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Warping Waldseemüller: A Cartometric Study of the Coast of South America As Portrayed on the 1507 World Map

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Abstract

In an attempt to shed some light on the problem of Martin Waldseemüller's portrayal of the shape of South America on his important 1507 world map, polynomial warping algorithms and regression analysis are applied to the coastlines of South America on his map and the known coastal outline. Correlation coefficients are calculated and regression curves are analyzed for inflection point behavior in order to quantitatively compare the geometries of both coastlines. High correlations have been found between both shapes suggesting the possibility of Waldseemüller's use of empirical data in mapping the South American continent rather than pure chance.

Editor's Note: For background information on Waldeemüller's 1507 map and its acquistion by the Library of Congress, see John Hébert, "The Map that Named America: Martin Waldeemüller's 1507 World Map" (*Coordinates*, Series B, No. 4). A high-resolution image of the map can be found at http://hdl.loc.gov/loc.gmd/g3200.ct000725C.

Keywords: Waldseemüller, polynominal warping, regression analysis, cartometry

> What any picture, of whatever form, must have in common with reality in order to depict it —correctly or incorrectly — in any way at all, is logical form, i.e. the form of reality[1].
>
> Ludwig Wittgenstein
> *Tractatus Logico-Philosophicus*

Introduction

The coast of South America portrayed in the large 1507 world map by Martin Waldseemüller (Hylacomylus) (Figure 1a and 1b) remains one of the great unsolved problems in the history of cartographic scholarship and has been the subject of scholarly comment and theorizing since the map's discovery by Fr. Joseph Fischer in the collections of the Wolfegg Castle in Germany in 1900[2]. The 1507 world map, that now resides in the Geography and Map Division of the Library of Congress, shows, for the first time, the continents of North and South America as separate entities detached from Asia and portrays South America with a shape that is geometrically similar to the outline of the continent as we recognize it today. The two aspects of shape and location of the continent, separated as it is on the map from Asia, are chronologically problematic in that in 1507, the map's supposed creation date, neither Vasco Nunez de Balboa nor Ferdinand Magellan had seen the Pacific Ocean. Waldseemüller discusses his portrayal of the New World in his *Cosmographiae Introductio cum Quibusdam Geometriae ac Astronomiae Principiis ad Eam Rem Necessarius* printed in four editions in St.

Dié, under the Duke of Lorraine, in 1507[3]. Waldseemüller writes:

> *Hunc in modu terra iam quadripartite cognscitiet; sunt tress prime partes cotinenentes Quarta est insula cu omni quaque mari circudata conspiciat[4].* (The earth is now known to be divided into four parts. The first three parts are continents, while the fourth part is an island, because it has been found to be surrounded on all sides by water.)

