Deuteron-nucleus collisions in a multiphase transport model

Zi-wei Lin
Physics Department, The Ohio State University, Columbus, Ohio 43210, USA

Che Ming Ko
Cyclotron Institute and Physics Department, Texas A&M University, College Station, Texas 77843, USA

(Received 8 January 2003; published 26 November 2003)

Using a multiphase transport model, we study pseudorapidity distributions and transverse momentum spectra in deuteron-gold collisions at RHIC. We find that final-state partonic and hadronic interactions affect the transverse momentum spectrum of protons more than those of kaons or pions. Relative to $p+p$ collisions at same center-of-mass energy per nucleon pair, the effect of final-state interactions on the charged particle transverse momentum spectra in $d+Au$ collisions is much smaller than observed in experimental data, indicating that initial-state effects such as the Cronin effect are important.

DOI: 10.1103/PhysRevC.68.054904 PACS number(s): 25.75.–q, 24.10.Lx

I. INTRODUCTION

To study heavy ion collisions at the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory, in which a deconfined plasma of quarks and gluons is expected to be formed, we have developed a multiphase transport (AMPT) model that includes both final-state partonic and hadronic interactions [1–3]. The model has been very useful for understanding various observables in Au+Au collisions at RHIC such as the rapidity and transverse momentum distributions of various particles [1–3] as well as charmonium [4] and strangeness [5] productions. In particular, it allows one to study both thermal and chemical equilibration in the partonic and hadronic matter formed in these collisions. The AMPT model has also been extended to include the string melting mechanism [6,7], in which soft strings produced from initial nucleon-nucleon interactions are converted directly to partons, in order to explain the measured elliptic flow [8] and two-pion correlation functions [9] at RHIC.

In spite of its success, the AMPT model has many uncertainties in its input physical parameters, particularly those related to the initial conditions introduced to the model. For example, the number of initial minijet partons in the AMPT model, that is given by the hard processes from the Heavy Ion Jet INteraction Generator (HIJING) model [10–12], depends on the nuclear shadowing, i.e., the modification of the parton distributions in a nucleon when it is in a nucleus. As a result, the final particle multiplicity produced in relativistic heavy ion collisions is affected by nuclear shadowing [2,3]. The production of high $p_T$ particles or hadrons made of heavy quarks, which is described by perturbative QCD processes, is even more sensitive to the nuclear shadowing effect. Furthermore, other nuclear effects such as the Cronin effect [13], which set in already in $p$-A collisions, need to be included. The uncertainties in the initial condition also exist in other transport models with partonic degrees of freedom [14,15] as well as theoretical models such as the hydrodynamical model [16–19] and the QCD saturation model [20–24].

Since final-state interactions (FSI) are expected to be less important in deuteron-gold collisions than in collisions between heavy nuclei, both the Cronin effect and the nuclear shadowing effect (e.g., through dilepton measurements [25]) can be better studied in these collisions. Improved knowledge on these effects are useful for making reliable theoretical interpretations of the observations in heavy ion collisions at RHIC. In this paper, we use the AMPT model to study the global observables in deuteron-gold collisions such as the pseudorapidity distributions and the transverse momentum spectra. Because of the small interaction volume in $d+Au$ collisions, we use the default AMPT model, i.e., without the string melting mechanism, in the present study, as the initial energy density produced in these collisions is expected to be small.

This paper is organized as follows. In Sec. II, we briefly review the AMPT model. Results from the AMPT model on deuteron-gold collisions at RHIC are then shown in Sec. III, with the charged particle pseudorapidity distributions and their centrality dependence given in Sec. III A, the transverse momentum spectra of different particles in Sec. III B, and the effects due to modifications of the string fragmentation parameters and nuclear shadowing in Sec. III C. A summary is then given in Sec. IV. Finally, the effect of centrality selection on the centrality dependence of charged particle pseudorapidity distributions is discussed in the Appendix.

