# Elastic and Inelastic Scattering of $\alpha$ Particles from ${ }^{138} \mathrm{Ba}^{\dagger}$ 

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#### Abstract

Differential cross sections for the elastic scattering and inelastic scattering to the low-lying states in ${ }^{138} \mathrm{Ba}$ have been measured using $50-\mathrm{MeV} \alpha$ particles. The angular distributions are analyzed in the distorted-wave Born approximation, employing collective-model form factors. Spin and parity assignments of $2^{+}, 4^{+}$, and $3^{-}$are verified for states at $1.436,1.898$, and 2.880 MeV . Assignments of $2^{+}$and $4^{+}$are made for states at 2.19 and 2.27 MeV . Tentative assignments of $2^{+}$and $4^{+}$are made for members of unresolved multiplets at 3.34 and 3.50 MeV . Values of $\beta_{l} R^{\prime}$ are obtained for all observed levels. Reasonable agreement for excitation energies and $J^{\pi}$ assignments has been obtained between the present results and previous experimental measurements and with recent theoretical calculations. A comparison of available isoscalar and electromagnetic transition rates in the mass region $115<A<150$ indicates that, for the lowest $2^{+}$states, the ratio of the neutron-to-proton contributions to the isoscalar matrix elements is in the ratio $N / Z$ as predicted by the collective vibrational model, except for the $N=82$ region. A similar result has previously been observed at the $N=28$ shell closure.


## I. INTRODUCTION

This investigation of the structure of ${ }^{138} \mathrm{Ba}$ using the $\alpha, \alpha^{\prime}$ reaction is an extension of our work on selected $N=82$ isotones. ${ }^{1}$ These isotones have been the subject of a large number of recent theoretical ${ }^{2-4}$ and experimental ${ }^{5-9}$ studies. These studies have been motivated to a large extent by the hope that the closed-shell nature of these nuclei will allow their low-lying excited states to be described by shell-model configurations involving only those protons outside the $Z=50$ closed shell.

Previous scattering data ${ }^{10-14}$ on ${ }^{138} \mathrm{Ba}$ are rather old and incomplete. Some recent $\gamma$-ray experiments ${ }^{15-18}$ have accurately determined the excita tion energies of a large number of excited states, but many of these states do not as yet have definite spin and parity assignments. In the present work, the inelastic scattering of $\alpha$ particles is used to determine spin and parity assignments and to measure isoscalar transition rates. The inelastic scattering of $\alpha$ particles, interpreted via the collective model, is a well-established method of determining the spins and parities of excited states of even-even nuclei. ${ }^{19.20}$ The isoscalar transition rates are obtained using the methods of Bernstein. ${ }^{21}$ We also present a comparison of our spin and parity assignments and isoscalar transition rates to other experimental ${ }^{17,18,22,23}$ and theoretical ${ }^{3,24}$ results.

## II. DESCRIPTION OF EXPERIMENT

Most of the experimental techniques have been described in the earlier paper. ${ }^{1}$ A beam of 49.6 $\pm 0.3-\mathrm{MeV} \alpha$ particles was scattered from ${ }^{138} \mathrm{Ba}$
targets that were prepared by vacuum evaporating metallic barium onto $75-\mu \mathrm{g} / \mathrm{cm}^{2}$ carbon foils. A $1500-\mu \mathrm{m}$ surface-barrier detector with conventional nuclear electronics was used to acquire the data for $\theta_{\text {c.m. }}>20^{\circ}$. In order to obtain better energy resolution, the data for $\theta_{\text {c.m. }}<20^{\circ}$ were obtained through the use of a $50-\mathrm{mm}$ by $10-\mathrm{mm}$ positionsensitive nuclear-triode detector ${ }^{25}$ which was located in the focal plane of an Enge split-pole spec trograph. ${ }^{26}$ In this latter case, the signals from the nuclear triode were processed by a technique ${ }^{27}$ which employs on-line digital division of the two detector outputs, $E$ and $x E$, in order to obtain more exact position information. The required division-plus -particle identification was performed on line through the use of a multipurpose experimental interface ${ }^{28}$ and an IBM 7094 computer. Figure 1 shows a typical spectrum obtained with the spectrograph. Over -all energy resolution for spectrograph data is 38 keV (full width at half maximum) while data taken at $\theta_{\text {lab }}>20^{\circ}$ with the surface-barrier detector have a typical resolution of 75 keV (full width at half maximum).