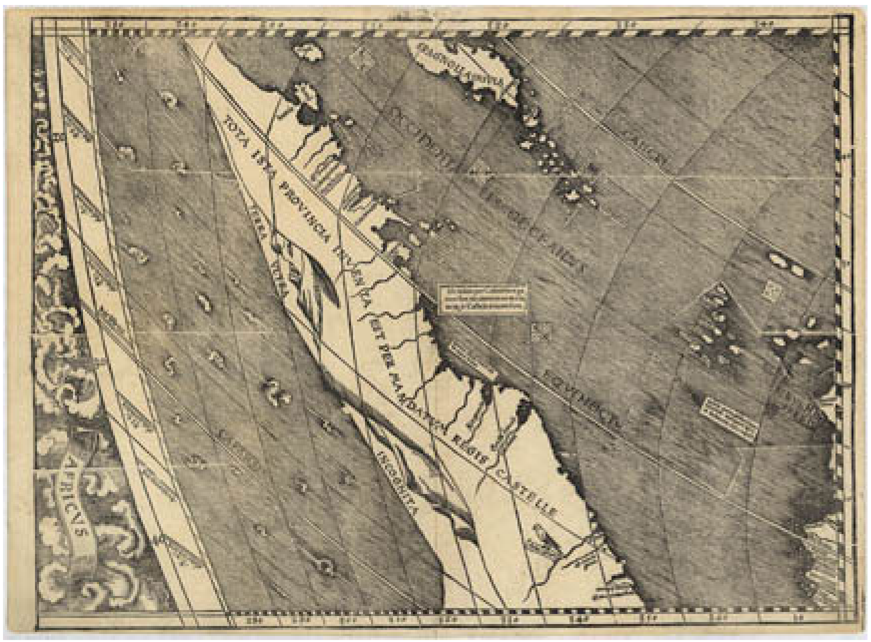

Figure 1a: Northern Part of South America on the 1507 World Map

Figure 1b: Southern part of South American on the 1507 World Map

The semantics of the Latin here are important. Waldseemüller uses the words “now known” and “has been found” both of which necessarily imply the existence of some form of empirical evidence rather than a case of pure speculation. Although there has been much discussion on the historical meaning and context of Waldseemüller’s portrayal no quantitative analysis has ever been performed that might give some indication of the map’s similarity with modern equivalents.

The following paper is a cartometric study of the coastline of South America as portrayed by Waldseemüller on the 1507 world map. We apply polynomial warping algorithms and polynomial regression analysis to the 1507 map in an effort to determine its geometric similarity to the known coast of South America. Polynomial warping algorithms are applied to the digitized world map and polynomial regressions are carried out on these results in the *Mathematica*[5] environment. The regression curves are then analyzed for inflection point behavior and global and local correlation coefficients are calculated. The application of these techniques to early maps, such as this, is difficult and fraught with inaccuracies, but this study attempts to take many of these into account and uses relative quantities to help reduce the inherent error generated by these calculations.

Description of Techniques and Results

The process of polynomial warping is essentially a mathematical transformation or mapping from a distorted image, such as a map with an unknown scale or geometrical grid, to a target image that is well known. The objective is to perform a spatial transformation or warp so that the corrected image or map can be measured or have a metric placed upon it relative to the known grid or map. The mathematical functions that are used in the process are general polynomials of arbitrary order and depend on the amount of distortion in the unknown map. The warping process includes all deformations that can be modeled by polynomial transformations of the form:

$$x = P_x(u, v) = \sum_{i=0}^{n-1} \sum_{r+s=i} a_{irs} v^s u^r$$

$$y = P_y(u, v) = \sum_{i=0}^{n-1} \sum_{r+s=i} b_{irs} v^s u^r$$

For the application to the Waldseemüller 1507 map, we required high order transformations— in other words, transformations that are not linear or affine. Affine transformations have only first order terms and can shift and rotate an image but cannot induce bending or stretching. Higher order terms allow much more flexibility and for much more image distortion than do affine forms. The *u* and *v* values in the equations above are the new coordinates in the warped image that are moved to the pixel values that were originally held in the x and y positions of the original image. The weighted constants *a 0….a n* must be calculated so that these equations are as close an approximation (i.e. preserve scale) to the targeted map as possible. In the case of a second-order approximation, six coefficients must be found and the error minimized and this value moves rapidly upward as the order of the transformation increases. In practice this involves the solution of matrix equations whose dimension is dependent on the number of tie points that are used to link the two maps together. This process allows us to reshape the one image map to the surface of the target map in any way we want, and to calculate the difference in alignment between the two as shown in Figure 2. This process of matrix transformations is similar to Waldo Tobler's bi-dimensional regression but allows for the warping of the actual map along with the underlying coordinate grid.[6]

Figure 2: Non-linear warp of the north sheet of South America

The results of third order polynomial warping on the Waldseemüller 1507 map of South America, shown in Figure 3, *is* for the northern part of the continent with the target image of South America overlaid. The prime meridian used in the calculation was that of Greenwich with corrections made to account for the difference between that location and the prime meridian of the Waldseemüller map, which goes through the Canary Islands. Waldseemüller's coordinates were also normalized to match those of modern maps. This was necessary because the 1507 map employs a coordinate scale that runs from 0 degrees to 360 degrees with the 280 degree longitude meridian representing the left hand or western border of the map (Figures 1a and 1b). The tie points used to link the two maps together were the intersections of the latitude and longitude lines on both images. As can be seen from figure 3, there is a great deal difference when comparing the outline of the coasts visually. This visual estimation is highly misleading and requires further processing in order to yield any analyzable results.