II. THE AMPT MODEL

The AMPT model is a hybrid model that consists of four components: the initial conditions, the parton cascade, the conversion from partonic to hadronic matter, and the hadron cascade. In the default AMPT model [1–5], the initial conditions are generated from the HIJING model [10–12] (version 1.383 for this study), which usually uses a Woods-Saxon radial shape for the colliding nuclei and introduces a parametrized nuclear shadowing function that depends on the impact parameter of the collision [10]. Interactions among minijet partons, which are produced from initial hard nucleon-nucleon interactions, are modeled by the Zhang’s parton cascade (ZPC) [14]. After partons stop interacting, they recombine with their parent strings, which are produced
from initial soft nucleon-nucleon interactions, and fragment to hadrons according to the Lund string fragmentation model [26,27]. Dynamics of resulting hadronic matter is then described by a hadronic cascade based on the relativistic transport (ART) model [28]. Final hadronic observables including contributions from the strong decay of resonances are determined when the hadronic matter freezes out.

In this study, we use the Hulthen wave function [29] to model the structure of a deuteron [30]:

$$u(r) = Ce^{-\alpha r}(1-e^{-\beta r}),$$

where $r$ represents the relative distance between the proton and neutron in the deuteron, the normalization constant $C$ is determined from $\int_0^\infty u(r)^2dr=1$, and the small $D$-wave contribution ($\sim 6\%$) to the deuteron wave function is neglected. With $\alpha=(4.38 \text{ fm})^{-1}$ and $\mu=(1.05 \text{ fm})^{-1}$, the root-mean-square radius of the deuteron, $\sqrt{(\alpha^2+\beta^2)/2}$, is 2.0 fm, consistent with the measured value.

III. AMPT RESULTS ON DEUTERON-GOLD COLLISIONS

We have studied deuteron-gold collisions at $\sqrt{s_{NN}}=200$ GeV with both deuteron and gold having the same energy of 100 GeV per nucleon. In this study, “minimum-bias” $d+Au$ events are defined as those within the impact parameter range between 0 and 12 fm, and they are separated into different centrality bins.

A. Pseudorapidity distributions of charged particles

Figure 1 shows the pseudorapidity distributions of charged particles in the six centrality bins of $0\text{--}10\%$, $10\text{--}20\%$, $20\text{--}40\%$, $40\text{--}60\%$, $60\text{--}80\%$, and $80\text{--}100\%$. Here, the centrality is determined according to the value of $N_{\text{part}}$, i.e., the total number of participants in both deuteron and gold nuclei, in each event. The pseudorapidity in this study is evaluated in the nucleon-nucleon center-of-mass frame, and negative rapidities correspond to the fragmentation region of the gold nucleus. Solid curves are results from the AMPT model, while dashed curves are those from the HIJING model without quenching. Note that the AMPT model without final-state partonic and hadronic interactions is equivalent to the HIJING model without jet quenching if the popcorn mechanism [31] in the PYTHIA/JETSET program [26] is treated in the same way. It is seen that the asymmetry in the pseudorapidity distributions, e.g., by comparing values of $dN_{ch}/d\eta$ at $\eta=-2$ and $\eta=+2$, decreases as collisions become less central, and the pseudorapidity distribution for the most peripheral bin is almost symmetric. As expected, final-state interactions in $d+Au$ collisions have a smaller effect on charged particle pseudorapidity distributions than in the case of central heavy ion collisions at RHIC [2,3].

We note that the appearance of small bumps around $\eta\sim-4$ is mainly due to interactions of produced particles with initial incoming nucleons in the gold nucleus, which has an initial half width of $R_{Au}/2$ in the longitudinal direction with $R_{Au}$ being the hard-sphere radius of the gold nucleus and $\gamma$ denoting the Lorentz boost factor of a nucleon in the nucleon-nucleon center-of-mass frame. If the initial half width is reduced by a factor of 5, the charged particle pseudorapidity distributions would be smooth around $\eta\sim-4$ and also have slightly lower values in the region of $-6<\eta<-3$.

