Conductance characteristics between a normal metal and a clean superconductor carrying a supercurrent

Degang Zhang,[1] C. S. Ting,[1] and C.-R. Hu[2]
[1]*Texas Center for Superconductivity and Department of Physics, University of Houston, Houston, Texas 77204, USA*
[2]*Department of Physics, Texas A&M University, College Station, Texas 77843, USA*
(Received 22 December 2003; revised manuscript received 14 May 2004; published 29 November 2004)

The effect of a transverse supercurrent I_s up to the thermodynamic critical current on the low-temperature conductance characteristics between a normal metal N and a clean s- or d-wave superconductor (S) is theoretically investigated, covering from metallic contact $(z=0)$ to the tunneling limit $(z \gg 1)$. For d-wave S both (100) and (110) contacts are studied. Many features found are due to current-induced gap anisotropy and requires S to be in the clean limit. Near critical I_s and for $z \simeq 1$, a three-humped structure appears for both s-wave S and d-wave S with (100) contact, signaling onset of current-induced gaplessness on the Fermi surface where gap originally exists.

DOI: 10.1103/PhysRevB.70.172508 PACS number(s): 74.45.+c, 74.25.Sv, 74.50.+r

It is well known that Andreev reflection plays a fundamental role in understanding the transport properties of a normal metal/superconductor junction (NSJ).[1] From the current-voltage $(I-V)$, or the differential conductance $[G(V) \equiv dI(V)/dV]$ characteristics of the junction, one can learn much information about S, including its elementary excitation spectrum and its order-parameter symmetry, etc. Blonder *et al.* have developed a general theory [2] for studying $I-V$ and $G(V)$ of an NSJ that allows a dimensionless barrier-strength parameter z to range from metallic contact, $z=0$, to the tunneling regime, $z \gg 1$. However, only conventional s-wave symmetry for S was considered by them. Recently, much attention has been paid to the conductance characteristics of d-wave, cuprate S in both theory and experiment[3–13] Due solely to the sign change of the d-wave gap-function order parameter $\Delta(\mathbf{k})$ on the Fermi surface, a zero-bias conductance peak (ZBCP) appears in the tunneling spectrum of an $N/(d-\text{wave } S)$ junction with non-$(n0m)$ contact.[4–6] The ZBCP arises from a sizable number of midgap states formed at the S side of the N/S interface and appears for all z, but is narrower and taller for larger z. In a large magnetic field, the ZBCP splits into two peaks.[7–11] It is interesting to also study the effect of a transverse supercurrent I_s in S on $G(V)$. Very recently, $G(V)$ for tunneling into a diffusive s-wave superconducting wire carrying an I_s was measured and compared with theory.[14] It was shown that the coherence peaks were suppressed and broadened with increasing I_s, and the effect is the same as that caused by a magnetic field. The positions of the coherence peaks in $G(V)$ were found to practically not shift with I_s, up to $\sim 4/5$ of the critical current. In this work, we investigate theoretically the conductance characteristics of a *clean* NSJ with an I_s in S parallel to the interface, by extending the theory of Blonder *et al.*[2] Unlike Ref. 14, this work is not limited to $z \gg 1$. We also consider d-wave S with (100) and (110) contacts.

