PHYSICAL REVIEW C **83**, 045802 (2011)

Energy release from hadron-quark phase transition in neutron stars and the axial w mode of gravitational waves

Weikang Lin and Bao-An Li*
Department of Physics and Astronomy, Texas A&M University–Commerce, Commerce, Texas 75429-3011, USA

Jun Xu and Che Ming Ko
Cyclotron Institute and Department of Physics and Astronomy, Texas A&M University, College Station, Texas 77843-3366, USA

De Hua Wen
Department of Physics, South China University of Technology, Guangzhou 510641, People's Republic of China
(Received 28 November 2010; published 15 April 2011)

Describing the hyperonic and quark phases of neutron stars with an isospin- and momentum-dependent effective interaction for the baryon octet and the MIT bag model, respectively, and using the Gibbs conditions to construct the mixed phase, we study the energy release from a neutron star owing to the hadron-quark phase transition. Moreover, the frequency and damping time of the first axial w mode of gravitational waves are studied for both hyperonic and hybrid stars. We find that the energy release is much more sensitive to the bag constant than the density dependence of the nuclear symmetry energy. Also, the frequency of the w mode is found to be significantly different with or without the hadron-quark phase transition and depends strongly on the value of the bag constant. Effects of the density dependence of the nuclear symmetry energy become, however, important for large values of the bag constant that lead to higher hadron-quark transition densities.

DOI: 10.1103/PhysRevC.83.045802 PACS number(s): 04.40.Dg, 97.60.Jd, 04.30.−w, 26.60.−c

I. INTRODUCTION

Neutron stars (NSs) are among the most mysterious objects in the Universe. With a typical mass of about $1.4M_{\odot}$ but a radius of only about 12 km, the average density in NSs is several times that in atomic nuclei. Their extreme compactness makes them a natural laboratory to test our knowledge about general relativity and the properties of dense neutron-rich nuclear matter [1]. Moreover, their generally large angular momentum and possible quadrupole deformation make them strong candidates among many possible sources to emit gravitational waves (GWs) [2]. It is well known that the estimated central density of some NSs may be higher than the predicted hadron-quark phase transition density, and thus there might be a quark core in these so-called hybrid stars; see, e.g., Ref. [3] for a recent review. Furthermore, for some hadronic NSs with central densities below but close to the hadron-quark transition density, their central densities might increase owing to, for example, their spindowns [4] or the accretion of masses from their binary companions [5]. Therefore, the hadron-quark phase transition may also occur in their cores, turning them into hybrid NSs as they evolve. Consequently, a microcollapse is expected to take place as a result of the softer equation of state (EOS) of quark matter than that of hadronic matter. Because of the difference in the binding energies of the pure hyperonic and hybrid configurations of NSs, some energies are released from the NS after the hadron-quark phase transition. To understand the mechanism of the hadron-quark phase transition and the associated energy release, their dependence on the properties of the dense hadronic and quark matter, the way the energy release is dissipated in NSs or carried away by gravitational waves, and the gravitational wave signatures of the EOS of dense matter and the expected hadron-quark phase transition are among the many interesting questions currently under intense investigation in neutron star physics and gravitational wave astronomy; see, e.g., Refs. [3,6,9,11–16].

In a recent work involving some of us [17], several model EOSs for hybrid stars were obtained from an isospin- and momentum-dependent effective interaction (MDI) [17,18] for the baryon octet, the MIT bag model for the quark matter [19,20], and the Gibbs construction for the hadron-quark phase transition [21,22]. Since there have been many other studies using similar approaches in the literature, it is especially worth mentioning that while the isospin-symmetric part of the hadronic EOSs used in this work is constrained by the experimental data on collective flow and kaon production in relativistic heavy-ion collisions [23], the symmetry energy term at subsaturation densities is constrained in a narrow range by the experimental data on isospin diffusion in heavy-ion collisions at intermediate energies [7]. The EOSs obtained with various values of the bag constant were then used to study the main features of the hadron-quark phase transitions and the mass-radius correlations of hybrid stars. In the present paper, we continue our recent work in Refs. [17,24–28], to make new contributions to the fast-growing field of nuclear astrophysics in two ways. First, our current knowledge about the EOS of dense matter in either the hadronic or quark phase, which is critical for understanding the physics of the hadron-quark phase transition, is still very poor, despite the great efforts made by both the astrophysics and the nuclear physics community. Specifically, for the neutron-rich hadronic matter the most uncertain part of its EOS is the density dependence of the nuclear symmetry energy which encodes the energy associated

*Corresponding author: Bao-An_Li@Tamu-Commerce.edu

with the neutron-proton asymmetry [7]. For the quark matter within the MIT bag model, the most uncertain part of its EOS is the bag constant, which measures the inward pressure on the surface of the bag to balance the outward pressure originating from the Fermi motion and interactions of quarks confined in the bag. It is thus of interest and also necessary to examine the relative effects of the density dependence of the nuclear symmetry energy and the bag constant on the total energy release due to the hadron-quark phase transition in NSs. Second, a part of the energy release in NSs may be carried away by GWs. The maximum amplitude of the latter can then be obtained by assuming that all the released energy is available for GWs. Besides testing a fundamental prediction of general relativity, gravitational waves hold the great promise to open up a completely new nonelectromagnetic window into the Universe. In addition, if detected in the future, the GWs may also help probe the properties of dense neutron-rich nuclear matter in NSs. Thus, it is important to examine the imprints of the nuclear symmetry energy and the bag constant on the frequency and damping time of GWs.

