Isoscalar giant dipole resonance in $^{90}$Zr, $^{116}$Sn, and $^{208}$Pb

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Strength functions for isoscalar dipole excitations in $^{90}$Zr, $^{116}$Sn, and $^{208}$Pb have been measured with inelastic scattering of 240 MeV α particles at small angles. The isoscalar $E1$ strength distribution in each nucleus is found to consist of a broad component at $E_x \sim 114/A^{1/3}$ MeV containing approximately 100% of the $E1$ EWSR and a narrower one at $E_x \sim 72/A^{1/3}$ MeV containing 15–28% of the total isoscalar $E1$ strength. The higher component is the compression mode $E1$ strength previously reported only in $^{208}$Pb, whereas the lower component may be a new mode not reported previously, but suggested by recent RPA-HF and relativistic mean field calculations.

Evidence for a broad isoscalar giant dipole resonance (ISGDR) at $E_x \sim 21$ MeV in $^{208}$Pb has been reported by several groups [1–4]. The structure, transition density and sum rule for this $3\hbar\omega$ resonance, a result of the operator $r^3 Y_{10}$, have been described by Deul [5], Harakeh [6], and Stringari [7]. It is of particular interest because it is a compression mode and like the isoscalar giant monopole resonance (GMR), its energy can be related to compressibility [7]. In the scaling model

$$E_1 = h \left[(7K_A + 27/25\epsilon_F)/3m(r^2)\right]^{1/2},$$

where $K_A$ is the nuclear compressibility and $\epsilon_F$ is the Fermi energy. Van Giai and Sagawa [8] using the Skyrme interaction in Hartree-Fock (HF) random phase approximation (RPA) calculations found the $r^3 Y_{10}$ $E1$ strength at about $E_x \sim 25$ MeV in $^{208}$Pb.

We have investigated the giant resonance regions in $^{90}$Zr, $^{116}$Sn, and $^{208}$Pb using inelastic scattering of 240 MeV α particles where excellent peak to continuum ratios are obtained [9–11] and where competing pickup-breakup reactions are well above the region where ISGDR strength is expected. GMR strengths extracted from this data have already been reported [12]. The experimental technique has been described thoroughly in Refs. [9–12]. Sample spectra obtained are shown in Fig. 1. The prominent giant quadrupole resonance (GQR) and GMR are obvious as is a weaker and broader peak at higher excitation that moves systematically with mass.

The transition density for the ISGDR is [6]

$$\rho(r) = -\beta_1 / R \sqrt{3} [3r^2 d/dr + 10r - 5(3r^2)/dr + \epsilon (r^2 + dr^2) ]^2 \rho_\gamma(r).$$

For one state which exhausts the energy weighted sum rule [6]

$$\beta_1^2 = (6\pi \hbar^2 / mAE_\gamma R^2 / (11r^4) - 25/3(r^2)^2 - 10\epsilon(r^2)),$$

where $\epsilon = (4/E_2 + 5/E_0) \hbar^2 / 3m A$ and $E_2$ and $E_0$ are quadrupole and monopole giant resonance energies, respectively. In these nuclei the isovector giant dipole resonance (IVGDR) is

![FIG. 1. Inelastic α spectra obtained near the peak of the ISGDR for $^{90}$Zr, $^{116}$Sn, and $^{208}$Pb. The solid lines show the continuum chosen for the analysis and the LEOR contribution for $^{90}$Zr and $^{116}$Sn.](image)

TABLE I. Parameters used in DWBA calculations.

<table>
<thead>
<tr>
<th>Target</th>
<th>$c$ (fm)</th>
<th>$(c_n-c_p)$ (fm)</th>
<th>$\alpha$ (fm)</th>
<th>$V$ (MeV)</th>
<th>$V_I$ (MeV)</th>
<th>$R_I$ (fm)</th>
<th>$A_I$ (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{90}$Zr</td>
<td>4.90$^b$</td>
<td>0.18$^b$</td>
<td>0.515$^b$</td>
<td>40.2</td>
<td>40.9</td>
<td>4.77</td>
<td>1.242</td>
</tr>
<tr>
<td>$^{116}$Sn</td>
<td>5.43$^a$</td>
<td>0.18$^a$</td>
<td>0.515$^a$</td>
<td>36.7</td>
<td>23.9</td>
<td>6.45</td>
<td>1.047</td>
</tr>
<tr>
<td>$^{208}$Pb</td>
<td>6.67$^b$</td>
<td>0.26$^b$</td>
<td>0.545$^b$</td>
<td>43.3</td>
<td>61.4</td>
<td>7.75</td>
<td>0.567</td>
</tr>
</tbody>
</table>

$^a$Reference [13].