When a uniform I_s passes through a conventional three-dimensional s-wave S, the phase of $\Delta(\mathbf{k})$ has a spatial variation of $2\mathbf{q}_s \cdot \mathbf{x}$, where $\mathbf{x}$ is the center-of-mass position of a Cooper pair, $\mathbf{q}_s = (m^*/2)\mathbf{v}_s$, with $\mathbf{v}_s$ the supercurrent velocity, and m^* the mass of a Cooper pair. (We assume $\hbar = 1$.) This spatially varying phase leads to an anisotropic quasiparticle excitation spectrum in a clean S. At temperature $T=0$, the magnitude of the order parameter Δ_q is unaffected by current until the Landau criterion is satisfied (i.e., $q = 0.5\Delta^0$, where $q \equiv q_s/k_F$ and $\Delta^0 \equiv \Delta_0/E_F$). Here Δ_0 is the superconducting gap when $I_s=0$, k_F and E_F are the Fermi momentum and energy, respectively. When $q \geq 0.5\Delta^0$, S becomes gapless, and quasiparticles are generated in a portion of the Fermi surface.[15] Without a current, the effect of increasing z on the Andreev-reflection-induced enhancement of G within the gap is to suppress it, more for lower bias, resulting in a double peaked structure.[2] We find that current-induced gap anisotropy has the effect of moving these peaks toward zero bias. At $z=1$, these peaks merge into one at zero bias when current-induced gaplessness sets in, resulting in a three-humped structure for G. (The two finite-energy peaks are quasiparticle coherence peaks.) At larger z, such as at $z=5$, only the coherence peaks appear. At smaller z, such as at $z=0.5$, the quasiparticle peaks are suppressed near the critical current, whereas a central peak appears practically at already $q/\Delta^0 = 0.45$ when $k_BT/E_F = 0.01$. Thus even though theoretically a ZBCP at zero temperature is a signal for current-induced gaplessness, in practice the three-humped structure at $z \simeq 1$ is a more sensitive signal. [Essentially the same physics occurs also when S is d-wave with (100) contact, but now the current-induced gaplessness occurs among the d-wave gap anisotropy, so the effect of the former is less prominent.]

As q is increased further, Δ_q gradually decreases to zero at $q = 0.67\Delta^0$ [Fig. 1(a)]. The supercurrent density quickly reaches a peak (the thermodynamic critical current density) at $q = q_c = 0.515\Delta^0$ [Fig. 1(c)].[16] The region $q > q_c$, in which I_s is a decreasing function of q, is unstable and cannot be observed experimentally. (For a two-dimensional s-wave S, superconductivity disappears immediately after the Landau criterion is met. Then $q_c = 0.5\Delta^0$.)

Different from that in an s-wave S, the Δ_q vs q relation in a d-wave S also depends on the direction of the supercurrent. (Here Δ_q denotes the maximum gap in the presence of I_s.)

For a two-dimensional d-wave S with a supercurrent, the gap-function order parameter at $T=0$ is described by[17]

$$\pi \ln\frac{\Delta^0}{\Delta^q} = \int_{\geqslant} d\theta \cos^2(2\theta)\ln(g + \sqrt{g^2 - 1}), \tag{1}$$

where

$$g \equiv \frac{2q}{\Delta^q}\left|\frac{\cos(\theta-\phi)}{\cos(2\theta)}\right|,$$

$\Delta^q=\Delta_q/E_F$, ϕ is the angle between the supercurrent and the antinodal direction and the integral in Eq. (1) is from 0 to 2π with the constraint $g^2-1\geqslant 0$.

Figure 1(b) shows the dependence of the d-wave Δ_q on q at $\phi=0$ and $\pi/4$. We can see that when q is less than $\sim 0.3\Delta^0$, the changes of the order parameter with q in both the antinodal and nodal directions are almost the same. However, a great difference exists for larger q. When I_s is along the antinodal direction, Δ^q has a sharp drop (from $0.883\Delta^0$ to $0.588\Delta^0$) between $q=0.384\Delta^0$ and $0.385\Delta^0$. After that it drops continuously to zero at $q=0.53\Delta^0$. When $\phi=\pi/4$, Δ^q gradually decreases to $0.689\Delta^0$ at $q=0.469\Delta^0$, and has no solution beyond.

