	1.0
24	631 ~ 690	0	0.02472	1	1.0
25	691 ~ 750	0	0.02068	1	1.0
26	751 ~ 810	0	0.01714	1	1.0
27	811 ~ 870	0	0.01409	1	1.0
Calculated Chi-square value, χ^2_{cal}					45.8
Table value of χ^2, α=0.05, df = 27-1-2=24					36.4

χ^2_{cal} is greater than χ^2 : H_O is rejected

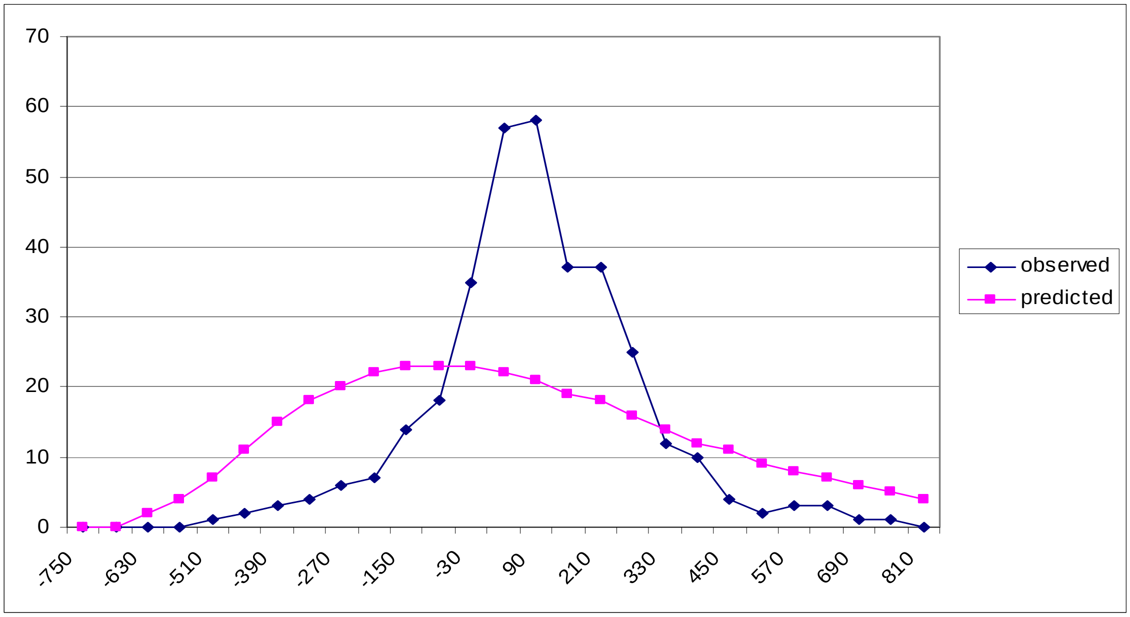

FIGURE 6-9 Observed and Predicted Schedule Adherence by Gamma Distribution (Non-Clustering)

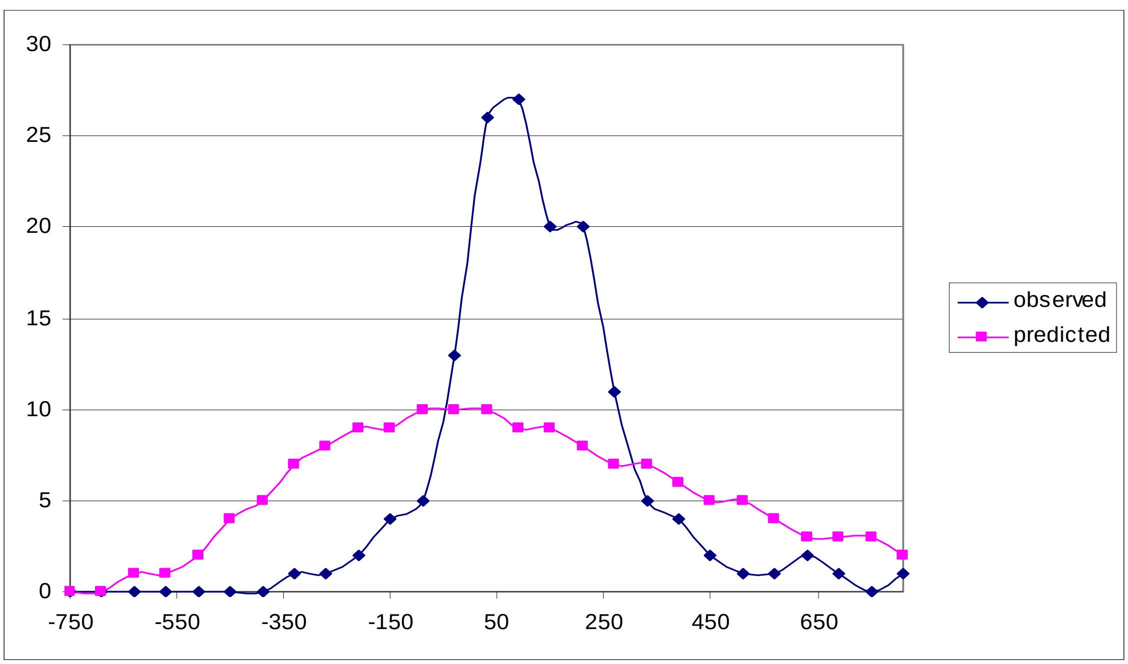

FIGURE 6-10 Observed and Predicted Schedule Adherence by Gamma Distribution (Weekend)

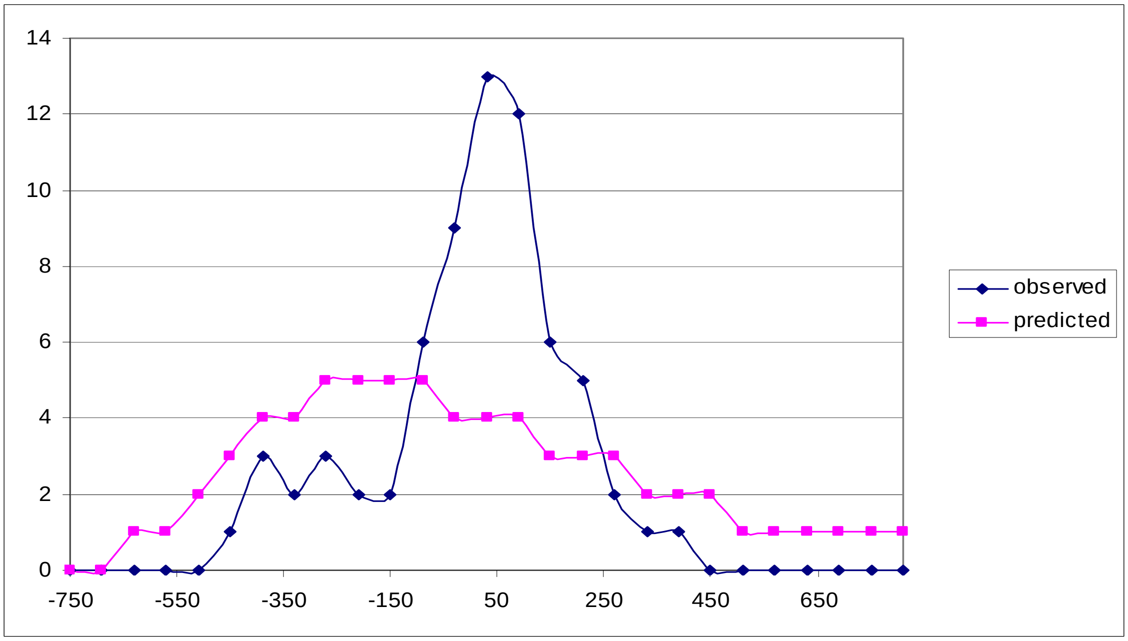

FIGURE 6-11 Observed and Predicted Schedule Adherence by Gamma Distribution (Weekday Peak)

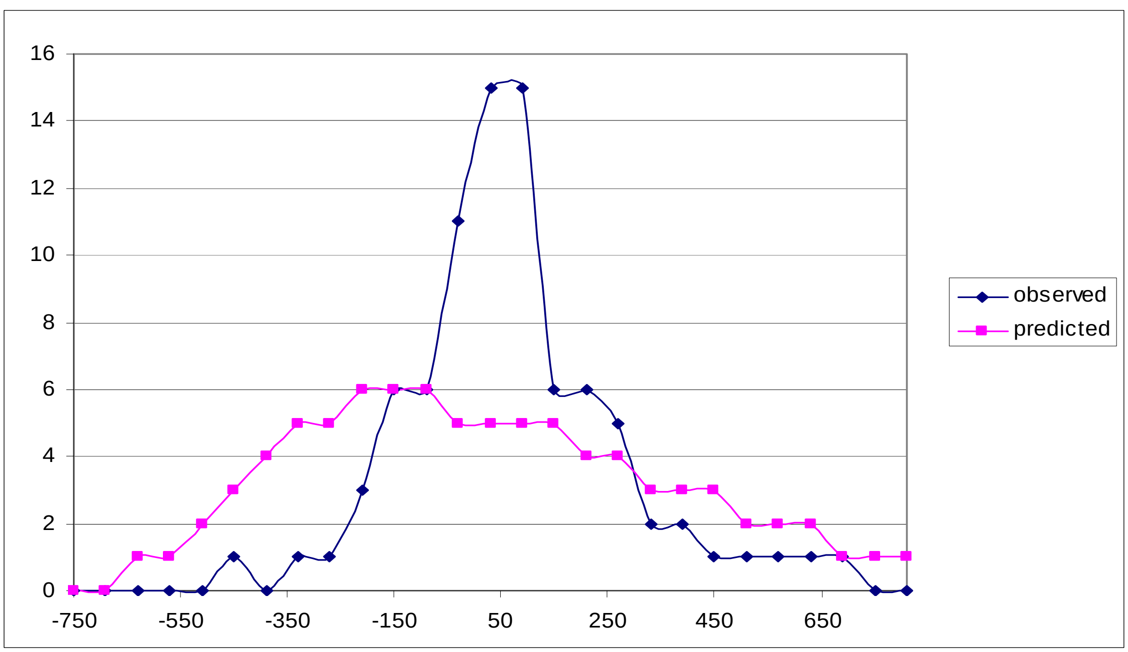

FIGURE 6-12 Observed and Predicted Schedule Adherence by Gamma Distribution (Weekday Non-Peak)

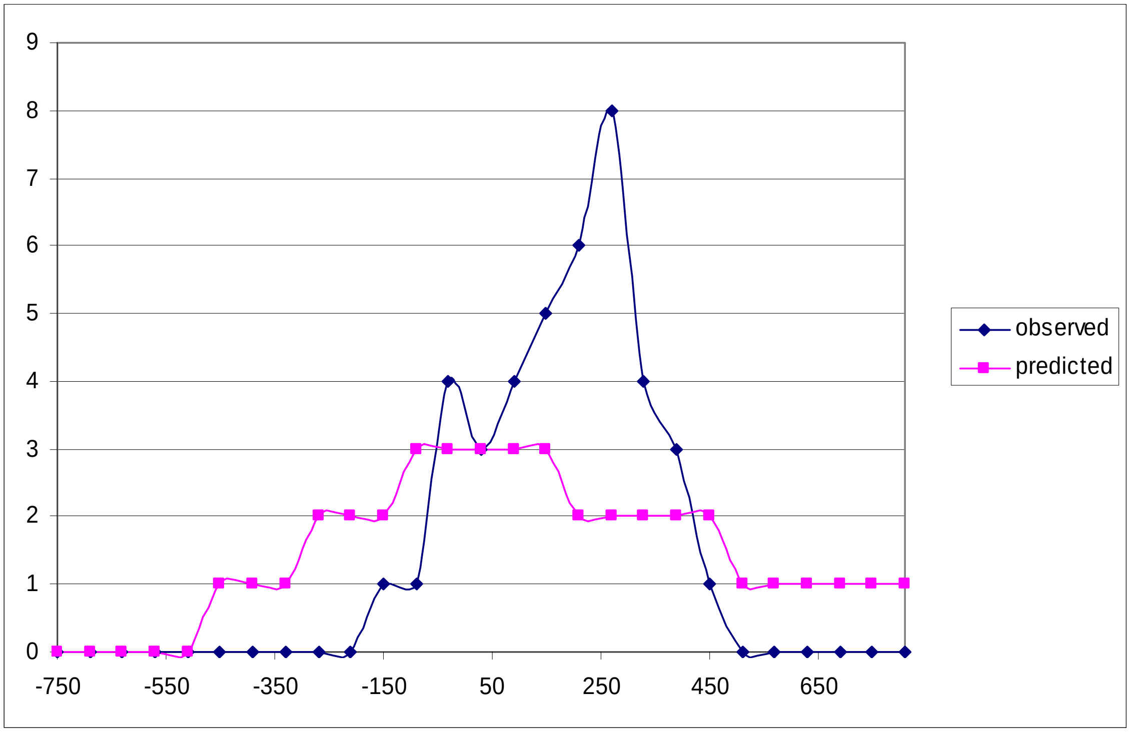

FIGURE 6-13 Observed and Predicted Schedule Adherence by Gamma Distribution (Weekday Evening)

6.2.3 Normal Distribution

The chi-squared goodness-of-fit test for normal distribution follows steps below (60).

Step 1: Calculate mean and standard deviation

Step 2: Calculate Z value using Equation 6-10

$$Z = \frac{x - \mu}{\sigma} \tag{6-10}$$

Step 3: Solve probability density function f(x) for various desired values of schedule adherence t using Equation 6-11

$$f(x;\mu,\sigma)=\frac{1}{\sqrt{2\pi\alpha}}e^{-(x-\mu)^2/(2\sigma^2)} \qquad -\infty < x < \infty \qquad (6\text{-}11)$$

Step 4: Calculate probability of grouped schedule adherence using Equation 6-12

$$P(t \le x < t+\Delta t) = F(t+\Delta t) - F(t) \qquad (6\text{-}12)$$

Step 5: Calculate frequencies of grouped schedule adherence using Equation 6-8

Step 6: Calculate chi-square value using Equation 6-9

Step 7: Compare the values of χ^2_{cal} and χ^2, where χ^2 is the table value from a statistics book, with two user-selected parameters α and degrees of freedom. The null hypothesis H_O is that the normal distribution gives good fit of schedule adherence. If χ^2_{cal} is greater than χ^2, H_O is rejected meaning poor fit. If χ^2_{cal} is less than χ^2, H_O is accepted meaning good fit.

The mean and standard deviation of schedule adherence are shown in TABLE 6-5. In Step 3 through 5, the probability density function, probability, and frequencies for grouped schedule adherence were calculated using Equation 6-11 through Equation 6-12 and Equation 6-8, respectively. These results are shown in TABLE 6-12 through TABLE 6-16 by time period. As can be seen in these tables, all five time periods give the same results, namely that χ^2_{cal} is less than χ^2, hence the H_O is not rejected. This means that the normal distribution gives a good fit of the schedule adherence variable. FIGURE 6-14 through FIGURE 6-18 shows the difference between observed schedule adherence and predicted schedule adherence by the normal distribution. There are no large differences between the two distributions. Consequently, the normal distribution adequately estimates schedule adherence.

In this chapter, schedule adherence was grouped by sixty seconds. In TABLE 6-12, group 13 means that schedule adherence is between minus 29 seconds and plus 30 seconds. In other words, group 13 means that the bus is on time. The definition of a bus being on time varies. Minus one minute to plus three minutes was used in previous research paper (61). Because the probability by the normal distribution was calculated by 60-second interval in this chapter, the probability of a bus being on time, being ahead of the schedule, or being behind schedule can be calculated. For example, if the definition of a bus being on-time is that the bus arrives 90 seconds early or 210 seconds late, according to TABLE 6-14, the probability of a bus being on time in the weekday peak period is the summation of the probability of group 11 through group 16, or 0.09210 + 0.11860 + 0.12910 + 0.12950 + 0.11990 + 0.09353, or 0.6827. This probability can be used by transit operators to manage their on-time performance. The operators should conduct this probability analysis at regular basis, and they can monitor the pattern of the on-time performance.

For transit users who are provided the real-time arrival time information by some devices such as internet or kiosk, the probability that a given bus will be on time at bus stop i+1, i+2, etc based on the predicted bus arrival time at bus stop i can be provided. For example, the prediction model will predict the predicted bus arrival time at stop i+1, i+2,⋯⋯,i+n, based on the arrival time at stop i. In addition to the predicted arrival time, the pre-defined bus schedule time already exists. If the predicted arrival time at next stops is in between minus one minute and plus three minutes of the scheduled time, the probability of the bus being on time is 1.0. If the prediction arrival time at next stops is not in between minus one minute to plus three minutes, the probability of the bus being on time is zero. For instance, if the scheduled time at bus stop i+n is 8:30 A.M. and the predicted arrival time at the stop is 8:45 a.m., the bus will arrive approximately 15 minutes after the scheduled time. This result means the probability of the bus is on time is effectively zero.

TABLE 6-12 Probability and Predicted Frequencies of Non-Clustering

Group	Schedule Adherence, x (sec)	Observed frequency, f_O	Probability, P(x)	Predicted Frequency, F(x)	$\frac{(f_o - f_t)^2}{f_t}$
1	-809 ~-750	0	0.00000	0	0.0
2	-749 ~ -690	0	0.00000	0	0.0
3	-689 ~ -630	0	0.00000	0	0.0
4	-629 ~ -570	0	0.00040	0	0.1
5	-569 ~ -510	1	0.00090	0	1.6
6	-509 ~ -450	2	0.00210	1	2.3
7	-449 ~ -390	3	0.00500	2	1.0
8	-329 ~ -270	4	0.01080	4	0.0
9	-269 ~ -210	6	0.02090	7	0.2
10	-209 ~ -150	7	0.03480	12	2.0
11	-149 ~ -90	14	0.05650	19	1.4
12	-89 ~ -30	18	0.08050	27	3.2
13	-29 ~ 30	35	0.10020	34	0.0
14	31 ~ 90	57	0.12040	41	6.3
15	91 ~ 150	58	0.12710	43	5.1
16	151 ~ 210	37	0.11760	40	0.2
17	211 ~ 270	37	0.10510	36	0.0
18	271 ~ 330	25	0.08200	28	0.3
19	331 ~ 390	12	0.05790	20	3.0
20	391 ~ 450	10	0.03600	12	0.4
21	451 ~ 510	4	0.02160	7	1.5
22	511 ~ 570	2	0.01130	4	0.9
23	571 ~ 630	3	0.00520	2	0.9
24	631 ~ 690	3	0.00240	1	5.8
25	691 ~ 750	1	0.00080	0	1.9
26	751 ~ 810	1	0.00050	0	4.1
27	811 ~ 870	0	0.00000	0	0.0
Calculated Chi-square value, χ^2_{cal}					42.3
Table value of χ^2, α=0.05, df = 27-1-2=24					36.4
Table value of χ^2, α=0.01, df = 27-1-2=24					43.0
Table value of χ^2, α=0.005, df = 27-1-2=24					45.6

With α=0.05, χ^2_{cal} is greater than χ^2 : H_O is rejected

However, with α=0.01 and α=0.005, χ^2_{cal} is less than χ^2 : H_O is not rejected

TABLE 6-13 Probability and Predicted Frequencies of Weekend

Group	Schedule Adherence, x (sec)	Observed frequency, f_O	Probability, P(x)	Predicted Frequency, F(x)	$\frac{(f_o - f_t)^2}{f_t}$
1	-809 ~-750	0	0.00000	0	0.0
2	-749 ~ -690	0	0.00000	0	0.0
3	-689 ~ -630	0	0.00000	0	0.0
4	-629 ~ -570	0	0.00000	0	0.0
5	-569 ~ -510	0	0.00000	0	0.0
6	-509 ~ -450	0	0.00080	0	0.1
7	-449 ~ -390	0	0.00170	0	0.2
8	-329 ~ -270	1	0.00440	1	0.2
9	-269 ~ -210	1	0.01010	1	0.2
10	-209 ~ -150	2	0.02050	3	0.3
11	-149 ~ -90	4	0.03890	6	0.5
12	-89 ~ -30	5	0.06150	9	1.8
13	-29 ~ 30	13	0.09170	13	0.0
14	31 ~ 90	26	0.11500	17	4.9
15	91 ~ 150	27	0.13550	20	2.5
16	151 ~ 210	20	0.13400	20	0.0
17	211 ~ 270	20	0.12160	18	0.3
18	271 ~ 330	11	0.10080	15	1.0
19	331 ~ 390	5	0.07010	10	2.7
20	391 ~ 450	4	0.04590	7	1.1
21	451 ~ 510	2	0.02530	4	0.8
22	511 ~ 570	1	0.01310	2	0.4
23	571 ~ 630	1	0.00560	1	0.0
24	631 ~ 690	2	0.00230	0	8.2
25	691 ~ 750	1	0.00090	0	5.7
26	751 ~ 810	0	0.00030	0	0.0
27	811 ~ 870	1	0.00000	0	0.0
Calculated Chi-square value, χ^2_{cal}					31.1
Table value of χ^2, α=0.05, df = 27-1-2=24					36.4
Table value of χ^2, α=0.01, df = 27-1-2=24					43.0
Table value of χ^2, α=0.005, df = 27-1-2=24					45.6

With α=0.05, α=0.01 and α=0.005, χ^2_{cal} is less than χ^2 : H_O is not rejected

TABLE 6-14 Probability and Predicted Frequencies of Weekday Peak

Group	Schedule Adherence, x (sec)	Observed frequency, f_O	Probability, P(x)	Predicted Frequency, F(x)	$\frac{(f_o - f_t)^2}{f_t}$
1	-809 ~-750	0	0.00000	0	0.0
2	-749 ~ -690	0	0.00000	0	0.0
3	-689 ~ -630	0	0.00000	0	0.0
4	-629 ~ -570	0	0.00090	0	0.1
5	-569 ~ -510	0	0.00230	0	0.2
6	-509 ~ -450	1	0.00580	0	0.9
7	-449 ~ -390	3	0.01230	1	5.6
8	-329 ~ -270	2	0.02480	2	0.1
9	-269 ~ -210	3	0.04200	3	0.0
10	-209 ~ -150	2	0.06770	5	1.5
11	-149 ~ -90	2	0.09210	6	2.9
12	-89 ~ -30	6	0.11860	8	0.5
13	-29 ~ 30	9	0.12910	9	0.0
14	31 ~ 90	13	0.12950	9	2.0
15	91 ~ 150	12	0.11990	8	1.8
16	151 ~ 210	6	0.09350	6	0.0
17	211 ~ 270	5	0.06930	5	0.0
18	271 ~ 330	2	0.04330	3	0.3
19	331 ~ 390	1	0.02570	2	0.3
20	391 ~ 450	1	0.01290	1	0.0
21	451 ~ 510	0	0.00610	0	0.4
22	511 ~ 570	0	0.00250	0	0.2
23	571 ~ 630	0	0.00090	0	0.1
24	631 ~ 690	0	0.00030	0	0.0
25	691 ~ 750	0	0.00000	0	0.0
26	751 ~ 810	0	0.00010	0	0.0
27	811 ~ 870	0	0.00000	0	0.0
Calculated Chi-square value, χ^2_{cal}					16.9
Table value of χ^2, α=0.05, df = 27-1-2=24					36.4
Table value of χ^2, α=0.01, df = 27-1-2=24					43.0
Table value of χ^2, α=0.005, df = 27-1-2=24					45.6

With α=0.05, α=0.01 and α=0.005, χ^2_{cal} is less than χ^2 : H_O is not rejected

TABLE 6-15 Probability and Predicted Frequencies of Weekday Non-Peak

Group	Schedule Adherence, x (sec)	Observed frequency, f_O	Probability, P(x)	Predicted Frequency, F(x)	$\frac{(f_o - f_t)^2}{f_t}$
1	-809 ~-750	0	0.00000	0	0.0
2	-749 ~ -690	0	0.00000	0	0.0
3	-689 ~ -630	0	0.00000	0	0.0
4	-629 ~ -570	0	0.00070	0	0.1
5	-569 ~ -510	0	0.00170	0	0.1
6	-509 ~ -450	1	0.00400	0	1.3
7	-449 ~ -390	0	0.00820	1	0.7
8	-329 ~ -270	1	0.01570	1	0.1
9	-269 ~ -210	1	0.02750	2	0.8
10	-209 ~ -150	3	0.04380	4	0.1
11	-149 ~ -90	6	0.06400	5	0.1
12	-89 ~ -30	6	0.08540	7	0.2
13	-29 ~ 30	11	0.10430	9	0.5
14	31 ~ 90	15	0.11640	10	2.6
15	91 ~ 150	15	0.11890	10	2.4
16	151 ~ 210	6	0.11090	9	1.2
17	211 ~ 270	6	0.09480	8	0.5
18	271 ~ 330	5	0.07620	6	0.3
19	331 ~ 390	2	0.05220	4	1.3
20	391 ~ 450	2	0.03400	3	0.3
21	451 ~ 510	1	0.02020	2	0.3
22	511 ~ 570	1	0.01110	1	0.0
23	571 ~ 630	1	0.00550	0	0.6
24	631 ~ 690	1	0.00250	0	2.9
25	691 ~ 750	1	0.00100	0	9.8
26	751 ~ 810	0	0.00060	0	0.1
27	811 ~ 870	0	0.00000	0	0.0
Calculated Chi-square value, χ^2_{cal}					26.4
Table value of χ^2, α=0.05, df = 27-1-2=24					36.4
Table value of χ^2, α=0.01, df = 27-1-2=24					43.0
Table value of χ^2, α=0.005, df = 27-1-2=24					45.6

With α=0.05, α=0.01 and α=0.005, χ^2_{cal} is less than χ^2 : H_O is not rejected

TABLE 6-16 Probability and Predicted Frequencies of Weekday Evening

Group	Schedule Adherence, x (sec)	Observed frequency, f_O	Probability, P(x)	Predicted Frequency, F(x)	$\frac{(f_o - f_t)^2}{f_t}$
1	-809 ~-750	0	0.00000	0	0.0
2	-749 ~ -690	0	0.00000	0	0.0
3	-689 ~ -630	0	0.00000	0	0.0
4	-629 ~ -570	0	0.00000	0	0.0
5	-569 ~ -510	0	0.00000	0	0.0
6	-509 ~ -450	0	0.00000	0	0.0
7	-449 ~ -390	0	0.00000	0	0.0
8	-329 ~ -270	0	0.00050	0	0.0
9	-269 ~ -210	0	0.00140	0	0.1
10	-209 ~ -150	0	0.00470	0	0.2
11	-149 ~ -90	1	0.01260	1	0.5
12	-89 ~ -30	1	0.03030	1	0.0
13	-29 ~ 30	4	0.05800	2	1.2
14	31 ~ 90	3	0.09580	4	0.2
15	91 ~ 150	4	0.13390	5	0.3
16	151 ~ 210	5	0.15880	6	0.3
17	211 ~ 270	6	0.15940	6	0.0
18	271 ~ 330	8	0.13850	6	1.1
19	331 ~ 390	4	0.09680	4	0.0
20	391 ~ 450	3	0.05880	2	0.2
21	451 ~ 510	1	0.03030	1	0.0
22	511 ~ 570	0	0.01330	1	0.5
23	571 ~ 630	0	0.00490	0	0.2
24	631 ~ 690	0	0.00150	0	0.1
25	691 ~ 750	0	0.00050	0	0.0
26	751 ~ 810	0	0.00000	0	0.0
27	811 ~ 870	0	0.00000	0	0.0
Calculated Chi-square value, χ^2_{cal}					5.0
Table value of χ^2, α=0.05, df = 27-1-2=24					36.4
Table value of χ^2, α=0.01, df = 27-1-2=24					43.0
Table value of χ^2, α=0.005, df = 27-1-2=24					45.6

With α=0.05, α=0.01 and α=0.005, χ^2_{cal} is less than χ^2 : H_O is not rejected

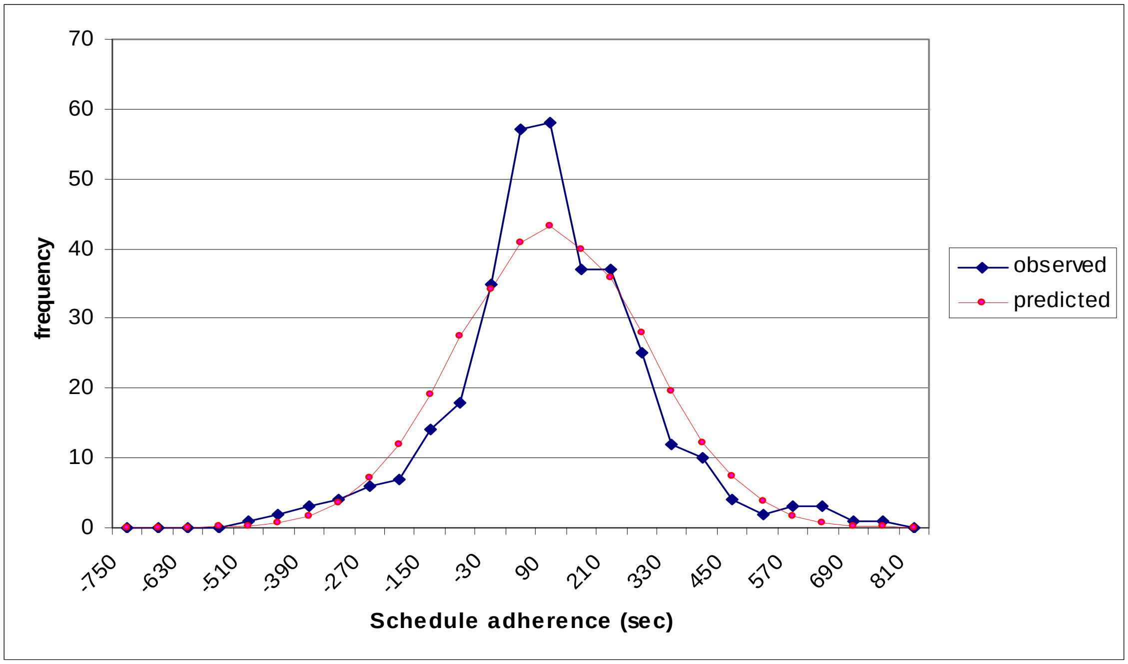

FIGURE 6-14 Observed and Predicted Schedule Adherence by Normal Distribution (Non-Clustering)