Figure 2(a) shows the pseudorapidity distributions of charged particles for minimum-bias $d+Au$ events. Solid curves are results from the AMPT model, dashed curves with open circles are AMPT results without final-state interactions, and dashed curves with filled circles are those without both final-state interactions and the popcorn mechanism for baryon-antibaryon production [1–3]. We see that both final-state interactions and the popcorn mechanism broaden the pseudorapidity distributions, especially in the fragmentation region of the gold nucleus, leading thus to a moderate increase of the particle multiplicity in the region of $-6<\eta<-3$. We note that without final-state interactions and the popcorn mechanism, results from the AMPT model are equivalent to those from the HIJING model (without jet quenching) as the same values of $a$ and $b$ in Eq. (2) are used.

One can separate events according to the number of participants from the deuteron, i.e., $N_{\text{part}}^d$. As an example, Fig. 2(b) shows the pseudorapidity distributions for minimum-bias $d+Au$ events with $N_{\text{part}}^d=1$ and $N_{\text{part}}^d=2$. Solid curves are results from the AMPT model without quenching. Note that the AMPT model without final-state partonic and hadronic interactions is equivalent to the HIJING model without jet quenching if the popcorn mechanism is treated in the same way. It is seen that the asymmetry in the pseudorapidity distributions, e.g., by comparing values of $dN_{ch}/d\eta$ at $\eta=-2$ and $\eta=+2$, decreases as collisions become less central, and the pseudorapidity distribution for the most peripheral bin is almost symmetric. As expected, final-state interactions in $d+Au$ collisions have a smaller effect on charged particle pseudorapidity distributions than in the case of central heavy ion collisions at RHIC [2,3].

FIG. 1. Pseudorapidity distributions of charged particles in $d+Au$ collisions at $\sqrt{s_{NN}}=200$ GeV with centralities determined from $N_{\text{part}}$. Solid and dashed curves represent results from the AMPT model and the HIJING model (without quenching), respectively.

FIG. 2. Pseudorapidity distributions of charged particles (a) for minimum-bias events when different interactions are included and (b) for events from AMPT with $N_{\text{part}}^d=2$ and with $N_{\text{part}}^d=1$.
Fig. 3. The average value of impact parameter (in fm) and $N_{\text{part}}^{d}$ (solid curve) as a function of the total $N_{\text{part}}$. See text for details.

Fig. 4. Centrality dependences of charged particle multiplicity and transverse energy per participant, the mean transverse energy (in GeV) per charged particle, and the mean $p_{T}$ (in GeV/c) of charged particles at midpseudorapidity.

Fig. 5. Centrality dependence of charged particle multiplicity per participant at different pseudorapidities is shown in Fig. 5, where the value at each $\eta$ represents the average value within the pseudorapidity range of $\eta \pm 0.5$. We see an increase of this quantity at the backward pseudorapidity $\eta = -4$ (curve with open diamonds) and a fast decrease at the forward pseudorapidity $\eta = +4$ (curve with filled diamonds). Furthermore, the decrease with centrality becomes stronger as the pseudorapidity changes from negative values to more positive values. These are consistent with the picture that multiple interactions in the gold nucleus push particle production in more central $d+Au$ collisions. Fig. 4 also shows that the ratio of the transverse energy and particle multiplicity per participant (solid curve with filled squares), i.e., the average transverse energy per charged particle, is rather flat. Also shown by the solid curve with triangles is the mean transverse momentum of charged particles, which is seen to change little with $N_{\text{part}}$ as well.

The centrality dependence of the charged particle multiplicity per participant at different pseudorapidities is shown in Fig. 5, where the value at each $\eta$ represents the average value within the pseudorapidity range of $\eta \pm 0.5$. We see an increase of this quantity at the backward pseudorapidity $\eta = -4$ (curve with open diamonds) and a fast decrease at the forward pseudorapidity $\eta = +4$ (curve with filled diamonds). Furthermore, the decrease with centrality becomes stronger as the pseudorapidity changes from negative values to more positive values. These are consistent with the picture that multiple interactions in the gold nucleus push particle production in more central $d+Au$ collisions. Fig. 4 also shows that the ratio of the transverse energy and particle multiplicity per participant (solid curve with filled squares), i.e., the average transverse energy per charged particle, is rather flat. Also shown by the solid curve with triangles is the mean transverse momentum of charged particles, which is seen to change little with $N_{\text{part}}$ as well.