## III. RESULTS

Figure 2 shows a comparison of the measured elastic scattering angular distribution and an opti-cal-model calculation based on an optical potential of the form,

$$
\begin{equation*}
U(r)=V_{C}(r)-V_{0}\left(e^{x}+1\right)^{-1}-i W\left(e^{x^{\prime}}+1\right)^{-1} \tag{1}
\end{equation*}
$$

where $x=\left(r-R_{0}\right) / a_{0}$ and $x^{\prime}=\left(r-R^{\prime}\right) / a^{\prime}$, with $R_{0}$ $=r_{0} A^{1 / 3}$ and $R^{\prime}=r^{\prime} A^{1 / 3}$, and where $V_{C}(r)$ is the Coulomb potential for a uniformly charged sphere of radius $R_{c}=r_{c} A^{1 / 3}$.


FIG. 1. Example of typical spectrum obtained from the spectrograph data.

The limited angular range of the experimental data allowed too much freedom in the choice of the optical potential, so the parameters derived from the more complete ${ }^{144} \mathrm{Sm}$ data ${ }^{1}$ were used in this study as well. The parameters are $V_{0}=185.0 \mathrm{MeV}$, $r_{0}=1.40 \mathrm{fm}, a_{0}=0.52 \mathrm{fm}, W=25.8 \mathrm{MeV}, r^{\prime}=1.33$ $\mathrm{fm}, a^{\prime}=0.49 \mathrm{fm}$, and $r_{c}=1.4 \mathrm{fm}$.


FIG. 2. Elastic scattering and odd-parity angular distributions for $49.6-\mathrm{MeV} \alpha$ particles incident on ${ }^{138} \mathrm{Ba}$.

The inelastic scattering angular distributions were analyzed with the conventional distorted -wave Born approximation (DWBA). The transition amplitude is calculated assuming that the projectile excites vibrational states described by a collectivemodel Hamiltonian. The analysis has been restricted to one -step transitions and has been discussed in detail by Bassel et al. ${ }^{19}$ and Rost. ${ }^{20}$ This analysis leads to a particularly simple relationship between experimental and theoretical cross sections for even-even nuclei, ${ }^{21}$

$$
\begin{equation*}
\frac{d \sigma}{d \Omega}(0 \rightarrow l)=\left(\beta_{l} R^{\prime}\right)^{2} \sigma_{l} \tag{2}
\end{equation*}
$$

in which the deformation length $\beta_{l} R^{\prime}$ is determined by a comparison of the measured cross section, $d \sigma / d \Omega$, and the DWBA prediction $\sigma_{l}$. The shape of the theoretical angular distribution is a function of the $l$ transfer and is completely determined by the choice of optical-model parameters.

The angular distributions (Figs. 2-5) show good agreement with the Blair phase rule in that the one observed odd-parity state at 2.88 MeV is in phase with the elastic scattering and the even-parity states are in phase with each other and out of phase with the elastic scattering. It should be noted that small-angle data are required in order