Several types of quasinormal mode, such as (1) f modes, associated with the global oscillation of the fluid, (2) g modes, associated with the fluid buoyancy, and (3) p modes, associated with the pressure gradient, may be excited in rotating NSs and subsequently emit GWs. The fraction of energy released by each oscillation mode is still an open question [8]. It has been estimated that the fractional energy release of each mode is approximately inversely proportional to its damping time [9]. The latter depends not only on the mass and radius but also on the rotational period of NSs. These modes exist in both Newtonian gravitational theory and general relativity. In this work, we focus on the first axial w mode, which exists only within general relativity. It is due to the disturbance of the space-time curvature and the motion of the fluid is negligible [29]. As we shall discuss in detail, both the frequency and damping time of the w mode is uniquely determined by the EOS of NS matter. A major characteristic of the axial w mode is its high frequency of above about 7 kHz accompanied by very rapid damping. These frequencies are outside the peak regions of the detector sensitivities of currently operating and planned gravitational wave detectors. For example, the bandwidth targeted by the ground-based laser interferometric detectors LIGO [30], VIRGO [31], and GEO [32] are in the range of about 100 to 350 Hz. The space-based detectors that are currently under planning or construction, such as the Laser Interferometric Space Antenna (LISA) [33], are designed to measure GWs at frequencies below 0.1 Hz. The highest frequencies to be measured are about 3.4 kHz using existing bar detectors, such as the Brazilian spherical antenna (having a frequency range of 3.0–3.4 kHz) and the Dutch Mini-GRAIL (2.8–3.0 kHz) [8]. Hopefully, this and other studies on the w mode will help stimulate more discussions on the possibility of detecting high-frequency GWs in the future.

We find in the present study that the energy release is much more sensitive to the bag constant than to the density dependence of the nuclear symmetry energy. Furthermore, the frequency of the w mode is found to be significantly different with or without the hadron-quark phase transition and depends strongly on the value of the bag constant for a given density dependence of the nuclear symmetry energy. However, when a larger bag constant is used such that the hadron-quark phase transition happens at a higher baryon density, effects of the density dependence of the nuclear symmetry energy also become very important.

This paper is organized as follows. In Sec. II, we summarize the MDI interaction, the MIT bag model, and the resulting model EOSs for hybrid stars obtained using the Gibbs construction for the mixed phase. In Sec. III, the total energy release from the hadron-quark phase transition in NSs and its dependence on the symmetry energy and the bag constant are presented. In Sec. IV, the frequency and damping time of the axial w mode of GWs and their dependence on the symmetry energy and the bag constant are studied. Finally, a summary and some concluding remarks are given in Sec. V.

II. MODEL EQUATIONS OF STATE FOR HYBRID STARS

Since we are mostly interested in examining the relative effects of the nuclear symmetry energy and the bag constant on the total energy release during the hadron-quark phase transition in neutron stars as well as the frequency and damping time of the w mode of emitted GWs, we first illustrate their effects on the EOS of hybrid stars by using the model introduced in Ref. [17]. For completeness and to facilitate the discussions, we briefly summarize the EOSs for hybrid stars obtained in Ref. [17] using the MDI interaction for the baryon octet, the MIT bag model for the quark matter, and the Gibbs construction for the hadron-quark phase transition. We refer the reader to Ref. [17] for details.

Assuming that the nucleon-hyperon and hyperon-hyperon interactions have the same density and momentum dependence as the nucleon-nucleon interaction with the interaction parameters fitted to known experimental data at normal nuclear matter density, the potential energy density of a hypernuclear matter due to interactions between two baryons is

$$V_{bb'} = \sum_{\tau_b,\tau'_{b'}} \left[\frac{A_{bb'}}{2\rho_0}\rho_{\tau_b}\rho_{\tau'_{b'}} + \frac{A'_{bb'}}{2\rho_0}\tau_b\tau'_{b'}\rho_{\tau_b}\rho_{\tau'_{b'}} + \frac{B_{bb'}}{\sigma+1}\frac{\rho^{\sigma-1}}{\rho_0^{\sigma}}(\rho_{\tau_b}\rho_{\tau'_{b'}} - x\tau_b\tau'_{b'}\rho_{\tau_b}\rho_{\tau'_{b'}}) + \frac{C_{\tau_b,\tau'_{b'}}}{\rho_0}\iint d^3p\,d^3p'\,\frac{f_{\tau_b}(\vec{r},\vec{p})f_{\tau'_{b'}}(\vec{r},\vec{p}\,')}{1+(\vec{p}-\vec{p}\,')^2/\Lambda^2}\right], \quad (1)$$

where b (b') denotes the baryon octet included in the present study, i.e., N, Λ, Σ, and Ξ. The conventions for the isospin are $\tau_N = -1$ for neutrons and 1 for protons, $\tau_\Lambda = 0$ for Λ, $\tau_\Sigma = -1$ for Σ^-, 0 for Σ^0, and 1 for Σ^+, and $\tau_\Xi = -1$ for Ξ^- and 1 for Ξ^0. The total baryon density is given by $\rho = \sum_b \sum_{\tau_b} \rho_{\tau_b}$, and $f_{\tau_b}(\vec{r},\vec{p})$ is the phase-space distribution function of particle species τ_b. For hyperons in symmetric nuclear matter at saturation density, their potentials are fixed at $U_\Lambda^{(N)}(\rho_N = \rho_0) = -30$ MeV, $U_\Xi^{(N)}(\rho_N = \rho_0) = -18$ MeV, and $U_\Sigma^{(N)}(\rho_N = \rho_0) = 30$ MeV for Λ, Ξ, and Σ, respectively. The parameter x is used to model the isospin dependence of

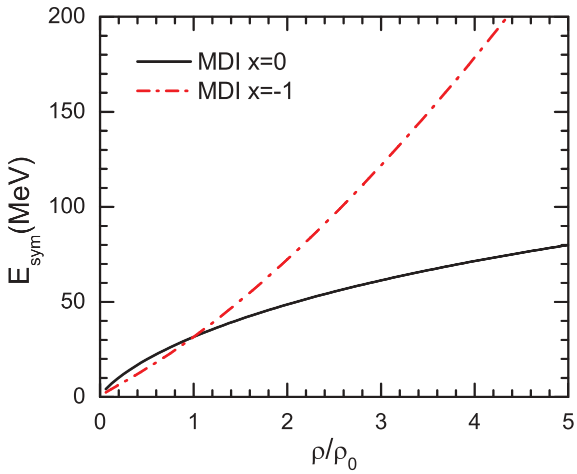

FIG. 1. (Color online) Density dependence of nuclear symmetry energy from the MDI interaction with parameter $x = 0$ and -1.