The centrality dependence of the charged particle multiplicity per participant at different pseudorapidities is shown in Fig. 5, where the value at each $\eta$ represents the average value within the pseudorapidity range of $\eta \pm 0.5$. We see an increase of this quantity at the backward pseudorapidity $\eta = -4$ (curve with open diamonds) and a fast decrease at the forward pseudorapidity $\eta = +4$ (curve with filled diamonds). Furthermore, the decrease with centrality becomes stronger as the pseudorapidity changes from negative values to more positive values. These are consistent with the picture that multiple interactions in the gold nucleus push particle production in more central $d+Au$ collisions. Fig. 4 also shows that the ratio of the transverse energy and particle multiplicity per participant (solid curve with filled squares), i.e., the average transverse energy per charged particle, is rather flat. Also shown by the solid curve with triangles is the mean transverse momentum of charged particles, which is seen to change little with $N_{\text{part}}$ as well.

The centrality dependence of the charged particle multiplicity per participant at different pseudorapidities is shown in Fig. 5, where the value at each $\eta$ represents the average value within the pseudorapidity range of $\eta \pm 0.5$. We see an increase of this quantity at the backward pseudorapidity $\eta = -4$ (curve with open diamonds) and a fast decrease at the forward pseudorapidity $\eta = +4$ (curve with filled diamonds). Furthermore, the decrease with centrality becomes stronger as the pseudorapidity changes from negative values to more positive values. These are consistent with the picture that multiple interactions in the gold nucleus push particle production in more central $d+Au$ collisions. Fig. 4 also shows that the ratio of the transverse energy and particle multiplicity per participant (solid curve with filled squares), i.e., the average transverse energy per charged particle, is rather flat. Also shown by the solid curve with triangles is the mean transverse momentum of charged particles, which is seen to change little with $N_{\text{part}}$ as well.
ductions toward the negative rapidity region. Since we find that the mean transverse energy per charged particle at a given pseudorapidity does not change much with centrality, the centrality dependence of \( \frac{dE_T}{dN_{\text{part}}} \), i.e., the transverse energy per participant, has a similar behavior at each pseudorapidity as \( \frac{dN_{\text{ch}}}{d\eta} \).

### B. Transverse momentum spectra

The transverse momentum spectra of charged particles in the pseudorapidity range of \(-1 < \eta < 1\) for both minimum-bias and 0–20% central \( d + Au \) collisions are shown in Fig. 6. Solid curves with circles are results from the AMPT model while dashed curves are those without final-state partonic and hadronic interactions. For comparisons, the AMPT results for minimum-bias \( p + p \) collisions are shown by the solid curve. In all transverse momentum distributions and their ratios shown later, we include the statistical errors in the AMPT model, which become large at moderately high \( p_T \).

To better examine the distributions shown in Fig. 6, ratios of the \( p_T \) spectra of charged particles from \( d + Au \) collisions over those from minimum-bias \( p + p \) collisions are shown in Fig. 7. We see that, even without final-state partonic and hadronic interactions, the ratios (dashed curves) are not flat but instead tend to increase with \( p_T \) in the range of 0–3 GeV/c shown in the figure. This behavior is mainly a result of different scalings for hard and soft processes with respect to the number of binary collisions in the initial conditions from the HIJING model. While particle production at high enough \( p_T \) scales with the number of initial binary collisions, low momentum particles have a weaker dependence. Final-state interactions further modify the ratios of \( p_T \) spectra, as seen from comparisons between the solid and dashed curves.