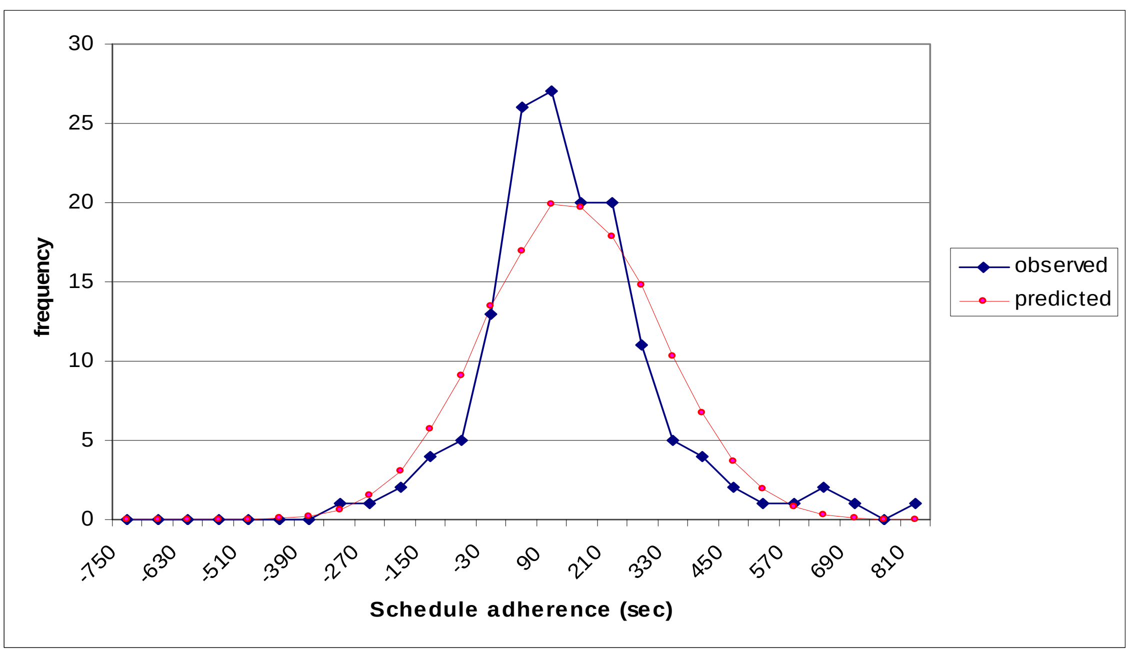

FIGURE 6-15 Observed and Predicted Schedule Adherence by Normal Distribution (Weekend)

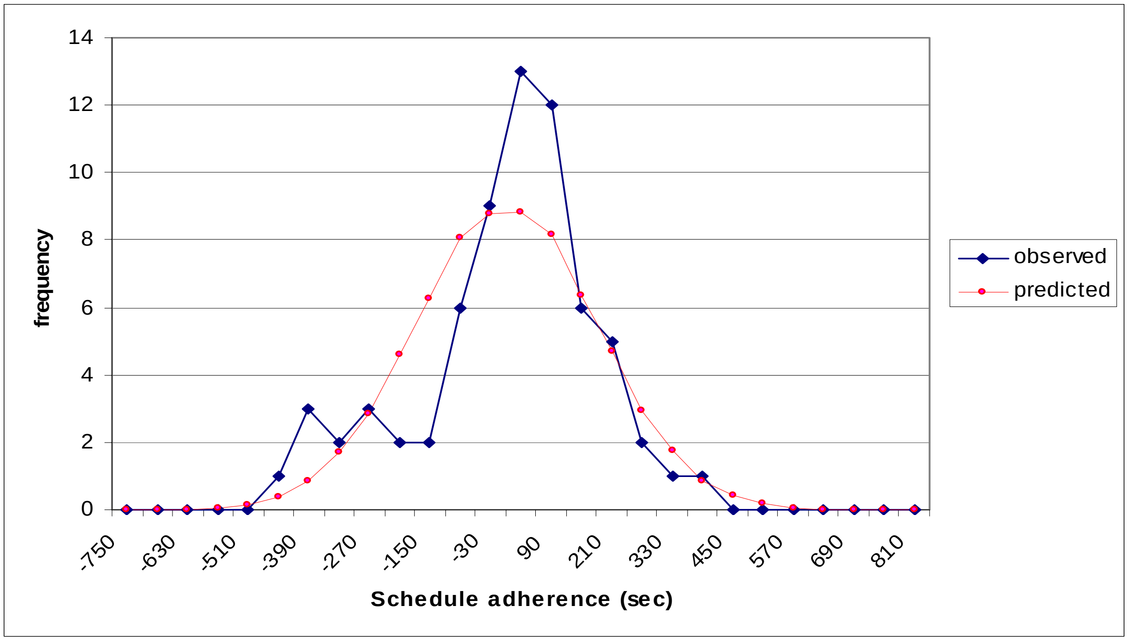

FIGURE 6-16 Observed and Predicted Schedule Adherence by Normal Distribution (Weekday Peak)

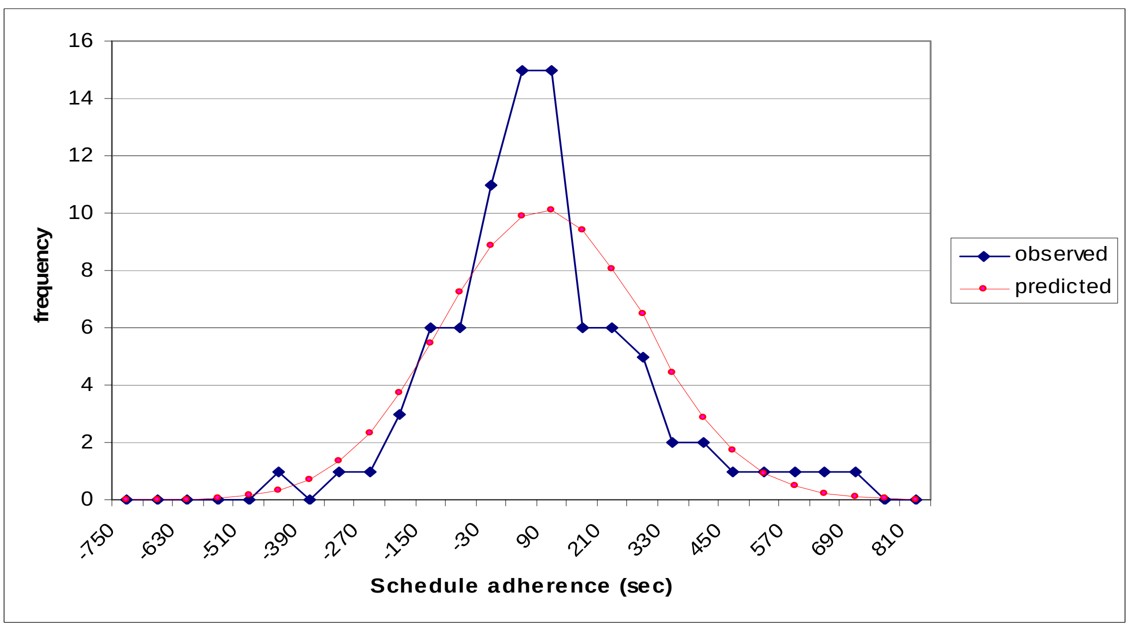

FIGURE 6-17 Observed and Predicted Schedule Adherence by Normal Distribution (Weekday Non-Peak)

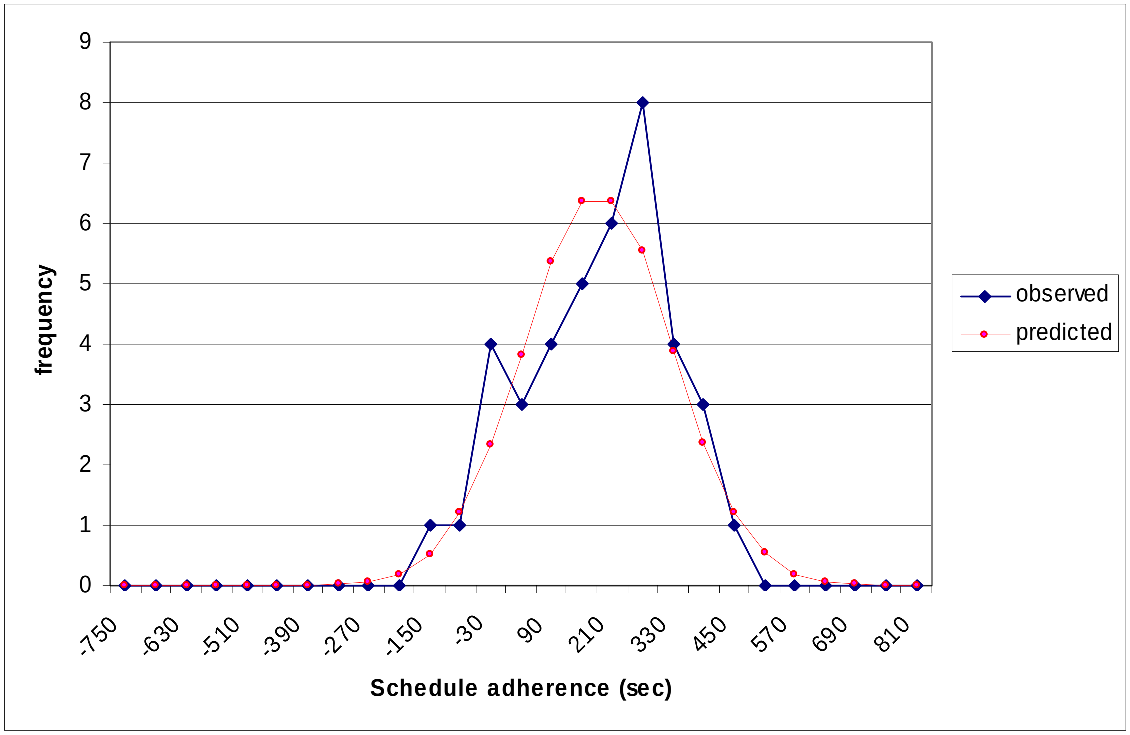

FIGURE 6-18 Observed and Predicted Schedule Adherence by Normal Distribution (Weekday Evening)

6.3 REAL-TIME APPLICATION

When using these models in real-time there are two fundamental questions that need to be answered. The first is what data should be input. The second is whether the model needs to be retrained as new data are obtained. In practice, because real-time vehicle location data from AVL systems can be obtained every second, the minimum update interval also is one second. In addition, the prediction model may need to have its parameters recalibrated at regular intervals. However, it would be prohibitively expensive and time consuming to retrain the model every second. It is also unlikely that there would be a noticeable benefit to transit agencies or their customers if this was done. In this section, the definition of real-time information from the user's viewpoint is defined. In addition, a methodology for updating the prediction model for real-time application is discussed. It should be noted that while the artificial neural network models developed in chapter IV are used as the prediction models for bus arrival time, the methodology could be applied to any model.

6.3.1 Real-Time Prediction

A real-time prediction model would have the process shown in FIGURE 6-19. The new or real-time AVL location data are added to the accumulated AVL data. The new accumulated AVL data would be used to train the ANN models. With the calibrated parameters from the new accumulated AVL data, the ANN models would be used to predict bus arrival time. Theoretically, the models could be updated or retrained every second because that is how often new AVL data could be obtained. Intuitively, this would be unrealistic because 1) calibrating the ANN models is a time consuming business and a one second update would be impossible, and 2) the marginal benefit from each successive model would be small. The focus of this following section is on identifying the best update interval. A fundamental question is how often the prediction models need to be retrained.

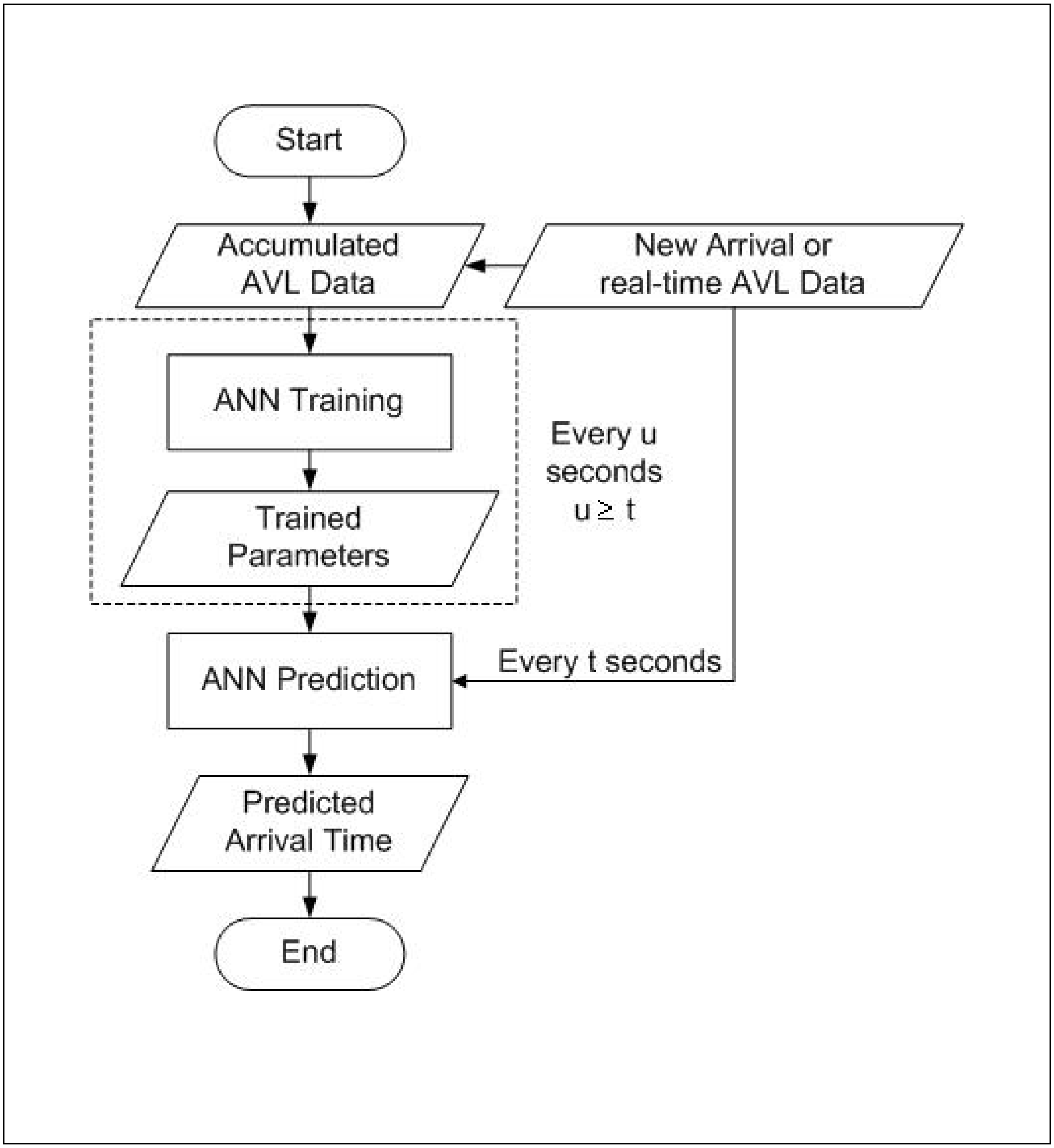

FIGURE 6-19 Process of Real-Time Prediction

6.3.2 Real-Time Service

From the perspective of the transit user, the real-time application should not to be retrained every second if the real-time application could give accurate (i.e. acceptable to transit user's tolerance) prediction of arrival time. Therefore, in this section an alternative concept is introduced, called real-time service. Real-time service indicates a prediction model that predicts the bus arrival time with pre-calibrated parameters and the pre-calibrated parameters can be updated at regular intervals. The exact interval, (i.e. 30 minutes or 3 hours or 3 days) would be obtained through a sensitivity analysis.

6.3.2.1 Real-Time Prediction vs. Real-Time Service

In this section, the benefits and costs of real-time service are studied. In this dissertation, concepts of real-time prediction and real-time service are examined. Real-time prediction means that the prediction model is retrained on a regular basis as AVL data are obtained. For illustrative purpose, t is set equal to one second. Once the model is retrained the arrival time is predicted. In contrast, real-time service means that the bus arrival time is predicted using the new AVL data and a calibrated ANN model that is updated every u seconds. Note that u is larger than t.

FIGURE 6-20 shows the difference between a real-time prediction model and a real-time service model. Model A is a real-time prediction model in that the ANN is retrained every t seconds. This model uses newly calibrated bias (B_n) and weight (W_n) parameters that are updated every t seconds. Model B is a real-time service model that predicts the arrival time with the previously calibrated bias (B_o) and weight (W_o).

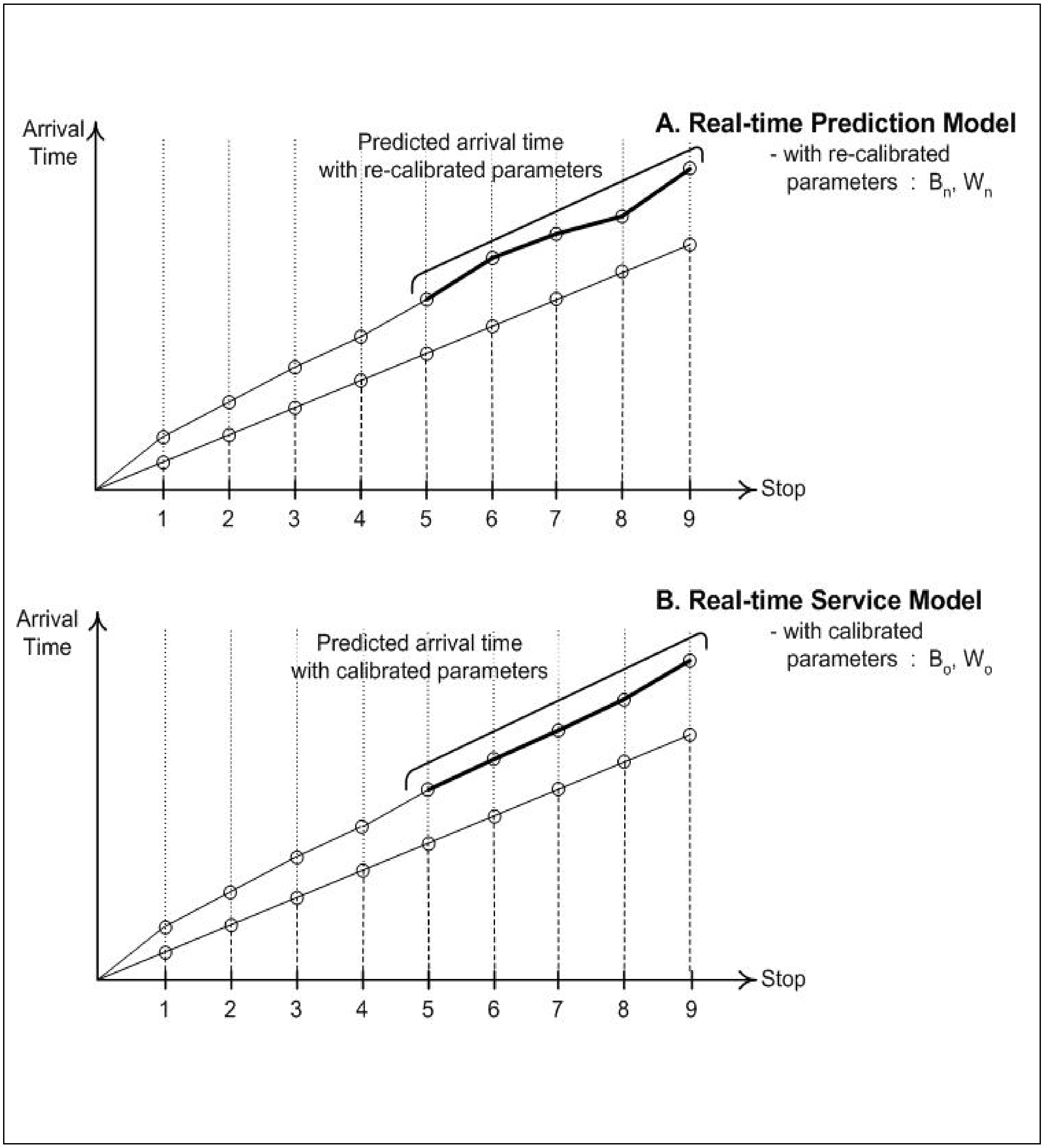

FIGURE 6-20 Real-time Prediction Model vs. Real-time Service Model

6.3.2.2 Simulation of Real-time Prediction Model and Real-time Service Model

In this section, a methodology for comparing the real-time prediction model and real-time service model is developed. In order to test the different approaches I observations from the test bed are used. Of the total, J observations are used in the database set while the remaining K observations represented "new" AVL data. In FIGURE 6-21, the right side of the flow chart represents the real-time prediction model, and the left side of the flow chart represents the real-time service model. In the case of real-time prediction, the real-time data are uploaded every second and therefore the training and prediction of the ANN models are repeated. In contrast, in the case of the real-time service model, the new AVL data are only used to predict arrival time using previously calibrated models. By comparing the predicted arrival time from these two models, the feasibility of the real-time service model can be identified. In order to identify the acceptable update interval, a specific update interval such as 30 minutes, 1 hour, 3 hours, etc can be tested. For example, the pre-calibrated parameters with real-time data obtained 30 minutes previously can be used to predict the bus arrival time and it can be accepted if the predicted arrival time from the real-time service model is not significantly different from that from the real-time prediction model. However, the results of this type of sensitivity analysis simulation are not included in this dissertation, because a sufficient amount of data was not available for making a valid comparison.

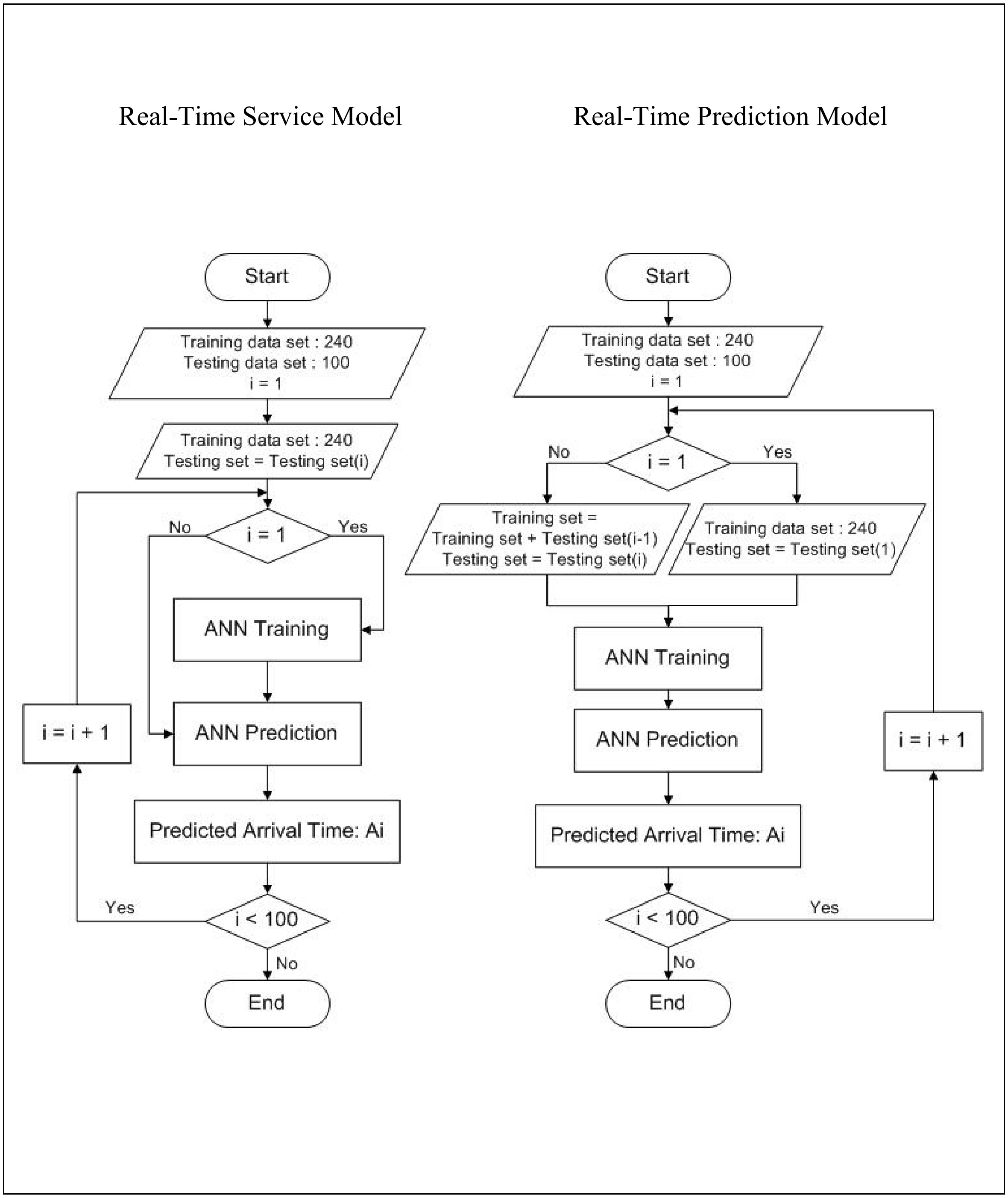

FIGURE 6-21 Real-Time Service Availability

6.4 CONCLUDING REMARKS

Having access to accurate and timely travel time information would be very useful to transit patrons as well as transit authorities. Because variability in travel time (both waiting and on-board) is extremely important for transit choice it would also be useful to extend the model to provide not only estimates of the travel time but also prediction intervals. In chapter V, three different prediction models for bus arrival time were evaluated and statistically tested. It was found that the artificial neural network models gave best results in terms of prediction accuracy. Subsequently, in this chapter, based on the artificial neural network (ANN) models, methodology for identifying the prediction interval of the bus arrival time and the probability of being on time for a given bus was developed. Because ANNs are non parametric models, conventional techniques for prediction interval can not be used. Consequently, a new computer-intensive method, a bootstrap technique, was used to obtain prediction interval of bus arrival time.

In this chapter, one hundred, two hundred, and one thousand bootstrap samples were used to obtain prediction interval of bus arrival time. It was found that the two hundred and one thousand bootstrap samples gave better results compared to a one hundred bootstrap sample. However, the difference between two hundred and one thousand was not significant. Consequently, a two hundred bootstrap sample was adopted. This prediction interval information could be provided to transit users with the bus arrival time. In addition, the transit agencies could also use this information to manage their on-time performance and to apply transit signal priority techniques.

In addition, on-time performance of a bus is very valuable to transit operators because customers use this to measure quality of service. It would be extremely important to identify, in real-time, whether a given bus is on schedule or not. To measure the on-time performance, the probability of a bus being on time is required. The prediction interval can be used to identify, in real-time, the probability that a given bus is on time. In addition to the prediction interval of bus arrival time, the probability that a given bus is on time was calculated. The probability density function of schedule adherence seemed to be the gamma distribution or the normal distribution. To determine which distribution is the best fit for schedule adherence, a chi-squared goodness-of-fit test was used. It was found that a normal distribution adequately estimates schedule adherence. With the normal distribution, the probability of a bus being on time, being ahead schedule, and being behind schedule can be estimated. Transit agencies could use also this information to manage their on-time performance and to apply new strategies such as transit priority signal. The information can be used to improve operating efficiency (i.e. better headway planning etc.) and increase revenue (resulting from becoming more attractive to users). It leads to improve schedule reliability resulting in better service quality. In addition, the passenger waiting time can be reduced.

In the last section, a methodology for using the techniques in a real-time application was discussed. The ANN models is said to be difficult to use in real-time applications because the running time for training ANN model is relatively long. Therefore an alternative method, a real-time service model was introduced and examined. Real-time prediction models retrain ANN models and predict bus arrival time on a regular basis as AVL data are obtained. Once the model is retrained the arrival time is predicted. In contrast, real-time service means that the bus arrival time is predicted using the new AVL data and the calibrated parameters from the previous ANN training. While a methodology for identifying the update interval was provided there was, unfortunately, insufficient data to do the analysis.

CHAPTER VII

CONCLUSIONS

7.1 SUMMARY

The problem statement of this research identified four main needs: 1) to develop a bus arrival time prediction model using AVL data; 2) to explicitly consider traffic congestion, dwell times at stops, and schedule adherence in this model; 3) to provide prediction intervals for the model; and 4) to provide probability of a bus being on time. A summary of how each of these issues was addressed, the main conclusions, and recommendations for further research is provided in the following sections.

This research developed a model to predict bus arrival times. Three different prediction models were developed and tested. It was found that the ANN models gave the smallest prediction errors in terms of prediction accuracy. Because the ANN models are non-parametric models, a bootstrap technique was used to calculate the prediction intervals of bus arrival time. To estimate the probability that a given bus is on time, a gamma and a normal distribution were examined with the chi-squared goodness-of-fit test. It was found that the normal distribution was the best at identifying the probability of a bus being on time, being ahead of schedule, and being behind of schedule. Finally, to provide real-time information, a real-time service model was introduced. Each of these steps is briefly detailed below.

7.2 BUS ARRIVAL TIME PREDICTION MODELS

To provide accurate and timely bus arrival time information for transit passengers, bus arrival time prediction models were developed. Historical data based models, multi linear regression models, and artificial neural network models were developed. These three models were calibrated and tested on a transit route in Houston, Texas. The input to the models consisted of arrival time, dwell time, and schedule adherence at each bus stop.

7.2.1 Prediction Intervals of Bus Arrival Time

The ability to provide accurate and timely travel time information would be very useful to transit patrons as well as transit authorities. Because variability in travel time (both waiting and on-board) is extremely important for transit choices, it would also be useful to extend the model to provide not only estimates of travel time but also prediction intervals. With the ANN models, the prediction intervals of bus arrival time were calculated. Because the ANN models are non parametric models, conventional techniques for prediction intervals can not be used. Consequently, a newly developed computer-intensive method, the bootstrap technique was used to obtain prediction intervals of bus arrival time. 100, 200, and 1000 bootstrap samples were used to obtain prediction interval of bus arrival time.

7.2.2 Probability of a Bus Being On-Time

On-time performance of a bus is very important to transit operators to provide quality service to transit passengers. To measure the on-time performance, the probability of a bus being on time is required. In addition to the prediction interval of bus arrival time, the probability that a given bus is on time was calculated. The probability density function of schedule adherence seemed to be the gamma distribution or the normal distribution. To determine which distribution is the best fit for the schedule adherence, a chi-squared goodness-of-fit test was used.

7.3 CONCLUSIONS

7.3.1 Bus Arrival Time Prediction Models

It was found that the artificial neural network models performed considerably better than either historical data based models or multi linear regression models. It was hypothesized that the ANN was able to identify the complex non-linear relationship between travel time and the independent variables and this led to superior results. To provide useful and trustworthy information, passenger demand, signals, delay due to traffic congestion or accidents, etc., must be considered. Especially in larger cities, the above factors can significantly influence the predicted arrival time. In this dissertation, traffic congestion, dwell time, and schedule adherence were considered, and this contributed to predicting bus travel time in larger cities. In order to consider traffic congestion and different travel patterns, the data set is clustered into four time periods: weekend, weekday peak, weekday non-peak, and weekday evening. The three input variables and link travel time were also clustered by time period.

7.3.2 Prediction Interval of Bus Arrival Time

Two hundred and one thousand bootstrap samples gave better result in comparison to a one hundred bootstrap sample. However, the difference between two hundred and one thousand was not significant. Consequently, a two hundred bootstrap sample was enough to obtain the prediction interval of bus arrival time with reasonable value.