The supercurrent densities along the antinodal and nodal directions are given by[17]

$$j_s = env_s\left[1 - \frac{\Delta^q}{2q\pi}\int_{\geqslant} d\theta|\cos(\theta-\phi)\cos(2\theta)|\sqrt{g^2-1}\right], \tag{2}$$

where n is the density of electrons. Figure 1(d) gives the corresponding dependences of the supercurrent density on q. It is seen that the thermodynamic critical current $j_{sc} = 0.238 env_F\Delta^0$ $(0.225 env_F\Delta^0)$ is reached at $q=q_c=0.35\Delta^0$ $(0.39\Delta^0)$ for current in the antinodal (nodal) direction.

The elementary excitations in S are governed by the time-independent Bogoliubov-de Gennes equations,[18]

$$Eu(\mathbf{x}) = h_0 u(\mathbf{x}) + \int d\mathbf{x}'\ \Delta(\mathbf{s},\mathbf{r})v(\mathbf{x}'), \tag{3a}$$

$$Ev(\mathbf{x}) = -h_0 v(\mathbf{x}) + \int d\mathbf{x}'\ \Delta^*(\mathbf{s},\mathbf{r})u(\mathbf{x}'), \tag{3b}$$

where $\mathbf{s}=\mathbf{x}-\mathbf{x}'$, $\mathbf{r}=\frac{1}{2}(\mathbf{x}+\mathbf{x}')$, and $h_0=-(\nabla^2/2m)+U\delta(x)-\mu$ with μ the chemical potential. It is useful to express the superconducting order parameter in the form $\Delta(\mathbf{s},\mathbf{r}) = \int d\mathbf{k}\ e^{i\mathbf{k}\cdot\mathbf{s}}\overline{\Delta}(\mathbf{k},\mathbf{r})e^{i2\mathbf{q}_s\cdot\mathbf{r}}$.[3] Neglecting the proximity effect near the N/S interface at $x=0$, we have $\overline{\Delta}(\mathbf{k},\mathbf{r})=\Delta(\mathbf{k})\Theta(x)$, where $\Theta(x)$ is a step function, and $\Delta(\mathbf{k})$ is the order parameter of a bulk S in the presence of I_s.

In the WKBJ approximation, Eqs. (3) have special solutions of the form

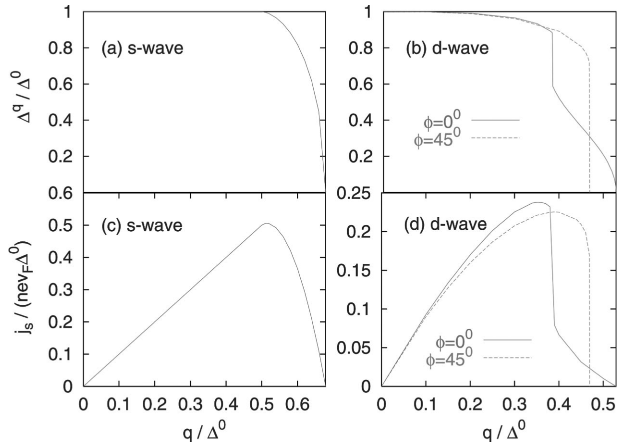

FIG. 1. Dependence of the superconducting order parameter on the normalized supercurrent velocity parameter q for (a) an s-wave and (b) a d-wave S. (ϕ is the angle between the supercurrent and the antinodal direction in the latter case.) In (c) and (d), the corresponding dependences of supercurrent density on q are given.

$$\begin{pmatrix} u \\ v \end{pmatrix} = e^{i\mathbf{k}_F\cdot\mathbf{x}}\begin{pmatrix} e^{i\mathbf{q}_s\cdot\mathbf{x}}\overline{u} \\ e^{-i\mathbf{q}_s\cdot\mathbf{x}}\overline{v} \end{pmatrix}, \tag{4}$$

where $\overline{u}(\mathbf{x})$ and $\overline{v}(\mathbf{x})$ obey the generalized Andreev equations,[1]