Recently, the transverse momentum spectra of charged particles in \( d + Au \) collisions have been measured at RHIC [32]. Compared with the experimental data at \( 0 < p_T < 3 \) GeV/c, the \( p_T \) dependence of the ratios from the AMPT calculations shown in Fig. 7, either with or without final-state interactions, are much weaker. Thus, interactions in the final state are not the main reason for the observed strong \( p_T \) dependence of the ratios of transverse momentum spectra, suggesting that initial-state effects such as parton momentum broadening due to the Cronin effect [13,33] (not included in the AMPT model so far) are important in \( d + Au \) collisions.

For identified particles such as \( \pi^+, K^+ \), and protons, their transverse momentum spectra within the rapidity range of \(-1 < \eta < 1\) from minimum-bias \( d + Au \) collisions are shown in Fig. 8, with the proton spectra scaled down by a factor of 10. Solid and dashed curves represent AMPT results with and without final-state interactions, respectively. Compared with results from the AMPT model on central heavy ion collisions, e.g., Fig. 2 of Ref. [3] for central Pb+Pb collisions at SPS, the effect of final-state interactions on the transverse momentum spectra in \( d + Au \) collisions are much weaker.

**FIG. 6.** Transverse momentum spectra of charged particles within \(-1 < \eta < 1\) for \( d + Au \) collisions from AMPT with (solid curves with circles) or without (dashed) final-state interactions. The solid curve is the AMPT result for \( p + p \) collisions at the same center-of-mass energy per nucleon pair.

**FIG. 7.** Ratios of the charged particle transverse momentum spectra for \( d + Au \) collisions over the spectra for \( p + p \) collisions from AMPT with (solid) or without (dashed) final-state interactions.

**FIG. 8.** Transverse momentum spectra of \( \pi^+, K^+ \), and protons from AMPT with (solid) or without (dashed) final-state interactions for minimum-bias \( d + Au \) collisions. The proton spectra are scaled down by a factor of 10.
Figure 9 shows the ratios of the $p_T$ spectra from AMPT over those from the AMPT model without final-state interactions for minimum-bias (dashed) and 0–20% central (solid) $d+Au$ collisions.

Figure 10. Ratios of the transverse momentum spectra from AMPT for $d+Au$ collisions over those for $p+p$ collisions. Solid and dashed curves correspond to 0–20% central and minimum-bias $d+Au$ collisions, respectively.

C. Effects of the string fragmentation parameters

and nuclear shadowing

In the default AMPT model, hadron production after the partonic phase is described by the Lund string fragmentation model. In this model, the longitudinal momentum distribution of a hadron with transverse mass $m_T$ produced from the string fragmentation is given by the following symmetric splitting function [27]:

$$f(z) = z^{-1}(1-z)^a \exp(-b m_T^2/z). \quad (2)$$

where $z$ denotes the light-cone momentum fraction of the produced hadron with respect to that of the fragmenting string. In the HIJING model, which reproduces the experimental charged particle multiplicities in high energy $p+p$ and $p\bar{p}$ collisions, the default values of $a=0.5$ and $b=0.9$ GeV$^{-2}$ are used. However, in order to reproduce the rapidity distributions of charged particles in central Pb+Pb collisions at the CERN-SPS energy using the AMPT model, we find that values of these parameters need to change to $a=2.2$ and $b=0.5$ GeV$^{-2}$ [2,3]. The increase of $a$ and decrease of $b$ relative to the default values in the HIJING model soften the splitting function and thus enhance the total charged particle multiplicity by about 20% in central Pb+Pb collisions at SPS. The change of these two parameters can perhaps be attributed to the modification of string fragmentation when multiple strings are produced and overlap in heavy ion collisions, although it is not known yet how these parameters should be modified in different colliding systems. Since peripheral $d+Au$ collisions are expected to behave similarly as $p+p$ collisions, and even in central $d+Au$ collisions the string density is much lower than in central heavy ion collisions, we have assumed that the parameters $a$ and $b$ of Eq. (2) for deuteron-gold collisions have the same values as in $p+p$ collisions, i.e., they take the default values in the HIJING model.