$^b$Reference [19].
excited in α scattering both by Coulomb excitation and by the nuclear force due to differences in neutron and proton distributions in the target nucleus. The nuclear transition density for the IVGDR is given by

\[ g_1(r) = \alpha_1 \gamma (N-Z) / A \left[ \frac{d}{dr} + \frac{1}{3} c a^2 / dr^2 \right] \rho(r), \]

where \( \alpha_1 = \pi \hbar^2 A / 2mNZe^2 \), \( \gamma = 3(c_n - c_p)A / 2c(N-Z) \) and \( c_n \), \( c_p \), and \( c \) are the respective matter, neutron, and proton radii. The transition densities and sum rules for the other multipolarities are described by Satchler [13] and the versions used in this work are given in Ref. [9]. Density dependent single folding calculations as described in Ref. [11] and by Satchler and Kwok [14] were carried out with optical model parameters obtained from elastic scattering [15,16] for each of the nuclei studied. Such calculations have been shown [11,14,15] to give cross sections that fit the data for low-lying states when using \( B(EL) \) values from electromagnetic measurements. The calculations were carried out with the code PTOLEMY [17]. Input parameters for PTOLEMY were modified [18] to obtain a relativistic kinematically correct calculation. Radial moments were obtained by numerical integration of the Fermi mass distributions for each nucleus. The folding model and Fermi parameters as well as \( c_n - c_p \) values used in this work are given in Table I.

The multipole components of the giant resonance peak were obtained by dividing the spectra into a peak and continuum [indicated by the solid lines in Fig. 1] then dividing the peak into multiple regions (bins) by excitation energy. The angular distributions obtained for each of these bins was then compared to distorted wave Born approximation (DWBA) calculations to obtain the multipole components [11]. This technique is described in Ref. [11] for 24Mg where isoscalar E0, E1, E2, and E3 strength distributions were

| Table II. Parameters obtained for upper and lower components of the ISGDR. |
|----------------|----------------|-----------------|----------------|----------------|
| \( A \)       | Lower Centroid (MeV) | Lower rms width (MeV) | Lower Strength %E1 EWSR | Upper Centroid (MeV) | Upper rms width (MeV) | Upper Strength %E1 EWSR |
| 208Pb          | 12.2 ± 0.6      | 1.9 ± 0.5       | 32 ± 12        | 19.9 ± 0.8      | 2.5 ± 0.6       | 115 ± 20 |
| 116Sn          | 14.7 ± 0.5      | 1.6 ± 0.5       | 38 ± 12        | 23.0 ± 0.6      | 3.7 ± 0.5       | 100 ± 15 |
| 90Zr           | 16.2 ± 0.8      | 1.9 ± 0.7       | 19 ± 6         | 25.7 ± 0.7      | 3.5 ± 0.6       | 103 ± 18 |
extracted up to $E_x \approx 40$ MeV. A sample of the angular distributions obtained for the giant resonance (GR) peak and the assumed continuum in $^{90}$Zr are shown in Fig. 2. Fits to the angular distributions were carried out with a sum of isoscalar $E_0, E_1, E_2, E_3, E_4$ strengths. The isovector giant dipole resonance (IVGDR) contributions were calculated from the known distributions [20] and held fixed in the fits. Sample fits obtained, along with the individual components of the fits, are shown superimposed on the data in Fig. 2.

The “continua” illustrated by the lines in Fig. 1 were obtained by matching the slope and strength of the continuum above the GR peak, and decreasing this smoothly to 0 near the particle threshold, then adding in the known low energy octupole resonance (LEOR). The effects of the continuum choice were explored by also carrying out analyses using two other continua, one higher and one lower, each at the limits of plausibility. Because of discrete states, the LEOR, and the detector threshold, the continuum is most poorly defined at low excitation and these other continua differed the least at the highest excitation.

In the $^{24}$Mg analysis [11] it was demonstrated that $E0$ strength in the peak and continuum could be identified, and that the total $E0$ strength obtained is essentially independent of continuum choice. That was also found to be true in these nuclei. However in the $^{24}$Mg analysis [11] it was also demonstrated that other processes in the continuum gave angular distributions that could be fit with a sum of $E1, E2, E3,$ and $E4$ multipole strengths and hence strength distributions for these multipoles could be very sensitive to assumptions about the continuum. In the present analyses, the $E1$ distributions were relatively insensitive to the different continua used, with the strength changing about 30% from the highest to lowest continuum because the continuum at high excitation is well defined. The $E2$ and $E3$ strengths were very sensitive to the continuum assumptions, and with the highest continuum chosen each decreased to less than half the expected strength. The centroids of all the distributions differed little with the different continuum assumptions, but the widths of the $E2$ and $E3$ distributions changed by up to about 20%.

<table>
<thead>
<tr>
<th>$A$</th>
<th>Lower component</th>
<th>Upper component</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Exp. (MeV)</td>
<td>Colo et al. (MeV)</td>
</tr>
<tr>
<td>$^{208}$Pb</td>
<td>12.2</td>
<td>10.9</td>
</tr>
<tr>
<td>$^{116}$Sn</td>
<td>14.7</td>
<td>12.5</td>
</tr>
<tr>
<td>$^{90}$Zr</td>
<td>16.2</td>
<td>14.5</td>
</tr>
</tbody>
</table>
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tained with a deformed potential analysis while the
t very similar to those reported in Ref.

FIG. 3 for 116 Sn and Fig. 4 for 90 Zr and 208 Pb are shown by the histograms. The error bars represent the uncertainty due to the fitting of the angular distributions as described in the text.