7.3.3 Probability of a Bus Being On-Time

In brief, the normal distribution estimates well the schedule adherence. With the normal distribution, the probability of a bus being on time, being ahead schedule, and being behind schedule can be estimated.

7.4 FUTURE STUDY

While the results are encouraging, there are still a number of extensions to the model that should be studied. It is hypothesized that if other real-time data were available, such as variability in passenger demand at any given bus stop, traffic congestion measures, signals including progression, delay due to traffic congestion or accident, incident information, exclusive HOV or bus lanes, etc., arrival time predictions could be improved. Note that this type of information is typically unavailable on urban street networks. However, as new ITS data collection techniques, such as cell phone monitoring improve, this will not be an impediment.

It is hypothesized that universal function approximator models such as ANN would work best for the arrival time prediction problem. However, these models require some effort for calibrating, and the best model for a given situation would have to be determined on a case by case basis. A process that would "self-calibrating" would be extremely useful for these types of applications.

Recently, the use of AVI data, has been rapidly increasing, itself. At some stage, the prediction technique of auto travel time using AVL data, not AVI data, could be used using the relationship between bus and auto travel time. There are current attempts to use AVL data as probe data for auto travel time data. Thus, there should be opportunities to extend bus travel time prediction model to use for the prediction of auto travel time.

The methodology for real-time application was discussed. The ANN models showed that it is difficult to predict for real-time application because the running time for training ANN is relatively long. Therefore, an alternative method, a real-time service model was introduced. While real-time prediction models retrain ANN models and predict bus arrival time when the new arrival of real-time data are uploaded, real-time service models do not retrain ANN and use parameters which are calibrated before. This dissertation did not include the results from the simulation of real-time prediction model and real-time service model because the data required for an adequate study were not available. Note that different data than that used for training and testing the ANN models are required to simulate the real-time prediction and real-time service model. How often these types of models should be recalibrated is a subject of future study.

GLOSSARY

Arrival Time: The time a bus arrives at a specific bus stop.

Artificial Neural Network Models (ANN Models): Information processing structure whose design is motivated by the design and functioning of the human brain and components thereof.

Advanced Public Transportation Systems (APTS): Intelligent Transportation Systems (ITS) technology that is designed to improve transit services through advanced vehicle operations, communications, customer service and market development.

Advanced Traveler Information Systems (ATIS): The use of intelligent transportation systems technologies and communication methods for providing information to travelers.

Automatic Vehicle Location (AVL) Systems: Technology that tracks the current location of fleet vehicles to assist in dispatching, maintaining schedules, answering specific customer inquiries, etc.

Bootstrap Technique: A computer-intensive method used to make statistical inferences

Bus Stop: A place where passengers can board or alight from the bus, usually identified by a sign.

Corridor: A broad geographical band that follows a general directional flow connecting major sources of trips that may contain a number of streets, highways and transit route alignments.

Dwell Time: The scheduled time a vehicle or train is allowed to discharge and take on passengers at a stop, including opening and closing doors.

Global Positioning Systems (GPS): A satellite-based navigation system, funded by and controlled by the U.S.

Headway: Time interval between vehicles moving in the same direction on a particular route.

Intelligent Transportation Systems (ITS): ITS applies state-of-the-art and emerging technologies to provide more efficient and effective solutions to current multimodal transportation problems, as well as anticipate and address future transportation demands through an intermodal, strategic approach.

Mean Absolute Percentage Error (MAPE): The average percentage difference between the observed value and the predicted value.

$$MAPE = \frac{1}{n}\sum_{i}^{n}\frac{|y_i - y_o|}{y_o} \times 100\%$$

where,

y_i	Predicted value;
y_o	Observed value;
n	Number of observations.

Model: An analytical tool (often mathematical) used by transportation planners to assist in making forecasts of land use, economic activity, travel activity and their effects on the quality of resources such as land, air and water.

Off-Peak Period: Non-rush periods of the day when travel activity is generally lower and less transit service is scheduled.

Peak Period: Morning and afternoon time periods when transit riding is heaviest

Ridership: The number of rides taken by people using a public transportation system in a given time period.

Schedule Adherence: Transit vehicles have a predefined schedule to follow. Schedule adherence can be calculated by subtracting the scheduled data from the actual arrival time.

Time Check Point: Transit bus has predetermined schedule along the route. On the schedule table, some specific bus stops have the scheduled arrival time. Transit passengers assume that the bus arrives at the time and the drivers should keep on schedule. These specific bus stops are called time check points.

Travel Time: Time to traverse a route between any two points of interest.

Validation: The process to determine whether a model provides an accurate representation of the real-world system under study. It involves comparing the model output to generated analytical solutions or to collected field data.

NOTATION

α_{jt} = Average arrival time at stop j departing during time period t

a_1 = Slope of the line connecting stop 1 and stop 9

a_2 = Slope of the line connecting point P1 and point P2

b = Test bed.

= 1 (Test bed 1: downtown area) and

= 2 (Test bed 2: north area)

b_1 = The point of contact of the line connecting stop 1 and stop 9

b_2 = The point of contact of the line connecting point P1 and point P2

A_{jkt} = Arrival time for bus k at bus stop j departing during time period t

dx = Difference between x coordinate of stop 1 and x coordinate of stop 9

dy = Difference between y coordinate of stop 1 and y coordinate of stop 9

D_{jk} = Departure time of bus k at bus stop j

L_{Mjk} = Distance from stop M to stop j for bus k

M = Current bus stop. i.e. from 1 to N-1

m = Model classification.

= 1 (Historical data based model),

= 2 (Regression model), and

= 3 (Artificial neural network model)

M_{mbtn} = Model n with m classification, b test bed, and t time period.

n = Model number

N_b = Number of last bus stop of test bed b

=1 (Downtown area) and

=2 (North area)

P_{jk} = Predetermined/Scheduled arrival time of bus k at bus stop j

S_{jkt} = Schedule adherence of bus k at bus stop j departing during time period t

τ_{Mjkt} = Travel time from current stop M to stop j for bus k

departing during time period t, j = M, N_b;

t = Time period.

= 1 (weekend period),

= 2 (weekday peak period),

= 3 (weekday non-peak period), and

= 4 (weekday evening period)

T = Number of time periods. For the test bed, this is equal to 4 ;

weekend, weekday peak, weekday off-peak, and weekday evening

Ti = GPS Time of data i

ϖ_{jt} = Average dwell time at stop j departing during time period t

W_{jkt} = Dwell time of bus k at bus stop j departing during time period t

x_{P2} = x coordinate of point P2

y_{P2} = y coordinate of point P2

$X_{i-1,\, i+1}$: Distance between data i-1 and data i+1.

$X_{i-1,\, i+1}$ is the summation of $X_{i-1,\, i}$ and $X_{i,\, i+1}$.

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APPENDIX A

BUS SCHEDULE BY BUS STOP FOR THE STUDY PERIOD

TABLE A- 1 Bus Schedule by Bus Stop for the Study Period (Downtown Area, Weekday)

Time Period	Stop 1	Stop 9
Non-Peak	5:25 A.M.	5:33 A.M.
	5:55 A.M.	6:03 A.M.
Peak	6:25 A.M.	6:34 A.M.
	6:55 A.M.	7:04 A.M.
	7:25 A.M.	7:34 A.M.
	7:55 A.M.	8:04 A.M.
Non-Peak	8:25 A.M.	8:33 A.M.
	9:10 A.M.	9:18 A.M.
	10:10 A.M.	10:18 A.M.
	11:10 A.M.	11:18 A.M.
	12:10 P.M.	12:18 P.M.
	1:10 P.M.	1:18 P.M.
	2:10 P.M.	2:18 P.M.
	3:08 P.M.	3:16 P.M.
	3:53 P.M.	4:01 P.M.
Peak	4:23 P.M.	4:32 P.M.
	4:53 P.M.	5:02 P.M.
	5:23 P.M.	5:32 P.M.
	5:53 P.M.	6:02 P.M.
Non-Peak	6:23 P.M.	6:31 P.M.
	6:53 P.M.	7:06 P.M.
Evening	7:23 P.M.	7:36 P.M.
	7:53 P.M.	8:06 P.M.
	8:53 P.M.	9:06 P.M.
	9:53 P.M.	10:06 P.M.
	10:53 P.M.	11:06 P.M.
	11:53 P.M.	12:06 A.M.

Weekday: Monday through Friday

Peak period: 6:15 A.M. ~ 8:15 A.M. and 4:15 P.M. ~ 6:10 P.M.

Non-peak period: Before 6:15 A.M., 8:15 A.M. ~ 4:15 P.M., and
6:10 P.M. ~ 7:15 P.M.

Evening period: After 7:15 P.M.

TABLE A- 2 Bus Schedule by Bus Stop for the Study Period (Downtown Area, Weekend)

Time Period	Stop 1	Stop 9
Saturday	5:15 A.M.	5:23 A.M.
	6:15 A.M.	6:23 A.M.
	7:15 A.M.	7:23 A.M.
	8:15 A.M.	8:23 A.M.
	9:15 A.M.	9:23 A.M.
	10:15 A.M.	10:23 A.M.
	11:15 A.M.	11:23 A.M.
	12:15 P.M.	12:23 P.M.
	1:15 P.M.	1:23 P.M.
	2:15 P.M.	2:23 P.M.
	3:15 P.M.	3:23 P.M.
	4:15 P.M.	4:23 P.M.
	5:15 P.M.	5:23 P.M.
	6:15 P.M.	6:23 P.M.
	7:24 P.M.	7:37 P.M.
	8:24 P.M.	8:37 P.M.
	9:24 P.M.	9:37 P.M.
	10:24 P.M.	10:37 P.M.
	11:24 P.M.	11:37 P.M.
	12:24 A.M.	12:37 A.M.
Sunday	7:17 A.M	7:25 A.M.
	8:17 A.M	8:25 A.M.
	9:17 A.M	9:25 A.M.
	10:17 A.M	10:25 A.M.
	11:17 A.M	11:25 A.M.
	12:17 P.M	12:25 P.M.
	1:17 P.M	1:25 P.M.
	2:17 P.M	2:25 P.M.
	3:17 P.M	3:25 P.M.
	4:17 P.M	4:25 P.M.
	5:17 P.M	5:25 P.M.
	6:24 P.M	6:37 P.M.
	7:24 P.M	7:37 P.M.

Weekend: Saturday and Sunday

TABLE A- 3 Bus Schedule by Bus Stop for the Study Period (North Area, Weekday)

Time Period	Stop 6	Stop 20
Non-Peak	5:11 A.M.	5:17 A.M.
	5:41 A.M.	5:47 A.M.
Peak	6:11 A.M.	6:17 A.M.
	6:39 A.M.	6:46 A.M.
	7:08 A.M.	7:15 A.M.
	7:38 A.M.	7:45 A.M.
	8:08 A.M.	8:15 A.M.
Non-Peak	8:53 A.M.	9:00 A.M.
	9:54 A.M.	10:01 A.M.
	10:54 A.M.	11:01 A.M.
	11:54 A.M.	12:01 P.M.
	12:54 P.M.	1:01 P.M.
	1:54 P.M.	2:01 P.M.
	2:52 P.M.	2:59 P.M.
	3:37 P.M.	3:44 P.M.
	4:06 P.M.	4:13 P.M.
Peak	4:36 P.M.	4:43 P.M.
	5:06 P.M.	5:13 P.M.
	5:36 P.M.	5:43 P.M.
	6:07 P.M.	6:14 P.M.
Non-Peak	6:38 P.M.	6:45 P.M.
	7:10 P.M.	7:16 P.M.
Evening	7:40 P.M.	7:46 P.M.
	8:40 P.M.	8:46 P.M.
	9:40 P.M.	9:46 P.M.
	10:40 P.M.	10:46 P.M.
	11:38 P.M.	11:45 A.M.

Weekday: Monday through Friday

Peak period: 6:15 A.M. ~ 8:15 A.M. and 4:15 P.M. ~ 6:10 P.M.

Non-peak period: Before 6:15 A.M., 8:15 A.M. ~ 4:15 P.M., and
6:10 P.M. ~ 7:15 P.M.

Evening period: After 7:15 P.M.

TABLE A- 4 Bus Schedule by Bus Stop for the Study Period (North Area, Weekend)

Time Period	Stop 6	Stop 20
Saturday	5:00 A.M.	5:07 A.M.
	6:00 A.M.	6:07 A.M.
	7:00 A.M.	7:07 A.M.
	8:00 A.M.	8:07 A.M.
	9:00 A.M.	9:07 A.M.
	10:00 A.M.	10:07 A.M.
	11:00 A.M.	11:07 A.M.
	12:00 P.M.	12:07 P.M.
	1:00 P.M.	1:07 P.M.
	2:00P.M.	2:07 P.M.
	3:00 P.M.	3:07 P.M.
	4:00 P.M.	4:07 P.M.
	5:00 P.M.	5:07 P.M.
	6:00 P.M.	6:07 P.M.
	7:09 P.M.	7:16 P.M.
	8:09 P.M.	8:16 P.M.
	9:09 P.M.	9:16 P.M.
	10:09 P.M.	10:16 P.M.
	11:09 P.M.	11:16 P.M.
	12:09 A.M.	12:16 A.M.

Weekend: Saturday

APPENDIX B

MAPE RESULT OF ARTIFICIAL NEURAL NETWORK MODELS WITH DIFFERENT TRAINING AND LEARNING FUNCTIONS

TABLE B-1 MAPE of ANN Models with Different Training Functions (Batch Training with Weight and Bias Learning Rule)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	287.36	339.03	339.99	341.69	319.33
Model 2	173.72	267.41	229.37	234.61	236.02
Model 3	140.58	183.44	221.59	169.01	151.71
Model 4	120.67	145.14	160.55	114.66	95.549
Model 5	49.07	52.677	59.055	64.105	70.361
Model 6	55.27	43.444	100.93	87.481	37.617
Model 7	25.67	30.516	38.478	58.595	19.722
Model 8	23.37	13.12	22.721	18.642	20.944
Average	109.46	134.35	146.59	136.10	118.91

TABLE B-2 MAPE of ANN Models with Different Training Functions (BFGS Quasi-Newton Backpropagation)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	23.69	20.98	19.08	20.56	20.92
Model 2	16.68	15.09	12.17	20.27	15.95
Model 3	15.85	12.18	11.42	17.01	16.12
Model 4	13.93	10.95	9.16	13.47	10.80
Model 5	8.87	8.37	9.25	10.15	8.25
Model 6	8.13	8.58	6.14	7.36	7.58
Model 7	6.39	6.87	6.16	7.32	6.10
Model 8	4.66	5.86	5.00	7.51	3.77
Average	12.28	11.11	9.80	12.96	11.19

TABLE B-3 MAPE of ANN Models with Different Training Functions (Bayesian Regularization)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	14.75	11.88	6.23	18.52	11.03
Model 2	5.56	4.40	6.41	8.36	10.27
Model 3	4.21	3.23	6.72	7.72	5.06
Model 4	4.95	2.64	4.45	5.94	4.11
Model 5	3.34	2.49	4.47	6.64	2.58
Model 6	2.50	2.96	3.58	6.97	2.87
Model 7	2.71	3.07	3.80	5.41	2.43
Model 8	2.66	2.31	2.91	3.49	2.29
Average	5.09	4.12	4.82	7.88	5.08

TABLE B-4 MAPE of ANN Models with Different Training Functions (Powell-Beale Conjugate Gradient Backpropagation)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	20.35	23.72	19.38	23.33	16.90
Model 2	16.81	14.18	13.73	19.19	16.64
Model 3	15.36	13.87	10.23	15.23	14.53
Model 4	12.62	10.37	9.33	14.65	11.76
Model 5	8.47	9.77	8.71	11.64	7.90
Model 6	7.88	7.43	6.53	9.62	6.10
Model 7	6.55	7.03	6.69	9.30	4.09
Model 8	5.13	6.02	5.31	7.77	5.12
Average	11.64	11.55	9.99	13.84	10.38

TABLE B-5 MAPE of ANN Models with Different Training Functions (Fletcher-Powell Conjugate Gradient Backpropagation)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	24.14	22.74	15.90	21.86	16.53
Model 2	17.72	14.95	12.02	15.85	16.17
Model 3	14.86	13.23	11.05	16.79	15.27
Model 4	13.45	9.99	9.45	15.09	11.61
Model 5	10.77	8.99	8.61	15.35	5.65
Model 6	8.98	8.79	6.52	11.77	8.46
Model 7	7.06	7.02	5.46	7.31	5.65
Model 8	6.58	6.22	5.33	7.31	5.43
Average	12.94	11.49	9.29	13.92	10.60

TABLE B-6 MAPE of ANN Models with Different Training Functions (Gradient Descent Backpropagation)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	381.93	402.29	318.94	317.39	409.05
Model 2	252.64	237.89	315.72	279.78	252.33
Model 3	147.96	177.15	155.59	218.48	180.74
Model 4	139.35	159.53	157.32	110.79	92.86
Model 5	124.48	116.82	128.65	110.46	89.12
Model 6	62.55	54.61	68.65	61.82	50.34
Model 7	45.96	74.88	90.47	34.71	55.67
Model 8	18.54	27.16	68.69	34.28	27.78
Average	146.68	156.29	163.00	145.96	144.74

TABLE B-7 MAPE of ANN Models with Different Training Functions (Gradient Descent with Adaptive Learning Rate Backpropagation)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	50.37	66.06	33.58	38.93	91.43
Model 2	37.71	42.08	54.24	42.32	28.80
Model 3	31.71	29.25	38.12	29.77	18.67
Model 4	25.93	26.75	36.06	21.14	21.59
Model 5	24.47	24.17	13.41	26.78	12.85
Model 6	24.73	18.66	11.67	21.17	12.96
Model 7	17.67	14.33	9.51	16.81	14.61
Model 8	10.99	10.88	9.77	15.30	12.28
Average	27.95	29.02	25.79	26.53	26.65

TABLE B-8 MAPE of ANN Models with Different Training Functions (Levenberg-Marquardt Backpropagation)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	13.62	10.88	8.76	18.55	12.50
Model 2	5.07	4.48	5.47	9.31	7.42
Model 3	4.71	4.05	6.10	8.55	9.15
Model 4	4.14	3.84	4.23	7.75	6.91
Model 5	4.89	2.82	5.29	6.26	6.16
Model 6	3.89	2.67	4.13	7.13	7.14
Model 7	3.25	2.57	3.61	6.34	5.03
Model 8	1.87	2.91	5.03	5.29	3.13
Average	5.18	4.28	5.33	8.65	7.18

TABLE B-9 MAPE of ANN Models with Different Training Functions (One Step Secant Backpropagations)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	25.41	24.46	21.43	21.83	23.32
Model 2	19.96	16.05	16.54	17.73	16.01
Model 3	16.36	15.00	12.84	18.23	18.28
Model 4	16.09	11.47	12.60	18.69	12.13
Model 5	13.70	12.30	11.10	16.29	9.95
Model 6	12.02	10.41	9.39	13.34	10.40
Model 7	8.24	9.11	7.82	9.98	9.60
Model 8	6.85	6.58	5.59	9.76	7.08
Average	14.83	13.17	12.16	15.73	13.34

TABLE B-10 MAPE of ANN Models with Different Training Functions (Resilient Backpropagation)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	25.80	23.73	14.99	24.22	18.38
Model 2	17.34	13.86	16.12	19.19	18.14
Model 3	15.38	14.17	8.86	16.43	15.87
Model 4	13.31	11.73	15.52	19.32	14.35
Model 5	10.07	8.12	11.14	15.76	10.68
Model 6	7.43	7.13	5.41	9.23	7.97
Model 7	7.09	8.19	6.24	9.58	9.29
Model 8	4.88	8.45	3.84	8.19	7.23
Average	12.66	11.92	10.26	15.24	12.74

TABLE B-11 MAPE of ANN Models with Different Training Functions (Sequential Order Incremental Update)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	599.15	502.43	563.42	501.99	348.05
Model 2	270.77	486.37	420.62	356.57	283.76
Model 3	342.42	383.90	355.89	441.58	230.71
Model 4	344.58	296.51	296.37	169.85	242.88
Model 5	304.68	322.91	230.17	277.07	95.94
Model 6	230.09	223.48	215.08	68.56	107.14
Model 7	146.95	195.73	153.60	143.75	96.17
Model 8	119.31	65.14	178.06	167.98	79.53
Average	294.74	309.56	301.65	265.92	185.52

TABLE B- 12 MAPE of ANN Models with Different Training Functions (Scaled Conjugate Gradient Backpropagation)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	23.74	24.37	17.86	22.29	20.38
Model 2	18.46	15.63	14.78	15.74	17.24
Model 3	16.24	13.86	12.42	19.28	15.33
Model 4	13.24	10.51	9.06	15.00	12.19
Model 5	8.92	7.84	9.71	13.73	8.46
Model 6	10.18	9.66	6.97	10.88	8.94
Model 7	7.41	8.19	7.95	6.31	5.99
Model 8	7.20	5.62	6.60	8.44	5.65
Average	13.17	11.96	10.67	13.96	11.77

TABLE B-13 MAPE of ANN Models with Different Training Functions (Batch Training with Weight and Bias Learning Rule)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	829.14	587.55	699.25	1012.40	766.14
Model 2	421.25	615.84	503.43	572.67	491.26
Model 3	352.05	393.34	457.98	516.53	429.25
Model 4	308.99	365.58	420.63	333.19	404.38
Model 5	318.38	382.15	293.15	363.55	382.82
Model 6	260.19	347.93	253.90	298.21	205.40
Model 7	279.90	301.22	264.70	298.37	305.08
Model 8	270.36	306.44	276.60	279.03	283.04
Model 9	256.62	285.71	222.30	243.49	238.66
Model 10	252.42	242.37	207.95	228.29	214.90
Model 11	206.01	200.28	228.89	220.99	238.77
Model 12	175.81	217.24	184.78	191.14	186.80
Model 13	188.87	212.13	153.96	195.18	222.16
Model 14	166.03	168.47	164.25	157.48	133.04
Model 15	174.19	196.93	140.96	147.03	179.98
Model 16	139.53	136.79	146.03	141.20	131.30
Model 17	127.54	97.94	109.72	141.74	121.91
Model 18	110.97	96.18	111.52	94.60	130.62
Model 19	81.40	105.83	93.87	86.41	97.76
Model 20	68.60	102.57	86.00	63.35	93.96
Model 21	51.80	81.485	94.87	61.79	53.14
Model 22	45.83	29.28	52.82	40.82	50.07
Model 23	29.93	28.57	14.45	23.66	30.70
Model 24	17.38	10.79	16.73	14.28	17.49
Average	213.88	229.69	216.61	238.56	225.36

TABLE B-14 MAPE of ANN Models with Different Training Functions (BFGS Quasi-Newton Backpropagation)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	17.56	13.32	16.17	17.63	18.08
Model 2	13.34	15.50	11.97	14.90	14.94
Model 3	11.51	13.47	11.44	15.14	14.17
Model 4	11.20	9.99	9.96	11.52	11.76
Model 5	10.23	10.78	8.25	12.01	10.90
Model 6	8.51	8.85	8.13	10.50	10.91
Model 7	8.55	10.64	9.69	12.10	11.24
Model 8	8.16	7.10	9.08	8.95	11.63
Model 9	9.69	8.93	7.58	10.04	9.49
Model 10	8.09	9.60	7.08	8.58	8.54
Model 11	11.37	9.91	7.52	8.17	8.39
Model 12	10.62	5.96	8.80	7.33	8.17
Model 13	8.47	5.70	5.92	7.03	8.21
Model 14	9.20	6.22	8.43	7.10	7.01
Model 15	8.10	6.05	6.39	7.52	6.37
Model 16	5.66	4.55	4.50	6.48	5.68
Model 17	5.89	4.72	5.34	7.17	5.28
Model 18	8.71	6.24	5.30	5.83	6.56
Model 19	6.05	4.12	3.69	5.52	6.19
Model 20	6.32	4.16	4.63	6.59	5.36
Model 21	5.27	4.31	5.60	6.15	3.90
Model 22	5.86	5.03	3.91	5.47	3.89
Model 23	4.56	4.29	3.88	4.54	3.51
Model 24	3.85	2.72	3.92	4.82	4.66
Average	8.62	7.59	7.38	8.80	8.53

TABLE B-15 MAPE of ANN Models with Different Training Functions (Bayesian Regularization)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	12.23	4.48	9.94	12.49	5.36
Model 2	8.07	2.24	2.86	10.78	3.50
Model 3	5.82	2.26	2.28	7.99	3.07
Model 4	4.92	1.82	1.97	6.94	2.68
Model 5	4.53	1.48	3.23	6.17	2.23
Model 6	5.18	1.50	1.29	6.34	2.73
Model 7	1.23	1.28	1.12	2.43	2.63
Model 8	0.82	0.69	1.00	0.70	2.17
Model 9	1.55	1.11	0.79	0.67	1.78
Model 10	1.14	0.63	0.66	0.25	1.33
Model 11	0.19	0.23	0.57	0.24	1.32
Model 12	1.24	0.07	0.51	0.19	1.14
Model 13	1.21	0.86	1.20	0.09	0.84
Model 14	1.01	0.06	0.46	0.22	0.82
Model 15	0.99	0.40	0.46	0.49	0.84
Model 16	0.95	0.53	0.46	0.26	0.79
Model 17	0.06	0.09	0.43	0.13	0.62
Model 18	0.75	0.70	0.50	0.96	0.65
Model 19	0.39	0.11	0.54	0.30	0.67
Model 20	0.62	0.42	0.51	0.27	0.64
Model 21	0.14	0.36	0.46	0.17	0.67
Model 22	0.70	0.12	0.32	0.05	0.61
Model 23	0.13	0.11	0.33	0.10	0.65
Model 24	0.18	0.05	0.32	0.06	0.61
Average	2.25	0.90	1.34	2.43	1.60

TABLE B-16 MAPE of ANN Models with Different Training Functions (Powell-Beale Conjugate Gradient Backpropagation)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	14.59	12.83	13.96	14.54	13.83
Model 2	12.26	11.68	10.71	13.97	11.65
Model 3	11.01	12.05	10.51	11.70	11.97
Model 4	9.08	8.35	8.31	9.19	11.89
Model 5	9.81	8.96	7.99	10.78	8.07
Model 6	8.09	7.85	7.98	11.03	8.69
Model 7	7.99	7.96	6.21	9.66	11.12
Model 8	6.85	8.32	6.15	7.34	8.97
Model 9	13.02	7.08	7.04	9.61	9.43
Model 10	7.68	6.65	6.30	8.45	8.27
Model 11	7.30	5.80	5.65	8.42	7.69
Model 12	6.60	7.16	6.52	6.57	6.63
Model 13	6.01	5.22	7.05	6.40	8.13
Model 14	7.26	6.29	5.40	6.37	7.74
Model 15	7.53	5.46	5.57	5.73	7.44
Model 16	8.77	5.08	5.88	6.17	6.42
Model 17	6.68	4.42	5.53	6.16	5.64
Model 18	6.68	4.22	5.57	5.70	4.77
Model 19	8.94	4.61	3.95	6.08	5.18
Model 20	6.21	4.82	3.62	5.63	6.27
Model 21	5.82	4.78	3.78	5.90	6.42
Model 22	6.08	4.66	6.66	5.54	3.73
Model 23	5.02	5.39	3.31	5.51	4.75
Model 24	4.36	3.71	3.63	5.71	4.23
Average	8.07	6.81	6.55	8.01	7.87

TABLE B-17 MAPE of ANN Models with Different Training Functions (Fletcher-Powell Conjugate Gradient Backpropagation)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	15.66	16.58	15.51	16.46	18.50
Model 2	13.49	15.19	12.59	12.78	14.37
Model 3	12.81	13.14	10.51	12.25	11.98
Model 4	10.38	10.70	10.64	13.36	12.70
Model 5	10.03	10.57	6.17	11.97	12.00
Model 6	9.71	9.73	10.09	9.18	12.19
Model 7	8.02	9.04	8.75	11.19	10.12
Model 8	8.80	7.26	8.62	10.24	8.90
Model 9	12.05	9.38	7.35	8.14	12.04
Model 10	9.75	8.36	6.19	8.32	10.30
Model 11	11.66	7.67	5.77	10.19	9.01
Model 12	10.16	9.44	6.19	7.80	9.63
Model 13	10.50	8.73	5.16	7.07	9.33
Model 14	8.45	6.61	5.63	6.32	8.16
Model 15	11.38	8.92	5.68	8.16	9.50
Model 16	5.50	5.73	6.80	8.85	7.18
Model 17	8.61	6.37	4.36	7.56	5.27
Model 18	8.10	6.57	5.05	7.14	7.99
Model 19	8.21	5.41	6.24	6.86	8.14
Model 20	5.81	5.78	5.73	5.68	5.03
Model 21	5.71	6.16	6.04	6.62	7.05
Model 22	5.62	4.12	4.49	6.30	3.96
Model 23	5.55	5.52	4.95	6.32	4.16
Model 24	5.08	4.54	4.56	5.31	2.55
Average	9.21	8.40	7.21	8.92	9.17

TABLE B-18 MAPE of ANN Models with Different Training Functions (Gradient Descent Backpropagation)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	835.79	587.92	704.73	1012.70	768.02
Model 2	423.10	616.79	504.03	576.45	493.78
Model 3	353.06	393.42	458.53	518.67	429.64
Model 4	310.18	367.96	422.46	333.70	404.94
Model 5	319.62	385.83	293.79	364.37	384.08
Model 6	264.17	350.42	254.37	300.30	206.82
Model 7	282.68	302.70	265.84	298.71	306.84
Model 8	278.97	315.59	280.99	279.36	283.48
Model 9	257.03	287.07	223.41	254.16	239.17
Model 10	265.46	256.35	208.71	241.61	215.14
Model 11	215.88	215.43	235.86	223.38	239.06
Model 12	178.02	221.64	186.06	193.12	190.03
Model 13	189.66	213.53	155.45	207.33	227.28
Model 14	170.84	173.07	166.29	163.63	134.18
Model 15	180.86	201.26	141.55	154.23	185.25
Model 16	151.65	144.67	149.50	152.25	131.82
Model 17	131.50	106.75	114.19	148.53	127.27
Model 18	120.58	100.65	113.59	94.54	137.17
Model 19	84.05	112.17	106.74	89.52	105.15
Model 20	79.29	108.13	88.95	74.86	106.58
Model 21	67.63	85.98	108.76	73.13	56.01
Model 22	56.46	38.51	54.08	44.28	53.82
Model 23	30.11	39.86	18.00	24.97	37.56
Model 24	23.22	11.31	16.75	14.28	26.44
Average	219.58	234.88	219.69	243.25	228.73