However, since the values of $a$ and $b$ parameters need to be modified in order to describe the total multiplicity in central heavy ion collisions, it is still possible that these parameters are also modified in nonperipheral deuteron-gold collisions. If in central deuteron-gold collisions the parameters $a$
and \( b \) already have the modified values as in heavy ion collisions, the pseudorapidity distribution for the 20% most central deuteron-gold collisions (selected from \( N_{\text{part}} \)) would be given by the dashed curve in Fig. 11, which shows a substantial increase compared with the previous AMPT result obtained with default \( a \) and \( b \) values in the HIJING model (solid curve). Comparison of these results for deuteron-gold collisions with upcoming experimental results at RHIC will thus allow us to learn how the string fragmentation process is modified in different colliding systems or centralities.

When the nuclear shadowing effect is turned off in the AMPT model, the pseudorapidity distribution for the 20% most central deuteron-gold collisions is shown by the dot-dashed curve in Fig. 11, which is not much different from the solid curve obtained with the nuclear shadowing. Thus, pseudorapidity distributions of charged particles in deuteron-gold collisions are not very sensitive to the nuclear shadowing effect. This is different from that seen in collisions between heavy nuclei, where the effect is much larger as nuclear shadowing affects the production of minijet partons, which scales with the number of binary collisions. The effects of nuclear shadowing on \( p_T \) spectra of \( \pi^+, K^+ \), and protons, shown in Fig. 12 for 0–20% central \( d+Au \) collisions, are also relatively small and do not have a strong \( p_T \) dependence within \( 0<p_T<3 \) GeV/c. Note that, although the nuclear shadowing effect is unimportant for global observables in small colliding systems such as deuteron-gold collisions, it affects significantly observables which are dominated by partonic interactions such as open charm production [25] or the yield of high \( p_T \) particles.

### IV. SUMMARY

Using a multiphase transport (AMPT) model that includes both final-state partonic and hadronic interactions, we have studied the pseudorapidity distributions of charged particles, their centrality dependence, and transverse momentum spectra of different particles in deuteron-gold collisions at \( \sqrt{s_{NN}} =200 \) GeV. Due to the asymmetry of these collisions, the centrality dependence of charged particle multiplicity per participant is very different at different pseudorapidities, and it goes from increasing with centrality at the backward pseudorapidity region (the fragmentation region of the gold nuclei) to decreasing with centrality at the forward region. The charged particle pseudorapidity distribution in central deuteron-gold collisions is also sensitive to the values of the parameters used in the string fragmentation function. Furthermore, we find that, although final-state partonic and hadronic interactions modify the transverse momentum spectra of charged particles in \( d+Au \) collisions relative to scaled \( p+p \) collisions, the \( p_T \) dependence of the modification due to final-state interactions is much weaker than observed at RHIC. Thus, initial-state effects such as parton momentum broadening due to the Cronin effect are important in deuteron-gold collisions. However, we find that final-state interactions have a much stronger effect on the \( p_T \) spectra of protons than those of kaons or pions. Comparison of these predictions with the experimental data will thus help us to learn more about initial-state effects on transverse momentum spectra and to study whether string fragmentation is modified in deuteron-gold collisions as in central heavy ion collisions.

### ACKNOWLEDGMENTS

We thank Ulrich Heinz, Peter Jacobs, and Michael Murray for valuable comments. This paper is based on work supported by the U.S. Department of Energy under Grant No. DE-FG02-01ER41190 (Z.W.L.) and by the U.S. National Science Foundation under Grant No. PHY-0098805 as well as the Welch Foundation under Grant No. A-1358 (C.M.K.).
APPENDIX: EFFECTS OF DIFFERENT CENTRALITY SELECTIONS

To extract useful information from the centrality dependence of global observables, it is helpful to study how the criteria of centrality selection affects the centrality dependence. The number of participants and the impact parameter are known in theoretical models, but they cannot be directly measured in experiments. Instead, experimental centrality selections usually involve cutting on the charged particle multiplicity or the transverse energy. Thus, in addition to separating events into different centrality bins using the total number of participants, we have also tried three other methods, using the impact parameter, the number of charged particles within $-1 < \eta < 1$, and the total number of charged particles. For example, when the total number of charged particles is used for the centrality selection, we order all 35,000 events from the AMPT model by the total number of charged particles in each event. The centrality bin of, say 0–10%, then consists of the first 3,500 events in that ordered list.