the interaction between two baryons. Its value can be adjusted to mimic the different density dependence of the nuclear symmetry energy predicted by various microscopic many-body theories [18]. Currently, experimental constraints on this parameter scatter within a large range [34]. For illustrations in the present study, we take the x parameter to be 0 or -1. Shown in Fig. 1 is the corresponding density dependence of the nuclear symmetry energy $E_{\rm sym}(\rho)$. It is worth noting that the latest experimental constraints on $E_{\rm sym}(\rho)$ at subsaturation densities are consistent with but span a region larger than the one covered by the $x = 0$ and $x = -1$ curves [34]. The experimental constraints on $E_{\rm sym}(\rho)$ at suprasaturation densities are, on the other hand, still very uncertain. In fact, the high-density behavior of $E_{\rm sym}(\rho)$ is considered the most uncertain part of the EOS of dense neutron-rich nuclear matter [35]. One of the purposes of this work is to study effects of the symmetry energy. As shown in Fig. 1, the choice of $x = 0$ and $x = -1$ allows the variation of the symmetry energy in a large range at suprasaturation densities.

In addition to the above EOS for hyperons, the MIT bag model for the quark matter [19,20] and the Gibbs construction for the hadron-quark phase transition [21,22] have been used in Ref. [17] to obtain the complete EOS of hybrid stars. As in previous studies (see, e.g., Refs. [36,37]), the hybrid star is divided into three parts from the center to the surface: the liquid core, the inner crust, and the outer crust. It is in the liquid core that the matter can be pure hadrons, quarks, or a mixture of the two. In the inner crust, a parametrized EOS of $P = a + b\epsilon^{4/3}$ is used as in Refs. [36,37]. The outer crust usually consists of heavy nuclei and the electron gas, where the Baym-Pethick-Sutherland (BPS) EOS [38] is used. The transition density ρ_t between the liquid core and the inner crust has been consistently determined in Refs. [36,37]. Taking the density separating the inner from the outer crust to be $\rho_{\rm out} = 2.46 \times 10^{-4}$ fm^{-3} [39], the parameters a and b can then be determined using the pressures and energy densities at ρ_t and $\rho_{\rm out}$. Shown in Figs. 2(a) and 2(b) are the EOSs with the symmetry energy parameter $x = 0$ and $x = -1$, respectively, and using the MIT bag constant $B^{1/4} = 180$ and 170 MeV. For comparison, the pure $npe\mu$ (labeled as nucleonic) and hyperonic (labeled as MDI Hyp-R interaction) EOSs are also included. As is well known, the appearance of hyperons and the hadron-quark phase transition softens the EOS of neutron star matter. Also, it is worth noting that the adiabatic coefficient $\gamma = d\log(P)/d\log(\rho)$ at saturation density is 2.63 and 2.57 for $x = 0$ and $x = -1$, respectively. The EOSs with $x = 0$ and $x = -1$ are thus about the same below and around the saturation density. However, it is interesting to see that the starting point and the degree of softening due to the appearance of hyperons are sensitive to the $E_{\rm sym}(\rho)$ at high densities. Moreover, the $E_{\rm sym}(\rho)$ also affects appreciably the starting point of the hadron-quark mixed phase especially with the larger bag constant. Nevertheless, it is obvious that the starting point is much more sensitive to the bag constant B for a given symmetry energy parameter x. These features are consistent with those first noticed by Kutschera and Niemiec [40]. It is also important to emphasize that, since the mixed phase is described using the Gibbs instead of the Maxwell construction, the energy density increases continuously with pressure across the mixed phase without a jump [41]. Consequently, the Seidov criterion that a stable hybrid star must satisfy [9,42], i.e., $\frac{\epsilon_q}{\epsilon_H} < \frac{3}{2}(1 + \frac{P}{\epsilon_H})$ with P, ϵ_q, and ϵ_H denoting, respectively, the pressure and the energy density of quarks and hadrons at the phase transition, is automatically satisfied.

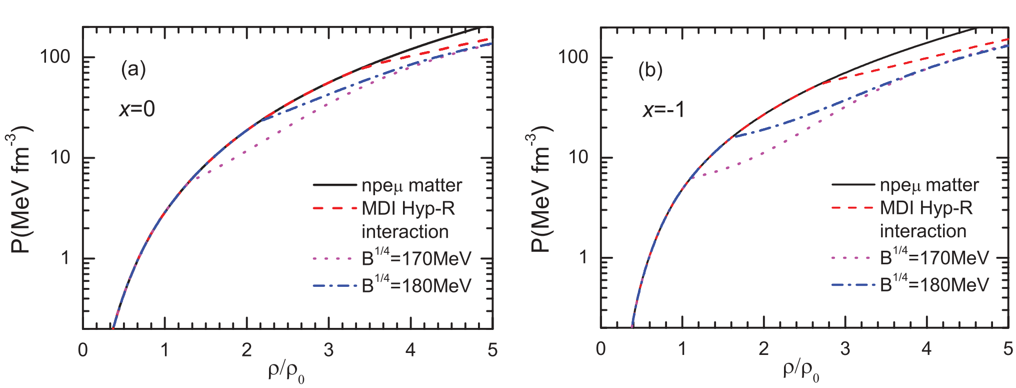

FIG. 2. (Color online) The EOSs of pure $npe\mu$ matter (nucleonic), hyperonic matter (MDI Hyp-R interaction), and hybrid stars for the bag constant $B^{1/4} = 180$ and 170 MeV, and the symmetry energy parameter $x = 0$ (a) and $x = -1$ (b).