TABLE B-19 MAPE of ANN Models with Different Training Functions (Gradient Descent with Adaptive Learning Rate Backpropagation)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	90.48	83.03	151.65	71.51	123.38
Model 2	65.57	61.51	80.73	76.94	82.86
Model 3	67.29	36.64	73.45	47.53	33.39
Model 4	59.37	51.27	66.33	41.57	28.15
Model 5	27.57	84.75	24.58	48.66	57.22
Model 6	36.21	54.36	49.35	28.40	38.41
Model 7	22.49	44.55	28.76	40.87	58.38
Model 8	21.76	44.66	40.98	19.28	49.44
Model 9	25.42	22.52	30.80	20.01	24.86
Model 10	33.85	42.23	18.36	29.78	49.58
Model 11	27.83	29.01	18.30	40.49	54.60
Model 12	28.18	36.34	23.49	18.34	22.99
Model 13	21.57	24.32	37.42	31.28	29.99
Model 14	33.38	20.86	17.52	27.04	23.20
Model 15	22.34	26.84	14.45	21.62	21.23
Model 16	28.19	21.15	19.38	20.25	30.64
Model 17	29.93	23.52	18.31	20.32	18.99
Model 18	22.38	14.10	14.65	19.52	21.60
Model 19	22.16	13.96	16.93	17.45	15.16
Model 20	13.53	16.79	12.33	16.94	17.70
Model 21	23.01	13.84	16.63	18.54	12.60
Model 22	14.45	11.06	6.57	18.25	18.44
Model 23	15.25	10.53	12.12	13.93	9.73
Model 24	10.33	11.91	7.62	12.16	8.27
Average	31.77	33.32	33.36	30.03	35.45

TABLE B-20 MAPE of ANN Models with Different Training Functions (Levenberg-Marquardt Backpropagation)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	10.53	5.14	9.00	12.67	10.33
Model 2	8.02	3.39	8.79	10.34	6.80
Model 3	6.43	3.22	5.46	9.41	6.34
Model 4	5.46	2.95	7.29	7.37	6.14
Model 5	5.94	2.70	6.91	7.75	4.73
Model 6	4.62	2.14	5.26	8.27	5.80
Model 7	1.78	2.11	4.53	3.43	7.06
Model 8	1.58	2.10	4.50	3.79	5.85
Model 9	2.00	1.94	4.72	4.85	5.49
Model 10	1.91	2.07	4.17	3.52	5.23
Model 11	1.74	1.82	4.04	3.04	5.20
Model 12	1.72	1.84	3.77	3.27	5.90
Model 13	1.78	1.50	4.23	3.41	5.20
Model 14	1.41	1.84	3.18	2.74	4.69
Model 15	1.44	1.65	4.53	3.53	4.00
Model 16	1.57	1.49	5.39	2.88	3.87
Model 17	1.13	1.23	3.60	1.87	4.30
Model 18	1.12	1.16	3.43	2.00	5.21
Model 19	1.24	1.38	4.06	2.64	3.59
Model 20	1.11	0.87	3.36	2.32	3.83
Model 21	1.06	1.11	3.97	1.89	4.39
Model 22	0.95	0.75	3.61	1.40	3.10
Model 23	0.70	0.66	2.36	0.88	2.04
Model 24	1.33	1.16	2.04	2.10	3.11
Average	2.77	1.93	4.68	4.39	5.09

TABLE B-21 MAPE of ANN Models with Different Training Functions (One Step Secant Backpropagations)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	18.99	18.25	16.29	22.00	19.94
Model 2	13.48	15.63	13.22	19.41	18.99
Model 3	15.31	15.60	11.47	16.15	16.03
Model 4	11.31	12.71	10.57	14.08	15.76
Model 5	10.79	11.30	9.34	14.17	12.05
Model 6	12.50	10.81	10.26	13.46	13.76
Model 7	11.69	10.63	8.64	12.20	11.17
Model 8	8.07	11.43	8.24	11.45	12.05
Model 9	15.63	11.08	8.99	11.57	12.01
Model 10	13.22	9.23	7.33	12.38	13.64
Model 11	12.74	10.00	7.42	12.83	12.14
Model 12	10.89	10.18	8.71	10.72	10.28
Model 13	12.45	7.47	8.13	8.55	9.91
Model 14	11.12	9.19	7.68	9.86	9.43
Model 15	9.64	9.11	7.84	9.08	12.20
Model 16	11.82	7.36	7.97	9.27	8.83
Model 17	9.57	6.38	7.16	8.98	8.85
Model 18	9.12	6.54	6.95	8.04	8.45
Model 19	9.85	8.31	5.60	8.66	9.03
Model 20	8.36	6.93	5.80	8.64	7.26
Model 21	8.95	8.29	6.28	7.41	7.66
Model 22	9.00	7.59	6.03	7.68	8.33
Model 23	7.88	5.58	4.56	7.30	6.61
Model 24	6.03	5.51	4.41	7.16	4.74
Average	11.18	9.80	8.29	11.29	11.21

TABLE B-22 MAPE of ANN Models with Different Training Functions (Resilient Backpropagation)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	14.61	17.50	15.94	16.19	15.05
Model 2	12.79	15.65	13.29	18.12	16.63
Model 3	11.48	15.06	9.74	12.29	15.08
Model 4	10.61	12.94	10.86	13.72	10.22
Model 5	8.39	12.34	7.07	12.98	9.34
Model 6	10.17	7.92	6.34	11.00	10.73
Model 7	7.61	11.09	6.72	8.26	11.04
Model 8	9.08	6.80	4.89	9.42	10.22
Model 9	13.04	5.32	5.83	7.12	7.48
Model 10	6.50	8.19	6.13	7.28	7.66
Model 11	8.37	5.79	6.04	12.25	7.46
Model 12	9.84	10.84	5.23	7.68	12.31
Model 13	5.79	6.98	5.48	7.70	7.20
Model 14	5.65	8.16	5.38	6.89	9.58
Model 15	5.41	4.84	6.51	7.07	6.72
Model 16	5.26	3.55	5.73	6.83	6.43
Model 17	6.12	4.74	4.19	6.27	9.45
Model 18	9.81	4.04	5.64	6.17	6.87
Model 19	6.61	4.45	4.02	6.38	7.33
Model 20	3.65	4.85	3.82	5.87	3.34
Model 21	5.45	4.10	5.31	5.66	3.88
Model 22	4.70	4.85	5.87	5.24	7.39
Model 23	6.52	6.52	6.63	6.21	5.70
Model 24	5.61	6.02	10.08	3.54	4.25
Average	8.05	8.02	6.95	8.76	8.81

TABLE B-23 MAPE of ANN Models with Different Training Functions (Sequential Order Incremental Update)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	886.79	1280.60	1117.90	740.08	902.45
Model 2	814.67	544.88	497.73	517.97	542.61
Model 3	634.86	431.22	438.00	601.41	526.46
Model 4	431.67	461.66	403.50	437.42	499.23
Model 5	487.94	418.64	328.58	323.74	415.57
Model 6	446.86	375.08	312.28	344.30	375.15
Model 7	294.04	338.98	229.07	433.96	371.89
Model 8	288.84	292.67	311.20	358.47	362.64
Model 9	277.57	350.38	313.04	352.61	282.14
Model 10	270.55	307.14	309.58	301.40	250.54
Model 11	233.40	302.52	197.04	328.05	261.07
Model 12	298.73	269.25	242.47	287.75	244.58
Model 13	246.08	324.43	217.37	272.99	194.21
Model 14	156.08	257.13	234.34	209.63	217.73
Model 15	255.75	272.16	221.49	197.75	184.23
Model 16	253.74	283.67	153.50	257.62	255.30
Model 17	236.62	197.60	193.29	213.14	235.27
Model 18	259.75	196.30	223.89	186.22	151.74
Model 19	133.90	124.68	155.25	244.24	247.00
Model 20	178.17	156.22	160.43	111.49	168.09
Model 21	186.70	121.38	133.51	135.13	120.00
Model 22	148.60	114.67	149.15	146.31	117.25
Model 23	76.56	60.95	104.23	97.78	99.64
Model 24	102.63	70.40	96.40	72.91	110.51
Average	316.69	314.69	280.97	298.85	297.30

TABLE B-24 MAPE of ANN Models with Different Training Functions (Scaled Conjugate Gradient Backpropagation)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	17.16	13.08	13.20	15.92	15.28
Model 2	17.29	10.99	12.69	13.00	15.04
Model 3	11.80	14.76	9.23	14.97	14.24
Model 4	11.11	13.84	10.47	10.82	11.57
Model 5	11.17	8.92	10.76	13.78	12.50
Model 6	9.99	11.71	9.68	11.16	10.69
Model 7	8.83	6.64	8.07	8.91	14.34
Model 8	9.27	6.60	7.11	10.13	11.53
Model 9	14.55	11.21	9.01	10.73	8.62
Model 10	12.52	9.41	7.46	8.12	8.60
Model 11	12.25	9.62	6.92	9.82	8.29
Model 12	11.52	8.30	6.66	7.65	9.67
Model 13	8.01	10.92	8.68	7.02	8.22
Model 14	12.10	8.70	7.23	9.38	11.39
Model 15	7.48	5.99	7.23	9.12	7.80
Model 16	8.62	8.06	6.72	8.67	9.78
Model 17	11.19	7.32	7.35	7.47	8.88
Model 18	8.50	6.98	6.65	8.34	5.61
Model 19	7.98	6.91	4.47	6.73	8.37
Model 20	6.61	7.25	4.12	6.78	7.40
Model 21	7.19	5.72	6.14	6.94	4.78
Model 22	5.13	5.95	5.37	6.76	6.67
Model 23	5.88	7.68	5.15	6.23	5.25
Model 24	5.19	5.26	4.05	5.88	4.90
Average	10.06	8.83	7.68	9.35	9.56

TABLE B-25 MAPE of ANN Models with Different Learning Functions (Conscience Bias Learning Function)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	14.39	10.91	9.23	16.72	11.33
Model 2	5.93	4.01	6.19	7.40	7.80
Model 3	4.63	3.90	6.45	7.37	11.86
Model 4	4.59	3.07	6.80	8.57	7.43
Model 5	3.24	2.95	4.90	6.35	5.15
Model 6	3.79	2.66	4.43	5.95	6.46
Model 7	2.69	3.00	4.01	5.91	5.13
Model 8	2.98	2.65	5.26	6.16	5.37
Average	5.28	4.14	5.91	8.05	7.57

TABLE B-26 MAPE of ANN Models with Different Learning Functions (Gradient Descent Weight/Bias Learning Function)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	14.97	10.51	6.18	17.08	11.60
Model 2	5.42	4.26	6.65	9.15	10.64
Model 3	5.07	3.89	5.78	8.41	8.97
Model 4	3.67	3.05	4.70	8.33	6.61
Model 5	3.21	3.28	4.95	6.77	6.95
Model 6	3.53	2.15	4.04	7.25	5.33
Model 7	3.26	2.17	5.30	6.63	4.70
Model 8	2.42	2.91	4.48	5.24	3.87
Average	5.19	4.03	5.26	8.61	7.33

TABLE B-27 MAPE of ANN Models with Different Learning Functions (Gradient Descent with Momentum Weight/Bias Learning Function)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	14.09	7.19	7.19	15.37	11.68
Model 2	6.06	6.02	6.02	8.36	10.39
Model 3	4.92	5.37	5.37	7.86	12.80
Model 4	3.54	5.08	5.08	9.22	8.89
Model 5	3.67	5.46	5.46	6.11	5.23
Model 6	3.26	5.32	5.32	7.60	6.29
Model 7	3.02	3.30	3.30	7.08	6.80
Model 8	2.73	3.87	3.87	5.35	3.83
Average	5.16	5.20	5.20	8.37	8.24

TABLE B-28 MAPE of ANN Models with Different Learning Functions (Hebb Weight Learning Function)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	13.76	12.82	7.83	17.85	10.99
Model 2	5.84	4.00	6.14	8.59	9.31
Model 3	4.53	3.90	5.68	8.00	9.47
Model 4	4.51	3.38	5.95	7.73	6.52
Model 5	3.80	3.55	4.46	6.43	7.84
Model 6	3.32	3.64	3.54	8.11	5.40
Model 7	2.88	2.21	5.46	5.50	5.45
Model 8	2.70	2.91	4.77	5.89	5.00
Average	5.17	4.55	5.48	8.51	7.50

TABLE B-29 MAPE of ANN Models with Different Learning Functions (Hebb with Decay Weight Learning Function)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	12.57	9.53	7.67	18.72	11.96
Model 2	5.72	4.12	6.56	8.19	9.74
Model 3	4.43	4.20	5.06	8.69	11.62
Model 4	3.84	3.47	4.71	7.99	8.07
Model 5	3.35	3.67	5.23	7.12	5.62
Model 6	2.60	2.94	4.08	7.52	6.70
Model 7	2.70	3.16	4.25	6.98	5.04
Model 8	2.53	2.91	3.64	4.10	5.22
Average	4.72	4.25	5.15	8.66	8.00

TABLE B-30 MAPE of ANN Models with Different Learning Functions (Instar Weight Learning Function)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	14.27	10.53	8.31	19.41	13.59
Model 2	5.38	4.27	6.67	8.94	10.93
Model 3	5.25	3.55	6.29	8.21	7.11
Model 4	4.14	3.83	4.92	7.63	7.67
Model 5	3.95	2.90	5.28	6.57	7.49
Model 6	3.74	3.35	3.84	7.36	6.52
Model 7	3.09	3.21	4.63	6.51	3.98
Model 8	2.84	2.91	5.08	7.17	4.01
Average	5.33	4.32	5.63	8.97	7.66

TABLE B-31 31 MAPE of ANN Models with Different Learning Functions (Kohonen Weight Learning Function)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	14.15	9.50	7.99	17.59	13.39
Model 2	6.31	3.95	5.01	7.60	8.67
Model 3	4.54	3.90	6.00	8.93	7.77
Model 4	4.09	2.85	5.22	10.04	10.17
Model 5	3.66	2.13	5.36	5.36	6.07
Model 6	3.71	2.21	3.57	7.98	6.11
Model 7	2.87	3.34	4.01	6.09	6.20
Model 8	2.80	2.91	4.09	5.04	4.21
Average	5.27	3.85	5.16	8.58	7.82

TABLE B-32 MAPE of ANN Models with Different Learning Functions (LVQ1 Weight Learning Function)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	13.62	10.88	8.76	18.55	12.50
Model 2	5.07	4.48	5.47	9.31	7.42
Model 3	4.71	4.05	6.10	8.55	9.15
Model 4	4.14	3.84	4.23	7.75	6.91
Model 5	4.89	2.82	5.29	6.26	6.16
Model 6	3.89	2.67	4.13	7.13	7.14
Model 7	3.25	2.57	3.61	6.34	5.03
Model 8	1.87	2.91	5.03	5.29	3.13
Average	5.18	4.28	5.33	8.65	7.18

TABLE B-33 MAPE of ANN Models with Different Learning Functions (LVQ2 Weight Learning Function)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	15.55	11.76	9.05	18.94	9.95
Model 2	5.24	3.81	6.54	8.07	9.79
Model 3	5.12	3.61	5.38	8.02	11.44
Model 4	5.01	3.41	5.44	9.69	7.58
Model 5	2.98	3.70	4.61	5.75	6.31
Model 6	3.22	3.16	4.92	8.07	6.89
Model 7	2.45	2.29	3.10	6.26	5.48
Model 8	2.86	2.91	3.33	5.49	4.77
Average	5.31	4.33	5.30	8.79	7.77

TABLE B-34 MAPE of ANN Models with Different Learning Functions (Outstar Weight Learning Function)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	14.65	9.06	6.94	18.25	11.38
Model 2	5.38	4.49	6.02	8.64	10.54
Model 3	5.32	4.59	5.59	7.21	11.99
Model 4	4.05	3.83	5.77	8.08	6.01
Model 5	3.77	3.47	5.00	6.80	7.32
Model 6	3.11	2.11	5.01	7.36	6.43
Model 7	2.51	3.21	3.30	6.92	4.02
Model 8	2.73	2.91	3.84	5.71	3.11
Average	5.19	4.21	5.18	8.62	7.60

TABLE B- 35 MAPE of ANN Models with Different Learning Functions (Perceptron Weight and Bias Learning Function)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	14.76	10.79	7.11	17.96	11.04
Model 2	5.58	4.11	5.75	7.65	9.92
Model 3	4.33	4.15	5.79	7.86	8.70
Model 4	4.36	3.73	5.01	7.07	7.64
Model 5	2.77	3.05	5.14	6.82	6.30
Model 6	3.61	2.52	3.70	5.52	4.70
Model 7	2.60	3.21	3.51	5.58	4.29
Model 8	3.02	2.91	4.30	4.52	4.74
Average	5.13	4.31	5.04	7.87	7.17

TABLE B- 36 MAPE of ANN Models with Different Learning Functions (Normalized Perceptron Weight and Bias Learning Function)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	14.42	12.61	8.55	18.90	11.90
Model 2	5.35	4.33	6.48	7.81	10.83
Model 3	5.02	4.14	4.99	8.01	9.65
Model 4	4.30	3.47	6.13	7.18	8.68
Model 5	3.10	3.77	5.01	6.40	7.62
Model 6	2.87	3.20	3.59	7.32	7.78
Model 7	2.82	3.17	3.07	5.44	7.63
Model 8	3.14	2.91	3.96	4.24	4.76
Average	5.13	4.70	5.22	8.16	8.61

TABLE B-37 MAPE of ANN Models with Different Learning Functions (Self-organizing Map Weight Learning Function)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	14.08	11.03	6.81	17.84	11.63
Model 2	5.67	4.09	6.71	7.65	9.81
Model 3	4.29	3.71	6.05	8.55	9.88
Model 4	3.81	3.55	6.83	7.78	7.21
Model 5	3.56	3.55	5.15	6.62	4.57
Model 6	2.79	3.09	2.94	5.60	6.45
Model 7	3.06	2.07	4.41	7.16	4.63
Model 8	2.57	2.91	3.53	4.90	5.20
Average	4.98	4.25	5.30	8.26	7.42

TABLE B-38 MAPE of ANN Models with Different Learning Functions (Widrow-Hoff Weight and Bias Learning Rule)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	14.65	14.04	6.99	18.61	11.03
Model 2	5.38	4.49	7.04	7.70	12.39
Model 3	4.82	3.17	5.63	9.39	10.83
Model 4	3.85	4.24	5.68	7.82	9.71
Model 5	3.53	2.76	5.95	7.71	5.62
Model 6	3.39	2.56	5.34	5.80	6.56
Model 7	2.31	2.21	3.31	7.80	7.31
Model 8	2.41	2.91	3.89	3.76	5.79
Average	5.04	4.55	5.48	8.57	8.65

TABLE B-39 MAPE of ANN Models with Different Learning Functions (Conscience Bias Learning Function)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	12.49	5.01	8.52	12.57	9.37
Model 2	8.63	3.79	7.54	11.10	7.04
Model 3	6.28	3.12	4.64	7.99	6.15
Model 4	5.68	2.94	5.79	7.12	6.31
Model 5	5.51	2.69	5.18	6.22	5.93
Model 6	4.94	2.28	4.25	6.59	5.73
Model 7	1.57	1.91	4.20	4.12	5.90
Model 8	1.63	2.11	4.10	3.31	5.01
Model 9	1.94	1.92	3.62	3.30	5.44
Model 10	1.64	2.17	6.97	4.09	5.71
Model 11	1.92	2.25	3.95	3.14	5.76
Model 12	1.58	1.67	4.61	4.20	5.24
Model 13	1.48	1.60	4.42	3.11	4.54
Model 14	1.25	1.40	3.05	3.03	4.39
Model 15	1.48	1.86	4.36	2.72	6.07
Model 16	1.22	1.51	4.05	3.17	3.36
Model 17	1.17	1.34	3.42	2.48	5.04
Model 18	1.24	1.15	3.25	1.43	5.65
Model 19	1.19	1.04	2.89	0.96	3.72
Model 20	1.14	1.52	3.45	1.03	3.00
Model 21	1.03	1.01	4.62	1.89	3.42
Model 22	1.12	0.98	2.68	1.40	3.59
Model 23	0.70	0.66	3.12	0.88	2.61
Model 24	1.37	1.47	3.36	2.66	2.53
Average	2.84	1.97	4.42	4.10	5.06

TABLE B-40 MAPE of ANN Models with Different Learning Functions (Gradient Descent Weight/Bias Learning Function)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	11.82	5.30	8.93	12.45	8.43
Model 2	7.44	4.08	6.60	12.66	7.97
Model 3	6.47	3.16	6.91	10.39	7.61
Model 4	5.75	3.15	5.01	6.98	6.52
Model 5	5.61	2.81	5.34	7.05	7.06
Model 6	4.99	2.24	4.71	6.21	6.48
Model 7	1.83	1.92	3.97	4.28	4.98
Model 8	1.64	2.09	4.31	3.35	6.04
Model 9	2.13	2.34	3.43	3.52	4.87
Model 10	1.75	2.14	4.21	4.38	5.01
Model 11	2.02	1.64	3.19	3.63	4.96
Model 12	1.82	1.81	3.23	2.37	5.21
Model 13	1.38	1.72	3.90	3.63	5.09
Model 14	1.86	1.84	4.20	3.84	5.10
Model 15	1.24	1.47	4.08	2.48	5.24
Model 16	1.22	1.68	3.42	2.63	5.20
Model 17	1.38	1.53	4.10	2.58	3.34
Model 18	1.46	0.94	2.48	2.61	4.69
Model 19	1.10	1.09	3.47	2.60	3.51
Model 20	1.23	1.07	3.11	2.29	4.41
Model 21	0.97	1.13	3.85	1.89	4.06
Model 22	0.86	0.53	3.94	1.39	3.72
Model 23	0.70	0.66	3.28	0.88	2.65
Model 24	1.20	1.52	2.41	2.02	2.80
Average	2.83	1.99	4.25	4.42	5.21

TABLE B-41 MAPE of ANN Models with Different Learning Functions (Gradient Descent with Momentum Weight/Bias Learning Function)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	11.75	5.37	7.94	14.60	8.16
Model 2	8.37	3.89	7.58	13.73	5.76
Model 3	6.79	3.10	5.80	10.12	7.14
Model 4	5.10	2.87	7.37	6.88	7.25
Model 5	5.18	2.46	4.11	8.70	5.62
Model 6	4.83	2.46	3.79	8.27	5.86
Model 7	1.85	2.23	5.75	3.87	5.74
Model 8	1.72	1.94	4.93	3.93	5.34
Model 9	1.56	2.04	3.68	3.47	4.12
Model 10	1.86	1.98	4.82	3.34	4.93
Model 11	1.73	1.83	4.02	3.54	4.89
Model 12	1.51	1.95	3.56	2.32	5.91
Model 13	1.58	1.45	5.34	2.57	5.26
Model 14	1.06	1.57	3.86	2.88	3.96
Model 15	1.41	1.50	4.68	3.14	6.95
Model 16	1.26	1.49	4.30	2.21	4.80
Model 17	1.09	1.43	3.12	2.62	4.18
Model 18	1.25	1.14	2.86	2.58	4.14
Model 19	1.04	1.43	3.44	2.36	4.16
Model 20	1.12	0.97	3.83	2.28	3.16
Model 21	0.85	0.93	3.23	1.89	4.00
Model 22	1.12	0.81	3.67	1.41	4.25
Model 23	0.70	0.66	3.21	0.88	3.74
Model 24	1.15	1.70	3.97	2.41	2.18
Average	2.75	1.97	4.54	4.58	5.06

TABLE B-42 MAPE of ANN Models with Different Learning Functions (Hebb Weight Learning Function)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	12.23	4.78	8.24	12.89	8.47
Model 2	7.91	3.87	6.93	12.10	6.77
Model 3	6.98	3.12	7.49	8.58	8.38
Model 4	5.48	2.96	5.84	6.94	6.17
Model 5	5.77	2.54	4.99	6.22	5.63
Model 6	4.53	2.43	5.32	6.64	8.55
Model 7	1.88	2.11	4.75	4.56	5.38
Model 8	1.59	2.07	4.10	2.83	5.91
Model 9	1.53	1.79	4.19	3.11	5.37
Model 10	2.10	1.96	5.07	4.73	4.63
Model 11	1.70	2.08	3.57	3.56	4.97
Model 12	1.70	1.85	3.72	3.25	4.80
Model 13	1.42	1.82	4.23	2.71	5.20
Model 14	1.44	1.83	4.06	3.23	5.32
Model 15	1.32	1.39	4.08	2.93	4.36
Model 16	1.42	1.46	3.22	3.20	4.15
Model 17	1.51	1.43	3.53	1.78	5.31
Model 18	1.05	1.22	3.44	2.55	4.70
Model 19	1.38	0.95	2.87	1.22	3.72
Model 20	1.31	1.07	3.99	2.34	2.69
Model 21	1.25	0.84	2.32	0.86	3.73
Model 22	0.75	0.91	3.78	1.40	3.02
Model 23	0.70	0.66	2.86	0.89	2.49
Model 24	1.20	1.58	2.79	2.35	1.56
Average	2.84	1.95	4.39	4.20	5.05

TABLE B-43 MAPE of ANN Models with Different Learning Functions (Hebb with Decay Weight Learning Function)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	12.49	5.33	8.02	15.57	9.02
Model 2	7.91	3.96	7.05	13.74	7.27
Model 3	6.33	2.97	6.59	8.81	6.37
Model 4	6.06	3.07	5.86	7.02	7.15
Model 5	6.17	2.72	4.66	6.32	5.98
Model 6	5.00	2.25	4.93	6.31	5.74
Model 7	1.79	2.04	6.47	4.16	5.77
Model 8	1.65	2.20	4.58	3.64	5.92
Model 9	2.26	1.67	3.90	3.27	4.70
Model 10	2.01	1.92	3.75	3.50	6.83
Model 11	1.90	2.11	5.15	3.35	6.34
Model 12	1.65	1.89	3.94	3.50	5.98
Model 13	1.35	1.89	3.58	3.41	5.40
Model 14	1.55	1.47	4.16	2.92	3.74
Model 15	1.37	1.39	3.96	2.89	4.49
Model 16	1.30	1.44	3.55	2.30	3.05
Model 17	1.20	1.51	3.77	1.75	3.31
Model 18	1.40	1.05	3.23	2.79	5.06
Model 19	0.89	1.08	3.37	2.64	3.65
Model 20	0.98	1.03	2.63	2.14	2.76
Model 21	1.15	1.03	3.67	1.21	3.73
Model 22	0.83	0.51	3.53	1.38	3.41
Model 23	0.70	0.66	2.28	0.88	3.73
Model 24	0.90	1.56	3.72	2.48	2.20
Average	2.87	1.95	4.43	4.42	5.07

TABLE B-44 MAPE of ANN Models with Different Learning Functions (Instar Weight Learning Function)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	11.81	5.16	9.17	12.45	10.85
Model 2	6.68	3.58	7.22	11.75	6.32
Model 3	6.36	3.02	5.30	10.61	7.32
Model 4	5.86	3.11	6.31	9.81	6.38
Model 5	5.55	2.72	5.59	6.41	5.47
Model 6	4.89	2.19	3.95	7.58	7.12
Model 7	1.81	1.78	4.02	4.19	7.30
Model 8	1.59	2.36	3.81	3.86	5.30
Model 9	1.73	2.11	4.52	4.21	5.12
Model 10	1.82	1.82	3.41	2.71	4.56
Model 11	1.73	1.74	4.18	4.49	6.27
Model 12	1.28	1.45	4.02	3.50	6.80
Model 13	1.44	1.51	4.37	3.47	4.66
Model 14	1.23	1.74	4.03	3.08	5.06
Model 15	1.55	1.23	5.00	2.79	4.95
Model 16	1.71	1.56	3.10	2.00	3.87
Model 17	1.51	1.36	4.98	2.13	3.94
Model 18	1.30	1.57	3.66	2.84	3.67
Model 19	1.24	0.91	2.62	2.64	3.90
Model 20	1.44	0.87	2.54	2.34	2.35
Model 21	0.94	0.89	4.23	1.05	3.65
Model 22	0.94	0.91	2.38	1.41	2.78
Model 23	0.70	0.66	2.77	0.88	1.98
Model 24	0.76	1.47	3.33	2.48	2.97
Average	2.75	1.91	4.35	4.53	5.11

TABLE B-45 MAPE of ANN Models with Different Learning Functions (Kohonen Weight Learning Function)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	11.56	4.93	9.39	14.47	10.01
Model 2	7.92	3.65	7.74	10.16	7.76
Model 3	7.35	3.32	5.61	9.87	6.77
Model 4	5.81	3.00	6.45	9.82	7.08
Model 5	5.15	2.60	4.91	6.87	5.17
Model 6	4.91	2.43	4.94	7.73	6.08
Model 7	1.66	2.23	4.16	3.85	6.73
Model 8	1.71	1.99	4.38	3.79	7.94
Model 9	2.06	1.80	5.63	5.06	5.64
Model 10	1.78	1.86	4.56	3.41	4.89
Model 11	1.68	2.41	3.68	4.06	4.35
Model 12	1.48	1.84	5.54	3.07	3.87
Model 13	1.95	1.74	2.72	2.64	4.97
Model 14	1.67	1.51	3.96	3.56	5.98
Model 15	1.54	1.41	3.82	2.82	4.64
Model 16	1.25	1.49	4.12	2.74	5.34
Model 17	1.16	1.70	3.12	1.55	3.89
Model 18	1.24	1.14	3.43	1.89	4.35
Model 19	1.17	1.49	2.91	1.46	3.74
Model 20	1.08	1.17	2.62	2.34	3.17
Model 21	1.19	1.01	4.27	1.88	4.01
Model 22	0.81	0.62	3.11	1.41	2.98
Model 23	0.70	0.66	2.08	0.88	3.33
Model 24	0.92	1.41	2.99	2.96	2.56
Average	2.82	1.98	4.42	4.51	5.22

TABLE B-46 MAPE of ANN Models with Different Learning Functions (LVQ1 Weight Learning Function)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	12.09	5.33	7.32	14.08	8.65
Model 2	7.11	3.68	8.58	11.34	8.80
Model 3	7.20	3.37	6.69	8.12	5.81
Model 4	5.40	2.92	4.48	7.52	5.43
Model 5	5.81	2.47	4.47	6.84	5.96
Model 6	4.81	2.31	5.14	8.06	5.73
Model 7	1.82	2.09	4.95	4.16	7.59
Model 8	1.61	2.17	3.41	4.61	5.24
Model 9	1.93	1.99	5.13	3.48	5.47
Model 10	1.66	1.84	3.70	5.00	4.42
Model 11	2.07	2.34	4.60	3.20	5.11
Model 12	1.78	1.81	3.44	3.17	4.35
Model 13	1.63	1.51	3.95	3.08	4.46
Model 14	1.47	1.99	4.27	3.39	5.62
Model 15	1.57	1.77	4.47	2.30	5.56
Model 16	1.46	1.17	3.71	3.08	5.32
Model 17	1.19	1.43	3.65	3.06	5.39
Model 18	1.35	1.28	2.28	2.14	4.27
Model 19	1.12	0.95	4.50	2.64	4.43
Model 20	0.99	0.68	2.68	2.34	2.77
Model 21	1.10	0.96	2.92	1.89	3.63
Model 22	1.05	0.95	4.82	1.40	3.98
Model 23	0.70	0.66	3.94	0.88	3.03
Model 24	1.18	0.46	3.29	2.45	3.02
Average	2.84	1.92	4.43	4.51	5.17