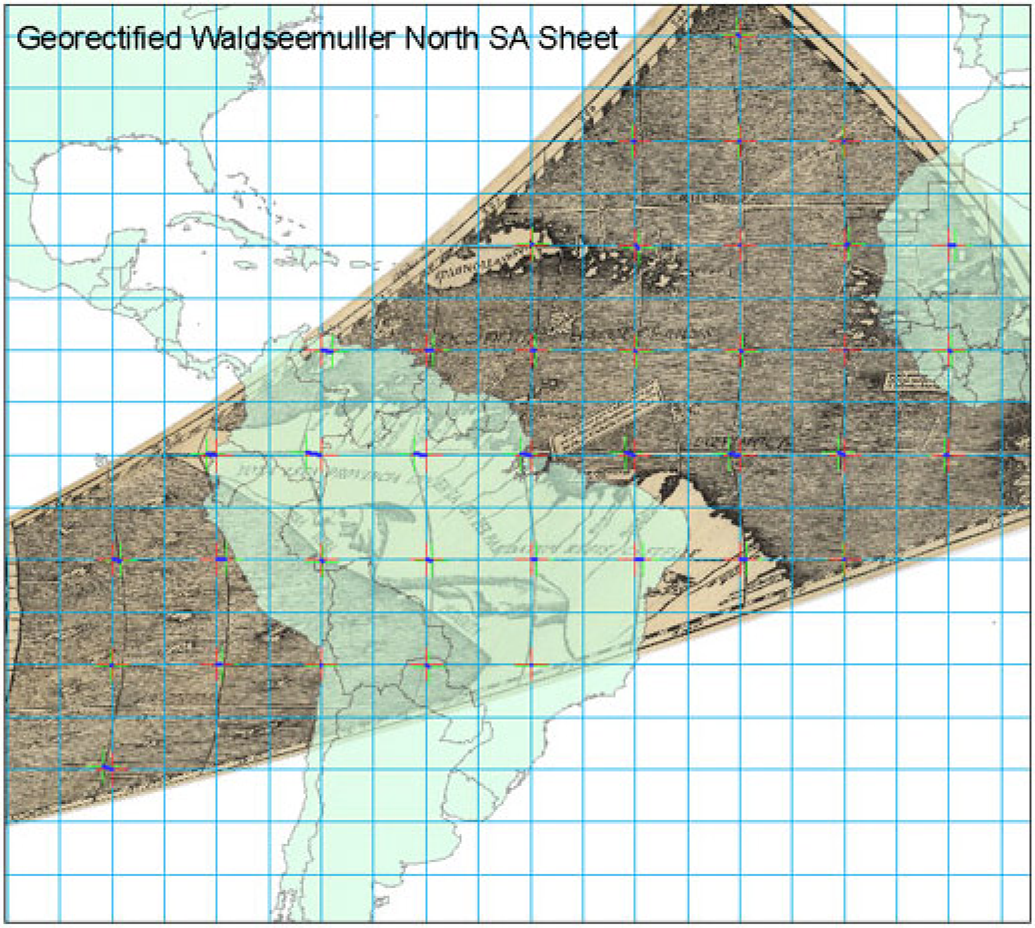

Figure 3: Third order polynomial warp of the north sheet with Modern image of South America superimposed

To compare the coastlines, one hundred points were selected at equal intervals along parallels of the two images. The distances between the two corresponding points on each side of the coastlines were then digitally measured. These measured values give an indication of the width of the continent at each of the one hundred points. The measurement process introduced some systematic error into the regression analysis. It is impossible to estimate this error because the measurements were made along the boundary lines of each of the coastlines, and these lines— especially on the Waldseemüller map— have a nonzero width. It is assumed that this error is low however, and because we are making relative comparisons between two sets of data, much of this error should cancel itself out, having little effect on the analysis. The distances along lines of constant latitude were selected as regression variables because we know that Waldseemüller knew these values accurately. In the *Cosmographiae Introductio* he provides us with a table of values, shown in figure 4[7], which give distances per degree of longitude at various intervals of latitude[8].