In Fig. 13, we plot the average value of impact parameters versus the average value of $N_{\text{part}}$ at six centrality bins of 0–10%, 10–20%, 20–40%, 40–60%, 60–80%, and 80–100% when the centrality is determined from $N_{\text{part}}$ for determining the centrality is quite large (around 1.2 fm) for these centrality bins. We also find that, when $N_{\text{ch}}$ within $-1 < \eta < 1$ or the total $N_{\text{ch}}$ is used for the centrality selection, these widths are even larger (between 1.3 and 1.8 fm), indicating that the correlation between $N_{\text{ch}}$ and the impact parameter is weaker than that between $N_{\text{part}}$ and the impact parameter.

Different centrality selections also lead to differences in both the shape and magnitude of the pseudorapidity distributions at the same centrality bin. Solid curves in Figs. 14(a) and 14(b) for determining the centrality. The distribu-
This is due to the compensating effect of slower selection. Comparing with Fig. 5, we find that the centrality dependence of $dN_{ch}/d\eta$ at $\eta=0$ grows slower with centrality when the impact parameter is used for the centrality selection. However, it is interesting to see, from comparing Fig. 15(a) with Fig. 5, that the centrality dependences of $(dN_{ch}/d\eta)/N_{\text{part}}$ for the two cases are quite similar. This is due to the compensating effect of slower growth of $N_{\text{part}}$ with centrality when the impact parameter is used to select the centrality (see Fig. 13). We note that in relativistic heavy ion collisions where the multiplicity is much higher, these different methods for the centrality selection lead to quite similar results, contrary to those shown in Figs. 13 and 14 for $d+$Au collisions.

Figure 15(b) shows the centrality dependence of charged particle multiplicity per participant at different pseudorapidities when $N_{ch}$ within $-1<\eta<1$ is used for the centrality selection. Comparing with Fig. 5, we find that the centrality dependence at $\eta=-4$ or $\eta=+4$ is similar to the case where $N_{\text{part}}$ determines the centrality, but the centrality dependence at $\eta=-2.0$ or $+2$ is totally different. For example, while in Fig. 5 [and in Fig. 15(a)] the curve of $(dN_{ch}/d\eta)/N_{\text{part}}$ at $\eta=0$ shows a small decrease with centrality, the corresponding curve in Fig. 15(b) increases significantly with centrality. We note that these differences from Fig. 5 exist even when the total number of $N_{ch}$ is used for selecting the centrality (instead of $N_{\text{part}}$ within $-1<\eta<1$). Part of this large difference in the centrality dependence is due to the stronger increase of $dN_{ch}/d\eta(\eta=0)$ with centrality in the case when $N_{ch}$ around midpseudorapidity is used to determine the centrality, as shown in Fig. 14(b). Also, since the average value of $N_{\text{part}}$ grows slower with centrality in this case (see Fig. 13), the difference is further enhanced after dividing $dN_{ch}/d\eta$ by $N_{\text{part}}$. Figures 14 and 15 thus show that the criteria of centrality selection can introduce appreciable differences in the centrality dependence of global observables in deuteron-gold collisions, and corresponding care must be taken when comparing theoretical results with experimental data on centrality dependences.

[30] This form of the Hulthen function has been implemented in version 1.383 of the HIJING model.
[31] The popcorn mechanism is controlled mainly by parameters MSTJ(12) and PAR3(5) in the PYTHIA/JETSET program.