III. TOTAL ENERGY RELEASE DUE TO HADRON-QUARK PHASE TRANSITION IN NEUTRON STARS

The amount of energy release during the microcollapse of a neutron star triggered by the hadron-quark phase transition is given by the change in its binding energy before and after the phase transition. The binding energy of a neutron star can be obtained from first solving the Tolman-Oppenheimer-Volkoff (TOV) equation with the corresponding EOS ($G = c = 1$),

$$\frac{dP(r)}{dr} = -\frac{[m(r) + 4\pi r^3 P(r)][\epsilon(r) + P(r)]}{r[r - 2m(r)]},$$
$$\frac{dm(r)}{dr} = 4\pi\epsilon(r)r^2, \quad (2)$$

where $P(r)$ and $\epsilon(r)$ are the pressure and the energy density at radius r. The binding energy of a neutron star is then given by [43]

$$E_b = M_g - M_{\rm bar}, \quad (3)$$

where M_g is the gravitational mass of the neutron star measured from infinity [44], i.e.,

$$M_g = \int_0^R 4\pi r^2 \epsilon(r) dr. \quad (4)$$

$M_{\rm bar}$ is the baryonic mass of the neutron star, namely, the mass when all the matter in the neutron star is dispersed to infinity [43]. It can be calculated from $M_{\rm bar} = N M_B$, where $M_B = 1.66 \times 10^{-24} g$ is the nucleon mass and N is the total number of baryons [43], i.e.,

$$N = \int_0^R 4\pi r^2 \left[1 - \frac{2m(r)}{r}\right]^{-1/2} \rho_B(r) dr \quad (5)$$

with $\rho_B(r)$ being the baryon density profile of the neutron star. While the total energy release E_g during the phase transition is the difference of Eq. (3) before and after the microcollapse, it reduces to the difference in the gravitational masses for the hadronic (h) and hybrid (q) configurations of the neutron star as a result of baryon number conservation, namely,

$$E_g = M_{g,h} - M_{g,q}. \quad (6)$$

Shown in Fig. 3 is the energy release as a function of the baryonic mass $M_{\rm bar}/M_\odot$ of a neutron star. It is seen that the energy release increases with $M_{\rm bar}/M_\odot$ and is higher with a smaller ($B^{1/4} = 170$ MeV) bag constant B but a stiffer ($x = -1$) symmetry energy. Effects of varying the bag constant B are obviously more significant than those of varying the symmetry energy parameter x, especially on the core mass and thus the energy release. The variation of the bag constant B also affects the radii of hybrid stars appreciably. On the other hand, the variation of the symmetry energy parameter x has appreciable effects only on the radii of both hyperonic and hybrid stars. Its effect on the energy release is much smaller than that of the bag constant B. It is worth noting that it was first shown in Ref. [45] that the binding energies of NSs consisting of pure $npe\mu$ matter depend strongly on the density dependence of the nuclear symmetry energy. However, the difference in binding energies before and after the hadron-quark phase transition becomes less sensitive to the symmetry energy.

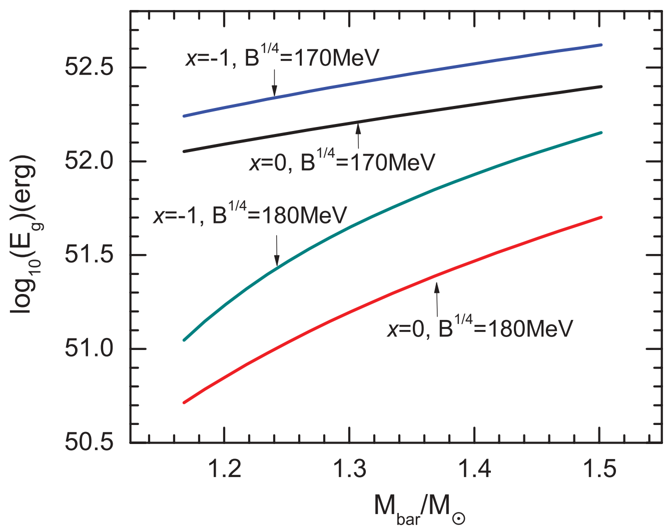

FIG. 3. (Color online) Total energy release due to the hadron-quark phase transition as a function of the baryonic mass of a neutron star.

In Tables I, II, III, and IV, we give more detailed comparisons of the baryon number (N), gravitational masses of hyperonic stars [$M_g(H)$], hybrid stars [$M_g(HQ)$], and the quark core including the mixed phase [M_g(core)], the radii of hyperonic [$R(H)$] and hybrid stars [$R(HQ)$], as well as the energy release (E_g).

Some of the features discussed above can be better understood from inspecting the gravitational mass-radius and mass–central-density correlations with and without the hadron-quark phase transition using different bag constants and symmetry energy functionals. Shown in Fig. 4 are the gravitational mass M_g as a function of radius R and central density ρ_c for several relevant cases. It is seen, on the one hand, that with $B^{1/4} = 170$ MeV the phase transition happens at a significantly lower density, resulting in a smaller maximum mass, than the case with $B^{1/4} = 180$ MeV. On the other hand, for a given bag constant B, the hyperonic EOS is stiffer with a stiffer symmetry energy, leading to a larger radius and thus a lower central density and a smaller maximum mass. The mass difference before and after the hadron-quark phase transition is therefore also larger.

To see how the total energy release is related to emitted GWs, we note that the magnitude of GWs is generally denoted by the gravitational strain amplitude in the following wave form:

$$h(t) = h_0 e^{-(t/\tau_{g\omega})} \sin(\omega_0 t), \quad (7)$$

where h_0 is the initial amplitude, ω_0 is the angular frequency, and $\tau_{g\omega}$ is the damping time scale. Assuming all energy release E_g is available for GW emission, the maximum value of the initial amplitude is then [46]

$$h_0 = \frac{4}{\omega_0 D} \left[\frac{E_g}{\tau_{g\omega}}\right]^{1/2}, \quad (8)$$

where D is the distance of the source from the detector. Although the maximum strain amplitude is directly determined by the energy release, not all the energy release is available for

TABLE I. The baryon number (N), gravitational masses of hyperonic stars [$M_g(H)$] hybrid stars [$M_g(HQ)$], and the quark core including the mixed phase [M_g(core)], the radii of hyperonic [$R(H)$] and hybrid stars [$R(HQ)$], as well as the energy release (E_g) for $x = 0$ and $B^{1/4} = 170$ MeV. All masses are in units of $M_\odot$ and radii in kilometers.