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	Gradus	Gradus.	Millia Ital.	Mil.Ger
Aequa	1	12	60	15
tor.	12	25	59	14 ½ ¼
Tropi	25	30	54	13 ½
cus.	30	37	50	12 ½
	37	41	47	11 ¼
	41 usq ad	51 faciunt	40	10
	51	57	32	8
	57	63	28	7
	63	66	26	6 ½
Circu.	66	70	21	5 ¼
Arcti.	70	80	6	1 ½
Polus Arcti.	80	90		0

Figure 4: Table of Latitudinal Measure from the *Cosmographiae Introductio* of Waldseemüller, 1507

In figure 5 we have graphed these intervals along with the known values of the distance per degree. The two graphs display a remarkably similar form, and are highly correlated, giving us some confidence that Waldseemüller could display distances along parallels at scale for any region in his map accurately, if indeed he knew their values.

The measured widths were subjected to polynomial interpolation analysis, then graphed, and are shown in Figure 6. There are many types of polynomial interpolation schemes, including cubic splines, Lagranges's method, and least squares minimization. Both Langrange's method and that of cubic splines are true interpolation techniques, and allow the inclusion of all data points on the fitted curve. Least squares methods are approximation techniques and were chosen because the nature of our data requires regressions of high order. Regression analysis of this type is an ad hoc method that attempts to identify trends and to bring forth correlations in closed variable systems.