$$\left(E - \frac{\mathbf{q}_s^2}{2m} - \frac{\mathbf{q}_s\cdot\mathbf{k}_F}{m}\right)\overline{u} = -\frac{i(\mathbf{k}_F+\mathbf{q}_s)}{m}\cdot\nabla\overline{u} + \Delta(\mathbf{k}_F)\Theta(x)\overline{v}, \tag{5a}$$

$$\left(E + \frac{\mathbf{q}_s^2}{2m} - \frac{\mathbf{q}_s\cdot\mathbf{k}_F}{m}\right)\overline{v} = \frac{i(\mathbf{k}_F-\mathbf{q}_s)}{m}\cdot\nabla\overline{v} + \Delta^*(\mathbf{k}_F)\Theta(x)\overline{u}. \tag{5b}$$

Obviously, the eigenenergy E is symmetric about $E=\mathbf{q}_s\cdot\mathbf{k}_F/m$ rather than zero. This leads to different energy gaps for different electron directions. As a result, many features appear in $G(V)$ of a NSJ carrying a transverse I_s, as we shall see below. With $\mathbf{q}_s$ parallel to the interface of the NSJ, we have

$$\begin{pmatrix} \overline{u}_\nu \\ \overline{v}_\nu \end{pmatrix} = e^{i\alpha_\nu x}\begin{pmatrix} u_\nu^> \\ v_\nu^> \end{pmatrix} \quad (\text{for } x>0), \tag{6a}$$

$$\begin{pmatrix} \overline{u}_\nu \\ \overline{v}_\nu \end{pmatrix} = \begin{pmatrix} e^{i\beta_\nu x}u_\nu^< \\ e^{i\gamma_\nu x}v_\nu^< \end{pmatrix} \quad (\text{for } x<0), \tag{6b}$$

where $\nu=\text{sign}(k_{Fx})$; $\alpha_\nu=[-\nu\mathbf{q}_s^2/2+mA_\nu]/|k_{Fx}|$, with $A_\nu \equiv \sqrt{(E+q_s k_{Fy}/m)^2-\Delta_\nu(\mathbf{k}_F)\Delta_\nu^*(\mathbf{k}_F)}$; $\beta_\nu=m\nu[-\mathbf{q}_s^2/(2m)+E+q_s k_{Fy}/m]/|k_{Fx}|$; $\gamma_\nu=-m\nu[\mathbf{q}_s^2/(2m)+E+q_s k_{Fy}/m]/|k_{Fx}|$; $u_\nu^{>(<)}$ and $v_\nu^{>(<)}$ are constants. For example, in S, we have $B_\nu \equiv u_\nu^>/v_\nu^> = \Delta_\nu(\mathbf{k}_F)/(E+q_s k_{Fy}/m-\nu A_\nu)$.

Following Ref. 2, we obtain the Andreev and normal reflection coefficients, $a(E)$ and $b(E)$,

$$a(E)=\frac{2q_+(k_++k_-)}{B_-(-k_-+q_++2imU)(k_+-q_-+2imU)-B_+(k_++q_++2imU)(-k_--q_-+2imU)}, \tag{7a}$$

$$b(E)=\frac{B_+(k_-+q_--2imU)(-k_++q_+-2imU)+B_-(k_+-q_-+2imU)(k_-+q_+-2imU)}{B_-(-k_-+q_++2imU)(k_+-q_-+2imU)-B_+(k_++q_++2imU)(-k_--q_-+2imU)}. \tag{7b}$$

Here $q_+=|k_{Fx}|+\beta_+$, $q_-=|k_{Fx}|+\gamma_+$, and $k_\nu=|k_{Fx}|+\nu\alpha_\nu$. The critical supercurrent velocity is much less than the Fermi velocity. So the Andreev approximation, $q_\pm\approx k_\pm\approx|k_{Fx}|$, also holds in the presence of a supercurrent. The normalized conductance can then be calculated according to a formula given in Ref. 2:

$$G=\frac{G_s}{G_n},$$

$$G_n=-\frac{e^2}{\pi}\int_{-\infty}^{+\infty}dE\int_{-\pi/2}^{\pi/2}d\theta\frac{\partial f(E-eV)}{\partial E}[1-|b(+\infty)|^2],$$