N	$M_g(H)$	$M_g(HQ)$	M_g(core)	$R(H)$	$R(HQ)$	log[(E_g)(erg)]
1.4×10^{57}	1.0861	1.0798	0.886659	11.2917	10.1679	52.0517
1.5×10^{57}	1.1570	1.1491	0.971092	11.3044	10.1256	52.1492
1.6×10^{57}	1.2270	1.2173	1.054172	11.3041	10.0691	52.2379
1.7×10^{57}	1.2960	1.2843	1.135890	11.2909	9.9935	52.3199
1.8×10^{57}	1.3641	1.3501	1.217399	11.2620	9.8796	52.3977

emitting gravitational waves as the energy can be dissipated by other mechanisms [9]. In fact, most of the energy release from the hadron-quark phase transition would be used to excite radial oscillations [47]. Moreover, pure radial modes do not generate any gravitational waves unless they are coupled with rotation [10]. However, because of angular momentum conservation, rotating neutron stars are more likely to be formed during supernova explosions. If there are some other dissipating mechanisms of time scale τ_d during the hadron-quark phase transition in neutron stars, the fraction of energy dissipated by gravitational waves is then approximately $f_g = 1/(1 + \tau_{g\omega}/\tau_d)$ [9]. Thus, the relative damping time scale of the GWs with respect to that due to other mechanisms determines the fraction of the total energy release that can be carried away by GWs. In the literature, time scales of various GW modes and various energy dissipation mechanisms have been considered; see, e.g., Refs. [9,10] and references therein. Except for the w mode for which the damping time is uniquely determined by the EOS of matter inside NSs, the damping times of other modes also depend on the rotational periods of NSs [10]. As for many other interesting questions in the field, the conclusions so far are still very model dependent.

IV. FREQUENCY AND DAMPING TIME OF THE AXIAL *w* MODE OF GRAVITATIONAL WAVES

GWs from nonradial oscillations of NSs carry important information about their internal structures [48–50]. One special type of normal mode of oscillations that only exists in general relativity is the so-called w mode associated with the perturbation of the space-time curvature and for which the motion of the fluid is negligible [29]. While the frequency of the w mode is significantly above the operating frequencies of existing GW detectors, it is nevertheless useful to further investigate what information about the internal structure of NSs may be revealed from a future detection of the w mode of GWs.

According to Chandrasekhar and Ferrari [29], the axial perturbation equations for a static neutron star can be simplified by introducing a function $z(r)$ that is constructed from the radial part of the perturbed axial metric components. It satisfies a Schrödinger-like differential equation

$$\frac{d^2z}{dr_*^2} + [\omega^2 - V(r)]z = 0, \tag{9}$$

where ω ($= \omega_0 + i\omega_i$) is the complex eigenfrequency of the axial w mode and r_* is the tortoise coordinate defined by

$$\frac{d}{dr_*} = e^{\lambda-\nu}\frac{d}{dr}, \tag{10}$$

where e^ν and e^λ are the metric functions given by the line element for a static neutron star [29]. Inside a neutron star, the potential function V is defined by

$$V = \frac{e^{2\nu}}{r^3}\{l(l+1)r + 4\pi r^3[\rho(r) - P(r)] - 6m(r)\}, \tag{11}$$

where l is the spherical harmonics index (used in describing the perturbed metric; only the case $l = 2$ is considered here), $\rho(r)$ and $P(r)$ are the density and pressure, and $m(r)$ is the mass inside radius r, respectively. Outside the neutron star, Eq. (11) reduces to

$$V = \frac{r - 2M_g}{r^4}[l(l+1)r - 6M_g], \tag{12}$$

where M_g is the total gravitational mass of the neutron star. The solutions to Eq. (9) are subject to a set of boundary conditions (BCs) constructed by Chandrasekhar and Ferrari [29]: regular BCs at the neutron star center, continuous BCs at the surface, and behaving as a purely outgoing wave at infinity. In a recent

TABLE II. As Table I but with $x = 0$ and $B^{1/4} = 180$ MeV.

N	$M_g(H)$	$M_g(HQ)$	M_g(core)	$R(H)$	$R(HQ)$	log[(E_g)(erg)]
1.4×10^{57}	1.0861	1.0858	0.335018	11.2917	11.0960	50.7137
1.5×10^{57}	1.1570	1.1564	0.457409	11.3044	11.0076	51.0370
1.6×10^{57}	1.2270	1.2259	0.582411	11.3041	10.8971	51.2965
1.7×10^{57}	1.2960	1.2942	0.708860	11.2909	10.7625	51.5124
1.8×10^{57}	1.3641	1.3612	0.838033	11.2620	10.5902	51.7018

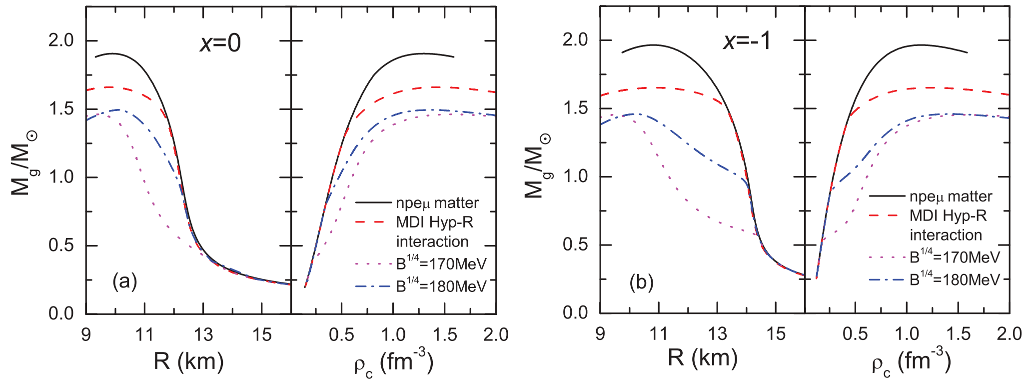

FIG. 4. (Color online) The mass-radius and mass–central-density correlations for pure $npe\mu$ matter (nucleonic), hyperonic matter (MDI Hyp-R interaction), and hybrid stars for the bag constant $B^{1/4} = 180$ and 170 MeV, and the symmetry energy parameter $x = 0$ (a) and $x = -1$ (b).

work [28] using the continued fraction method [51,52], some of us have studied the frequency and damping time of the axial w mode for NSs containing only the $npe\mu$ matter. It was found that the density dependence of the symmetry energy has strong imprints on both the frequency and damping time of the axial w mode. In this section, we examine how the appearance of hyperons and the hadron-quark phase transition may affect the frequency and damping time of the axial w-mode. As mentioned earlier, we focus on the relative effects of the symmetry energy and the bag constant on the first w mode.