The multipole distributions obtained for 116Sn are shown in Fig. 3. For each nucleus, the E0 distributions obtained are very similar to those reported in Ref. [12], which were obtained with a deformed potential analysis while the E2 distributions agreed with those in the literature. The E3 distributions obtained contained approximately 2/3 of the E3 strength in the ISGDR as expected, with the remainder in the 1h0 excitation.

The E1 strength distribution in each nucleus (shown in Fig. 3 for 116Sn and Fig. 4 for 90Zr and 208Pb) consists of two components, a broad component at $E_x \sim 114/\Delta^{1/3}$ MeV containing approximately 100% of the E1 EWSR and a narrower one at $E_x \sim 72/\Delta^{1/3}$ MeV containing 19–38% of the E1 EWSR. The energies and widths for the components were obtained by fitting a Gaussian to the lower peak in each spectrum then subtracting that Gaussian and obtaining the centroid and root-mean-square (rms) width for the remaining strength. The parameters obtained for the two components are summarized in Table II. The errors include the uncertainties of the fits and the continuum choices. For 208Pb, the centroid and width (FWHM) obtained for the upper component (19.9 ± 0.8 MeV, $5.9 \pm 1.4$ MeV) are in agreement with the values from Morsch et al. [1] (21.3 ± 0.8 MeV, $5.9 \pm 0.8$ MeV) but not with those from Davis et al. [4] (22.4 ± 0.5 MeV, 3.0 ± 0.5 MeV). Because the cross section of the high energy octupole resonance (HEOR) is increasing toward larger angles, the subtraction technique used by Davis et al. [4] should result in an over subtraction in the region of the HEOR, obscuring some of the lower excitation part of the ISGDR, qualitatively consistent with what is observed.

The lower component contains 15–28% of the total ISGDR strength and is near the IVGDR, which was included in the analysis at the known strength. In order to account for the strength of the low energy component of the ISGDR in 116Sn (as the IVGDR), $c_n - c_p$ would have to be increased by a factor of 5 to 0.95 fm from the expected 0.18 fm [13]. A similar increase would be required for the other nuclei.

Recent calculations of the $r^3Y_{10}$ response function by Kolomiets et al. [21] and Colo et al. [22] using HF-RPA with Skyrme interactions and by Vretenar et al. [23] using relativistic RPA show the ISGDR strength split into upper and lower components qualitatively similar to that observed here. The upper component in these calculations is the compression mode, whose energy depends on the compression modulus, while the energy of the lower component changes with mass but is essentially independent of compression modulus. Vretenar et al. [23] describe this lower mode as a ‘‘kind of toroidal motion’’ and a surface effect, not a volume compression effect. Colo et al. [22] did not report transition densities for the two modes, but Vretenar et al. [23] show that their transition density for this lower component is very different from the transition density for the higher mode.

There are substantial differences between theory and experiment for the energies of the two modes. The centroid energies for both modes are compared to the predicted values in Table III. The centroids of the higher (compression) mode calculated with interactions which reproduce GMR energies are about 4 MeV higher than the experimental centroids while the calculated centroids for the lower mode lie 1–2 MeV below the experimental values. Thus the experimental ‘‘splitting’’ of the ISGDR from the present data is 5–7 MeV less than that predicted in the calculations.

Poelhekken et al. [24], using small angle $\alpha$ scattering in coincidence with $\gamma$ decay, identified isoscalar 1$^+$ strength in 40Ca, 58Ni, 90Zr, and 208Pb, lying almost entirely below the $E_x = 10$ MeV threshold of the present experiment and below the strength predicted by Vretenar et al. and Colo et al. They identified 8% of the E1 EWSR in 90Zr and 14.7% in 208Pb. Because our effective solid angle changes rapidly below $E_x = 10$ MeV, we do not extract multipole strength there. However if we combine the 1$^+$ strength identified by Poelhekken et al. [24] with the lower component we identify, the centroids for the lower strength in 90Zr and 208Pb are 14.4 MeV and 10.7 MeV, respectively, in agreement with the centroids calculated by Colo et al. (14.5 MeV, 10.9 MeV) and by Vretenar et al. for 208Pb only (10.35 MeV). The combined strength in the lower peak is 24% and 42% for 90Zr and 208Pb, respectively, compared to 34% and 36% in the Colo et al. calculation. However, neither calculation shows strength at the energies where it is reported in Ref. [24].

In conclusion, an isoscalar dipole giant resonance has been identified in 90Zr, 116Sn, and 208Pb having two components at approximately $E_x \sim 72/\Delta^{1/3}$ MeV and $E_x \sim 114/\Delta^{1/3}$ MeV. The higher component is identified as the compression mode predicted in RPA calculations, but lies about 4.5 MeV lower than expected. The lower component, containing 15–28% of the total E1 strength, lies 1–2 MeV above a new surface mode resonance suggested in recent RPA calculation. If low-lying 1$^+$ strength seen by Poelhekken et al. [24] in 90Zr and 208Pb is combined with our observed strength, then reasonable agreement is obtained with predicted centroids and fair agreement with predicted strengths for the lower components.
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