$$G_s=-\frac{e^2}{\pi}\int_{-\infty}^{+\infty}dE\int_{-\pi/2}^{\pi/2}d\theta\frac{\partial f(E-eV)}{\partial E}\times[1+|a(-E)|^2-|b(E)|^2], \tag{8}$$

where $|k_{Fx}|=k_F\cos\theta$, $f(E)$ is the Fermi distribution function, G_n and G_s are the differential conductance for S in the normal and superconducting states, respectively.

S-wave superconductor. In this case, the superconducting order parameter $\Delta_\nu(\mathbf{k}_F)=\Delta_q$ is independent of ν.

In Fig. 2, $G(V)$ at various q and $z\equiv 2mU/k_F$ are plotted (for $k_BT=0.01E_F$ and $\Delta_0=0.1E_F$). At $z=q=0$, electrons incoming with all momenta $\mathbf{k}_F$ with $k_{Fx}>0$ can enter S and equal number of holes at opposite momenta are retroreflected into N if $|eV|<\Delta_0$. So $G=2.0$ within the superconducting gap at $T=0$. As q increases, the range of $G=2.0$ diminishes and the $G(V)$ curve turns into a nearly triangular peak due to the current-induced gap anisotropy [Fig. 2(a)]. At $z\gg 1$ [Fig. 2(d)], the coherence peaks are suppressed and broadened as q increases, but unlike the case of a dirty S,[14] here the peaks of $G(V)$ move *outward* while the gap shrinks, again because of the gap anisotropy. The intermediate-z results are even richer in behavior [Figs. 2(b) and 2(c)]: A three-humped structure including a peak at zero bias appears at nearly critical I_s and $z\simeq 1$ because the system has become gapless. This central peak disappears at $z\gg 1$ because it is due to Andreev reflection. It splits into two weak peaks at weaker currents when the system is not gapless. These features are characteristically different from the ZBCP induced by the midgap surface states in d-wave S with non-$(n0m)$ contacts.[4]

Electrons entering at a fixed incident angle θ contributes to $G(V)$ a central peak if the gaplessness condition $2q|\sin\theta|>\Delta^0$ is satisfied. This is possible only for $q>0.5\Delta^0$. For $0.5\Delta^0<q<0.67\Delta^0$, there is a critical angle $|\theta_c|=\arcsin(\Delta^0/2q)$, which decreases from 90° to 48.3° in this range. No central peak is induced by electrons with $|\theta|<|\theta_c|$. However, only a small portion of this q range can be observed, because only the region $q\leqslant 0.515\Delta^0$ is stable.

D-wave superconductor. In this case, the pair potential has the form $\Delta_\nu(\mathbf{k}_F)=\Delta_q\cos(2\theta_\nu)$. Here, $\theta_\nu=\theta+\nu\alpha$, α is the angle between the antinodal direction and the positive x axis.

Figure 3 presents the normalized conductance at different z and q for a d-wave S with (100) contact (i.e., $\alpha=0°$). For $z=0$ [Fig. 3(a)], the central peak due to Andreev reflection is gradually suppressed by increasing q. For $z\gg 1$ [Fig. 3(d)], one sees mainly the filling up of the central dip as q increases. For intermediate z [Figs. 3b and 3c], one sees intricate behavior with some similarity to the corresponding cases in Fig. 2, as the only remaining effect of current-induced gap anisotropy and the eventual gaplessness, which are here largely obscured by the d-wave anisotropy.

Figure 4 is like Fig. 3 but with (110) contact (i.e., $\alpha=45^0$). It is seen that the ZBCP induced by the midgap surface states is suppressed, broadened, and eventually split at sufficiently large z when q is increased.