TABLE B-47 MAPE of ANN Models with Different Learning Functions (LVQ2 Weight Learning Function)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	12.04	5.28	8.76	12.45	10.97
Model 2	8.88	3.84	6.26	12.19	7.36
Model 3	5.94	3.26	5.37	10.74	6.63
Model 4	5.68	3.09	6.38	7.39	5.98
Model 5	5.76	2.51	4.83	6.25	5.97
Model 6	5.49	2.17	4.59	7.59	5.22
Model 7	1.83	2.36	3.95	3.45	8.34
Model 8	1.57	1.92	4.86	3.80	4.91
Model 9	2.09	2.24	4.40	3.48	5.65
Model 10	1.96	2.15	3.01	2.93	6.22
Model 11	1.78	1.91	3.96	3.96	4.09
Model 12	1.30	1.48	3.65	2.84	5.41
Model 13	1.36	1.68	4.00	3.40	4.13
Model 14	1.80	1.52	3.62	3.16	4.22
Model 15	1.86	1.83	3.99	2.68	6.07
Model 16	1.44	1.42	4.42	2.28	5.21
Model 17	1.22	1.45	3.93	1.62	6.40
Model 18	1.18	1.20	4.10	1.85	4.14
Model 19	1.16	0.99	3.68	2.34	3.30
Model 20	1.27	0.82	2.26	2.00	3.78
Model 21	1.18	0.78	3.06	1.89	3.32
Model 22	0.78	0.64	2.50	1.40	2.65
Model 23	0.70	0.66	2.65	0.87	3.45
Model 24	1.27	1.30	4.09	2.58	2.88
Average	2.90	1.94	4.26	4.30	5.26

TABLE B-48 MAPE of ANN Models with Different Learning Functions (Outstar Weight Learning Function)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	11.87	5.31	7.73	14.70	7.06
Model 2	7.69	4.14	6.50	12.63	6.76
Model 3	5.91	3.10	5.11	10.04	8.20
Model 4	5.90	2.92	4.79	9.82	5.65
Model 5	5.41	2.76	4.14	8.70	4.46
Model 6	4.82	1.96	4.17	6.24	9.48
Model 7	1.86	2.27	4.50	3.94	6.31
Model 8	1.68	1.91	3.92	3.79	5.99
Model 9	2.05	2.33	3.51	3.95	4.44
Model 10	1.94	2.28	4.49	3.32	6.65
Model 11	1.60	1.91	3.07	3.02	6.23
Model 12	1.73	1.58	4.18	3.81	4.85
Model 13	1.57	1.43	3.87	3.18	4.03
Model 14	1.54	2.31	5.27	2.93	3.22
Model 15	1.60	1.74	5.24	2.95	4.77
Model 16	1.51	1.55	3.48	3.15	4.45
Model 17	1.24	1.60	4.15	2.32	3.93
Model 18	1.16	1.14	2.76	1.89	4.04
Model 19	1.02	1.16	3.11	2.48	3.09
Model 20	0.97	1.31	5.06	2.35	3.86
Model 21	0.92	1.29	2.40	1.90	3.30
Model 22	0.60	0.84	2.95	1.39	3.91
Model 23	0.70	0.66	2.84	0.89	3.82
Model 24	1.20	1.78	2.66	1.88	2.80
Average	2.77	2.05	4.16	4.64	5.05

TABLE B-49 MAPE of ANN Models with Different Learning Functions (Perceptron Weight and Bias Learning Function)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	11.77	4.93	8.52	14.33	8.09
Model 2	8.05	3.53	5.89	10.37	8.58
Model 3	6.69	3.08	6.05	10.15	6.48
Model 4	5.90	3.11	5.48	8.57	5.72
Model 5	5.09	2.50	5.05	6.18	5.84
Model 6	4.80	2.14	5.02	6.08	5.25
Model 7	1.77	2.22	4.82	4.16	5.21
Model 8	1.66	1.91	4.28	4.83	6.65
Model 9	1.74	1.95	4.78	3.49	5.05
Model 10	1.77	1.98	5.11	3.35	6.50
Model 11	1.79	1.78	3.33	3.63	4.87
Model 12	1.38	1.57	3.83	3.09	5.23
Model 13	1.29	1.78	4.22	2.78	5.12
Model 14	1.60	1.64	3.85	2.77	4.58
Model 15	1.51	1.56	6.12	3.17	4.90
Model 16	1.40	1.58	3.30	2.27	4.83
Model 17	1.73	1.22	4.22	2.05	5.15
Model 18	1.17	1.24	2.89	1.74	3.63
Model 19	1.53	1.35	4.36	2.64	3.72
Model 20	0.92	1.30	3.18	2.34	4.43
Model 21	0.93	0.93	3.39	1.89	3.72
Model 22	1.08	0.70	2.92	1.39	3.07
Model 23	0.70	0.66	2.44	0.89	4.04
Model 24	1.36	1.37	2.27	2.53	2.15
Average	2.82	1.92	4.39	4.36	5.12

TABLE B-50 MAPE of ANN Models with Different Learning Functions (Normalized Perceptron Weight and Bias Learning Function)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	12.32	4.97	8.87	13.21	8.26
Model 2	8.82	3.63	6.69	10.09	6.84
Model 3	6.02	3.16	6.69	8.05	7.42
Model 4	5.42	2.42	5.62	9.82	7.87
Model 5	5.37	2.58	4.77	6.15	8.06
Model 6	4.66	2.25	3.52	6.13	7.97
Model 7	1.76	2.06	4.38	4.16	4.46
Model 8	1.62	1.93	4.56	3.93	6.13
Model 9	1.91	1.95	3.58	3.47	4.71
Model 10	1.64	1.81	4.55	3.17	6.05
Model 11	1.95	2.06	3.62	3.41	4.56
Model 12	1.74	1.98	4.21	2.98	4.85
Model 13	1.60	1.57	4.48	3.82	5.34
Model 14	1.60	1.49	4.10	2.86	5.12
Model 15	1.46	1.80	4.76	3.08	6.21
Model 16	1.54	1.57	3.75	2.06	3.62
Model 17	1.10	1.28	4.36	1.84	5.12
Model 18	1.31	1.69	3.86	2.84	3.75
Model 19	1.20	1.65	3.42	2.30	3.15
Model 20	1.22	1.20	3.56	2.34	3.48
Model 21	1.15	0.88	3.45	1.55	4.04
Model 22	0.63	0.75	4.38	1.41	2.70
Model 23	0.70	0.66	3.08	0.87	3.51
Model 24	1.19	1.75	3.24	2.08	3.44
Average	2.83	1.96	4.48	4.24	5.28

TABLE B-51 MAPE of ANN Models with Different Learning Functions (Self-organizing Map Weight Learning Function)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	12.56	4.89	9.41	14.11	9.53
Model 2	8.15	3.87	6.27	12.52	7.32
Model 3	6.75	3.22	5.96	9.04	6.47
Model 4	5.60	2.94	6.38	6.94	5.41
Model 5	5.58	2.49	4.89	6.32	5.77
Model 6	5.43	2.21	5.67	6.15	6.43
Model 7	1.83	1.99	6.20	4.17	5.63
Model 8	1.58	2.23	5.66	4.88	5.42
Model 9	1.92	2.15	3.71	3.43	6.37
Model 10	1.78	1.87	4.95	4.33	5.22
Model 11	2.07	1.98	4.48	3.86	4.89
Model 12	1.67	1.70	3.65	2.97	5.43
Model 13	1.62	1.54	3.88	2.80	5.57
Model 14	1.44	1.34	5.05	2.65	4.68
Model 15	1.26	1.64	4.61	3.07	4.50
Model 16	1.57	1.49	3.71	3.20	4.37
Model 17	1.23	1.44	3.30	2.39	3.98
Model 18	1.16	1.16	3.49	2.24	4.06
Model 19	1.06	0.99	3.02	2.38	3.43
Model 20	1.42	1.00	2.69	1.67	4.10
Model 21	1.18	1.14	2.71	1.89	3.61
Model 22	0.83	0.76	2.48	1.39	4.44
Model 23	0.70	0.66	2.85	0.89	2.50
Model 24	1.02	1.73	3.04	2.23	4.13
Average	2.89	1.93	4.50	4.40	5.14

TABLE B-52 MAPE of ANN Models with Different Learning Functions (Widrow-Hoff Weight and Bias Learning Rule)

	Non-Clustering	Weekend	Weekday Peak	Weekday Non-Peak	Weekday Evening
Model 1	10.81	5.14	9.00	12.10	9.07
Model 2	8.57	3.62	6.38	10.48	7.50
Model 3	6.39	3.16	6.05	7.92	7.41
Model 4	5.83	2.82	6.19	7.22	7.40
Model 5	5.02	2.59	5.58	6.21	5.67
Model 6	4.86	2.03	7.40	7.62	5.54
Model 7	1.82	2.09	3.89	4.17	5.88
Model 8	1.67	2.35	3.32	2.85	5.28
Model 9	1.73	2.26	4.95	3.09	4.55
Model 10	1.81	1.99	3.74	3.22	5.55
Model 11	1.67	1.77	3.72	2.98	4.00
Model 12	1.50	1.77	3.82	3.14	6.14
Model 13	1.52	1.74	3.85	3.07	4.80
Model 14	1.69	1.71	3.55	3.06	5.17
Model 15	1.44	1.76	3.41	2.72	5.59
Model 16	1.28	1.57	2.77	3.02	3.73
Model 17	1.58	1.58	4.15	2.15	3.28
Model 18	1.20	1.24	2.67	1.89	6.31
Model 19	1.34	0.93	2.96	2.15	3.25
Model 20	1.01	1.05	3.02	1.92	3.56
Model 21	1.30	1.10	3.30	1.89	3.25
Model 22	0.83	0.78	2.54	1.40	3.89
Model 23	0.70	0.66	3.12	0.90	3.17
Model 24	0.92	1.31	3.03	2.77	2.76
Average	2.77	1.96	4.27	4.08	5.11

APPENDIX C

SCHEDULE ADHERENCE BY TIME PERIOD AND BUS STOP

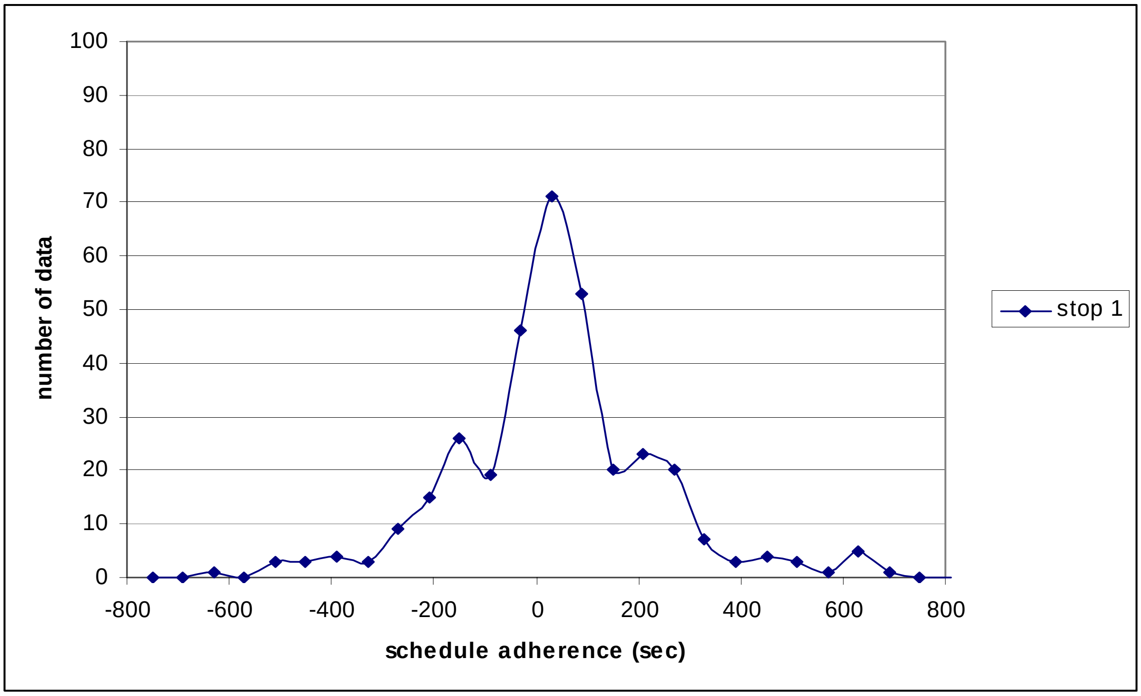

FIGURE C- 1 Schedule Adherence of Non-Clustering (stop 1)

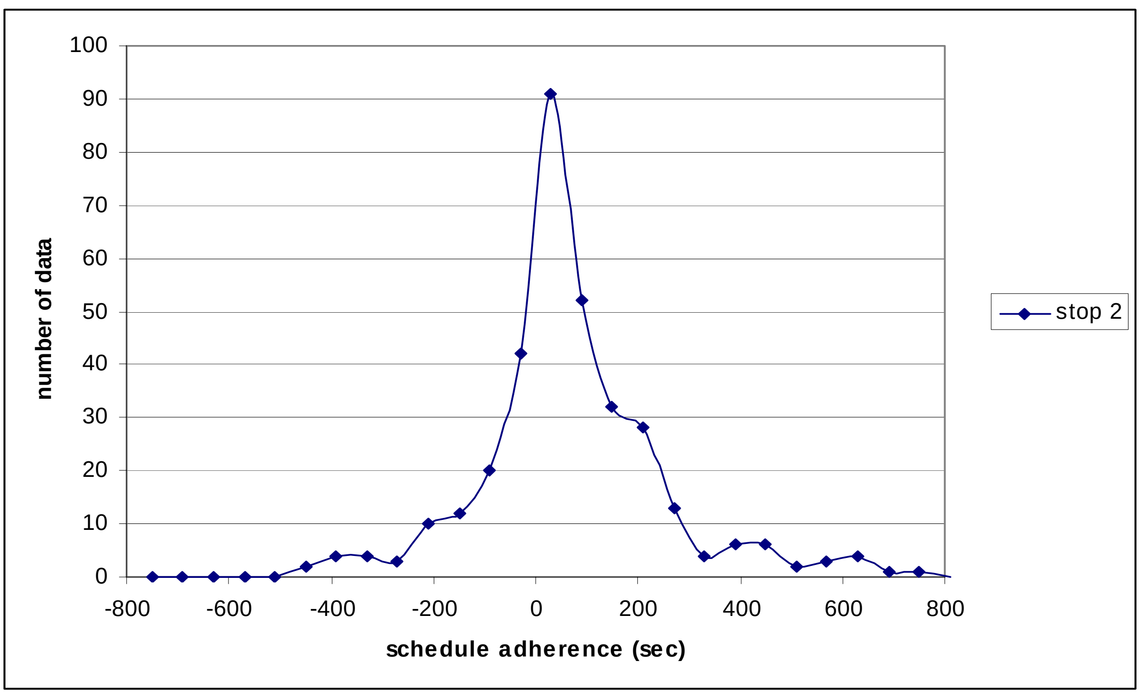

FIGURE C- 2 Schedule Adherence of Non-Clustering (stop 2)

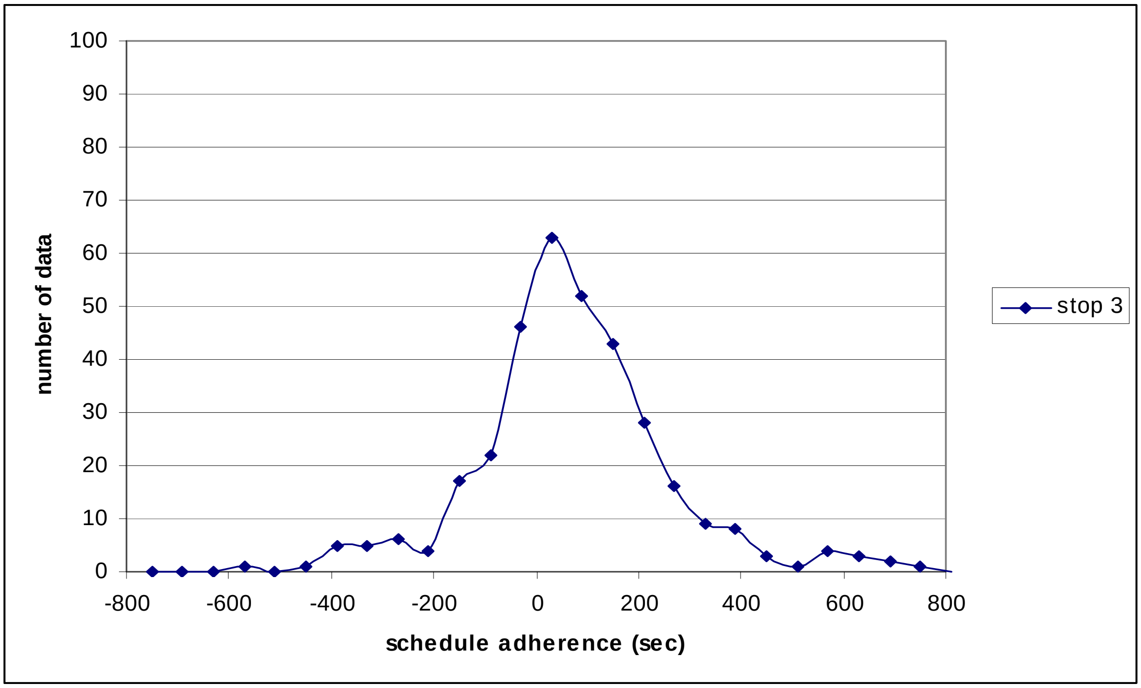

FIGURE C- 3 Schedule Adherence of Non-Clustering (stop 3)

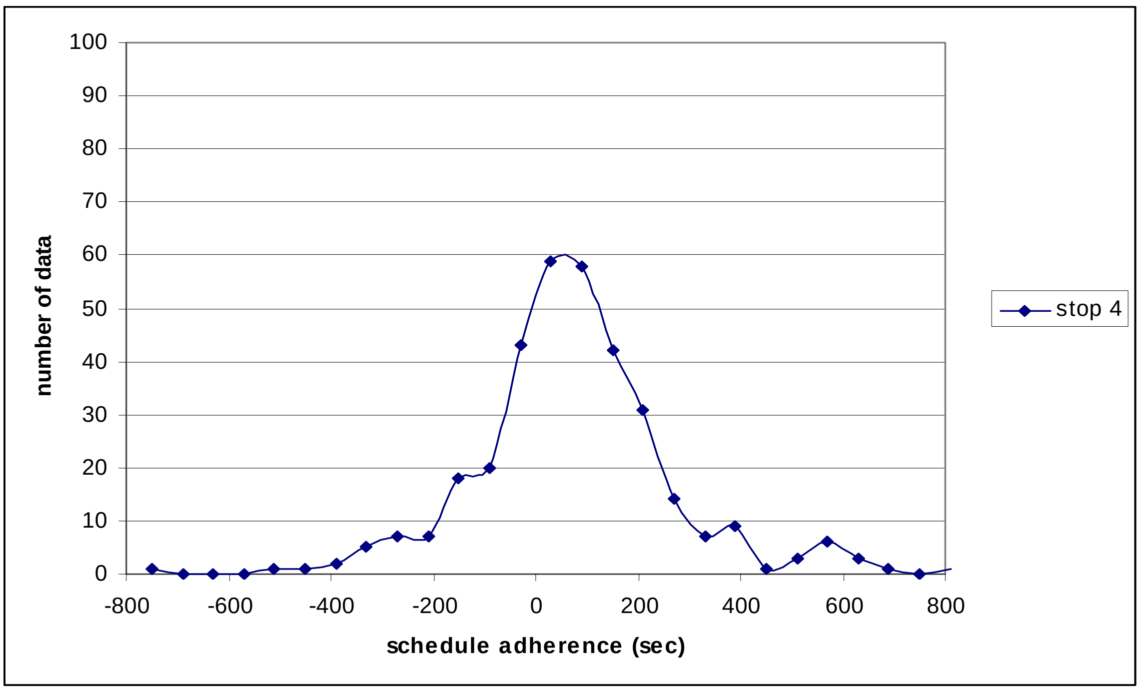

FIGURE C- 4 Schedule Adherence of Non-Clustering (stop 4)

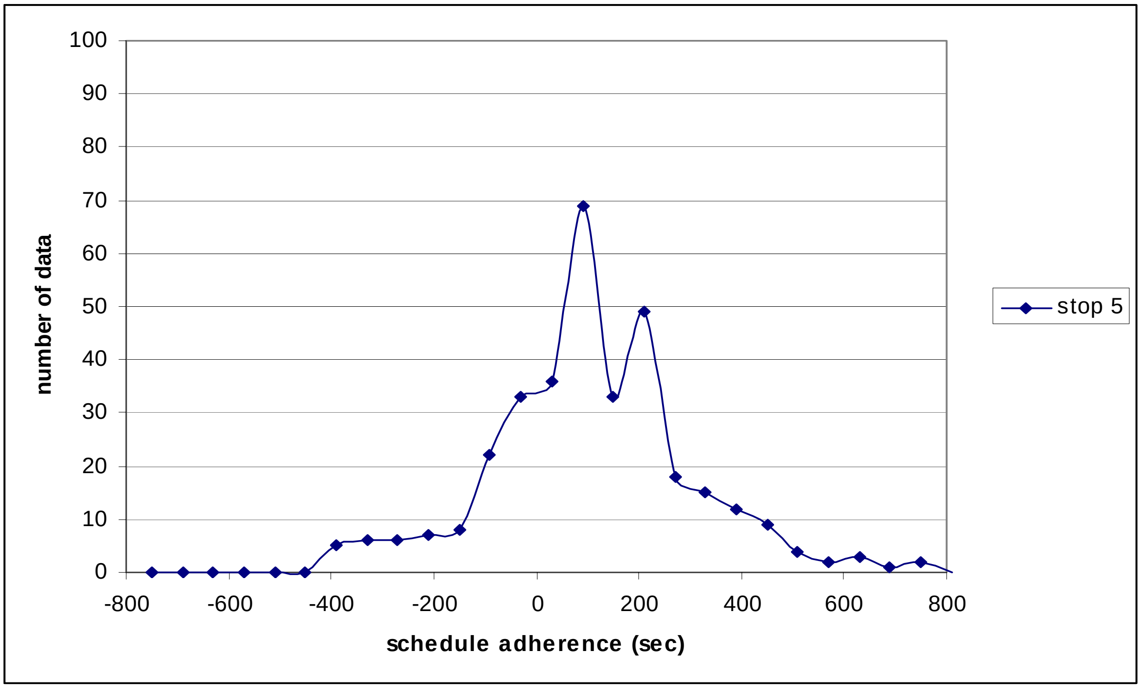

FIGURE C- 5 Schedule Adherence of Non-Clustering (stop 5)

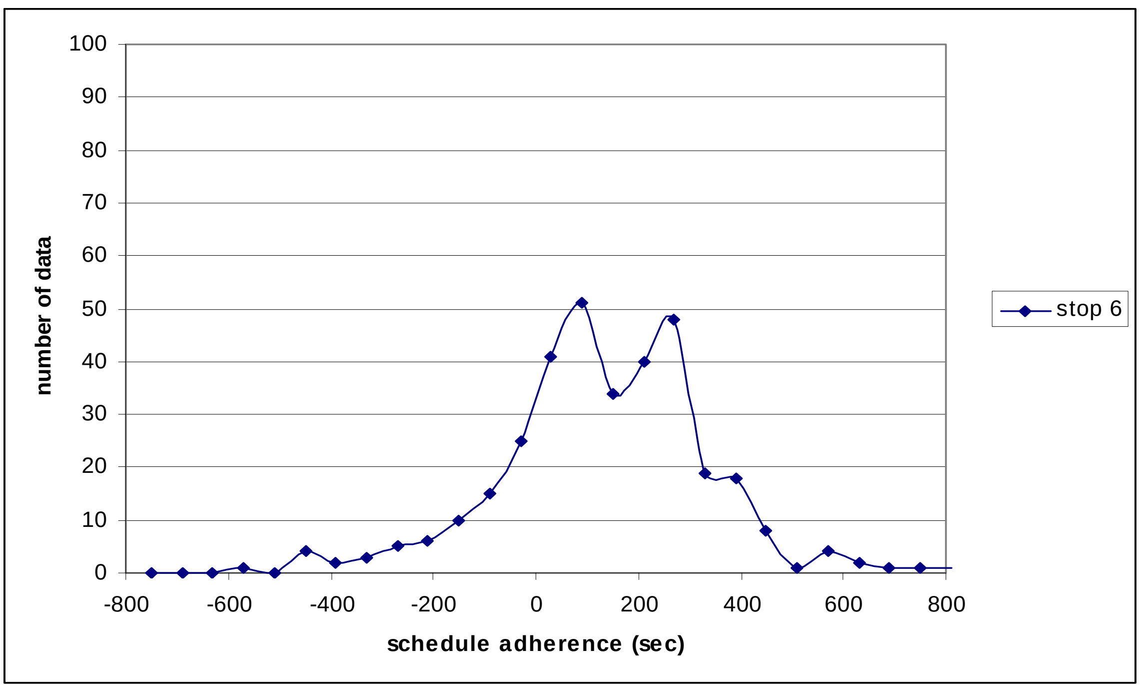

FIGURE C- 6 Schedule Adherence of Non-Clustering (stop 6)

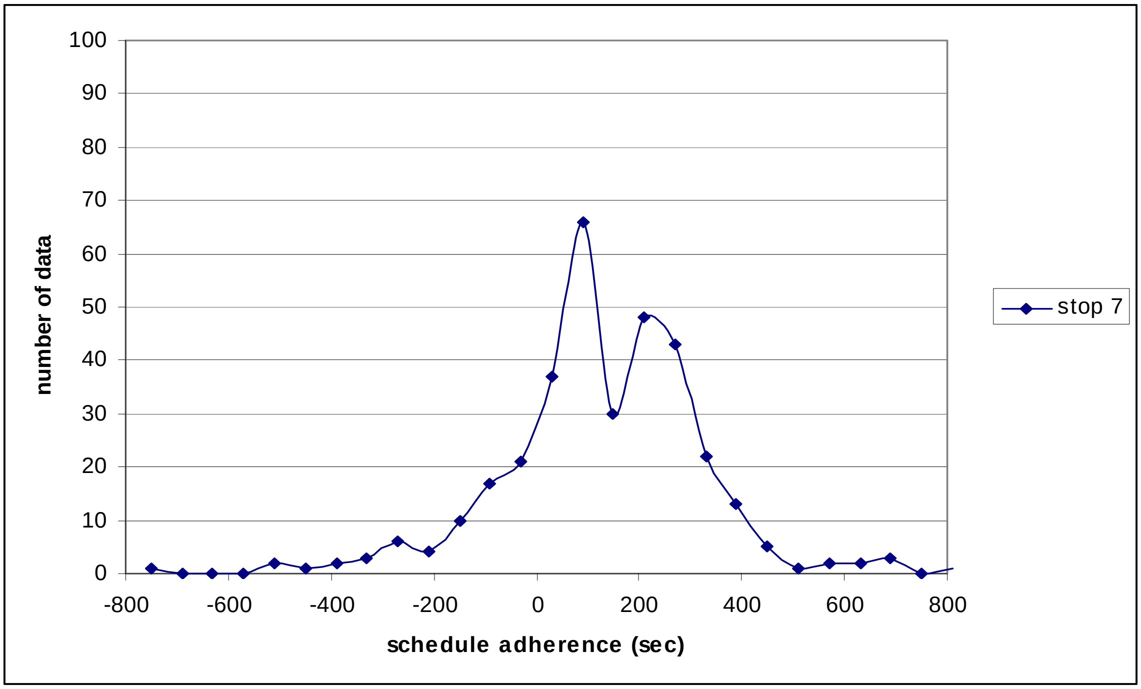

FIGURE C- 7 Schedule Adherence of Non-Clustering (stop 7)

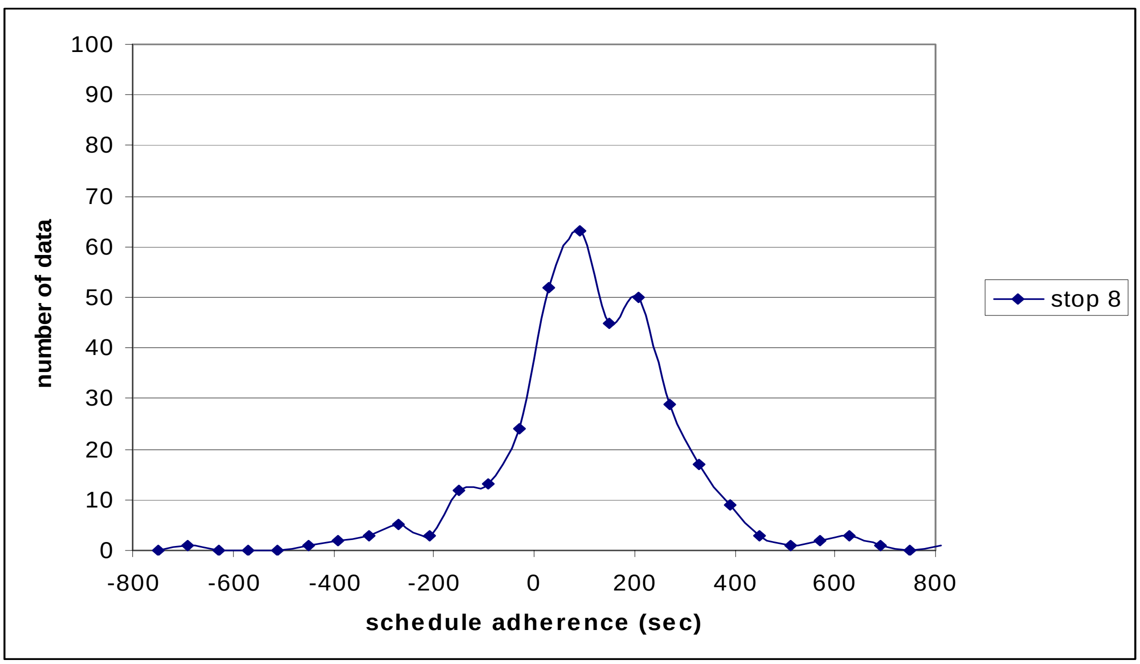

FIGURE C- 8 Schedule Adherence of Non-Clustering (stop 8)