FIG. 3. Angular distributions for ( $\alpha, \alpha^{\prime}$ ) transitions to $2^{+}$levels in ${ }^{138} \mathrm{Ba}$.
to differentiate uniquely between an $l=2$ and an $l=4$ angular distribution.
The odd-parity level at 2.88 MeV has only recently been proposed as a $J^{\pi}=3^{-}$state based on thermal-neutron-capture experiments, ${ }^{17}{ }^{138} \mathrm{Cs}$ decay studies, ${ }^{16,18}$ and ( $p, p^{\prime}$ ) reaction studies. ${ }^{29}$ The ( $\alpha, \alpha^{\prime}$ ) results confirm this assignment as the DWBA prediction for an $l=3$ transfer, shown in Fig. 2, is in excellent agreement with the measured angular distribution.
The first excited state of ${ }^{138} \mathrm{Ba}$, at 1.436 MeV , has been established as a $2^{+}$state through ( $n, n^{\prime}$ ) studies, ${ }^{14}$ Coulomb excitation measurements, ${ }^{23,30}$ ( $\alpha, \alpha^{\prime} \gamma$ ) studies, ${ }^{31}$ and the recent decay scheme work. ${ }^{16-18}$ The $l=2$ DWBA prediction compared with the measured angular distribution for transitions to this state is shown in Fig. 3. The agreement between experiment and theory is not particularly good for $\theta_{\text {c.m. }}<20^{\circ}$ where the interference between Coulomb and nuclear excitation produces rapid oscillations. The errors quoted below on the measured transition strength for this reaction have been obtained by normalizing the DWBA prediction to the small-angle data separately and to the large-angle data separately. The quoted value was obtained by normalizing to all the data simultaneously, as shown in Fig. 3.


FIG. 4. Angular distributions for ( $\alpha, \alpha^{\prime}$ ) transitions to $4^{+}$levels in ${ }^{138} \mathrm{Ba}$.

A group excited at 3.34 MeV contains an unresolved multiplet of states with a separation of less than 40 keV . The measured angular distribution for the reactions populating these states is shown in Fig. 3, together with an $l=2$ prediction. Com parisons of the data with $l=3,4$, and 5 predictions and also with all combinations of any two $l$ values have been made and the only reasonably good fit achieved is the $l=2$ case shown. Thus, we make the rather speculative conclusion that there is at least one state at $3.34 \pm 0.04 \mathrm{MeV}$ having $J^{\pi}=2^{+}$. The $\gamma$-decay study of Hill and Fuller ${ }^{18}$ assigns $J$ $=1,2$ to each member of a triplet of states at $3.340,3.352$, and 3.366 MeV . We conclude that at least two of these states are excited in the present ( $\alpha, \alpha^{\prime}$ ) study and that the spin assignment for those states is possibly $J^{\pi}=2^{+}$.
Measured angular distributions identified as $l=4$ transitions are shown in Fig. 4. The state at 1.898 MeV has recently been confirmed ${ }^{14,18}$ as having $J^{\pi}$ $=4^{+}$and the present data certainly verifies this assignment. The angular distribution identified with $E^{*}=3.50 \mathrm{MeV}$ again corresponds to the excitation of a multiplet of states with a separation of 40 keV or less. Based on a comparison with other pure and mixed $l$ predictions, we conclude


FIG. 5. ${ }^{138} \mathrm{Ba}\left(\alpha, \alpha^{\prime}\right)$ angular distributions showing separate components of a doublet at $2.24-\mathrm{MeV}$ average energy.


FIG. 6. Energy level diagram for ${ }^{138} \mathrm{Ba}$ : (a) Mariscotti et al. (Ref. 17); (b) Hill and Fuller (Ref. 18); (c) Waroquier and Hyde (Ref. 4); and (d) Wildenthal (Ref. 24).
that at least one of the members of the multiplet probably has spin and parity given by $J^{\pi}=4^{+}$.
A group with an average energy of 2.24 MeV (Fig. 5) is not resolved in the large-angle data. The small-angle data taken on the spectrograph showed it to be a doublet with a separation of 80 keV . The individual members have energies of $2.19 \pm 0.04$ and $2.27 \pm 0.04 \mathrm{MeV}$. The level at 2.19 MeV is tentatively assigned $J^{\pi}=2^{+}$on the basis of the agreement between the measured angular distribution and the DWBA prediction of an $l=2$ angular distribution. A similar comparison for the level at 2.27 MeV leads to a $J^{\pi}=4^{+}$assignment for the spin and parity of this level.