In summary, the order parameter and the critical current density of a d-wave superconductor (S) carrying a supercurrent I_s are obtained. The differential conductance of a (normal metal)/(clean s- or d-wave S) junction carrying a transverse I_s is theoretically investigated, covering barrier strengths from metallic contact ($z=0$) to the tunneling regime ($z\gg 1$). For s-wave S, several features result from current-induced gap anisotropy, distinguishing this clean-

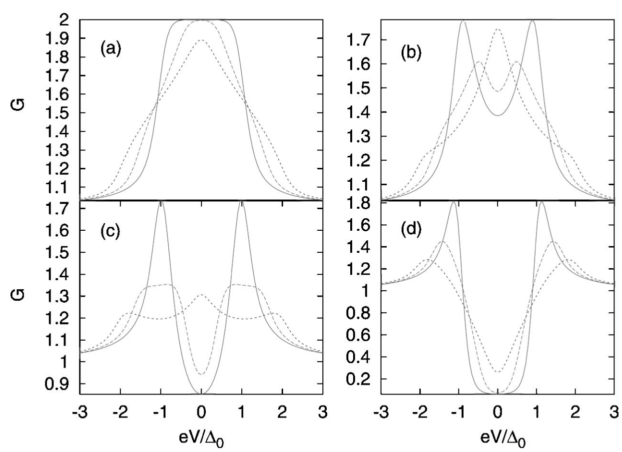

FIG. 2. The normalized differential conductance vs voltage for an $N/(s\text{-wave } S)$ junction: (a) $z=0$, (b) $z=0.5$, (c) $z=1.0$, and (d) $z=5.0$. Solid, $q=0$; dash, $q=0.3\Delta^0$; and dot, $q=0.515\Delta^0$, at which the thermodynamic critical current is reached. Here $q\equiv q_s/k_F$ and $\Delta^0\equiv\Delta_0/E_F$.

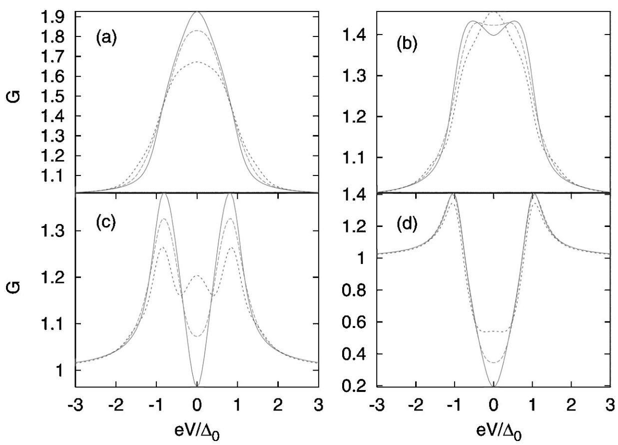

FIG. 3. The normalized differential conductance vs voltage for an $N/(d$-wave $S)$ junction with (100) contact: (a) $z=0$, (b)$z=0.5$, (c) $z=1.0$, and (d) $z=5.0$. Solid, $q=0$; dash, $q=0.2\Delta^0$; and dot, $q=0.35\Delta^0$, at which the thermodynamic critical current is reached.

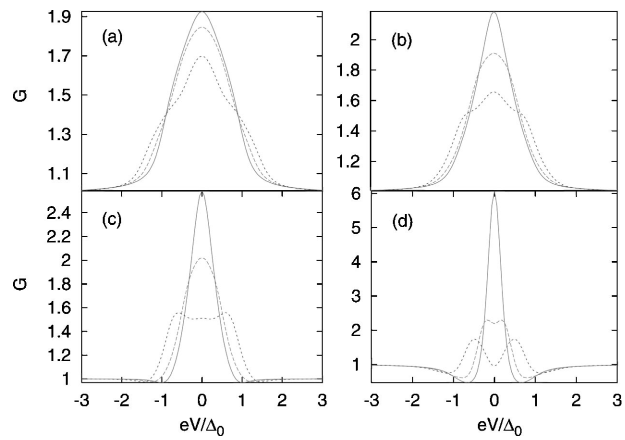

FIG. 4. Same as Fig. 3 except with (110) contact, and the blue curve is for $q=0.39\Delta^0$, where the critical current is reached.