Shown in Figs. 5(a) and 5(b) is the frequency as a function of the neutron star compactness M_g/R for the four cases of pure $npe\mu$ star, hyperonic star, and hybrid stars with two different values of the bag constant. It is interesting to see that the frequency is very sensitive to the EOS used. This is consistent with the earlier findings in Refs. [28,50] where a large ensemble of hadronic EOSs was used to describe the $npe\mu$ matter. Our results show, however, that the frequency of the w mode increases rapidly as the EOS becomes softened by hyperons or the hadron-quark phase transition. Moreover, the effect of the hadron-quark phase transition is dramatic. Comparison of results in the two figures for $x = 0$ and $x = -1$ further shows that the bag constant B plays a much stronger role than the symmetry energy parameter x.

Shown in Figs. 6(a) and 6(b) is the damping time as a function of M_g/R for the same four cases. It is seen that the damping time is longer for more compact NSs. Effects of the EOS on the damping time are appreciable but not as strong as those on the frequency. This might turn out to be an advantage for extracting information about the EOS of neutron star matter from analyzing the GW signals.

To assess more clearly the relative effects of the symmetry energy and bag constant, we present in Fig. 7 the correlation between the damping time and the frequency for hybrid stars. For the softer EOS with $B^{1/4} = 170$ MeV, the effect of the symmetry energy is small. The density dependence of the symmetry energy can, however, significantly affect the frequency of the w mode for the stiffer EOS with $B^{1/4} = 180$ MeV as a result of the higher hadron-quark transition

TABLE III. As Table I but with $x = -1$ and $B^{1/4} = 170$ MeV.

N	$M_g(H)$	$M_g(HQ)$	M_g(core)	$R(H)$	$R(HQ)$	log[(E_g)(erg)]
1.4×10^{57}	1.0911	1.0813	0.865139	12.5570	10.3024	52.2408
1.5×10^{57}	1.1632	1.1507	0.953706	12.5975	10.2280	52.3516
1.6×10^{57}	1.2346	1.2189	1.040034	12.6178	10.1446	52.4498
1.7×10^{57}	1.3052	1.2859	1.124924	12.6185	10.0418	52.5385
1.8×10^{57}	1.3749	1.3516	1.209269	12.5998	9.8992	52.6205

TABLE IV. As Table I but with $x = -1$ and $B^{1/4} = 180$ MeV.

N	$M_g(H)$	$M_g(HQ)$	M_g(core)	$R(H)$	$R(HQ)$	log[(E_g)(erg)]
1.4×10^{57}	1.0911	1.0904	0.389260	12.5570	11.8321	51.0467
1.5×10^{57}	1.1632	1.1616	0.563910	12.5975	11.4949	51.4673
1.6×10^{57}	1.2346	1.2314	0.716042	12.6178	11.2027	51.7548
1.7×10^{57}	1.3052	1.3999	0.855991	12.6185	10.9207	51.9731
1.8×10^{57}	1.3749	1.3670	0.990235	12.5998	10.6140	52.1525

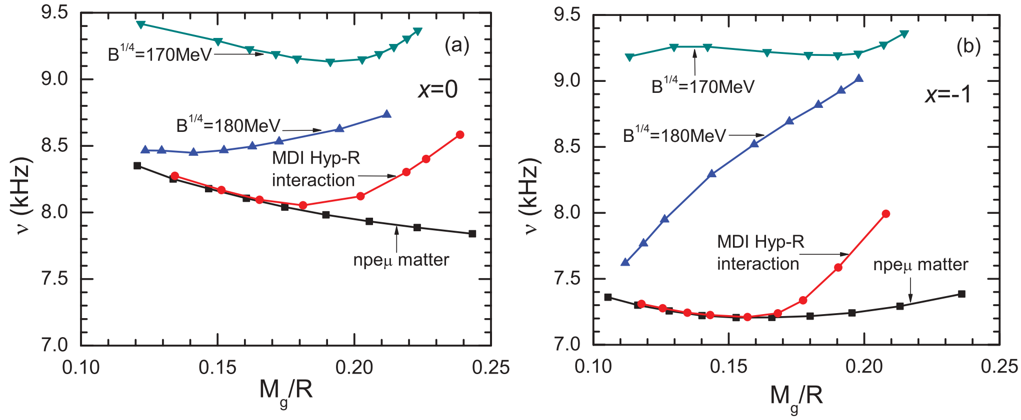

FIG. 5. (Color online) Frequency of the first w mode as a function of the neutron star compactness M_g/R for the symmetry energy parameter $x = 0$ (a) and $x = -1$ (b).

density as shown in Fig. 2. In this case, the symmetry energy in the hadronic phase then also plays an important role in determining the structure of NSs. Nevertheless, comparing the frequencies of the w mode for the same x parameter but different values of B, it is clear that the bag constant has a much stronger effect as it changes the underlying EOS of dense matter more significantly

V. SUMMARY AND CONCLUDING REMARKS

Using an isospin- and momentum-dependent effective interaction for the baryon octet and the MIT bag model to describe, respectively, the hadronic and quark phases of neutron stars, we have investigated the maximum available energy for gravitational wave emission owing to the microcollapse triggered by the hadron-quark phase transition in neutron stars. Moreover, the frequency and damping time of the first axial w mode of gravitational waves have been studied for both hadronic and hybrid NSs. Since the most uncertain part of the EOS of a neutron star is the density dependence of the nuclear symmetry energy in the neutron-rich nucleonic matter and the bag constant in the quark matter within the MIT bag model, we have studied their effects on the energy release as well as the frequency and damping time of the first axial w mode of GWs from neutron stars. We have found that the energy release is much more sensitive to the bag constant than to the density dependence of the nuclear symmetry energy. Also, the frequency of the w mode has been found to be significantly different with or without the hadron-quark phase transition and to depend strongly on the bag constant. Moreover, the effects of the symmetry energy and bag constant on the damping time have been found to be appreciable but not as strong as those on the frequency. We have further found that the effect of the symmetry energy on the frequency becomes stronger with a larger value of the bag constant that leads to a higher hadron-quark transition density. While the predicted frequency of the w mode is significantly above the bands of operating frequencies of the existing GW detectors, our results have indicated that the frequency of the w mode can indeed carry important information about the internal structure of NSs and the properties of dense neutron-rich matter.