## IV. DISCUSSION

## Spin and Parity Assignments

Figure 6 shows a comparison of the spin and parity assignments made in the present study with previous experimental data. The $2^{+}, 4^{+}$, and $3^{-}$ levels located at $1.336,1.898$, and 2.880 MeV have previously been established. The results of this experiment are in agreement with these assignments. The doublet observed at 2.19 and 2.27 MeV is of more interest. We believe that it should be identified with the levels observed at 2.2179 and 2.3074 MeV in the $\gamma$-decay studies. ${ }^{18}$ The apparent energy shift of approximately 40 keV is attributed to two causes. First, the present data were taken prior to a systematic calibration of the Enge spectrograph. Second, there was a rather substantial nonlinearity in the nuclear triode position signal. The failure to accurately compensate for this nonlinearity is probably responsible for the


FIG. 7. Measured values of $G(I S, 2) / G(E 2)$ for the first $2^{+}$states of nuclei in the mass region $115<A<150$. The data used are listed in Table II.
observed energy shift. On the basis of the foregoing assumption we make a tentative $J^{\pi}=4^{+}$as signment for the previously reported level at 2.3074 MeV and we confirm the $J^{\pi}=2^{+}$assignment for the level at 2.2179 MeV .

The energy of a group at 3.34 MeV is in agreement with a similar group seen in previous experiments. The peak width of this group was such that at least two members are known to be contributing. Because of the reasonably good $l=2$ shape of the angular distribution one may tentatively conclude that at least two of the three levels previously seen ${ }^{18}$ with energies of $3.340,3.352$, and 3.366 MeV have a spin and parity given by the $J^{\pi}=2^{+}$.

One of the members of the multiplet observed at $E^{*}=3.50 \mathrm{MeV}$ may be the same as the level previously reported by Mariscotti et al. ${ }^{17}$ The shape of the angular distribution obtained in this work indicates that at least one of the levels has $J^{\pi}=4^{+}$.

Levels were also seen at $2.12,2.65$, and 4.17 MeV . Data were incomplete on these levels and no spin or parity assignments could be made.
A comparison of the experimental energy level scheme to the theoretical calculations of Waroquier and Hyde ${ }^{4}$ and to those of Wildenthal ${ }^{24}$ leads to the same observations as were made ${ }^{1}$ in the case of ${ }^{144} \mathrm{Sm}$. There appears to be good agreement for low-lying natural-parity states with the exception of the $3^{-}$level in the calculation of Waroquier and Hyde. These authors point out that the octupole vibrations are not adequately represented in the two-quasiparticle space included in their calculation.

## Transition Rates

The isoscalar transition rates, $G($ IS, $l$ ), presented in Table I have been obtained using the methods described by Bernstein. ${ }^{21}$ First, following the sug gestion of Bernstein and others ${ }^{32,33}$ an "equivalent mass value" is obtained for $\beta_{m}$ using the equation

$$
\begin{equation*}
\beta_{l} R^{\prime}=\beta_{m} R \tag{3}
\end{equation*}
$$

where $\beta_{l}$ is defined in Eq. (2), $R^{\prime}$ is the radius in the imaginary geometry of the optical potential (1), $\beta_{m}$ is the deformation of the mass distribution, and $R$ is the half-density radius of a Fermi distribution. Using this formalism, the isoscalar transition rate, expressed in single-particle units, is obtained as

$$
\begin{equation*}
G(\mathrm{IS}, l)=F(R, a)\left(Z \beta_{m}\right)^{2}(3+l)^{2} / 4 \pi(2 l+1) \tag{4}
\end{equation*}
$$