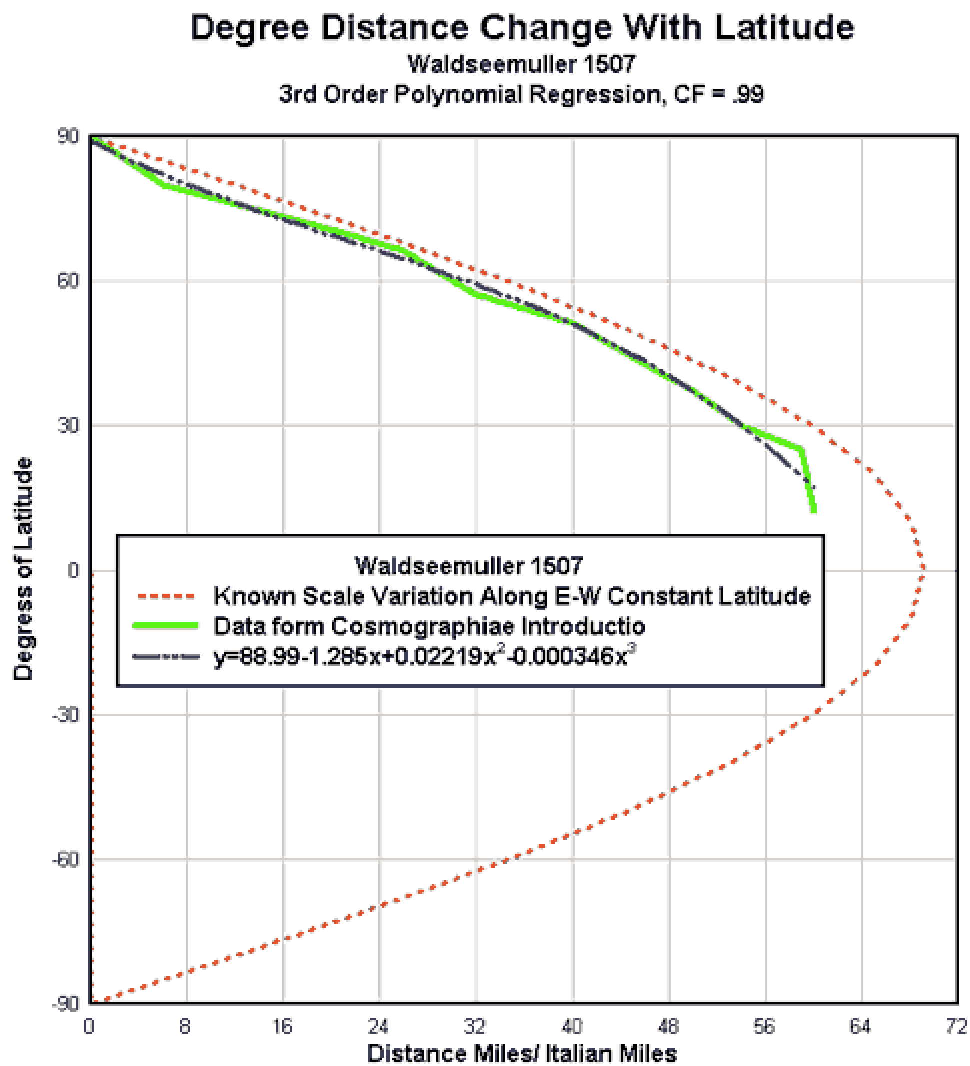

Figure 5: Comparison of Length of Degrees of the Parallel from the 1507 Cosmographiae Introductio and known modern values

In this case, the variables we are interested in are the widths of the two coasts of South America along parallels displayed in our warped figure and measured by the techniques outlined above. The raw data for each map was subjected to a seventh and tenth order polynomial regression and yielded a remarkably good fit with correlation coefficients of .99 and .97 for the known coast of South America and for the Waldseemüller coast, respectively. This result is of course not unexpected. Given any data, a polynomial fit of reasonable accuracy can be generated if one uses a high enough order regression. Seventh and tenth order polynomials were used here because they were the lowest order regression to give such high correlations. Normally one wants to use the lowest order regression possible, but in this case because we are going to compare the two curves with each other, the order of the individual curves was unimportant as long as we arrived at smooth enough functions to fit the actual raw data.

The two resultant curves shown in figure 6 are remarkably similar in structure and display some interesting features. First, the overall correlation coefficient between the two sets of data is .76, which is fairly high if one is attempting to hold to the theory that Waldseemüller simply drew the outline of South America with no spatial reference points or geographical data. Second, and more mportantly, are the points in which the two curves intersect each other. These intersections represent locations where the width of the coastline on the Waldseemüller map and the known coast of South America are equal and hence have correlations of 1.0. One particularly interesting intersection point is in the region of the equator and takes place at a value of –1.9 degrees latitude. This yields a correlation at the actual equator and Waldseemüller’s map of .88. This is again a very high correlation and occurs in a geographical region that would have been of great interest to the mapmaker.