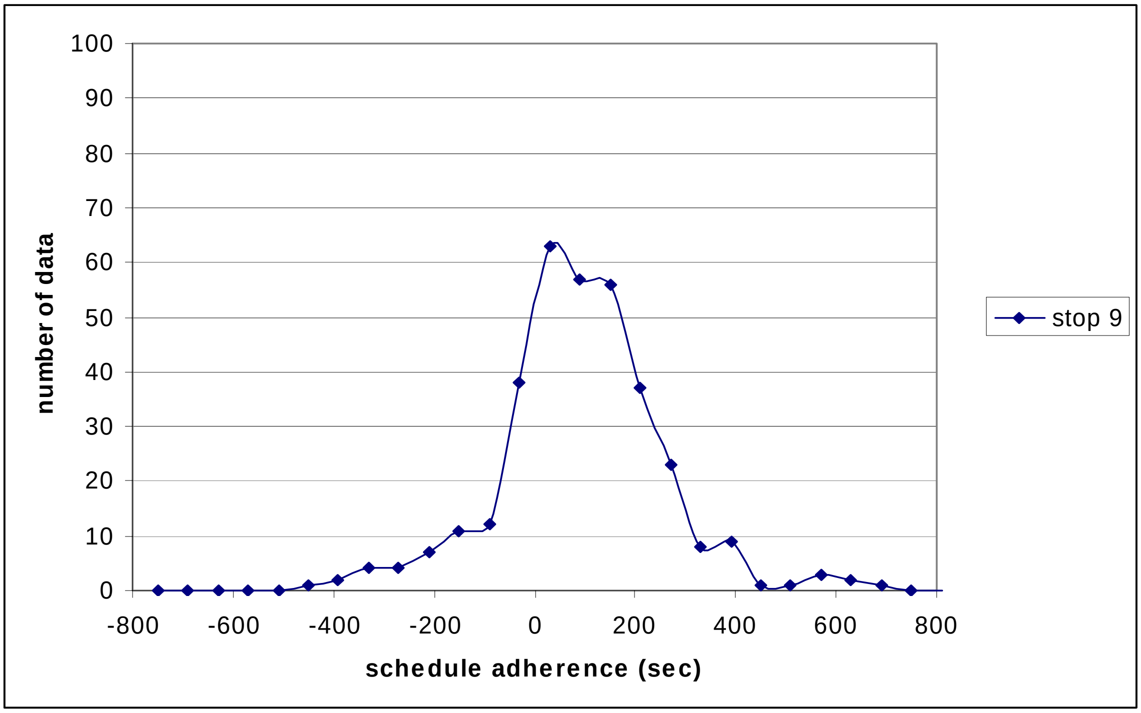

FIGURE C- 9 Schedule Adherence of Non-Clustering (stop 9)

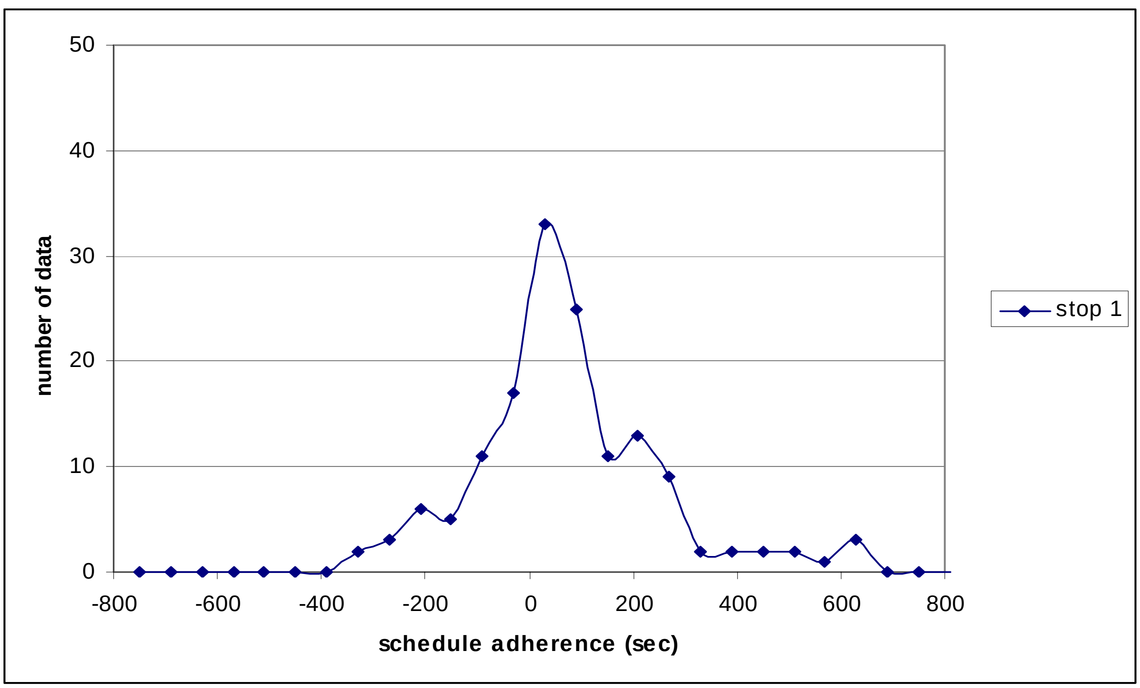

FIGURE C- 10 Schedule Adherence of Weekend (stop 1)

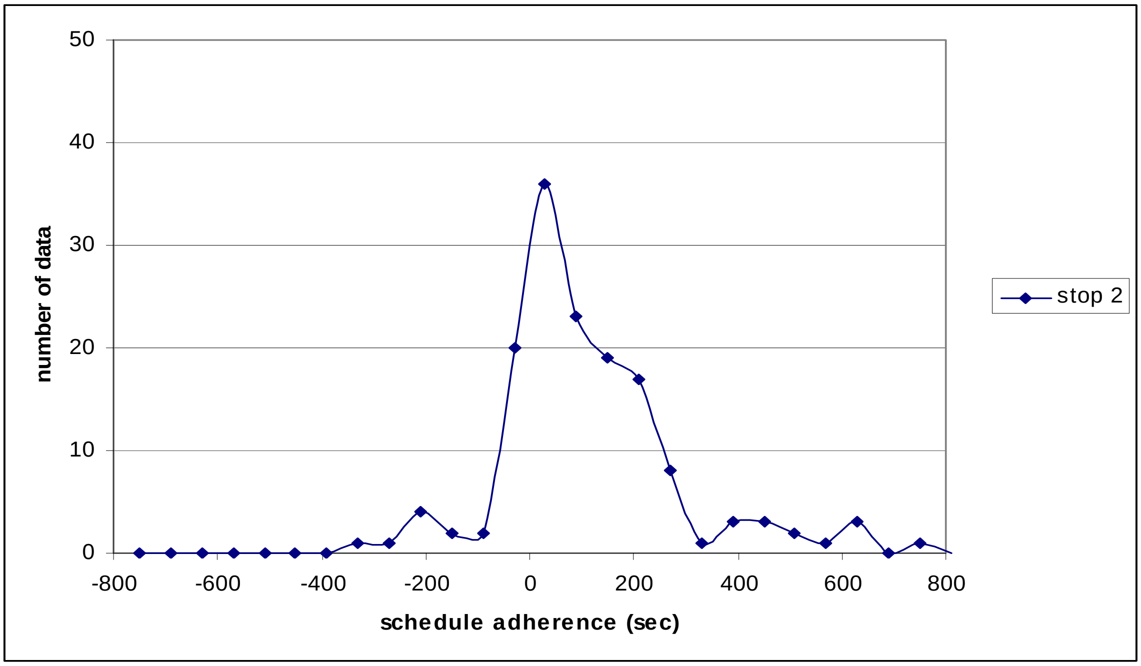

FIGURE C- 11 Schedule Adherence of Weekend (stop 2)

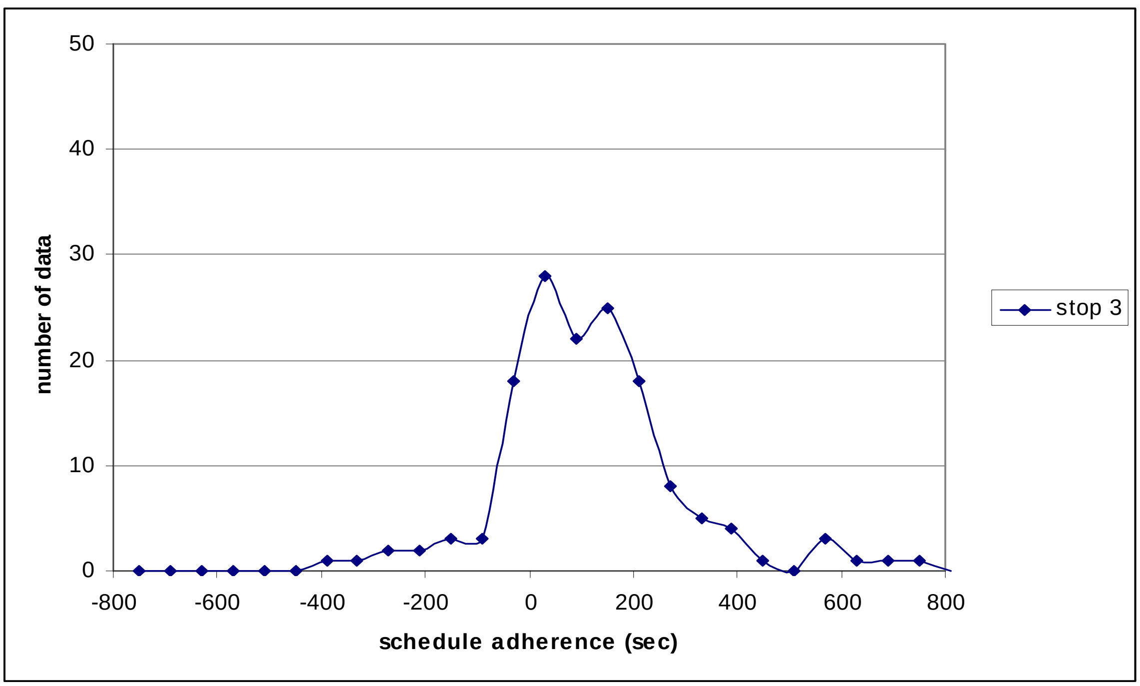

FIGURE C- 12 Schedule Adherence of Weekend (stop 3)

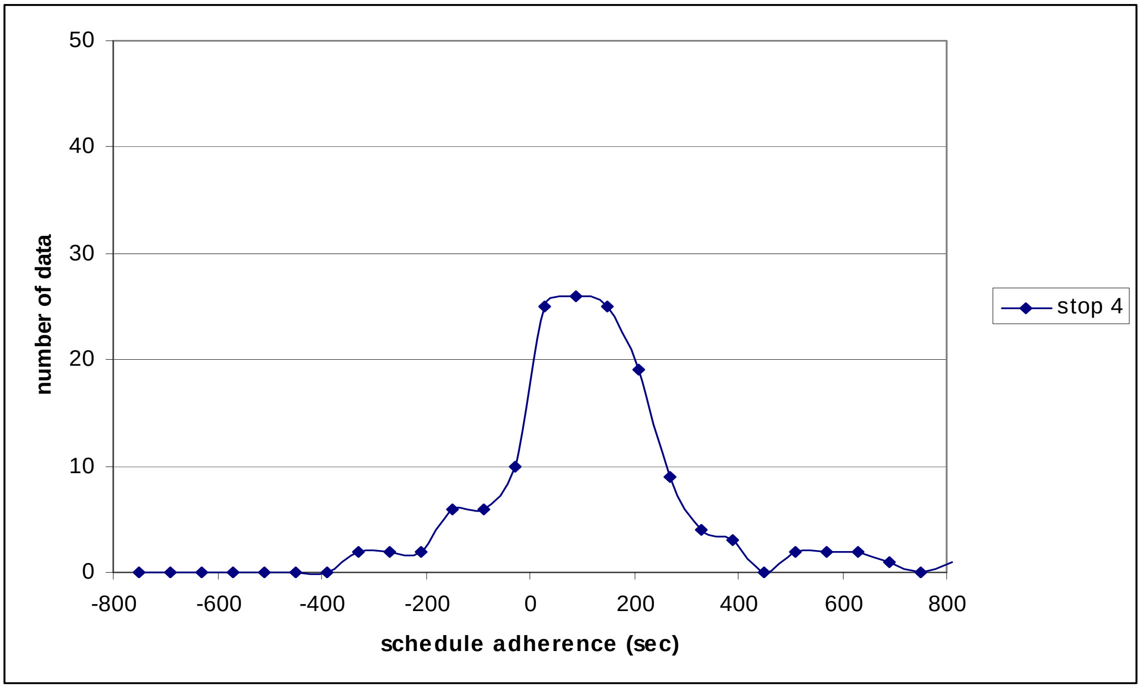

FIGURE C- 13 Schedule Adherence of Weekend (stop 4)

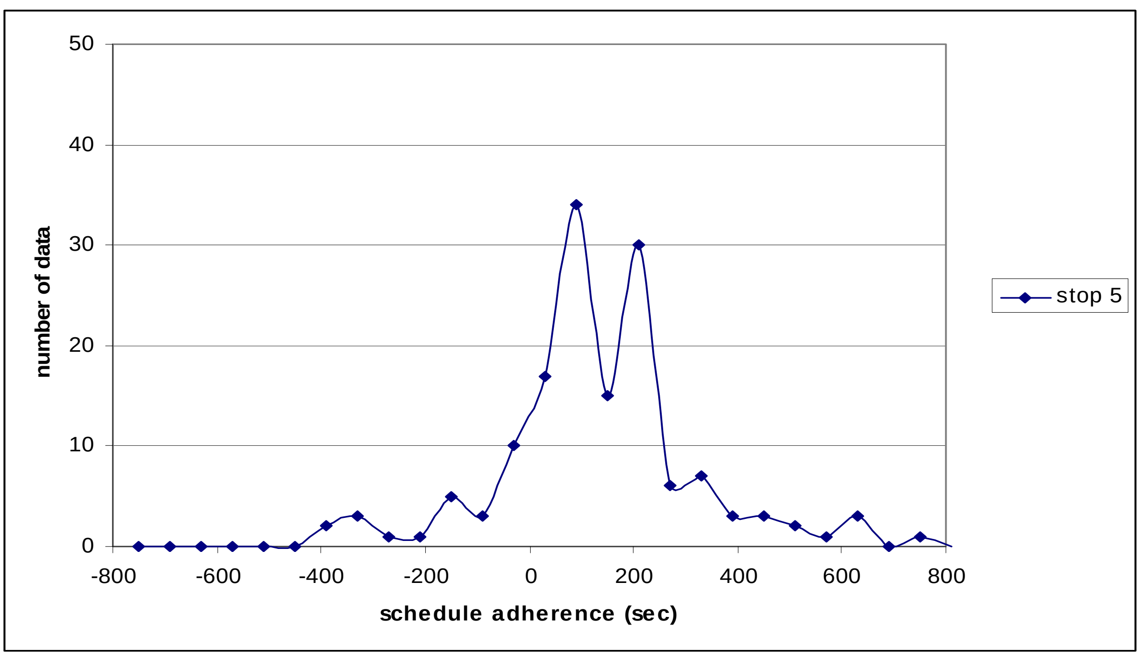

FIGURE C- 14 Schedule Adherence of Weekend (stop 5)

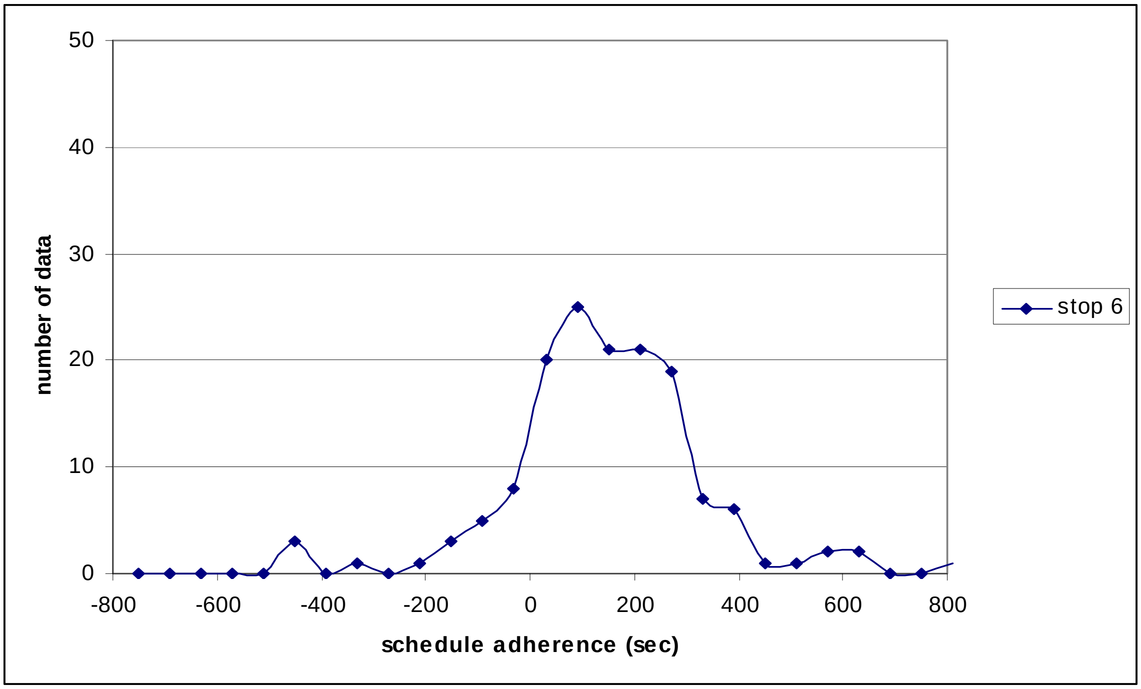

FIGURE C- 15 Schedule Adherence of Weekend (stop 6)

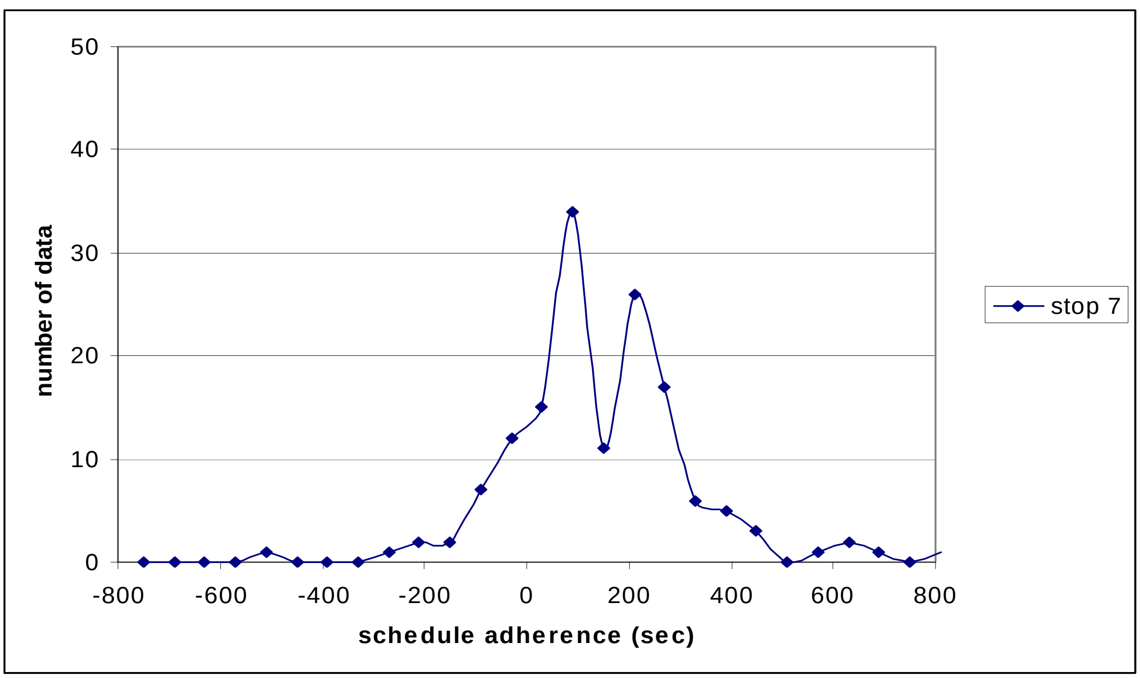

FIGURE C- 16 Schedule Adherence of Weekend (stop 7)

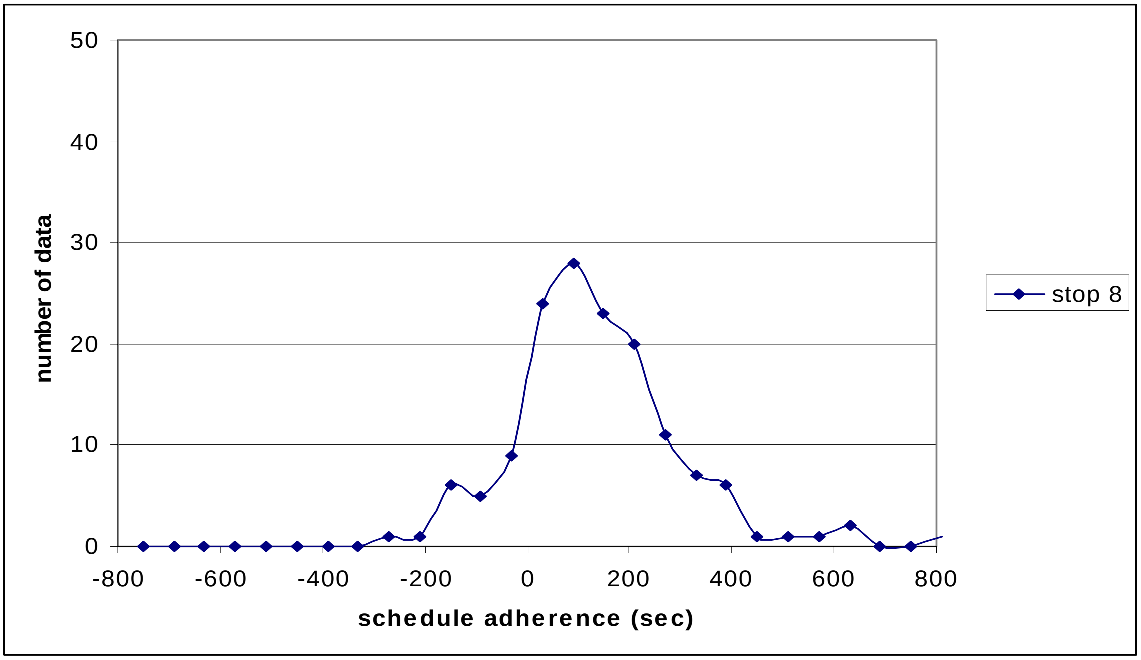

FIGURE C- 17 Schedule Adherence of Weekend (stop 8)

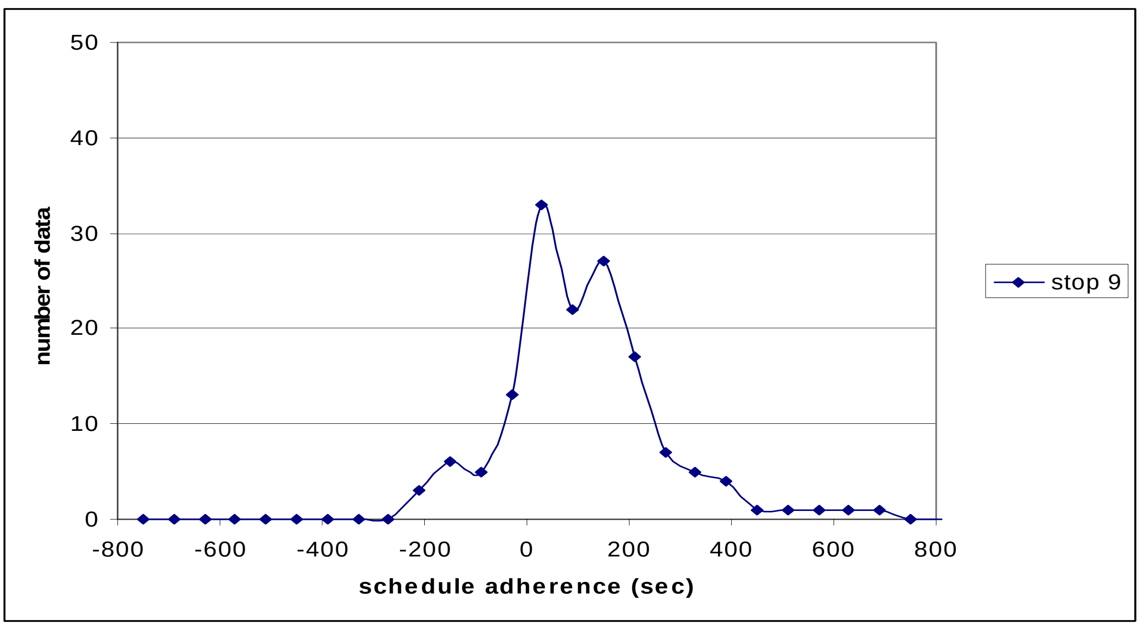

FIGURE C- 18 Schedule Adherence of Weekend (stop 9)

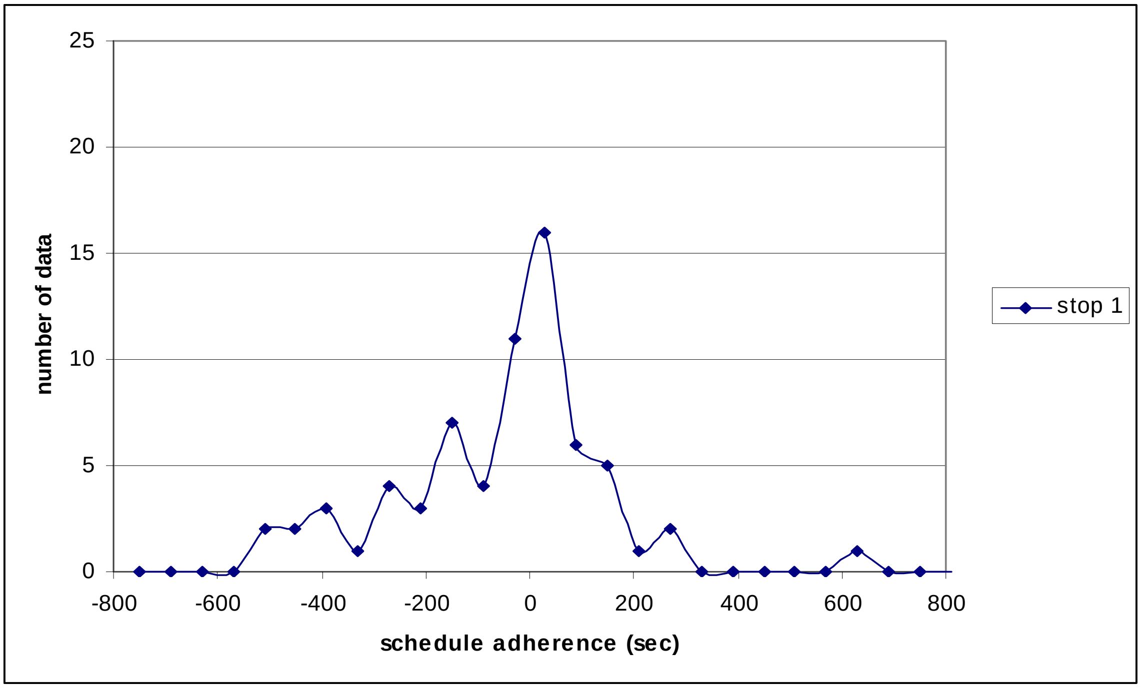

FIGURE C- 19 Schedule Adherence of Weekday Peak (stop 1)

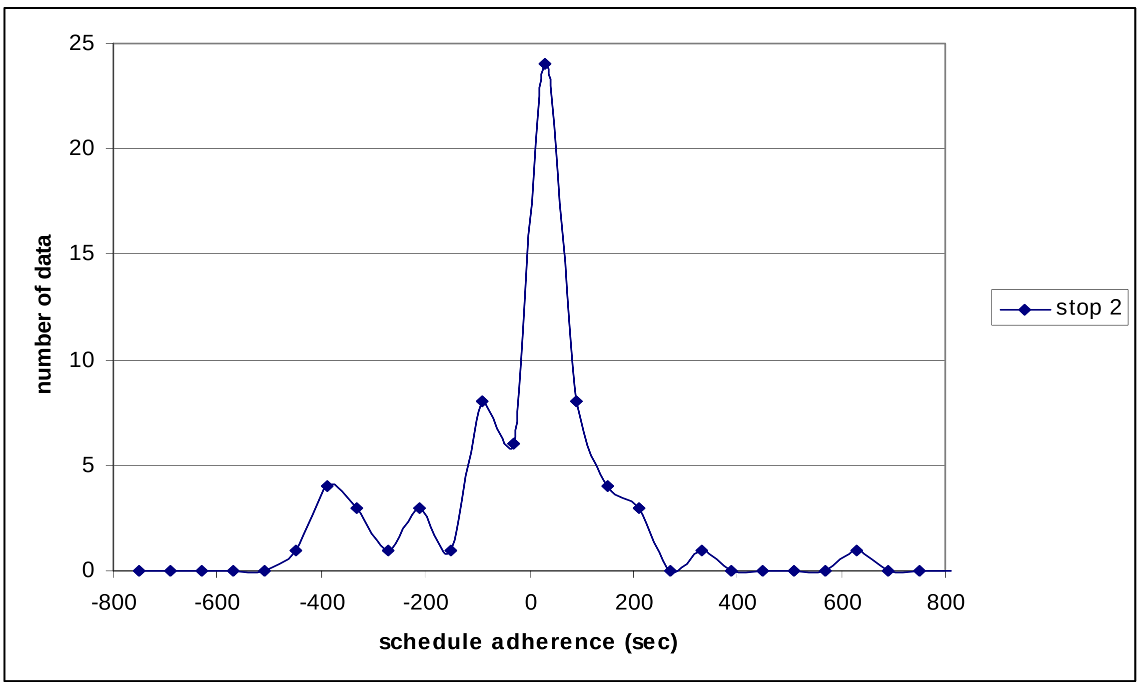

FIGURE C- 20 Schedule Adherence of Weekday Peak (stop 2)