where values for $F(R, a)$ are given in Bernstein's tables. ${ }^{21}$ These tables are based on the earlier work of Owen and Satchler ${ }^{34}$ in which they calculat-
ed the multipole moments of a Fermi density distribution.
One does not expect $G($ IS, $l)$ and $G(E l)$ to be equal, in general. By definition, ${ }^{21}$ the isoscalar transition rates, as measured in the collective analyses of ( $\alpha, \alpha^{\prime}$ ) experiments, will equal measured electromagnetic transition rates if the relative neutron and proton contributions to the isoscalar transition rate are in the ratio $N / Z$. For $N>Z$ nuclei this condition is satisfied in the collective vibrational model, assuming that the mass and charge distributions of the nuclear ground state have exactly the same form. The ratio of the electromagnetic and isoscalar transition rate may in fact be a probe of the basis for the introduction of effective charges in models that attempt to calculate electromagnetic rates. ${ }^{21} \mathrm{~A}$ comparison of the ratio $G($ IS, 2$) / G(E 2)$ for the excitation of the lowest $2^{+}$states in nuclei in the mass region $115<A<150$ is shown in Fig. 7. The experimental data used in constructing the figure are listed in Table II. Except as noted in the table, the data have been taken from Bernstein's review article. ${ }^{21}$ The error bars shown in Fig. 7 are constructed from the experimental errors quoted in each case and do not contain any additional estimate of relative errors. In spite of the relatively large errors in the ratios ( $17-31 \%$ ), the data do indicate that perhaps one is observing shell effects, since for those nuclei with $N \cong 82$ the isoscalar transition rate is significantly less than the electromagnetic transition rate. It appears that the region of the $N=82$ shell closure is an exception to the generally observed equality ${ }^{21}$ between isoscalar

TABLE I. Transition strengths in ${ }^{138} \mathrm{Ba}$.

| $J^{\pi}$ | $\begin{gathered} E^{*} \\ (\mathrm{MeV}) \end{gathered}$ | Experiment |  |  | Theory (Ref. 3) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\begin{gathered} (\alpha, \\ \beta_{l} R^{\prime} \end{gathered}$ | $\alpha^{\prime}$ ) | Coulomb excitation |  | Gauss ${ }^{\text {b }}$ |
|  |  |  | $G(\mathrm{IS}, \ell)^{\mathrm{a}}$ |  | $G(E l)$ | $G(E l)$ |
| 2 | 1.436 | 0.42 | 5.6 | $18.0{ }^{\text {c }}$ | 8.3 | 10.0 |
|  |  |  |  | $13.0{ }^{\text {d }}$ |  |  |
|  |  |  |  | $10.0{ }^{\text {e }}$ |  |  |
| $4^{+}$ | 1.898 | 0.30 | 3.6 |  | $\ldots$ | $\ldots$ |
| $2^{+}$ | 2.19 | 0.19 | 1.2 |  | $\cdots$ | 1.1 |
| $4^{+}$ | 2.27 | 0.19 | 1.4 |  | $\cdots$ | ... |
| $3^{-}$ | 2.88 | 0.58 | 11.6 |  | $\ldots$ | 1.1 |
| $\left(2^{+}\right)$ | 3.34 | 0.18 | 1.0 |  | $\cdots$ | ... |
| $\left(4^{+}\right)$ | 3.50 | 0.31 | 4.0 |  | $\cdots$ | $\ldots$ |

[^0]and electromagnetic transition rates of $2^{+}$states. The fact that similar trends have not been observed at other major shell closures, such as $N$ $=126$, is most probably due to the lack of accurate measurements of the isoscalar rates. These observations can only be considered suggestive, however, until more accurate data and more complete analyses have been performed.

## SUMMARY

The scattering of $50-\mathrm{MeV} \alpha$ particles from ${ }^{138} \mathrm{Ba}$ has been analyzed via the DWBA employing a vibrational collective-model form factor. Complex potentials with different real and imaginary geometries were used in the analysis. Current shellmodel calculations ${ }^{4,24}$ are in reasonable agreement with the present results with regard to placement and spin and parity assignments for the strongly excited states. Isoscalar transition rates in ${ }^{138} \mathrm{Ba}$ as well as those in ${ }^{144} \mathrm{Sm}$ and ${ }^{140} \mathrm{Ce}$ have a consis tently lower value than the electromagnetic transition rates measured for the same transitions. A first conclusion is that the vibrational model does not provide a good description of these states. The microscopic analysis of the ${ }^{140} \mathrm{Ce}\left(\alpha, \alpha^{\prime}\right)$ data ${ }^{34}$ further indicates that core polarization effects are very important in ${ }^{140} \mathrm{Ce}$ and presumably the same conclusion holds for the other $N=82$ nuclei. The suppressed isoscalar transition rates could be evidence that the $N=82$ shell closure may cause a significant difference between the core excitations of protons and neutrons in these nuclei.
A $4^{+}$assignment of the $1.898-\mathrm{MeV}$ level in ${ }^{138} \mathrm{Ba}$