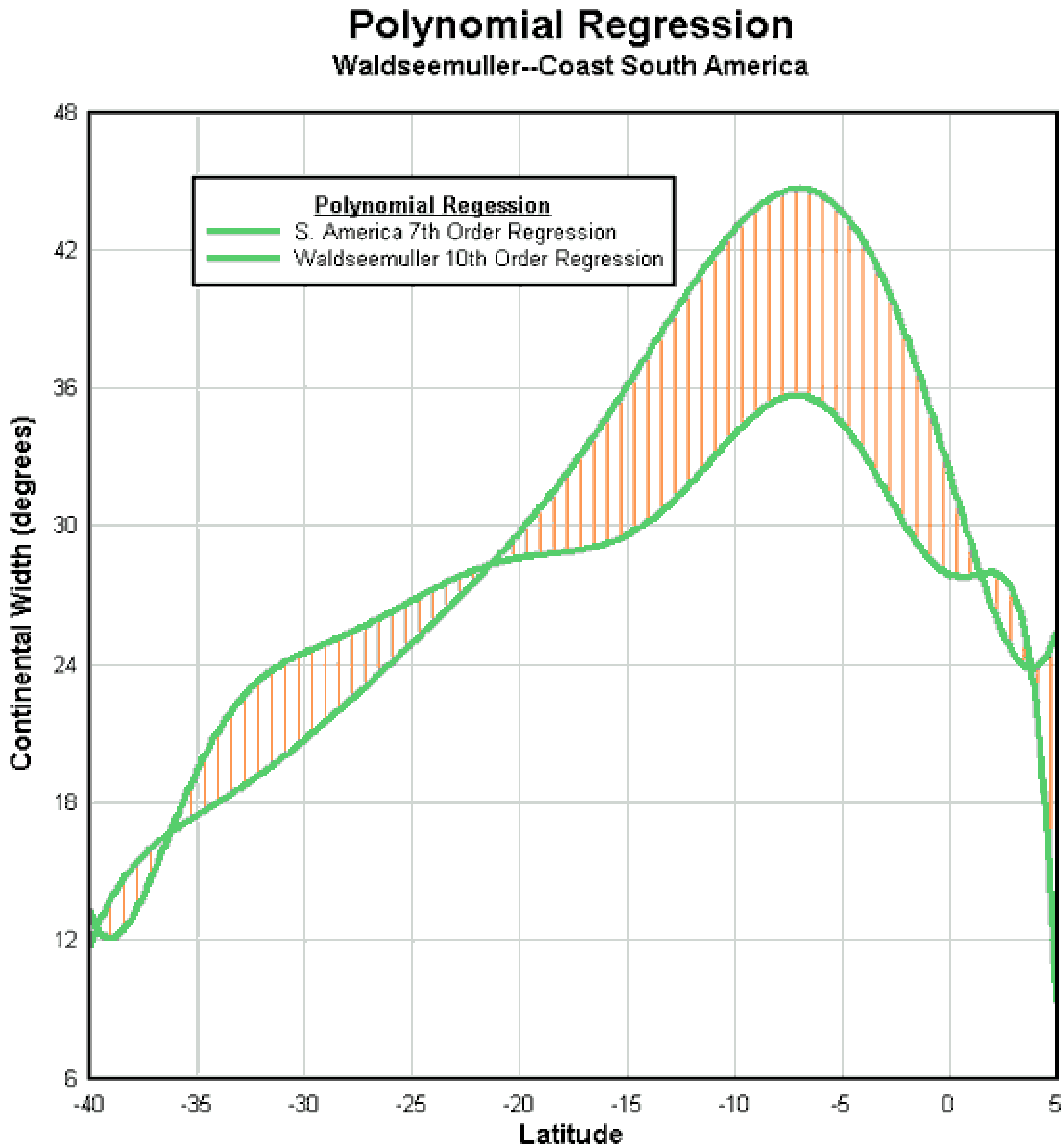

Figure 6: Polynomial Regression Curves for the Coast of South America

The other point of interest is the intersection that occurs at 21.7 degrees latitude. This point closely corresponds to the place at which the western coast of South America changes from its north-south direction and expands westward. The point at which this occurs is the location of the modern city of Arica, Chile. The correlation between the Waldseemüller map and this actual location is .91. Again we have a high correlation at a geographically significant point. It is also important to observe the inflection point behavior of the two curves. Both curves are remarkably similar in shape and this similarity gives us an indication of the geometrical correspondence between the two representations of the continent. Looking at the inflection points, which are the local maximum and minimums of the curves, we can see that the maximum width of the continent, represented by the maximum point on both curves, occurs at nearly the same point and has a correlation of .98.