It is worth noting that for this exploratory study we have used probably the simplest methods to construct the hadron-quark phase transition and to describe the quark phase, while the description of the hadronic phase is more accurate and

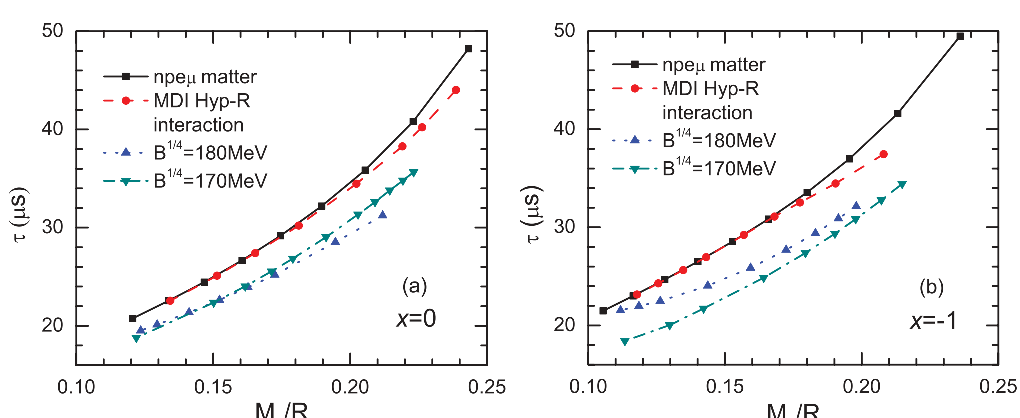

FIG. 6. (Color online) Damping time of the first w mode as a function of the neutron star compactness M_g/R for the symmetry energy parameter $x = 0$ (a) and $x = -1$ (b).

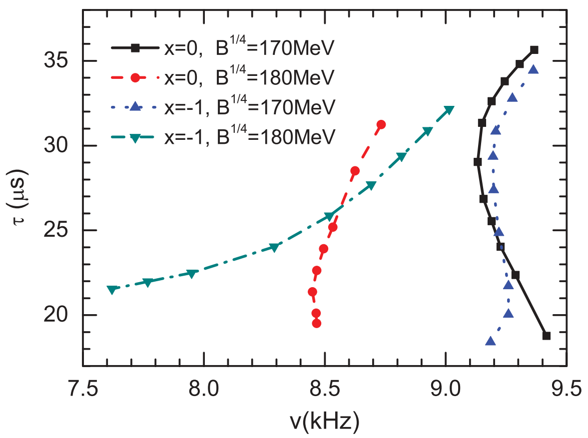

FIG. 7. (Color online) Damping time versus frequency of the first w mode for hybrid stars.

complete. A more realistic EOS of the mixed phase can be obtained from Wigner-Seitz cell calculations (see e.g., Refs. [53,54]) by taking into account both the Coulomb and surface effects that are neglected in the present work. The energy release during the hadron-quark phase transition and thus the frequency and damping time of the w mode are expected to depend on how the phase transition is modeled. How these different descriptions of the hadron-quark mixed phase may quantitatively affect the energy release and what their imprints may have on gravitational waves are interesting questions to be investigated. We also notice that significant progress has been made in calculating the EOS of interacting quark-gluon matter using more advanced QCD-based models; see, e.g., Refs. [55–57]. For the current exploration, we used the popular MIT bag model for its ease of use. For quantitative studies, it would be more fruitful to merge the accurate and complete EOS for the hadronic phase with an approximately equally accurate EOS for the quark phase. We expect that the observed sensitivity of the energy release as well as the frequency and damping time of the w mode to the MIT bag constant will show up in their dependence on the model parameters that describe the interactions in the quark matter. We plan to carry out such studies in the near future.

ACKNOWLEDGMENTS

This work was supported in part by the U.S. National Science Foundation Grants No. PHY-0757839 and No. PHY-0758115, the National Aeronautics and Space Administration Grant No. NNX11AC41G issued through the Science Mission Directorate, the Welch Foundation Grant No. A-1358, the Texas Coordinating Board of Higher Education Grant No. 003565-0004-2007, the Young Teachers' Training Program from China Scholarship Council Grant No. 2007109651, the National Natural Science Foundation of China Grant No.10947023, and the Fundamental Research Funds for the Central University, China Grant No.2009ZM0193.