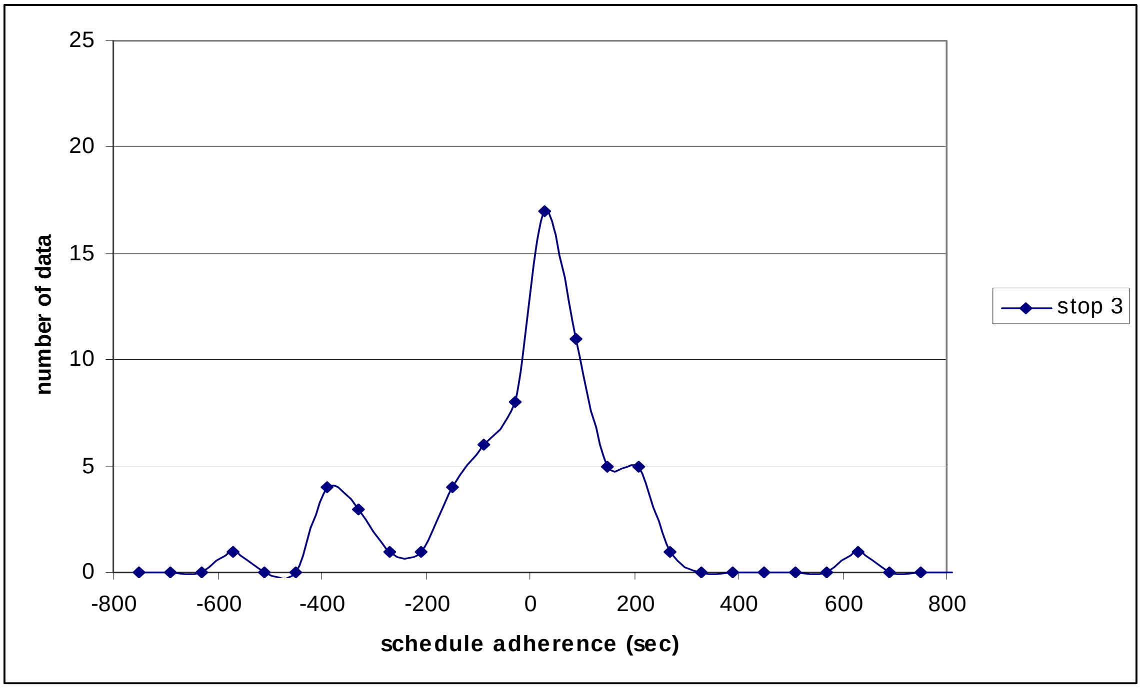

FIGURE C- 21 Schedule Adherence of Weekday Peak (stop 3)

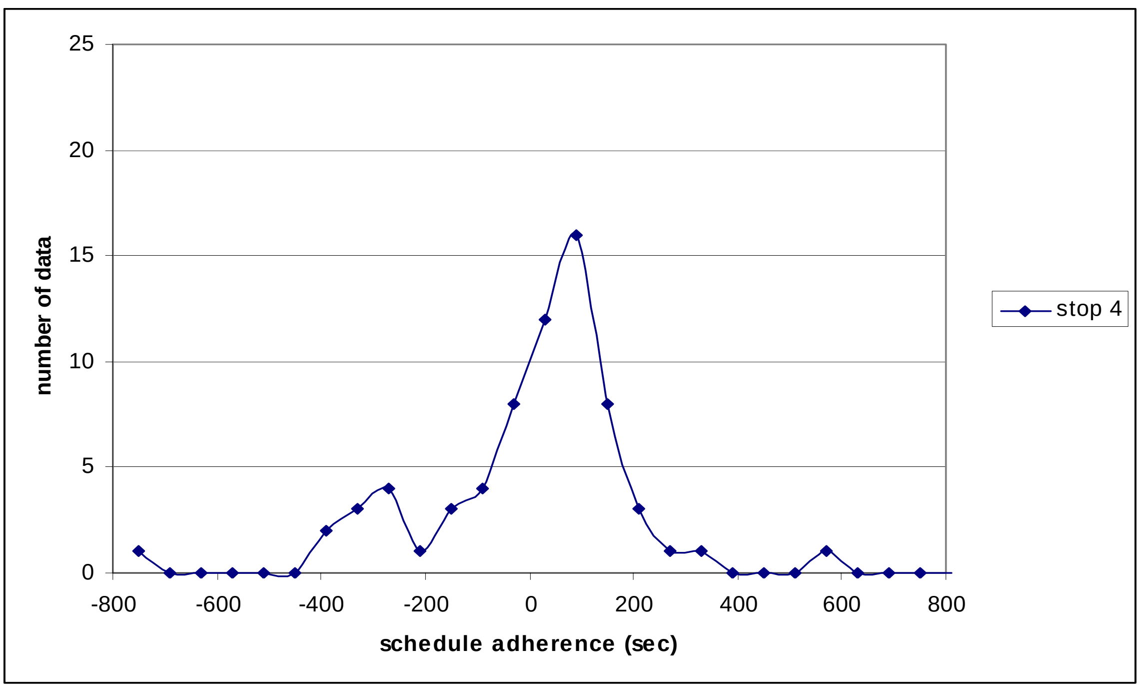

FIGURE C- 22 Schedule Adherence of Weekday Peak (stop 4)

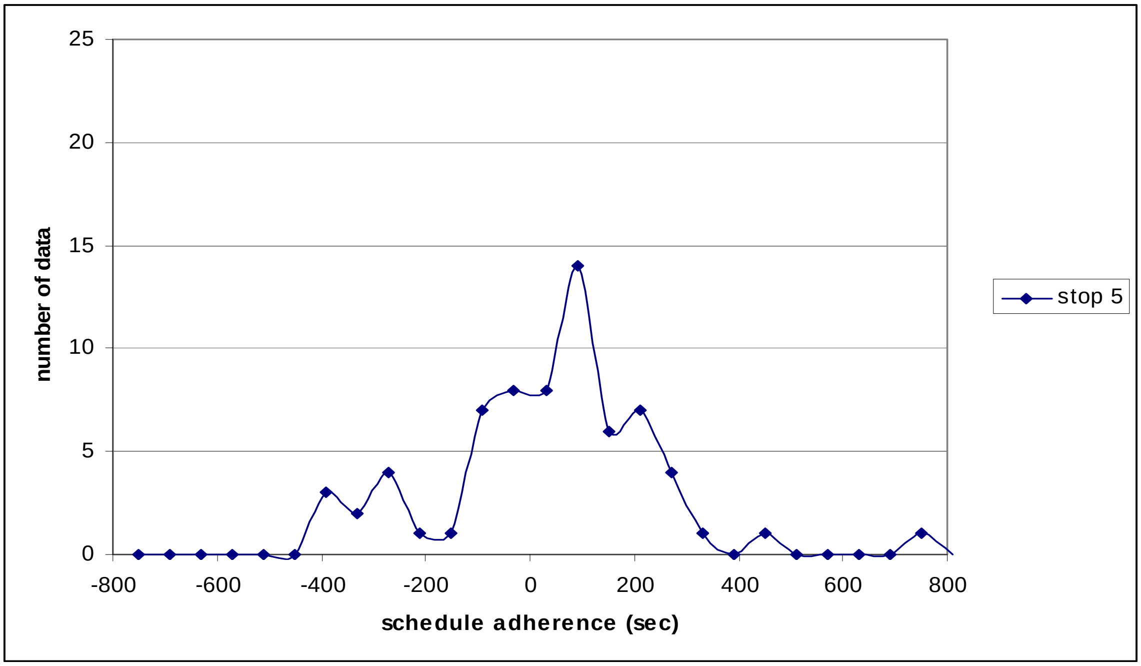

FIGURE C- 23 Schedule Adherence of Weekday Peak (stop 5)

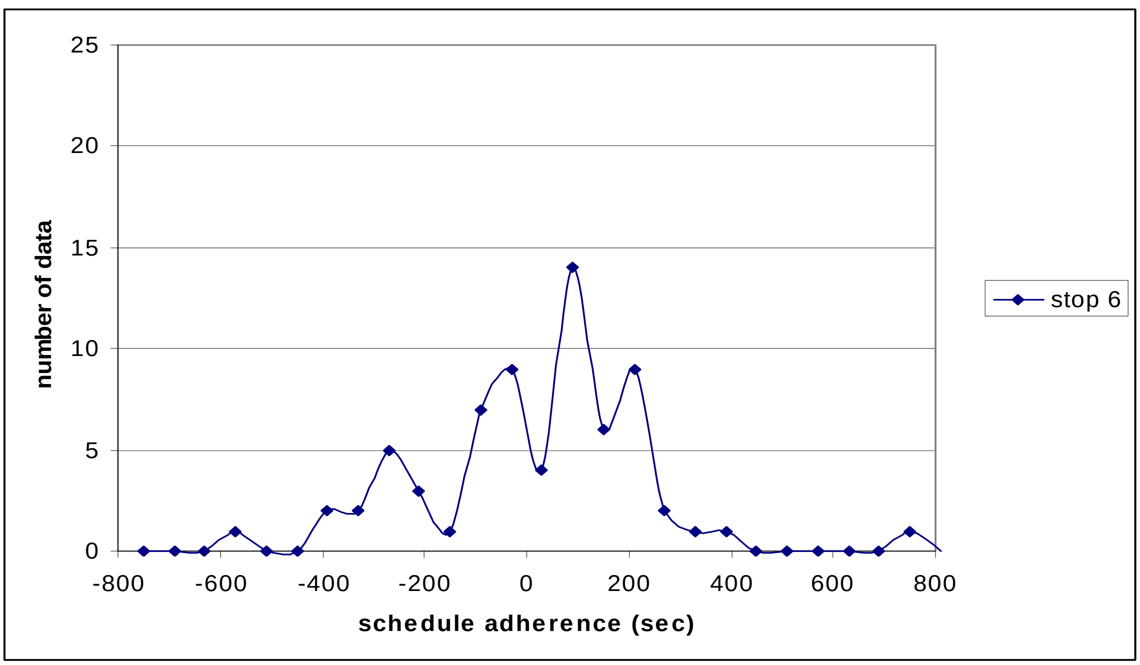

FIGURE C- 24 Schedule Adherence of Weekday Peak (stop 6)

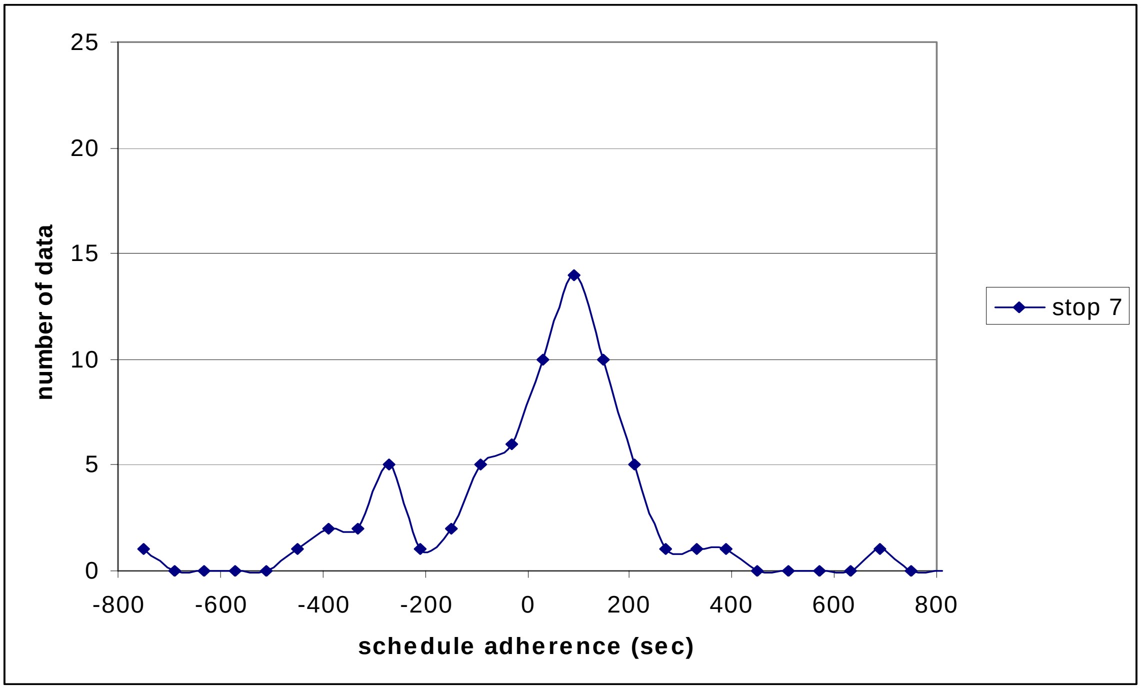

FIGURE C- 25 Schedule Adherence of Weekday Peak (stop 7)

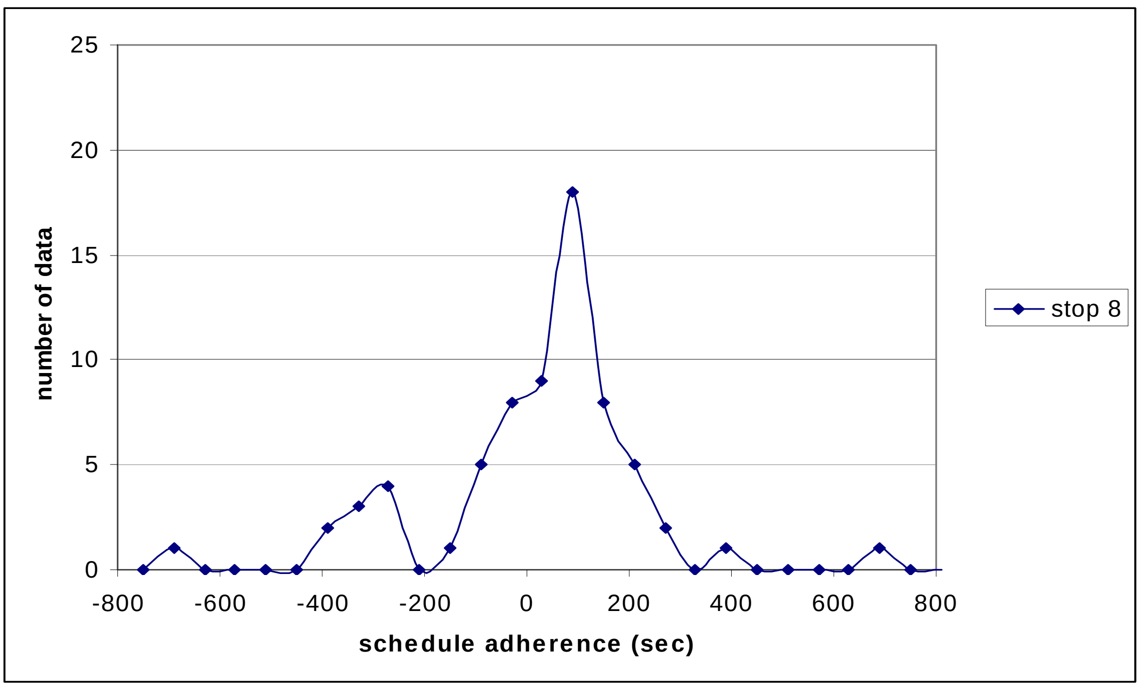

FIGURE C- 26 Schedule Adherence of Weekday Peak (stop 8)

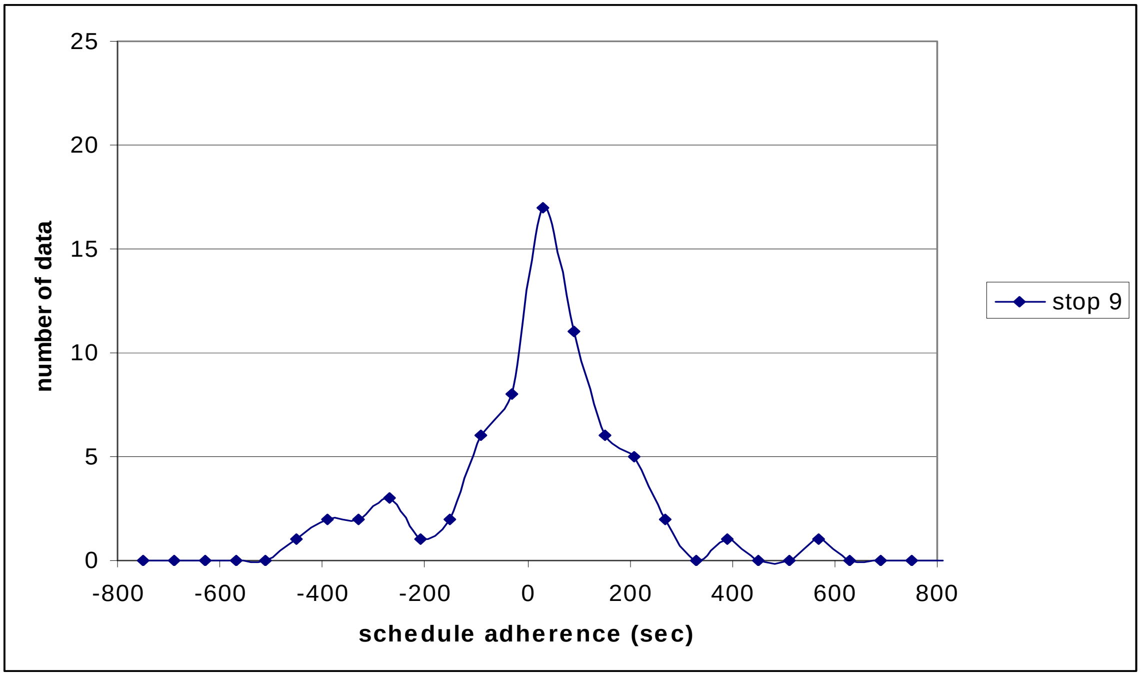

FIGURE C- 27 Schedule Adherence of Weekday Peak (stop 9)

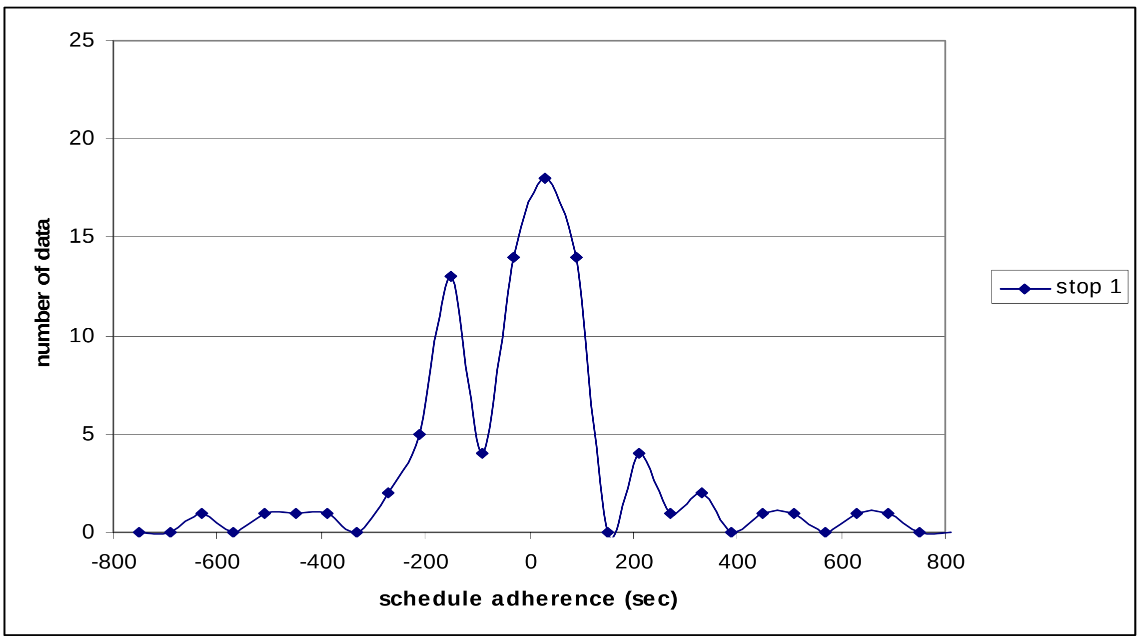

FIGURE C- 28 Schedule Adherence of Weekday Non-Peak (stop 1)

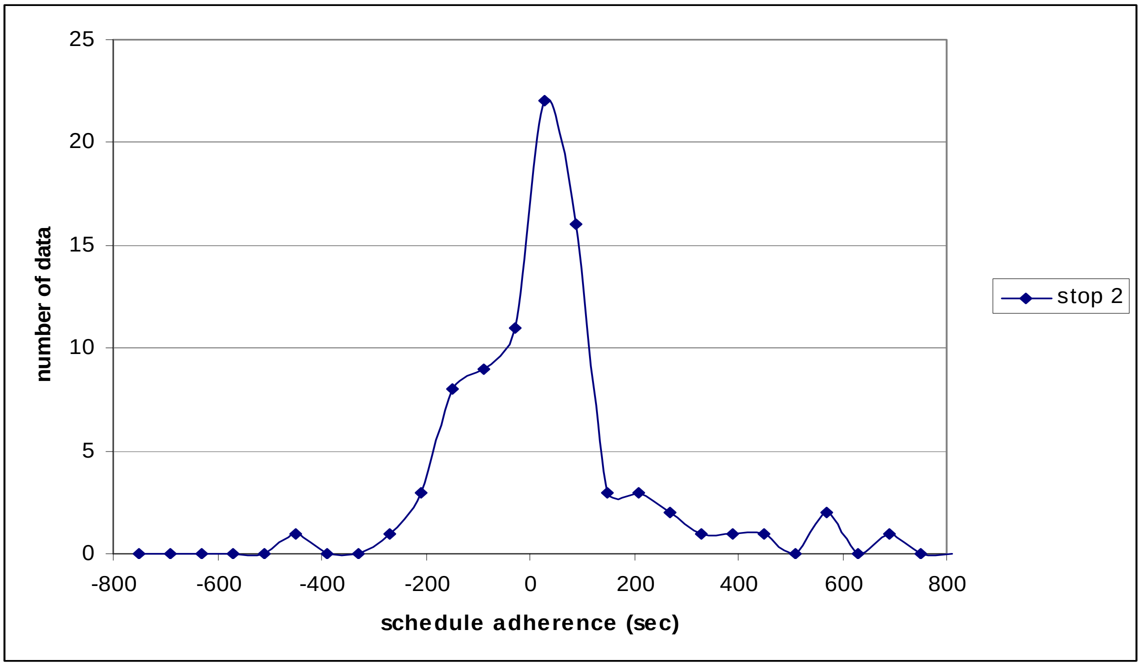

FIGURE C- 29 Schedule Adherence of Weekday Non-Peak (stop 2)

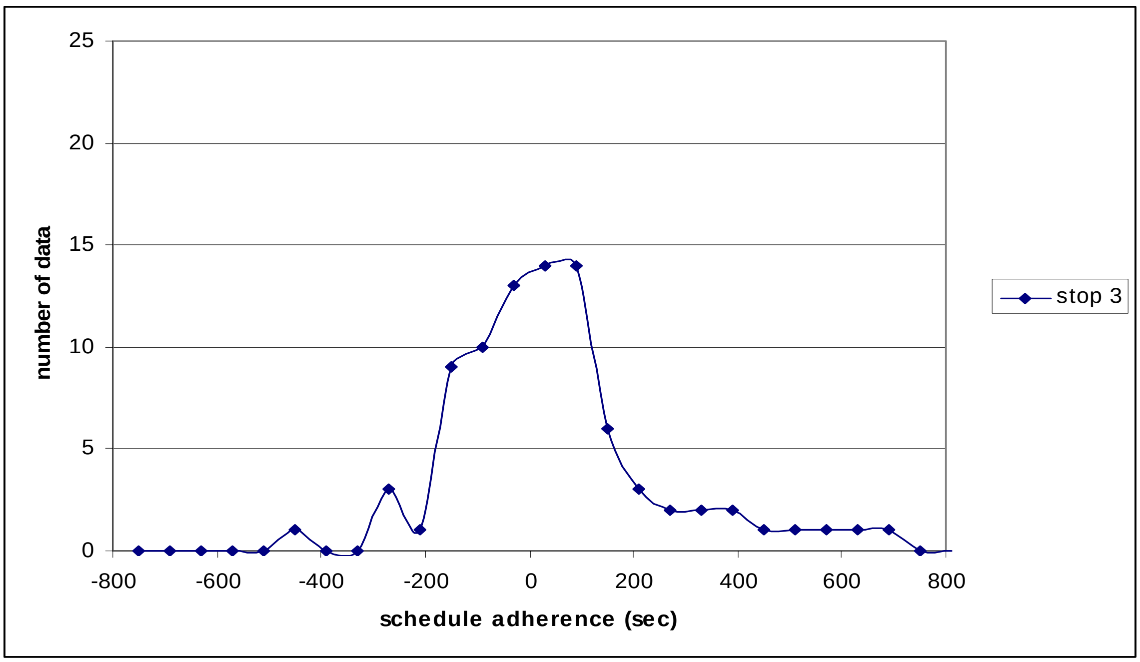

FIGURE C- 30 Schedule Adherence of Weekday Non-Peak (stop 3)

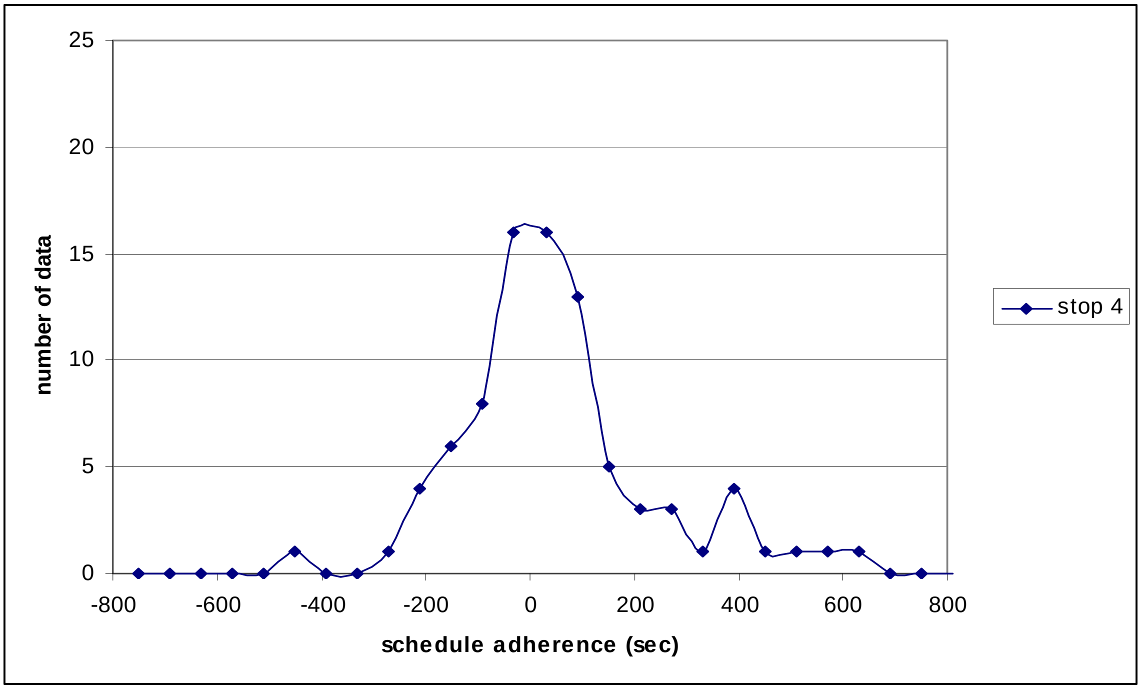

FIGURE C- 31 Schedule Adherence of Weekday Non-Peak (stop 4)

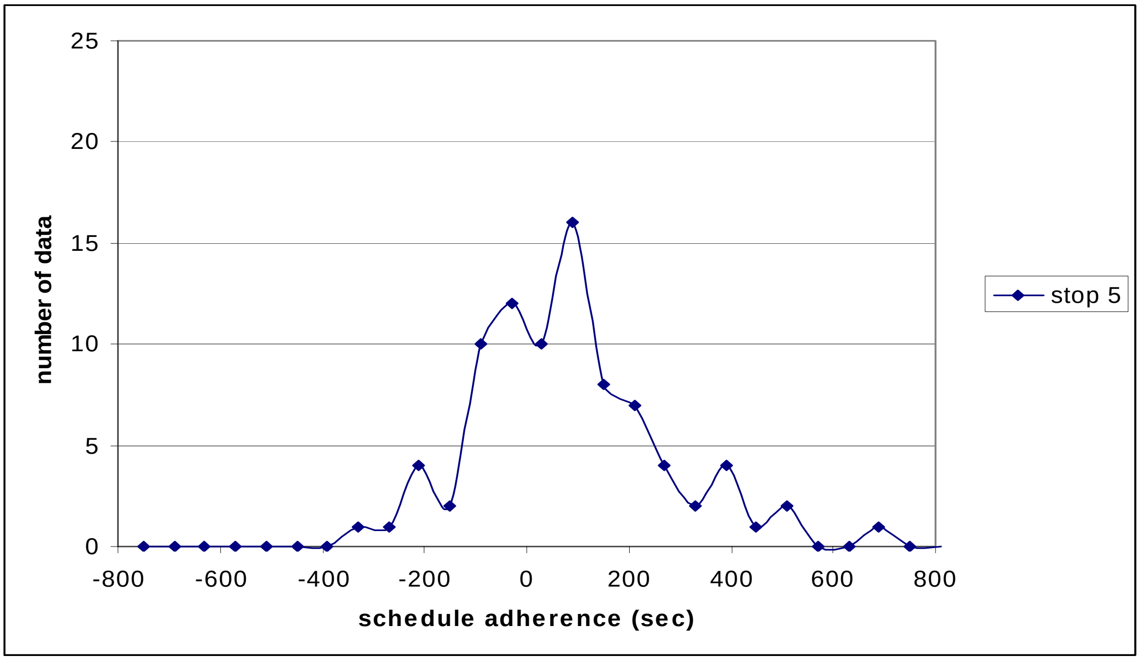

FIGURE C- 32 Schedule Adherence of Weekday Non-Peak (stop 5)

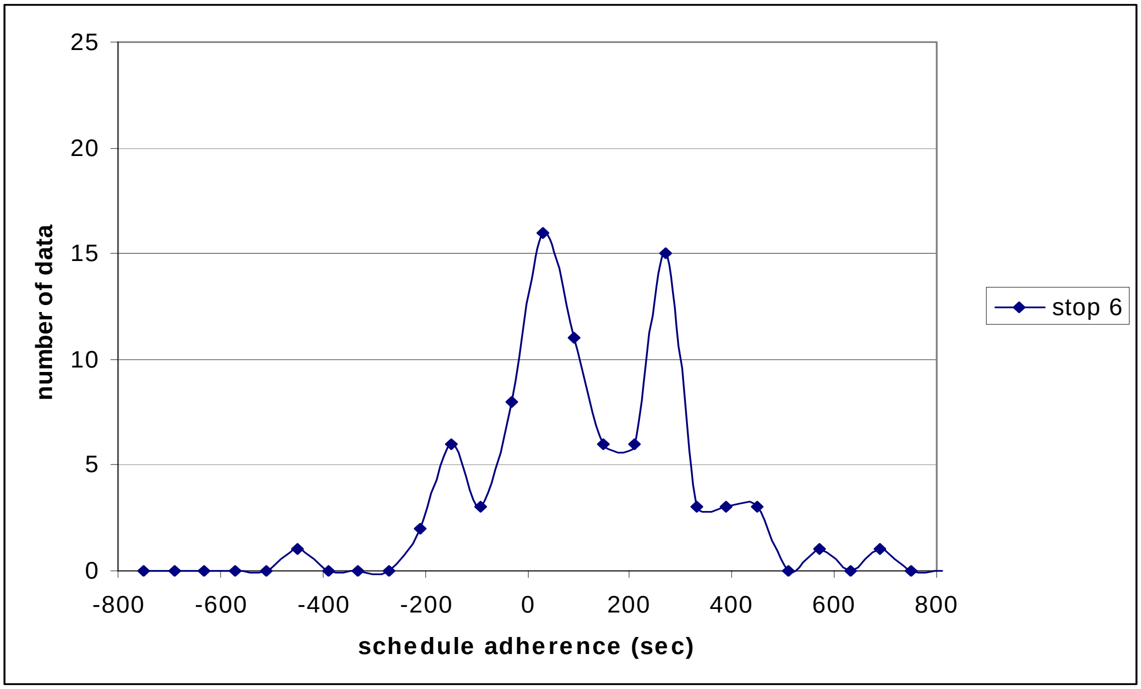

FIGURE C- 33 Schedule Adherence of Weekday Non-Peak (stop 6)