Conclusions

The results of this study are highly compelling. The overall correlation of the coastal shapes, the structure and form of the polynomial regression curves, and the individual latitudinal points of high correlation, while not conclusive individually, together do represent something that is probably not attributable to pure coincidence and show geometric similarity between Waldseemüller's representation of South America and the modern one. This leads us to three possible conclusions regarding the portrayal of South America on the 1507 world map. First, is that Waldseemüller got lucky and drew a highly correlated geometrically similar coastline for the South American continent with little or no spatial or geographical data. Second, that the date of 1507 for the large wall map is not correct, and it was conceived of and printed at a later date.[9] This is possible but not probable, as there is evidence that the world map existed by the late summer of 1507. In a letter dated 12 th of August 1507, the humanist Johnannes Trithemius wrote to Veldicus Monapius that he had "a few days before purchased cheaply a handsome terrestrial globe of small size lately printed in Strasbourg, and at the same time a large wall map".[10] There is also a letter from Waldseemüller to his collaborator on the *Cosmographiae Introductio*, Mathias Ringmann, that is dated February 1508 and describes Waldseemüller's moving to Strasbourg "where you know, we formerly composed, drew and printed a representation of the world both as a globe and as a map…" The third possibility is that Waldseemüller had access to geographical or spatial data regarding the coastline of South America and that this data is no longer extant. This possibility of is made more probable *by* the results of this study, but no documentary or historical evidence exists that supports this conclusion. We are left with only probabilities and directions for further research. Cartometric research on the map is continuing, focusing on Africa and the region around the Caspian Sea, and on comparisons with Waldseemüller's other cartographic works, such as the *Carta Marina* of 1516 and his Ptolemy of 1513.

Acknowledgements

I would like to thank Ginny Mason, Cartographer in the Geography and Map Division of the Library of Congress for her helpful discussions and suggestions regarding the methodology and theory of polynomial warping and rubber sheeting.

Notes

1. Ludwig Wittgenstein, *Tractatus Logico-Philosophicus* (London: Routledge Keagan and Paul, 1921), p. 7.

2. There are many good reviews of scholarship on the Waldseemüller Map and the output of

the group of humanists surrounding the Gymnasium Vosagense in St. Dié. For the best modern treatment of what is known, see Robert W. Karrow's *Bio-Bibliographies of the Cartographers of Abraham Ortelius, 1570* (Chicago: Speculum Orbis Press for the Newberry Library, 1993), 568-583.

3. Armand d'Avezac-Macaya in his *Martin Hylacomylus Waltzemuller Ses Ouvages et Ses Collaborators* published anonymously in 1867 in Paris makes an argument for four editions. He distingushes them by taking the first line of the title and the colophon (page 112). Henry C. Murphy in the catalogue of the Carter-Brown Library (i., 35) thought that two of these are simply made up from the original May and September editions of 1507. Harrisse differs in opinion from Murphy maintaining that there are in fact three real editions from 1507, see his *Bibl. Amer. Vet., Additions*, no.24.

4. [Martin Waldseemüller and Mathias Ringmann] *Cosmographiae Introductio…,Rudimenta* (St. Dié, 1507), [p.30]. Reprinted by Joseph Fischer and Franz von Wieser, *The Cosmographiae Introductio of Martin Waldseemüller in Facsimile, Followed by the Four Voyages of Amerigo Vespucci, with their Translation into English; to which are added Waldseemüller's Two World Maps of 1507 with an Introduction*, ed. Charles George Hebermann (New York: United States Catholic Historical Society, 1907).

5. Data were analyzed using the FIT function in the Mathematica mathematics software program produced Wolfram Associates. Multiple regression comparison of the two data sets was carried out using the Multivariate Descriptive Statistics Package in Mathematica.

6. Waldo Tobler, "Bi-dimensional Regression", *Geographical Analysis* 26(1994): 186-212.

7. Waldseemüller, *Cosmographiae*, [p.36].

8. The values given by Waldseemüller and Ringmann yield a circumference of the earth much larger than the Ptolemaic value of 5000 stades. This is not surprising since there are many texts that use much higher values closer to that of Eratosthenes value of 7800.

9. Elizabeth Harris in her seminal study "The Waldseemüller World Map: A Typographic Appraisal", *Imago Mundi* 37(1985): 30-53, has concluded that the only surviving copy now in the Library of Congress is in at least its second state and could not have been printed before 1515. The place of the printing of both the world map and Waldseemüller's globe gores, also thought to be from 1507, remains uncertain.

10. Johnannes Trithemius, *Epistlarum familarium* (Hagenau: 1536).

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