TABLE II. Experimental transition strengths. Data taken from Ref. 21 unless otherwise noted.

| Nucleus | $G(\mathrm{IS}, 2)$ | $G(E 2)$ | $G(\mathrm{IS}, 2) / G(E 2)$ |
| :---: | :---: | :---: | :---: |
| ${ }^{116} \mathrm{Sn}_{66}$ | $17.0 \pm 2.5$ | $14.0 \pm 0.7$ | $1.21 \pm 0.26$ |
| ${ }^{122} \mathrm{Te}_{70}$ | $34.0 \pm 3.4$ | $35.6 \pm 3.3$ | $0.95 \pm 0.21$ |
| ${ }^{124} \mathrm{Te}_{72}$ | $27.8 \pm 2.8$ | $32.3 \pm 6.5$ | $0.86 \pm 0.33$ |
| ${ }^{126} \mathrm{Te}_{74}$ | $24.5 \pm 2.5$ | $28.4 \pm 2.0$ | $0.86 \pm 0.16$ |
| ${ }^{128} \mathrm{Te}_{76}$ | $19.9 \pm 2.0$ | $21.3 \pm 1.7$ | $0.93 \pm 0.19$ |
| ${ }^{130} \mathrm{Te}_{78}$ | $11.8 \pm 1.2$ | $17.2 \pm 1.5$ | $0.69 \pm 0.14$ |
| ${ }^{138} \mathrm{Ba}_{82}$ | $5.6 \pm 1.1^{\text {a }}$ | $10.0 \pm 0.4^{\text {b }}$ | $0.56 \pm 0.14$ |
| ${ }^{140} \mathrm{Ce}_{82}$ | $7.4 \pm 0.8^{\text {c }}$ | $18.0 \pm 2.0^{\text {d }}$ | $0.41 \pm 0.10$ |
| ${ }^{144} \mathrm{Sm}_{82}$ | $7.0 \pm 1.1{ }^{\text {e }}$ | $11.0 \pm 1.8^{\text {f }}$ | $0.64 \pm 0.24$ |
| ${ }^{148} \mathrm{Sm}_{82}$ | $26.5 \pm 4.0$ | $38.0 \pm 4.3$ | $0.70 \pm 0.21$ |
| ${ }^{\text {a }}$ Present work. <br> ${ }^{\mathrm{b}}$ Reference 30. <br> ${ }^{c}$ Reference 35. |  |  | ${ }^{\text {d }}$ Reference 36. <br> ${ }^{e}$ Reference 1. <br> ${ }^{f}$ Reference 37. |

has been confirmed. A $2^{+}$assignment is confirmed for a level at $2.19 \pm 0.04 \mathrm{MeV}$, and a tentative $4^{+}$ assignment is made for the level at $2.27 \pm 0.04$ MeV . These two levels are believed to be the 2.2179- and $2.3074-\mathrm{MeV}$ levels previously assigned $J=2$ and $J=(3,4)$, respectively. ${ }^{18}$ A tentative $2^{+}$assignment is made for at least one member of an unresolved multiplet located at 3.34 $\pm 0.04 \mathrm{MeV}$. A tentative $4^{+}$assignment is made for at least one member of an unresolved multiplet located at $3.50 \pm 0.04 \mathrm{MeV}$.

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[^0]:    ${ }^{\text {a }}$ Estimated uncertainty in quoted values is $\pm 20 \%$.
    ${ }^{\mathrm{b}}$ Two forms for the nucleon-nucleon residual interaction were used, either a surface $\delta$ interaction (SDI) or a Gaussian interaction (Gauss).
    ${ }^{\text {c }}$ See Ref. 22.
    ${ }^{\mathrm{d}}$ See Ref. 23.
    ${ }^{\mathrm{e}}$ See Ref. 30.