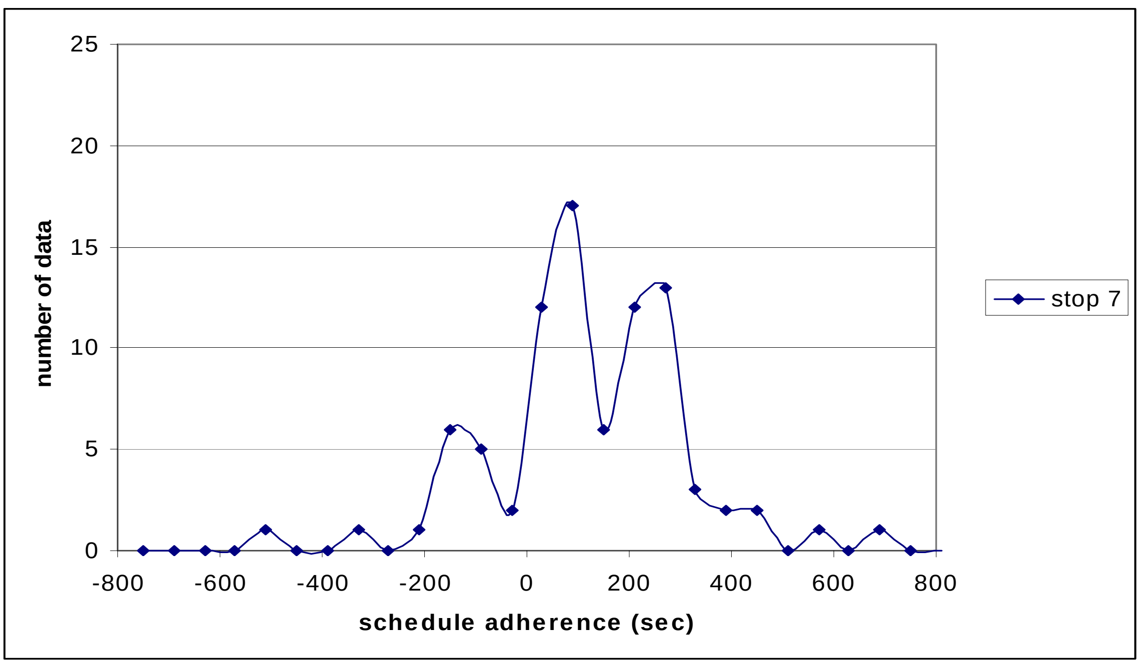

FIGURE C- 34 Schedule Adherence of Weekday Non-Peak (stop 7)

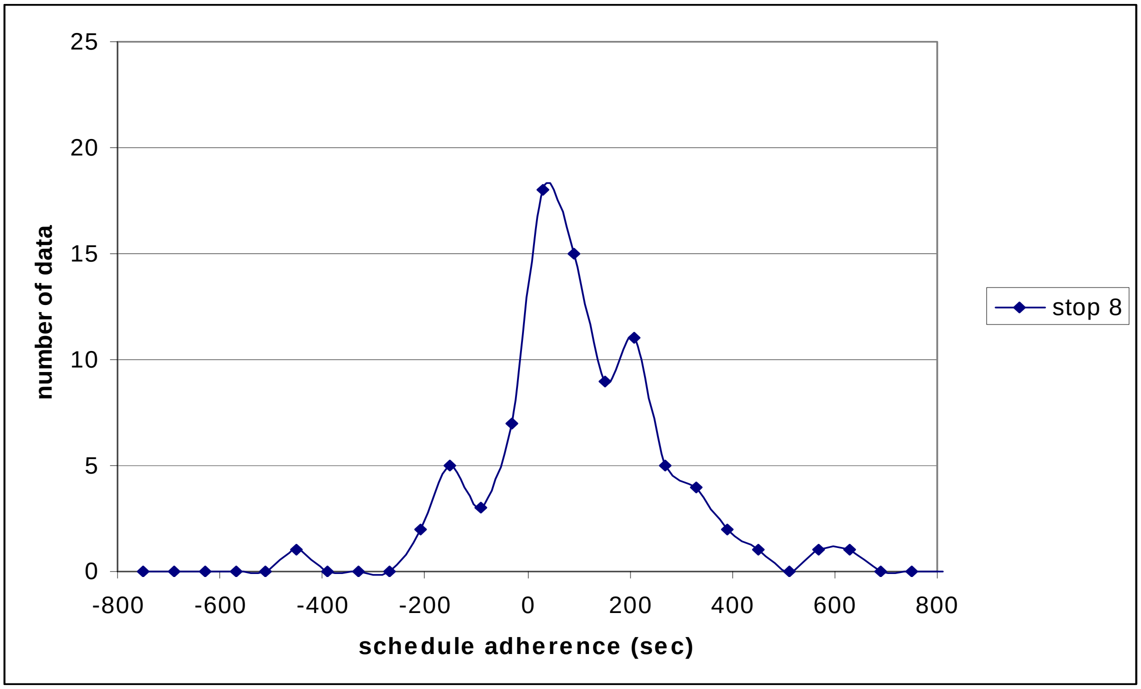

FIGURE C- 35 Schedule Adherence of Weekday Non-Peak (stop 8)

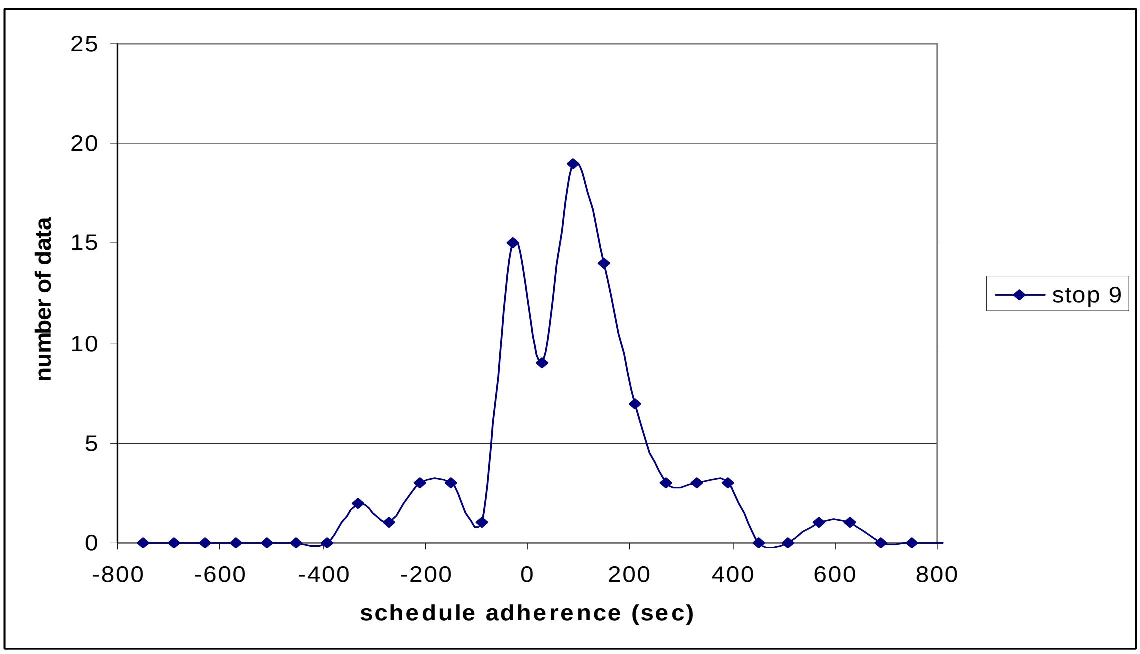

FIGURE C- 36 Schedule Adherence of Weekday Non-Peak (stop 9)

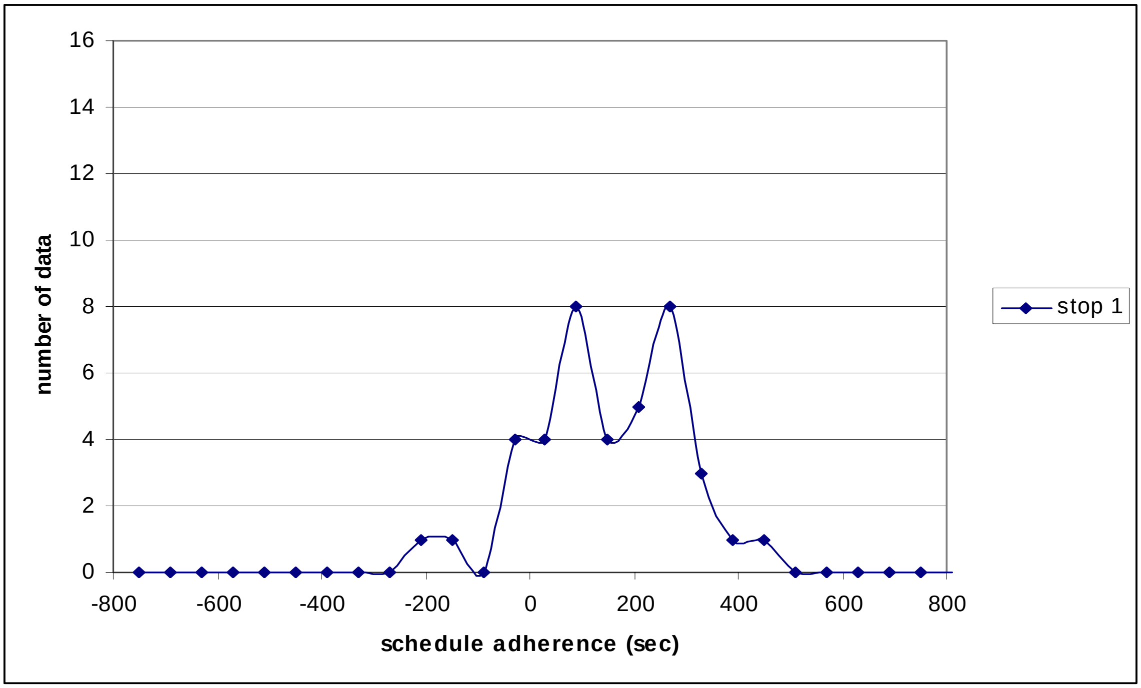

FIGURE C- 37 Schedule Adherence of Weekday Evening (stop 1)

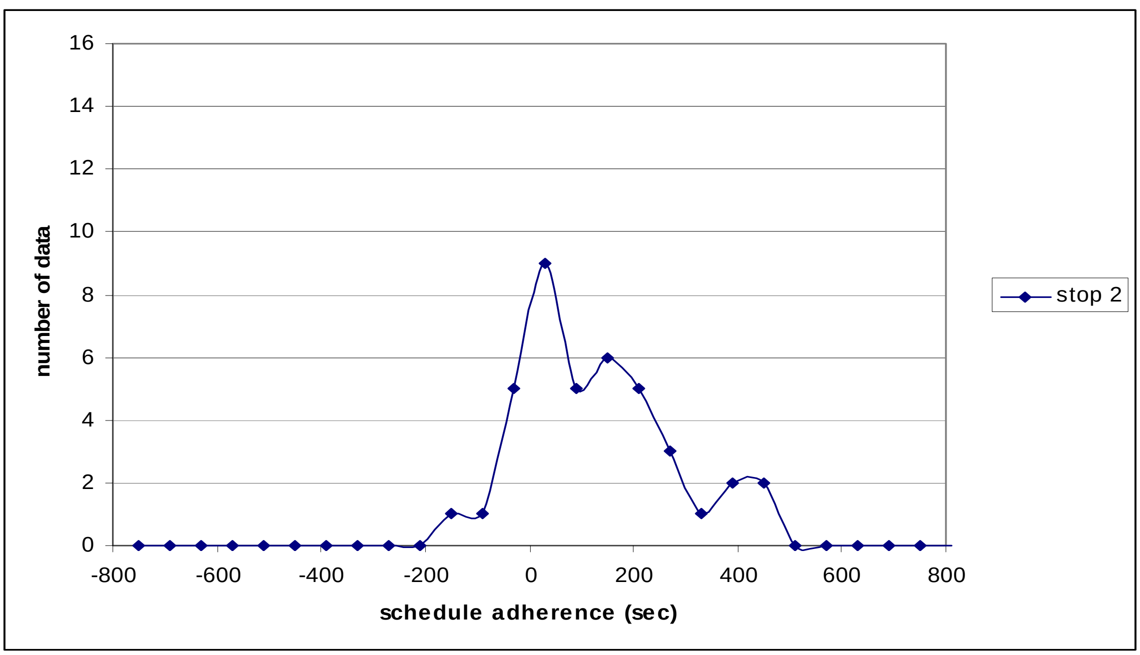

FIGURE C- 38 Schedule Adherence of Weekday Evening (stop 2)

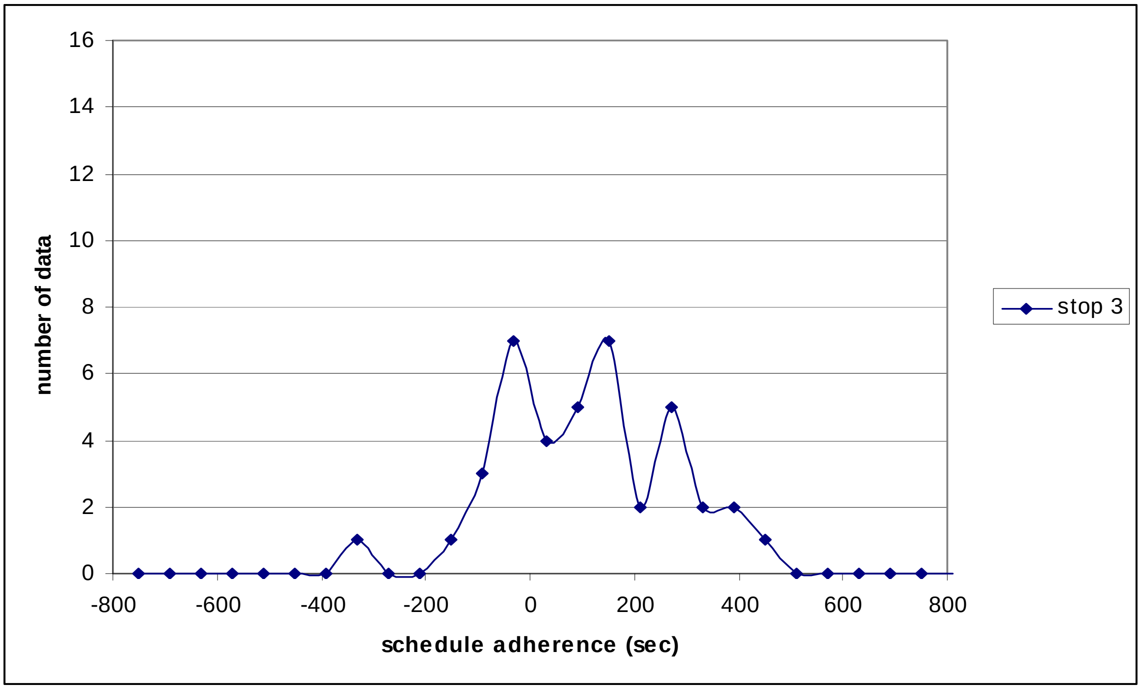

FIGURE C- 39 Schedule Adherence of Weekday Evening (stop 3)

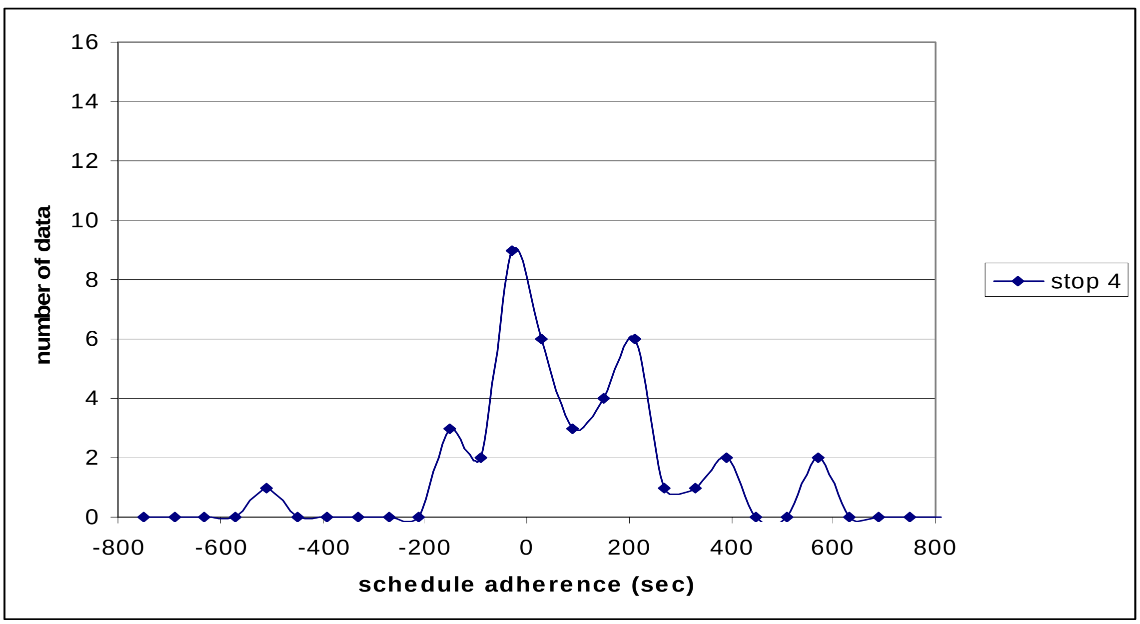

FIGURE C- 40 Schedule Adherence of Weekday Evening (stop 4)

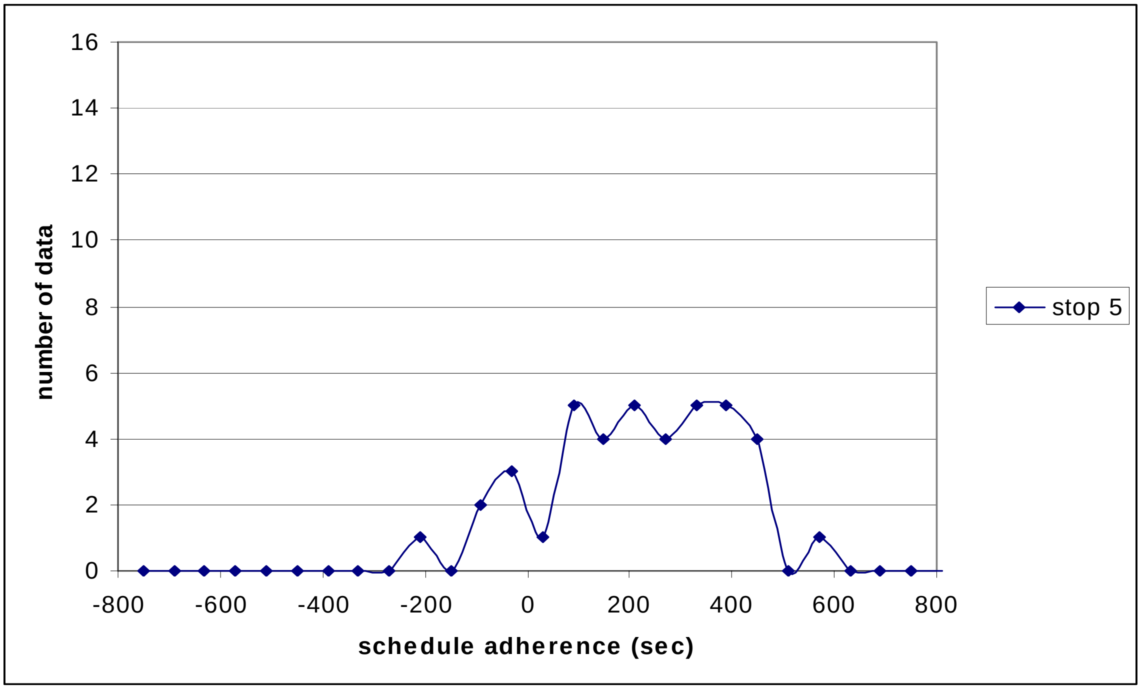

FIGURE C- 41 Schedule Adherence of Weekday Evening (stop 5)

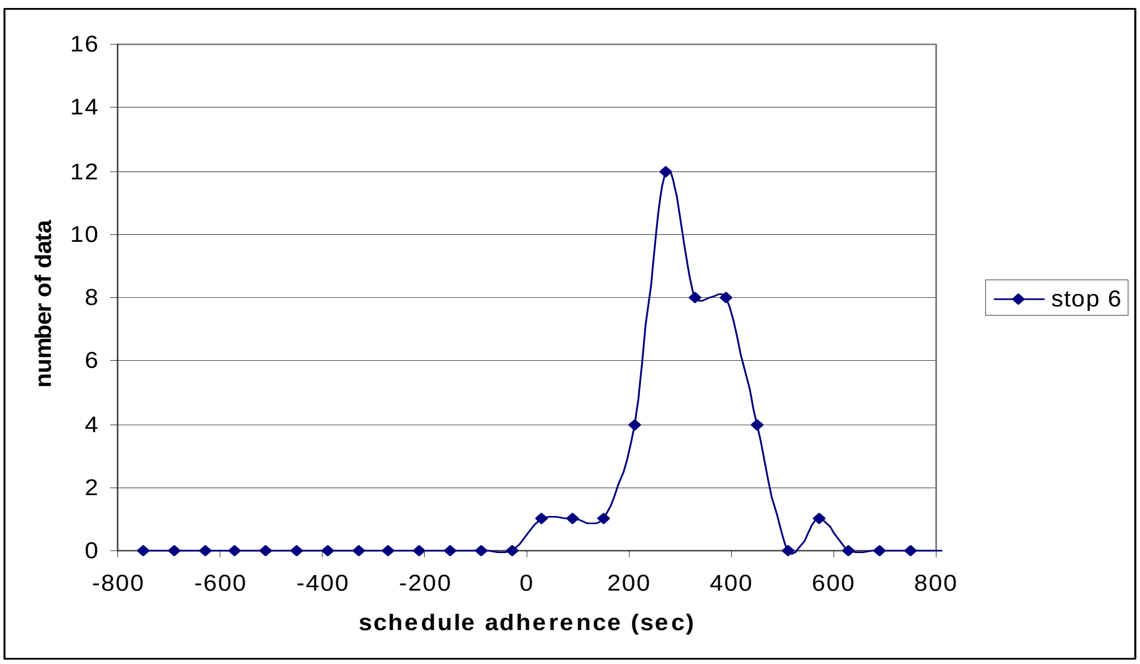

FIGURE C- 42 Schedule Adherence of Weekday Evening (stop 6)

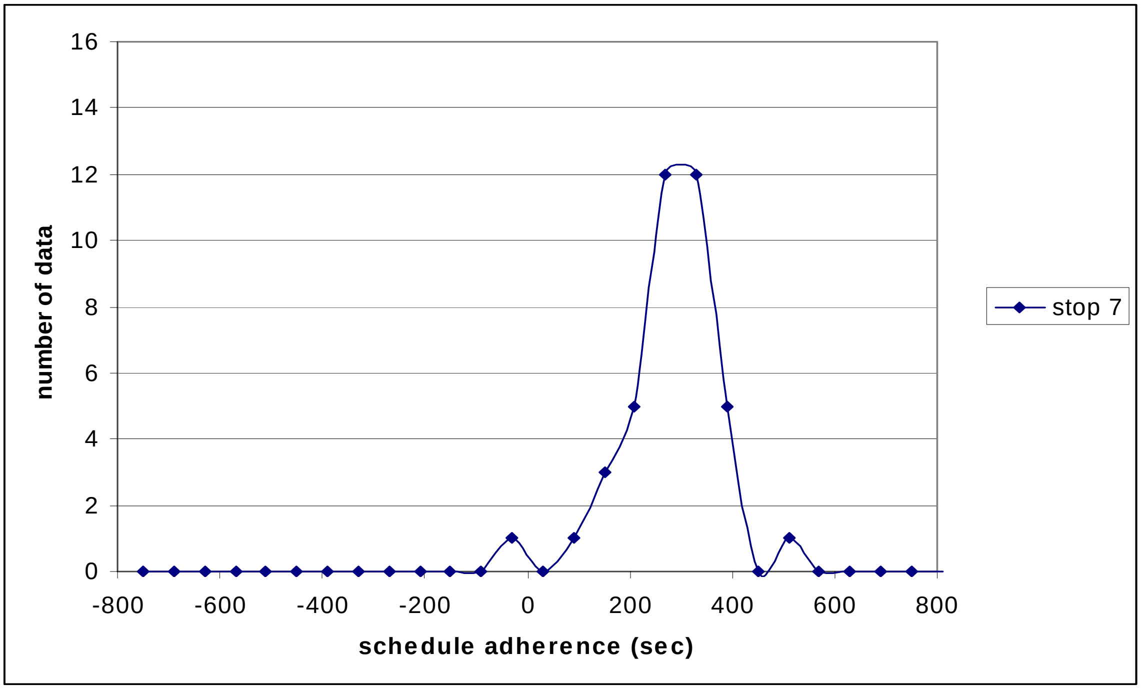

FIGURE C- 43 Schedule Adherence of Weekday Evening (stop 7)

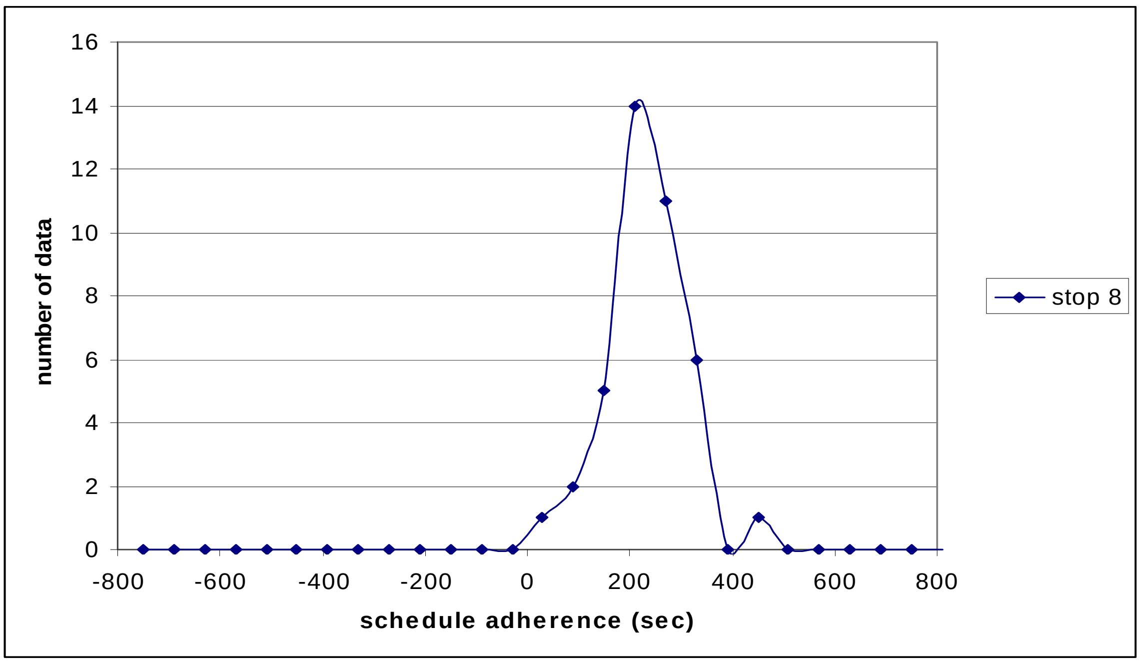

FIGURE C- 44 Schedule Adherence of Weekday Evening (stop 8)

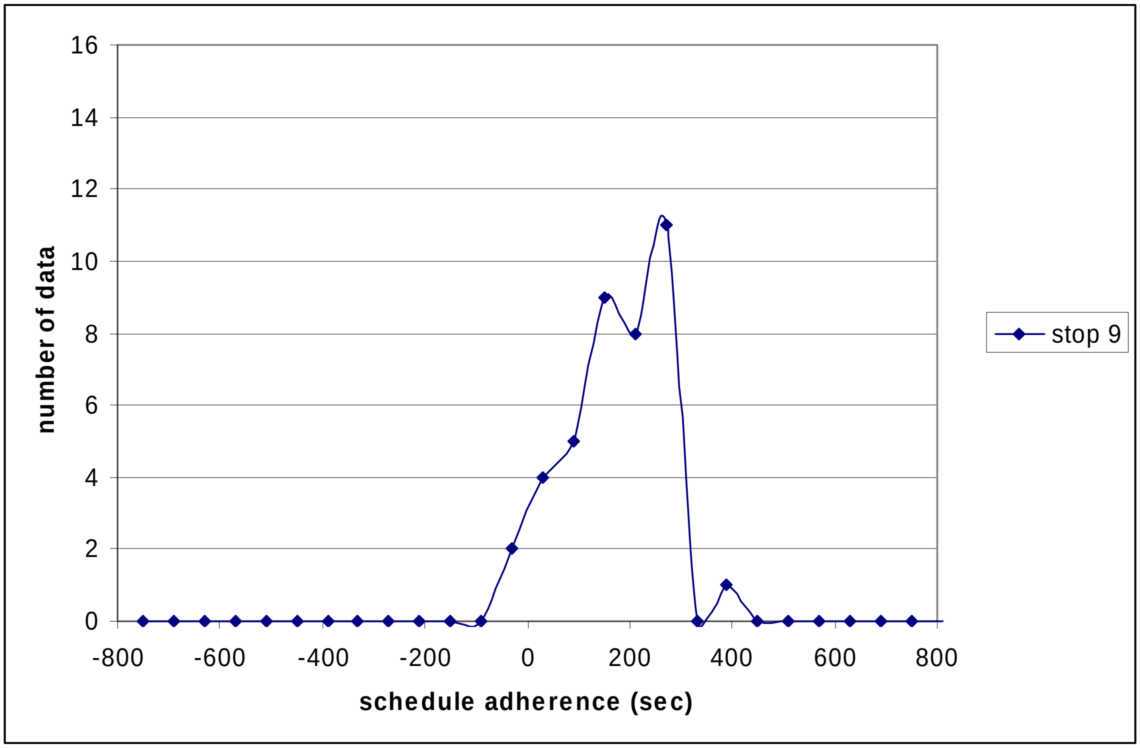

FIGURE C- 45 Schedule Adherence of Weekday Evening (stop 9)

VITA

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PUBLICATIONS AND PRESENTATIONS

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