[1] J. M. Lattimer and M. Prakash, Science **304**, 536 (2004).
[2] M. Maggiore, *Gravitational Waves* (Oxford University Press, Oxford, 2008), Vol. 1.
[3] I. Bombaci, in *Proceedings of the Eleventh Marcel Grossman Meeting on General Relativity*, edited by H. Kleinert, R. T. Jantzen, and R. Ruffini (World Scientific, Singapore, 2008), pp. 605–628.
[4] J. C. N. de Araujo and G. F. Marranghello, Gen. Relativ. Astrophys. **41**, 1389 (2008).
[5] K. S. Cheng and Z. G. Dai, Astrophys. J. **492**, 281 (1998).
[6] P. Haensel, J. L. Zdunik, and R. Schaeffer, Astron. Astrophys. **217**, 137 (1989).
[7] B. A. Li, L. W. Chen, and C. M. Ko, Phys. Rep. **464**, 113 (2008).
[8] G. F. Marranghello and J. C. N. de Araujo, Class. Quantum Grav. **23**, 6345 (2006).
[9] G. F. Marranghelo, C. A. Z. Vasconcellos, and J. A. de Freitas Pacheco, Phys. Rev. D **66**, 064027 (2002).
[10] W. Y. Chau, Astrophys. J. **147**, 664 (1967).
[11] N. Andersson, V. Ferrari, D. I. Jones, K. D. Kokkotas, B. Krishnan, J. Read, L. Rezzolla, and B. Zink, Gen. Relativ. Gravit. **43**, 409 (2011), special issue on Einstein Telescope.
[12] J. C. N. de Araujo and G. F. Marranghello, J. Phys.: Conf. Ser. **122**, 012041 (2008), and references therein.
[13] Omar Benhar, Valeria Ferrari, Leonardo Gualtieri, and Stefania Marassi, Gen. Relativ. Gravit. **39**, 1323 (2007).
[14] E. B. Abdikamalov, H. Dimmelmeier, L. Rezzolla, and J. C. Miller, Mon. Not. R. Astron. Soc. **394**, 52 (2009).
[15] J. A. de Freitas Pacheco, Res. Astron. Astrophys. **10**, 1071 (2010).
[16] S. Schettler, T. Boeckel, and J. Schaffner-Bielich, Phys. Rev. D **83**, 064030 (2011).
[17] J. Xu, L. W. Chen, C. M. Ko, and B. A. Li, Phys. Rev. C **81**, 055803 (2010).
[18] C. B. Das, S. Das Gupta, C. Gale, and B. A. Li, Phys. Rev. C **67**, 034611 (2003).
[19] A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, and V. F. Weisskopf, Phys. Rev. D **9**, 12 (1974).
[20] U. Heinz, P. R. Subramanian, H. Stocker, and W. Greiner, Nucl. Phys. **12**, 1237 (1986).
[21] N. K. Glendenning, Phys. Rev. D **46**, 1274 (1992).
[22] N. K. Glendenning, Phys. Rep. **342**, 393 (2001).
[23] P. Danielewicz, R. Lacey, and W. G. Lynch, Science **298**, 1592 (2002).
[24] B. A. Li and A. W. Steiner, Phys. Lett. B **642**, 436 (2006).
[25] P. G. Krastev, B. A. Li, and A. Worley, Astrophys. J. **676**, 1170 (2008).
[26] A. Worley, P. G. Krastev, and B. A. Li, Astrophys. J. **685**, 390 (2008).
[27] P. G. Krastev, B. A. Li, and A. Worley, Phys. Lett. B **668**, 1 (2008).
[28] D. H. Wen, B. A. Li, and P. G. Krastev, Phys. Rev. C **80**, 025801 (2009).
[29] S. Chandrasekhar and V. Ferrari, Proc. R. Soc. London, Ser. A **432**, 247 (1991); **434**, 449 (1991).
[30] B. Abbott *et al.* [LIGO Scientific Collaboration (LSC)], Phys. Rev. Lett. **94**, 181103 (2005).
[31] F. Acernese *et al.*, Class. Quantum Grav. **24**, S491 (2007).
[32] B. Abbott *et al.* (LSC), Nucl. Instrum. Methods Phys. Res., Sect. A **517**, 154 (2004).

[33] [http://lisa.nasa.gov/].
[34] C. Xu, B. A. Li, and L. W. Chen, Phys. Rev. C **82**, 054607 (2010).
[35] M. Kutschera, Phys. Lett. B **340**, 1 (1994).
[36] J. Xu, L. W. Chen, B. A. Li, and H. R. Ma, Phys. Rev. C **79**, 035802 (2009).
[37] J. Xu, L. W. Chen, B. A. Li, and H. R. Ma, Astrophys. J. **697**, 1549 (2009).
[38] G. Baym, C. Pethick, and P. Sutherland, Astrophys. J. **170**, 299 (1971).
[39] S. B. Ruster, M. Hempel, and J. Schaffner-Bielich, Phys. Rev. C **73**, 035804 (2006).
[40] M. Kutschera and J. Niemiec, Phys. Rev. C **62**, 025802 (2000).
[41] A. Bhattacharyya, I. N. Mishustin, and W. Greiner, J. Phys. G **37**, 025201 (2010).
[42] Z. Seidov, Astron. Zh. **48**, 443 (1971).
[43] Steven Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity* (John Wiley & Sons, New York, 1972), p. 303.
[44] James B. Hartle, *Gravity* (Addison-Wesley, Reading, MA, 2003).
[45] W. G. Newton and B. A. Li, Phys. Rev. C **80**, 065809 (2009).
[46] J. de Freitas Pacheco, in *Proceedings of the VIth International Workshop on Relativistic Aspects of Nuclear Physics*, edited by T. Kodama *et al.* (World Scientific, Singapore, 2001), p. 158.
[47] H. Sotani, K. Tominaga, and K. I. Maeda, Phys. Rev. D **65**, 024010 (2002).
[48] K. D. Kokkotas and B. F. Schutz, Mon. Not. R. Astron. Soc. **255**, 119 (1992).
[49] N. Andersson and K. D. Kokkotas, Mon. Not. R. Astron. Soc. **299**, 1059 (1998).
[50] O. Benhar, Mod. Phys. Lett. A **20**, 2335 (2005).
[51] M. Leins, H. P. Nollert, and M. H. Soffel, Phys. Rev. D **48**, 3467 (1993).
[52] O. Benhar, E. Berti, and V. Ferrari, Mon. Not. R. Astron. Soc. **310**, 797 (1999).
[53] H. Heiselberg, C. J. Pethick, and E. F. Staubo, Phys. Rev. Lett. **70**, 1355 (1993).
[54] T. Maruyama *et al.*, Phys. Lett. B **659**, 192 (2008).
[55] Mark Alford, Matt Braby, Mark Paris, and Sanjay Reddy, Astrophys. J. **629**, 969 (2005).
[56] Aleksi Kurkela, Paul Romatschke, Aleksi Vuorinen, and Bin Wu, arXiv:1006.4062v1.
[57] Aleksi Kurkela, Paul Romatschke, and Aleksi Vuorinen, Phys. Rev. D **81**, 105021 (2010).