limit study from the dirty-limit result.[14] They include triangular low-bias conductance curve at $z \ll 1$ due to Andreev reflection, and outward shift of coherence peaks while gap shrinks at $z \gg 1$. A three-humped structure including a peak at zero bias occurs at $z \simeq 1$ near the critical I_s, signaling current-induced gaplessness. For d-wave S, (100) contact shows mainly weakening at small z of the low-bias enhancement of G due to Andreev reflection, and gap filling at large z, with a weaker current-induced central peak for $z \simeq 1$ near the critical current, as the current-induced gaplessness is now obscured by the d-wave gap anisotropy. With (110) contact, when the dominant feature at zero current is a midgap-surface-states-induced zero-bias conductance peak, this peak is shown to split by I_s at large z, much like the effect of an external magnetic field.[7,8,10,11] We remark that this formulation can also be applied to the case of a $d+s$ superconductor. Since the critical current for an s-wave superconductor is about a factor of two higher than that for a d-wave one, the existence of an s component can be verified by a supercurrent reaching a magnitude between the two critical values.

The authors wish to thank J. Wei for helpful discussions. This work was supported by the Texas Center for Superconductivity and Advanced Materials at the University of Houston and by the Robert A. Welch Foundation (C.S.T).

[1] A. F. Andreev, Zh. Eksp. Teor. Fiz. **46**, 1823 (1964) Sov. Phys. JETP **19**, 1228 (1964)].
[2] G. E. Blonder, M. Tinkham, and T. M. Klapwijk, Phys. Rev. B **25**, 4515 (1982).
[3] C. Bruder, Phys. Rev. B **41**, 4017 (1990).
[4] C.-R. Hu, Phys. Rev. Lett. **72**, 1526 (1994).
[5] Y. Tanaka and S. Kashiwaya, Phys. Rev. Lett. **74**, 3451 (1995).
[6] J. H. Xu, J. H. Miller, Jr., and C. S. Ting, Phys. Rev. B **53**, 3604 (1996).
[7] M. Covington, M. Aprili, E. Paraoanu, L. H. Greene, F. Xu, J. Zhu, and C. A. Mirkin, Phys. Rev. Lett. **79**, 277 (1997).
[8] M. Fogelström, D. Rainer, and J. A. Sauls, Phys. Rev. Lett. **79**, 281 (1997).
[9] J. Y. T. Wei, N.-C. Yeh, D. F. Garrigus, and M. Strasik, Phys. Rev. Lett. **81**, 2542 (1998).
[10] M. Aprili, E. Badica, and L. H. Greene, Phys. Rev. Lett. **83**, 4630 (1999).
[11] R. Krupke and G. Deutscher, Phys. Rev. Lett. **83**, 4634 (1999).
[12] H. Aubin, L. H. Greene, S. Jian, and D. G. Hinks, Phys. Rev. Lett. **89**, 177001 (2002).
[13] A. Kohen, G. Leibovitch, and G. Deutscher, Phys. Rev. Lett. **90**, 207005 (2003).
[14] A. Anthore, H. Pothier, and D. Esteve, Phys. Rev. Lett. **90**, 127001 (2003).
[15] A. M. Zagoskin, *Quantum Theory of Many-Body Systems: Techniques and Applications* (Springer, New York, 1998).
[16] J. Bardeen, Rev. Mod. Phys. **34**, 667 (1962); K. Maki, in *Superconductivity*, edited by R. D. Parks (Marcer Dekker, New York, 1969).
[17] D. Zhang *et al.*, Dissertation.
[18] P. G. de Gennes, *Superconductivity of Metals and Alloys* (Benjamin, New York, 1966).